Mind the gap: the economic problem as an interplay among desires, matter, and energy

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Abstract

There is a rich literature on the economic importance of energy. Yet, little has been achieved to harmonize core economic theory with energetic principles. This paper proposes a theoretical framework that might prove valuable to do so, based on a slight conceptual modification of the neoclassical economic problem. By conceiving its necessary condition not as desires but as gaps between desired and spontaneous states of material reality, an extension of economic imperialism towards the realm of energetics is enabled. When the origin of the economic problem is placed on physical divergences, goods are exposed as specific material configurations that close gaps. And as material rearrangements can only be achieved through energy transferred by prime movers (e.g. workers, horses, engines), such transfers are revealed as the essence of economic activity. Thus, whenever energy and power constrained agents are analyzed, the derivation of optimization procedures follows intuitively: consumer's constraints are energetic, where economic energies play the role of prices, and firm's profits are energy surpluses, where prime movers play the role of factors of production. This leads to familiar refutable hypothesis expressed in energy terms. Furthermore, equilibrium is characterized by the schedule of energy assignments that lead to utility maximization, for any set of economic energies and energy contents of goods. Although only autarchic and single-period agents are analyzed, the stage is set to analyze exchanging and multi-period agents, which could allow for a general interpretation of economic fundamentals (e.g. prices, capital) as visible social expressions of invisible energetic dynamics.

Keywords

Desires; matter; energy; prime movers

JEL Classification:

D11, D21, O13, Q40

Nomenclature

φ	The World
A	All constituent elements of φ
C	Elements of φ that agents can influence
z_k	one element of φ
μ	Spontaneous state of φ at $t^{\circ}+\Delta t$
A'	Constituent elements of μ
θ	Desires state of φ at $t^{\circ}+\Delta t$
A^{*}	Constituent elements of θ
G	Total energy expenditure
eB	Energy budget
eI	Energy income
eA	Energy endowment
x	Prime movers
x_l	Quantity of prime mover of type l
ω	Energy dissipated at full workload
p_l	Power of prime mover l
p_a	Power of an agent
δ_i	Energy content of energy good
δ_j'	Energy assignment for non-energy good
γ_i	Marginal embodied energy of good
γ_k^A	Average embodied energy of good k
β_k	Marginal economic energy of good k
β_k^A	Average economic energy of good k
Q_k	Good
Q_i	Energy good
$Q_{i,k}$	Energy goods used to produce Q_k
$Q_{i,l}$	Energy goods used by one x_l
Q_j	Non-energy good
α	Mg energy surplus
E	Total energy surplus
$EROI$	Energy return over investment
$MEROI$	Mg energy return over investment
PES	Primary energy sources
eBC	Energy budget constraint
EBC	Economic budget constraint
W_p	Physical work
W_x	Non-useful energy transfer
W_m	Physical minimum amount of work
W_n	Non-useful work
$\frac{W_p}{G}$	First-law-efficiency
$\frac{W_m}{W_p}$	Work-efficiency
$\rho=\frac{W_m}{G}$	Total-efficiency
ϕ_k	Mg energy cost of production of good k
ϕ_l	Mg energy surplus of prime mover l
λ	Marginal utility of energy
Ω	Steady-state of the agent

1. Introduction

There is an extensive literature highlighting the importance of energy for economic systems. These range from biological studies showing the role it plays in maintaining living organisms (Mcnab, 2002; Spicer & Gaston, 2000), ecological ones pointing to its centrality in ecosystems (Miller & Spoolman, 2010), to physical ones that stress its role in production (Halliday & Resnick, 1966; Hewitt, 2007). Others use econometric techniques to show causation and/or correlation between energetic and economic variables (Asafu-Adjaye, 2000; Liu et al., 2008; Soytas & Sari, 2003; Tsani, 2010; Warr & Ayres, 2010), or give detailed historical accounts of the role energy played in the transition from hunter/gatherers tribes up to the globalized economy (Scheidel & Sorman, 2012; Smil, 1994; Warr et al., 2010).

Many authors have criticized neoclassical economics for failing to properly account for it. These can be grouped into those concerned with the absence of thermodynamics (Georgescu-Roegen, 1971, 1975), the neglect of the particularity of an economy ran on non-renewable energy sources (Murphy & Hall, 2011), of the fact that literally and figuratively energy “makes the world go round”, and more generally, the inconsistency between economics and well-established natural science (H. Daly, 1992; C. Hall et al., 2001; Jackson, 2009).

The response from the profession’s core has been cold, remaining mostly impervious as compared to strong reactions to other critiques evidenced by the rich history of economic thought (Backhouse, 2004). It has analyzed energy markets extensively (Bhattacharyya, 2011), giving valuable insights into the workings of specific energy markets (Bhattacharyya, 2007; Edwards, 2003), and conditions for optimal transitions paths from non-renewable to renewable resource base (Hartwick, 1977). Yet it has generally done so considering energy as another factor of production or another commodity (Asafu-Adjaye, 2000; Bhattacharyya, 2011), and thus bypassing deeper considerations of thermodynamics, the role of energy as the vehicle of material change, and the inconsistency between neoclassicism and relevant features of biology, ecology and physics.

Accordingly, actual responses have come mostly from outside economic mainstream, as energy-theories of value (Alessio, 1981; Costanza, 1980; Huettner, 1982), and streams of biophysical and ecological economics (Hall & Klitgaard, 2012; Herrmann-Pillath, 2015). These proposals have generally been ignored , and when not so, heavily criticized by economic mainstream (Varian, 1991). Likely, this has to do somewhat with the inflexibility of the profession, but mostly to lack of rigor, procedures that seem noneconomical, or the failure to deliver the crucial produce of scientific endeavors: refutable hypotheses (Bridgman, 1927; Friedman, 1953).

The central question of this paper, while arguing why energy has a particular role in economic systems, is how can it be included into economic analysis in a convincing way. In brief, it can be done by extending economic imperialism (Becker, 1993; Lazear, 1999) to the realm of energetics.

To do so, it is highlighted that the satisfaction of desires is intrinsically associated with specific materialities (Meskell et al, 2005; Slater, 2002). This is the basis for a slight precision to Robbin's (1932) famous definition of the economic problem, as the notion of *"ends"* is specified as the closure of gaps: a complex formed by desires and divergent materialities between spontaneous states of material reality and those consistent with the satisfaction of desires, instead of desires alone.

The first implication of this is that formed by two elements, gaps can be closed in two distinct ways: a subjective approach based on managing desires, and an objective one based on rearranging mater. As the latter deals with material configurations, it naturally connects with physics, and the key insight taken from it is that the unique source of material change is energy transfers (Halliday & Resnick, 1966; Hewitt, 2007). Moreover, energy is not transferred at will, but requires prime movers to do so, system capable of taking energy sources and transferring them to achieve desired change (Smil, 2013).

If the objective approach deals with material rearrangements that close gaps, which are argued to be goods, and these are fundamentally produced by energy transfers, then the notion of *"scarce means"* can be specified as limited energy sources to transfer energy (e.g. rice, hay, oil), and limited prime movers to perform such transfer (e.g. workers, horses, engines). Therefore, optimization procedures are enabled by recognizing that an agent's production of goods is constrained by energy and power, and that the closure of gaps enabled by those goods is its objective. This allows for the *"relentlessly and unflinchingly"* (Becker, 1976) application of neoclassical procedures, and the derivation of refutable hypotheses (Samuelson, 1965).

This paper, given space-constraints, only develops optimization procedures for autarchic and single-period agents in (mostly) long-run equilibrium, and therefore is mainly a benchmark from where future research starts-off. Other papers are underway extending this framework to 1) exchanging agents, thus dealing with markets, market prices, and among other issues, inequality, 2) multi-period agents, where intertemporal considerations of non-renewable energy becomes central, 3) full analysis of short-term equilibrium, where energy expenditure and its intrinsic opportunity cost differ, and 4) issues of choice under uncertainty and path dependency that derive from this perspective.

In the basic framework developed here, the producer problem is set to minimize their total (direct plus indirect) energy expenditure, given a target level of production. This leads to, among other results, tangency conditions, conditional demand functions, and unambiguously downward-sloping own-substitution effects and upward-sloping cross-substitution effects.

Alternatively, the producer problem is specified as producers of a good seeking to maximize the energy surplus it provides. This leads to the equating of marginal energy income and expenditure, such that total energy surplus is maximized. Also, to the same tangency conditions found in the minimization setting, unconditional demand functions, unambiguous downward-sloping own-substitution effects and upward-sloping supply curves. Moreover, this setting implies that non-energy goods (e.g. books, cars), can only be produced given the energy surplus secured by the production of energy goods (e.g. rice, oil).

Yet these procedures are only necessary conditions for energy-constraint agents to maximize their utility. Specifying the consumer problem as utility (*a la* Fisher[1]) maximization subject to an energy budget constraint, tangency conditions are found, together with conditional demand functions for non-energy goods, unambiguously downward-sloping own substitution effects for non-energy goods, and ambiguous price effects for energy goods.

Merged into an equilibrium setting that equates production and consumption, optimal marginal energy expenditure on each non-energy good equals the marginal rate of substitution between the good and energy. Also, energy goods in the long-run yield, at the margin, zero energy surplus and utility, yet in the short-run they yield positive marginal energy surplus and negative marginal utility. This creates internal dynamics that drive agents to transition from the short to the long-run, where the steady-state is characterized by no possible additional energy surplus. During this transition, total energy surplus is transferred progressively from consumers to producers, and ends up being totally assigned to the latter.

The remainder of this paper is organized as follows. Section 2 develops the perspective of the gap in greater detail, and sections 3 and 4 presents the subjective and objective approach respectively. Section 5 focuses on the objective one. Section 6 presents concluding remarks and future research.

[1] In his doctoral dissertation *Mathematical Investigations in the theory of Value and Prices* (1892), Irwin Fisher uses the concept of utility explicitly independent from any psychological consideration related to pleasure and pain. He uses it "merely as a way to describe individual's behavior" (Backhouse, 2004).

2. The gap

This paper is based on the proposition that gaps between spontaneous arrangement of material reality and those associated with the satisfaction of desires constitute the necessary condition of the economic problem. They, as a complex formed by desires and materiality, have internal dynamics that will be overlooked: Desires can lead to material divergences between spontaneous and desired states, or certain spontaneous material configurations can lead to desires. Both possibilities have merits and limitations, yet both sustain the core proposition of this paper: gaps are the necessary condition for the economic problem to arise.

The fact that gaps are less perceived than desires is irrelevant. While desires are subjectively felt, and thus arguably part of the economic problem, both spontaneous and desired material configurations are often invisible and ignored. Nonetheless, what agents perceive or think is not of relevance for economics, but only how they behave. The proposition of the gap as the necessary condition of the economic problem implies assuming that consciously or not, agents behave as if closing gaps.

Gaps emerge as a contingent phenomenon dependent on the desires of agents, their environment, and the time period under consideration, which implies that neither their existence nor inexistence is guaranteed. Yet the complete inexistence of gaps implies Paradise, where all desires are taken care of even before they are felt. Thus, the existence of at least a few gaps is virtually assured after some reasonably short period of time. As Nature is not bound to be kind to humans, matter is unlikely to arrange itself spontaneously according to its desires. The remainder of this section formalizing this.

