SYSTEM OF DEMAND EQUATIONS: THEORY AND APPLICATIONS

Abebe Shimeles[*]
Department of Banking and Finance
Addis Ababa College of Commerce

ABSTRACT: *This paper notes the rapid advances that have been made in the modelling, estimation and applications of systems of demand equations in the literature. It attempts to empirically estimate for Ethiopian data sets using a two period price information following Pollak and Wales [57], Dybrig [31], and a saving information as in Lluch [46] and Howe [41] to identify all parameters of the Linear Expenditure System (LES) and The Extended Linear Expenditure System (ELES), respectively. In spite of the theoretical and empirical limitations of the LES , parameter estimate were found to accord with the right sign and magnitude required by the choice theory. Of particular interest is the process of estimating directly the Frisch Parameter and its welfare implications. The paper also discusses the estimation of compensated and uncompensated own-and-cross price elasticities in the context of Slutsky decomposition. Some areas of future research are pointed out in exploiting fully budget studies so far generated in Ethiopia. Notably, the possibility of estimating some generalized expenditure systems may stimulate interest for applications to some policy issues.*

BACKGROUND OF THE STUDY

The construction of system of demand equations from consistent utility maximizing exercise using the powerful programming concepts such as duality has gone through remarkable improvements over the past three decades or so. Stone's pioneering work [66], Frisch's investigative result [35] set the important stage that was later to produce alternative systems of demand equations in line with a certain preference structure underlining the consumer's utility. Stone's approach was to search for a demand function that would satisfy certain regularities like adding up and subsistence consumption level when current income is zero. His persistence produced the famous Linear Expenditure System. Frisch took up the utility implication of Stone's demand analysis for empirical works. Notably his instructive observation regarding the relationship between income elasticities and own-and-cross price elasticities gave an opportunity to estimate complete system of demand equations in the context of additive preferences or what is better known as additively separable utility functions. The whole body of the literature in the 1960s and 1970s was devoted to the

expounding and derivation of demand functional forms that satisfy parametric restrictions implied by the maximization of quasi-concave utility functions subject to a linear budget constraint by an individual consumer.

The works of Theil [68, 67] produced the famous Rotterdam model, of Houthakker [40] indirect addi-log utility function, and the class of flexible demand functional forms by Christensen et.al.. [16] and Diewart [29], and Almost Ideal Demand System of Deaton and Muellbauer [27] marked significant developments in the whole episode.

This paper makes an attempt at estimating demand systems in line with the neoclassical choice theory from available data sets for Ethiopia, and is organized as follows: Section I notes in brief important developments in the modelling, estimation and application of system of demand equations, section II discusses previous attempts to estimate demand systems in Ethiopia and the estimation of system of demand equations in the context of the Linear and Extended Linear Expenditure Systems based on budget studies for Addis Ababa for the years 1975,1979 and 1980 and for rural Ethiopia for the year 1980. Section III analyses empirical parameter estimates and makes concluding remarks on the uses of systems of demand equations estimated from budget studies for important policy issues.

I: Modelling, Estimation and Applications of Demand Systems.

In this section we shall make a brief attempt to note the process of refining the concept of choice from the following perspectives:

1.1. The search for an appropriate demand functional form in the context of a priori set parametric, semi-parametric and non-parametric tests

1.2. Major problems in the process of estimating demand systems;

1.3. The empirical applications and limitations of demand systems.

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1.1 Parametric Restrictions and Demand Functional Forms.

Consumer behaviour has been systematically captured in the economics literature through two different approaches that just converge in the conclusions they finally arrive at: The Utility and Revealed Preference approaches. While the former dates as far back as to Marshall's Principle, the latter is a sole contribution made by Samuelson [62]; and it is this first one that we review in this section. Hicks'[39] remarkable contribution in interpreting Marshall's concept of utility (particularly marginal utility of money) in the context of substitution and income effect for price changes brought forth the famous Slutsky decomposition named after Slutsky's impressive theoretical exposition [45][1]. The work after that was the incorporation of Sheppard's Lemma into the utility maximization process to produce the notion of duality in consumer behaviour. In fact, it has been proved that given preferences are monotonic,strictly convex, continuous and twice differentiable, there always exists a utility function that satisfy second-order condition for maximum[2]. Thus a utility function does exist not because of its psychological connotation as used to be implied in the early neoclassical literature,but, as a matter of mathematical necessity, implied by preference structure. More formally, given a utility function f *defined over exhaustive set of quantity vector X , i.e.($x_1, x_2, \ldots x_n$),* a typical consumer's choice problem for given income level y *and price vector P.i.e., ($p_1, p_2, \ldots p_n$)* can be written as:

