UNIVERSITY OF CALIFORNIA DIVISION OF AGRICULTURAL SCIENCES
GIANNINI FOUNDATION OF AGRICULTURAL ECONOMICS

A Stochastic Approach To Replacement Policies For Plum Trees

Lionel E. Ward and J. Edwin Faris

Giannini Foundation Monograph Number 22 • October, 1968

CALIFORNIA AGRICULTURAL EXPERIMENT STATION

This monograph presents an analytical model and its use in determining the optimum replacement pattern for plum trees taking into account certain stochastic elements. In the replacement decision the most important determinants are the yields expected from the present and the replacement trees. Future yields are not known with certainty but probabilities exist of obtaining certain yield levels. In this study probability matrices were developed for orchards of three capacity levels defined by the productive ability of the trees. The analysis was placed in a dynamic programming framework, and—considering the appropriate costs and returns—an optimum replacement policy was determined for each of the three representative orchards.

The low-producing trees for orchards 24 years and older required replacement when predicted total yields for the next three years fell to about 13,000 pounds per acre. This value was 21,500 pounds for the medium-capacity orchard, and about 30,000 pounds for the top-capacity orchard. These figures are based upon replacement of an orchard with another orchard of similar productive capacity.

A deterministic replacement model, which requires much less in terms of data and computational effort, was also constructed and the results compared with the stochastic model. The replacement policies forthcoming from both models were essentially the same. Therefore it appears that the efforts of constructing the more sophisticated stochastic replacement model may not be worthwhile if the same results can be obtained from a simpler deterministic model.

THE AUTHORS:

Lionel E. Ward is Postgraduate Research Agricultural Economist, Department of Agricultural Economics, University of California, Davis.

J. Edwin Faris is Professor of Agricultural Economics and Agricultural Economist in the Experiment Station and on the Giannini Foundation, University of California, Davis.

TABLE OF CONTENTS

Lionel E. Ward and J. Edwin Faris

A Stochastic Approach to Replacement Policies For Plum Trees[1]

INTRODUCTION

A COMMON PROBLEM confronting the the orchardist is the decision of when to replace an orchard. Normally, the following production cycle can be expected. (1) no yield for the first few years, (2) yields rapidly increasing to a maximum, (3) yields constant or slowly declining, and (4) yields decreasing (at a faster rate). Decreasing net return results from decreasing yield. A substantial capital expenditure and a number of years of negative annual net return results from the replacement of trees. The orchardist needs to determine the point where the expected net returns from the present trees drop so low that it becomes profitable to replace the trees.

The replacement point has usually been determined by a deterministic model, in which the probability of an event occurring equals 1. Single valued yield estimates are projected into the future and their net value discounted for present-day comparison (Faris, 1960). However, for most orchardists the problem is complicated by uncertainty surrounding yields and prices for any given year or age of trees. Probabilities of obtaining a specified yield following a poor yield may be different than the probabilities following a good yield for trees of the same capabilities. This could be critical in making the appropriate replacement decision.

The plum industry in California is representative of many tree fruits and nuts which make a considerable contribution to the State's large agricultural output. Information helpful in the decision-making process will be valuable to the fruits and nuts industries because a valid replacement policy provides a sound basis for capital expenditures.

OBJECTIVES

This investigation studied some of the controllable and uncontrollable factors affecting plum yields and analyzed their influence on the replacement decision. Its major objective, however, was to develop an analytical model for replacing plum trees taking into account certain stochastic elements. Replacement policies are set forth to aid orchardists in making economically sound decisions. These, of necessity, are stated as general guidelines because it is impossible to determine policies applicable to all situations. In fact, it can be argued that each orchard is a separate case and would require a specific analysis.

[1] Submitted for publication June 13, 1968.

Replacement policies using the stochastic model and the deterministic model of Faris are compared to determine if the inclusion of uncertainty in the model increases the usefulness of the model in the decision-making process.

To formulate a stochastic replacement policy, an estimate of the probability distribution of future plum yields is required. If the yield pattern were deterministic it would be necessary only to establish a single yield curve and ignore any distribution of future actual yields around this estimate. However, the emphasis of this study is placed on setting forth a probabilistic framework which recognizes the importance of the random variables that are present. Developing the appropriate probabilities is one of the subobjectives of this investigation.

FRAMEWORK FOR ANALYSIS

The replacement problem requires a multistage decision process—a sequence of decisions that maximizes (or minimizes) some objective function. The sequence of decisions is the policy: replace an orchard of type i every N_i years with an orchard of type j. The objective function is expressed in terms of maximum discounted net returns. Therefore, the replacement model is formulated as a multistage decision process designed to find a sequence of decisions that maximizes the present value of the stream of net returns to the plum grower over the entire planning horizon.

This investigation presents the appropriate model and data within a stochastic framework. Although much of the following discussion would be appropriate for a deterministic model, providing appropriate probabilities were inserted in the model, a much less mathematically complicated model is used for obtaining optimum policies in a deterministic framework (Faris, 1960). Therefore, this monograph, except for the section beginning on page 26, presents the stochastic approach.

The study of this type of process led to the development of dynamic programming (Bellman, 1955).[2] The entire problem is characterized by a recurrence relation. Of the several methods that can be used to solve this recurrence relation, we selected the policy improvement method (Howard, 1960), in which the problem is considered in its entirety from the start and at each step the initial choice of a policy is improved until the optimum policy is found.

The Markov Chain Process and The Replacement Model

The Markov chain process is expressed in matrix form and is composed of all the transition probabilities of moving from any state s_i to any other state s_j. A state is defined by specific variables that uniquely describe it—in the plum replacement problem a state is defined in terms of age and yield. The matrix of transition probabilities, together with an initial starting state, completely defines the Markov chain process.

An example of a probability matrix (P) is shown in table 1 which provides for transition from any yield-age combination to all possible yields in future time periods. A net return, r_{ij}, is associated with the transition from one state

[2] However, the study of multistage decision processes does not represent the sole application of dynamic programming. Conversely, not all such processes are amenable to the use of dynamic programming.

TABLE 1

PROBABILITY MATRIX FOR TRANSITION FROM YIELD AT A GIVEN AGE TO ANY YIELD AT A FUTURE TIME PERIOD*

Age			t			$t+1$			→	N†		
	Yield		y_1	y_2	y_3	y_1	y_2	y_3	→	y_1	y_2	y_3
		State j / i	4	5	6	7	8	9	→	$n-2$	$n-1$	n
$t-1$	y_1	1	P_{14}	P_{15}	P_{16}							
	y_2	2	P_{24}	P_{25}	P_{26}							
	y_3	3	P_{34}	P_{35}	P_{36}							
t	y_1	4				P_{47}	P_{48}	P_{49}				
	y_2	5				P_{57}	P_{58}	P_{59}				
	y_3	6				P_{67}	P_{68}	P_{69}				
↓	↓	↓							↘			
$N-1$	y_1	$n-5$								$p_{n-5,\,n-2}$	$p_{n-5,\,n-1}$	$p_{n-5,\,n}$
	y_2	$n-4$								$p_{n-4,\,n-2}$	$p_{n-4,\,n-1}$	$p_{n-4,\,n}$
	y_3	$n-3$								$p_{n-3,\,n-2}$	$p_{n-3,\,n-1}$	$p_{n-3,\,n}$

* Cells outside those with defined probabilities on the matrix diagonal have transition probabilities equal to zero.
† "N" is equal to the number of *stages*, i.e., the age of the tree. "n" refers to the number of states defined by age and yield.

to another (s_i to s_j). The components of reward matrix (r_{ij}) correspond identically to the positive elements of the probability matrix. The r_{ij}'s are gross returns less operating and replacement costs and may be negative.

The earnings possible at each stage—a discrete time period, e.g. one year—are weighted by the probability p_{ij} of making that particular transition. That is, the expected immediate reward (q_i) for any state i is the sum of the expected rewards for all states j which can be reached by a transition out of state i. Thus

$$q_i = \sum_{j=1}^{n} p_{ij} r_{ij} \, . \qquad (1)$$

Knowledge is required, in the replacement problem, of progressive total returns at successive periods during the asset's productive life. $V(N)$ is a return vector (column vector) with n components $v_i(N)$ in which $v_i(N)$ is the present value of expected total net returns in the next N transitions when the process is in state i. The total expected return can be expressed as a recurrence relation in vector form as

$$V(N) = Q + P \cdot V(N-1) \qquad (2)$$
$$N = 1, 2, 3, \ldots .$$

Q is a vector of expected immediate rewards and $P \cdot V(N-1)$ is the expected future returns from stage $i+1$ to N. It is necessary to discount the expected future returns in order to obtain the present value of the expected flow of returns. If the rate of interest is I, the discount factor is expressed as

$$\beta = \frac{1}{I+1}, \qquad (3)$$

and equation (2) can be expressed as

$$V(N) = Q + B[P \cdot V(N-1)]. \quad (4)$$

The decision required at each sequential interval of the Markov process is to select one of the two alternatives on the basis of maximizing expected future returns, $V(N)$. In this study the grower has the choice of (1) keeping the present block of trees or (2) replacing the trees with new replants. The first decision is made at stage 1 when there is one time period remaining in the enterprise. The best possible policy is selected on the basis of maximization of expected immediate rewards (q_i^k) for all alternatives (k) and states (i) in this time period, assuming no salvage value for the enterprise. In stage 2 (two time periods from the end of the enterprise) the maximization procedure requires the selection of that path out of all states i in stage 2 that gives highest expected immediate rewards plus the reward associated with the policy selected for stage 1. This pattern is continued through N stages so that the vector equation (4) can be expressed in algebraic terms to give the total expected return v_i (N) in N stages.

$$v_i(N) = \max_k \left[q_i^k + \beta \sum_{j=1}^{n} p_{ij} v_j(N-1) \right]. \quad (5)$$

The above recursive relation indicates the alternative to select in each state at each stage, and also provides an estimate of expected future returns at every stage in the process (Howard, 1960, p. 29).

The iterations are continued until the policies converge. The criterion used for convergence is that the policies for three successive iterations must be identical. For example, three successive iterations indicating that the optimum replacement age is 37 years.

Application of Dynamic Programming—Simple Replacement Illustration

An illustration follows of the technique of dynamic programming and/or its application to a replacement. It assumes that the trees begin bearing fruit in the second year but cease producing after the fourth. Table 2 summarizes the yields (and rewards) possible in each year. Yield (or reward) and age define the state of the system. There are seven such states—one associated with the first year ($t = 1$) and two each with the second, third, and fourth years. A probability exists for transition from a state in one year (state i) to another state (state j) in the succeeding year (table 3). For example, two-year-old trees having just yielded 8 tons of fruit per acre have a probability $p_{24} = 0.4$ of producing 6 tons and a probability of $p_{25} = 0.6$ of producing 8 tons as three-year-old trees. However, if the two-year-old trees had yielded 12 tons of fruit per acre the probability of producing 6 tons would be 0.7 (p_{34}) and the probability of producing 8 tons would be 0.3 (p_{35}) as three-year-old trees. It is possible to go from any state to state 1 (the trees can be replaced at the end of any year). Trees in states 6 and 7 must be replaced and have a unit probability of returning to state 1.

The expected immediate rewards out of each state are obtained by the application of equation (1). The alternatives, k, are to keep ($k = 1$) or replace ($k = 2$) and i is the present state of the trees. Deriving q_i^k for a keep alternative ($i = 1$, $k = 1$)

TABLE 2
YIELDS AND RETURNS BY AGE OF TREES ILLUSTRATIVE EXAMPLE

Age	Yield	Net revenue from yield	State
years	*tons per acre*	*dollars*	
1	0	−100	1
2	8	150	2
2	12	250	3
3	6	100	4
3	8	150	5
4	4	50	6
4	6	100	7
5	0	−100	1 or 8

$$q_1^1 = p_{12}r_{12} + p_{13}r_{13}$$
$$= 0.5\ (150) + 0.5\ (250)$$
$$= \$200.00.$$

The expected immediate return is $200 per acre from transition out of state 1 to states 2 and 3 in time period 2 (for the decision to keep). The replacement alternative q_i^2 always results in an expected immediate cost of $100. Table 4 presents all of the q_i^k of the column vector Q.

The dynamic programming technique is an iterative procedure. It determines the best policy available at successive stages until an optimum policy is obtained after N stages. An optimum policy to follow is one with an infinite planning horizon. The best policy at stage 1 is that policy which should be followed if the orchardist expects to discontinue operations after one more year. The best alternative at each state is determined by maximizing the present value of total expected returns $v_i\ (N)$ at each state.

Total expected returns to state i in stage 1 are equal only to the immediate rewards for state i. Thus, from equation (5),

$$v_i\ (1) = \max_k \left[q_i^k \right]. \qquad (6)$$

Table 5 presents the returns for the two alternatives in each state and selects that policy which maximizes the returns. This first policy suggests keeping all trees except the four year olds (states 6 and 7).

TABLE 3
TRANSITION OR PROBABILITY MATRICES ILLUSTRATIVE EXAMPLE

State i \ State j	Keep (years) 2	3	4	5	6	7	1*
	probability						
1	0.5	0.5	...	...	...	...	...
2	...	...	0.4	0.6	...	...	...
3	...	...	0.7	0.3	...	...	...
4	...	...	...	...	0.5	0.5	...
5	...	...	...	...	0.8	0.2	...
6	...	...	...	...	...	...	1.0
7	...	...	...	...	...	...	1.0

* From states 6 and 7 the only choice is to replace, but these replacement probabilities must be included in this matrix to demonstrate the accessibility of all states.