An agent is defined as any person or groups of people, being an individual agent a single person, and a collective one any group, from a family to humankind. Also, "The World" φ is defined as the entirety of the external material reality of an agent, comprising natural elements such as wind and soil, and social elements such as roads and other agents. Such elements, which for an agent can be relevant (e.g. food) or irrelevant (e.g. distant star), are characterized by their precise velocities and locations, as conceived by Laplace (1951), which determine their quantities and qualities.

Thus φ is unique for every specific set of its constituent material elements A, which is in turn unique for every specific combination of constituents $\boldsymbol{z} = (z_1, \dots, z_K)$ (where K is the total amount of elements in A). Furthermore, each z_k is unique for every specific combination of velocity and location.

While defining $\boldsymbol{z}$ there is a need for an arbitrary definition of scale. Thus $\boldsymbol{z}$ is $\boldsymbol{z}^{s=1,\dots,S}$, where 1 and S respectively represent the smallest and largest level of organization of matter. Hereafter only two levels will be of interest: the molecular level $s = 1$, and the object level $s = r$ (where r is the jumps in scale from the former to the latter). Also, z_k will be a constituent of z_k^r whenever the referred molecule is part of the object, and hereafter "material configuration" will refer to precise velocity and location of $\boldsymbol{z}$.

Material configuration changes as time t changes exogenously[2], which redefines φ as $\varphi\{A[z(t)]\}$, but to simplify notation it will be referred as $\varphi(A,t)$. Note that for every agent at any t there is a unique $\varphi(A,t)$, which is almost the same across agents. If $\varphi(A,t)$ completely describes the material configuration of reality (including irrelevant $\boldsymbol{z}$), and assuming that material reality is the same at any t regardless of the observer, then the sole difference in $\varphi(A,t)$ among agents is that others are part of it, while the agent itself is not. By defining the subset of $\boldsymbol{z}$ that the agent can influence as C, then $\varphi(C,t)$ can vary widely given how different the within-reach $\boldsymbol{z}$ can be.

For any agent there are three fundamental states of $\varphi(A,t)$: The first is the "Present State" $\varphi(A^\circ, t^\circ)$ which corresponds to $\varphi(A,t)$ at a particular time $t = t^\circ$. This is what the agent recognizes as its present material reality, and from where it perceives the future material configuration of φ. The second is the "Spontaneous state" $\mu = \varphi(A', t^\circ + \Delta t)$ which corresponds to $\varphi(A,t)$ after certain time Δt when the Agent has not interfered in The World, where A' represents the spontaneously modified set of $\boldsymbol{z}$. This spontaneous change makes the Present State $\varphi(A^\circ, t^\circ)$ follow a "Spontaneous Trajectory" towards μ. Note that by "spontaneous" any notion of natural trajectory or state is purposefully avoided. The Spontaneous Trajectory, and thus μ, could be fundamentally shaped by other Agents.The third is the "Desired State" $\theta = \varphi(A^*, t^\circ + \Delta t)$ which corresponds to material configuration consistent with the satisfaction of the agent's desires at $t^\circ + \Delta t$, where A^* represents the desired material configuration.

The intuition of the perspective of the gap is worth emphasizing. A gap exists whenever agents wish for a tree to order itself as a wooden table, for mixed minerals to be refined, for knowledge to enlighten the workings of a given process, or for a cure to remedy a certain illness. Given this generality, gaps can be as different between agents as are their desires, nature, and values.

General and particular gaps respectively refer to the distance between Spontaneous and Desired *general* states of φ, and between Spontaneous and Desired *particular* configurations of $\boldsymbol{z}$. Thus, general gaps between μ and θ are due to one or more particular gaps between the configuration of $\boldsymbol{z}$ in A' and A^*. Hereafter, when talking plainly about gaps, particular ones will be referred to.

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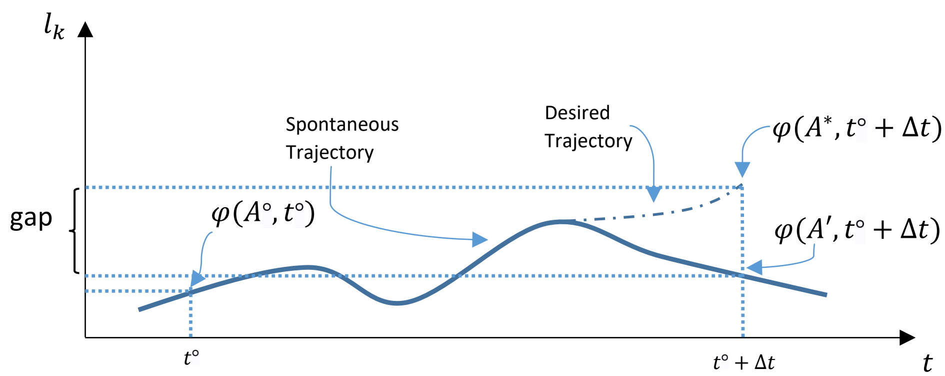
[2] Given that our focus is on economics, it seems appropriate to use Euclidean instead of Minkowski space (Naber, 2012). This implies that we will obviate the fact that time is in itself a measurement of changes in velocity and location.

To represent gaps as in figure 1 the following is done: First, the three dimensions of space are collapsed into the variable location l (as commonly done in any phase space (Prigogine, 1996)), and put in the y-axis. Then, velocity is held constant[3], and the exogenous passing of time is measured in the x-axis. Finally, φ is mapped in terms of the location l_k of one $\boldsymbol{z}$ and its evolution over time. This could be done mapping the particular $\boldsymbol{z}$ under consideration, but this presentation is more meaningful: it directly implies how particular gaps, *ceteris paribus*, lead to general ones. To depict a general gap, one dimension per diverging z_k has to be added to figure 1.

Fig. 1



The concept of the gap represented in figure 1 is only a first approach, and as such, clearly insufficient. To fully develop the complexity of this concept, seven distinct characteristics are recognized. Gaps are unique to time, velocities, and location of $\boldsymbol{z}$, where location also determines quantity and quality. Quantity refers to the amount of the same z_k at a specific location (e.g. amount of H_2O in a pot), while quality refers to differences in otherwise identical z_k^r due to marginal modifications of their constituent z_k (e.g. changes in temperature of water in a pot). Quantity allows reinterpreting figure 1 by mapping φ not to time and location l_k of one z_k, but to time and quantity of z_k^r, *ceteris paribus*. By doing this, gaps are reduced to divergences between spontaneous and desired quantities of z_k^r, determined by divergences in their $\boldsymbol{z}$. Hereafter particular gaps will be considered like this.

A second characteristic is that under the vast majority of circumstances, agents face multiple gaps simultaneously. As suggested, this means that general gaps can only be fully represented by a hyperspace representing time and one additional dimension by each divergent z_k^r in terms of quantity, *ceteris paribus*.

[3] Velocity is a fundamental part of the material configuration of φ, but to illustrate gaps it is reasonable to omit it. Location could be hold constant instead, but this would be less intuitive.

Importantly, multiplicity of gaps, whenever an agent has scarce resources to close them all, implies the problem of choice. This will be explored in section 5.

A third characteristic is bearability, which refers to the possibility of keeping a gap open over time. Some gaps can be carried forward only for a limited time, in which case they are understood to be urgent and thus unbearable. These can be easiest understood as related to physiological needs. Others that can be carried into the future indefinitely are understood as non-urgent and thus bearable. For most gaps this distinction is not clear, varying between agents according to its environment, culture, and other factors.

Fourth, gaps can be more or less stable according to how frequently otherwise identical gaps emerge. Unstable gaps force agents to close the same type over and over again. Considering enough time, all gaps are unstable, because as long as φ does not spontaneously follow θ, the organization of matter will eventually tend to disorganize itself.

Gaps can have up to three more characteristics: uncertainty, indirectness and interdependency. While all are relevant, only the four detailed up to here are necessary for the scope of this paper. More details on all characteristics are presented in part A of the online appendix.

Made of desires and divergent materialities, the closure of gaps can be conceived from two approaches: focusing on desires by moving θ towards μ -the subjective approach-, or dealing with material configurations by stirring μ towards θ -the objective approach-. Both imply changing A, which is ultimately dependent on a change of its constituent $\boldsymbol{z}$.

3. The subjective approach: managing desires

The subjective approach to closing gaps consists of an agent managing its desires regarding the future material configuration of φ, achieving a change of A^* with A', and thus allowing for $\theta \to \mu$.

Nonetheless, θ can be adjusted only to a limited extent: if agents face unbearable gaps, the acceptance of μ is not an option. For example, if the gap under consideration is having food or not, accepting μ means death. Unfortunately, distinguishing which gaps can or cannot be closed subjectively is as difficult as differentiating between bearable and unbearable gaps, and thus contingent upon many factors.

With this in mind, two assertions can be made. One is that the subjective approach seems to have extensive applications today. Whether it be for societies where the “Lebow doctrine” (Lebow, 1955) or the "machinery for consumer-demand creation" (Galbraith, 1998) has been consolidated[4], where groups are driven by conspicuous consumption and pecuniary emulation (Veblen, 1899), or where cultures show highly sophisticated and/or damaging patterns of consumption (Brown & Cameron, 2000; Spangenberg & Lorek, 2002). In all these contexts, in line with buddhist economics (Brown, 2017; Sivaraksa, 2016), this approach can be useful in closing some gaps and narrowing down others.

Yet, the other assertion is that no agent can close *all* gaps subjectively for any significant timespan. Certain unavoidable ones emerge as a consequence of physical and biological dynamics that force agents to always do some effort: as a bare minimum, they must perform basal work (Mcnab, 2002; Spicer & Gaston, 2000). Human organisms, as well as all living beings, are systems far from thermal and entropic equilibrium (Morowitz, 1968; Proops, 1983; Schrodinger, 1944) and as such require constant efforts to neutralize entropy increase, sustain thermal imbalances, and maintain highly complex organizations (Boltzmann, 1886; Glansdorff & Prigogine, 1971; Nicolis & Prigogine, 1977). In short, agents cannot completely solve their economic problem subjectively, because doing so is incompatible with the maintenance of the thermal disequilibrium state of life (Schrodinger, 1944).

Thus, the subjective approach can be useful to close and/or narrow some gaps sometimes, but it is not enough to close all gaps all the time. Also, it does not lead to refutable hypothesis. Given this, the objective approach turns out to be unavoidable for the agent, and of higher interest to the economist.

4. The objective approach: doing work

The objective approach to closing gaps consists of agents rearranging future material configurations, achieving a change of A' with A^*, and thus allowing for $\mu \rightarrow \theta$. Given that by definition the subset of $\boldsymbol{z}$ that agents can modify is C, it will be used instead of A from now on.

Rearrangements of matter to close gaps is work, and by emphasizing that it consists specifically of rearrangements of $\boldsymbol{z}$ –of their location and velocity that leads to specific z^r-, this notion of work is compatible with how both economics and physics understand it. It is economical in the sense that it is done to satisfy a desire, and it is physical in the sense that it deals with movements of matter. Taking this perspective, before becoming a theory of choice economics starts of as a theory of change; of how purposeful change comes to pass.

[4] Constant displacement of θ away from μ, thereby creating economic problems in an attempt to stimulate economic activity.