Max. $f(X)$

sub. $y \geq P'X$ $i = 1, \ldots, n$. ....[1]

Alternatively, duality theory has made it possible to specify the above problem as:

Min. $e = P'X$

Sub. $f \leq f(X)$ ....[2]

where, e stands for total expenditure.

By manipulating the relevant Lagrangian function of (1) and (2) and observing that the Khun-Tucker necessary and sufficiency conditions ensure an interior and unique optimal solution, we have, respectively, vectors of demand functions $x_i(p_1, \ldots p_n, y)$ and $h_i(p_1, \ldots p_n, u)$ that maximize utility or minimize expenditure for an appropriately specified preference structure. Thus the vector of demand functions $X(P, y)$ is known as the Marshallian demand function, and of $H(P, u)$ as the Hicksian, or compensated vector of demand functions. If we

plug $X(P,y)$ and $H(P,u)$, respectively, into the objective functions (1) and (2) we get the Indirect Utility function $v(p_1,...,p_n,y)$ defined over prices and income, and the Expenditure function, $e(p_1,...,p_n,u)$. This synthesization of consumer preference generated important identities that evolved into Propositions and Lemmas of considerable analytical importance in applied demand analysis.[3]

When the inequality constraint in equation (1) is removed the following host of restrictions emerge:

1. The homogeneity of degree zero of demand functions.

Given that expenditure functions [inverse of $V = f(P,Y)$] are homogeneous of degree 1, then demand functions become homogeneous of degree zero in prices. Thus, using Euler's Theorem on homogeneity , it can be shown that,

$$\sum E_{ij} + E_i = 0 \qquad i,j=1,...,n. \qquad ....[3]$$

where, the first term stands for the uncompensated cross price elasticity of demand for commodity i as a result of a change in the price of commodity j; and the second term is income elasticity of commodity i.

2. Engle's Aggregation.

The direct differentiation of the budget constraint is shown to lead to the following identities:

$$\sum_{i=1}^{n} v_i E_i = 1 \qquad [4]$$

where, v_i=average budget share of the ith commodity, and further more,

3. Cournot's Aggregation.

The principle underlining utility maximization results in the restriction:

$$v_j \cdot \sum_j v_j E_{ij} = 0 \qquad i, j = 1,...,n \qquad [5]$$

4. Symmetry and negativity of the Slutsky Substitution Matrix

Using the results of the problem in (1), it is shown to hold true that:

$$\frac{\delta X_i(P,y)}{\delta p_j} = \frac{\delta h_i(P,u)}{\delta p_j} - X_j \frac{\delta X_i(P,y)}{\delta y} \qquad [6]$$

Where, (6) is known as the Slutsky Equation*, which decomposes the effect of a change in the jth commodity price on the demand for the ith commodity into two. The first term is the Hicks' compensated demand function that measures the substitution effect, and the last term is the income effect (see.e.g., Green [38]). The system of differential equations in [6] require symmetry of the substitution matrix which is given by the first term of the Right Hand side of [6] to be solved as is discussed in Deaton and Muellbauer [27]. Also, since a direct differentiation of the expenditure function with respect to p gives the Hicksian Compensated demand function by Shephard's Lemma it follows that $h_i(p,...,p_n,u)/\delta p_i \leq 0$, that is the Substitution Matrix of the Slutsky equation is negative semidefinite. For a rigorous demonstration of the negativity condition see, for instance, Barten and Geyskens [8]. Further the Matrix that contains price and income responses of the Marshallian demand functions as its elements will have to be symmetric and negative semidefinite. This can be seen to hold by simply moving the income-effect term of the Slutsky equation in (6) to the Left Hand side. The literature in this regard has concentrated in the formulation of either a convenient utility function (usually indirect utility function) or demand function that satisfy these restrictions implied by the theory. Both approaches amount to the same thing as far as the integrability condition by an appropriate choice of parameters is met. That means, a system of demand equation that satisfy the above restrictions simultaneously implies that, it has a corresponding utility function and .vice