Stage 2 indicates that there are two years remaining in the planning horizon. The total expected return to each state over this period consists of the sum of expected immediate returns at stage 2 plus the expected returns for the following year. For convenience, equation (5) is applied with a discount factor of $\beta = 1$. Adopting a "keep" policy in state 1, the total return to that state in two stages is given by

$$v_1(2) = q_1^1 + p_{12}^1 \cdot v_2(1) + p_{13}^1 \cdot v_3(1)$$
$$= 200 + 0.5\ (130) + 0.5\ (115)$$
$$= \$322.50.$$

If the trees in state 1 were replaced with two stages remaining, the total return would be

$$v_i(2) = q_1^2 + p_{11}^1 \cdot v_1(1)$$
$$= -100 + 1.0\ (200)$$
$$= \$100.00.$$

TABLE 4
EXPECTED IMMEDIATE REWARDS TO EACH STATE

State	Alternative values of q_i^k	
	Keep	Replace
	dollars	
1	200	−100
2	130	−100
3	115	−100
4	75	−100
5	00	−100
6	..	−100
7	..	−100

The returns to state 1 are, therefore, maximized by choosing the first alternative. Figure 1 presents the possible steps starting from state 1 that are possible in two stages (time periods). If the orchardist keeps his one-year-old trees, the expected immediate reward is \$200. At the end of the year the probability is 0.5 that the trees will be in state 2, and 0.5 that they will be in state 3. If the trees are in state 2 the expected reward for the following year is \$130, and if in state 3 the expected reward is \$115. Assume now that the trees produced 12 tons of fruit the second year (the trees would be in state 3 and stage 2). Would the best policy be to keep the trees for another year or replace them if the planning horizon is only three years? Using equation (5) the total expected reward for keeping the trees is

$$v_3(3) = q_3^1 + p_{34} \cdot v_4(2) + p_{35} \cdot v_5(2)$$
$$= 115 + 0.7\ (100) + 0.3\ (100)$$
$$= \$215.00.$$

The value of 100 for v_4 and v_5 are obtained from following the best policy in stage 2 (see table 6). Replacing the trees at this point would bring a total expected reward of

TABLE 5
MAXIMIZATION OF RETURNS AND SELECTION OF BEST POLICY AT STAGE 1

State	Keep	Replace	Maximum	Policy*
	dollars			
1	200	−100	200	K
2	130	−100	130	K
3	115	−100	115	K
4	75	−100	75	K
5	60	−100	60	K
6	..	−100	−100	R
7	..	−100	−100	R

* K = Keep; R = Replace.

$$v_3(3) = q_3^2 + p_{31} \cdot v_1(2)$$
$$= -100 + 1.0\ (322.50)$$
$$= \$222.50.$$

The above indicates that an orchardist remaining in business for three more years should replace two-year-old trees that have first yielded 12 tons of fruit per acre if he is to maximize expected net returns, assuming no difference in salvage value of trees of different ages. The best policies for stages 2 and 3 for trees in each of the states are presented in table 6.

As the complete policies for stages 2 and 3 are not alike it is obvious that the solution has not converged. It is apparent from the above illustrations that the age of the trees at the beginning of the period exerts considerable influence on the policy chosen when the number of stages is small. As the number of stages is increased beyond four, the complete life of the tree will be included in any plan chosen. This means that all states of the system are accessible—a condition necessary to progress to a "steady-state" equilibrium. A larger number of stages will establish state probabilities. At this point the policies converge and the optimum policy holds for an infinite horizon.

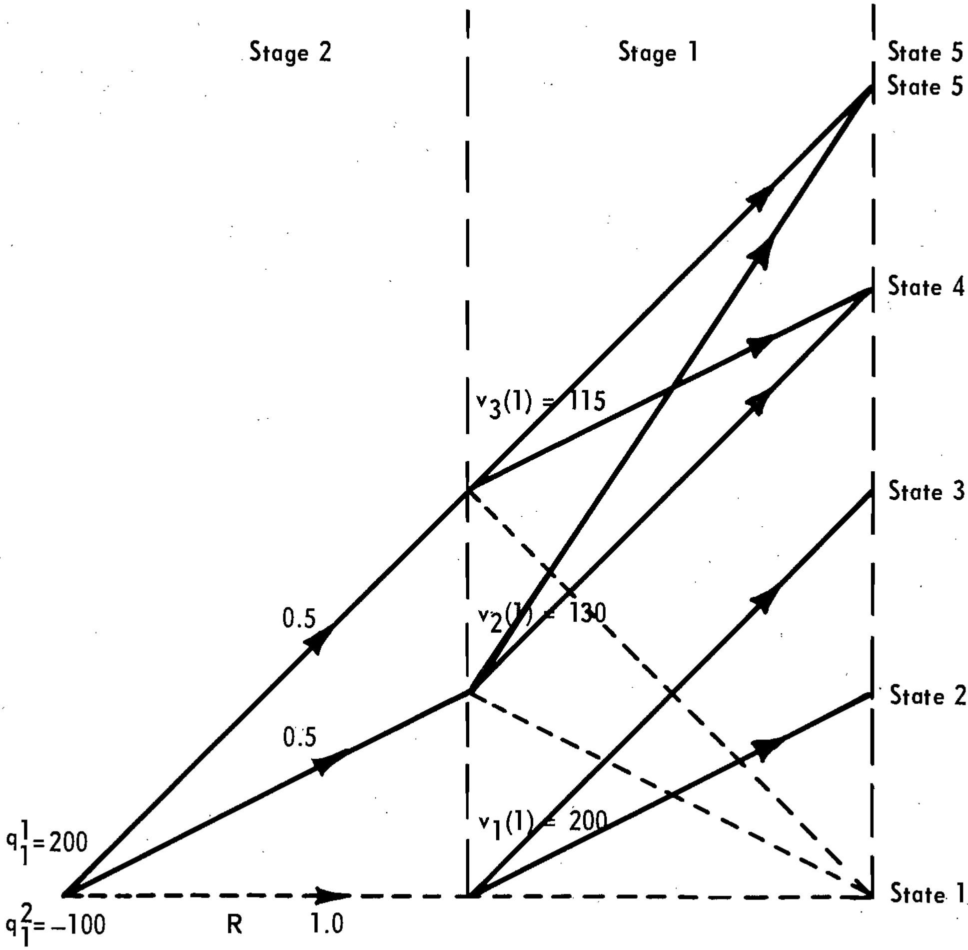

Fig. 1. Possible steps in two stages from state 1. The broken lines indicate replacement (k = 2).

TABLE 6
MAXIMIZED TOTAL EXPECTED RETURNS IN TWO AND THREE STAGES AND SELECTION OF BEST POLICIES

State	Stage 2		Stage 3	
	Maximum	Policy*	Maximum	Policy*
	dollars		*dollars*	
1........	322.5	K	390.75	K
2........	196.0	K	230.00	K
3........	185.5	K	222.50	R
4........	100.0	R	222.50	R
5........	100.0	R	222.50	R
6........	100.0	R	222.50	R
7........	100.0	R	222.50	R

* K = Keep; R = Replace.

TABLE 7
MAXIMIZED TOTAL EXPECTED RETURNS AND BEST POLICY AT STAGE 5 AND STAGE 14 (optimum)

State	Stage 5		⟶	Stage 14	
	Maximum	Policy*	⟶	Maximum	Policy*
	dollars			*dollars*	
1......	545.00	K		1,212.50	K
2......	423.45	K		1,094.63	K
3......	410.47	K		1,079.82	K
4......	365.75	K		1,039.39	K
5......	350.75	K		1,035.77	R
6......	326.25	R		1,035.77	R
7......	326.25	R		1,035.77	R

* K = Keep; R = Replace.

To achieve policy convergence the example was continued to 15 iterations, at which point three successive stages, 13, 14, and 15, indicated the same policy. Table 7 contains the optimum policy whereby all four-year-old trees and those three-year-old trees with a low predicted yield are replaced. All younger trees are kept. The state gain (increase in total expected rewards to a state from stage N to stage $N + 1$) for all states should also converge asymptotically to a constant value. Although convergence was not distinct at 14 stages, an additional year of operation increases net returns by approximately $75 per acre.

DATA—SOURCE AND SCOPE

One of the most important, and difficult, facets of the plum tree replacement problem is the construction of a yield transition or probability matrix. This requires information from orchards with similar physical attributes over a series of years.

Source and Type of Data

For this study, the area in Fresno and Tulare counties bounded by Hanford, Fresno, Sanger, and Exeter was selected as the area of data acquisition. This area produces more than 50 per cent of the total plum production in California. Data were collected only for the Santa Rosa variety to avoid the problem of yield difference between varieties. The late Santa Rosa variety was excluded. Santa Rosa plums represent approximately 30 per cent of all varieties grown in California and appear to be assured of continued favor.

Packing sheds in Fresno and Tulare counties provided gross (orchard-run) and net (pack-out) production from a number of orchards for up to nine years (1955–1963). Questionnaires, sent to the orchardists (a total of 112 separate properties), asked for information on the present age of the trees, total number of trees per age group, number of trees per acre, rootstock used, and soil type. If the yields obtained from the packing shed referred to trees of more than one age group, the orchardists were asked to estimate the yields applicable to each group. It was necessary to personally contact about one-half of the orchardists to obtain the desired information. A total of 580 usable observations were obtained for the analysis. The unit of observation was yield per acre per year. For the observation to be usable it had to be in a series of two or more years.

Production costs were obtained from farm advisors in Tulare and Fresno counties, and this information was supplemented by interviews with a few orchardists. The Federal-State Market News Service at Fresno was used as the source for prices of plums.

Factors Influencing Data Reliability

A major difficulty encountered in the yield data was the existence of cross-sectional bias in favor of trees above 20 years old. In many instances the less productive of these older trees are removed as their productive ability declines. Hence, the observed pattern does not accurately indicate the decline in yields that normally would be expected for older trees. Figure 2 illustrates the strong tendency for the mean yield to stabilize, or even rise again, in later years.

The sparsity of observations for the older trees is also a problem. Yields were recorded for trees up to 29 years old, but

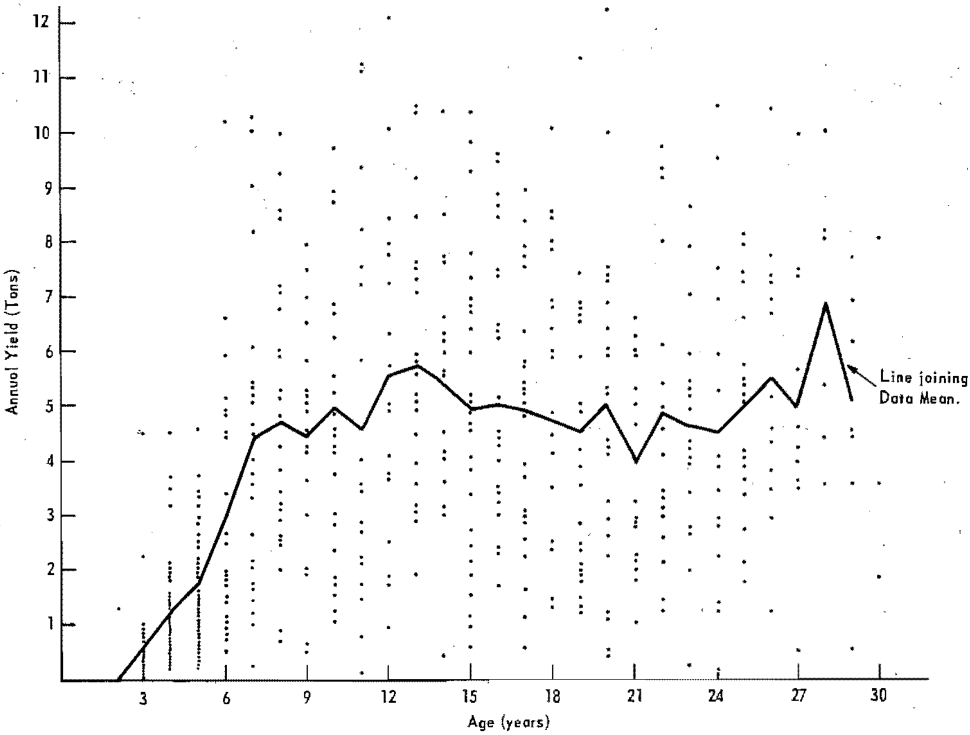

Fig. 2. Incidence of Actual observations and their means.

only seven properties had trees of this age. From annual data on bearing, nonbearing, and yearly plantings of Santa Rosa plums since 1937, it was estimated by simple statistical inference that at present Santa Rosa plums are, on the average, replaced every 26 years.[3] For these reasons the functional relationships for age and yields are based only upon the data for trees up to 24 years of age. Extrapolation of the data will be discussed in the section starting on page 22.

Another source of bias in the data results from the practice of successively replanting single trees within an aging block rather than replanting the entire block at one time. As this procedure is continued through time it becomes increasingly difficult to attribute yield from that block to trees of a known age group.

