Production Functions and Linear Programming Models for Dairy Cattle Feeding

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This publication discusses possibilities of increased efficiency and profitability of dairy production through improved feed formulation and feeding programs. Production response functions, estimated from experimental data centering on feed energy-milk output relationships, maximum voluntary intake lines, and tests of the linearity of isoquants, were combined with standard linear programming techniques into an operational computer model designed to provide feeding programs which optimize a total dairy-cattle feeding program. The study concludes with a report on field tests of the computer program for dairy herds under controlled experimental conditions and under commercial conditions.

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PRODUCTION FUNCTIONS AND LINEAR PROGRAMMING MODELS FOR DAIRY CATTLE FEEDING[1]

INTRODUCTION

BECAUSE OF THE IMPORTANCE of the dairy industry in California and the U. S., a substantial body of research literature dealing with many important phases of dairying, including breeding, disease control, efficient milking and milk handling systems, and milk marketing and pricing has developed. One aspect of dairy production which has attracted the attention of dairy nutritionists and agricultural economists is the question of optimal rations and feeding programs. Since feed costs typically comprise about 50 to 65 per cent of total dairy production costs, concern with feeding appears to be well taken.

Previous research in determining optimal dairy rations has generally taken one of two different approaches. One approach has been to empirically estimate the underlying feed-milk production response function, usually from regression analysis of experimental data. Then, for given prices of milk and feed, differential calculus is employed to find the levels of concentrate, roughage, and milk which maximize net returns over feed costs. A second approach involves use of standard linear programming techniques to select least-cost dairy rations from a wide variety of different feeds. The production response approach provides a method for determining the optimal level of feeding but generally considers only a limited number of feeds (usually one concentrate or concentrate mix and one roughage). The programming approach considers a wide variety of feeds but the optimal level of feeding is not determined. The basic purpose of this report is to combine these two approaches into an operational model designed to increase the efficiency and profitability of dairy production through improved feed formulation and feeding programs. More specifically, the objectives are: (1) to quantify the functional relationships of milk production to feed inputs, cow ability, weight, and stage of lactation, based on regression analysis of experiments carried out at the University of California at Davis which have particular relevance to the economics of dairy feeding; (2) to incorporate these production relationships into a linear programming computer model designed to provide feeding programs which maximize economic returns over feed costs; and (3) to report tests of the linear programming computer model against conventional feeding programs under field conditions.

[1]

PRODUCTION FUNCTION MODELS

The production function as defined in this study represents a more macroscopic view of the milk production process than is ordinarily taken by scientists dealing with fundamental nutritional complexities. The principal justification for the macroscopic approach is that the variables assumed to influence milk production (feed inputs, cow characteristics, and environmental factors) are those which the dairyman can readily observe and which, to a greater or lesser degree, he can control in his attempt to attain efficient production. Formalizing this concept, as shown in equation (1), milk production per cow (M) in any time period depends on the level of concrete fed (C), the level of roughage fed (R), the cow's inherent production capability or "ability" (A), the breed (B), age (Y), and size or body weight of cow (W), the stage of lactation (T), environmental variables such as temperature (F) and humidity (D) and many other variables represented by a random variable (u).

$$M = f\ (C, R, A, B, Y, W, T, F, D, u) \qquad (1)$$

In experimental work only a limited number of independent variables can be treated as endogenous, the assumption being that the other variables are held essentially constant by the experimenter. The variables in a typical experiment may be represented as in equation (2), where breed (B), age (Y), and environmental conditions (F and D) are held constant at some predetermined levels, while concentrate and roughage (C and R), ability (A), weight (W) and stage of lactation (T) are endogenous variables. E

$$M = f\ (C, R, A, W, T|\ |B, Y, F, D, u) \qquad (2)$$

The dairy cattle feeding problem *per se* is generally represented by a still further restricted subset of variables as shown in equation (3) in which

$$M = f\ (C, R|A, W, T|\ |B, Y, F, D, u) \qquad (3)$$

the function relating feed inputs to milk production also assumes A, W and T fixed at specific levels. In this form, the production function shows milk output from various feed inputs for a cow of a given ability and weight in a specific month of lactation.

The general nature of the biological relationship of feed inputs to milk output is well established from previous work, as summarized in figure 1. The upper portion of this figure depicts the relation in three dimensions, while the lower portion projects the produce surface onto a 2-dimensional plane showing milk response as milk isoquants or contour lines. The diagrams suggest diminishing marginal productivity to each feed individually and in combination, and indicate near-linear isoquants. The relevant portion of the production surface is bounded by four restrictions: (1) the maximum voluntary intake line (MVIL) often referred to as the "stomach line" or "appetite line" in previous research; (2) the minimum roughage requirement line representing a lower limit of roughage in the ration below which physiological disturbances often occur, resulting in severe fat depression in the milk produced; (3) a maintenance requirement which represents the lower limit of total energy intake required for maintenance of body weight and general health; and (4) a minimum level (zero) of concentrate feeding (the roughage axis).

The general effects of ability, time of lactation and body weight are also known.
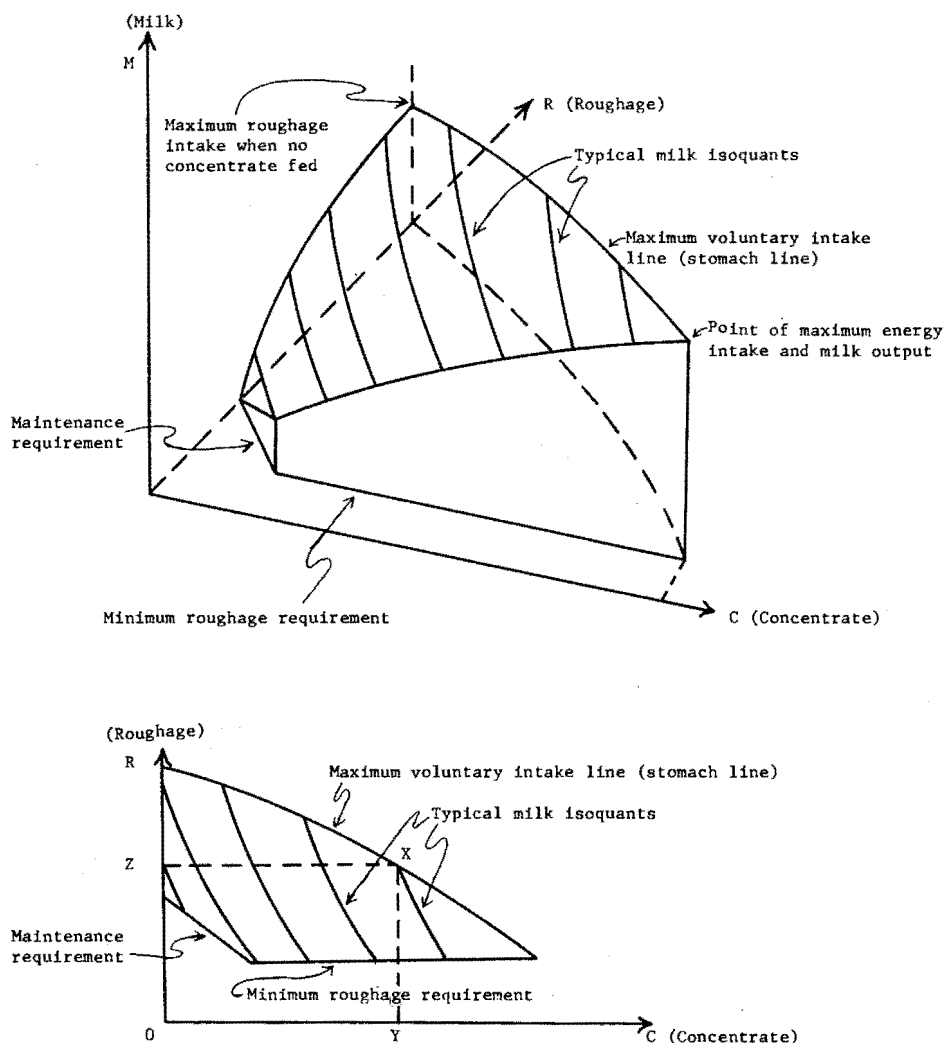
Fig. 1. Theoretical production relationships of feed inputs to milk output.

Cow ability causes important shifts of the entire production surface and often shows an interaction with the feed variables such that cow ability not only changes the level of the surface but the shape of the surface. As the stage of lactation or time increases, the production surface shifts downward. Again, time may interact with feed variables to change the shape as well as the level of the surface over time. Cow size or weight is a rough measure of the cow's capacity for feed intake, other things being equal. Thus, an important effect of cow size is to shift the maximum voluntary intake line, and therefore maximum energy intake, as well as to affect the body maintenance requirement. The effects of breed, age and environment have also been investigated [Heady, *et al.*, 1964a] but are less easily summarized briefly.

Perhaps the most complete studies to date incorporating most of the variables

of equation (1) in a single experiment and subsequent production function estimation have been reported by Heady and co-workers at Iowa State University [Heady, *et al.*, 1964a, 1964b] and by Hoover, *et al.* [1967]. Earlier studies by Heady, *et al.* [1956a, 1956b] included a somewhat more restricted subset of variables similar to those in equation (2). The California work reported herein also involves the subset of variables in equation (2). Still earlier work, such as the original study by Jensen, *et al.* [1942], was restricted to fewer variables and often involved a limited area of the production function near the maximum voluntary intake line.

Relevance of Alternative Feeding Systems

From a general scientific point of view a "complete" production function of the type indicated in equation (1) and a "complete" feed-milk surface as shown in figure 1 are required to express the full range of variables influencing milk production. However, the economically relevant portion of the feed-milk surface may be much more restricted, depending on the particular type of feeding and milking system used. Most dry-lot dairies in California have mechanized milking parlors in which each cow is fed a specified amount of concentrate mix twice a day while she is being milked. The cow is then returned to a common corral with other cows where roughage (usually alfalfa hay) is available on an unrestricted basis. Thus, for example, if a cow is fed quantity OY of concentrate (bottom portion fig. 1), she will voluntarily eat OZ of hay, arriving at the maximum voluntary intake line (MVIL) at point X. In such a system, only the MVIL quantities are relevant for determining the optimal economic feeding program. Since cows are not fed controlled amounts of roughage individually, there is no feasible way of operating at feed combinations below the MVIL. One possible alternative would be to restrict the quantity of roughage fed on a group basis. However, experience shows that more aggressive animals will consume more than their proportionate share and others less, so that poor control is maintained over the feed combinations for individual cows.

In some stanchion-type operations, both concentrate and roughage can be fed on an individual basis. The current and probable future trend is toward feeding complete rations (all-in-one rations) where all feed components are mixed and fed as a single feed. The complete ration may also be packaged (e.g., cubed or wafered). These feeding systems permit complete control over the proportion of roughage and concentrates fed, as well as control over the total feed intake. In these cases where all feed inputs for the individual cow are under control of the manager, the entire feed-milk surface becomes relevant.

Derivation of Economic Optima

Because of the dominance of the milking parlor system in California, a considerable proportion of the experimental work reported herein has concentrated on estimation of the MVIL portion of the production surface. Some previous studies, such as the work by Redman, *et al.* [1965], have also concentrated on MVIL feeding experiments. In such cases, the relevant feed-milk relationships become those indicated in the upper portion of figure 2 and can be summarized in equations (4) and (5).[2]

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[2] The question of whether the relationships summarized in equations (4) and (5) should be represented as a simultaneous system is reserved until later in the report.

Maximum voluntary intake line (MVIL):

$$R = f(C, A, W, T) \qquad (4)$$

Milk production with MVIL feeding:

$$M = f(R, C, A, W, T) \qquad (5)$$

where R and C are in a relationship dictated by equation (4). The objective is to maximize returns over feed costs (π) subject to the side condition that R and C must be fed in combinations along the MVIL. Letting P_M, P_C and P_H represent the prices of milk, concentrate and roughage respectively, the problem is:

$$\text{Max } \pi = P_M \cdot M - (P_C C + P_R R) \qquad (6a)$$

subject to $R = f(C, A, W, T)$ from equation (4).

Letting λ represent a Lagrange multiplier and substituting equation (5) for M in (6a), the problem can be rewritten as maximizing the Lagrangian expression L:

$$\begin{aligned}\text{Max } L = &P_M f(R, C, A, W, T) \\ &- (P_C C + P_R R) \\ &+ \lambda \, [R - f(C, A, W, T)]\end{aligned} \qquad (6b)$$

$$\begin{aligned}\text{Max } \pi = &P_M f(R, C, A, W, T) \\ &- (P_C C + P_R R) \\ &+ \lambda \, [R - f(C, A, W, T)]\end{aligned} \qquad (6)$$

Given values of A, W, T and P_M, P_C and P_R the profit maximizing levels of R, C (and indirectly M) are given by simultaneous solution of the three equations in (7). The second order conditions for a maximum are assumed to hold.

$$\frac{\partial \pi}{\partial C} = 0$$

$$\frac{\partial \pi}{\partial R} = 0 \qquad (7)$$

$$\frac{\partial \pi}{\partial \lambda} = 0$$

The above assumes that a particular concentrate mix and a particular roughage or roughage combination have been specified *a priori*. However, questions of the optimum concentrate mix and of the optimum roughage mix are in themselves relevant economic questions. Thus, one approach to the optimum feeding program would be to proceed in two stages: (1) Solve two separate linear programming problems to find the least-cost combinations of feeds to produce 1 pound (ton) of concentrate mix and 1 pound (ton) of roughage mix;[3] (2) given the (least-cost) price per pound of the concentrate mix and of the roughage mix, apply the maximization procedure indicated above in equations (6) and (7) to find the optimum levels of concentrate and roughage feeding.

However, the entire feeding problem can be solved more efficiently and accurately by casting it in a more general linear programming (LP) framework. The objective function of the LP model is to maximize income above feed costs (value of milk production minus feed costs). This program simultaneously selects components of the concentrate mix, components of the roughage portion of the ration, the roughage-concentrate ratio, the levels of feeding, and the quantity of milk production which maximizes income over feed costs. The LP method is made operational by three basic assumptions: (1) the curvilinear MVIL can be approximated by linear segments; (2) the curvilinear energy-milk output relationship can be approximated by linear segments; and (3) within specified limits all feeds exhibit constant marginal rates of substitution for one another in terms of estimated net energy (ENE) and digestible protein (DP) given by recent nutritional standards. The lower portion of figure 2 shows the linearization of the curvilinear relationships. Obviously, a greater degree of accuracy

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[3] Linear programming (LP) is now a fairly standard procedure used by feed companies and some large dairies in formulating least-cost concentrate mixes.

can be obtained by including more linear segments. Thus, the first two assumptions are easily met; only the third assumption need be examined in detail. Previous studies [e.g., Heady *et al.*, 1956*a*, 1956*b*, 1964*a*, 1964*b*, and Hoover, *et al.*, 1967] have concluded that the milk isoquants are slightly curvilinear. However, over the relevant portion of the production surface as defined in figure 1, the estimated isoquants have been nearly linear (constant marginal rates of substitution between roughage and concentrate). A possible hypothesis is that the isoquants are truly linear over this range, and that curvilinearity has been observed only because observations outside this range were included. Such "unusual" feeding combinations have led to physiological disturbances, lowered fat content and the dubious conversion of the non-homogenous product to a single standardized product via the fat-correction formula. Further, in order to represent diminishing marginal productivity of milk output to increases in each feed input (an accepted relationship), equation forms (quadratic, logarithmic) have been used which force non-linear isoquants
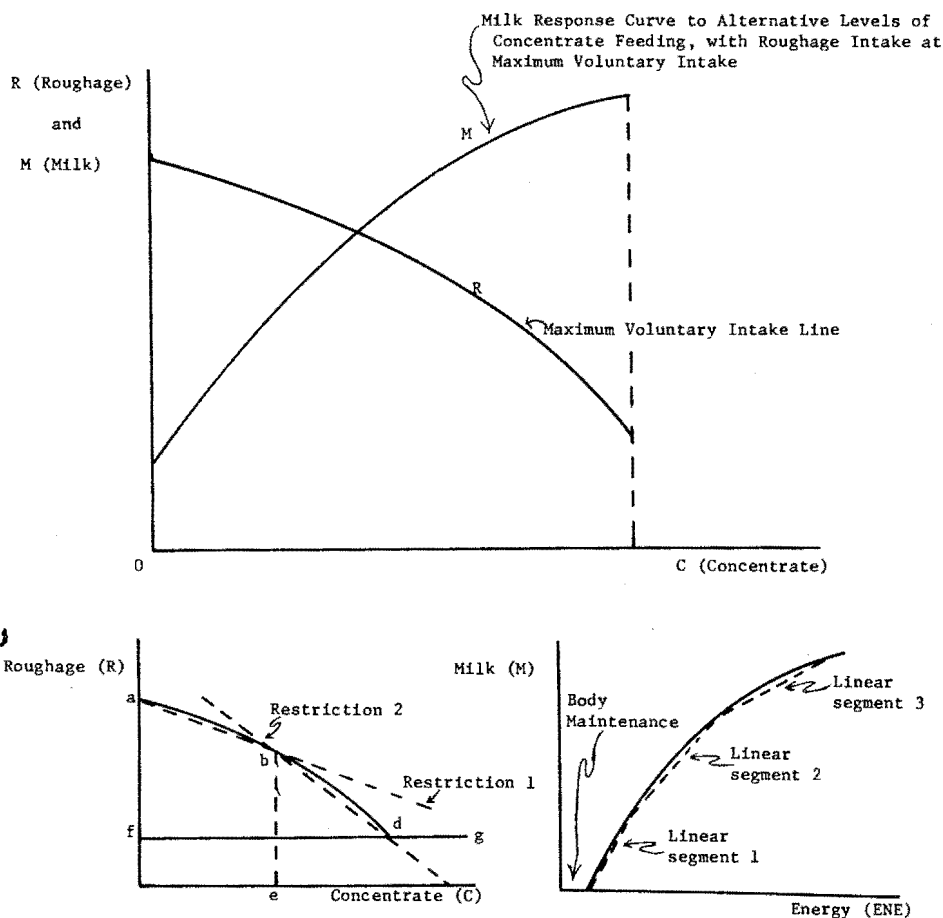

Fig. 2. Maximum voluntary intake line (MVIL) and milk response curve with MVIL feeding.

as well. Thus, one specific objective of this report is to present tests of the hypothesis of linear isoquants, even when diminishing productivity of individual feed inputs is accepted.

In summary, the first major section of the report provides estimates of production relationships derived from experimental data, centering on feed energy-milk output relationships, maximum voluntary intake line estimates and tests of the linearity of isoquants. The second major section of the report builds on the relationships examined in the first section to construct a computer linear programming model for optimizing a total dairy cattle feeding program. The study concludes with a report on field tests of the computer program for dairy herds under controlled experimental conditions and under commercial conditions.