versa. Among the list we have the noted Linear Expenditure System, Rotterdam Demand Model, the class of flexible transcendental demand functions, the addi-log indirect utility function (constant elasticity demand function), and Almost Ideal Demand System. The specific functional forms of each is given in Appendix B. As empirical studies proliferated, a growing attention has been given to the type of data needed to justify evidences and to the econometric procedures that improve efficiency of estimates. The works of Barten [6], Barten and Geyskens [7], Lluch [47], and Keifer [43] offered important and valuable insights to the empirical procedures for testing the theoretical restrictions indicated above in the context of the Rotterdam model. Beyond the exercise of testing parameter restrictions of a demand system generated from a specific utility function , a series of attempts were made to specify a demand system that would approximate to any direct utility function. The class of flexible demand systems initiated by Diewart [29],and, Christensen et.al. [16], in the context of the Translog system opened the way for the subsequent development of a family of flexible semi-parameteric demand systems, including the Almost Ideal Demand System of Deaton and Muellbauer [27]. In spite of all such advances, one issue however remained elusive all along. As Deaton often remarked [26, 24, 23], and in fact many others too (e.g Barten [7], Lluch [47], Blundell [9]) there is so far no adequate general system of demand equation that readily lends itself for nested parametric tests ,i.e., specific restrictions would lead to specialized system of demand equations. We may however note the Restricted Non-linear Demand System estimated for instance by Ray [61], the Quadratic Expenditure System suggested and estimated by Pollak and Wales [57, 58], and Normalized Quadratic Expenditure Systems of Diewart and Wales [30] as exceptions. But most of these reduce to the Linear Expenditure System for a certain restrictions without much of a change in the underlying preference structure. As a result there is no a unified approach that make comparisons easier among alternative functional forms. Deaton [23], and Pesaran and Deaton [56] developed and applied a technique to compare non nested demand systems. However, some quite disagree, albeit in a different sense, the justice of using demand systems with parametric restrictions (e.g. Varian [71]); they argue that the theory of revealed preference, a la Samuelson, is much more flexible and easily lends itself for non-parametric tests. The flexible semi-parameteric demand systems referred to in the foregoing and others that were developed later were meant, as mentioned above, to approximate any

direct utility function, and were justified on the grounds that the parametric restrictions stated in the neoclassical choice theory would have to be imposed and tested empirically not built in the demand systems. Therefore, if restrictions are rejected by a data, then the neoclassical choice theory is said to be empirically invalid, and vice versa. We may however observe the reservations shown and the comments given, for instance, by Deaton and Muellbauer [27] on the same kind of claim Christensen et.al [16] made after they found that the Translog demand system had been rejected in almost all empirical cases they investigated. That is, there is no apriopi reason to suppose that preferences are translog and that the flexible demand systems have not yet satisfactorily resolved the empirical and theoretical issues surrounding the neoclassical choice theory. In fact, Diewart and Wales [30: 98] strongly contended that all demand systems in the semi-parametric family, including the popular Almost Ideal System, do not satisfy the desired flexibility and concavity they claimed to have possessed. They suggested a Normalized Quadratic Expenditure Linear and Quadratic Spline models that they showed satisfy important properties that have been missing for so long in most demand functional forms. In their own words [30, 98] " First they [i.e., these systems] are flexible in the sense of being able to locally approximate an arbitrary twice continuously differentiable preference functions to the second degree. Second, the concavity in prices property required by well behaved expenditure functions is imposed during the estimation. Third, a semi-flexible version of the model requiring fewer parameters may be estimated if there is a shortage of degrees of freedom, or if convergence difficulties are encountered during estimation ". It remains for subsequent empirical works to substantiate these claims and perhaps soon there shall be a single framework good enough for those looking for one. When Diewart and Wales [30] applied these demand systems for Japanese data, results turned out to be quite satisfactory and in accordance with the neoclassical choice theory. The trick that made it all possible was the Spline technique they aptly employed. A brief discussion of this technique's application to econometric procedures is given in Johnston [42].