Rootstock may be an important factor in influencing plum yields in older trees. However, many growers were unable to indicate the rootstock with certainty, particularly where orchards had recently changed hands. Soil type is another factor believed to have an effect upon plum yields. Topsoil and subsoil are both relevant in this regard, but most orchardists were unable to furnish sufficiently de-

[3] Consider two consecutive years A and B. The change in nonbearing acreage from A to B ($= v$) and the trees planted in B ($= w$), indicates the area of trees added to the bearing acreage in B ($= x$). ($w - v = x$.) Statistics provide the actual change in this bearing acreage from A to B ($= y$). Hence, x less y gives an estimate of the area replaced between years A and B. The average proportion of bearing acreage replaced annually is 4.38 per cent—complete replacement every 23 years. Trees have a nonbearing life of three years giving an average tree life of 26 years.

tailed information. Hence, soil types were eventually distinguished according to a heavy, medium, or sandy texture of the topsoil.

The prevailing price influences the proportion of the crop that is harvested. When prices are depressed the grower is apt to leave more fruit unharvested than when prices are higher (this trend is noticeable, apart from current marketing orders). It was not possible to empirically measure this effect. As low prices are largely a function of high yields it is suggested that this reduces the amount of within-orchard yield variability since heavy crops are not totally recorded.

ESTIMATING STOCHASTIC YIELD RELATIONSHIPS

Future variability of plum yields is assumed to be similar to that in the past. Yields vary because of a number of factors; important among them are weather, pests, and diseases. Another group of factors influencing yield are "systematic" variables such as age of tree and the previous year's crop.

The initial purpose of the analysis is to isolate the predictable or systematic variables that can be quantified, and to derive a regression equation that best fits the data available. Random disturbances from this predicted line will then be assumed to occur in a normally distributed manner. Probabilities assigned to these yield disturbances enable preparation of the transition matrix. Subsequent derivation of the reward matrix and definition of the alternatives completes the empirical framework.

Deriving a Function for Yield

A number of approaches were investigated with functional relationships formulated and regressions run. Soil type and rootstock were included as independent variables in several of these formulations. As a result of the low t-test values obtained, it is concluded that there is little empirical support for the belief that data available for soil and rootstock are significant in determining yield patterns for plums.[4] They are therefore eliminated from further considerations.

The singularly most important factor associated with the yield (Y) of plum trees at age t is the yield at age $t - 1$. However, the relative magnitude of Y_{t-1} (relative to other blocks of trees of the same age) is expected to be dependent on the normal capacity of the trees of that particular block. The large variations in yields for trees of the same age (see figure 2) give some indication that the capacity of the trees is an important explanatory variable. To define Y_{t-1} with respect to this capacity, the properties were arbitrarily separated into three capacity levels according to the following limits for mature trees:

$C =$ (less than 8,000 pounds per acre average),
$C = 1$ (8,000 to 13,000 pounds per acre average), and
$C = 2$ (more than 13,000 pounds per acre average).

C is used as a "shift" variable to permit a functional relationship to be determined for all of the data collectively.[5]

[4] If more complete information on rootstock and soils had been available it is possible that the t-tests would have proved significant.

[5] The advantage of using 0, 1, and 2, rather than average yields for the group, is that it dissociates the orchard capacity from its present age. Instead of a single-shift variable, C could have been defined as two dummy variables (retaining the intercept term) with the appropriate

Plum trees tend to alternate bearing, but this is not their strict pattern. This inconsistency creates a problem when using last year's production (Y_{t-1}) to estimate the present year's production (Y_t). Several regressions verified the importance of this inconsistency—in the t-tests run on these regressions, Y_{t-1} was not significant at the 10 per cent level. Consequently, it is necessary to make several adjustments in the approach to estimating yield relationships.

First, yields are combined in three-year periods. Thus, $t = 1$ includes the aggregate yield for years 1, 2, and 3; $t = 2$ includes the aggregate yield for years 4, 5, and 6, and so on. For example, if $t = 5.67$, then Y_t represents aggregate yields in years 15, 16, and 17, and Y_{t-1} is yield in years 12, 13, and 14. Combining yields restricts the number of observations available, but it allows Y_{t-1} to act as a determinant of production potential. It also restricts the amount of within-orchard variation. This is a disadvantage because it masks a considerable part of the uncertainty that the dynamic programming technique can handle explicitly. However, the approach used approximates reality where replacemend decisions are based upon yields for several previous years rather than on just one year's results.

The next step is to introduce a variable to take into account alternate bearing. Over a period of years most blocks of trees establish a yield pattern which tends to define an approximation to "normal" capacity for the trees. It is logical to reason that a deviation from the mean in one period will be at least partly compensated for in the following period. Thus a difference variable ($\bar{Y} - Y_{t-1}$) is included to account for alternate bearing. $\bar{Y}$ is the average annual yield for the number of years available (six to nine years) expressed as a three-year mean for each orchard.[6] This difference variable permits Y_{t-1} and C to adjust a prediction to the appropriate orchard capacity.

Age (T) is included as an independent variable. It is included in both first and second order form to account for the parabolic tendency of the yield curve.[7] Because of the yield response patterns over the life of the tree, the yield relationships were estimated for two separate periods; the period $t = 0.00$ to $t = 4.33$ (a period of rapidly increasing yields) and the period $t = 3.67$ to $t = 8.00$ (a period when yields reach a maximum and then decline). Fitting of the latter relationship is discussed first. Placing the above variables in an equation resulted in the following:

$$Y_t = -11.4087 + \underset{(7.154)}{0.7595\, Y_{t-1}} + \underset{(10.097)}{1.3052\, (\bar{Y} - Y_{t-1})} + \underset{(3.096)}{4.3896C} \qquad (7)$$

$$+ \underset{(1.902)}{5.3931T} - \underset{(2.001)}{0.4783T^2}$$

$$R^2 = 0.9432 \qquad S_{1.23456} = 3.3005$$

0 and 1 values for each. Undoubtedly, this would have been a better statistical procedure. However, the limits were selected so that the average values for $C = 0$ and $C = 2$ are approximately equal distance from $C = 1$ in order to eliminate the need for two variables. See figure 3.

[6] The period Y_t was not included in $\bar{Y}$ as Y_t is the dependent variable.

[7] "t" is capitalized (T) only with respect to the independent variable in the regression equation. Both notations refer to the age of the tree.

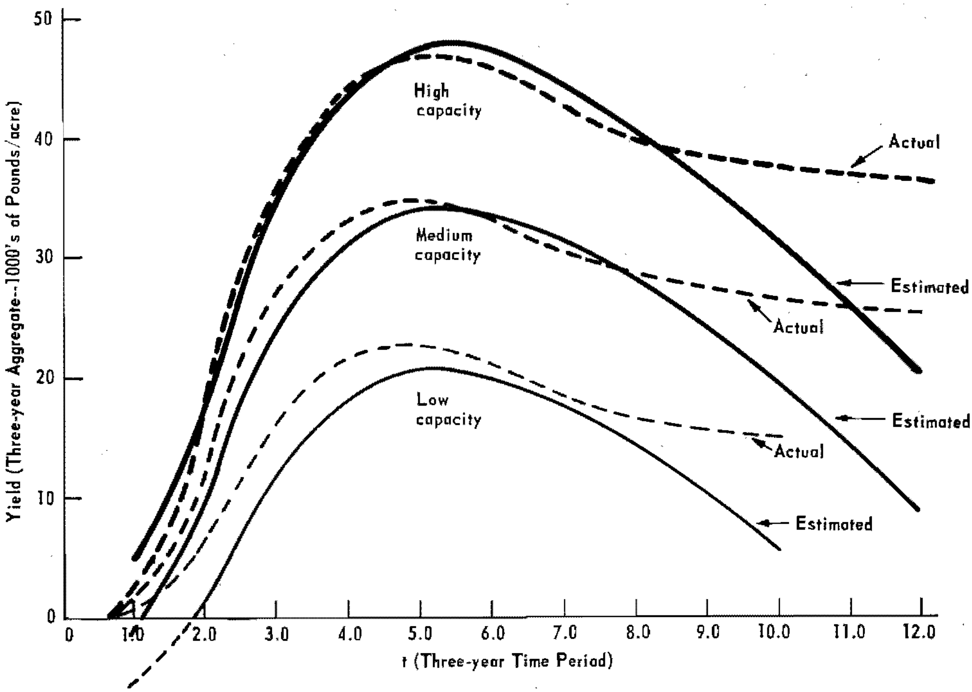

Fig. 3. Actual and estimated yields (three-year aggregate) for representative orchards for three different capacity levels.

This equation is marked by a high coefficient of determination (R^2) and significant t-tests (t-ratios are shown in parentheses below the regression coefficients) on all the independent variables. Of particular importance is the greatly reduced standard deviation of the estimate of Y_t ($S = 3.3005$) relative to most of the other formulations attempted.

A verbal interpretation of the function for Y_t indicates that the yield of Santa Rosa plums in the next three years will rise by 759.5 pounds for every 1,000 pounds increase in the yield of the previous three years, and by 1,305 pounds for every 1,000 pounds that Y_{t-1} is less than the average three-year yield for that property. Similarly, a shift in tree capacity by one level will bring a positive influence in the next three years' yield of 4,389.6 pounds. The value of the coefficient for any three-year age period T will be determined by the combined effect of $5.3931T - 0.4783T^2$. The maximum contribution by T to current yield is for trees between 15 and 18 years old ($t = 5.0$ to 6.0). With other factors constant, this would represent the upper curvature of the parabola.

The average yield, by age of trees, was calculated for each of the three tree capacity levels. Free hand curves were fitted to these observed averages (dash lines in figure 3). The yields obtained from the smooth curves were inserted into equation (7) to visually test the fit (continuous lines in figure 3). The projection of the estimate to $t = 12$ (age 36 years) underestimates Y_t. However, this compensates for the cross-section bias in the actual data. The curve is not extrapolated beyond $t = 12$ since replacement is anticipated before this time.

Although equation (7) is based upon

observations for trees 11 years and older ($t = 3.67$), figure 3 illustrates the flexibility of the equation and the successful extrapolation of the curve back to the earlier years of growth. Several other functional relationships were formulated for the yield curve for the years $t = 0.00$ to $t = 4.33$. However, these functions did not appear to be much more satisfactory than the extrapolation of equation (7) in terms of the fitted lines.[8] Consequently, equation (7) is used to estimate the yield relationships throughout the entire life of the orchards.

The Probability Matrix

The Markov chain to be developed is composed of states defined by the age of the plum tree and yield per acre over a three-year period. The probability matrix constructed differs from that used in the hypothetical example only because its dimensions are larger. Three complete matrices are formed for each capacity level; one for each set of time periods. Thus one matrix is constructed for $t = 0.00$ to 12.00, a second for $t = 0.33$ to 12.33, and a third for $t = 0.67$ to 12.67. A single time period equals 1.00, or three years.

A medium-producing orchard ($C = 1$) with time periods running from 0 to 12 is used to illustrate the construction of the probability matrix. Using the regressions equation (7), the estimate of yield in age t (Y_t) was derived for values of Y_{t-1} in orchards of medium capacity.[9] The yields accessible from any given Y_{t-1} were assumed to be normally distributed about a mean of Y_t and standard deviations $S_{1\cdot 2-6} = 3.3005$.[10] The yield range of each state is 6,000 pounds (1 ton per year variation in yield between states) and the probability of reaching yields falling outside the 99.90 per cent fiducial limits of the normal curve are assumed equal to zero. For each starting state i there exists a row vector of transition probabilities to a number of terminating states j.

State 1 represents that short period of time during which the tree is planted; zero yield, zero age. State 2 is defined by zero yield (less than 3,000 pounds) at $t = 1$ (aggregate of age 1, 2, and 3 years), state 3 by 6,000 pounds (3,000 to 9,000 pounds) at $t = 1$, and state 4 by 12,000 pounds (9,000 to 15,000 pounds) at $t = 1$ (see table 8). Similarly, states 5 to 9 are associated with $t = 2$, and probability vectors exist for reaching these states from states 2, 3, and 4.

All the probability vectors are formulated into a single stochastic matrix table. Each set of transitions defined by the time periods ($t - 1$) and t forms a rectangular segment which is singularly stochastic with all other elements in the corresponding rows equal to zero. This insures ordered Markovian progression period by period from $t = 0$ to $t = 12$. Lower (higher) yields in time period $t - 1$ are associated with larger (smaller) probabilities of obtaining relatively higher (lower) yields in time period t. Thus if an orchard has relatively low yields in

---

[8] The reason that all of the observations are not lumped together, regardless of age to estimate the yield function, is that C is defined in terms of mature trees. When a functional relationship for $t = 0.00$ to $t = 4.33$ was plotted with equation (7), it resulted in a discontinuous curve for one or more of the three yield curves because of the tree-capacity variable.

[9] See Appendix tables A-1 and A-2 for yields of representative orchards in three-year aggregates and the shifting-mean yield ($\bar{Y}$).

[10] This assumption is crucial for the determination of the probability matrix. A chi-square test was run to determine the normality of the distribution about a mean of Y_t. $X^2_{95} = 16.151$ for the book value while the calculated value is 14.8872; thus the probability that the distribution is normal is greater than 95 per cent.

a three-year period (Y_{t-1}) the expected immediate yield in the following three years (Y_t) is greater than average for trees of that age and quality.

The Reward Matrix

The reward matrix expresses the net return from being in each state. Yields, age, and capacity of orchard are the important determinants of the net returns associated with each state.

Replacement costs are presented in Appendix table A-3. Costs for pulling, subsoiling, fumigation, and planting trees occur only in the first year and are assumed constant for all orchards.