PRODUCTION RELATIONSHIPS
FROM EXPERIMENTAL DATA

Milk Response with Unrestricted Roughage Intake (MVIL Feeding)

The experiment analyzed in this section was designed to generate observations on the maximum voluntary intake line (MVIL)—the relevant portion of the feed-milk surface for milking parlor systems. The experiment provided each cow with a measured quantity of a standard concentrate ration, followed by free choice of hay. The data generated were used (1) to determine the slope and position of the MVIL for cows of various liveweights (W), productive potentials (A), at different stages of lactation (T); and (2) to measure the rate of increase in milk output with respect to total energy fed along the MVIL.

These two relationships may be estimated by analyzing the experimental data within a recursive system: the predetermined concentrate level first determines the voluntary hay intake of the cow; the aggregate energy of the resulting total feed intake then determines milk output. This method gives results which afford comparison with previous estimates of both MVIL and milk-energy response relationships.

It may be, however, that milk output and roughage intake are simultaneously determined once the concentrate level is fixed. An alternative analytical method, then, is a system of two simultaneous equations in which concentrate is an exogenous variable and roughage and milk are endogenous variables. The main results from the Davis trial are presented below in terms of single equations, but the results from a simultaneous model are also offered for purposes of comparison.

Experimental procedures

The experiment to be described was conducted in 1967 by the Department of Animal Science at the University of California, Davis. For the first 7 weeks of their lactation 26 Holstein heifers calving between November and the beginning of February were fed an estimated 60 per cent of daily energy requirements in a pelleted concentrate mix, then cubed alfalfa hay was fed *ad libitum*. This 7-week pre-trial period was used to index the animals according to production ability (A). At the end of the 7 weeks, the heifers were divided randomly into six groups, of four (or five) each, and each group was assigned to one of six feed treatments for the remainder of the lactation. The first five treatments were defined so that 20 per cent, 35 per cent, 50 per cent, 65 per cent and 80 per cent, respectively, of

estimated net energy requirements were derived from the concentrate mix; cubed hay was fed *ad libitum*. The sixth treatment was a ration comprised wholly of the concentrate mix fed *ad libitum*.

The concentrate mix was comprised, by weight, as follows:

Steam-rolled barley ...... 80 per cent
Cottonseed meal .... 15 per cent
Molasses .... 3 per cent
Oyster shell meal .... 1 per cent
Salt .... 1 per cent

Total   100 per cent

The concentrate mix was supplemented with 1,110 international vitamin units A per pound of mix. The energy level per pound of the concentrate mix is approximately 0.76 Mcal of estimated net energy (ENE).

The animals were fed in individual stanchions and concentrate and hay intake were recorded on an individual cow basis. *Ad libitum* portions of the ration were dispensed twice daily in weighed amounts which were about 10 per cent more than would be consumed, and refusals were weighed to obtain estimates of feed intake. For cows in the first five treatments groups, the concentrate intakes each day were pre-determined on the basis of the percentage (20 per cent to 80 per cent) of the total estimated energy requirements to be met from concentrates. For cows in the sixth group, concentrate was fed *ad libitum*, and hay was not offered. Thus, although all of the treatments ensured that each animal would eat to MVIL capacity, the last group was on a different feeding system from the other five groups. Figure 3 shows the pattern of observations for a hypo-
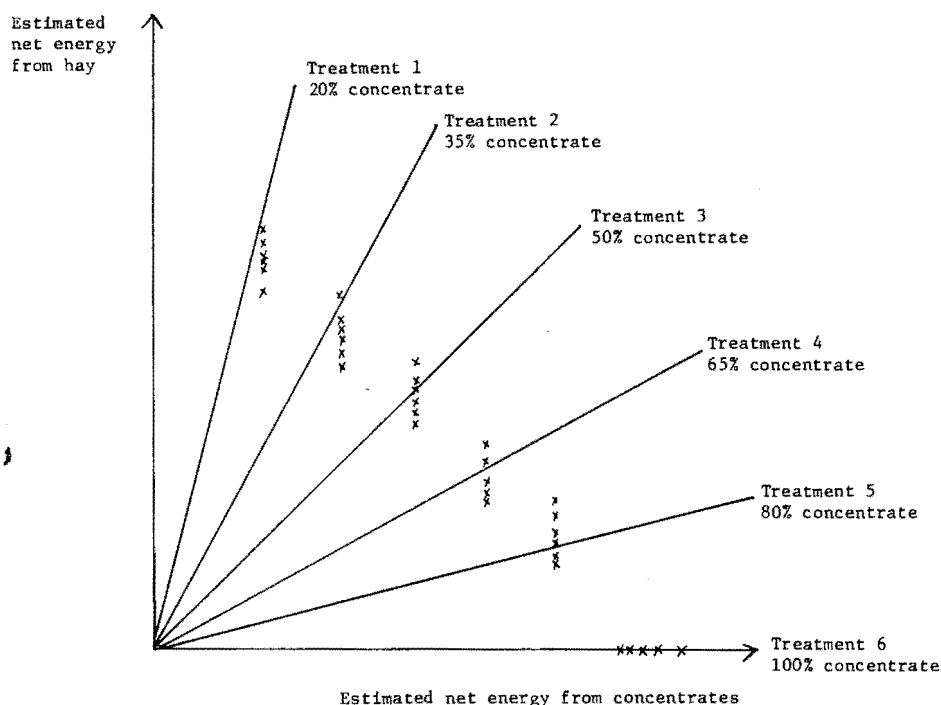

Fig. 3. Points showing hypothetical scatter of observations for 1967 trial, for cows of identical estimated energy requirements.

thetical group of cows of identical estimated energy requirements.

The cows were milked twice daily, yields recorded and samples taken for weekly composite analyses for solids and fat. The animals were weighed daily prior to the morning milking, and weekly means recorded. Feed intake, milk, and butterfat were all averaged on a weekly basis in the analysis of the data. Records were kept for the full lactation, but only the first 35 weeks were used, giving 28 weeks in the trial period following the 7-week pre-trial period.

The following variables are defined:

$T_i, i = l, \ldots, 28 =$ Time, in weeks from the end of the pre-trail period.

$A_{1j}, j = l, \ldots, 26 =$ Mean weekly milk yield over 7 pre-trial weeks, for the jth cow.

$A_{2j}, j = l, \ldots, 26 =$ Mean weekly milk yield over last 4 weeks of pre-trial period, for the jth cow.

$A_{3j}, j = l, \ldots, 26 =$ Mean weekly fat-corrected milk over 7 pre-trial weeks, for the jth cow.

$A_{4j}, j = l, \ldots, 26 =$ Mean weekly fat-corrected milk over last 4 weeks of pre-trial period, for the jth cow.

$W_{0j} =$ Liveweight of jth cow at the beginning of the trial period (in pounds).

$W_{ij} =$ Liveweight of jth cow in the ith week of the trial period (in pounds).

$\triangle W_{ij} = W_{ij} - W_{(i-l)j} =$ Change in liveweight from the previous period (in pounds).

$C_{ij} =$ Concentrate consumed by jth cow in the ith period (pounds per week).

$H_{ij} =$ Hay consumed by jth cow in the ith period (pounds per week).

$M_{ij} =$ Milk yield of jth cow in the ith period (pounds per week).

$FCM_{ij} =$ Fat-corrected milk yield of the jth cow in the ith period (pounds per week).

In selecting a measure of productive potential from the four alternative ability (A) variables defined, the simple correlation coefficients between milk output in the pre-trial period (the ability index) and total milk output over the total trial period were highest for A_2 and A_3 (0.56 and 0.60, respectively). Variables A_2 and A_3 were therefore selected as indexes of differences in inherent production capability among cows.

Determinants of the MVIL

The general form used to describe the MVIL relationship is a single regression equation with hay as the dependent variable. While this is unobjectionable for the observations on the first five treatments, inclusion of the all-concentrate group is questionable since the "dependent" variable in this case is fixed at a zero level. Regressions were therefore obtained both including and excluding the 100 per cent concentrate group.

A basic hypothesis is that liveweight and ability are the major determinants of the MVIL at any point in time. Equations which include time as an explicit variable are used to measure shifts in the relationship through the lactation. Previous studies have suggested that a quadratic form generally describes the relation between hay intake and concentrate.

Tables 1 and 2 present, respectively, the estimated maximum voluntary in-

TABLE 1

MAXIMUM VOLUNTARY INTAKE EQUATIONS WITH HAY INTAKE (POUNDS PER WEEK) AS THE DEPENDENT VARIABLE (BASED ON ALL OBSERVATIONS)

Equation		Constant term (k)	Independent variables								R^2
			Concentrate		Ability	Liveweight	Time	Interactions			
Number	Description		(C)	(C^2)	(A_2)	(W)	(T)	(CA_2)	(CW)	(CT)	
8a.........	Quadratic	105**	−0.658**	−0.00247**	0.246**	0.0477**					0.780
9a.........	Quadratic, with interactions	92.1**	−0.544†	−0.00249**	0.277**	0.0640**		0.000219	−0.000157		0.780
10a.........	Quadratic, with interactions and time	54.8†	−0.673*	−0.00176**	0.220**	0.121**	−1.35*	0.000503	−0.000182	−0.00816**	0.828
Units of measurement of variables		lbs/wk	lb/wk	—	lb/wk	lb	wk	—	—	—	—
Mean values of variables		—	96.2	11,900	317	1,190	14.5	30,800	114,000	1,370	—

** = significant at 1 per cent level.
 * = significant at 5 per cent level.
 † = significant at 10 per cent level.

TABLE 2

MAXIMUM VOLUNTARY INTAKE EQUATIONS WITH HAY INTAKE (POUNDS PER WEEK) AS THE DEPENDENT VARIABLE (EXCLUDING 100% CONCENTRATE OBSERVATIONS)

Equation		Constant term (k)	Independent variables								R^2
			Concentrate		Ability	Liveweight	Time	Interactions			
Number	Description		(C)	(C^2)	(A_2)	(W)	(T)	(CA_2)	(CW)	(CT)	
8b.........	Quadratic	136**	−0.528**	−0.00162†	0.214**	0.0203*					0.515
9b.........	Quadratic, with interactions	6.92	1.17**	−0.000134	0.364**	0.0959**		−0.00249**	−0.000949**		0.529
10b.........	Quadratic, with interactions and time	−39.2	1.11**	−0.00400	0.294**	0.167**	−0.641†	−0.00112†	−0.00123**	−0.0165**	0.663
Units of measurement of variables		lb/wk	lb/wk	—	lb/wk	lb	wk	—	—	—	—
Mean values of variables		—	80.0	7,711	318	1,210	14.5	25,900	97,600	1,140	—

** = significant at 1 per cent level.
 * = significant at 5 per cent level.
 † = significant at 10 per cent level.

take equations for the case where all observations are included, and where the observations on the cow fed 100 per cent concentrate are excluded. As hypothesized, the main effects of liveweight and ability are significant in every case, and show that hay intake is positively related to size and inherent production capacity of the cow.

When all observations are included (table 1) the MVIL is strongly curvilinear, as indicated by the significant quadratic term for concentrate (C^2). When the 100 per cent concentrate observations are excluded (table 2) the quadratic concentrate term is smaller in absolute magnitude, but is still significant at the 10 per cent level.

The addition of time (T) and concentrate-time interaction (CT) to the quadratic equations increases the coefficient of determination significantly for both sets of observations (tables 1 and 2). As expected, the time variable is negative, showing that appetite and feed intake diminish during the lactation. The negative CT interaction term shows that the reduction in feed intake over time is more pronounced at higher levels of concentrate in the ration.

Figures 4, 5, and 6 present graphically some of the more important relationships derived from the equations in tables 1 and 2. Figure 4 shows the position of the MVIL over the entire lactation when calculated from all observations (equation 8a) or from all observations except the 100 per cent concentrate levels (equation 8b). (The dashed lines show the MVIL equations extended beyond the range of the observations in each case.) It appears that inclusion of the 100 per cent concentrate observations may distort the shape of the MVIL over more usual ranges of concentrates fed.

Figure 5 shows the modest effects of different levels of liveweight (W) and ability (A_2) on the position of the MVIL (from equation 10b where 100 per cent concentrate observations are excluded). The levels of W and A_2 plotted are plus and minus one standard deviation from the means of the variables owing to the negative interaction terms with concentrate (CA_2) and (CW) the slopes of the MVIL change with changes in W and A_2.

Figure 6 shows the position of the MVIL at different stages of the lactation. The shift through time is substantial and the slope of the line becomes steeper as a result of the significant negative interaction term (CT). From figures 5 and 6 it appears that time (T) is a more important shifter of the MVIL than are ability (A_2) or liveweight (W). Stage of lactation, then, will obviously be a critical variable in developing feeding recommendations.

Determinants of milk response

The appropriate statistical model of milk response from MVIL feeding is open to debate. Three alternative formulations are examined in this section: (a) a single equation in which milk output is considered a function of hay, concentrate, and other variables (a conventional production function); (b) a single equation in which milk output is considered a function of estimated net energy (to be used in conjunction with an MVIL equation in a recursive system); and (c) a simultaneous equations system in which hay intake and milk output are jointly determined. Properties and problems related to each system are discussed along with the empirical results.

There is in addition a general problem of measurement associated with the output variable, milk production, owing to the variation in fat content. The butterfat percentage of milk decreases when an all-concentrate ration is fed. For this and other reasons, it is more meaningful to standardize output by expressing it in terms of fat-cor-

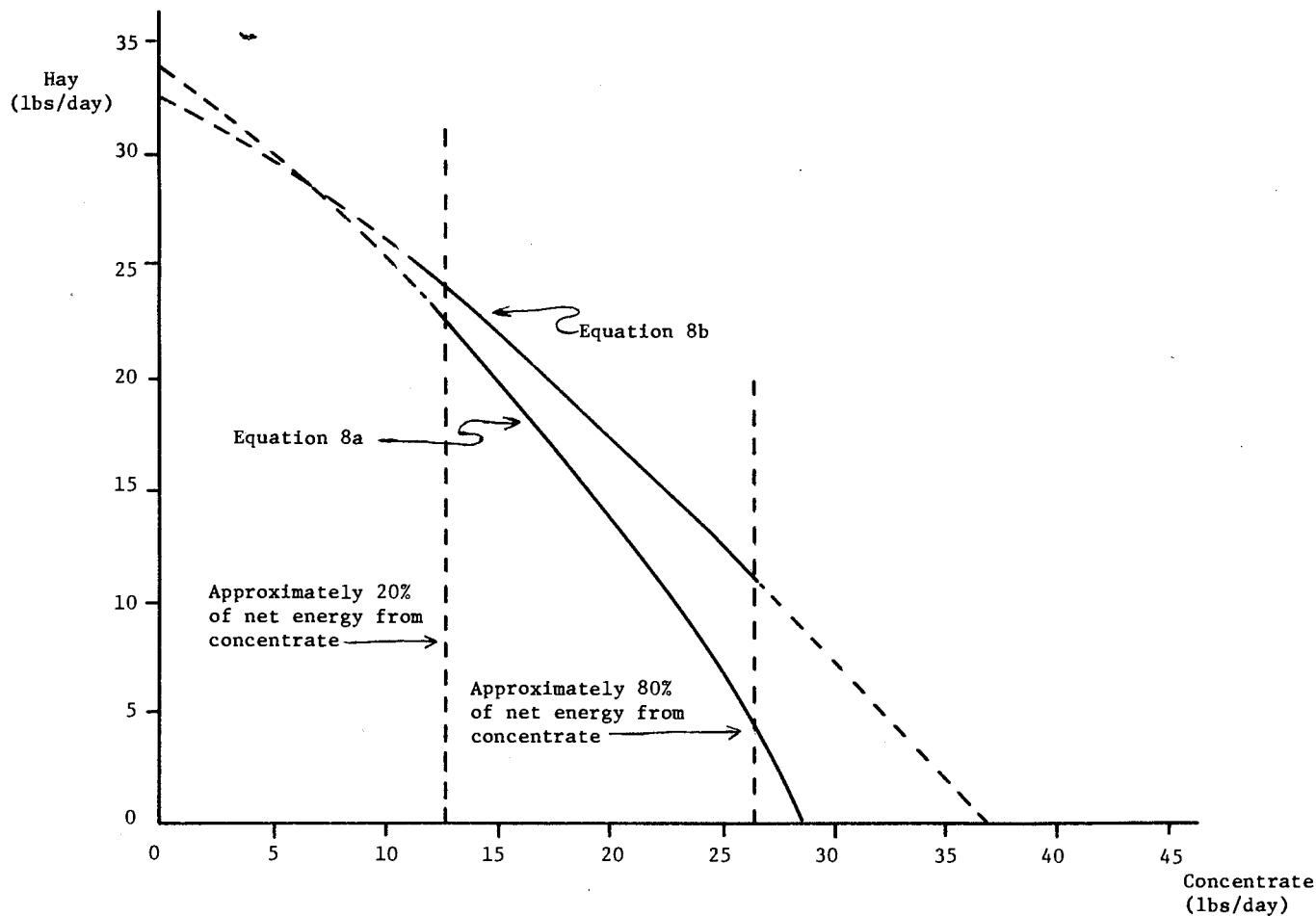Fig. 4. Maximum voluntary intake lines from equations 8a and 8b, with ability and weight fixed at mean levels.
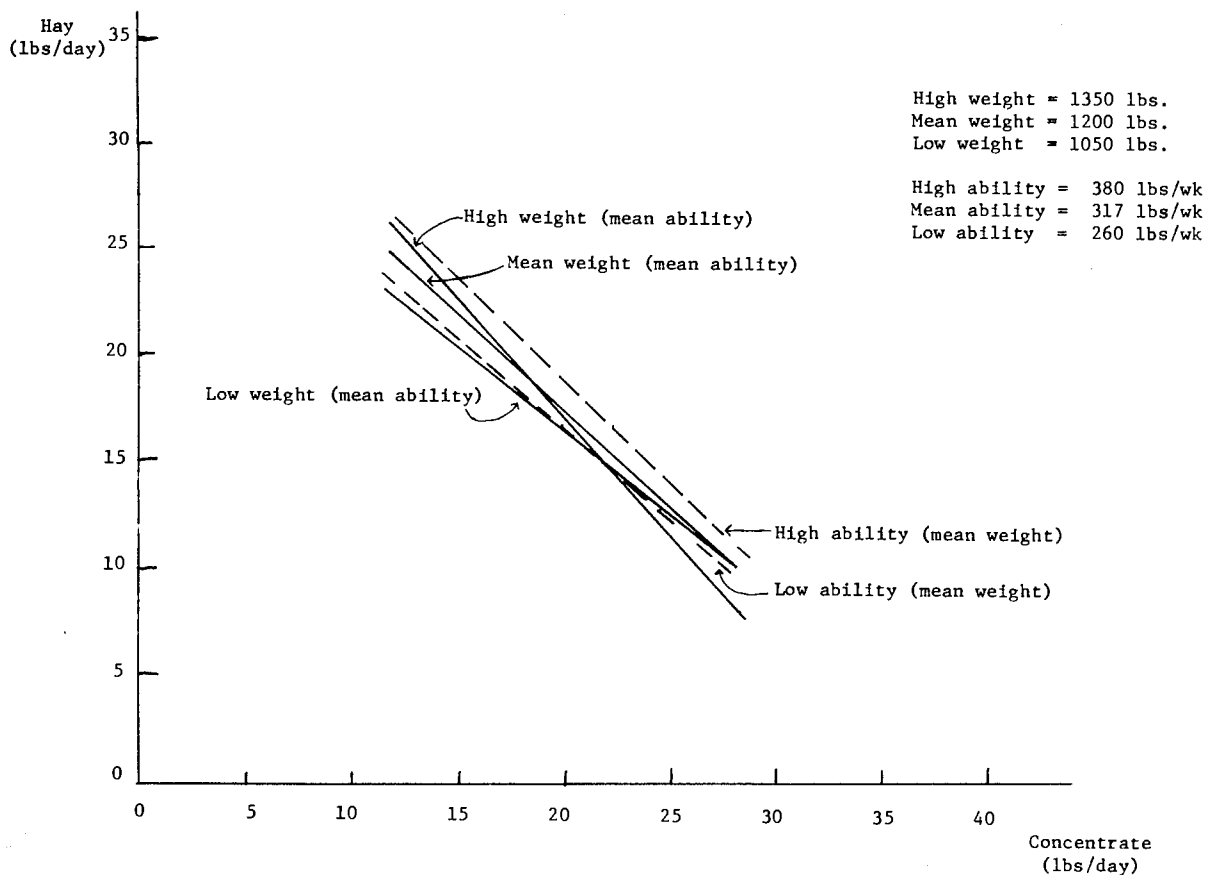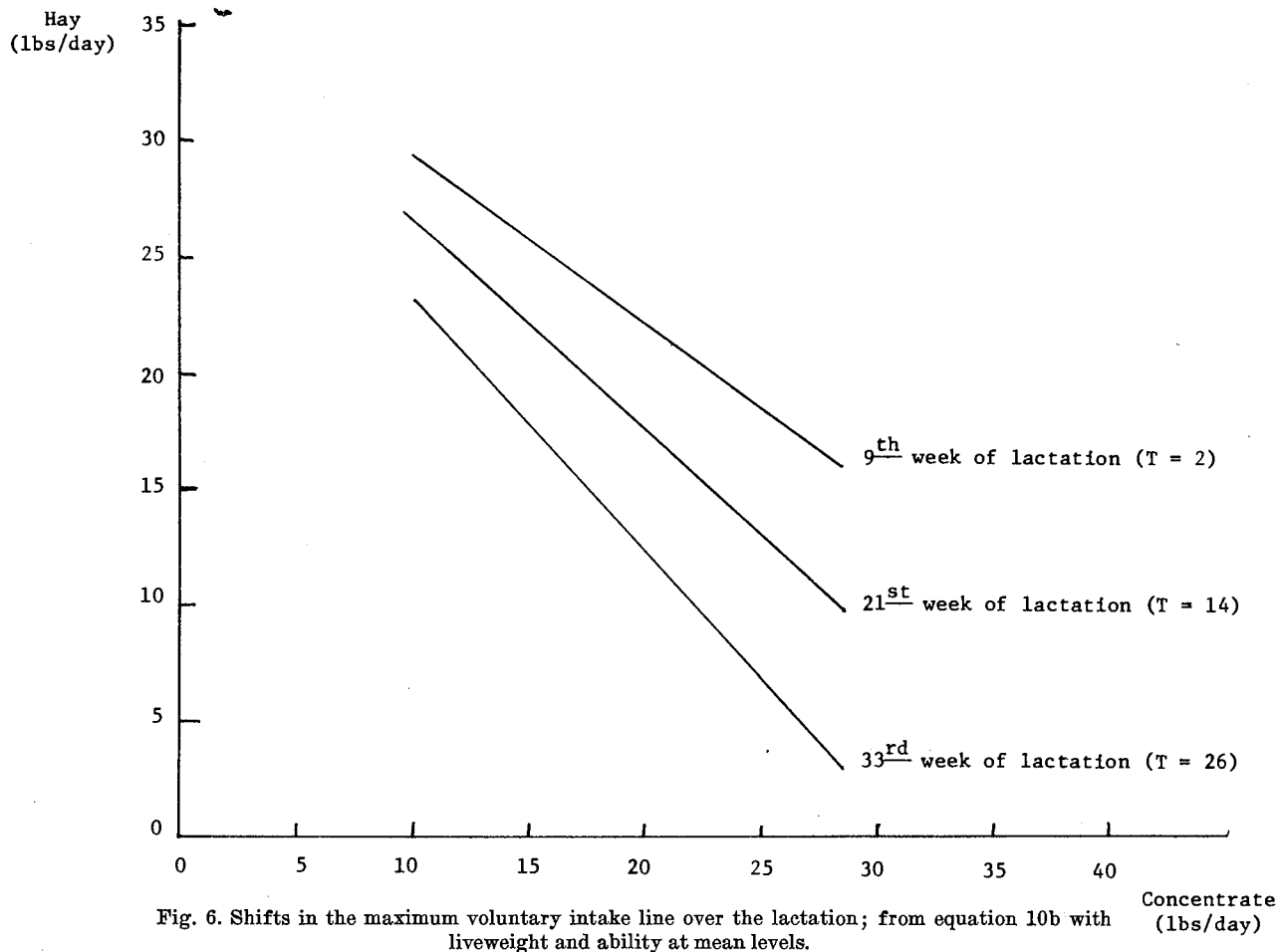
Fig. 5. Maximum voluntary intake lines from equation 10b, showing effects of changes in live-weight (W) and ability (A_2); twenty-first week of lactation (T = 14).