1.2. Major Problems In Estimating Demand Systems

Apart from the theoretical problems of specifying an appropriate demand system,

the estimation process is beset with a number of statistical problems. Noteworthy are the identification and the aggregation problems.

The identification problem refers to whether or not the parameters of a given demand systems are estimable given the number of equations, number of commodities entering the demand system and number of independent parameters. This is an issue basically similar with the statistical problems regarding structural equations discussed in Jonhston [42]. As is the custom in econometrics, identification problem in system equations can be overcome either by imposing restrictions on parameters like the ones on demand systems implied by the choice theory, or use already available knowledge about the value of certain parameters estimated may be from previous studies. We have in section III an extension of this discussion in solving the identification problem in the context of the Linear Expenditure System by Pollak and Wales [57] , Howe [41], and generally for all additive preferences by Dybrig [31].

Aggregation over households, commodities and prices constitute broadly the essentials of the problem. Demand systems in most cases have been estimated using data from national accounts or budget studies. The construction of system of demand equations in the context of choice theory however derives its fine results from the assumption of individuality of the consumer, compactness, continuity and convexity of the budget set, and a price vector that forms a separating hyperplane with the preference structure. Consistency, therefore, requires imposition of restrictive assumptions in applied demand analysis. Some of these are stringent, and others are quite tolerable. Aggregation problem over households arises by the common knowledge that observed expenditure aggregates in national accounts, or budget surveys represent mean expenditure values of households whose heterogeneity is evident at least with respect to family size, and age,not to mention those running through religious, ethnic and cultural grounds. From the theory, aggregation is allowed if only homogeneity is maintained across all households, and this is known in the literature as the assumption of exact linear aggregation, (e.g., Deaton and Muellbauer [27]). The Engel curves implied by this assumption are homothetic and thus linear; and individual preferences can be mapped into groups of households if only there is linearity in Engel

curves. Exact aggregation is a stringent restriction in application. Recent works reported that this restriction was summarily rejected by the data, (e.g. Nicol [53]) , even though other regularity conditions were satisfied. Alongside with this, numerous attempts have been made to incorporate and capture demographic effects in demand system estimation. The works of Prais and Houthakker [59] and of Muellbauer [50] in the context of general and specific equivalent scales to bring compatibility between adult and child consumption produced impressive results. Pollak and Wales [58] introduced various ways of incorporating family size and age factors in demand systems. Nicol's result was a further indication that demographic variables play significant role in consumer behaviour determination. In cases where the data fits well into linear Engel curves, Seade [63] demonstrated its straightforward implications to the problem of measuring consumer surplus via Hick's compensated demand system. To avoid the severity of Exact Linear aggregation, subsequent works introduced Generalized Linearity of Engel curves or exact Non-linear aggregation with different special cases: Price Independent Generalized Logarithmic cost functions (PIGLOG), and, Generalized Gorman Polar Form discussed in Deaton and Mullebauer [27], Blundell [27], and the Modified Price Independent Generalized Logarithmic (MPIGLOG) form of Cooper and MacLaren [17]. As a result of such modifications most demand systems commonly used in empirical works satisfy exact non linear aggregation, at least locally.

Aggregation problem is manifested also in the nature of price data used in empirical application. Most studies take price indices reported in national accounts as representatives to individual prices. The uses of panel data, e.g., as in Keifer [43] indicated its advantage over other alternative sources in providing asymptotically consistent and efficient parameter estimates. The fact that market information is asymmetric, and that in most cases markets are fragmented and therefore are imperfect entail the prevalence of multiple prices at a point of time. The chance, therefore that an individual consumer may face different prices for the same commodity is quite substantial. Anglin [3] demonstrated the serious threat such diverse prices would bring on the efficiency of parameters estimated in either system Least Square Estimators or Feasible Generalized Estimators. Quota systems, price controls and other instruments of discrimination result in duplicity or multiplicity of commodity prices

causing kinks in the budget line. If one is provided with the data modifying the budget constraint, or the convexification assumption applied in production (see,e.g.Varian [70]) may be used to overcome the problem of aggregation in prices.