Operating cost estimates were developed for trees one to five years of age (Appendix table A-3). Costs for irrigation, fertilizer application, and pruning are varied according to the orchard capacity level. Slight adjustments are also made for cultivation and spraying costs. Yields are recorded from three years of age and are rounded to the nearest ton. Thinning and harvesting costs are varied according to yield.

Operating costs for trees six years old and older are assumed to vary with yield, but not with age. Pruning, spraying, thinning, and harvesting costs are those costs that vary with yield (see Appendix tables A-4 and A-5).

Fixed costs. Costs of equipment, buildings, and other types of capital investment are irrelevant in reaching a final decision on replacement policy because in an infinite planning horizon they are independent of any such decisions.

Plum prices. The crucial assumption in determining returns is that of constant prices. The familiar shifts in prices from year to year are largely due to shifts in supply of Santa Rosa plums on the market. When the final analysis uses yields in three-year aggregates, much of this year-to-year variation is avoided. Also, an overall light (or heavy) crop in California is not necessarily found in all orchards, and therefore the price received by a grower for his plums is a reflection of California's total crop and not his own.

A weighted mean price was calculated on the basis of daily sales and prices at shipping point (f.o.b.) for the years 1960–1962. (Free on board prices are not available prior to 1960.) The average price was $3.76 per 28-pound crate. Costs deductible from this figure include packing charges ($1.20), 10 per cent commission, and a negligible assessment charge. The final net return figure used was $2.17 per crate, or $155.00 per ton of fruit. This value was used to determine net returns per acre regardless of orchard quality, size of crop, or age.

This latter assumption requires further justification. Information on pack-out by size was provided by one packing company for some 30 growers. Regression analysis indicated some limited dependence of size of fruit on orchard quality, but very little on age of tree or size of crop. In another analysis, the percentage pack-out was calculated for the same properties and rearranged in calendar years. Again, specific variables of age and orchard capacity failed to significantly explain the level of fruit cullage. Some unexplained factor, such as weather, remains the dominating influence.This is supported by a pattern of culling rates which vary according to the particular year rather than to age or tree quality.

Net returns. The net returns were calculated for each state by orchard capacity and the three sets of time period intervals. An example of these net returns for the reward matrix is presented in Appendix table A-6. The cost of replacement is debited to the states in time period 1 and not to the initial transitory state (state 1).

Obtaining an Annual Replacement Policy

With yield aggregated in three yearly totals, any conclusion reached will only amount to a decision every three years. However, it is necessary to make the replacement decision each year. This is accomplished by running three separate programs, one for each set of three-year aggregates. Thus, in the example above, the value of t used is a complete integer. That is, decisions are made at three, six, nine years, and so on. Similarly, if t = 5.33, the decision is made for 13-year-old trees (since Y_t is a prediction of total yields from 14 to 16 years of age); t = 6.33 when the trees are 16 years old. The main problem confronted in programming with t starting at 0.33 and 0.67 (again state 1 is t = 0) is that the first time period is less than three years. This introduces a minimal amount of inaccuracy which becomes negligible after a number of iterations. The important aspect is that this method permits development of a more specific *annual* replacement policy.

OPTIMUM REPLACEMENT PATTERNS

Derivation of an *annual* replacement policy for one orchard requires three separate programs commencing at zero, one, and two years of age. Three representative orchards (high-, medium-, and low-producing orchards) were programmed in this manner. From the results of these analyses a generalized replacement policy is possible which is applicable to all levels of orchard potential. The analysis is extended to include a replacement of medium-producing with high-producing orchards The method of analysis and interpretation of results is the same for each of the programs.

Medium-Producing Orchard

Medium-level trees (C = 1) requiring a triennial policy commencing at zero years of age (t = 0.00) comprised 68 states in a total of 12 three-year time periods. Using the probability and reward matrices, the expected immediate rewards to each state (q_i^k) are calculated by application of equation (1). Reviewing their interpretation and meaning, the anticipated immediate reward q_1^1 from any transition out of state 1 (zero yield, zero age) is given (see table 8) by

$$q_1^1 = 0.8883\ (-579) + 0.1105\ (-224) + 0.0012\ (36)$$

$$= -\$539.034.$$

Hence from state 1 the expected immediate loss is about \$539 per acre from the trees for years 1 through 3. Likewise, the immediate reward for q_2^1 is determined by

$$q_2^1 = 0.0465\ (-365) + 0.5068\ (3) + 0.4211\ (312) + 0.0256\ (651)$$

$$= \$132.597.$$

Therefore, the reward expected from state 2 is a positive return of about \$133 per acre earned from years 4 through 6. Similarly, values are derived for q_3^1 (−\$46.50) and q_4^1 (−\$219). This in-

Table 8

THE PROBABILITY MATRIX FOR A MEDIUM-CAPACITY ORCHARD ($C = 1$), TIME PERIOD 0.00 TO 12.00

Time period			1.0			2.0				3.0					→	12.0	0.0
	Yield*		0	6,000	12,000	6,000	12,000	18,000	24,000	12,000	18,000	24,000	30,000	36,000	→	0	0
		State j / i	2	3	4	5	6	7	8	9	10	11	12	13	→	68	1
0	0	1	0.8883	0.1105	0.0012												
1.0	0	2				0.0465	0.5068	0.4211	0.0256								
	6,000	3				0.2445	0.6263	0.1276	0.0016								
	12,000	4				0.6179	0.3651	0.0170									
2.0	6,000	5									0.0212	0.3963	0.5288	0.0537			
	12,000	6								0.0022	0.1493	0.6308	0.2130	0.0047			
	18,000	7								0.0314	0.4518	0.4792	0.0376				
	24,000	8								0.1895	0.6358	0.1683	0.0028				
↓	↓	↓													↘		
11.0	0	61														1.00	
	6,000	62														1.00	
	12,000	63														1.00	
	18,000	64														1.00	
	24,000	65														1.00	
	30,000	66														1.00	
	36,000	67														1.00	
12.0	0	68															1.00

* Yield is pounds per acre in three years and is midpoint of a 6,000 pound range, e.g., $Y = 12,000$ represents 9,000 to 15,000 pounds.

TABLE 9

EXPECTED IMMEDIATE REWARDS TO ALL STATES FOR THE MEDIUM-CAPACITY ORCHARD, t = 0.00 TO 12.00

Time period	State	$\overset{k}{q_i}$*	Time period	State	$\overset{k}{q_i}$*
three-year units		*dollars*	*three-year units*		*dollars*
0	1	− 539.00		34	1,089.30
				35	902.40
	2	132.60	7.0	36	722.10
1.0	3	− 46.50		37	539.30
	4	− 219.10		38	360.60
				39	184.00
	5	730.70			
2.0	6	549.80		40	1,040.30
	7	370.80		41	855.70
	8	194.10		42	673.10
			8.0	43	493.00
	9	1,435.90		44	314.40
	10	1,236.40		45	138.40
3.0	11	1,045.30		46	− 27.40
	12	860.10			
	13	666.30		47	963.50
				48	760.40
	14	1,604.20		49	579.90
	15	1,401.40	9.0	50	400.60
4.0	16	1,202.80		51	222.60
	17	1,012.80		52	49.80
	18	828.80		53	− 102.80
	19	646.40			
				54	786.00
	20	1,632.60		55	605.00
	21	1,429.60		56	425.70
	22	1,230.60	10.0	57	247.40
5.0	23	1,040.60		58	73.60
	24	854.10		59	− 83.20
	25	672.30		60	− 209.70
	26	493.90			
				61	− 600.00
	27	1,409.60		62	− 600.00
	28	1,210.80		63	− 600.00
	29	1,020.30	11.0	64	− 600.00
6.0	30	836.10		65	− 600.00
	31	654.00		66	− 600.00
	32	477.00		67	− 600.00
	33	319.90			
			12.0	68	0

* The expected immediate rewards presented are associated with keeping the orchard for another three years. In addition, there is an expected immediate reward of zero associated with each state if the orchard is replaced.

crease in expected net loss is due to the alternate bearing effect. The higher-than-average past yields of state 4 result in lower-than-average yields in the next time period. Within each time period the values for q_i characteristically reflect this drop in immediate rewards (table 9). This is not true of time period 11 where there is unit probability of zero yield and return for all states in their transition to time period 12.

Replacement at any state requires transition to state 1. Therefore, the immediate reward to replacement (q_1^2) is always zero.

The policy at stage 1 should be chosen with one time period (three years) remaining before the termination of the enterprise. The total rewards to each state are simply the immediate rewards found in table 9 (see equation 2). The policy indicates a choice of an alternanative based on maximization of returns. This would require replacement in states 3, 4, 46, 53, and 59 through 68. It would be more profitable to go to state 1 which has an immediate expected reward of zero rather than incur negative net returns for the next three years. (It is assumed that the trees have no salvage value. Thus the orchard operation will be discontinued.)

The policy at stage 2 was derived from expected total returns to each state over the next six years. Determination requires application of equation (5). For the current problem an annual discount rate (r) of 6 per cent was chosen. The time unit in this program is three years with returns derived from an aggregate of single years. The discount factor in equation (3) then becomes determined by the expression

$$\beta = \frac{1 + (1+r) + (1+r)^2}{3(1+r)^3}.$$ [11]

Thus for an annual interest rate of 6 per cent, $\beta = 0.891004$ over the three-year period.

Consider, as an example, the determination of the present value of future rewards for state 5 in stage 2 $= v_5(2)$. Stage 2 means that six years are left before termination of the enterprise. From state 5 there is a probability of transition to each of the four states 10, 11, 12, and 13 in the next time period (see table 8). With only one period remaining, the future returns to these states would be equal to their maximum q_1^k values. These returns are weighted by the transition probabilities and discounted by β. The expected immediate reward q_5^1 is \$730 so that

$$\begin{aligned} v_5(2) &= 730 + 0.891\,(0.0212 \times 1{,}236 \\ &\quad + 0.3963 \times 1{,}045 + 0.5288 \\ &\quad \times 860 + 0.0537 \times 666) \\ &= \$1{,}560. \end{aligned}$$

This return of \$1,560 is the reward for choosing a policy of keeping the trees for at least another six years. The alternative of replacing the trees would have amounted to the following evaluation

$$\begin{aligned} v_5(2) &= -539 + 0.891\,(0.8883 \times 132 \\ &\quad + 0.1105\,(-46.5) + 0.0012 \\ &\quad (-219)) \\ &= -\$439.338. \end{aligned}$$

The first alternative can easily be chosen as the one that will maximize returns.

The value of $v_6(3)$ is \$2,483. Explained in words this means that a property that has just produced a total of 3 tons of fruit per acre during the age period four through six years can be expected to yield a net return (discounted to the present) of \$2,483 per acre over the next nine years provided an optimal policy is chosen at each stage. Table 10 presents the maximized returns and the best replacement policy available. Replacement is indicated for states 60 through 68—all the 33-year-old trees and the lowest yielding (predicted) 30 year olds.

Policies derived for these planning horizons of short duration vary from one stage to the next. Hence the policy to be used is dependent on the number of years the grower plans to continue producing plums. This dependence is eliminated as the number of stages in the planning horizon is extended to a large number N. At 12 stages (36 years) all trees will be replaced at some time during this period.

[11] This is the same as $\dfrac{\frac{1}{1+r} + \frac{1}{(1+r)^2} + \frac{1}{(1+r)^3}}{3}$ or the average of the present values for the three-year period.

Table 10

MAXIMIZED TOTAL EXPECTED RETURNS AND CHOICE OF POLICY IN STAGES 1, 2, AND 3 FOR THE MEDIUM-CAPACITY ORCHARD, t = 0.00 TO 12.00