Fig. 6. Shifts in the maximum voluntary intake line over the lactation; from equation 10b with liveweight and ability at mean levels.

rected milk. The standard fat-correction procedure has limitations both from the standpoint of the basic production relationships derived and in the context of the economic problem of optimal feeding. Paris, *et al.* [1970] argue that the milk standardization procedure obscures fundamentally different production relationships for each of the main two components of milk—skim milk and butterfat. Their work suggests the possibility that conventional convex-to-the-origin isoquants for standardized milk may be simply an average of isoquants of completely different shapes for each of the components of milk. Since milk prices are often calculated on the basis of the separate components of milk, the standardization also obscures information required for determination of optimal feeding practices. For example, the milk price to producers in California is calculated from pounds of milk fat, nonfat solids, and fluid carrier separately, rather than on the basis of a fat percentage correction. Although the 4 per cent fat-correction formula implies a rate of substitution between milk and butterfat which is close to the present California price ratio between fluid carrier plus nonfat solids and milkfat, these prices are not immutable. The proportion of the total price accounted for by the milkfat component has in fact dropped over time, and with an increasing household consumption of low-fat milk a further fall in the butterfat price component may be forseeable.

Basic research should ideally attempt to establish production relationships which are applicable for economic analysis under any pricing system. Thus, it appears desirable that future research follow the lead of Paris *et al.* [1970] in attempting to specify production functions by milk components such as butterfat and solids-not-fat. As a partial test for the possible influence

of the fat-correction formula, production relationships in this study are estimated for uncorrected milk as well as for 4 per cent fat-corrected milk.

The response surface with hay and concentrate as independent feed variables. A general functional relationship between milk output and the individual feed inputs can be obtained from the experimental data, but this milk response surface has a restricted interpretation when derived solely from observations on cows fed at the MVIL. Since the scatter of observations is restricted to a narrow range in the vicinity of selected points on the MVIL, extrapolation to other regions is unwarranted. In particular, the shapes of isoquants and the nature of substitution rates between hay and concentrate cannot be implied from this function. Also, the function is estimated from ex-post observations of hay consumption. When the resulting milk response surface is used predictively in determining the optimum concentrate levels in a system involving free choice hay, the actual hay intake is unknown. Predicted hay intake rather than actual hay consumed must inevitably form the basis for decisions in practice. In this respect, the apparent reliability of the response surface estimated from ex-post hay intake is deceptive, and statistical measures such as the coefficient of determination are misleading.

Table 3 shows empirical results for alternative specifications of this model. Equations 11, 12 and 13 use uncorrected milk as the dependent variable. Although the R^2 value of the relationship is not raised dramatically by addition of quadratic and interaction terms, the reductions in sum of squared residuals between equations 11 and 12 and between 12 and 13 are significant at the 1 per cent level according to the F-tests. In addition, all of the second-order terms are statistically significant except the C^2 term in equation 12. The un-

TABLE 3

MILK RESPONSE SURFACE: EQUATIONS WITH HAY AND CONCENTRATE AS INDEPENDENT VARIABLES, BASED ON ALL OBSERVATIONS[a]

| Equation | | Constant term (k) | Independent variables | | | | | | | | | | | | R^2 |
Number	Description		(C)	(H)	(A_2)	(T)	(C^2)	(H^2)	(T^2)	(CH)	(CT)	(HT)	(CA_2)	(HA_2)	
11........	Uncorrected milk, linear	−69.5**	+0.871**	+0.513**	+0.638**	−3.56**									0.720
12........	Uncorrected milk, quadratic	−130†	+1.59*	+1.77**	+0.658**	−12.8**	−0.00157	−0.00383**	+0.0913**	−0.00564**	+0.0304**	+0.0248**			0.754
13........	Uncorrected milk, with ability interactions	+154†	+0.820	+0.950†	−0.594**	−12.6**	−0.00528**	−0.00436**	+0.0967**	−0.00935**	+0.0273**	+0.0240**	+0.00663**	+0.00431**	0.767
14........	Fat corrected milk, quadratic	−4.85	+0.826†	+0.743*	+0.404†	−9.26**	−0.00142	−0.000344	+0.0925**	−0.00244†	+0.0197**	+0.0124**			0.767
Mean values of variables		—	96.2	147	317	14.5	11,900	27,000	276	11,000	13,700	2,080	30,800	47,100	—

[a] H = hay in lb./wk. Other variables and units of measure as defined in table 1.
** = significant at 1 per cent probability.
* = significant at 5 per cent.
† = significant at 10 per cent.

expected negative sign of ability (A_2) in equation 13 is outweighed by the positive interaction effects of ability with the two feed inputs $(CA_2$ and $HA_2)$. The main effects of hay and concentrate in this equation must logically be retained, even though they are not highly significant.

Comparison of equations 12 and 14 for uncorrected and fat-corrected milk show that the individual regression coefficients have the same signs, although their absolute and relative magnitudes vary somewhat between the two equations. More detailed work remains to be done in considering milk output as a multiple product in terms of its basic components.

Although the isoquants derived from these equations would be misleading if extended over the entire production surface, they do give independent estimates of the substitution rates between feeds in the neighborhood of the MVIL. A comparison between the substitution rates from equation 13 and results based on a fixed net energy value for feeds is made in a later section.

Response curves with estimated net energy as an independent feed variable. Since there is only one genuinely independent feed variable in this particular experiment, an alternative approach is to combine the two feed inputs (concentrate and hay) into an estimated total energy variable. The use of a single net-energy variable (E_{ij}) in deriving response curves implies milk isoquants with a slope equal to the ratio of the standard net-energy values of the feeds. E_{ij} is thus a linear combination of the individual feeds consumed, where the energy values for individual feeds are derived from Morrison's standards, modified by more recent estimates of the Animal Science

Department at Davis. The energy variable is defined as follows:

$$E_{ij} = .764\,C_{ij} + .415\,H_{ij},$$

where C_{ij} and H_{ij} are pounds of concentrate and hay consumed, respectively, by the jth cow in the ith period, and E_{ij} is net energy calculated in megacalories.[4]

Table 4 shows the statistical results from alternative milk response equations, with energy as an independent variable. Alternative definitions of the dependent variable (uncorrected and 4 per cent fat-corrected milk) and of the observation set (all observations, and observations excluding the cows on 100 per cent concentrate) were attempted. In all cases the equations show a diminishing marginal productivity to higher energy levels (a positive linear term E and a negative quadratic term E^2). Both energy variables are consistently significant statistically except in equations 15 and 16 where uncorrected milk is used as the dependent variable. Figure 7 shows the marked difference in response to increasing levels of energy intake with and without the fat correction (equations 16 and 17a). The severe butterfat depression experienced at high concentrate levels in the experiment accounts for the sharp divergence of the plotted functions. However, even when the observations on cows fed 100 per cent concentrate were excluded, the milk-response curves for FCM were much the same shape as those shown for FCM in figure 7.

In every equation in table 4, cow ability is shown to be an important variable shifting the milk-energy response curve. The A variable is highly significant and positive in all equations in table 4 except equations 16 and 16C in which A is significant but negative; however, in the latter case the energy-

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[4] The coefficient 0.764 was derived by multiplying the individual feed components of the concentrate ration shown on page 8 by the standard net energy values of each feed component. The coefficient 0.415 is the net energy value for alfalfa hay.
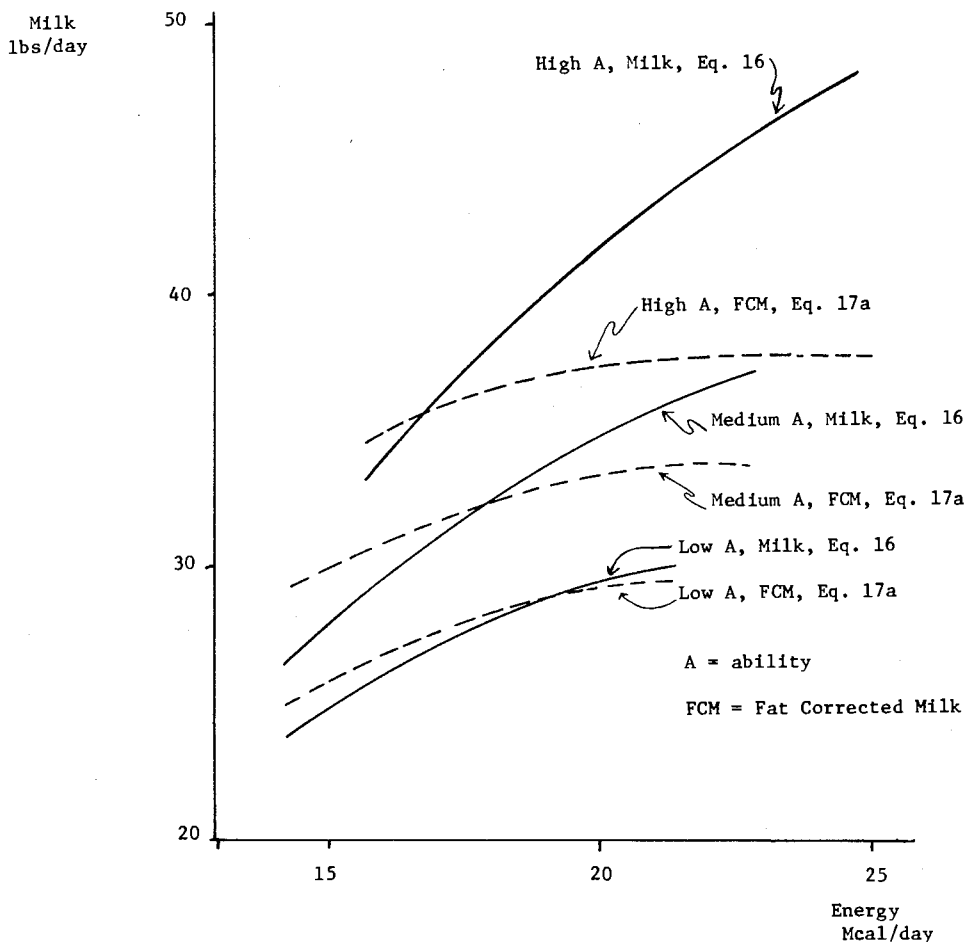
Fig. 7. Comparison of uncorrected milk and 4 percent fat-corrected milk response to energy.

ability interaction term (EA) includes most of the ability effect. Figure 7 shows the shifts in the response curves for three ability levels (mean ability and plus or minus one standard deviation in ability).

The time variable also shifts the response function strongly over the period of the lactation; T and T^2 are significant at the 1 per cent or 5 per cent level in all equations in table 4. The negative sign on T and the positive sign on T^2 in the table indicate a decreasing rate of shift over time. The

time variables in the various equations thus contain implicit measures of persistency in milk production over the lactation which vary somewhat from equation to equation. A direct measure of persistency was obtained by aggregating milk observations by months, and expressing milk in one month as a function of milk in the previous month. The average decline per month for all cows was 6.93 per cent, which agrees closely with established estimates of persistency.

Since the experiment was conducted

TABLE 4

MILK RESPONSE WITH ENERGY AS AN INDEPENDENT VARIABLE

	Equation		Constant term (k)	Independent variables[2]								
Number	Description	Definition of dependent variable and observation set		(E)	(A)	(T)	(E^2)	(T^2)	(ET)	(EA)	(W_o)	(R^2)
15.......	With time interaction	Uncorrected milk, all observations	15.9	0.709	0.637**	−11.5**	−0.000410	0.0865**	0.0414**			0.729
16.......	With time and ability interactions	Uncorrected milk, all observations	220**	0.219	−0.540**	−10.5**	−0.00864**	0.0845**	0.0330**	0.0890**		0.747
16c.......	With time and ability interactions, including initial weight	Uncorrected milk, all observations	161**	2.46**	−0.420**	−6.6**	−0.0140**	0.0758**	0.0062	0.0082**	−0.135**	0.794
Mean values of variables			—	135	317	14.5	18,600	276	1,910	43,100	1,080	—
17a.......	Without interaction	4 per cent fat corrected milk, all observations	−103**	2.67**	0.574**	−5.06**	−0.00823**	0.0598**				0.681
18a.......	With time and ability interaction	4 per cent fat corrected milk, all observations	−39.1	2.01**	0.506**	−6.81**	−0.00687**	0.0666**	0.0118	0.0005		0.682
17c.......	With time and ability interaction, including initial weight	4 per cent fat corrected milk, all observations	−78.3*	2.88**	0.587**	−5.08**	−0.00884**	0.0611**			−0.0424**	0.690
Mean values of variables			—	135	273	14.5	18,600	276	1,910	37,100	1,080	—
17b.......	Without interaction	4 per cent fat corrected milk, excluding 100 per cent conc. observations	−77.5*	2.24**	0.601**	−4.62**	−0.00675**	0.0445*				0.761
18b.......	With time and ability interaction	4 per cent fat corrected milk, excluding 100 per cent conc. observations	−18.9	1.50*	0.624**	−6.55**	−0.00445†	0.0508**	0.0134†	−0.00018		0.762
Mean values of variables			—	133	276	14.5	18,200	276	1,890	37,200	—	—

[a] A defined as A_2 in equations 15 and 16, and as A_3, in equations 17a, 18a, 17b, and 18b. E = estimated net energy in Mcal per week. Other variables and units of measure as defined in Table 1.
** = significant at 1 per cent level.
* = significant at 5 per cent level.
† = significant at 10 per cent level.

with heifers, liveweight gains of up to 300 pounds were experienced during the lactation. There is a logical basis for considering liveweight gain as a variable affecting milk output, but a satisfactory specification of the function was not found. Recorded liveweights showed great fluctuation from week to week, apparently including a large random component since change in weight ($\triangle W$) was not a significant variable when added as a regressor.[5] Total liveweight per period (W) was strongly correlated with time, confounding the two effects when both were included. A redefinition of liveweight which eliminated the monthly fluctuations from the upward trend of liveweight also failed to improve on the result obtained with unadjusted liveweight. Nevertheless, there is a significant independent effect of body weight on milk production, of which initial liveweight was found to be a satisfactory expression. Equations 16C and 17C show that initial body weight is a significant variable whose addition as a regressor raises the R^2 substantially, in the case of uncorrected milk. The negative sign of W_o in these equations implies that larger animals require more energy for maintenance and therefore that a given energy level will provide less milk. Of course, the larger animal has a higher MVIL which permits a greater feed intake and higher production potential, other things being equal.