Finally, aggregation problem over commodities led to various interesting specifications of the preference structure. The familiar concepts such as want independence, utility tree and separability are known for immensely simplifying estimation procedures. The task of classifying commodities would have been inexhaustible without a certain degree of aggregation. In fact, all system demand analysis assume a certain degree of independence in the choice process that works through either separability or two-stage budgeting.

1.3. Empirical Application of Demand Systems

A growing body of the literature on optimal commodity taxes have used extensively the concept of neoclassical choice theory. We may cite the voluminous work made on the issues of marginal versus non-marginal tax reform in a revenue neutral setting. For example, Ahmad and Stern [1], Diamond and McFadden [28], Ray [61, 60], Deaton [22, 21], King [44], Cragg [19] applied system of demand equations to the problem of finding an optimal commodity tax rate that would maximize social welfare functions of the community subject to the government revenue constraint. We may elaborate this by appealing to propositions familiar in normative welfare economics. Cragg [19, p.126] explains that " the government is modelled as having preferences given by a symmetric, strictly concave, additively separable Social Welfare Function [SWF] which is increasing in the utility of each household ". This SWF is specified in terms of indirect utility functions, $V^h(p_1,...,p_n, y^h)$ of the hth household, $h= 1,...,H$, such that the government's problem will be to maximize this SWF subject to its fiscal revenue constraint." Such studies bring out the degree of income inequality inherent in the indirect system, and whether reform is necessary or not. Not so less important is also the role systems of demand equations play to understand and capture the process of economic growth. Tsujimura and Sato [69] and Lluch and Powell [69] for instance used systems of demand equations to investigate the changes in the structure of consumption of households in the

growth process. System of demand equations are also very useful in many planning exercises to project demand aspects and are helpful to cast growth strategies behind the structure of demand. In another sphere too, a system demand equations can be used to study the problem of financing such important services as education and health in the context of a federal government political structure. For instance, Coyte and Landon [18] analyzed and compared the method of cost-sharing with block-funding in financing health and education services for Canada. System of Demand equations are quite useful in measuring inflationary impacts on different income groups. The concepts of compensated and equivalent variation are easily captured and measured through the expenditure function. This brief description indeed suffers completeness without the statement that there is no area of economics, applied or theoretical, that does better without the principles of optimization and duality theory.

II. Estimation of The LES and ELES For Ethiopia's Budget Studies

Estimation of system of demand equations is one of those research areas that received little attention in Ethiopia so far, especially its empirical significance to policy formulations and to the understanding of patterns of consumption by households. Of the few contributions that have been made so far, some have more or less restricted the functional forms from reflecting the choice theory, because the estimated demand system were not types which can be generated from any direct utility function[7], e.g. Asmerom [5,4], or used indirect ways to estimate price responses as in Getachew [36] following Frisch's decomposition. The only one that directly estimated price responses from cross-section data using the neoclassical choice theory is the study by the consultancy group better known by their acronym CESEN for ministry of Mines and Energy which reported its findings on the demand for energy for urban and rural twelve regions of Ethiopia including Addis Ababa [34].

This section discusses the attempts made in this study to estimate the parameters of the Linear Expenditure System [LES] and The Extended Linear Expenditure System [ELES] for Ethiopia.

2.1. Description of The Models

2.1.1 The Linear Expenditure System [LES]

The LES is the earliest empirically estimated demand system and in fact the most popular one in the literature. It is specified generally as,