State	Stage 1		Stage 2		Stage 3		State	Stage 1		Stage 2		Stage 3	
	Max $v_i(1)$	Policy*	Max $v_i(2)$	Policy*	Max $v_i(3)$	Policy*		Max $v_i(1)$	Policy*	Max $v_i(2)$	Policy*	Max $v_i(3)$	Policy*
	dollars		*dollars*		*dollars*			*dollars*		*dollars*		*dollars*	
1	− 539.00	K	− 434.10	K	− 54.13	K	35	902.40	K	1.321.50	K	1,664.50	K
2	132.60	K	554.70	K	1.410.20	K	36	722.10	K	1,227.40	K	1,528.70	K
3	0	R	461.90	K	1,273.20	K	37	539.30	K	1,133.50	K	1,392.30	K
4	0	R	367.60	K	1,139.20	K	38	360.60	K	1,043.10	K	1,260.30	K
5	730.70	K	1,560.30	K	2,483.90	K	39	184.00	K	955.00	K	1,131.40	K
6	549.80	K	1,470.60	K	2,349.20	K	40	1,040.30	K	1,161.00	K	1,386.20	K
7	370.80	K	1,383.90	K	2,217.80	K	41	855.70	K	1,057.00	K	1,244.70	K
8	194.10	K	1,300.50	K	2,090.40	K	42	673.10	K	960.10	K	1,108.70	K
9	1,435.90	K	2,233.90	K	3,196.30	K	43	493.00	K	866.70	K	978.90	K
10	1,236.40	K	2,124.40	K	3,041.40	K	44	314.40	K	775.30	K	855.60	K
11	1,045.30	K	2,024.70	K	2,896.70	K	45	138.40	K	686.80	K	740.80	K
12	860.10	K	1,932.90	K	2,760.10	K	46	0	R	610.10	K	643.90	K
13	666.30	K	1,825.40	K	2,599.20	K	47	963.50	K	969.20	K	861.00	K
14	1,604.20	K	2,351.10	K	3,171.90	K	48	760.40	K	785.90	K	729.80	K
15	1,401.40	K	2,238.30	K	3,014.70	K	49	579.90	K	645.80	K	625.00	K
16	1,202.80	K	2,130.90	K	2,863.30	K	50	400.60	K	530.70	K	526.80	K
17	1,012.80	K	2,033.00	K	2,721.90	K	51	222.60	K	433.70	K	433.30	K
18	828.80	K	1,942.60	K	2,588.50	K	52	49.80	K	346.40	K	346.40	K
19	646.40	K	1,856.60	K	2,459.40	K	53	0	R	280.40	K	280.40	K
20	1,632.60	K	2,189.70	K	2,864.20	K	54	786.00	K	786.00	K	358.00	K
21	1,429.60	K	2,074.00	K	2,705.90	K	55	605.00	K	605.00	K	177.10	K
22	1,230.60	K	1,963.50	K	2,552.50	K	56	425.70	K	425.70	K	− 2.20	K
23	1,040.60	K	1,862.30	K	2,408.70	K	57	247.40	K	247.40	K	− 180.50	K
24	854.10	K	1,766.90	K	2,270.10	K	58	73.60	K	73.60	K	− 354.30	K
25	672.30	K	1,677.00	K	2,137.40	K	59	0	R	− 83.20	K	− 386.80	R
26	493.90	K	1,591.60	K	2,009.60	K	60	0	R	− 209.70	K	− 386.80	R
27	1,409.60	K	1,800.00	K	2,374.00	K	61	0	R	− 480.30	R	− 386.80	R
28	1,210.80	K	1,688.60	K	2,219.60	K	62	0	R	− 480.30	R	− 386.80	R
29	1,020.30	K	1,586.40	K	2,074.30	K	63	0	R	− 480.30	R	− 386.80	R
30	836.10	K	1,490.40	K	1,935.60	K	64	0	R	− 480.30	R	− 386.80	R
31	654.00	K	1,396.60	K	1,799.50	K	65	0	R	− 480.30	R	− 386.80	R
32	477.00	K	1,307.10	K	1,668.60	K	66	0	R	− 480.30	R	− 386.80	R
33	319.90	K	1,229.30	K	1,553.70	K	67	0	R	− 480.30	R	− 386.80	R
34	1,089.30	K	1,421.30	K	1,806.70	K	68	0	R	− 480.30	R	− 386.80	R

* K = Keep; R = Replace.

That is, a complete life cycle is experienced regardless of the starting point. However, complete influence of the initial age of the tree is not eliminated at this point. Convergence is reached at stage 29; that is, the best policy at stage 29 is equal to that at stage 30 and all succeeding stages (table 11)—assuming replacement with an orchard of similar capacity.

The optimum policy at convergence specifies "keep" at states 1 to 43, 47 to 49, 54, and 55; "replace" at states 44 to 46, 53, and 56 to 68. The optimum policy suggests keeping all those trees 21 years old and under; trees 24, 27, and 30 years old should be kept if the predicted yield for the next three years is greater than 10½ to 11 tons. Trees with yield predictions lower than this will be replaced. All trees 33 years and older require replacement.

The decision whether to keep or replace 24-year-old trees is determined by the yield of the previous three years. This illustrates the influence of the alternate bearing effect contained in the functional yield relationships. All 33-year-old trees are replaced due to the assumption of zero returns. However, the predicted yields at this age have fallen to an extent (figure 3) that only a low probability exists of realizing profitable yields in the next three years.

The policy described is for an infinite horizon. Due to the influence of the discount factor the total expected rewards to each state in this infinite period converge to a constant value.

The program used for the time periods 0.00 to 12.00 provides a decision for trees aged 3, 6, 9, . . . 33 years. The same program is repeated for aggregates commencing at 0.33 and 0.67.[12] The first maximizes returns for trees 1, 4, 7, . . . 34 years of age; the second offers a policy every three years for trees from 2 to 35 years old.

Interpolation of results from the three time periods provides an annual replacement policy for trees up to 35 years old. A summary of the three programs for a medium-capacity orchard appears in

TABLE 11

OPTIMUM-POLICY AND MAXIMIZED RETURNS FOR ALL STATES OF THE MEDIUM-CAPACITY ORCHARD WITH AN INFINITE PLANNING HORIZON, t = 0.00 TO 12.00

State	Stage 30		State	Stage 30	
	Max $v_i(30)$	Policy*		Max $v_i(30)$	Policy*
	dollars			*dollars*	
1....	4,267.8	K	35....	4,337.9	K
2....	5,419.6	K	36....	4,220.0	K
3....	5,296.8	K	37....	4,113.4	K
4....	5,175.3	K	38....	4,017.5	K
5....	6,106.8	K	39....	3,927.0	K
6....	5,988.5	K	40....	4,398.9	K
7....	5,871.8	K	41....	4,214.8	K
8....	5,759.1	K	42....	4,037.3	K
9....	6,383.4	K	43....	3,882.9	K
10....	6,243.1	K	44....	3,789.7	R
11....	6,113.2	K	45....	3,789.7	R
12....	5,991.4	K	46....	3,789.7	R
13....	5,813.6	K	47....	4,389.3	K
14....	6,042.7	K	48....	4,119.1	K
15....	5,898.5	K	49....	3,938.6	K
16....	5,760.0	K	50....	3,789.7	R
17....	5,631.7	K	51....	3,789.7	R
18....	5,511.5	K	52....	3,789.7	R
19....	5,396.1	K	53....	3,789.7	R
20....	5,556.6	K	54....	4,144.6	K
21....	5,412.0	K	55....	3,963.7	K
22....	5,273.7	K	56....	3,789.7	R
23....	4,145.9	K	57....	3,789.7	R
24....	5,024.6	K	58....	3,789.7	R
25....	4,909.8	K	59....	3,789.7	R
26....	4,800.4	K	60....	3,789.7	R
27....	5,001.2	K	61....	3,789.7	R
28....	4,850.2	K	62....	3,789.7	R
29....	4,710.3	K	63....	3,789.7	R
30....	4,579.6	K	64....	3,789.7	R
31....	4,454.9	K	65....	3,789.7	R
32....	4,339.2	K	66....	3,789.7	R
33....	4,241.3	K	67....	3,789.7	R
34....	4,479.3	K	68....	3,789.7	R

* K = Keep; R = Replace.

[12] As in the first program, state 1 is zero yield, zero age. Therefore, state 2 is associated with time period 1 which is not a full three-year period.

table 12. Trees 33 years and older and those 24 to 32 years old with a predicted annual average yield of less than 3½ to 4 tons per acre should be replaced. All other trees should be retained at least until the next policy decision.

High-Producing Orchard

A pattern of average yields was developed for a high-capacity orchard $C = 2$ (figure 3) to an age of 36 years old. Net returns were adjusted according to estimates of costs and gross returns made in the previous major section. The program procedure used was the same as for the medium-capacity orchard.

Table 13 contains the complete policy for the high-capacity orchard. Earliest replacement indicated is for 26-year-old trees with an estimated yield for the next three years of 12½ tons. Trees 27 years and older should be replaced if their future yield estimates fall below about 5 tons per acre per annum. Using this criterion most high-capacity orchards will be replaced by 33 years of age.

Low-Producing Orchard

The regression equation derived to fit the observed data is least suited to predicting yields on low-capacity orchards $C = 0$ (figure 3). This is mainly because the yield for young trees is underestimated. Hence the cost of replacement is too high as a result of the lower gross returns expected.

However, the prediction appears to be sufficiently consistent to provide a satisfactory guide for determining a replacement policy. The program was computed in three sections, terminating at $t =$ 10.00, 10.33, and 10.67. The lower yields and net returns available from the low-producing orchard made it unnecessary to extend the program beyond the tenth time period. The optimum policy is summarized in table 14. Predicted annual yields as low as 2 tons per acre would necessitate replacement when trees are 19 or 20 years old. As trees become older this minimum requirement is nearer 3 tons at 26 years. Low-capacity trees 27 years and older are unlikely to produce yields that would warrant a "keep" policy.

Replacing with an Improved Orchard

It is a reasonable presumption that replacement of a block of plum trees will often bring about an improvement of yield capacity in the new block of trees. To illustrate the type of outcome to be expected, a medium-capacity orchard is assumed to be replaced by a high-capacity orchard. The programming principle is the same, but slight modifications are required to accommodate the different characteristics of the problem.

The transition matrix for $C = 1$ (medium capacity) occupies states 1 to 54 (for $t = 0.00$ to 10.00)[13] and for $C = 2$ (high capacity) states 55 to 122 ($t =$ 0.00 to 12.00). The reward matrices are of the same dimensions. There are three policy alternatives for most states: keep, replace with state 1 ($C = 1$), or replace with state 55 ($C = 2$).[14] There are two alternatives at states 1 and 55 (keep, or replace with 55) and at states 54 and 122 (replace with 1 or 55).

The program was repeated for all three sets of time periods. Table 15 presents

[13] The number of time periods was reduced to ten because replacement was anticipated at an earlier age and higher yield.

[14] Although with an infinite horizon the alternative of replacing with state 1 obviously would not be chosen, it was included as an alternative to keep all states accessible; without this provision the $C = 2$ matrix would act as a "trap."

TABLE 12

OPTIMUM REPLACEMENT POLICY FOR THE MEDIUM-CAPACITY ORCHARD, i = 0.00 TO 12.00*

Age	Predicted yields ($\hat{Y}_i$) next three years (pounds)†									
	0	6,000	12,000	18,000	24,000	30,000	36,000	42,000	48,000	54,000
years										
1	K	K	K	K						
2	K	K	K	K	‡					
3		K	K	K	K					
4		K	K	K	K	K				
5			K	K	K	K	K			
6			K	K	K	K	K			
7				K	K	K	K	K		
8				K	K	K	K	K	K	
9				K	K	K	K	K	K	
10				K	K	K	K	K	K	
11				K	K	K	K	K	K	K
12				K	K	K	K	K	K	K
13				K	K	K	K	K	K	K
14				K	K	K	K	K	K	K
15				K	K	K	K	K	K	K
16				K	K	K	K	K	K	K
17				K	K	K	K	K	K	
18				K	K	K	K	K	K	
19			K	K	K	K	K	K	K	
20			K	K	K	K	K	K	K	
21			K	K	K	K	K	K	K	
22			K	K	K	K	K	K	K	
23			K	K	K	K	K			
24		R	R	R	K	K	K	K		
25		R	R	R	K	K	K	K		
26		R	R	R	K	K	K	K		
27	R	R	R	R	K	K	K			
28	R	R	R	R	K	K	K			
29	R	R	R	R	K	K	K			
30	R	R	R	R	R	K	K			
31	R	R	R	R	R	K	K			
32	R	R	R	R	R	K	K			
33	R									
34	R									
35	R									
36	R									

* K = Keep; R = Replace.

† Divide by 6,000 to obtain predicted annual yield in tons.

‡ Where the policy has not been specified such yields are unlikely from medium-producing trees of that particular age.

the optimum policy: 18- and 19-year-old trees in medium-capacity orchards should be replaced if their predicted annual yield is less than 4 tons per acre. This minimum requirement is 4½ to 5 tons for older trees. These values reflect the return expectation of the higher-yielding orchard.

GENERAL GUIDELINES TO REPLACEMENT

The purpose of this section is to use the optimal replacement policies determined in the previous section to develop a more general framework for replacement decisions. Although the replacement decision must be made on an orchard-by-orchard basis, general guidelines should furnish some useful information regarding the replacement decision.