The idea that work consists of rearrangements of matter from one (given) configuration to another (desired) one is wholly natural. As examples, a farmer moves soil and water from one place to another and places fertilizers and pesticides. A teacher modifies neural connections in student's brains. A mining company separates non-useful elements from useful minerals. A construction company puts in place all sorts of materials to form a house, a bridge, or a hospital. Moreover, this same logic applies to families, governments, and any other type of aggregate agent. They do work by rearranging matter towards a specific configuration consistent with the satisfaction of their desires, and by doing so they close gaps.

4.1 Goods and services

The work agents perform leads to particular material arrangements that are different from those crystallizing in their absence: to changes in $\boldsymbol{z}$ that lead to changes in quantities of $\boldsymbol{z}^r$, and ultimately to C^* as opposed to C'. Whenever this new configuration of matter can or cannot be unquestionably observed by an outside observer, it will respectively be a good or a service. Nevertheless, without loss of generality, both will be referred to as goods.

This notion of goods is not artificial. A car (z_k^r) is a set of minerals and other materials (z_k) taken from underground, rearranged, and assembled in a precise way. A book (z_k^r) is cellulous and hydrocarbons (z_k) rearranged in a way that assembles pages with ink. Even healthcare (z_k^r) creates neural connections (z_k) that provide knowledge about healing, and then the material conditions for actual healing, and consultancy (z_k^r) configures neurons (z_k) to achieve knowledge about a certain topic. Hereafter $\boldsymbol{Q_k}$ will be used to refer to the quantity of z_k^r produced during Δt, where $\boldsymbol{Q_k} = (Q_1, \dots, Q_K)$.

Regardless of their nature, all goods have two common features: they are a precise configuration of matter resultant from an agent's work, and they are produced to close gaps. Thus, something that is not the result of an agent's work is not a good, as are not material configurations useless to close gaps.

As goods are produced to close gaps, the characteristics of gaps (section 2) lead to precise characteristics of goods. Specificity of gaps makes goods unique to time, velocity, and location, just as Debreu (1959) argues. Multiplicity to the issue of choice when agents do not have, for whatever reason, the capacity to produce enough goods to close them all. Bearability to the notion of basic and luxury goods. Other characteristics are presented in part B of the online appendix.

By taking this perspective, the "black box" of production is enlightened. Production of goods is no longer an opaque process that combines labor and capital to obtain outputs given certain technology, but a series of movements that change material configuration from μ to θ. Nonetheless, the specific set of movements to produce each good, remains relevant to the industrial engineer but not to the economist. In that sense, the black box is replaced with a grey one, which enables a better recognition of the nature of the productive process while avoiding overwhelming details.

4.2 Work and energy

A significant implication of the perspective of the gap is that if work consists of material rearrangements, then it turns out that its essence is nothing else than energy transfers, as they are the unique source of material change (Halliday & Resnick, 1966; Hewitt, 2007).

While this claim might be controversial for the mainstream of the economics profession, it is only natural for other disciplines. As Smil (1999) states, *“Energy is the only universal currency: one of its many forms must be transformed to another in order for stars to shine, planets to rotate, plants to grow, and civilizations to evolve. Recognition of this universality was one of the great achievements of nineteenth-century science, but, surprisingly, this recognition has not lead to comprehensive, systematic studies that view our world through the powerful prism of energy”*. Being unfamiliar to economists, this is orthodoxy in physics: movement and change are the testimony of invisible energy transfers.

When agents do work to produce goods, they are transferring energy to achieve material reconfigurations. This is why energy reveals itself as the fundamental economic resource, which in turn implies a particular theory of cost explored in sections 4.3 and 5.1. This has been suggested before (Alessio, 1981; Fermi, 1937; Gillett, 2006; Hannon et al., 1986; Patterson, 1998), but not with an economic logic. By virtue of this, work is measurable in energy, in particular, in joules[5].

4.3 Energy expenditure, embodied energy, and prime movers

The energy an agent transfers to reconfigure matter and produce goods is its energy expenditure[6] eG. To make such transfers it must control mechanisms capable of doing so called prime movers (Smil, 1994, 2013). Thus, agents produce goods transferring eG using an array of prime movers denoted by $\boldsymbol{x}$.

An agent’s eG is the total energy transferred by the sum of all its L types of $\boldsymbol{x}$ to produce $\boldsymbol{Q_k}$, such that $eG = \boldsymbol{\omega} \cdot \boldsymbol{x}$, where $\boldsymbol{\omega} = (\omega_1, \dots, \omega_L)$, $\boldsymbol{x} = (x_1, \dots, x_L)$, x_l is the quantity of prime movers of type l, and ω_l

[5] One Joule (J) is the force of one newton acting on an object over one meter.

[6] The word “expenditure” could be problematic, given that energy is neither created nor destroyed. Nevertheless, for an agent the energy it uses to perform work is in fact spent in the proper sense of the word.

is the energy dissipated at full workload[7] by it. Accordingly, the total energy spent to produce one particular good Q_k will be $eG_k = \boldsymbol{\omega_k} \cdot \boldsymbol{x_k}$, which is similar in structure but distinct in meaning to Fisher and Kaysen's (1962) model of energy consumption**.** The problems of aggregation derived from energy and prime movers types and qualities will be overlooked (see Cleveland et al., (2000), Podobnik (2005), Bhattacharyya (2011), and Smil (2013, 2016)).

The average energy cost of good k, called its average embodied energy, is $\frac{eG_k}{Q_k} = \gamma_k^A$. This defines the triple equality $eG_k = \boldsymbol{\omega_k} \cdot \boldsymbol{x_k} = \gamma_k^A \cdot Q_k$, and by extension, $eG = \boldsymbol{\omega} \cdot \boldsymbol{x} = \boldsymbol{\gamma^A} \cdot \boldsymbol{Q}$**.** Moreover, the marginal energy cost of such good, called its marginal embodied energy, is $\frac{\partial eG_k}{\partial Q_k} = \gamma_k$, which in turn defines $deG_k = \gamma_k dQ_k + \boldsymbol{x_k} d\boldsymbol{\omega_k} + \boldsymbol{\omega_k} d\boldsymbol{x_k}$. A reader interested in embodied energy analysis can read IFIAS (1974) Bullard & Herendeen (1975) Costanza (1981), Huettner (1982), and Machado et al. (2001).

All $\boldsymbol{x}$ have finite power $\boldsymbol{p}$[8], which corresponds, in Watts, to its rate of energy transfer. As rough examples, humans show p of 100 W, horses of 700 W, and gas turbines surpass $6 \cdot 10^8$ W (6 million times that of a human). The overall power of an agent corresponds to $p_a = \boldsymbol{p} \cdot \boldsymbol{x}$. In 2013 humanity's p_a was estimated at 16.8 TW (Schramski, Gattie, & Brown, 2015), equivalent to 168 billion humans. The additional power of 161 billion humans comes primarily from Otto and Diesel engines, and steam and gas turbines. The recognition that agents have more than their bare hands to perform work is crucial, otherwise, this perspective would be useful to interpret hunter/gatherer tribes, but little more.

4.4 Energy budget, energy content, and primary energy sources

Agents draw eG from their energy budget eB, which contains exclusively free energy (i.e. energy that can do work, see Georgescu-Roegen (1975)), which is a concept intimately conditioned on the $\boldsymbol{x}$ the agent controls (waste heat is energy, yet not part of an agent's eB, and gasoline is not part of aboriginal tribe´s eB, yet it is for a car owner). Also, eB can have, following Lotka (1925), either endosomatic and exosomatic energy, which is again a concept intimately conditioned on the $\boldsymbol{x}$ the agent controls. The former is defined by being transferred by an agent's own biological organism, and the latter by any other $\boldsymbol{x}$. Thus, endosomatic energy is stored chemical energy in the form of glucose (e.g. rice), while exosomatic energy

7 This is estimated as the potential output (in Joules) of the prime mover times its availability factor, plus the energetic depreciation of the prime mover.

8 One Watt is the rate at which work is done when an object's velocity is held constant at one meter per second against constant opposing force of one newton.

can be widely more diverse (e.g. rice, hay, oil) in relation to the diversity of $\boldsymbol{x}$ an agent controls (e.g. workers, horses, internal combustion engines).

Defining $eB = eI + eA$ allows for an exogenous energy endowment eA, and for an endogenous energy income $eI = \boldsymbol{\delta_i} \cdot \boldsymbol{Q_i}$, where $\boldsymbol{\delta_i} = (\delta_1, \dots, \delta_m)$, $\boldsymbol{Q_i} = (Q_1, \dots, Q_m)$, Q_i is energy good of type i, and δ_i is its unitary caloric content in joules. The issues of energy types and qualities will also be overlooked. $\boldsymbol{Q_i}$ are characterized by being compatible with an agent's $\boldsymbol{x}$, and provide a positive marginal energy surplus $\boldsymbol{\alpha_i} = \boldsymbol{\delta_i} - \boldsymbol{\gamma_i}$. As examples, δ_i (in GJ/ton) is 46 for oil, 28 for coal, and 15 for cereal (Smil, 1999)), while their γ_i is less precise, and time and space specific, yet as reference 2.3 for oil, 0.6 for coal (J. Lambert et al, 2012), and 10 for cereal (Zahedi et al., 2015).

The idea of $\alpha_i > 0$ might be puzzling given the first law of thermodynamics. Yet such law is not violated, as $\boldsymbol{\delta_i}$ rarely comes from $\boldsymbol{\gamma_i}$, but from primary energy sources (PES). These are the originally untouched sources of energy (IEA, 2003), that are converted into secondary energy forms (here called $\boldsymbol{Q_i}$), to make them compatible with agent's $\boldsymbol{x}$, and thus the external sources of energy which allows for the existence of $\boldsymbol{\alpha_i}$ without breaking any laws of physics. If there is anything as a free lunch, PES are it.

Goods that show $\delta_k > 0$ but $\alpha_k < 0$ are quasi-energy goods and those with $\delta_k = 0$ non-energy goods. Without loss of generality, both will be referred to as non-energy goods and denoted by $\boldsymbol{Q_j} = (Q_1, \dots, Q_n)$. What allows for their production is the total energy surplus $E = \sum_{i=1}^{m} \int_0^{Q_i} \alpha_i(Q_i)\, dQ_i$. Such is the importance of $\boldsymbol{\alpha_i}$, as argued by the Energy Return Over Investment (EROI) literature (Hall, Balogh, & Murphy, 2005; Hall, Lambert, & Balogh, 2014; Lambert et al., 2013). Note that $\boldsymbol{\alpha_i}$ and EROI are intimately related, as $\boldsymbol{EROI_i} = \frac{\boldsymbol{\delta_i}}{\gamma_i^A}$ and marginal EROI is $\boldsymbol{MEROI_i} = \frac{\boldsymbol{\delta_i}}{\gamma_i}$.

Another important difference between $\boldsymbol{Q_i}$ and $\boldsymbol{Q_j}$ assumed hereafter, is that $\boldsymbol{\gamma_i}$ depends positively on the level of production, and therefore $\boldsymbol{\gamma_i}(\boldsymbol{Q_i})$ (and thus $\boldsymbol{\alpha_i}(\boldsymbol{Q_i})$). This is justified given that $\boldsymbol{Q_i}$ stem from PES, and their heterogeneous characteristics leads agents to tap first the most convenient ones, while leaving the least convenient ones for when the "lowest-ganging fruits" have been saturated. This fact of any $\boldsymbol{Q_i}$ has been used to explain differential rent back in the work of Ricardo, Malthus, West, and Torrens (Backhouse, 2004). On another hand, $\boldsymbol{\gamma_j}$ will be assumed constant for any $\boldsymbol{Q_j}$, and thus $\boldsymbol{\gamma_j} = \boldsymbol{\gamma_j^A}$.