$$p_i x_i = \gamma_i p_i + \beta_i (y - \sum_{k=1}^{n} \gamma_k p_k)$$

$$\sum_{i=1}^{n} \beta_i = 1. \qquad [7]$$

Where, γ_i and β_i stand for constant parameters. γ_i is quantity of commodity i necessary for subsistence, and β_i is known as marginal budget share, i.e., it is equal to the value of the derivative of (7) with respect to income. Normally the bracketed term in [7] is understood as "supernumerary expenditure" (Goldberger and Gamaletsos [37]). As shall be explained later, subsequent manipulations of (7) imply a linear Engel curve, i.e., no inferior goods, and quasi-homothetic utility function. Indeed there is a proportional relationship between income elasticities and price elasticities implied by the LES; this is shown in different forms by Frisch [35] and Deaton [23]. Empirical evidences shade mixed results. The surveys (Brown and Deaton [10], Deaton [25,23]) suggest that in most cases the income data fitted well and price responses tended to be unstable and statistically less significant. In general however, it performed quite satisfactorily for broadly grouped commodities. A cross-country comparison by Golderberger and Gamaletsos [37] , and Lluch and Powell [48] seem to support the LES in explaining consumption patterns. When one considers the visible data limitations in developing countries and the limited substitution possibilities in the specifically poor ones the empirical limitation of the LES is less restrictive.

The LES in [5] has 2n-1 independent parameters to be estimated (n-1 β_is and n γ_is) owing to the budget constraint. Pollak and Wales [57] argued that a one period budget study identify all but one of the independent parameters. Without additional information,

or restrictions therefore, the parameters of LES are under-identified. A two-period budget study however exactly identifies all parameters of LES. In other words, two budget studies give two Engel curves each with β_i estimates. Their intersection uniquely determines the n y_is (Pollak and Wales [57, P.350]. This line of reasoning has been taken up later on to formally prove the proposition that " knowing two Engel curves is typically sufficient to determine [the parameters of] an additive utility function" Dybrig [31, P. 379]. Therefore, one can estimate the parameters of other general systems as the Quadratic expenditure system and the Translog expenditure system from just two budget studies. In fact, since Frisch's demonstration of the possibilities of estimating cross-and-own price elasticities from one budget study, various studies emphasized the wider and deeper implication of Engel curves. Tsujimura and Sato [69] used Engel curves not exceeding six to recover price responses using the notion of numerical preference in the spirit of Dusenberry's Theory of Interdependent Preferences for Japan's Budget studies. The works of Deaton [57] in proving formally the implication of additive preferences with regard to the relationship between income and cross elasticities, and the insightful observations Pollak and Wales [57] made on the empirical properties of Engel curves opened new venues in the area of the Identification and Recoverability theory discussed in Dybrig [31]. In this context the crucial role the Frisch parameter plays needs mentioning in identifying all parameters of demand systems generated from additive preferences. If one justifiably fixes a value to this parameter ,then, one cross-section study is enough to generate estimates of price and income elasticities(see Getachew [36] for the application to Ethiopia). In this particular model, however, two period price information is sufficient to directly estimate the Frisch parameter from the data, instead of fixing it from outside of the model. Details of the computation are given in Appendix A.

2.1.2 The ELES

Lluch [46] suggested the Extended Linear Expenditure System in an attempt to capture inter temporal decision problems in influencing consumption behaviour (this is an area of great controversy and numerous research work in economics literature; one need only be reminded of Patinkin's and Pigou's real balance effects). Howe [41], developed

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the ELES by adding savings as the n+1 th commodity with the assumption that $\gamma_{n+1} = 0$, i.e., the subsistence saving amount is zero. With these modifications, the LES in (7) may be rewritten as:

$$p_i x_i = p_i \gamma_i + \beta_i (Z - \sum_{k=1}^{n} p_k \gamma_k) \qquad [8]$$

$$S = \beta_{n+1} (Z - \sum_{k=1}^{n} p_k \gamma_k) \qquad [9]$$

Where, Z stands for total income earned over a certain period of time; and S is for savings, and, (9) results because $\gamma_{n+1} = 0$. Further more, we may consider β_{n+1} as marginal propensity to save.

From (8) and (9), we infer that,

a. The budget constraint with equality is satisfied only if we make necessary modifications;

b. On the basis of (a), all parameters are exactly identified from a single cross-section study if there is information on savings. Case (b) is seen to hold as follows:

If we let

$$\mu = \sum_{i=1}^{n} \beta_i \qquad [10]$$

then,

$$Z = \frac{(1 - \mu) \sum_{k=1}^{n} p_k \gamma_k + S}{(1 - \mu)} = \sum_{k=1}^{n} p_k \gamma_k + \frac{S}{1 - \mu} \qquad [11]$$

Therefore, if we reformulate [7] as,

$$p_i x_i = p_i \gamma_i + \beta_i^* (Z - \sum_{k=1}^{n} p_k \gamma_k) \qquad [12]$$

where,

then, by using a single regressand, Z and 1 for intercept term for the n goods we shall be

$$\beta_i^* = \frac{\beta_i}{\mu} \rightarrow \sum_{i=1}^{n} \beta_i^* = 1 \qquad [13]$$

able to have 2n independent parameters which is the exact number of independent parameters in the ELES: μ, n-1 β_i s and n γ_i s. The inclusion of savings therefore played the crucial role of solving the identification problem in system equations.