Comparison of milk response curves using hay and concentrate as separate variables versus net energy as a single feed variable. A comparison can now be made between the milk response curves where hay and concentrate are considered as separate independent variables and where hay and concentrate are combined into a single variable on the basis of standard feed energy values. Equations 13 and 16 are comparable in the

sense that both measure milk response to feed inputs (or energy) with time, ability and interaction terms. Although equation 13 uses 12 independent variables while equation 16 uses only 7, the coefficient of determination of the former is only slightly higher (0.767 versus 0.747) indicating that the assumption of linear substitution between hay energy and concentrate energy only slightly impairs the explanation of milk output in the region of the MVIL.

The relationship between these two equations may be further demonstrated by converting the individual feed inputs of equation 13 into Mcals, using standard energy coefficients. The response of milk to increasing levels of total energy is then calculated where the feeds are in the fixed proportions of each individual treatment (fig. 8). The relevant portions of this family of curves form an envelope response curve at MVIL feeding: all other segments of the curves are hypothetical extrapolations above or below the MVIL. The envelope-response curve is thus derived from an equation which allows for separate hay and concentrate energy effects, which are combined by standard energy coefficients. The comparable response curve from the single energy variable (equation 16) is then compared with this composite curve. The close agreement of the two response curves in figure 8 suggests that an aggregated energy intake variable can be substituted for the separate feed variables in accurately describing milk response at the MVIL. Little can be said about the comparisons at non-MVIL levels since no observations are available in that region from this particular experiment.

Log-linear functions. Previous studies have not established clearly whether milk response to energy is better described by a quadratic or by a long-lin-
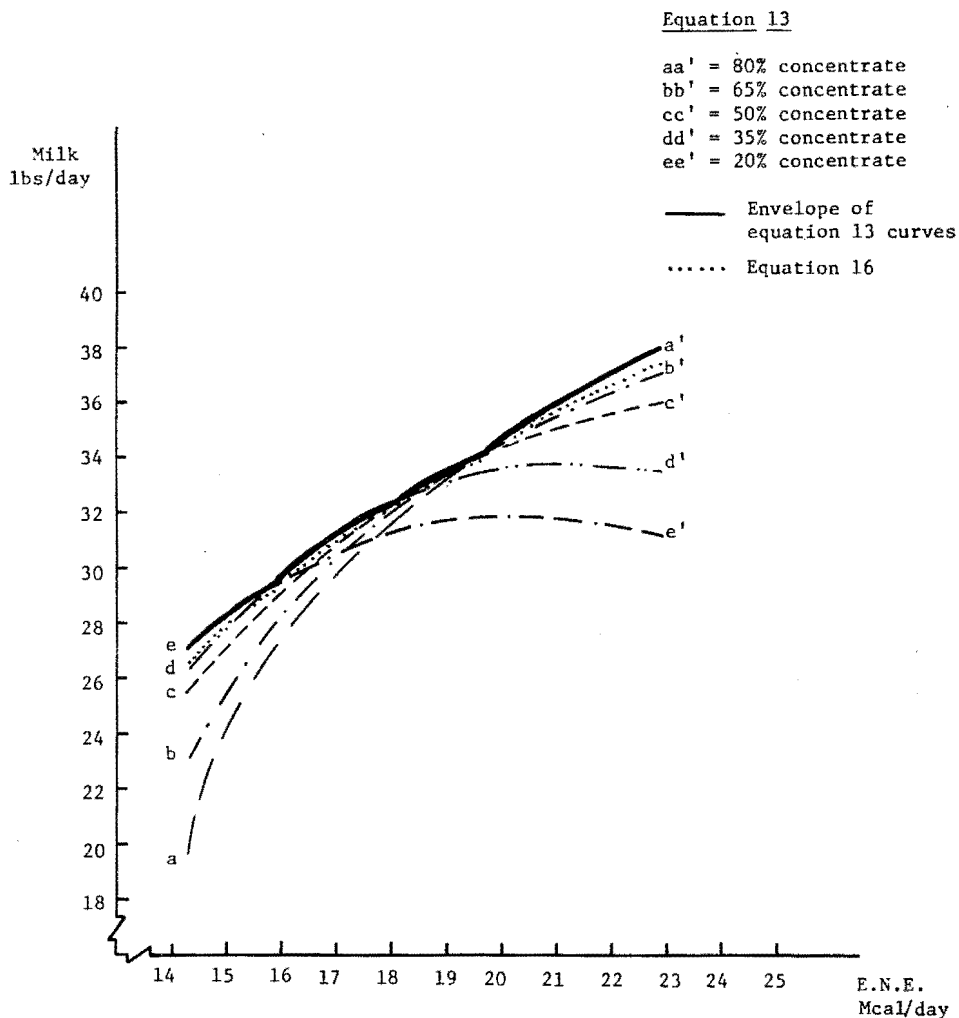

Fig. 8. Comparison of milk response to energy with milk response to various hay-concentrate proportions converted to energy according to standard energy coefficients (T = 14).

ear function. While the exponential form has the disadvantage that the response function is constrained to pass through the origin, the origin may be redefined to correspond to nutritional logic.[6] Since the logical energy origin is at the body maintenance requirement, the origin was defined as 9 Mcal per day—the estimated body maintenance of 7.5 Mcal per day, plus an additional 1.5 Mcal for body growth of first-calf heifers. Comparisons of coefficients of determination showed that shifting the energy origin from zero to 9 Mcal per day markedly improved the goodness of fit. However, the R^2 values from a formulation including the log values of energy, ability, time and weight were considerably below those obtained earlier from the quadratic formulations. Nevertheless, the log functions allow comparison with earlier work such as Blaxter's [1962], as shown in figure 9. The general shapes and positions of the curves are quite similar when comparable ability levels are considered.

An alternative framework: simultaneous determination of hay intake and milk output. In the feeding system used in the experiment, concentrate was the predetermined variable. Since hay was fed *ad libitum*, neither hay intake nor total energy intake were directly controlled. Hence, to treat hay or total energy as independent variables, as in the single equation models, is logically less satisfactory than a system of two simultaneous equations in which hay intake and milk output are jointly determined dependent variables. From a nutritional point of view, it would be logical to also include liveweight gain as an endogenous variable along

with hay intake and milk output in a simultaneous system. Empirical results from such a three-equation model, however, were entirely unsatisfactory because of the large random element associated with liveweight changes noted earlier. Thus, the results below are limited to two-equation systems.

The following sets of variables were used: (a) endogenous variables—M and H; (b) exogenous variables—C, A, W, T, and appropriate second order and interaction terms. To simplify estimation, a just-identified system was formulated, requiring that one exogenous variable be excluded from each equation. Since prior results (fig. 5) showed little if any relationship between current liveweight and milk output, liveweight was excluded from the milk equation. However, the model does allow for an indirect effect of liveweight on milk production via the MVIL. The omission of an exogenous variable from the hay equation has little *a priori* justification. However, the effect of ability, although statistically significant, was seen earlier to have a minor impact on the MVIL; therefore, ability was eliminated from the hay equation.

The reduced form equations based on all observations were as follows:

$$M = 94.3^{**} + 0.680C^{**}$$
$$- 0.00196C^{2**} \qquad (19a)$$
$$+ 0.744A_2{}^{**} - 3.78T^*$$
$$- 0.645W^*$$
$$(R^2 = 0.672)$$

$$H = 74.7^{**} - 0.882C^{**}$$
$$- 0.00155C^{2**} \qquad (19b)$$
$$+ 0.262A_2{}^{**}$$
$$- 2.12T^{**} + 0.104W^{**}$$
$$(R^2 = 0.824).$$

---

[6] The same reasoning may in principle be applied to the other variables. A zero origin for the ability and weight coordinates is logically unobjectionable. However, for the time variable, the point $T = 0$ does have a real meaning, since this may be interpreted as 1 week before the trial begins, or the 6th week of lactation. The negative time exponent means that milk converges to infinity as T tends to zero. Provided the other coordinates are correctly defined, this logically unsatisfactory property of the function does not appear to impair the predictions given for values of T greater or equal to unity. There is no intuitive basis for defining the time origin, but a trial and error procedure in which the origin was defined successively as −10 and −15 revealed that coefficients of the other regressors remained stable with changes in the time origin, and that there was little improvement in the goodness of fit.
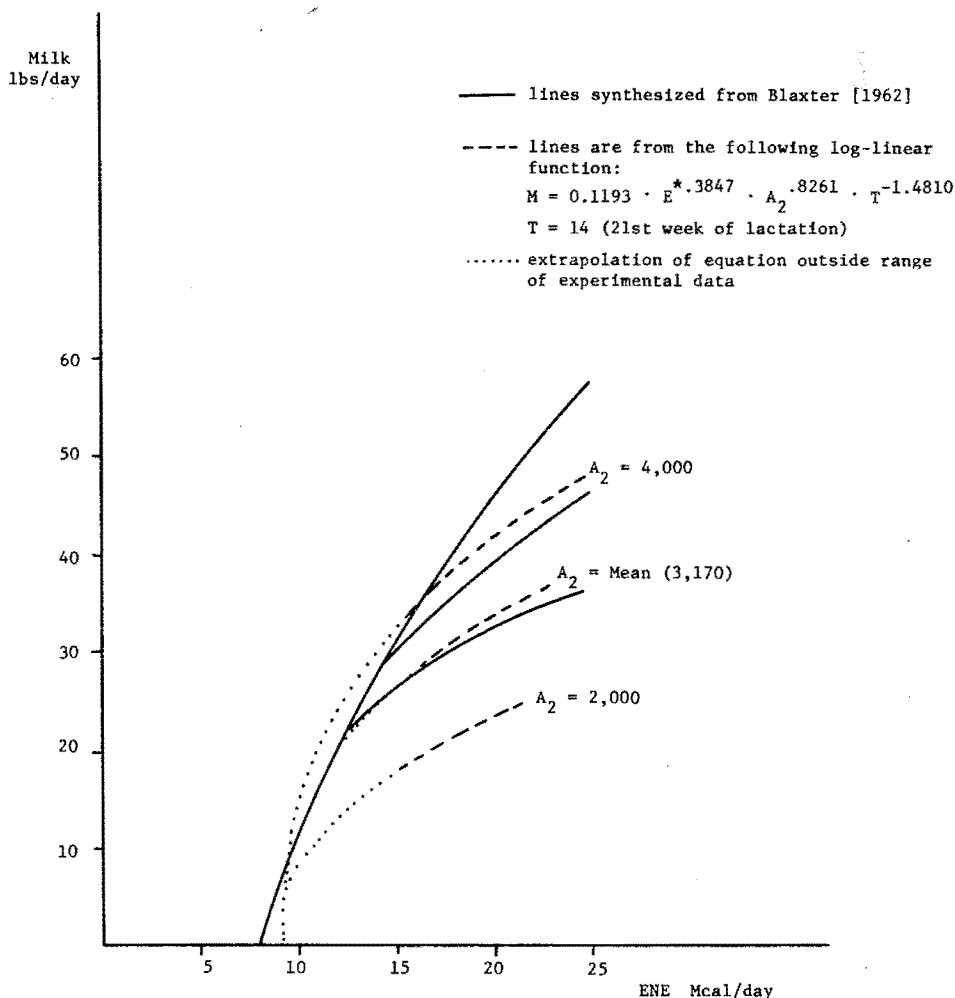
Fig. 9. Comparison of milk response to energy as synthesized from Blaxter [1962] and from log-linear function, University of California, Davis trial.

The resultant structural equations were then as follows:

$$M = 140.7 - 0.620H + 0.133C$$
$$- 0.00292C^2 + 0.925A_2 \quad (19c)$$
$$- 5.10T$$

$$H = 41.6 + 0.352M - 1.12C$$
$$- 0.000855C^2 \quad (19d)$$
$$- 0.790T + 0.133W.$$

When the 100 per cent concentrate group was excluded, the reduced forms were not significantly different from linear. The corresponding set of reduced form equations was:

$$M = 118.8** + 0.567C**$$
$$+ 0.732A_2** \quad (20a)$$
$$- 3.37T** - 0.0884W**$$
$$(R^2 = 0.776)$$

$$H = 41.6 + 0.352M - 1.12C$$
$$+ 0.236A_2** \quad (20b)$$
$$- 1.86T** + 0.0707W**$$
$$(R^2 = 0.625).$$

The structural forms were as follows:

$$M = 259.4 - 1.25H - 0.55C$$
$$- 5.70T + 1.03A_2 \qquad (20c)$$

$$H = 74.2 + 0.322M - 1.08C$$
$$- 0.776T + 0.0992W \qquad (20d).$$

The variables in the reduced form equations were highly significant and of expected sign (with the possible exception of liveweight, where the direction of causation is not clear). However, the derived structural forms contain several surprising coefficients. In both structural milk equations, 19c and 20c, hay has a negative sign; in the later equation concentrate also has a negative sign, contrary to expectations. Since the equations constitute a simultaneous system, however, a better test of "reasonableness" is the graphic comparison of predicted milk response with

previous single equation estimates as shown in figure 10. The two sets of milk responses are nearly parallel up to high energy levels. At high-energy levels the simultaneous system based on all observations (19a and 19b) shows more rapidly diminishing marginal productivity than the single equation system (16); the simultaneous system based on the observations excluding 100 per cent concentrate shows a linear response (extrapolated) and, therefore, no diminishing marginal productivity.

These comparisons suggest that even if the relationships are logically simultaneous, the recursive single equations method gives a close approximation to the results of a simultaneous model. It is still an open question which set of equations would yield better predictions in practice.

Milk Response with Restricted Roughage Intake

The experimental results analyzed above have been limited to MVIL feeding. It was argued that this conforms to widely-used commercial feeding practices. However, there are cases in which both roughage and concentrate levels are controlled and can be restricted to levels below the MVIL. Optimal feeding practices in such cases require knowledge of the complete production response surface. Heady, *et al.* [1956, 1964] have conducted several studies designed to estimate the complete surface.

*Production economics theory suggests that curvilinear relationships characterize production functions—diminishing marginal productivity of milk output to increased levels of feed (or energy) intake, and diminishing marginal rates of substitution between roughage and concentrate. The experimental results reported above support the considerable evidence available that there is diminishing marginal productivity of milk output to additional energy intake. However, past studies have

shown rather weak evidence for curvilinearity in milk isoquants over broad ranges of roughage-concentrate combinations. Curvilinearity has appeared to be marked only at extreme feed combinations where the observations are often non-existent or are suspect because of significant changes in body weight, gastric disorders, or changes in the composition of milk output toward lower fat content.

In view of the past evidence, three hypotheses relevant to the shape of milk isoquants are posed. The hypotheses are that, above a minimum level of hay intake (1.5 per cent of body weight) milk isoquants are (1) linear, (2) parallel, and (3) have a slope equal to the ratio of the standard net-energy values of the feeds. These hypotheses are tested below, using the results of feeding trials at the University of California at Davis.

Experimental design

The Department of Animal Science at Davis designed an experiment to evaluate the productive energy of hay
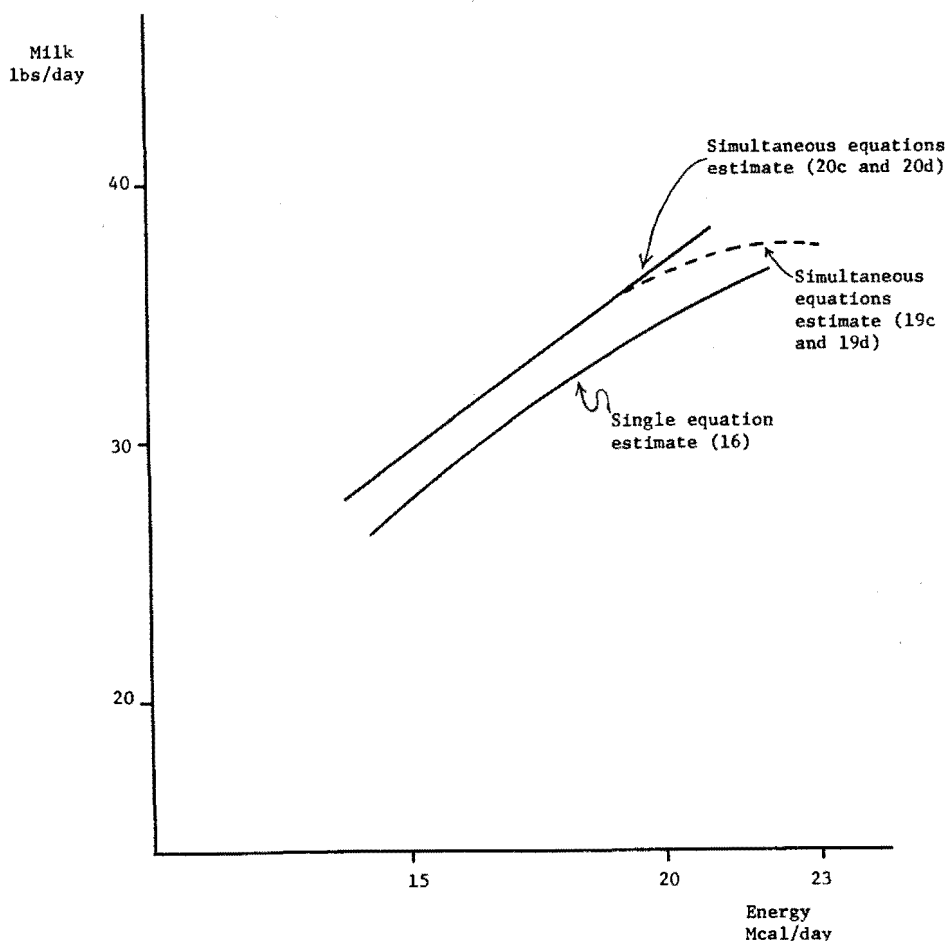
Fig. 10. Comparison of milk-energy response under single equation and simultaneous equations estimation (mean A, W, and T).

and concentrate. Although the range of observations in the region below the MVIL is perhaps too limited for the derivation of a complete isoquant map, the experiment does permit a test of the above hypotheses over a sizeable portion of the total surface. Laboratory evaluations were made of total energy for all feeds fed, thus permitting an ac-

curate comparison with relative feed values implicit in the isoquant relationships.