The idea that agents produce goods by transferring energy using prime movers can now be further specified as them transferring the $\boldsymbol{\delta_i}$ of their $\boldsymbol{Q_i}$. Thus a quadruple identity is found, such that $eG_k = \boldsymbol{\omega_k} \cdot \boldsymbol{x_k} = \gamma_k^A \cdot Q_k = \boldsymbol{\delta_{i,k}} \cdot \boldsymbol{Q_{i,k}}$, where $\boldsymbol{Q_{i,k}}$ are all the energy goods used to produce Q_k (e.g. tons of rice and/or oil). Given this, $\boldsymbol{\omega_k} = \boldsymbol{\delta_i} \cdot \boldsymbol{Q_{i,l}}$, where $\boldsymbol{Q_{i,l}}$ are the energy goods used by each x_l (e.g. tons of rice per worker, tons of oil per steam turbine).

It is a physical imperative that $eB \geq eG$, because agents cannot transfer energy they do not have. Such condition will be referred to as the energy budget constraint (eBC). Thus, an energy-constraint agent specifically implies that $eB = eG$, or $eA + \boldsymbol{\delta_i} \cdot \boldsymbol{Q_i} = \boldsymbol{\gamma^A} \cdot \boldsymbol{Q}$.

The eBC must hold regardless if it is built through producing, exchanging, borrowing, or stealing $\boldsymbol{Q_i}$. The last two ways are irrelevant in the long-run, and only production and exchange are activities of interest to economists, precisely because they provide the way for agents to satisfy their eBC in the long-run. The eBC can be violated in some periods, yet only when it is observed at a larger and encompassing one. For example, for two periods, if $eI_1 > eG_1$ then $eI_2 < eG_2$ can occur if $eI_{1+2} \geq eG_{1+2}$ holds. This is possible through eA, which is built in t_1 and used in t_2. Note that this could not happen the other way around (there is no such thing as – eA) because for energy to be transferred it must first be available. Borrowing does not change this, as it only changes the distribution of $\boldsymbol{Q_i}$ between agents over time.

4.5 Embodied energy and economic energy

Although the capacity to produce $\boldsymbol{Q_k}$ is governed by energy constraints, agents' behavior is not determined by energy *per se*, but by its intrinsic opportunity cost. This is how much energy is spent securing an additional unit of energy: the inverse of a $\boldsymbol{Q_i}$'s EROI. Naturally, if an agent spends higher (lower) amounts of energy to secure an additional unit, it will value energy expenditures more (less).

As energy expenditure and its intrinsic opportunity cost are the same only under precise circumstances, agents actually face two parallel energy budgets. One is the eBC, based on energy expenditures, embodied energy, and energy income, while the other is an economic budget constraint (EBC), based on the intrinsic opportunity cost of energy expenditure, economic energy, and economic income. These parallel budgets must be satisfied simultaneously and are linked by $\boldsymbol{Q_i}$'s EROI.

Assuming $\boldsymbol{Q_i}$ only uses the same $\boldsymbol{Q_i}$ to be produced (e.g. oil to produce oil as done in Gagnon, Hall, & Brinker (1992)), then $EROI_i = Q_i / Q_{in}$. This leads to $\gamma_k^A = \frac{\boldsymbol{\delta_{i,k}} \cdot \boldsymbol{Q_{i,k}}}{Q_k} = \frac{\delta_i \cdot Q_{in}}{Q_i} = \delta_i \cdot \frac{1}{EROI_i} = \beta_k^A$, which is its economic energy: the intrinsic opportunity cost of the energy used in its production. Given that marginal analysis is required to properly analyze opportunity costs, the previous expression can be re-specified as $\gamma_i = \delta_i \cdot \frac{1}{MEROI_i} = \beta_i$, where $MEROI_i = mQ_i / Q_{in}$ represents the marginal EROI of Q_i: the marginal quantity of Q_i produced (mQ_i) by each unit of Q_i used (Q_{in}).

Although $\boldsymbol{\gamma_i} = \boldsymbol{\beta_i}$, this does not stand for $\boldsymbol{Q_j}$**.** Whereas their embodied energy is $\gamma_j = \frac{\boldsymbol{\delta_{i,k}\cdot mQ_{i,k}}}{Q_k}$, their economic energy is $\beta_j = \frac{\boldsymbol{\delta_{i,k}\cdot mQ_{i,k}}}{\boldsymbol{MEROI_i}} \cdot \frac{1}{Q_k} \le \gamma_j$. Thus the intrinsic opportunity cost of the energy spent in the production of a good is by definition smaller, or at most equal, to the energy spent. No distinction is made between average and marginals given $\boldsymbol{\gamma_j^A} = \boldsymbol{\gamma_j}$ implies $\boldsymbol{\beta_j^A} = \boldsymbol{\beta_j}$.

The eBC ($eI \geq eG$), which can be expressed as $\boldsymbol{\delta_i} \cdot \boldsymbol{Q_i} \geq +\sum_{i=1}^{m} \int_0^{Q_i} \gamma_i(Q_i)\, dQ_i + \boldsymbol{\gamma_j} \cdot \boldsymbol{Q_j}$, underlies the EBC ($I \geq G$), where I and G are respectively the economic income and expenditure. The EBC, expressed as $\sum_{i=1}^{m} \gamma_i \cdot Q_i (2 - \frac{1}{MEROI_i}) \geq \sum_{i=1}^{m} \int_0^{Q_i} \beta_i(Q_i)\, dQ_i + \boldsymbol{\beta_j} \cdot \boldsymbol{Q_j}$, is linked to the eBC by $\boldsymbol{Q_i}$'s EROI. While G is straightforward to understand, I was not derived from theory, and was found inductively based on simulations (see part C of the online appendix). Both budgets must be satisfied simultaneously, which can be seen in the special case when $\boldsymbol{\gamma_i} \rightarrow \boldsymbol{\delta_i}$. As this implies that $\boldsymbol{MEROI_i} \rightarrow \mathbf{1}$ Then $\boldsymbol{\beta_j} \rightarrow \boldsymbol{\gamma_j}$ and $\sum_{i=1}^{m} \gamma_i \cdot Q_i (2 - \frac{1}{MEROI_i}) \rightarrow \boldsymbol{\delta_i} \cdot \boldsymbol{Q_i}$**.** Under such condition, which happens in long-run equilibrium (shown in section 5.1.2), the EBC converges towards the eBC.

Lifting the assumption of no sectoral interlinkages does not change any of the previous results. MEROI remains the link between eBC and EBC which converge in long-run equilibrium. The only difference is that the estimation of MEROI and $\boldsymbol{\gamma_j}$ becomes cumbersome as feedbacks must be accounted for using input-output tables (Casler & Wilbur, 1984; Herendeen, 1978; Karkacier & Goktolga, 2005).

4.6 Efficiency

Not all eG is effectively converted into physical work W_p used to close gaps. The Second Law of Thermodynamics (SLT) states that all energy transfers are subject to inefficiencies, and thus eG is always larger than W_p. The difference between both will be called useless energy transfer W_u, which usually takes the form of waste heat, and thus $eG = W_p + W_u$.

Given this, the first-law-efficiency of an agent can be defined as $\frac{W_p}{eG}$, bounded between 0 and 1. The first limit implies that all energy transferred is converted into waste heat, while the second that all is used to perform physical work. Because of the SLT, this second boundary can never be reached, and in many cases it becomes impossible to come close[9]. Nevertheless, given the aims of this paper and without loss of generality, it will be assumed that such boundary can be approached.

[9] Modern steam turbines and diesel engines have thermal-efficiencies of 40 and 50% respectively (Smil, 2013).

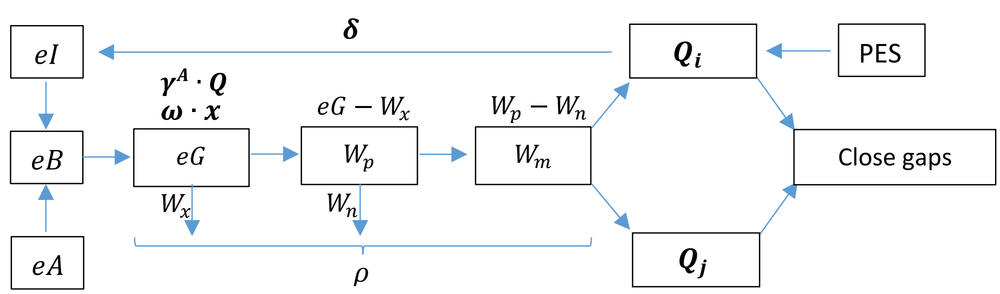
Moreover, W_p is usually larger than the physical minimum amount of work required to achieve desired change W_m. To perform only W_m agents have to be fully diligent, when in many cases they are not. Also, they need to do "negative work"[10], which in many cases is not known, possible, or convenient. W_m can be estimated with physical equations, either using Newtonian or quantum mechanics, entropy, or thermodynamics, as shown in part D of the online appendix.

The difference between W_p and W_m is non-useful work W_n, and thus $W_p = W_m + W_n$. This defines the work-efficiency of an agent as $\frac{W_m}{W_p}$, bounded between 0 and 1. The first limit implies that all physical work uselessly rearranges matter, closing no gaps. On the contrary, the second means that all physical work reconfigures matter that closes gaps. Given how diligent, faultless, and knowledgeable agents must be to achieve this second boundary, it will be assumed it can be approached but not reached.

By multiplying these two types of efficiencies, an agent's total-efficiency (efficiency hereafter) $\rho = \frac{W_m}{eG}$ is obtained, which is naturally bounded between 0 and 1. This implies that total-efficiency is influenced both by the physics governing the material reconfigurations under consideration, and by the magnitude of agent's diligence, mistakes, and random events. Naturally, there is a distinct ρ_k for the production of each Q_k, yet for simplicity only a common ρ will be assumed for all k.

A depiction of how agents objectively close gaps is presented in figure 2.

Fig. 2



5. Reinterpreting economics

Unbearable and unstable gaps, together with a limited eB, leads to energy-constrained agents such that $eB = eG$. As these agents cannot, by definition, objectively close all the gaps they face, they have clear

[10] An example of this is a Kinetic Energy Recovery Systems (KERS) that allows kinetic motion to be transformed into electricity instead of waste heat when a car uses its breaks.

incentives to minimize eG, or alternatively, to maximize E**.** Otherwise, they would not be pareto efficient, as spending less energy or securing more energy surplus would allow them to, *ceteris paribus*, produce more goods and thus close additional gaps.

Yet agents do not only face a technical-productive problem. As they face multiple gaps, they must not only deal with the problem of change, but also with the problem of choice: of choosing which goods will be produced, and thus which gaps will be close. Therefore, energy-constraints leads to the opportunity cost of producing goods, which are the goods not produced, yet more importantly leads to the opportunity cost of closing gaps, which are the gaps not closed.

The following analysis will be limited to autarkic, single-period, and mostly long-run equilibrium. As such, exchange and intertemporal issues will be excluded, and the energetic and economic budget will be exactly equivalent. Moreover, issues of instability, uncertainty, indirectness and interdependency will be overlooked.