Each state is defined by a certain age

TABLE 13

OPTIMUM REPLACEMENT POLICY FOR THE HIGH-CAPACITY ORCHARD, t = 0.00 TO 12.00*

Age	Predicted yields ($\hat{Y}_t$) next three years (pounds)†											
	0	6,000	12,000	18,000	24,000	30,000	36,000	42,000	48,000	54,000	60,000	66,000
Years												
1	K	K	K	K	‡							
2	K	K	K	K								
3		K	K	K	K	K						
4			K	K	K	K	K					
5				K	K	K	K	K				
6				K	K	K	K	K	K			
7					K	K	K	K	K	K		
8						K	K	K	K	K		
9						K	K	K	K	K	K	
10						K	K	K	K	K	K	
11						K	K	K	K	K	K	
12						K	K	K	K	K	K	K
13						K	K	K	K	K	K	K
14							K	K	K	K	K	K
15						K	K	K	K	K	K	K
16						K	K	K	K	K	K	K
17						K	K	K	K	K	K	
18					K	K	K	K	K	K	K	
19					K	K	K	K	K	K	K	
20					K	K	K	K	K	K	K	
21					K	K	K	K	K	K		
22					K	K	K	K	K	K		
23				K	K	K	K	K	K	K		
24				K	K	K	K	K	K	K		
25				K	K	K	K	K	K	K		
26				R	R	K	K	K	K	K		
27				R	R	K	K	K	K			
28				R	R	K	K	K	K			
29			R	R	R	K	K	K				
30			R	R	R	K	K	K				
31			R	R	R	K	K	K				
32			R	R	R	K	K	K				
33	R											
34	R											
35	R											
36	R											

* K = Keep; R = Replace.
† Divide by 6,000 to obtain predicted annual yield in tons.
‡ Where the policy has not been specified such yields are unlikely from good high-producing trees of that particular age.

and yield in the previous time period. These state variables also contribute to determination of an expected future yield. Indication of "keep" or "replace" for a particular state can therefore be identified with these future yield estimates. Thus, for each age and capacity of orchard, the maximum predicted yield for the next three years which would result in a "replace" decision for the orchard can be determined. For example, a 24-year-old medium-capacity orchard includes seven states (see table 9). Thus, the person making the decision would select the maximum of the yields associated with the states that would result in a "replace" decision (state 42) and could also determine the minimum yield from these seven states which would result in a "keep" decision. In a similar manner, the minimum yield estimate required for "keeping" the orchard can also be determined. Using the programs for the different age periods, the "maxi-

TABLE 14

OPTIMUM REPLACEMENT POLICY FOR LOW-CAPACITY ORCHARDS, t = 0.00 TO 10.00*

Age	Predicted yields ($\hat{Y}_t$) next three years (pounds)†							
	0	6,000	12,000	18,000	24,000	30,000	36,000	42,000
years								
1	K	K	‡					
2	K	K						
3	K	K	K					
4	K	K	K					
5	K	K	K	K				
6	K	K	K	K	K			
7	K	K	K	K	K	K		
8		K	K	K	K	K		
9		K	K	K	K	K	K	
10		K	K	K	K	K	K	
11		K	K	K	K	K	K	
12		K	K	K	K	K	K	
13		K	K	K	K	K	K	
14		K	K	K	K	K	K	
15		K	K	K	K	K	K	
16		K	K	K	K	K	K	
17		K	K	K	K	K	K	
18		K	K	K	K	K		
19	R	R	K	K	K	K		
20	R	R	R	K	K	K		
21	R	R	R	K	K	K		
22	R	R	R	K	K			
23	R	R	R	K	K			
24	R	R	R	K	K			
25	R	R	R	K	K			
26	R	R	R	R	K¶			
27	R							
28	R							
29	R							
30	R							

* K = Keep; R = Remove.
† Divide by 6,000 to obtain predicted annual yield in tons.
‡ Where the policy has not been specified such yields are unlikely from low-capacity trees of that particular age.
¶ Although a policy of "keep" is still proposed for trees of this age with a predicted yield of 4 tons, the probability of trees being in this situation is very remote.

mum replacement yields" and the "minimum keep yields" were determined (see table 16). These figures can be used to approximate a "replacement margin" for known age and quality level. For trees between 24 and 30 years old this margin is approximately 13,000 pounds for low-capacity orchards, 21,500 pounds for medium-capacity orchards, and 30,000 pounds for high-capacity orchards. These figures were obtained by selecting the maximum values of the "maximum replace" figures and the minimum of the "minimum keep" figures in table 16.[15] The values are plotted against orchard capacity (figure 4) and form a linear relationship. This line defines the "replacement margin" for the assumed price and production relationships. If the predicted yield for the next three years

[15] The figures in table 16 underestimate the maximum yield for replacement and overestimate the minimums for keeping the trees because these figures are based on predetermined yield estimates associated with each state. By selecting the maximum yields associated with "replacement" and the minimum yields associated with "keeping" over a period of a few years, a close approximation of the "replacement margin" can be obtained.

TABLE 15
OPTIMUM REPLACEMENT POLICY FOR MEDIUM-CAPACITY TREES REPLACED BY POTENTIALLY HIGH-CAPACITY TREES*

Age	Predicted yield ($\hat{Y}_t$) next three years (pounds)†									
	0	6,000	12,000	18,000	24,000	30,000	36,000	42,000	48,000	54,000
years										
1	K	K	K	K	‡					
2	K	K	K	K						
3	K	K	K	K	K					
4	K	K	K	K	K					
5			K	K	K	K	K			
6			K	K	K	K	K			
7				K	K	K	K	K	K	
8				K	K	K	K	K	K	
9				K	K	K	K	K	K	
10				K	K	K	K	K	K	
11				K	K	K	K	K	K	K
12				K	K	K	K	K	K	K
13				K	K	K	K	K	K	K
14				K	K	K	K	K	K	K
15				K	K	K	K	K	K	K
16				K	K	K	K	K	K	K
17				K	K	K	K	K	K	K
18				R	R	K	K	K	K	K
19				R	R	K	K	K	K	
20				R	R	K	K	K	K	
21			R	R	R	K	K	K	K	
22			R	R	R	K	K	K	K	
23		R	R	R	R	K	K	K		
24		R	R	R	R	K	K	K		
25		R	R	R	R	R	K	K		
26	R	R	R	R	R	R	K			
27	R									
28	R									
29	R									
30	R									

* K = Keep; R = Replace.
† Divide by 6,000 to obtain predicted annual yield in tons.
‡ Where the policy has not been specified such yields are unlikely from high-capacity trees of that particular age.

lies below the line, the appropriate policy is to replace the trees; otherwise, the policy is to keep.

It is necessary to determine the capacity of the orchard, however, before the "replacement margin" presented in figure 4 can be readily interpreted. The orchard capacity is based upon the *average yield* of an orchard. The relationships between average yield and capacity of orchard, as defined in this study by age of trees, can be estimated using figure 3 and envisaging a linear relationship between orchard capacity and yield. For example, a 23-year-old orchard ($t =$ 7.67) presently producing 29,000 pounds in a 3-year period is determined to be of orchard quality 0.9.

Having estimated orchard capacity, the replacement margin in figure 4 can be used to interpolate a replacement policy for any orchard of Santa Rosa plums. The important determinant is not only the magnitude of the predicted yield, but also the current average yield with respect to this expectation. Hence, trees in the medium-capacity range at their prime may decline rapidly and warrant replacement at an earlier age than indicated by the above results.

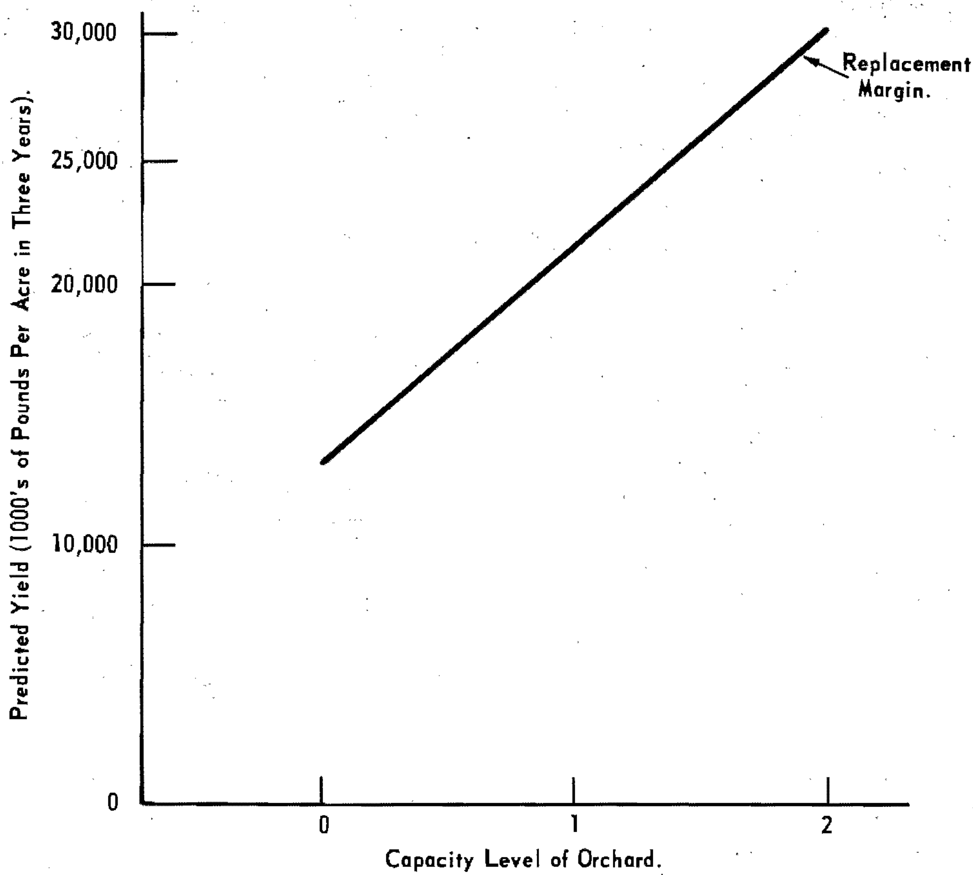

Figure 4. Minimum Predicted Yields Necessary to Warrant a Policy of Keeping Trees of Different Capacity Levels, Trees, 24 to 30 Years Old.

The modification for rapidly declining (or prematurely declining) trees is that a replacement policy can be estimated for trees that do not conform to the pattern used in the representative programs. As figure 3 shows, yields gradually declined in the sample orchards. Future yields from the next orchard are an important factor in determining optimum policy. Thus, trees that experience a rapid (or premature) decline in yields will be replaced when their yield expectation is equal to the replacement margin of the orchard planned for the future.

This concept amounts to a comparison of the average capacity of the present trees with the expected future capacity of the new orchard. The yields predicted for the next three years from the current trees must be greater than the replacement margin of the next block of trees. In the example presented above the trees should be kept if the predicted yield is greater than approximately 20,500 pounds of fruit.

DETERMINISTIC REPLACEMENT

Replacement problems can be approached using a deterministic rather than a stochastic model. The deterministic model is less "sophisticated" in the sense that the probability of moving from one state to the next state is 1.0. Thus, the construction of a probability or transition matrix is not necessary. This makes the model simpler in terms of both the information required and the computational techniques. The simple production relationships, costs, and returns required for the stochastic modle are also required (or can be used) for the deterministic model.

This section compares the results using a deterministic replacement model with those of the stochastic model. The reason for this comparison is to check the usefulness of the stochastic model. A more sophisticated model does not necessarily guarantee that the results will be improved.

Deterministic Model

The deterministic model used (Faris, 1960) does not have the same format as a dynamic programming model. However, its formulation and consequently the results are the same (Burt, 1963). The principle of optimum replacement is stated as follows: "The optimum time to replace is when the marginal net revenue from the present enterprise is equal to the highest amortized present value of expected net revenues from the enterprise immediately following."

The marginal net revenue, in this instance, is the annual net return. A three-year average could be used but the results would be approximately the same. It is less difficult to interpret the replacement decision based on annual costs and returns. The enterprise immediately following will, in its first years, consist of newly planted plum trees. Because revenue from the enterprise immediately following is received in the future, account must be taken of the orchardists' time preference for these returns in the future (Y). An appropriate discount rate (i) is used to convert Y to an equivalent present value (PV) so that

$$PV_n = Y_n \left[\frac{1}{(1+i)^n}\right]$$

where n refers to the year or age of the trees in the orchard immediately following (Faris, 1960).

The present values are accumulated which gives the total returns (discounted) to any point in time following replacement. This is a lump-sum value and must be converted to an annual equivalent to allow an appropriate comparison with the marginal net revenue from the present orchard. This is accomplished by calculating the amortized present value (A) of expected net returns from the replacement orchard;

$$A = \sum_{1}^{n} PV \left[\frac{i(1+i)^n}{(1+i)^n - 1}\right].$$

The amortized value from the replacement orchard is compared with the annual net return from the present orchard. When the former becomes larger than the latter, the orchard should be replaced because it is more profitable to replace than keep the present orchard.

Optimum Replacement

Yield estimates for the three qualities of orchards are similar to those used in the stochastic model except the yields are annual yields. The gross returns and costs are calculated using the same data as in the stochastic model. The discount rate (i) is 6 per cent, as in the stochastic model.

The calculations indicating the optimum replacement for medium-capacity orchards are presented in table 17. Assuming that the present trees are replaced with trees of the same capacity and that prices and costs do not change, the optimum replacement is after the crop harvest in year 28. The expected net returns from the present trees are $163 in year 29 which is slightly less than the highest amortized value of $164. In terms of yields it would be profitable to replace when expected yields decrease below 7,800 pounds per year (or 3.9 tons).

Similar calculations are made for high-capacity orchards and low-capacity orchards. For high-capacity orchards the optimum time to replace is after the harvest in year 29. In terms of yields it is profitable to replace when yields decrease below approximately 10,800 pounds (5.4 tons). For low-capacity orchards the optimum time to replace is after the harvest in year 24. In terms of yields a low-capacity orchard should be replaced when expected yields decrease to approximately 5,200 pounds per year (2.6 tons).

Replacement with a different quality of orchard can easily be determined using the tables constructed for the deterministic model. For example, assume that the expectation is to replace a medium-capacity orchard with a high-capacity orchard. The largest amortized value for a high-capacity orchard is $287. This is slightly less than the expected annual net return of $286 for the medium-capacity orchard in year 24. Thus, the medium-capacity orchard would be replaced at the end of year 24 or when the expected yield decreases to slightly less than 10,050 pounds per year.