Three groups of eight first-calf Holsteins were each assigned to four treatments in a Latin-square change-over design [Patterson and Lucas, 1962]. The trial commenced 7 weeks after the beginning of the lactation; all cows

TREATMENTS

A_1, B_1, C_1 = Basal ration (full energy requirement)
                    70% hay; 30% barley

$\begin{cases} A_2 \\ A_3 \\ A_4 \end{cases}$ A_2 = Restricted basal (75% of full energy requirement)
A_3 = A_2 + barley, to full requirement
A_4 = A_2 + hay, to full requirement

$\begin{cases} B_2 \\ B_3 \\ B_4 \end{cases}$ B_2 = Basal hay + 85% of basal barley
B_3 = Basal hay + 70% of basal barley
B_4 = Basal hay + 55% of basal barley

$\begin{cases} C_2 \\ C_3 \\ C_4 \end{cases}$ C_2 = Basal barley + 85% of basal hay
C_3 = Basal barley + 70% of basal hay
C_4 = Basal barley + 55% of basal hay

EE' = Estimated energy requirement
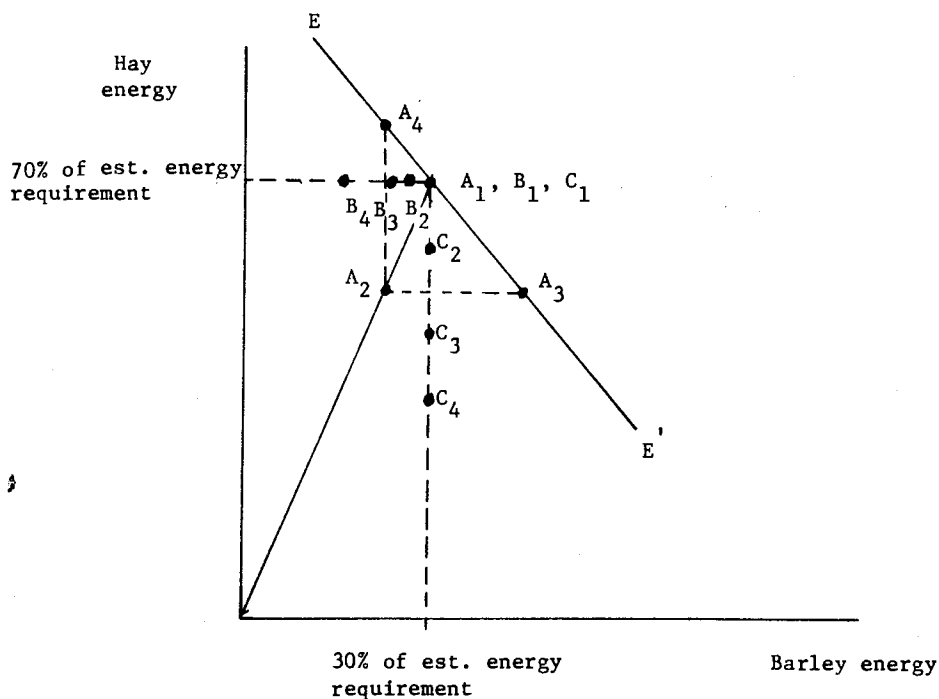
Fig. 11. Schematic treatment pattern for trial treatments.

being fed a high level of barley, with hay fed *ad libitum* in the pre-trial period. Figure 11 shows the treatments in each group. The alfalfa hay contained approximately 26 per cent crude fiber on a dry basis. Protein intake was adequate even at the restricted feed levels, but liveweight decreased significantly on certain treatments.

The 24-week trial period was divided into five treatment periods, each including a 28-day observation period. The first week following a change-over was allowed for adjustment, and was not included in the observations; thus, the carry-over effects were not statistically significant. The treatments assigned in the fourth period were extended to the fifth treatment period.

The cows were fed twice daily at milking. Concentrate was fed in the milking barn, and weighed amounts of hay were fed in individual stanchions after milking. Milk from each cow was weighed at each milking, and a sample withdrawn for inclusion in a weekly butterfat analysis. The cows were weighed one day in each week, and in addition on three successive days at the beginning and end of each treatment period. The data were aggregated for individual cows in each period, giving a total of 120 observations. Milk, hay and barley were measured in pounds per day, and the cow's liveweight in pounds. Time was defined in weeks, comparable to the MVIL experiment reported above.

Analytical techniques

In the experiment under consideration, the range of milk output is relatively restricted. Within this limited range, diminishing marginal productivity to increased levels of concentrate and hay are not likely to be apparent. If so, in a quadratic equation, the terms C^2 and H^2 would not differ significantly from zero. However, if the isoquants in this restricted range are curvilinear,

the interaction term CH would be positive. Thus, the test for linear and parallel isoquants requires that the terms C^2, H^2 and CH are not statistically different from zero.

Empirical results

Table 5 gives equations obtained for uncorrected milk and for 4 per cent fat-corrected milk. Equations 22a and 22b, linear in the feed inputs and quadratic in time, provide the basic equations. In both formulations, all variables are highly significant and with the expected signs. Equations 23a and 23b show that the added quadratic feed variables C^2 and H^2 alone are non-significant at the 10 per cent level. The R^2 value is scarcely increased, the F-tests showing that the reductions in sums of squared deviations is not significant at the 25 per cent level. Equations 24a and 24b add the CH interaction term to the quadratic equations 23a and 23b. The interaction term (CH) and the quadratic terms $(C^2$ and $H^2)$ are not significant, the R^2 values increase almost imperceptibly and the F-tests show no significant reduction in the sums of squared deviations compared to the equations linear in the feed variables (equations 22a and 22b). Therefore, the hypothesis that the isoquants are linear and parallel is not rejected.

Figure 12 contrasts the milk isoquants for the linear and quadratic equational forms and figure 13 those for milk and fat-corrected milk. The quadratic terms in the fat-corrected milk equation 23b, although not significant, would imply isoquants slightly concave to the origin, whereas the uncorrected milk isoquants would be slightly convex to the origin. Since the degree of curvilinearity is extremely slight and not statistically significant, limited inferences can be drawn. The work supports recent findings by Paris *et al.* [1970] that the shape of isoquants may be importantly affected by fat correction, but

TABLE 5

MILK RESPONSE EQUATIONS FROM UNIVERSITY OF CALIFORNIA DAVIS FEED TRIALS

Equation			Constant term (k)	Independent variables[a]							R^2
Number	Description	Definition of dependent variable		C	H	T	C^2	H^2	T^2	CH	
21a......	Linear	Uncorrected milk	3.81	1.54**	0.998**	−0.346**					0.709
22a......	T^2 included	Uncorrected milk	6.72	1.52**	1.01**	−0.997**			0.0250**		0.734
23a......	Quadratic (no interaction)	Uncorrected milk	6.65	1.75†	0.913†	−1.00**	−0.0142	0.00284	0.0253**		0.734
24a......	Quadratic (with interaction)	Uncorrected milk	1.15	2.54†	1.20†	−1.01**	−0.0194	−0.0038	0.0255**	−0.0408	0.736
Mean values of variables			—	7.59	16.99	13.00	61.29	302.64	219	128	—
21b......	Linear	4 per cent fat-corrected milk	2.09	1.54**	0.959**	−0.341**					0.746
22b......	T^2 included	4 per cent fat-corrected milk	4.90*	1.52**	0.971**	−0.970**			0.0242**		0.772
23b......	Quadratic (no interaction)	4 per cent fat-corrected milk	11.7*	0.979	0.410	−0.969**	0.0326	0.0161	0.0241**		0.775
24b......	Quadratic (with interaction)	4 per cent fat-corrected milk	13.56†	0.708	0.316	−0.968**	0.0343	0.0158	0.0240**	0.0136	0.775
Mean values of variables			—	7.59	16.99	13.00	61.29	302.64	219	128	—

[a] Variables and units of measure as defined in tables 1, 2, and 3.
** = significant at 1 per cent level.
* = significant at 5 per cent level.
† = significant at 10 per cent level.
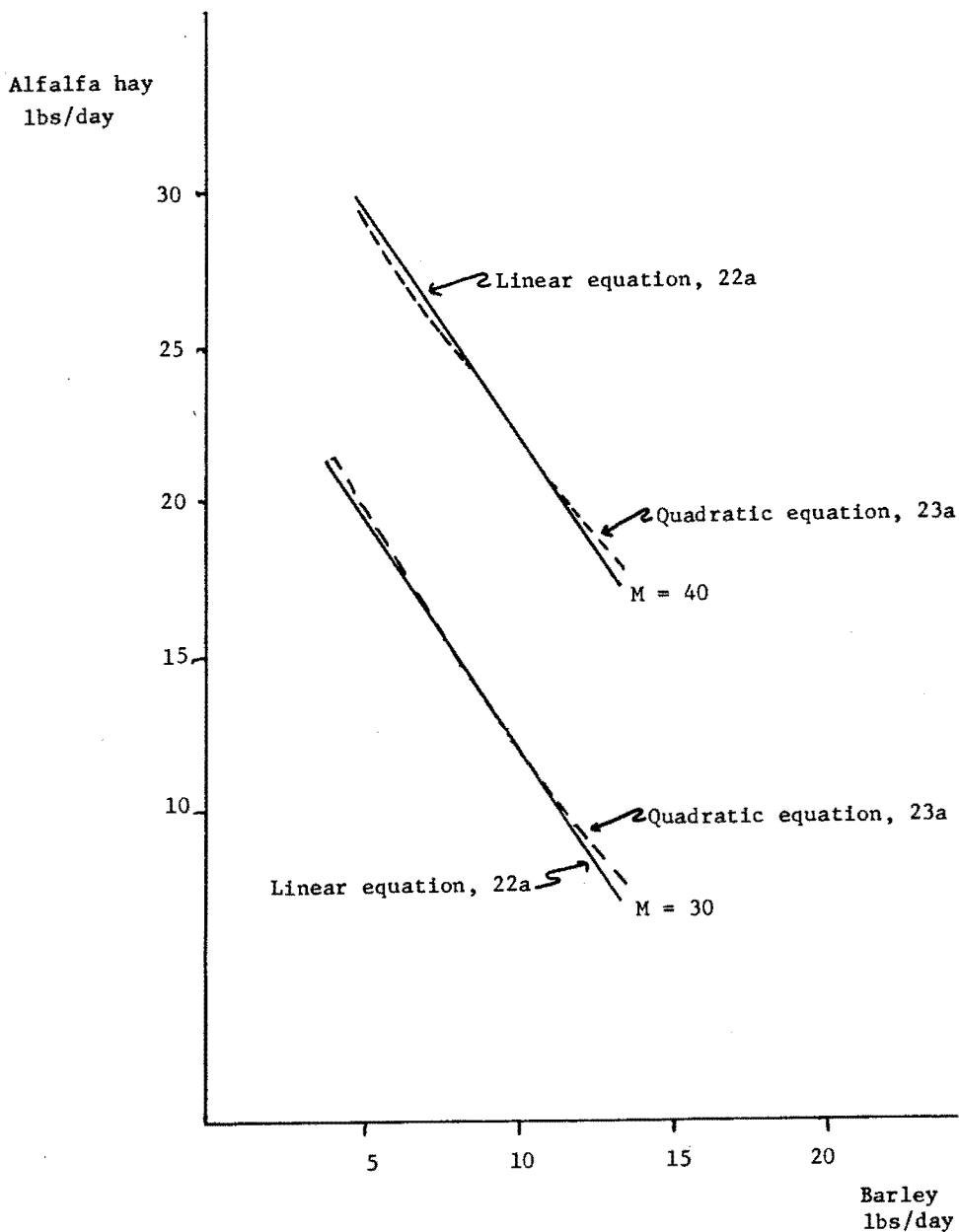
Fig. 12. Comparison of milk isoquants for linear and quadratic response functions $(T = 5)$.
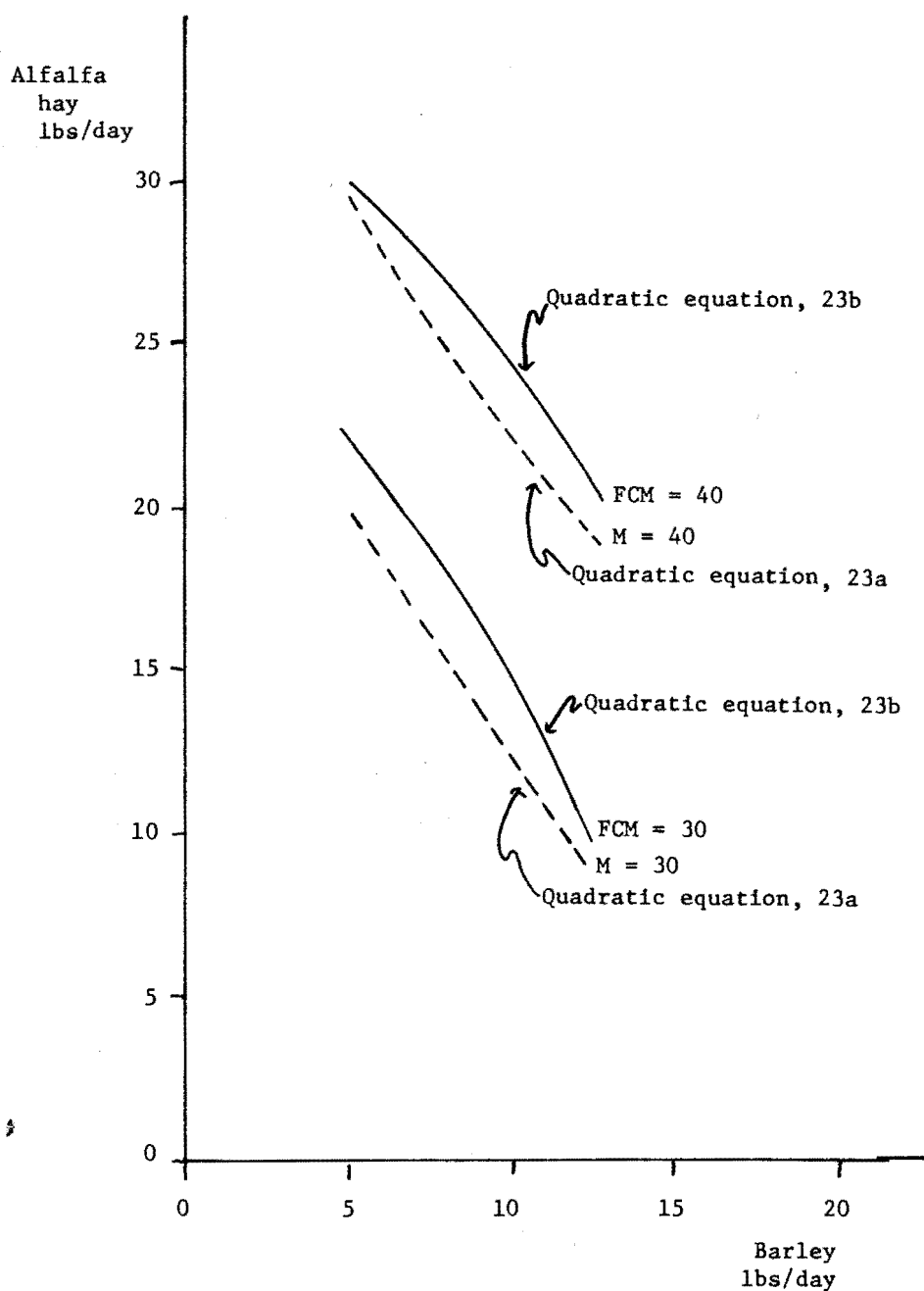
Fig. 13. Comparison of milk and fat-corrected isoquants from quadratic equations ($T = 5$).

our evidence differs in the sense that Paris *et al.* found uncorrected milk isoquants concave to the origin. More detailed work is apparently required if the components of milk are to be successfully treated as separate variables.

Table 6 compares the slopes of the isoquants obtained from milk and fat-corrected milk equations with the substitution ratios implied by (a) the revised Morrison standard energy values of barley and alfalfa hay and (b) the

total energy of the two feeds as evaluated in the laboratory. These results suggest that standard energy values are a good approximation to the slope of milk isoquants, at least over the range of feed inputs used in this experiment. In summary, the empirical tests support the three hypotheses proposed earlier: that isoquants are linear, parallel, and have slopes equal to the ratio of standard energy values for concentrate and hay.

TABLE 6

SLOPES OF ISOQUANTS FROM REGRESSION EQUATIONS AND RATIOS OF TOTAL ENERGY VALUES (C/H)

Source of Estimate	Milk	FCM
Equations 21a and 21b linear equation	1.54	1.60
Equations 22a and 22b with T^2	1.50	1.56
Equations 23a and 23b quadratic (at mean H and C)	1.52	1.56
Standard energy values (from Morrison) $H = 0.473$ Mcal $\quad C = 0.755$ Mcal	1.60	
Laboratory evaluation $H = 0.52$ Mcal $\quad C = 0.72$ Mcal	1.38	

LINEAR PROGRAMMING MODELS OF DAIRY CATTLE FEEDING

The production relationships derived from experimental data reported in the previous sections have established with some degree of confidence the following basic points:

• The technical rate of substitution between concentrates and hay is essentially constant over the relevant range (linear and parallel isoquants), and equal to the ratios of estimated net energy (ENE) of the two feed components.

• The feed-milk response is curvilinear, and individual feed inputs can be combined into a single ENE variable with little loss in accuracy.

• The maximum voluntary intake line is only slightly curvilinear over the relevant range.

• Cow ability, cow weight, and stage of lactation (time) are significant variables influencing the position of the feed-milk response function and maximum voluntary intake line.

These findings suggest that milk production relationships can be closely approximated by linear or linearly-segmented functions. Thus, the economic feeding problem can be cast in a linear programming (LP) framework with little loss of accuracy in expressing the basic production relationships while providing the possibility of including many alternative feeds and incorporating great computational advantages.