5.1 The producer problem

Any given ρ implies that there is a maximum Q_k that can be produced with a given set of $\boldsymbol{x_k}$ used at full workload. This relation between $\boldsymbol{x_k}$ and Q_k, dependent on ρ, corresponds to an agent's production function f, such that $Q_k = f(\boldsymbol{x_k})$. As $\rho \to 1$, f will increase the quantity produced given $\boldsymbol{x_k}$, and as it reaches one, thermodynamic limits to efficiency enhancements are met. f is assumed to be quasiconvex, continuous, and twice differentiable.

5.1.1 Energy expenditure minimization

When attempting to produce $\boldsymbol{\bar{Q}_k}$ with minimum energy expenditure, agents face the following problem:

$$\min eG \qquad\qquad s.t{:}\ Q_k = \bar{Q}_k \qquad \forall\ k = 1 \ldots K$$

Recalling that $eG = \boldsymbol{\omega} \cdot \boldsymbol{x}$, and $Q_k = f(\boldsymbol{x_k})$:

$$\min\ \boldsymbol{\omega} \cdot \boldsymbol{x} \qquad\qquad s.t{:}\ f(\boldsymbol{x_k}) = \bar{Q}_k \quad \forall\ k = 1 \ldots K$$

$$x_l \leq \bar{x}_l \qquad \forall\ l = 1 \ldots L$$

By assuming long-run the last restriction is relaxed. Thus the Lagrangian is:

$$\mathcal{L} = \boldsymbol{\omega} \cdot \boldsymbol{x} + \sum_{k=1}^{K} [\phi_k(\bar{Q}_k - f(\boldsymbol{x_k}))]$$

(i): $\frac{\partial \mathcal{L}}{\partial x_l} = K \cdot \omega_l - \sum_{k=1}^{K} \phi_k f_{l,k} = 0 \qquad \forall\, l = 1 \dots L$

(ii): $\frac{\partial \mathcal{L}}{\partial \phi_k} = \sum_{k=1}^{K} [\bar{Q}_k - f(\boldsymbol{x_k})] = 0 \qquad \forall\, k = 1 \dots K$

This setup is equivalent to conventional neoclassical specification of producer cost minimization problem. The difference is that factors of production are replace with $\boldsymbol{x}$, and factor payments with $\boldsymbol{\omega}$. Thus well-known mechanics can be used to derive FOC and refutable hypothesis, yet obtaining solutions that have distinct interpretations. Given the equivalent setting, tangency conditions, refutable hypothesis, and other solutions from the producer problem will only be stated and interpreted. Detailed procedures can be found in Silberberg & Suen (2001) or any other advanced microeconomic textbook.

Setting $K = 1$ and denoting any two x by 1 and 2, the tangency conditions are:

$$\frac{f_1}{f_2} = \frac{\omega_1}{\omega_2} \; and \; \frac{f_1}{\omega_1} = \frac{f_2}{\omega_2}$$

What the first form tells us is that the MRTS between two x must equal their relative energy expenditure. The second one that each unit of energy spent by each x must report the same amount of additional production.

Given a production function, we can use the tangency conditions to solve this system of equations, and obtain optimal demand function $\boldsymbol{x}^*(\boldsymbol{\omega}, \bar{Q}_k)$, as well as the minimum marginal energy cost of production $\phi^*(\boldsymbol{\omega}, \bar{Q}_k)$. By setting $L = 2$, totally differentiation the FOC, and applying Kramer's rule, we derive refutable hypothesis that yield the following unambiguous proposition:

$$\frac{\partial x_1^*}{\partial \omega_1} = \frac{-f_1^2}{|H|} < 0 \qquad \frac{\partial x_2^*}{\partial \omega_1} = \frac{f_1 f_2}{|H|} > 0$$

Where $|H| = \begin{vmatrix} f_{11} & f_{12} & f_1 \\ f_{21} & f_{22} & f_2 \\ f_1 & f_2 & 0 \end{vmatrix}$

eG can be evaluated at its optimum to obtain the energetic cost function $C^*(\omega_1, \omega_2, \bar{Q}_k) = \omega_1 x_1^* + \omega_2 x_2^*$, derive Sheppard's Lemma to recover optimal $\boldsymbol{x}$ demands and the energy cost of production, find that C^* is homogeneous of degree r in ω_1 and ω_2, and symmetric in $\frac{\partial x_1^*}{\partial \omega_1} = \frac{\partial x_2^*}{\partial \omega_2}$. Also, with C^* is possible to identify optimal average and marginal embodied energy, γ_k^{*A} and γ_k^*respectively.

5.1.2 Energy surplus maximization

Whereas energy expenditure minimization seems the most intuitive process followed by energy-constraint agents, energy surplus maximization also seems a reasonable objective. Moreover, given that energy expenditure minimization implies energy surplus maximization, such process should be expected to exists. To simplify, $\boldsymbol{Q_i}$ and $\boldsymbol{Q_j}$ will be analyzed separately.

There are two ways to present the problem for $\boldsymbol{Q_i}$. The first is that agents maximize the difference between energy income $eI = \boldsymbol{\delta_i} \cdot \boldsymbol{Q_i}$ and $eG = \sum_{i=1}^{m} eG_i(Q_i)$ associated with such production. Thus:

$$\max E = eI - eG = \boldsymbol{\delta_i} \cdot \boldsymbol{Q_i} - \sum_{i=1}^{m} eG_i(Q_i)$$

$$\frac{\partial E}{\partial Q_i} = \delta_i - \frac{\partial eG_i(Q_i)}{\partial Q_i} = 0 \qquad\qquad \forall\ i = 1 \dots m$$

$$\delta_i = \gamma_i(Q_i) \tag{1}$$

Q_i^* is such that marginal energy income (δ_i) equates marginal energy expenditure ($\gamma_i(Q_i)$). Note that Q_i^* is determined on technical terms, influenced by ρ_i, and independent from subjective characteristic of agents (except from those that lead them to be energy constraint in the first place). If $eG_i(Q_i)$ is known, Q_i^* can be obtained, and with it the maximum total energy surplus $E^* = \sum_{i=1}^{m} \int_0^{Q_i^*} [\delta_i - \gamma_i(Q_i)] dQ_i$.

The second way to present this problem focuses on the use of $\boldsymbol{x}$, such that $Q_i = f(\boldsymbol{x_i})$. Thus:

$$\max E = \sum_{i=1}^{m} \delta_i f(\boldsymbol{x_i}) - \sum_{i=1}^{m} \boldsymbol{\omega_i} \cdot \boldsymbol{x_i}$$

$$\frac{\partial E}{\partial x_l} = \sum_{i=1}^{m} \delta_i f_{l,i} - m \cdot \omega_l = 0 \qquad\qquad \forall\ l = 1 \dots L$$

Furthermore, by setting $m = 1$ and $L = 2$, the same tangency conditions presented in energy expenditure minimization, so in fact $\gamma_i(Q_i) = \gamma_i^*(Q_i)$, and thus optimality actually implies $\delta_i = \gamma_i^*(Q_i)$. Also, familiar refutable hypotheses are found:

$$\frac{\partial x_1^*}{\partial \omega_1} = \frac{f_{22}}{\delta_i(f_{11}f_{22} - f_{12}^2)} < 0 \qquad\qquad \frac{\partial x_2^*}{\partial \omega_1} = \frac{-f_{21}}{\delta_i(f_{11}f_{22} - f_{12}^2)} = ?$$

$$\frac{\partial x_1^*}{\partial \delta_i} = \frac{f_{12}f_2 - f_{22}f_1}{\delta_i(f_{11}f_{22} - f_{12}^2)} = ? \qquad\qquad \frac{\partial Q_i^*}{\partial \delta_i} = \frac{-\delta_i(f_1^2 f_{22} - 2f_1f_2f_{12} + f_2^2 f_{11})}{\delta_i(f_{11}f_{22} - f_{12}^2)} > 0$$

Own-energy expenditure effect is always negative, and cross energy expenditure effect is ambiguous and determined by the degree of complementary between $\boldsymbol{x_i}$. Also, the effect of an increase of δ_i on the use of $\boldsymbol{x_i}$ is ambiguous, and its effect on the quantity produced is always positive.

To perform energy surplus maximization process on Q_j an ad'hoc setup is needed. Define a sub-agent as a subset of $\boldsymbol{x_j}$ under an agent's control that produce a given Q_j, and an energy assignment δ'_j as certain amount of energy that the agent will assign to the sub-agent per unit produced. From the perspective of the sub-agent, δ'_j is no different than δ_i: it is the marginal amount of energy obtained by the production of the good. By following this artifice, and assuming sub-agents are also energy-constrained, the same results for Q_i are found: sub-agents produce Q_j^* such that:

$$\delta'_j = \gamma_j^* \tag{2}$$

How agents define δ'_j responds to the desired level of Q_j^*, mediated by the solution $\delta'_j = \gamma_j^*$. If such good shows an elastic marginal cost curve, higher Q_j^* will require higher δ'_j, given higher γ_j^*. If such curve is perfectly elastic, as is assumed here, any level of Q_j^* can be achieved with the same level of δ'_j. Yet this is only a partial solution that does not incorporate the trade-offs that increasing δ'_j implies for the production of other $\boldsymbol{Q_j}$. A complete answer is provided in section 5.3.1, showing that $\boldsymbol{\delta}'$ is optimally determined according to individual's desires and the scarcity of energy in general.

The usefulness of $\boldsymbol{\delta}'$ becomes evident when analyzing aggregate agents, such that sub-agents can be thought of as groups of their constituent agents endowed with an array of $\boldsymbol{x}$. In this context, $\boldsymbol{\delta}'$ becomes a crucial decision tool to signal to sub-agents how much to produce. In a modern economy, these sub-agents are firms, and the decision mechanism to define $\boldsymbol{\delta}'$ is either a central planner or the market mechanism, as will be discussed in section 6.

The definition of $\boldsymbol{\delta}'$ is endogenous to the agent, and thus $\boldsymbol{Q_j^*}$ is not determined in technical terms by an objective optimality condition (as Q_i^* given an exogenous δ_i). To find $\boldsymbol{Q_j^*}$, the producer problem must be complemented with the consumer problem. It is worth remembering that an agent's economic problem is the existence of gaps, and their objective is to close them, not to minimize energy expenditure nor maximize energy surplus *per se*. These two optimizing procedures are just behaviors consistent with energy-constraint contexts, which cannot by themselves determine which gaps are to be closed, and thus which goods are to be produced.

5.2 The consumer problem

As agents face energy-constraints in relation to unstable, unbearable, and multiple gaps, they are forced to choose which to close, and by taking such decisions reveal the subjective relative importance of each gap. This decision takes the material form of choosing which associated good to produce, which under autarchy implies which good to consume. Consumption of a good, from the perspective of the gap, is the use of a specific configurations of matter achieved through an agent's work (i.e. a good) to close a gap.

With a binding eBC, and defined $\boldsymbol{Q_k}$ and $\boldsymbol{\gamma_k}$, virtually all the elements to infer such decision process using the TRP are present (Houthakker, 1950; Samuelson, 1938). Preference relations are derived by demand decisions, which are obtained using eB instead of money-income, and $\boldsymbol{\gamma_k}$ (or $\boldsymbol{\beta_k}$) instead of prices. Lastly, leaving some mathematical subtleties aside, quasiconcave utility functions are guaranteed to exist by invoking the Strong Axiom of Revealed Preferences, given the implied negative semi-definiteness and symmetry of the (transformed) Slutsky terms (Silberberg & Suen, 2001).