2.2. The Theoretical Basis of LES , and ELES, and Other Important Derivations.

The LES and ELES are generated , respectively from the Stone-Geary type utility functions, or any of their positive monotone transform which are generally given as:

$$U(X) = \sum_{i=1}^{n} \beta_i \ln(X_i - \gamma_i) : \quad \sum_{i=1}^{n} \beta_i = 1. \qquad [14]$$

and,

$$U(X) = \sum_{i=1}^{n} \beta_i \ln(X_i - \gamma_i): \quad \sum_{i=1}^{n} \beta_i = 1, \gamma_n + 1 = 0. \qquad [15]$$

Using Roy's identity, and Shephard's Lemma it is easy to verify that the Indirect Utility and Expenditure functions corresponding to the above utility functions, respectively, are given by:

$$v(P,y) = \sum_{i=1}^{n} \beta_i [\ln \beta_i (y - \sum_{k=1}^{n} p_k \gamma_k) - \ln p_i]$$

$$e(P,u) = - \frac{U \prod_{k=1}^{n} p_k^{\beta_k}}{\prod_{k=1}^{n} \beta_k^{\beta_k}} + \sum_{k=1}^{n} p_k \gamma_k \qquad [16]$$

Regularity conditions that must be exhibited by the cost function (like concavity, homogeneous of degree 1) in line with concave optimization (duality) impose the following restrictions on LES parameters:

1. $0 < \beta_i < 1$,

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2. $\gamma_i \le X_i$

3. $\sum \beta_i = 1.$

The above restrictions imply a linear Engel curve, i.e., no inferior goods, and leads to the following derivations the details of which are given in Goldeberger and Gamaletsos [37], and also in Lluch and Powell [48] :

a. $E_i = \dfrac{\beta_i}{v_i}$

b. $E_{ij} = \phi E_i - E_i v_j (1 + \phi E_i),$
$\qquad \qquad i = j$
$\qquad = -E_i v_j (1 + \phi E_j)$
$\qquad \qquad i \ne j$

c. $E_{ij}^* = E_i (1 - \beta_i) \phi,$
$\qquad \qquad i = j$
$\qquad = -E_i \beta_j \phi,$
$\qquad \quad i \ne j$ [17]

Where, E_{ij} ($i = 1, ..., n$)= Uncompensated price elasticity of demand for commodity i with respect a change in the price of commodity j;

E_{ij}^* ($i = 1, ..., n$)= Compensated price elasticity of demand for commodity i with respect to commodity j.

ϕ = " supernumerary" ratio, or the reciprocal of the Frisch Parameter(ω), or the Money Flexibility of income , the expression of which is given by the value of the Lagrangian Multiplier in constrained maximization of the relevant direct utility function.[5]

It is not difficult to see the important role ϕ plays in linking price elasticities with income elasticities. As a measure of welfare , Frisch argued that as real expenditure grows, ω declines from a bigger negative value to a smaller one, i.e., a fall in the absolute value of ω overtime is regarded as an improvement in the standard of living of households.

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2.3 The Data and Method of Estimation.

2.3.1. The Data

Ethiopia does not have the practice of reporting aggregate expenditure on selected groups of commodities in the national accounts. Estimation of Systems of Demand Equations had to rely on data generated through budget studies. To the author's knowledge, published budget studies exist only for the years 1975, 1979, and 1980 for Addis Ababa region; and for the years covering 1980-1981 for the Ethiopian rural regions, excluding Tigrai. All these are published in the Statistical Bulletins [12],[13], and [14] of the Central Statistics Authority, respectively.