Production relationships derived from University of California experiments provides the rationale for using

TABLE 7

LINEAR PROGRAMMING MODEL FOR A 1400-POUND DAIRY COW OF MEDIUM MILK PRODUCTION POTENTIAL

Item	Constraint description	Constraint abbreviation	Constraint level	Sign	X_1 Barley, 46–48 lb.	X_2 Beet pulp, dried	X_3 Cotton-seed meal, 41% solution	X_4 Dicalc. phosphate	X_5 Lime-stone, ground	X_6 Milo, Calif.	X_7 Molasses, cane	X_8 Urea, 45% N	X_9 Wheat mixed feed	X_{10} Wheat soft, PCS	X_{11} Alfalfa hay, 21% MCF	X_{12} Corn silage, 30% DM 0.5% urea	X_{13} M_1	X_{14} M_2	X_{15} M_3	X_{16} M_4	X_{17} M_5	X_{18} M_6	X_{19} M_7	X_{20} M_8	X_{21} M_9	X_{22} M_{10}	X_{23} M_{11}	X_{24} M_{12}	X_{25} M_{13}	X_{26} Milk sales
	Row 0				−.0308	−.0271	−.0446	−.0550	−.0150	−.0297	−.0184	−0.500	−.0289	−.0302	−.0184	−.0067	0	0	0	0	0	0	0	0	0	0	0	0	0	.0450
Nutritional restrictions	1 Estimated net energy (megcal)	ENE	8.5	≤	.778	.767	.633	0	0	.778	.713	0	.706	.800	.460	.163	−.183	−.420	−.580	−.782	−.980	−1.080	−1.230	−1.430	−1.680	−1.980	−2.280	−2.68	−5.73	
	2 Digestible protein (lb.)	DP	.87	≤	.069	.041	.325	0	0	.089	0	2.109	.131	.083	.162	.023	−.055	−.058	−.061	−.061	−.063	−.063	−.063	−.063	−.063	−.063	−.063	−.063	−.063	
	3 Crude protein (13% minimum)	CP	0	≤	−.043	−.042	.281	−.13	−.13	−.021	−.100	2.682	.028	−.031	.092	−.010														
	4 Crude fiber (17% minimum)	CF	0	≤	−.113	.026	−.060	−.17	−.17	−.147	−.170	−.170	−.087	−.143	.030	.0133														
	5 Calcium (0.6% minimum)	Ca	0	≤	−.0054	.0009	−.0045	.234	.374	−.0057	.0006	−.0060	−.0049	−.0057	.0101	−.0011														
	6 Phosphorus (0.4% minimum)	P	0	≤	−.0007	−.0032	.0070	.181	−.004	−.001	−.0032	−.0040	.0069	−.0006	−.0016	−.0007														
	7 Nonprotein N (0.45% maximum)	NPN	0	≥	−.0045	−.0045	−.0045	−.0045	−.0045	−.0045	−.0045	.4455	−.0045	−.0045	−.0045	.0008														
MVIL and R min	8 Maximum voluntary intake 1 (lb.)	MVI-1	49.0	≥	.33	.33	.33	.33	.33	.33	.33	.33	.33	.33	1.0	.467														
	9 Maximum voluntary intake 2 (lb.)	MVI-2	53.5	≥	.68	.68	.68	.68	.68	.68	.68	.68	.68	.68	1.0	.467														
	10 Minimum roughage intake (lb.)	R MIN	25.2	≤											1.0	.33														
Milk response restrictions	11 Milk production segment 1 (lb.)	M-1	30	≥													1.0													
	12 Milk production segment 2 (lb.)	M-2	10	≥														1.0												
	13 Milk production segment 3 (lb.)	M-3	5	≥															1.0											
	14 Milk production segment 4 (lb.)	M-4	5	≥																1.0										
	15 Milk production segment 5 (lb.)	M-5	1	≥																	1.0									
	16 Milk production segment 6 (lb.)	M-6	1	≥																		1.0								
	17 Milk production segment 7 (lb.)	M-7	1	≥																			1.0							
	18 Milk production segment 8 (lb.)	M-8	1	≥																				1.0						
	19 Milk production segment 9 (lb.)	M-9	1	≥																					1.0					
	20 Milk production segment 10 (lb.)	M-10	1	≥																						1.0				
	21 Milk production segment 11 (lb.)	M-11	1	≥																							1.0			
	22 Milk production segment 12 (lb.)	M-12	1	≥																								1.0		
	23 Milk production segment 13 (lb.)	M-13	2	≥																									1.0	
Palatability restrictions	24 Barley maximum (75% of concentrate)	X_1 MAX	0	≥	.25	−.75	−.75	−.75	−.75	−.75	−.75	−.75	−.75	−.75																
	25 Beet pulp maximum (25% of concentrate)	X_2 MAX	0	≥	−.25	.75	−.25	−.25	−.25	−.25	−.25	−.25	−.25	−.25																
	26 Cottonseed meal maximum (25% of concentrate)	X_3 MAX	0	≥	−.25	−.25	.75	−.25	−.25	−.25	−.25	−.25	−.25	−.25																
	27 Dicalcium phosphate maximum (4% of concentrate)	X_4 MAX	0	≥	−.04	−.04	−.04	.96	−.04	−.04	−.04	−.04	−.04	−.04																
	28 Limestone maximum (4% of concentrate)	X_5 MAX	0	≥	−.04	−.04	−.04	−.04	.96	−.04	−.04	−.04	−.04	−.04																
	29 Milo maximum (50% of concentrate)	X_6 MAX	0	≥	−.50	−.50	−.50	−.50	−.50	.50	−.50	−.50	−.50	−.50																
	30 Molasses maximum (8% of concentrate)	X_7 MAX	0	≥	−.08	−.08	−.08	−.08	−.08	−.08	.92	−.08	−.08	−.08																
	31 Urea 45% maximum (2% of concentrate)	X_8 MAX	0	≥	−.02	−.02	−.02	−.02	−.02	−.02	−.02	.98	−.02	−.02																
	32 Wheat mixed feed maximum (25% of concentrate)	X_9 MAX	0	≥	−.25	−.25	−.25	−.25	−.25	−.25	−.25	−.25	.75	−.25																
	33 Wheat mixed feed and wheat maximum (50% of concentrate)	$X_9 + X_{10}$ MAX	0	≥	−.50	−.50	−.50	−.50	−.50	−.50	−.50	−.50	.50	.50																
Sales	34 Milk production and sales	M SALE	0	=													−1.0	−1.0	−1.0	−1.0	−1.0	−1.0	−1.0	−1.0	−1.0	−1.0	−1.0	−1.0	−1.0	1.0

the LP approach for determination of optimal dairy cattle feeding. However, the specific empirical estimates obtained must be modified somewhat for direct application to commercial conditions. The major limitation of the experimental data for direct commercial application is that the cows used generally were first-calf heifers. These animals therefore had not reached mature weight nor full productive capacity. Typically, the production performance of a first-calf heifer is nearly 40 per cent below its performance in later lactations.[7] This is due in part to the smaller feed intakes (lower MVIL) for immature cows, and to the substantial portion of the energy intake required for weight gain rather than milk production. Also, the inherent productive capacity (cow ability) of the animals averaged somewhat lower than the high potential cows used in commercial operations following rigid culling standards. Therefore, extrapolations beyond the range of the experimental data frequently would be required for relevance to commercial operations. For these reasons, the empirical production relationships used in the LP models to follow were synthesized from several sources, including past studies as well as the current trials.

Formulation of the General LP Model

Our purpose is to develop a commercially feasible computerized LP model for income maximization above feed cost for dairy cattle.[8] In its general form, the model simultaneously selects components of the concentrate mix, components of the roughage portion of the ration, the roughage-concentrate ratio, levels of feeding, and the quantity of milk production which maximizes net return. (Net return is defined here as value of milk production minus feed costs.) The program is adaptable to cows of different production abilities, weights and stage of lactation, as well as to various economic situations of feed prices, milk contracts, and milk prices.

Description of the model

Table 7 shows the LP model, in which all data are developed on a per cow per day basis. The model has six main structural features to be explained: (a) the specification of milk production response to increase in estimated net energy and in digestible protein; (b) specifications of minimum percentages of calcium, phosphorus, crude fiber, and other nutrients in the ration; (c) specification of the maximum voluntary intake of roughage for alternative levels of concentrate feeding; (d) specifications of the maximum percentage of the concentrate or roughage portions of the ration which can be met by each individual feed; (e) specification of the price received for milk (blend price); and (f) specification of the objective function (net return equation) to be maximized.

Milk production response curves. Estimates of milk produced in response to increasing levels of total estimated net energy intake were based primarily on the work of Blaxter [1962] since these estimates were quite consistent with the University of California trials (fig. 10) but covered a wider range of cow ability and energy levels. These synthesized estimates (fig. 14) show separate production responses for dairy cows of low, medium, and high production potential. The curves shift in response to the cow ability (A) factor used in the experimental work reported in the first part of this report. However,

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[7] The mature equivalent factor for 2-year-old heifers in California is 1.37 [McDaniel, *et al.*, 1967].

[8] The LP model presented here closely follows that model reported by Dean, *et al.*, [1969].
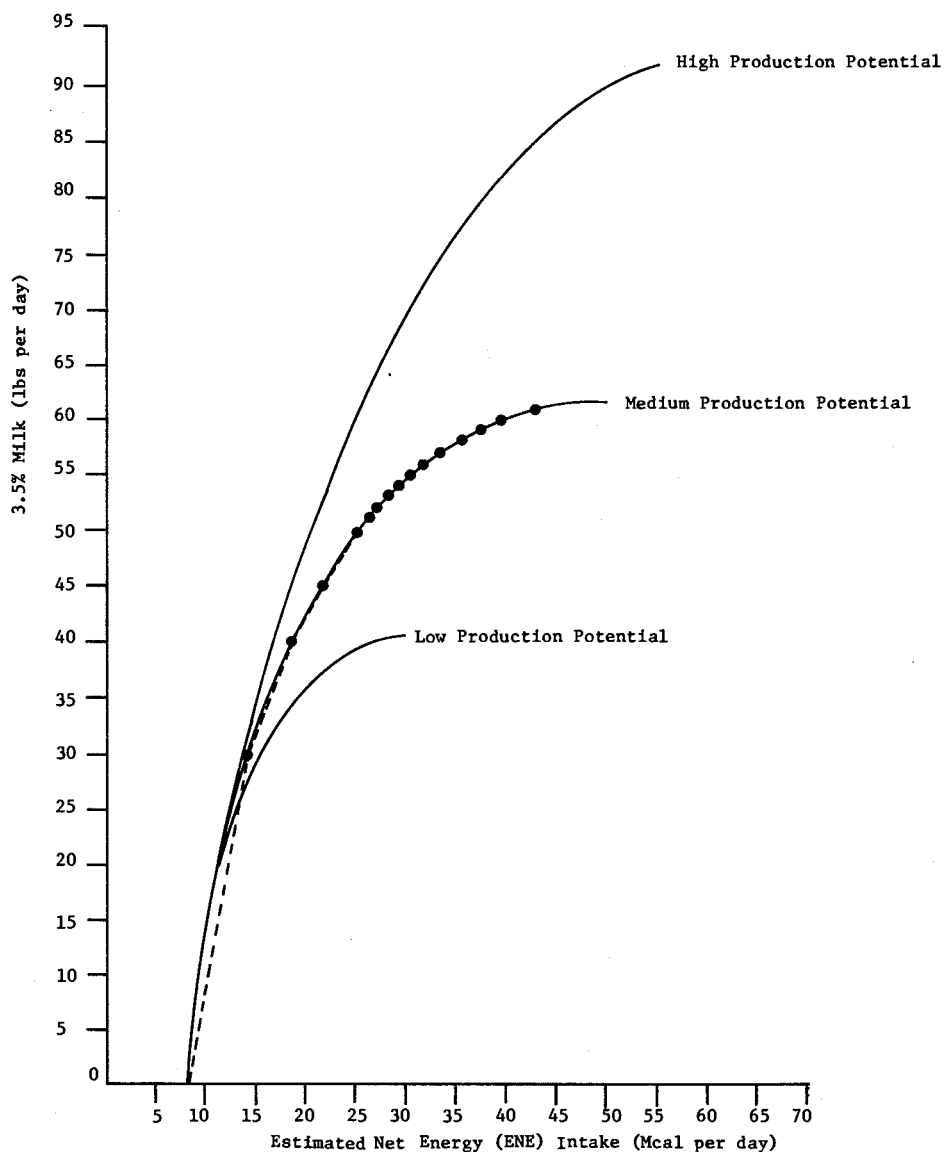
Fig. 14. Estimated milk response of a 1,400-pound cow of varying energy intake.

the high production potential curve implies an A value beyond the range of abilities available in the university trials. Response curves in figure 14 are based on 3.5 per cent FCM.

The procedure for incorporating a selected curvilinear milk production response into the linear programming model can be illustrated using the medium curve of figure 14. The curve is approximated by a series of linear segments which are then specified as a system of inequalities in table 7. Define M_1 as an activity representing a pound of milk production between 0 and 30 pounds (the first dashed linear segment in figure 14); M_2 as an activity representing a pound of milk production between 30 and 40 pounds (the second linear segment in figure 14); M_3 as the range from 40 to 45 pounds, etc. The estimated net-energy allowance per pound of milk produced in the first linear segment is 0.183 Mcal (the inverse slope of the segment), 0.420 Mcal for the second linear segment, etc., indicating increasing total energy intake per pound of milk as the milk production level increases. For the empirical work to follow we assume a 1400-pound cow. Estimated net energy for maintenance for a 1400-pound cow is 8.5 Mcal, read as the intercept of the milk response curve with the horizontal axis. The first row of table 7 is, therefore, an equation showing that the estimated net-energy allowance for body maintenance plus milk production is equal to the energy from all of the feeds included in the ration. Activity X_1 represents 1 pound of barley with 0.778 Mcal of estimated net energy; X_2 represents dried beet pulp with 0.767 Mcal of estimated net energy, etc. The equation is as follows:

$$8.50 + 0.183 \, M_1 + 0.420 \, M_2 \ldots + 5.73 \, M_{13}$$
$$\leq 0.778 \, X_1 + 0.767 \, X_2 + 0.633 \, X_3$$
$$+ \ldots + 0.800 \, X_{10} + 0.460 \, X_{11}$$
$$+ 0.163 \, X_{12}.$$

Rewriting we obtain:

Row 1: $8.50 \leq 0.778 \, X_1 + 0.767 \, X_2$
$$+ \ldots \ldots + 0.163 \, X_{12}$$
$$- 0.183 \, M_1 - \ldots \ldots$$
$$- 5.73 \, M_{13}.$$

Rows 11 to 23 of the model insure that the values of the M's are restricted to the relevant range,

Row 11: $M_1 \leq 30$, or $30 \geq M_1$
Row 12: $M_2 \leq 10$, or $10 \geq M_2$

$$\vdots \qquad \vdots \qquad \vdots$$

Row 23: $M_{13} \leq 2$, or $2 \geq M_{13}$

Since the incremental energy allowance per unit of milk produced by activity M_1 is less than that for activity M_2, M_2 less than that for M_3, etc., the program will always force M_1 to its maximum (30 pounds) before selecting M_2; M_2 will be forced to its maximum (10 pounds) before selecting M_3, etc. Thus rows 1 plus 11 through 23 completely specify the production response relationship shown in figure 14. The curvilinear relationship can be approximated as closely as is desired by adding a greater number of linear segments.

The digestible protein allowance (row 2, table 7) is analogous to the energy specification above. Digestible protein for maintenance is 0.87 pounds per day for the 1400-pound cow, while the incremental allowance of DP per pound milk production between 0 and 30 pounds is 0.055 (coefficient for activity M_1, row 2), .058 pounds per pound of milk for activity M_2, etc. As was the case for estimated net energy, the incremental protein allowance per unit of milk production increases with the milk production level [National Academy of Sciences, 1966]. The other coefficients in row 2 (table 7) show the pounds of digestible protein per pound of each feed.

Minimum percentage restrictions on nutrients. In addition to energy and protein, the nutrient restrictions in-

clude minimum allowance of crude protein, crude fiber, calcium, and phosphorus, and a maximum limitation on nonprotein nitrogen. The allowances cannot be entered directly in absolute amounts, because they are specified as minimum or maximum percentages of the total (unknown) quantity of the ration to be fed. For example, calcium must constitute at least 0.6 per cent by weight of the final ration. The calcium content of barley (X_1) is 0.06 per cent, of beet pulp (X_2) 0.69 per cent, etc. Hence, the left side of the following inequality shows the total calcium content of the ration fed, whereas the right side represents 0.6 per cent of all feeds fed:

$$0.0006\,X_1 + 0.0069\,X_2 + \ldots + 0.003\,X_{10}$$
$$+ \;0.0161\,X_{11} + 0.0009\,X_{12}$$
$$\geq 0.0060\,(X_1 + \ldots\ldots + X_{10}$$
$$+ \;X_{11} + 0.3333\,X_{12}).$$

An explanation of the coefficient .3333 of X_{12} on the right side of this equation is required. The restriction states that calcium must constitute at least 0.6 per cent by weight of the final ration, where total weight of the ration is expressed in terms of feeds standardized to 90 per cent dry matter. Since corn silage contains only 30 per cent dry matter compared to approximately 90 per cent for all other feeds, the correct coefficient for X_{12} is 30 per cent $\div$ 90 per cent $= .3333$. Combining coefficients for the X's in the above expression and rewriting we obtain row 5 of table 7:

$$0 \leq -\,0.0054\,X_1 + 0.009\,X_2 - \ldots\ldots$$
$$-\;0.0011\,X_{12}.$$

A similar procedure is used to determine the form of the restrictions for crude protein, crude fiber, phosphorus, and nonprotein nitrogen in rows 3, 4, 6, and 7 in table 7.

Maximum voluntary intake. Roughage fed per cow can either be limited by the dairyman or fed on an *ad libitum* basis, given a specified level of concen-

trate feeding. In either case it is necessary to specify the maximum quantity of roughage cows will voluntarily consume when fed various levels of concentrate. Estimates of the MVIL are shown in figure 15, as synthesized by the authors from work by Heady *et al.* [1954a, 1964b, 1956a, 1956b], Redman and Olson [1956], Mather *et al.* [1960], Kesler and Spahr [1964], and the University of California trials. Since such estimates are affected by the body weight and productive capacity of the animal as shown earlier in this report, two underlying assumptions are made on roughage intake. First, the maximum quantity of an excellent-quality-all-roughage diet (pounds of hay plus silage expressed in terms of 90 per cent dry-matter content) that a dairy cow can voluntarily consume is taken as 3.5 per cent of body weight. Secondly, a minimum level of roughage that should be consumed by a dairy cow to prevent depression in milk fat is taken to be 1.5 percent of body weight.

The curvilinear MVIL for medium and high producers in figure 15 is approximated by two linear segments, expressed as equations in rows 8 and 9 of table 7. More than two linear segments cannot be used in this case, because of problems of linear dependence. However, since the maximum voluntary intake is nearly linear over the relevant range, as confirmed by experimental evidence presented earlier in the report, approximation by only two linear segments is quite accurate. Aside from this limitation, the procedure of linear approximation is similar to that explained for the milk response curve. For example, row 8 of table 7 represents the first linear segment shown as a dashed line in figure 15. The equation of this line is: $R \leq 49.0 - .33\,C$ where R is roughage and C is concentrate. Rewriting the equation we obtain:

$$X_{11} + 1.4(0.3333\,X_{12}) \leq 49.0$$
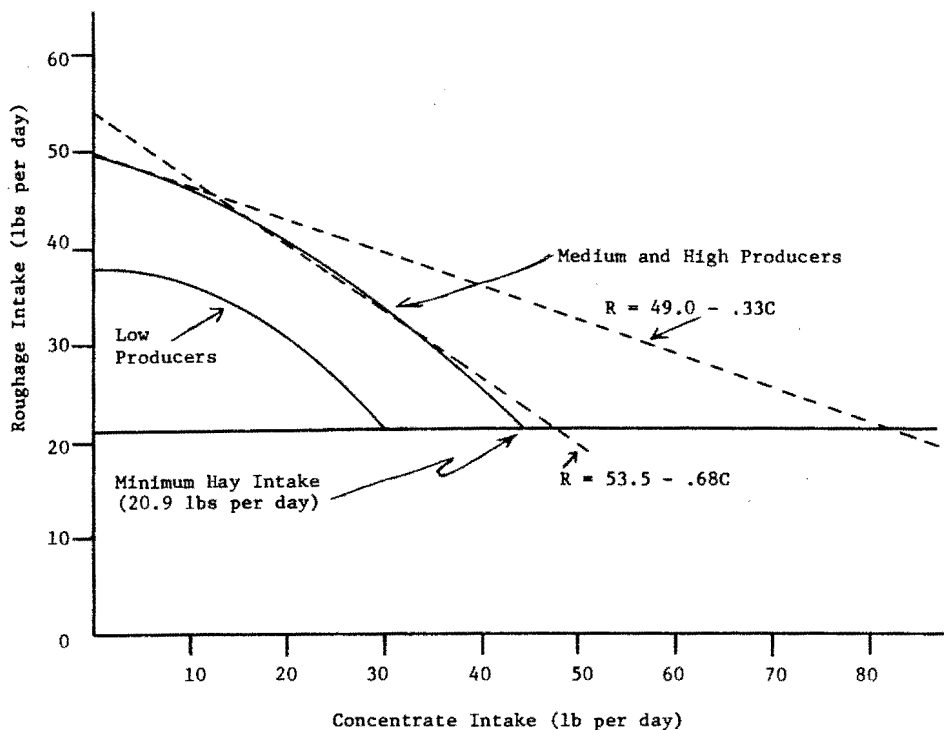$$-\;.33(X_1 + \ldots\ldots + X_{10}).$$

Fig. 15. Estimated maximum voluntary intake of roughage by cows having low, medium, and high production potential.