Although utility functions are used, from the perspective of the gap utility itself is not necessarily associated with a psychological process related to pain and pleasure. Utility is only a representation of ordered and purposeful decision making under scarcity, which can be associated with pain and pleasure, but also with other drivers of human actions. Thus, utility is conceived *a la* Fisher (see footnote 2).

One implication of this is that utility functions are not a natural property of agents, as those that are not energy-constrained fail to adhere to the requirements of the TRP. Another is that comparisons of wellbeing are harder to do. Agents with few gaps require fewer $\boldsymbol{Q_k}$, and might be better off than others that face many. This caveat, highlighted with the subjective approach and from heterogeneous external realities that make spontaneous trajectory move closer or farther from desired ones, takes utilitarism one step further. It suggests that utilities between agents cannot be compared, but that neither can utilities between the same one in different contexts.

Provided energy-constraint agents ($eB = eG$), a modified utility maximization problem is followed. Utility functions are assumed quasiconcave, continuous, and twice differentiable.

5.2.1 Exogenous energy budget (only $\boldsymbol{eA}$)

An agent only has eA and therefore only produces $\boldsymbol{Q_j}$. Given $\boldsymbol{\gamma_j} = \boldsymbol{\gamma_j^A}$:

$$\max U(\boldsymbol{Q_j}) \qquad s.t: \boldsymbol{\gamma_j} \cdot \boldsymbol{Q_j} \leq eA$$

When the constraint is met with equality given an energy-constraint agent, the Lagrangian is defined as:

$$\mathcal{L} = U(\boldsymbol{Q_j}) + \lambda[eA - \boldsymbol{\gamma_j} \cdot \boldsymbol{Q_j}]$$

This setup is equivalent to conventional neoclassical specification of consumer utility maximization problem. The difference is that money-income is replaced with eA, and prices with γ. The implication of this is that we can apply the same mechanics to derive FOC and refutable hypothesis, yet attach to the resulting solutions distinct interpretations.

Given that the mechanics are the same, direct reinterpretation of tangency conditions, refutable hypothesis, and other solutions from the consumer problem are provided. The detailed procedures can be found again in Silberberg & Suen (2001) or any other advanced microeconomic textbook.

The tangency condition between any two Q_j (denoted by 1 and 2) can be expressed as:

$$\frac{U_1}{U_2} = \frac{\gamma_1}{\gamma_2} \; and \; \frac{U_1}{\gamma_1} = \frac{U_2}{\gamma_2}$$

The first form implies that for optimality the Marginal Rate of Substitution (MRS) between Q_j must equalize their relative γ. The second that optimality requires that each unit of energy spent on producing each Q_j must report the same amount of additional utility. Replacing this condition on the eBC yields the Energetic-Marshallian demand functions for these goods $Q_k^*(\gamma_j, \gamma_i, \delta_i)$, as well as the marginal utility of energy $\lambda^*(\gamma_j, \gamma_i, \delta_i)$: the marginal capacity of energy to close gaps.

Furthermore, by reducing the problem to $n = 2$, totally differentiation the FOC, and applying Kramer's rule, refutable hypothesis are derived which yield the following proposition:

$$\frac{\partial Q_1^*}{\partial \gamma_1} = \frac{\lambda \begin{vmatrix} U_{11} & -\gamma_2 \\ -\gamma_2 & 0 \end{vmatrix}}{|H|} - Q_1^* \frac{\partial Q_1^*}{\partial eA} \qquad \frac{\partial Q_2^*}{\partial \gamma_1} = \frac{\lambda \begin{vmatrix} U_{21} & -\gamma_2 \\ -\gamma_1 & 0 \end{vmatrix}}{|H|} - Q_1^* \frac{\partial Q_2^*}{\partial eA}$$

Where $|H| = \begin{vmatrix} U_{11} & U_{12} & -\gamma_1 \\ U_{21} & U_{22} & -\gamma_2 \\ -\gamma_1 & -\gamma_2 & 0 \end{vmatrix}$

These equations contain the equivalent of own and cross price effects, but expressed as own and cross embodied energy effects. Each is decomposed into their respective own and cross energy-substitution effect, as well as their own and cross energy-endowment effects. Own energy-substitution effects are always negative, cross energy-substitution effects are equal and always positive (when $n = 2$), and energy-endowment effects depend on the normal or inferior nature of each good.

As becomes clear by now, a complete reinterpretation of the consumer problem is enabled. The marginal utility of an extra unit of energy $\frac{\partial \lambda}{\partial eA}$ will be negative given quasiconcavity of U if $U_{11} < 0$ is assumed, but ambiguous otherwise. Indirect Utility Functions can be specified and energetic versions of Roy's Identity as expressed using envelope theorem.

The dual of this problem yields the Energetic-Hicksian demand functions, own and cross Energetic-Slutsky equations, and the Energetic-Expenditure function. This last function is homogeneous of degree one, concave, and reciprocal in $\boldsymbol{\gamma_j}$. Such function also allows for the energetic version of Sheppard's Lemma. The Energetic-Engel Aggregation ($\varepsilon_{1,A}S_1 + \varepsilon_{2,A}S_2 = 1$)[11], Energetic-Cournot Aggregation ($\varepsilon_{1,\gamma_2}S_1 + \varepsilon_{2,\gamma_1}S_2 = -S_1$), energetic version of Hick's third law ($\varepsilon^c_{1,\gamma_2}Q_1^{*c}\gamma_1 + \varepsilon^c_{1,\gamma_2}Q_1^{*c}\gamma_2 = \varepsilon^c_{1,\gamma_1}Q_1^{*c}\gamma_1 + \varepsilon^c_{2,\gamma_1}Q_1^{*c}\gamma_2$), and other relations ($\varepsilon^c_{1,\gamma_1} + \varepsilon^c_{1,\gamma_2} = 0$, $\varepsilon^c_{1,\gamma_1}S_1 + \varepsilon^c_{2,\gamma_1}S_2 = 0$) can also be specified.

5.2.2 Endogenous energy budget (only $\boldsymbol{eI}$)

An agent only has eI and therefore builds its eB exclusively with $\boldsymbol{Q_i}$. Thus:

$$\max U(\boldsymbol{Q_k}) \qquad s.t\colon \boldsymbol{\gamma_k} \cdot \boldsymbol{Q_k} \leq eI$$

As the constraint is met with equality, and recalling that $eI = \boldsymbol{\delta_i} \cdot \boldsymbol{Q_i}$, the eBC is specified as:

$$\boldsymbol{\gamma_j} \cdot \boldsymbol{Q_j} + \sum_{i=1}^{m} \int_0^{Q_i} \gamma_i(Q_i)\, dQ_i = \boldsymbol{\delta_i} \cdot \boldsymbol{Q_i}$$

This eBC implies that $E = \boldsymbol{\gamma_j} \cdot \boldsymbol{Q_j}$: as stated in section 4.4 and 5.1.2, total energy surplus derived from $\boldsymbol{Q_i}$ is what allows for the production (and consumption) of $\boldsymbol{Q_j}$. The Lagrangian is:

$$\mathcal{L} = U(Q_{k=1\ldots K}) + \lambda[\sum_{i=1}^{m} [\delta_i \cdot Q_i - \int_0^{Q_i} \gamma_i(Q_i)\, dQ_i] - \boldsymbol{\gamma_j} \cdot \boldsymbol{Q_j}]$$

This setup is different from the standard consumer problem. The FOC are:

(i): $\frac{\partial \mathcal{L}}{\partial Q_j} = U_j - \lambda\gamma_j = 0 \qquad \forall\, j = 1 \ldots n$ (3)

(ii): $\frac{\partial \mathcal{L}}{\partial Q_i} = U_i - \lambda[\gamma_i(Q_i) - \delta_i] = 0 \qquad \forall\, i = 1 \ldots m$ (4)

(iii): $\frac{\partial \mathcal{L}}{\partial \lambda} = \sum_{i=1}^{m}[\delta_i \cdot Q_i - \int_0^{Q_i} \gamma_i(Q_i)\, dQ_i] - \boldsymbol{\gamma_j} \cdot \boldsymbol{Q_j} = 0$

[11] With $\varepsilon_{1,A} = \frac{\partial Q_1^*}{\partial eA}\frac{eA}{Q_1^*}$, $S_x = \frac{\gamma_1 Q_1^*}{eA}$

If agents are not energy-constrained, $\lambda = 0$ and $U_k = 0$. This is why energy-constraints are critical for utility functions to exist. If energy is freely available, agents can produce $\boldsymbol{Q_k}$ up to the point they closed all gaps. As no tradeoffs would exist, no choices would be required, and economics would be irrelevant.

Rearranging (4) leads by definition to $U_i = \lambda[\gamma_i(Q_i) - \delta_i] \leq 0$. Thus, optimality requires that the marginal utility of $\boldsymbol{Q_i}$ be *zero or negative.* Given utility *a la* Fisher, negative marginal utility need not be associated with pain. What it does imply is that agents can increase their utility (close more gaps) by reducing it in absolute value. A plausible interpretation of this is that the actual utility that $\boldsymbol{Q_i}$ provide is derived from their E which enables to produce $\boldsymbol{Q_j}$ that close other gaps. Thus negative marginal utility reflects that $\alpha > 0$ and that E is not maximized. This creates incentives, as will be detailed in section 5.3.1, to reduce α to zero by increasing its production.

Tangency conditions between any two Q_j have already been presented in the previous section. Leaving aside that at optimum $\alpha_i = \delta_i - \gamma_i(Q_i) = 0$, and expressing $\boldsymbol{\gamma_i(Q_i)}$ as $\boldsymbol{\gamma_i}$ hereafter, the tangency condition between any two Q_i (denoted by 3 and 4) can be expressed as:

$$\frac{U_3}{U_4} = \frac{\alpha_3}{\alpha_4} \; and \; \frac{U_3}{\alpha_3} = \frac{U_4}{\alpha_4}$$

The first form implies that for optimality the MRS between Q_i must equalize their relative α. The second that optimality requires that each unit of energy surplus generated while producing each Q_i must report the same amount of additional utility, which could be used as a way to grasp heterogeneous quality between the energy provided by different Q_i (density, cleanness, safety, etc). Furthermore, the discovery of a new high α_i not only has the effect of increasing an agent's eI, but also of altering the schedule of consumption of $\boldsymbol{Q_i}$. If one α_i rises, *ceteris paribus*, the other $\boldsymbol{Q_i}$'s marginal utility has to fall. Finally, as $\boldsymbol{\alpha} \rightarrow 0$, both forms become undefined.

The tangency condition between one Q_i and Q_j (denoted by 1 and 3 respectively) is:

$$\frac{U_{j=1}}{U_{i=3}} = -\frac{\gamma_{j=1}}{\alpha_{i=3}} \; and \; \frac{U_{j=1}}{\gamma_{j=1}} = -\frac{U_{i=3}}{\alpha_{i=3}}$$

The first form implies that the MRS between a Q_i and an Q_j must equal the negative of the ratio between the embodied energy of the former and the marginal energy surplus of the latter. This MRS states how much *less* of one Q_j must be consumed by the reduction in the consumption of one Q_i (utility being held constant by the sign of the marginal utility of Q_i). The second form implies that the marginal utility per unit of energy spent on Q_j must equal the marginal utility (in absolute value) per unit of energy surplus provided by the Q_j. Here again, as $\alpha \to 0$, both forms become undefined.