TABLE 16

GUIDE TO REPLACEMENT FOR ORCHARDS OF DIFFERENT CAPACITY LEVELS

Present and replacement orchard	Age of present orchard	Predicted yield next three years: Maximum replace	Predicted yield next three years: Minimum keep
	years	*pounds per acre*	
	19	7,905	11,179
	20	9,594	12,868
Low-capacity orchard replaced by a low-capacity orchard	21	11,359	14,635
	22	13,020	16,294
	23	11,436	14,710
	24	9,937	13,211
	25	11,609	14,883
	26	13,137	16,411
	24	20,102	23,376
	25	18,564	21,838
Medium-capacity orchard replaced by a medium-capacity orchard	26	20,092	23,366
	27	21,682	24,956
	28	19,797	23,071
	29	17,726	21,000
	30	22,143	25,417
	31	19,909	23,183
	32	20,788	24,062
	26	24,900	28,201
High-capacity orchard replaced by a high-capacity orchard	27	29,660	32,940
	28	27,740	31,015
	29	28,900	32,200
	30	30,102	33,370
	31	27,850	31,120
	32	28,860	32,136
	18	26,307	29,581
	19	24,683	27,957
Medium-capacity orchard replaced by a high-capacity orchard	20	26,372	29,646
	21	30,771	34,045
	22	26,523	29,797
	23	28,372	31,651
	24	26,650	29,924
	25	25,112	28,386
	26	26,040	29,914

Comparison of Results—Stochastic and Deterministic Models

To compare the results obtained from the stochastic model with those from the deterministic model, it is necessary to select either a three-year period or a one-year period with respect to future expected yields. The former is used in table 18. Using the criterion that orchards should be replaced when expected (or predicted) yields for the next three years fall below a certain level, it is possible to compare the results from each model. The replacement yield, using the deterministic model, falls between the limits

TABLE 17

EXPECTED YIELDS, COSTS, AND RETURNS PER ACRE FOR MEDIUM-CAPACITY ORCHARD USING THE DETERMINISTIC REPLACEMENT MODEL*

Age of trees	Yield	Gross returns	Annual costs	Net returns	Present value of net returns†	Accumulated present values	Amortized present value of net returns†
		dollars					
years	*cwt.*						
1	0	0	340	−340	−340	− 340	−340
2	0	0	105	−105	− 93	− 433	−236
3	15.0	116	160	− 44	− 37	− 470	−176
4	25.0	194	240	− 46	− 36	− 507	−146
5	37.5	291	300	− 9	− 7	− 514	−122
6	61.0	473	400	73	51	− 462	− 94
7	78.5	608	440	168	112	− 349	− 63
8	90.5	701	466	235	148	− 202	− 33
9	100.5	779	490	289	171	− 31	− 5
10	107.0	829	504	325	182	151	21
11	110.0	853	510	343	180	331	42
12	113.5	872	516	356	177	508	61
13	116.0	899	521	378	177	686	77
14	117.5	911	526	385	170	856	92
15	117.5	911	526	385	160	1,016	105
16	117.0	907	525	382	150	1,166	115
17	116.3	901	522	379	141	1,307	125
18	115.0	891	519	372	130	1,438	133
19	113.5	880	516	363	120	1,558	140
20	111.8	866	514	352	110	1,668	145
21	109.7	850	510	340	100	1,768	150
22	107.0	829	504	325	90	1,858	154
23	104.0	806	498	304	80	1,938	158
24	100.5	779	490	289	71	2,009	160
25	96.5	748	481	267	62	2,071	162
26	92.5	717	472	245	54	2,125	163
27	88.0	682	461	221	46	2,171	164
28	83.0	643	449	*194*	38	2,209	*165*
29	77.5	601	438	163	30	2,239	165
30	72.0	558	427	131	23	2,262	164
31	65.5	508	412	96	16	2,278	164

* Figures rounded to the nearest dollar; the physical relationships, costs, and returns are those used in the stochastic model.

† The rate "*i*" used in making these calculations is 6 per cent per annum.

obtained using the stochastic model (table 18). Thus, the results from the models are consistent. Also the results for the deterministic model are very close to those obtained in calculating the "replacement margin" for the stochastic model (see figure 4).

The above analysis indicates that the deterministic model is as appropriate a model as the stochastic model in determining the replacement of plum orchards. In addition it is simpler to compute and does not require the data for the construction of the transition matrix. A major advantage of the stochastic model is to account for the uncertainty of future yields. In this study the stochastic model offered difficulties because of (a) the problem of defining an appropriate yield function and (b) the need to group yields into three-year aggregates. A second advantage of the stochastic model is that expected yields are defined by yields in the previous time period(s) as well as age of the trees, while in the deterministic model yields are defined by only the age of the trees. This does make some difference in the replacement de-

TABLE 18

COMPARISON OF REPLACEMENT YIELDS FOR STOCHASTIC AND DETERMINISTIC MODELS

Present and replacement orchard	Optimum replacement age for deterministic model	Predicted yield next three years: Stochastic model*		Predicted yield next three years: Deterministic model†
		Maximum replace	Minimum keep	Replacement yield
	years	*pounds per acre*		
Low-capacity replaced by low-capacity	24	9,937	13,211	12,380
Medium-capacity replaced by medium-capacity	28	19,797	23,071	21,500
High-capacity replaced by high-capacity	29	28,900	32,200	29,650
Medium-capacity replaced by high-capacity	24	26,650	29,924	27,700

* Source: Table 16.
† Calculated by summing the yields for the three-year period immediately following the optimum replacement year. See table 17.

cision (see tables 12, 13, and 14). However, this advantage is lost when an attempt is made to generalize the results. Thus, in conclusion, it appears that the work needed for the stochastic model is not compensated for by the results achieved, when compared with the deterministic model.

SUMMARY

In attempting to maximize returns, the orchardist is faced with the problem of when, and with what, to replace aging trees. This study has been oriented toward the production of Santa Roas plums. A replacement policy was sought for these trees with the assumption that future plantings would be of the same variety. The primary technique applied was that of dynamic programming, with the problem placed in a stochastic framework. To illustrate this method of analysis, a simple replacement problem was formulated and an optimum policy found.

From the data collected on the relationship between yields per acre and age, a yield function was derived which enabled prediction of future yields from the present trees. To accommodate the large variability of plum yields, three representative orchards were considered depicting three different levels of orchard capacity—low, medium, and high. The yields established conform to smooth deterministic functions of yield on age. These values provided an expected average yield capacity of each of the three orchards at any given age. Estimates of future yield depended largely on this average, the previous yield, and the age of the tree. Determination of a stochastic relationship permitted a distribution of expectations around any estimate; a factor which allows for the influence of random variables.

Disturbances were assumed to be normally distributed about the yield estimates, with a standard deviation equal to the standard error of the estimates. Probabilities were then determined for a Markov chain of transitions from any state defined by immediate past yields at a certain age to terminating states of expected yields in the next time period.

From an analysis of costs and returns a reward matrix was constructed which corresponded identically to the dimensions of the transition matrix. Two alternatives were available at each decision point: "keep" present trees or "replace" with young trees of the same variety. The purpose of the program is to maximize the total expected returns. The alternative, offering the maximum return, is chosen at each stage over a number of stages until an optimum policy is provided which is applicable to an infinite planning horizon.

An optimum replacement policy was determined for each of the three representative orchards. The criterion for replacement is the magnitude of predicted yields for the next three years. The minimum prediction required to warrant a "keep" policy was slightly higher for older trees, but as a generalized principle it was possible to arrive at a replacement policy for each capacity level which could be applied to trees 24 years of age and over. The low-producing trees around this age required replacement when predicted total yields for the next three years fell to about 13,000 pounds per acre. This value was 21,500 pounds for the medium-capacity orchard, and about 30,000 pounds for the top-capacity orchard.

A deterministic replacement model was also used to compare the results and the models. The results from the deterministic model lead to essentially the same replacement decisions as those obtained using the stochastic model. The deterministric model allows much simpler computations because the transition matrix is not required. The data required for the transition matrix is one of the most demanding aspects of the stochastic model. The elimination of this aspect in a replacement model reduces the computations and data requirements considerably.

Replacement of the present orchard with one of a higher production level was also studied. This analysis showed that the important factor determining the level of replacement is the yield capacity of the replacement orchard. Thus, in using the simple replacement guide the capacity is based upon that of the future orchard and yield for the next three years is predicted for the present orchard. This distinctive feature can also be employed in determining the policy to choose when an orchard changes quality. The change may be for the better because the present orchard is rehabilitated, or worse, because of rapid or premature decline.

The most demanding requirement of this approach is the need to evaluate what represents the orchard's average yield. It must be derived from a reasonably long history of previous yields and the present condition of the trees in respect to this past production. It has been demonstrated that the yield of the past three years influences expectations for the next three years. If this previous yield is low in comparison to the average, the predicted yield is high. Mature trees usually have a fairly long stable period of production at their peak level, and the average yield is easily definable. When yields are declining, this may be because (1) the average is falling or (2) the decline is a short-run fluctuation. The answer must lie largely in the orchardist's appraisal of the condition of his trees. This appraisal is especially critical in judging the worth of a rehabilitated orchard.

Another judgment is required which also needs considerable foresight. The new orchard must be visualized in terms of its expected future production. Numerous factors, many not controlled by man, will contribute to disturbing these predictions of the future. However, the problem is always present whenever trees are to be replaced and the best predictions available must be relied upon to make a decision.

Returns to the grower have been based on constant prices. This need not necessarily hold true for the immediate future although for the infinite horizon it is a reasonable assumption. A grower's replacement decision may be influenced by a forecast of future price movements. A good example is the grower with old trees producing near the margin at good prices. Heavy plantings of recent years foretell of a considerable increase in the quantity of fruit to be marketed. In such circumstances the grower would be inclined to retain the present trees until the prices decline.

Replacement of Santa Rosa plum trees need not require planting new trees of the same variety. This opportunity to choose a new crop is important for the orchardist because a decision to plant trees will commit the occupied land for 20, 30, or more years. The consideration is largely economic and, therefore, the net returns per acre must be predicted for the various alternatives. Further analysis is required before a definite policy in this direction could be proposed. The work would involve replacing Santa Rosas with a new plum variety or new type of fruit just as one quality was replaced with another quality orchard.

CONCLUSIONS

An economically sound replacement policy has been derived which will maximize returns to the grower over a long period of time. An important feature of these maximum returns is that it has taken into account the uncertainty that surrounds the estimate of future yields. Random influences are predominant in the production of plums and no policy can guarantee a return equal to that maximized. It does, however, maximize the probability of otaining the largest return possible. The fundamental rule in replacement theory remains dominant: Trees should be kept until the expected future returns from such a policy are less than the future returns likely if the trees are replaced.

The particular methodology used in the major portion of the analysis proved successful in arriving at a general replacement policy for Santa Rosa plum trees. However, certain difficulties were encountered in the course of the analysis which may be of less concern in replacement studies of assets of a different nature. Firstly, the stochastic approach to a replacement problem requires derivations of transition probabilities through all ages of the asset under study. Determining reliable probabilities was the main problem met in applying this method to replacement of plum trees. Random variables cause a wide variation of plum yields and their influence makes it difficult to express plum yields in functional form. Some of the random factor effect is absorbed by establishing the concept of an average-yield capacity. The possible unreliability of this procedure has been discussed. High-yield variance will be a feature of many replacement studies of various types of tree fruits. However, the problem should be not as big in a similar study with animals or capital equipment, and, therefore, many of the attending difficulties outlined above would be avoided.

Another problem less likely to be encountered in studies other than fruit trees is that of data collection. Apart from the questionable accuracy of many of the observations, good data are not available for plums over a long period of time (eight years is short relative to normal life of a tree). Improved data and smaller-yield variances would considerably enhance the successful application of this type of dynamic programming to the replacement problem.

It is suggested that the actual dynamic program used could itself be modified by using an improved routine. The present technique is described by Howard, 1960, as the value-iteration method and its principal disadvantage is the large number of iterations required to achieve policy convergence. The policy-iteration routine is an efficient program which is much more economical in computer time (Howard, 1960). Despite these problems, the work undertaken has demonstrated the flexibility of the use of dynamic programming in replacement theory. With respect to plum trees, general conclusions have been reached which can generally be applied as a guide to the determination of an optimum replacement policy.

Finally, it appears that a deterministic model may be the most appropriate model to use in a number of instances. The replacement decisions resulting from the application of the deterministic model were essentially the same as those obtained using the stochastic model. In this study the extra work needed for the stochastic model was not compensated for by the results achieved relative to the deterministic model.