The coefficient of .3333 for X_{12} (corn silage) again converts pounds of "as fed" silage to an equivalent number of pounds of roughage at 90 per cent dry matter. The coefficient –.33 for concentrates is the slope of the dashed linear segment and 49.0 is its intercept with the vertical axis. Rewriting this inequality we obtain:

Row 8: $49.0 \geq 0.33 X_1 + \ldots + 0.33 X_{10} + X_{11} + 0.467 X_{12}.$

The coefficient of 1.4 for corn silage in these equations is derived as follows: The maximum voluntary intake of excellent-quality alfalfa hay was set at 3.5 per cent of body weight. The maxi-

mum voluntary intake of 90 per cent dry matter equivalent of silage was set at 2.5 per cent of body weight ($3.5 \div 2.5 = 1.4$). Therefore, each unit of 90 per cent dry matter from corn silage uses up 1.4 units of the maximum voluntary intake allowed for excellent-quality alfalfa hay.

Row 9 in table 7 is derived from the other linear segment bounding the maximum voluntary intake in figure 15. Rows 8 and 9, taken together, closely approximate the curvilinear restriction of figure 15. Equation 10 specifies the minimum hay consumption requirement of 25.2 pounds (1.8 per cent of 1400 pounds live body weight).

Percentage restrictions on individual feeds. Feeds making up the concentrate portion of the ration are limited individually to specified maximum percentages of the total concentrate fed, in order to ensure palatability. For example, barley (X_1) is limited to less that 75 per cent of all concentrates fed, as follows:

$$X_1 \leq 0.75\,(X_1 + X_2 + \ldots + X_{10})$$

Rewriting we obtain: Row 24:

$$0 \geq 0.25\,X_1 - 0.75\,X_2 - \ldots - 0.75\,X_{10}.$$

Milk Contracts. Milk marketing arrangements are often very complex. The most common arrangement in California is for the operator to have a milk contract which stipulates a base quantity per month, with specified percentages of that base sold as Class 1, Class 2, etc. The blend price calculated from terms of the contract is used as the milk price in the solution of the problem.

Value of Objective Function. The objective function for the maximizing model of this report can be expressed mathematically as follows:

Maximize $\pi = pq - \Sigma_j\,C_j\,X_j$

Where: $\pi =$ returns over feed costs in dollars

$p =$ price per pound of milk

$q =$ quantity in pounds of milk sold

$Cj =$ cost per pound of the jth feed

$Xj =$ quantity in pounds of the jth feed

Row 0 of table 7 shows the values in the objective function for a particular example problem. The price of barley (X_1) is \$0.0308 per pound and milk sells for \$0.0450 per pound. The objective function is thus:

Maximize $\pi = 0.0450\,M - 0.0308\,X_1$
$- 0.0271\,X_2 - \ldots$
$- .0067\,X_{12}.$

The objective function treats the other costs of the dairy operation (labor, depreciation, interest, taxes, etc.) as fixed over a specified short planning period. Consequently, it is unnecessary to incorporate these cost components into the model to select the optimum feeding program.

Empirical Results from LP Models

The purpose of this section is to illustrate potential uses and flexibility of the general LP model presented above. By presenting several methods of deriving economic optima for dairy feeding, the advantages of a general LP formulation will become apparent. Five alternative formulations of the LP problem, each representing a more general structuring of the model, will be applied to three different economic situations in California dairying. The five formulations of the model are:

1. Assuming a standard concentrate mix and alfalfa hay, maximize the value of milk production over feed costs per cow. Assume MVIL feeding where the curvilinear MVIL and ENE-

milk response relationships are approximated by linear segments.

2. Formulate a least-cost LP model for the concentrate portion of the ration, given alfalfa hay fed at the optimum level in method 1. Assume MVIL feeding as in formulation (1). A comparison of models (2) and (1) illustrates the advantages and additional profits possible in using a least-cost concentrate mix rather than a standard concentrate mix.

3. Formulate an LP model, allowing selection of both the concentrate mix and level of concentrate mix and alfalfa hay which maximizes profit. Assume MVIL feeding. This model is identical to model (2) only when the

level of alfalfa hay in (2) happens to be optimal. Thus, this solution will provide a profit at least equal to that in (2).

4. Same as (3) except to permit non-MVIL feeding. This step shows the additional profits possible from a complete mix feeding system (such as wafering) which permits more precise control of the concentrate and alfalfa hay inputs.

5. The same as (4) except that roughages other than alfalfa hay are considered. (In our example, only corn silage will be included in addition to alfalfa hay, but the number of roughages to be considered can be expanded to include the relevant set in any situation.) This step illustrates the additional profits possible from selecting optimally all components of the concentrate and roughage portions of the ration along with the optimum level of milk output. Because non-MVIL feeding is permitted, it represents the most general formulation of the model.

The above description indicates that the profit obtained from each successive more general model should equal or exceed that from the previous model. To illustrate the principles involved and the type of results obtainable, these five models are applied to three different economic situations which correspond generally to feeding conditions in the Sacramento Valley, the San Joaquin Valley, and the southern dairy areas of California. We assume dairy herds in each area comprised of cows weighing approximately 1400 pounds and capable of the medium milk-response curve in figure 14. The MVIL is assumed to correspond to that shown for medium and high producers in figure 15. These production relationships are represented by the linear approximations of the general LP model presented in table 7. The price relationships in the three geographic areas correspond approximately to those pre-

vailing in 1970 (table 8). Feed prices in the Sacramento Valley and San Joaquin Valley are very similar; however, the milk price in the latter is generally higher due to the larger percentage of milk output which is marketed as fluid milk. Both feed prices and milk prices in southern California are higher than those prevailing in the Central Valley. Hay and concentrate prices include a higher component for transportation costs in this area, and a high percentage of the milk is marketed as fluid milk.

Solution 1: A standard-concentrate ration (solution forced to MVIL)

This solution corresponds to the most common feeding practice among dairymen in California. A standard concentrate mix is purchased, fed at a selected level, and the cows are allowed to eat alfalfa *ad libitum* to the MVIL. A typical standard concentrate mix is shown below, by weight:

Barley, 46 to 48 pounds per bushel	40 per cent
Milo, soft, PCS	40 per cent
Cottonseed meal, 41 per cent sol	16 per cent
Molasses, cane	3 per cent
Dicalcium phosphate	1 per cent
Total concentrate mix . .	100 per cent

The costs of this concentrate mix, at the prices for the components shown in table 8, are $64.88 per ton ($0.0324 per pound), $65.28 per ton ($0.326 per pound), and $68.43 per ton ($0.0342 per pound) in the Sacramento Valley, San Joaquin Valley, and southern California, respectively. This concentrate mix is entered as a single activity in the LP model (with calculations of coefficients based on a weighted average of the individual feeds). Dicalcium phosphate was also entered as a separate activity to permit the calcium and phosphate requirements to be met by

<center>Table 8</center>

ESTIMATED FEED AND MILK PRICES REPRESENTING THREE DAIRY AREAS OF CALIFORNIA

Feed	Price per ton*			Price per pound		
	Sacramento Valley	San Joaquin Valley	Southern California	Sacramento Valley	San Joaquin Valley	Southern California
X_1 Barley, 46–48 lb./bu..............	61.60	62.60	64.40	.0308	.0313	.0322
X_2 Beet pulp, dried.................	54.20	54.20	60.20	.0271	.0271	.0301
X_3 Cottonseed meal, 41 per cent sol...	89.20	89.20	93.40	.0446	.0446	.0550
X_4 Dicalcium-phosphate............	110.00	110.00	110.00	.0550	.0550	.0550
X_5 Limestone, ground...............	30.00	30.00	30.00	.0150	.0150	.0150
X_6 Milo, California..................	59.40	59.40	63.80	.0297	.0297	.0319
X_7 Molasses, cane....................	36.80	36.80	36.80	.0184	.0184	.0184
X_8 Urea, 45 per cent N...............	100.00	100.00	100.00	.0500	.0500	.0500
X_9 Wheat mixed feed................	57.80	57.80	56.40	.0289	.0289	.0282
X_{10} Wheat, soft, PCS................	60.40	63.00	67.20	.0302	.0315	.0336
Roughages						
X_{11} Alfalfa hay, 21 per cent MCF.....	36.80	35.40	41.80	.0184	.0177	.0209
X_{12} Corn silage, 30 per cent DM, 0.5 per cent urea.................	10.00	10.00	13.40	.0050	.0050	.0067
Milk (3.5 per cent) FOB dairy........	4.50	5.00	5.50	.0450	.0500.	.0550

* Price of milk in dollars per CWT.

SOURCES: Grain prices (X_1, X_6, X_{10}) equal to 1970 net prices to growers plus $10 per ton processing and transportation. Wheat mixed feed price (X_9) equal to 1970 wheat mill run plus $5 per ton transportation. Beet pulps price (X_2) equal to 1970 beet pulp, F.O.B. plant, plus $10 per ton transportation. Molasses (X_7) equal to 1970 cane feeding molasses, F.O.B. tank car at California ports, plus $8 per ton transportation. Other concentrates (X_4, X_5, X_8) at "normal" prices estimated by authors. Alfalfa hay (X_{12}) equal to 1970 U.S. No. 1 net to growers plus $2 per ton premium for quality and $2 per ton transportation in Valley areas. U.S. No. 1 (delivered) price, Chino, plus $2 per ton premium for quality, in southern California. Corn silage, urea (X_{12}) at "normal" prices estimated by authors. Milk prices estimated by authors to be fairly typical of milk contracts in each area. For detailed sources of prices see Federal-State Market News Service [1970a, 1970b].

small additions of this supplement if necessary. Alfalfa hay was entered as the only roughage. To insure that the solution is optimal and lies on the MVIL, two solutions were obtained: (1) where maximum voluntary intake line 1 (MVI-1, row 8 of table 7) is an exact equality, and (2) where MVI-2 (row 9, table 7) is an exact equality. The higher income solution of these two is the optimal MVIL solution.

Table 9 (solution 1) shows that the optimal solutions for this model are on MVI-1. As expected, given the feed-milk price relationships, the level of concentrate feeding increases as we move from the Central Valley areas to southern California (9.62 pounds to 10.05 pounds). Since the solutions are

forced to lie on the MVIL, the amount of alfalfa hay declines correspondingly. Feed costs and net returns over feed costs also increase.[9]

These solutions are taken as the base from which successive improvements in profit can be obtained by more general formulations and solutions of the feeding problem. It should be recognized that this base solution may, in general, be too high in that it assumes that the producer feeds the optimal level of standard concentrate mix.

Solution 2: Least-cost concentrate mix ration (solution forced to MVIL)

The purpose of this solution is to show the improvement in net return

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[9] Details of these and the subsequent solutions to be discussed can be obtained from the authors. However, the Appendix (page 53) shows the computer printout for one of the 15 solutions (LP solution 5, southern California) which form the basis of this section of the report. The detailed solutions show (a) the range within which prices of feeds in the solution can change before the solution changes (b) the price at which feeds not in the solution would enter the solution (opportunity price), and (c) the estimated nutrient analysis of the ration.

TABLE 9

ILLUSTRATION OF IMPROVED RATION FORMULATION WITH SUCCESSIVELY MORE GENERAL LP MODELS FOR THREE AREAS OF CALIFORNIA WITH DIFFERENT PRICE RELATIONSHIPS

(PER COW—PER DAY BASIS)

Solution	Description	Area	Financial results			Physical results				Maximum voluntary intake restriction	Exact value of objective function	Exact increase in value of objective function	
			Total revenue	Total feed costs	Net return over feed costs	Milk production	Conc. fed	Alfalfa hay fed	Corn silage fed			Over preceding solution	Over base solution (No. 1)
			dollars			*pounds*						*dollars*	
1.........	Standard concentrate	Sacramento Valley	2.38	1.16	1.21	52.8	9.62	45.82	—	Solution on MVI-1	1.21398	—	—
	ration, alfalfa hay, MVI	San Joaquin Valley	2.64	1.13	1.51	52.8	9.62	45.82	—	Solution on MVI-1	1.50814	—	—
	feeding	Southern Calif.	2.92	1.30	1.61	53.0	10.05	45.68	—	Solution on MVI-1	1.61054	—	—
2*........	Least-cost concentrate	Sacramento Valley	2.38	1.12	1.25	52.8	9.64	45.82	—	Solution on MVI-1	1.25394	0.03996	0.03996
	ration, alfalfa hay MVI	San Joaquin Valley	2.64	1.09	1.55	52.8	9.64	45.82	—	Solution on MVI-1	1.54839	0.04025	0.04025
	feeding	Southern Calif.	2.91	1.26	1.65	53.0	10.06	45.68	—	Solution on MVI-1	1.65452	0.04398	0.04398
3.........	Optimum concentrate-	Sacramento Valley	2.39	1.13	1.25	53.0	9.98	45.71	—	Solution on MVI-1	1.25408	0.00014	0.04010
	alfalfa hay ration, MVI	San Joaquin Valley	2.65	1.10	1.55	53.0	10.08	45.67	—	Solution on MVI-1	1.55003	0.00164	0.04189
	feeding	Southern Calif.	2.97	1.31	1.66	54.0	12.45	44.89	—	Solution on MVI-1	1.65643	0.00191	0.04589
4.........	Optimum concentrate-	Sacramento Valley	2.34	1.07	1.27	52.0	12.76	38.34	—	Solution off MVI	1.26843	0.01435	0.05445
	alfalfa hay ration, non-	San Joaquin Valley	2.65	1.09	1.56	53.0	13.28	40.36	—	Solution off MVI	1.55602	0.00599	0.04788
	MVI feeding	Southern Calif.	2.92	1.24	1.68	53.0	14.70	37.98	—	Solution off MVI	1.67958	0.02315	0.06904
5.........	Optimum concentrate.	Sacramento Valley	2.32	0.94	1.38	51.6	14.02	9.10	74.66	Solution on MVI-2	1.37729	0.10886	0.16331
	roughage ration, non-MVI	San Joaquin Valley	2.59	0.95	1.64	51.8	13.94	11.15	70.40	Solution on MVI-2	1.64089	0.08487	0.13275
	feeding	Southern Calif.	2.90	1.17	1.73	52.8	15.93	12.46	64.69	Solution on MVI-2	1.72981	0.05023	0.11927

*Concentrate and hay levels forced to equal those in solution 1. Slight difference in concentrate level due to rounding error.

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possible from feeding a least-cost concentrate mix rather than a standard concentrate ration. Therefore, the levels of concentrate and hay were forced into the ration at the same levels as derived in solution 1; the only difference is that in solution 2 the concentrate portion of the ration was selected from among the 10 alternative individual concentrate feeds (table 8) in proportions which meet the nutritional requirements at minimum cost. Table 9 (solution 2 versus solution 1) shows that net returns increase from $1.21 to $1.25 per cow per day in the Sacramento Valley, from $1.51 to $1.55 in the San Joaquin Valley and from $1.61 to $1.65 in southern California.[10] The exact values of the objective function (net return over feed cost) and the increases from solution to solution are shown in the last three columns of table 9. The increases in net returns are only indicative of the improvement possible from a least-cost concentrate mix. Obviously, the critical question is to what extent the "standard" concentrate mix is formulated with relative prices in mind. Some indications of the reduction in feed costs likely to occur from least-cost rations are available from other studies. Bath, *et al.* [1968] found that least-cost concentrate mixes gave equal performance with a standard control concentrate mix, but at reduction in costs of $4.49, $1.05 and $2.99 per metric ton, in three separate trials. Howard *et al.* [1968] found that two types of least-cost ration formulations provided the same levels of milk production as a standard control ration, but with increases in daily income of $0.09 and $0.10 per cow, respectively.

Solution 3: Optimum concentrate-alfalfa hay ration (solution forced to MVIL)

This solution is designed to simultaneously select the concentrate mix and the level of concentrate feeding which

maximize profits when alfalfa hay is fed free choice. The only difference in this model compared with solution 2 is that the levels of concentrate and hay are not predetermined. It is sometimes not fully appreciated by those who formulate least-cost concentrate mixes that the nutrient specifications of the mix depend directly on the intended level of concentrate feeding, and therefore, on the amount of alfalfa hay (or other roughage) which will voluntarily be consumed along with the concentrate. In other words, the least-cost concentrate mix problem cannot be satisfactorily solved in isolation from the intended over-all feeding program.

Thus, solution 3 might be viewed as a general solution to the least-cost concentrate mix problem, wherein the optimum level of concentrate feeding, as well as the corresponding concentrate mix, are jointly determined. Table 9 shows that solution 3 raises net returns slightly as compared with solution 2.

Solution 4: Optimum concentrate-alfalfa hay ration (solution not forced to MVIL)

This solution is relevant to dairymen who employ a feeding system permitting complete control over all components of the total ration (e.g., pelleting, cubing, chopping, etc.), thereby permitting feeding below the MVIL if it is more profitable to do so. Again, alfalfa hay is considered the only relevant roughage available. Solution 4, table 9, shows that net returns again increase slightly compared with solution 3; compared with the base solution 1, daily returns over feed costs per cow have increased by approximately $0.05, $0.05, and $0.07, respectively. In every area solutions 4 and 3 indicate that, if only alfalfa hay is available as a roughage, feeding is slightly more profitable at a point below the MVIL.

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[10] The increase in net returns in each area is equal to the reduction in concentrate costs in this case, since the quantities of hay fed and milk produced are held constant and prices are constant.

Solution 5: Optimum concentrate-roughage ration
(solution not forced to MVIL)

This formulation of the problem can be considered the most general since alternative roughages (corn silage and alfalfa) as well as alternative concentrates (X_1 through X_{10}, table 8) are combined in proportions and levels to maximize returns over feed costs. Again, it is assumed that the dairyman is using a feeding system under which MVIL feeding is not mandatory.

Because corn silage was relatively favorably priced in 1970, dairymen in those areas where it was produced generally found it advantageous to include it in their rations. Solution 5, table 9, shows that corn silage in various amounts entered the optimum feeding program in all areas. Compared with the solution in which only alfalfa hay was available (solution 4), net returns increased markedly in every area. Compared with base solution 1, returns over feed costs per cow per day increased approximately $0.16 in the Sacramento Valley, $0.13 per day in the San Joaquin Valley, and $0.12 in southern California. All solutions in this case lie on the maximum voluntary intake line 2, even though this condition was not forced by the solution (probably because cows cannot eat as much dry matter equivalent of corn silage as they can of excellent quality alfalfa hay).