Given a utility function, these tangency conditions and the eBC solves the system of equations, and again Energetic-Marshallian demand function $Q_k^*(\gamma_j, \delta_i)$, as well as the marginal utility of energy $\lambda^*(\gamma_j, \delta_i)$ can be found.

Furthermore, replacing Q_k^* and λ^* in the FOC yields identities that can be differentiate. To make this manageable, only one Q_j ($j = 1$) and one Q_i ($i = 3$) are considered and expressed in matrix form:

$$\begin{bmatrix} U_{11} & U_{13} & -\gamma_1 \\ U_{31} & U_{33} - \lambda^* \frac{\partial \gamma_3}{\partial Q_3} & -(\gamma_3 - \delta_3) \\ -\gamma_1 & -(\gamma_3 - \delta_3) & 0 \end{bmatrix} \begin{bmatrix} dQ_1^* \\ dQ_3^* \\ d\lambda^* \end{bmatrix} \equiv \begin{bmatrix} \lambda^* d\gamma_1 \\ \lambda^*(d\gamma_3 - d\delta_3) \\ Q_1^* d\gamma_1 + Q_3^*(d\gamma_3 - d\delta_3) \end{bmatrix}$$

Full details of the derivation of comparative statics using Kramer's rule is presented in part F of the online appendix. The resultant refutable hypotheses are:

1) Q_j own-embodied-energy effect

$$\frac{dQ_1^*}{d\gamma_1} = \frac{1}{Q_3^*}\left[\frac{\lambda^*(\delta_3 - \gamma_3)[\gamma_1 Q_1^* - (\delta_3 - \gamma_3) Q_3^*]}{|H|} - Q_1^* \frac{dQ_1^*}{d\delta_3}\right] \tag{5}$$

This effect is decomposed into an own-substitution effect $\frac{\lambda^*(\delta_3 - \gamma_3)[\gamma_1 Q_1^* - (\delta_3 - \gamma_3) Q_3^*]}{|H|}$, which is negative or zero given the EBC, and an own-energy-income effect $Q_1^* \frac{dQ_1^*}{d\delta_3}$, which is signed according to normal or inferior nature of the Q_j. Note that the total effect is attenuated according to the importance of the Q_i.

Whenever the eBC is met with equality ($\gamma_1 Q_1^* = (\delta_3 - \gamma_3) Q_3^*$), which is implied when analyzing energy-constraint agents, substitution effect is zero. The same is true when no power-constraints are binding ($\delta_w = \gamma_w$). Thus, under those conditions only income effects matter:

$$\frac{dQ_1^*}{d\gamma_1} = -\frac{Q_1^*}{Q_3^*} \frac{dQ_1^*}{d\delta_3}$$

The absence of a substitution effect between Q_i and Q_j suggests that there is, for energy-constraint agents and in the long-run, a qualitative difference between both. This difference can be grasped by the second FOC, which implies that in the long-run $U_i = 0$. It seems interesting to analyze the problem for an agent with $n = 2, m = 2, K = 4$, such that substitution between two Q_i and two Q_j is enabled. Although promising, such exercise is left for future research.

2) Q_i cross-embodied-energy effect

$$\frac{dQ_3^*}{d\gamma_1} = \frac{1}{Q_3^*}\left[\frac{\lambda^*\gamma_1[\gamma_1 Q_1^* - (\delta_3 - \gamma_3)Q_3^*]}{|H|} - Q_1^*\frac{dQ_3^*}{d\delta_3}\right] \tag{6}$$

This expression is similar to (5). The full effect is decomposed into a negative or zero cross-substitution effect, and a cross-energy-income effect dependent on the normal or inferior nature of the good.

3) Marginal utility of energy embodied-energy effect

$$\frac{d\lambda^*}{d\gamma_1} = \frac{\lambda^*}{{Q_3^*}^2}\left[\frac{\lambda^*\gamma_1[(\delta_3 - \gamma_3)Q_3^* - \gamma_1 Q_1^*]}{|H|} + Q_1^*\frac{dQ_3^*}{d\delta_3} - Q_3^*\frac{dQ_1^*}{d\delta_3} - \frac{Q_1^* Q_3^*}{\lambda^*}\frac{d\lambda^*}{d\delta_3}\right] \tag{7}$$

This has many elements. A positive or zero term given by the EBC, a crossed energy income effect of Q_i, a negative crossed energy income effect of the Q_j, and lastly a positive term given that $\frac{d\lambda^*}{d3}$ is negative by definition. This expression implies that $\frac{d\lambda^*}{d\gamma_1}$ will tend to be positive, yet this hypothesis is not unambiguous. It will depend on the normality or inferiority nature of the goods under consideration.

The dual of this problem allows to derive the Energetic-Hicksian demand functions, the energy cost of utility, as well as the Energetic-Slutsky equations, Sheppard's Lemma, and precise properties of the Energetic-Expenditure function. Detailed derivations are available upon request.

5.3 Equilibrium

The equilibrium for an autarchic, single period, and long-run agent is found by setting the production and consumption of each good equal. For any Q_j this can be done recalling from equations (2) and (3) that for any $Q_j > 0$, $\delta'_j = \gamma_j$ and $U_j = \lambda\gamma_j$. Rearranging and combining both terms yields:

$$\delta'_j = \frac{U_j}{\lambda} \qquad \forall\, j = 1 \ldots n$$

Optimal δ'_j for good j is the ratio between its marginal utility and the marginal utility of energy, thus to the MRS between good j and energy itself: $\boldsymbol{\delta'_j}$ represents the ratio between the marginal contribution of a good to an agent's objectives (U_j), and energy's marginal extrinsic opportunity cost (λ). Note that as energy becomes less scarce (λ falls), $\boldsymbol{\delta'_j}$ will increase. Likewise, if $\boldsymbol{U_j}$ rises, so does $\boldsymbol{\delta'_j}$.

Furthermore, by recognizing that λ is the same for all goods, we arrive at a familiar expression which states that equilibrium requires that each unit of energy assigned for each good must yield the same marginal utility:

$$\frac{U_{j=1}}{\delta'_{j=1}} = \frac{U_{j=2}}{\delta'_{j=2}} = \cdots = \frac{U_{j=n}}{\delta'_{j=n}}$$

This procedure can be done for $\boldsymbol{Q_i}$ recalling from equations (1) and (4) that for any $Q_i > 0$, $\delta_i = \gamma_i(Q_i)$ and $U_i = \lambda[\gamma_i(Q_i) - \delta_i]$. Rearranging and combining yields, again, $U_i = 0$. In long-run equilibrium, $\boldsymbol{Q_i}$ not only yield no marginal energy surplus, but also no marginal utility. Thus, while they are the basis of an agent's energy income, and therefore the source from where it produces and consumes $\boldsymbol{Q_j}$, in the long-run, at the margin, they yield nothing. This result is conditional on the long-run implying no power-constraints. Thus, this paper will finish by exploring the short-run.

5.3.1 Short-run equilibrium and its evolution towards the long-run

In the short-run agents are power-constrained. This can be illustrated with an agent's energy surplus maximization process.

$$\max E = \boldsymbol{\delta_i} \cdot \boldsymbol{Q_i} - \textstyle\sum_{i=1}^{m} G_i(Q_i) \qquad s.t{:}\, x_l \leq \bar{x}_l \quad \forall\, l = 1 \ldots L$$

Where the Lagrangian is:

$$\mathcal{L} = \boldsymbol{\delta_i} \cdot \boldsymbol{Q_i} - \sum_{i=1}^{m} G_i(Q_i) + \sum_{l=1}^{L} \phi_l[\bar{x}_l - x_l]$$

By assuming that the agent lacks enough power of only one $\boldsymbol{x}$, say $x_1 \leq \bar{x}_1$, then $\phi_l = 0\ \forall\ l \neq 1$, $f(\boldsymbol{x_i}) = Q_i$ can be rewritten as $x_{i,1} = f^{-1}(Q_i, \boldsymbol{x}_{-1})$, and $\sum_{i=1}^{m} x_{i,1} = x_1$. The Lagrangian then is:

$$\mathcal{L} = \boldsymbol{\delta_i} \cdot \boldsymbol{Q_i} - \sum_{i=1}^{m} G_i(Q_i) + \phi_{l=1}[\bar{x}_1 - \sum_{i=1}^{m} f^{-1}(Q_i, \boldsymbol{x}_{-1})]$$

Taking FOC results in:

(i): $\frac{\partial \mathcal{L}}{\partial Q_i} = \delta_i - \gamma_i(Q_i) - \phi_{l=1} \cdot f^{-1\prime} = 0$

Which implies that in optimum $\delta_i = \gamma_i(Q_i) + \phi_{l=1} \cdot f^{-1\prime}$. Given $\phi_{l=1} > 0$ and $f^{-1\prime} > 0$, then $\delta_i > \gamma_i(Q_i)$. Thus, the effects of power-constraints are that at optimum, marginal energy income is greater than marginal energy cost.

Recalling from the previous section that $U_i = \lambda[\gamma_i(Q_i) - \delta_i]$ and replacing, a shadow equation is obtained where $U_i = -\lambda \cdot \phi_{l=1} \cdot f^{-1\prime}$. Given that λ, $\phi_{l=1}$, and $f^{-1\prime}$ are positive, the marginal utility of $\boldsymbol{Q_i}$ is, again, negative, but now it is inversely proportional to the magnitude of these shadow values and the marginal technical requirements of production. This provides a formal incentive for agents to reduce their energy and power constraints, and to increase ρ. More energy reduces λ, and more power directly reduces $\phi_{2,1}$

and indirectly λ. Likewise, increases in ρ reduces the amount of $\boldsymbol{x}$ required to increase production by one unit, and thus decreases $f^{-1\prime}$. All this drives the negative marginal utility of Q_i towards zero.

To visualize this only one Q_i is considered. Its demand curve is argued to be perfectly elastic at the level of its δ_i up to its power-constraint: The maximum energy that an agent will be willing to spend on a Q_i is, by definition, the energy it will provide, and it will be zero if it cannot use it at all. Given this, the demand for Q_i will be horizontal until it reaches the power-constraint, where it will fall vertically (figure 3.a). Moreover, the level of γ_i determines the E available to produce and consume Q_j, which corresponds entirely to consumer surplus. At Q° (the equilibrium level), $\delta_i > \gamma_i{}^{\circ}$.