APPENDIX

Appendix Table A-1
YIELD OF REPRESENTATIVE ORCHARDS IN THREE-YEAR AGGREGATES ACCORDING TO CAPACITY AND AGE

Age	Orchard capacity		
	Low	Medium	High
	yield per acre		
three-year units	*thousands of pounds*		
1.00	1.00	1.50	2.00
1.33	2.50	4.00	6.00
1.67	4.00	7.50	12.00
2.00	6.50	12.50	18.50
2.33	9.50	19.50	25.50
2.67	13.00	24.50	31.50
3.00	16.50	27.50	36.00
3.33	19.00	30.00	39.00
3.67	20.50	31.75	42.00
4.00	21.50	33.00	44.00
4.33	22.25	34.00	45.50
4.67	22.75	34.50	46.50
5.00	23.00	35.00	47.00
5.33	22.75	34.75	46.75
5.67	22.25	34.25	46.50
6.00	21.50	33.50	46.00
6.33	20.50	32.50	45.00
6.67	19.50	31.50	44.00
7.00	18.75	30.75	42.75
7.33	18.00	30.00	41.75
7.67	17.50	29.50	40.75
8.00	17.00	29.00	40.00
8.33	16.50	28.50	39.25
8.67	16.00	28.00	38.75
9.00	15.75	27.75	38.25
9.33	15.50	27.50	37.75
9.67	15.25	27.25	37.25
10.00	15.00	27.00	37.00
10.33		26.50	36.75
10.67		26.00	36.50
11.00		25.50	36.25
11.33		25.25	36.00
11.67		25.00	35.75
12.00		24.75	35.50

Appendix Table A-2
SHIFTING MEAN YIELD ($\bar{Y}$ = THREE-YEAR AGGREGATE) OF REPRESENTATIVE ORCHARDS BASED ON SIX YEARS' YIELD, CAPACITY, AND AGE

Age span of six years	Orchard capacity		
	Low	Medium	High
	yields per acre		
three-year units	*thousands of pounds*		
0.00– 1.00	0.50	0.75	1.00
0.33– 1.33	1.25	2.00	3.00
0.67– 1.67	2.00	3.75	6.00
1.00– 2.00	3.75	7.00	10.25
1.33– 2.33	6.00	11.75	15.25
1.67– 2.67	8.50	16.00	21.75
2.00– 3.00	11.25	20.00	27.25
2.33– 3.33	14.25	24.75	32.25
2.67– 3.67	16.75	28.125	36.75
3.00– 4.00	19.00	30.25	40.00
3.33– 4.33	20.625	32.00	42.25
3.67– 4.67	21.625	33.125	44.25
4.00– 5.00	22.25	34.00	45.50
4.33– 5.33	22.50	34.375	46.125
4.67– 5.67	22.50	34.375	46.50
5.00– 6.00	22.25	34.25	46.50
5.33– 6.33	21.625	33.625	45.825
5.67– 6.67	20.875	32.875	45.25
6.00– 7.00	20.125	32.125	44.375
6.33– 7.33	19.25	31.25	43.375
6.67– 7.67	18.50	30.50	42.375
7.00– 8.00	17.875	29.875	41.375
7.33– 8.33	17.25	29.25	40.50
7.67– 8.67	16.75	28.875	39.75
8.00– 9.00	16.375	28.325	39.125
8.33– 9.33	16.00	28.000	38.500
8.67– 9.67	15.625	27.625	38.000
9.00–10.00	15.375	27.375	37.625
9.33–10.33		27.00	37.125
9.67–10.67		26.625	36.875
10.00–11.00		26.250	36.625
10.33–11.33		25.875	36.375
10.67–11.67		25.500	36.125
11.00–12.00		25.125	35.875

APPENDIX TABLE A-3

COSTS ASSOCIATED WITH REPLACEMENT OF OLD TREES AND SUBSEQUENT CARE OF YOUNG TREES TO THE FIFTH YEAR

Item	Capacity of orchard, trees one–five years														
	Low — age and yield					Medium — age and yield					High — age and yield				
Years:	1	2	3	4	5	1	2	3	4	5	1	2	3	4	5
Tons:	0	0	0	1	2	0	0	0	1	2	0	0	1	2	3
	dollars per acre														
Pulling	60					60					60				
Subsoiling	20					20					20				
Tree preparation	20					20					25				
Fumigation	80					80					100				
Trees, planting, etc.	100					100					100				
Cultivation	15	17	20	20	20	17	19	23	23	23	17	20	25	25	25
Irrigation	12	15	18	22	25	15	20	25	27	30	20	25	30	35	40
Fertilizer		5	7	10	12		6	10	14	16	5	10	15	18	20
Pruning		10	20	30	50		15	30	50	80		20	50	80	90
Vermin protection	10	10				10	10				10	10			
Failure replacement		5	3	3			5	3	3			5	3		
Cover crop	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3
Spraying	5	12	20	28	35	5	15	25	35	40	5	15	30	40	45
Thinning					20					35				20	35
Miscellaneous	5	8	12	15	15	10	12	15	18	18	10	12	15	18	20
Harvesting costs				30	60				30	60			30	65	95
TOTAL COSTS	330	85	103	161	240	340	105	134	203	305	375	120	201	304	373
Returns (net of packing costs)				155	310				155	310			155	310	465
NET RETURNS	−335	−85	−103	−6	70	−340	−105	−134	−48	5	−375	−120	−46	6	92

APPENDIX TABLE A-4

PRUNING, THINNING, AND HARVESTING COSTS BY ANNUAL YIELD AND ORCHARD CAPACITY*

Annual yield	Orchard capacity								
	Low			Medium			High		
	Prune	Thin	Harvest	Prune	Thin	Harvest	Prune	Thin	Harvest
tons per acre	*dollars per acre*								
0	60			60			60		
1	65		43	70		44	70		45
2	70	20	67	80	20	67	80	20	75
3	75	40	91	85	35	94	90	35	103
4	80	60	110	90	60	118	100	60	126
5	85	75	131	95	75	140	105	75	146
6	90	90	149	100	90	160	110	90	161
7				102.50	100	178	115	100	179
8				105	110	194	120	110	195
9				107.50	120	207	125	120	208
10							130	125	219
11							135	130	228
12							140	135	234

* Pruning costs are applicable only for trees six years of age and older.

Appendix Table A-5
OPERATING COSTS AND RETURNS FOR A MATURE ORCHARD OF LOW, MEDIUM OR HIGH CAPACITY

Item	Capacity of orchard		
	Low	Medium	High
	*dollars per acre**		
Pre-harvest:			
Pruning	90	100	110
Brush disposal	7	8	8
Fertilizer	15	17	22
Spraying	40	44	52
Irrigation	30	30	40
Cultivation	23	23	25
Thinning	90	90	90
Cover crop	3	3	3
Taxes	22	22	22
Miscellaneous labor	10	10	10
Sundries	20	23	25
Harvest:			
Picking	120	132	132
Roadside	23	23	23
Miscellaneous	6	6	6
TOTAL CASH COSTS	499	531	568
Returns (less packing)	930	930	930
NET RETURNS	431	399	372

* Based on an annual yield of six tons.

Appendix Table A-6
SUMMARY OF NONZERO ELEMENTS OF THE REWARD MATRIX FOR A MEDIUM-CAPACITY ORCHARD TIME PERIOD 0.00 TO 12.00, WHEN A POLICY OF "KEEP" IS CHOSEN*

Time period $t-1$	States i	Possible yield in year t (thousands of pounds)									
		0	6	12	18	24	30	36	42	48	54
		dollars per acre									
0	1	−579	−224	36							
1	2–4	−365	3	312	651						
2–10	5–60		−300	−124	200	526	858	1,197	1,570	1,945	2,327
11	61–67	−600									
12	68	Replaced									

* The return to all states choosing a policy of "replace" is zero.

LITERATURE CITED

Bellman, Richard
1955. Notes in the theory of dynamic programming III: Equipment replacement policy. RAND Report P-632, January, 7 pp.

Burt, Oscar R.
1963. Economic replacement, Siam Review, **5**(3), July.

Burt, Oscar R., and John R. Allison
1963. Farm management decisions with dynamic programming. Journal of Farm Economics, **45**(1), February.

Dreyfus, Stuart
1961. Dynamic programming, Chapter 5 of Progress in Operations Research, Vol. I, ed. Russell L. Ackoff, New York: John Wiley and Sons.

Faris, J. Edwin
1960. Economics of replacing cling peach trees. Davis: University of California, Agric. Exp. Sta., Giannini Foundation Mimeo **232**, June, 125 pp.

Foote, Richard J.
1958. Analytical tools for studying demand and price structure, U. S. Department of Agriculture, Handbook **146**, August.

Howard, Ronald A.
1960. Dynamic programming and Markov processes. Cambridge, Mass.: M.I.T. Press, 136 pp.

Parzen, Emanuel
1962. Modern probability theory and statistics, New York, N. Y.: John Wiley and Sons.

1.5*m*-10,'68 (J2910) J.F. 141

GIANNINI FOUNDATION MONOGRAPHS TO DATE

The first titles in the Giannini Foundation Monograph Series were published in *Hilgardia* and are listed below. Numbers 18 *et seq.* are published in the present format. See back cover for information on obtaining copies.

No. 1. Major Economic Forces Affecting Agriculture, with Particular Reference to California, by S. V. Ciriacy-Wantrup. (*Hilgardia,* Volume 18, Number 1, December, 1947) 76 pages.

No. 2. Characteristics of Demand for California Plums, by Jerry Foytik. (*Hilgardia,* Volume 20, Number 20, April, 1951) 121 pages.

No. 3. Pricing Efficiency in the Manufactured Dairy Products Industry, by James B. Hassler. (*Hilgardia,* Volume 22, Number 8, August, 1953) 100 pages.

No. 4. Statistical Analysis of Supply Response in Late Spring Potatoes in California, by Chester O. McCorkle, Jr., and Yair Mundlak. (*Hilgardia,* Volume 24, Number 16, April, 1956) 39 pages.

No. 5. Economic Efficiency in Plant Operations With Special Reference to the Marketing of California Pears, by B. C. French, L. L. Sammet, and R. G. Bressler. (*Hilgardia,* Volume 24, Number 19, July, 1956) 179 pages. (Out of print—available on microfilm only.)

No. 6. Soil Variables for Use in Economic Analysis, by David Weeks and J. Herbert Snyder. (*Hilgardia,* Volume 26, Number 11, April, 1957) 24 pages.

No. 7. Economies of Scale for Evaporated Milk Plants in California, by James N. Boles. (*Hilgardia,* Volume 27, Number 21, October, 1958) 102 pages.

No. 8. Income, Price, and Yield Variability for Principal California Crops and Cropping Systems, by H. O. Carter and G. W. Dean. (*Hilgardia,* Volume 30, Number 6, October, 1960) 44 pages.

No. 9. The Impact of Irrigation on Farm Output in California, by Vernon W. Ruttan. (*Hilgardia,* Volume 31, Number 4, July, 1961) 43 pages.

No. 10. Interregional Competition in the Frozen Strawberry Industry, by C. C. Dennis and L. L. Sammet. (*Hilgardia,* Volume 31, Number 15, December, 1961) 113 pages.

No. 11. Econometric Analysis of the Market for California Early Potatoes, by Pinhas Zusman. (*Hilgardia,* Volume 33, Number 11, December, 1962) 120 pages.

No. 12. Orderly Marketing for California Avocados, by Stephen H. Sosnick. (*Hilgardia,* Volume 33, Number 14, December, 1962) 70 pages.

No. 13. Regional Location of Cattle Feeding—A Spatial Equilibrium Analysis, by G. A. King and L. F. Schrader. (*Hilgardia,* Volume 34, Number 10, July, 1963) 86 pages.

No. 14. Optimal Cooperative Pools for California Avocados, by Stephen H. Sosnick. (*Hilgardia,* Volume 35, Number 4, September, 1963) 38 pages.

No. 15. The Economics of Conjunctive Use of Ground and Surface Water, by Oscar R. Burt. (*Hilgardia,* Volume 36, Number 2, December, 1964) 81 pages.

No. 16. Size and Location Factors Affecting California's Beef Slaughtering Plants, by S. H. Logan and G. A. King. (*Hilgardia,* Volume 36, Number 4, December, 1964) 50 pages.

No. 17. Interregional Competition in the United States Turkey Industry, by D. Lee Bawden, H. O. Carter, and G. W. Dean. (*Hilgardia,* Volume 37, Number 13, June, 1966) 95 pages.

No. 18. World Trade in Fresh Oranges: An Analysis of the Effect of European Economic Community Tariff Policies, by Gerald W. Dean and Norman R. Collins. (*Giannini Foundation Monograph,* January, 1967) 70 pages.

No. 19. Conditional Projections of California Economic Growth, by Ivan M. Lee. (*Giannini Foundation Monograph,* February, 1967) 120 pages.

No. 20. Models of Commodity Transfer, by Duran Bell, Jr. (*Giannini Foundation Monograph,* October, 1967) 52 pages.

No. 21. Decision Models for California Turkey Growers, by Vernon R. Eidman, Harold O. Carter, and Gerald W. Dean. (*Giannini Foundation Monograph, July,* 1968) 80 pages.

GIANNINI FOUNDATION MONOGRAPH SERIES

What it is

The Giannini Foundation Monograph Series is comprised of technical research reports relating to the economics of agriculture. The series, introduced in 1967, is published by the California Agricultural Experiment Station. Similar technical economic research studies formerly were published in *Hilgardia.*

Each Monograph is a separate report of research undertaken in the California Experiment Station by staff members of the Department of Agricultural Economics and the Giannini Foundation of Agricultural Economics in the University of California. Topics covered range from analyses of farm and processing firms to broader problems of interregional resource use and public policy.

The Monographs are written in technical terms with professional economists as the intended audience. No attempt is made to reduce the writing to terms understandable to the layman. Each Monograph carries an abstract on the inside front cover.

Monographs are published at irregular intervals as research is completed and reported.

How to obtain copies

In general, copies will be sent free on request to individuals or organizations. The limit to California residents is 20 titles; the limit to nonresidents is 10. There is no distribution through agencies or stores.

A list of available Monographs in the series is published annually and may be obtained by writing to Agricultural Publications (address below). The list also explains how some out-of-print issues, including reports that formerly appeared in *Hilgardia,* may be obtained on microfilm or as record prints. To obtain the *Giannini Foundation Monograph Series* regularly, certain minimum qualifications must be met:

As a gift. Some libraries, educational institutions, or agricultural experiment stations may receive Monographs as issued where there is a definite need for the material and it will be made available to a considerable number of interested economists. Address requests to Agricultural Publications. Please give particulars.

As an exchange for similar research material. Address requests to Librarian, Giannini Foundation of Agricultural Economics, University of California, Berkeley, California 94720.

With the exception of communications about exchange agreements (see above), address all correspondence concerning the *Giannini Foundation Monograph Series* to:

Agricultural Publications
University Hall
University of California
Berkeley, California 94720