FIELD TESTS OF LP MODELS

The LP computer program was tested under field conditions in two separate trials: under controlled experimental conditions in the U.C. Davis dairy herd, and under commercial conditions using the dairy herd at Deuel Vocational Institution (DVI), Tracy, California. Results of these trials, modifications in the computer program implemented as a result of the trials, and recommendations for nutrient and other ration constraints for complete rations, are discussed below.

U.C. Davis Trial and Results

Twelve cows were paired according to age, stage of lactation, and previous milk production. One cow from each pair was assigned to the computer-ration treatment at the beginning of the trial, while its pair-mate received the control ration. Cows were rotated between treatments at 5-week intervals in a double-reversal design so that all cows were on both treatments during the experiment. The first week of each period was used as a change-over interval with data from the last 4 weeks of each period used in the analysis of the results.

Each group was fed a complete-ration outside of the milking parlor, with roughage and concentrate portions of the rations weighed separately but fed together twice each day. Ration components and amounts for both treatments remained constant during the trial. Roughage and concentrate amounts for the control treatment were based upon previous management practices in the herd, whereas the computer specified amounts for the group receiving the computer-formulated ration. Rations for both groups are listed on page 45. Milk from each cow was weighed and sampled twice daily; composite weekly samples were analyzed for milk fat content.

Milk production and composition from cows on the two treatments are listed on page 45. Production on both treatments was relatively low. How-

ever, cows fed the computer-formulated ration produced 0.70 pounds more milk with 0.10 per cent higher fat test, resulting in 0.08 pounds more milk fat and 1.40 pounds more 4 per cent fat-corrected-milk (FCM). The probabilities of the above differences being due to chance alone are listed in the column labelled P, with FCM at less than 8 per cent and milk fat per cent and pounds of milk fat at less than 5 per cent probability. Therefore, except for pounds of milk, the chances that the above differences were due to the feed treatments *per se* were very high.

EXPERIMENTAL RATIONS
(U.C. DAVIS TRIAL)

	Per cent of concentrate	
	Computer	Control
Barley	65	75
Beet pulp, dried	25	10
Wheat mixed feed	..	14
Molasses	8	..
Sodium tripoly-phosphate	1	..
Dicalcium phosphate	1	..
Salt	..	1
Total	100	100
Concentrate price (dollars per ton)	57.14	60.14
Concentrate fed (pounds per cow per day)	12.33	10.22
Alfalfa cubes price (dollars per ton)	30.00	30.00
Alfalfa cubes fed (pounds per cow per day)	23.29	25.76
Total ration cost (dollars per day)	0.70162	0.69372

Costs of the two rations are listed here (above) along with the composition

and amounts fed. Milk production and composition are shown below. It should be noted that the computer-formulated ration actually cost more per day, being about $0.70 compared with $0.69 per cow for the control ration. This was due to a higher level of concentrate and lower level of roughage that was recommended by the computer model than was fed to the control group. Even though the computer-formulated concentrate mix was less expensive than the control mix ($57.14 per ton versus $60.14 per ton), the higher level of concentrate fed resulted in a slightly greater total daily feed cost per cow.

MILK PRODUCTION AND COMPOSITION (U.C. DAVIS TRIAL)

	Ration		
	Computer	Control	P
Milk (pounds per day)	29.88	29.18	>.10
Milk fat (per cent	3.30	3.20	<.05
Milk fat (pounds per day)	1.00	0.92	<.05
4% FCM (pounds per day)	26.94	25.54	<.08

The price received for milk with 4 per cent fat during the trial averaged $5.64 per cwt. Therefore, with a difference of 1.4 pound per day of 4 per cent FCM, daily milk returns from the cows fed the computer-formulated ration amounted to $.079 more per cow than the control group. Feed costs were $.008 higher, however, making a net difference of $.071 per milking cow per day in favor of the computer-formulated ration. On a yearly basis, this would amount to $21.66 more income above feed cost per milking cow.

Cows used in the above experiment were relatively low producers and the trial lasted only 15 weeks and therefore results probably should not be extrapo-

lated to higher-producing cows nor to longer periods of time. However, the trial served the purpose of testing the computer program under actual feeding conditions and indicated that it was worthwhile to test the program further with larger numbers of higher-producing cows under commercial dairying conditions. Therefore, a field trial was set up on a large dairy to test and modify the program as necessary to make it of practical use to the dairy industry.

EXAMPLE OF A COMPUTER RATION (DVI TRIAL)

Specifications

Average cow weight	1400	pounds
Average milk fat percentage	3.5	per cent
Milk blend price	$ 5.25	per cwt (fob farm)

Feeds used in ration	Pounds per day	Per cent concentrate	Price per cwt	Range lower	Range upper
Corn, dent no. 2	9.77	40.63	3.05	2.74	3.19
Beet pulp, dried	6.01	25.00	2.72	2.33	2.99
Wheat mixed feed	6.01	25.00	2.77	2.38	3.13
Molasses, cane	1.92	8.00	1.36	0.13	2.54
Salt	0.24	1.00	1.00	−0.22	3.57
Dicalcium phosphate	0.09	0.37	6.00	4.03	7.89

Total concentrate—24.06

	Pounds per day	Per cent roughage	Price per cwt	Range lower	Range upper
Corn silage, 28 per cent DM	21.18	30.10	0.55	0.47	0.59
Alfalfa hay, 24 per cent MCF	16.39	69.90	1.77	1.59	1.83

Total roughage—23.45 (90 pct. DM)

Total feed cost is $1.07 per cow per day

Optimum Daily Milk Production Per Cow Under Present Conditions

Amount	Price	Income

53.0 lb. at $5.25 per CWT = $2.78

Milk Income − Feed Cost = Income Above Feed Cost/Cow

$2.78   − $1.07   = $1.71

Estimated analysis of ration (90 per cent DM Basis)

Estimated net energy	601.15 Kcal per pound	Ash	6.01 per cent
Digestible protein	8.46 per cent	Calcium	0.773 per cent
Crude protein	12.00 per cent	Phosphorus	0.351 per cent
Crude fat	2.61 per cent	NPN	0.000 per cent
Crude fiber	15.00 per cent		

	Price per cwt	
Feeds not used in ration	At formulation	Opportunity price
Barley, 46–48 pounds	3.05	3.03
Cottonseed meal, 41 per cent sol.	3.47	3.30
Milo, Calif. or MW	4.35	3.00
Urea, 45 per cent N	4.35	4.04
Limestone, ground	1.50	−0.22
Sodium tripolyphosphate	12.00	9.41

Deuel Vocational Institution Trial

The DVI herd of 180 milking cows was split into two groups with approximately equivalent previous milk production. One group was fed according to previous management practices on the dairy, with concentrates fed in the milking parlor and roughages fed separately outside. Alfalfa hay made up part of the ration year-round whereas corn silage was fed in the winter and spring and oat silage in the summer and fall.

The other group was fed a complete ration (roughages and concentrates mixed together), with the ration formulated by computer based on the variables previously discussed (feed prices, milk price, average cow size, average milk production, maximum voluntary feed intake, etc.). A small amount of concentrates were fed in the milking parlor during part of the trial to encourage cows to come into the parlor more rapidly. New ration formulas were developed as feed prices and availability of feed ingredients varied throughout the trial. The ration that was fed during a major portion of the trial is shown on page 46. Average analyses of feed ingredients sampled periodically throughout the trial are also shown on page 46.

The trial lasted 13 months. Milk weights were recorded twice daily for each cow in the trial, which is a routine practice in the DVI herd. Milk fat percentages determined monthly by the local DHIA were multiplied by the total of the daily milk weights to obtain monthly milk fat and 3.5 per cent FCM production for each cow.

Lactation records (305 days) were calculated for 64 cows in the control group and for 65 cows fed the computer ration. Records from cows which left the herd during the trial were not included in the milk production analysis. When a continuous, full lactation was not completed within the 13-month experimental period, the latter part of the previous lactation completed during the trial period was added to the current incomplete lactation to obtain a 305-day record. This was possible because cows which dried up and left an experimental group were returned to the same group when they freshened again. An attempt was made to balance both groups according to age at the beginning of the trial but this was difficult to maintain because of culling and other management practices during the 13-month period of the trial. Therefore, all production records were converted to a mature-equivalent (ME) basis to remove any bias due to differences in ages of cows in the two groups. Production data from the trial were subjected to an analysis of variance using a completely randomized design.

Results of D.V.I. Trial

Milk production and composition (ME basis) are shown on page 48. None of the small differences between groups approached statistical significance, with probabilities (P) of differences being due to chance alone greater than 50 per cent in all cases. Therefore, for all practical purposes, production from the two groups can be considered approximately equal.

Average ME milk production of 18,963 and 18,747 pounds, respectively, from the computer-formulated and control rations was excellent, indicating that both rations were nutritionally well balanced. Actual milk production for the 129 cows which completed the trial averaged 17,101 pounds in 305 days.

Fat test in both groups was lower

MATURE EQUIVALENT MILK PRODUCTION AND COMPOSITION (D.V.I.) TRIAL)

	Ration		
	Computer	Control	P
Milk (pounds per 305 days)	18,963	18,747	> .50
Milk fat (per cent)	3.18	3.28	> .50
Milk fat (pounds per 305 days)	601	611	> .50
3.5% FCM (pounds per 305 days)	17,932	17,537	> .50

AMOUNTS AND COSTS OF EXPERIMENTAL RATIONS (D.V.I. TRIAL)

	Ration	
	Computer	Control
Daily concentrate (pounds per cow)	25.88	21.86
Concentrate price (dollars per pound)	.0275	.0275
Daily concentrate cost (dollars per cow)	.7117	.6012
Daily hay (pounds per cow)	18.62	27.94
Hay price (dollars per pound)	.0177	.0177
Daily hay cost (dollars per cow)	.3296	.4945
Daily silage (pounds per cow)	23.40	23.40
Silage price (dollars per pound)	.0055	.0055
Daily silage cost (dollars per cow)	.1287	.1287
Total daily feed cost (dollars per cow)	1.1700	1.2244

than normal, being 3.18 per cent and 3.28 per cent, respectively. Low fat tests have been a problem in this herd from time to time and may be due partially to the relatively high level of concentrates fed, resulting in a low level of fiber in the ration. At one point during the trial, fat test dropped to 2.73 per cent for the group receiving the computer-formulated ration. This was corrected by reformulating the ration with a minimum of 17 per cent crude fiber (90 per cent DM basis) instead of 15 per cent, as originally specified. This resulted in a formula with less concentrates and more silage and a return to a more normal fat test from the group. Subsequently, this minimum level of crude fiber was made a permanent part of the computer-ration program constraints.

Accurate measurement and allotment of feed under the commercial conditions of a field trial are difficult even under the best of conditions. It is even more difficult in a large herd, as is the case at D.V.I. Recognizing this as a limitation on the accuracy of the feeding data, the average amounts and costs of the various feeds that were fed during the trial are shown above.

Both groups received the same amount of silage per cow, as the amount recommended for the computer-ration group also was fed to the control group. However, the computer-ration contained more concentrates and less hay than the control ration. The amount of hay fed to the control group was based on the number of bales fed multiplied by the average bale weight. When hay quality was poor, excess amounts were fed to allow the control cows to select the better portions and refuse the coarse stems, resulting in great wastage at times. However, the total amount fed was charged to the control group as

there was no provision for measuring hay refusals and this is considered a normal cost of milk production when baled hay is fed in this manner. Conversely, cows fed the computer-formulated complete-ration received the same baled hay after it was shredded and mixed with concentrates and water. Hay refusals in this form were negligible—a fringe benefit of feeding complete-rations. However, complete rations can be detrimental to high milk production when poor-quality chopped hay is included because the cows cannot select leafy portions and refuse the coarse, less nutritious stems.

The amount and composition of concentrate mix was specified by the LP program. The concentrate mix fed to the control group was similar but not identical to the computer-mix, although the feed company charged the same for both mixes. Therefore, the difference in concentrate mix costs was due to the amounts fed rather than a difference in feed prices.

Cows fed the computer ration received daily an average of 25.88 pounds concentrates, 18.62 pounds shredded alfalfa hay, and 23.4 pounds silage (corn or oats, as available) compared with 21.86 pounds concentrates, 27.94 pounds alfalfa hay, and 23.4 pounds silage fed to the control group (page 48, top right).

Total daily feed costs were $1.1700 and $1.2244, respectively, for the computer ration and control ration—a difference of $0.0544 per cow per day in favor of the computer ration. This amounts to $16.59 per cow per year, or $2,986 lower feed cost per year in a 180-cow herd, the approximate size of the D.V.I. herd. Since milk production was at least as high (page 48, top left), the $2,986 lower feed cost from feeding the computer ration would result in an equivalent amount of additional income above feed cost compared with feeding the control ration.

Conclusions and Comments on LP Models

Both the small-scale, well-controlled, double-reversal trial at U.C.D., and the large-scale field trial under commercial conditions at D.V.I., resulted in economic advantages when cows were fed the computer-formulated rations. This was accomplished even though all cows in the computer ration group were fed the same complete ration regardless of production level or stage of lactation. Subdividing the computer-ration group into subgroups ranking according to production levels (strings) and feeding complete rations with varying roughage concentrate ratios, would probably have resulted in even greater efficiency of feed utilization from this group.

For large-scale commercial use (such as by a feed company, dairy management consultant, or large dairyman) the general models of profit maximization presented here can be easily adapted to new conditions. If economic conditions differ only feed and milk prices need be changed. If other feeds are available they can be added directly to those already included, along with the necessary nutrient coefficients derived from standard tables. Of course, any palatability restrictions on these new feeds must also be entered, following the principles set forth earlier. If the model is to be adapted for cows of different production capabilities (e.g., low or high production potential curves such as those in figure 15) the required matrix changes are somewhat more complex. Again, however, the principles presented earlier, whereby the curvilinear response is divided into linear segments and new coefficients calculated in rows 1 and 2 for M_1 to M_{13}, can be

easily followed to formulate the relevant model. If cows of different liveweight are to be analyzed, the maximum voluntary intake lines (MVI-1 and MVI-2) must be shifted using the principles outlined earlier, as well as recalculation of the ENE and DP for maintenance (constraint column, rows 1 and 2 table 7) and the minimum roughage level required (constraint column, row 10); all these relationships are directly dependent on body weight.

While each of the above modifications could be presented and solutions derived, it is assumed that the technical reader is interested primarily in the principles involved and can adapt the basic model to suit his needs. For large-scale commercial use, it would probably be most efficient to prepare several basic matrices for cows of various common weight-production and potential-stage-of-lactation combinations. Then, given the characteristics of the various milking subgroups (strings) in the herd, the relevant list of feeds, and the type of feeding condition (MVIL or non-MVIL feeding), the appropriate models could be quickly selected and solutions efficiently obtained.

The LP models have some limitations. The concept of maximizing the value of milk production over feed costs assumes that all other costs remain constant, but this assumption need not hold in all cases. For example, feed-mixing costs for formulating rations are not explicitly considered and may vary depending on the ration selected. If silage is included in the ration, handling and storage costs may be higher. Feeding below the MVIL may involve control measures such as longer periods in the milking parlor or packaging feeds, with consequent increase in costs. Milk contracts may be such that a single-blend price for milk should be replaced by more complex contract terms. And, of course, the models have been formulated for strictly dry-lot feeding situations. Where pasture comprises a substantial portion of the feed intake, a more general model involving such considerations as seasonal pasture availability and opportunity costs for alternative uses of the land needs to be developed. In all likelihood such a model would need to encompass both the cropping and livestock system to be realistic, and therefore is less amenable to a more-or-less standard format.

ADDITIONAL RESEARCH NEEDED

Probably the major weaknesses in the LP models presented can be traced to deficiencies in basic knowledge of the production function relationships explored in the first portion of this report. Considerably more experimental work is needed, particularly on ENE-milk relationships and maximum voluntary intake curves. As these improved relationships are developed from experimental data they can be readily incorporated in the LP framework. Another weakness is that there is considerable variability in the nutrient composition of different batches of the same individual feed. Therefore, nutrient and palatability specifications of the models must be set at relatively safe levels (so-called fat coefficients in LP terminology) to insure acceptable rations from widely differing feed sources. Methods for quickly and cheaply determining feed quality would allow more precise ration formulation. Perhaps, also the use of stochastic programming techniques holds promise for more adequately handling the variability problem.

Finally, more precise methods of relating the production capacity of animals in the field to particular expected milk response curves are needed. Per-

haps the "challenge feeding" concept whereby a cow is fed at maximum energy intake in the first few weeks of lactation is sufficient for establishing which is the relevant production response curve for that cow. Commercially, of course, cows of similar production response must then be grouped into subgroups (strings) with each string fed the appropriate ration.

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APPENDIX

Computer Printout of LP Solution 5, Southern California: Optimum (Profit Maximizing)
Concentrate-Roughage Ration, Feeding at Maximum
Voluntary Intake Not Required

Average Cow Weight	1400 lb.				
Average Milk Fat Percentage	3.5 %				
Milk Blend Price	$5.50 per cwt. (FOB farm)				

Feeds Used In Ration	Lb/Day	% Rough.	Price Per Cwt.	Range Lower	Upper
Urea-Corn Silage 30% DM	64.69	63.38	0.67	0.64	0.74
Alfalfa Hay 21% MCF	12.46	36.62	2.09	1.93	2.16
Total Roughage — 34.02	(90% DM)				

		% Conc.			
Barley, 46–48 lbs.	6.36	39.95	3.22	2.65	3.28
Beet Pulp, Dried	3.98	25.00	3.01	−1.79	3.44
Wheat Mixed Feed	3.98	25.00	2.82	−1.98	3.31
Molasses	1.27	8.00	1.84	−26.06	2.60
Dicalcium Phosphate	0.33	2.05	5.50	−0.56	29.98
Total Concentrate — 15.93*					

* Provide salt free-choice or as 0.5% of concentrate.

Total feed cost is $1.17 per cow per day

Optimum Daily Milk Production Per Cow Under Present Conditions

Amount	Price	Income
52.8 lb. at $5.50 per cwt. = $2.90		

Milk Income – Feed Cost = Income Above Feed Cost/Cow

$2.90	–	$1.17	=	$1.73

Estimated Analysis of Ration (90% DM Basis)

Estimated Net Energy	560.57 KCAL/lb.	Crude Fiber	17.00 %
Digestible Protein	9.27 %	Ash	6.05 %
Crude Protein	13.00 %	Calcium	0.763 %
Crude Fat	2.27 %	Phosphorus	0.400 %
		NPN	0.291 %

Feeds Not Used In Ration	Price Per Cwt. At Formulation	Opportunity Price
Cottonseed Meal, 41% Sol.	4.67	3.47
Limestone	1.50	−0.70
Milo	3.19	3.12
Urea, 45% N	5.00	3.20
Wheat	3.36	3.23

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