Fig. 3.a Fig. 3.b

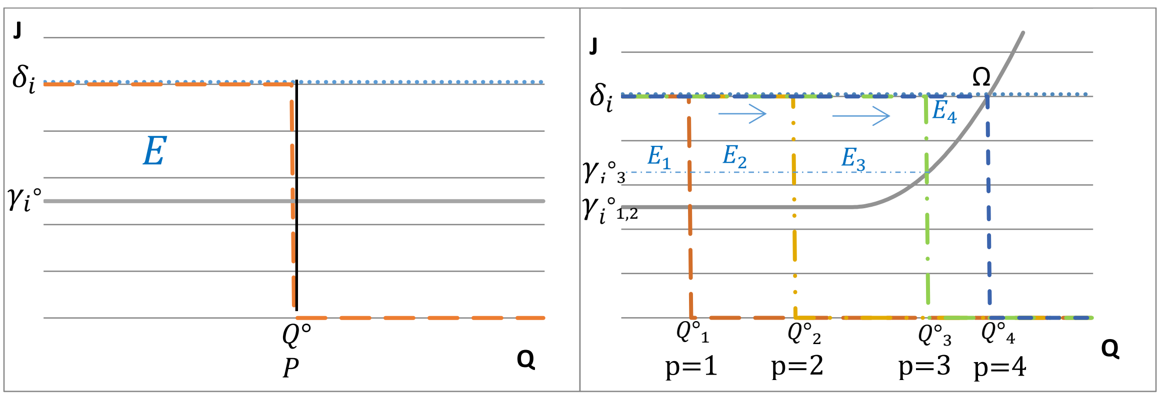


Figure 3.b shows the short-run equilibrium evolving towards the long-run. In $t = 1$ the agent has limited power, thereby producing $Q^{\circ}{}_1$ at $\gamma^{\circ}{}_1$, and $\delta_i > \gamma_i{}^{\circ}{}_1$. This agent secures E_1, which it can use to produce and consume $\boldsymbol{Q_j}$. The relation $U_i = -\lambda \cdot \phi_{l=1} \cdot f^{-1\prime}$ leads the agent to, among the $\boldsymbol{Q_j}$ it produces, build more $\boldsymbol{x}$ to relax its power-constraint. Thus at $\mathrm{t} = 2$ the agent produces more Q_i and additionally secures E_2. All the rest is the same. With new $\boldsymbol{x}$ at $\mathrm{t} = 3$, production leads to a marginal increase of $\gamma_i{}^{\circ}$. This increase is small as to keep $\delta_i > \gamma_i{}^{\circ}{}_3$. Finally, at $\mathrm{t} = 4$, Q_i is such that $\delta_i = \gamma_i{}^{\circ}$, and thus there are no incentives to produce any more $\boldsymbol{x}$ as there is no additional surplus energy to do work with. This point represents the steady-state Ω for this agent.

Three features stand out about this dynamic. 1) Any technological change that increases ρ and thus reduces γ_i, shifts the supply curve down and thus expands Ω to the right, yet this is bounded as $\rho \to 1$. 2) Discovery of new $\boldsymbol{Q_i}$ with high $\boldsymbol{\alpha}$, will lead to expansive dynamics, yet limited in the short-run by the availability of $\boldsymbol{x}$. 3) Along the transition from the short to the long-run, as γ increases, E is gradually shifted from consumer to producer surplus. Although this is irrelevant for an individual agent, it has potentially significant distributional implications for aggregate ones.

Another relevant analysis in the short-run, as $\delta_i > \gamma_i(Q_i)$ ($\mathrm{MEROI_i} > 1$), is how the eBC and EBC differ. As embodied and economic energy stop being equivalent, distinct results are expected from performing cost minimization, surplus maximization, and utility maximization to one or the other. Because of space constraints, such analysis will be left for further research, stressing only that by differing in the short-run, performing such procedures according to the schedules of $\boldsymbol{\gamma}$ and $\boldsymbol{\beta}$ must yield different results, which must nevertheless comply simultaneously with the EBC and eBC.

6. Concluding remarks

This paper explores the idea of extending economic rationale to energetics, and does so by considering gaps, instead of desires, as the necessary condition for the economic problem. The first implication of this is that as gaps are formed by desires and material divergences, the economic problem can be addressed either managing the first -the subjective approach-, or rearranging the second -the objective approach-. Although the subjective approach is argued to be a valuable way of simplifying the economic problem, its biological and cultural limitations, as well as the fact that it does not lead to any refutable hypothesis, leads to focus on the objective approach.

Such approach consists of material reconfigurations to close gaps, and therefore bridges economics with the issue of physical movements. In particular, goods are highlighted to be specific rearrangements of matter, and as what enables material changes are energy transfers, the objective approach reveals the central role of energy and prime movers for economics. The transfer of the former is the essence of change, and the latter is what concretely performs such transfers.

As agents spend energy to produce goods, they secure an energy income to satisfy an energy budget constraint. In parallel, agents face an economic budget constraint related to the former by energy good's Energy Return Over Investment (EROI), which takes into consideration that energy expenditures are usually not the same as the opportunity cost of such expenditures. In long-run equilibrium both budget constraints are identical.

Agents striving to close gaps objectively (*ends*) subject to energy and power constraints (*scarce means*), enables a straightforward development of optimization procedures. Under this context the problem of material change encompasses the problem of choice, as orthodox economics is normally thought of. Accordingly, the setting of producer and consumer problem from the perspective of the gap yields familiar results with distinct interpretation as well as wholly new ones.

Agents produce goods minimizing the energy spent on them. This leads to familiar comparative statics and conditions known from producer cost minimization, but with the precision that cost is energy and factors of production are prime movers. Also, agents produce energy-goods to maximize total energy surplus, which implies equating marginal energy income and expenditure, as well as an array of familiar comparative statics. This energy surplus provides the means to produce non-energy goods.

Agents consume goods to close gaps, and given they have limited energy income and each good has a defined embodied energy, the Theory of Revealed Preference can be used to argue for the existence of utility functions *a la* Fisher (see footnote 1). With them, the consumer problem can be defined with an exogenous, endogenous, or general (with both) energy budget, each leading to particular results. Whenever an exogenous one is used, refutable hypothesis and relevant relations are closely familiar to their neoclassical counterparts, yet they hold distinct interpretations. It is only as energy goods are considered that altogether new results appear.

In long-run equilibrium agents assign energy to the production of non-energy goods according to the MRS between each good and the marginal utility of energy, which corresponds to the highest opportunity cost of energy in general. Also, while energy goods are the basis of an agent's energy income, and therefore the source from where it produces and consumes non-energy goods, in the long-run, at the margin, they yield nothing.

In short-run equilibrium, power-constraints leads to positive marginal energy surplus, allocation of total energy surplus as consumer surplus, incentives to accumulate prime movers, and to increase efficiency. Yet, along the transitions to the long-run by the accumulation of prime movers, increases in the marginal costs of producing energy goods leads marginal energy surplus towards zero, allocation of total energy surplus as producer surplus, and the dissipation of incentives to accumulate more prime movers and increase efficiency provided by the shadow equation. This final state is the agent's steady-state, fundamentally defined by having no positive marginal energy surplus. This state can change with technological developments that reduce energy expenditures, or discoveries of new energy goods and prime movers. This implies a position in-between technological-optimist and neo-malthusian views, where efficiency has thermodynamic limit as efficiency reaches thermodynamic limits, yet the possibility remains of tapping new high energy surplus-yielding energy goods.

Whereas the importance of energy and prime movers for economies and societies are remarkably abundant (Ayres, 1998; Berndt, 1983; Costanza, 1980; Fermi, 1937; Huettner, 1982; Liu et al., 2008; Lotka, 1922; Odum & Odum, 1976; Podolinsky, 1880; Smil, 1994; Soddy, 1933), the perspective of the gap and the analytical development it enables is new. Given this, critiques made against basing economic analysis on energetic terms become generally irrelevant, as the perspective of the gap responds to concerns about one fundamental resource, non-energetic determinants of economics, and the shortcomings of energy theories of value.

From this perspective energy is the only fundamental resource as it is the exclusive source of material change. As such, it is the basis of all other resources, yet only so in the long-run, as in the short-run prime movers are also limiting factors. Moreover, this perspective does not imply that only energy matters. Constituted by desires, gaps themselves originate by determinants that are fully or partly independent from energetic considerations (e.g. tastes, traditions). Moreover, the way agents create and manage prime movers is influenced by an array of psychological, historical, and institutional factors that escapes the energetic realm. Finally, the perspective of the gap is not an energy theory of value, as these are axiomatic propositions. Here the axiom is that the necessary conditions of the economic problem are gaps. That energy transfers are become the unique and unquestionable way to close them objectively is not axiomatic, it is physics.

As agents strive to close gaps, it is clear why energy *per se* is not the source of economic value, but only the opportunity cost of energy spent in the production of goods that close gaps. On one hand this is consistent with research that finds a tendency towards maximum energy throughput and power (Odum, 1995), while others do not (Common & Stagl, 2005). On another, it is coherent with the idea that this analysis is useful to identify how energy shapes economic dynamics under energy-constrained contexts, but fails when energy surpluses are permanently large. If energy becomes “too cheap to meter”, utility functions break down, and the shadow value of energy and economic energies tend to zero.

This paper has only scratched the surface of the potential that this perspective has to understand and interpret economic phenomena. With the aim of presenting it along the array of associated concepts (energy goods, prime movers, efficiency, etc.), a limited analysis to autarchic, single-period, and mostly long-run agents has been developed here. While this was necessary because of space-constraints, it ruled out two of the most salient features of modern economies: exchange and non-renewable primary energy sources. Moreover, it avoided dealing with the short-run divergence between energy and economic budget constraints, and the consideration of uncertain and interdependent gaps and goods. Also because of space-constraints, this paper does not present any formal empirical evidence, which stands out as a crucial future step as extraordinary claims demand extraordinary evidence.

Dealing with exchanging agents might shed new insights into the nature of complex economic systems. Relative market prices might be social symbols of relative economic energies (likely the central refutable hypothesis of this perspective). Economic fundamentals (e.g. interest rates, wages) might be visible social symbols representing invisible energetic dynamics, and markets social arrangements that under perfect competition reveal economic energies, and define energy assignments according to aggregated preferences and general energy scarcity. Perhaps growth is the transition from the short to the long-run, and complex human organization, with its unparalleled adventure into religion, warfare, and science, could be the expression of underlying energy surpluses. This implies that the perspective of the gap not only leads to specific refutable hypothesis, but also to a fresh reinterpretation of what is economics: humans using scarce energy to reconfigure matter according to their desires.

Exchanging agents might not only lead to refutable hypothesis and a new economic paradigm, but also to understand the phenomena of extreme inequality (Jacobs, 2015). This broadly escapes the scope of this paper, but it is suggestive that total energy surplus, by its very nature, might be the essence of economic rent and thus lie at the core of extreme inequality. If correct, this could greatly enhance the inequality literature based on rent (Council of Economic Advisors, 2016; George, 1879; Sørensen, 2000) and rent-seeking (Dabla-Norris & Wade, 2001; Krueger, 1974; Tullock, 1967, 1993), by giving a more precise definition of what rent is and where it comes from. The result that in the long-run total energy surplus is entirely allocated as producer surplus is indicative of this possibility.

Moreover, multi-period agents lead to the issue of intertemporal optimal paths of energy use, which is a central challenge of any agent producing energy goods from non-renewable primary energy source. This might allow for further refutable hypothesis related to optimal transition paths in the spirit of Hartwick (1977). Again allowing for speculation, suboptimal intertemporal paths of energy use derived from suboptimal dissipation of fossil energy might be at the core of “The Great Acceleration” (Steffen et al., 2004;2015) and the onset of “The Anthropocene” (Crutzen, 2002; Zalasiewicz et al., 2008). This would allow for a systematic explanation of the current exponential growth dynamic of population and production, and the consequent trespassing of the biosphere’s limits (Rockström et al., 2009) and generalized ecological breakdown (Barnosky et al., 2012; Millennium Ecosystem Assessment, 2005).

7. References

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