All the budget studies use the household as baseline , and the details of the method of data collection and the sampling techniques utilized are given in the Bulletins mentioned. The important information provided in the Bulletins are: expenditure on broadly aggregated groups of commodities by different expenditure and income groups ; savings in the form of Iqub (the traditional practice of holding cash on a non-interest bearing fashion), and bank accounts; household size and age by expenditure group. The demographic-expenditure matrix is incomplete to offer an opportunity to incorporate family size by including number of children in the models. The 1975 data is part of a series of well intended attempt by the Central Statistics Office (CSO) and the then National Revolutionary Campaign and Central Planning Council(NRDCCPC) to conduct a national budget study survey that would cover 30 towns of Ethiopia, including Addis Ababa. Thus far, only the budget study for Addis Ababa was published in Statistical Bulletin 19 of the CSO [12]. The uses into which the data were meant to be put could not better be explained than what was stated in the Bulletin itself [12, P.2], " i. To examine the pattern of consumer behaviour and provide up-to-date and accurate weighting diagram for the compilation of consumer price indices; ii. To utilize the result of Addis Ababa survey in the revision of the 1963 based Addis Ababa Cost of Living Index; iii. To compare consumption expenditure per household per month for the towns included in the survey; iV. To supplement the sources of information used in compiling the official estimates of national expenditure in order to improve national income estimates". Eventhough evidence is lacking to what extent these objectives were met,

Getachew [36] employed this data quite satisfactorily in his investigation of demand pattern for Addis Ababa households. The sample units covered in this survey were employees of industrial establishments and the civil service[9]. The 1979 and 1980 data were generated for quite specific purpose of analyzing energy demand and on this basis to draw energy policy options for the 1990s and beyond by making demand forecasts. The sampling units in these data include self-employed households in the business of trade, handicrafts and other household industrial units, apart from employees of government offices and the public sector.

The relevant price data utilized were taken from Addis Ababa Retail Price Index published in the Statistical Abstract [15]. The entire data set employed in this study and details of estimation procedures undertaken are available from the author on request.

2.3.2. Estimation Methods.

The identification problem briefly explained in 2.1.1 and in 2.1.2, and discussed in Appendix A suggest also the possible method of estimation. For model in 2.1.2, the only efficient method of estimation is Indirect Least Squares[ILS], i.e. estimates of β are obtained by Ordinary Least Squares method equation by equation , and structural estimates are derived from reduced ones. It is a matter of simple manipulations to show that the ILS is BLUE (see, e.g. Jhonston [42, P.469]). Generalized Least Square methods are just unnecessary because of the identical regressand in all equations. The Model in 2.1.1.(i.e., LES) however can be estimated alternatively by ILS and, following the procedure developed by Parks [54] by Maximum Likelihood Estimator [MLE]. The ILS procedure involves estimating the marginal budget shares for two periods by OLS and using the price information from outside of the model to derive the estimates of the structural parameters. The MLE procedure obviously uses the price information within the model, though it can not be in the Full Information MLE sense because of the very limited price information available. Therefore equation by equation estimation of error terms may have to be undertaken in line with SURE (Serially Uncorrelated Regression Estimates) models. In this paper the procedure followed in this case too is the ILS for the additional advantage it has

in computation simplicity, notwithstanding its appropriateness when system equations are exactly identified.

III. Analysis of Empirical Results.

The LES and ELES parameters were estimated for the groups of commodities given in table 1 below. The commodity groups were selected on the basis of the convenience provided in having complete information to compare the two models. Further more the level of aggregation is in line with the preference structure implied by LES, which is additive by construction. Price elasticities tend to be dominated largely by the income effect, rather than by the substitution effect when commodities are highly aggregated. The explanatory variable used for estimation purposes in the case of ELES was total income per person, and prices and total expenditure per person were used for LES; the dependent variable in both cases being money expenditure on the ith commodity per person.

Table 1:**Key To Commodity Classifications.**

Commodity Groups	Description
1. Food	Food, Drinks, Tobacco
2. Housing Items	Housing Durable,Fuel,Water and light.
3. Clothing	Clothing and Foot Wear.
4. Transport	Same
5. Personal Care	Personal Care & Effects
6. Recreation	Recreation & Reading.
7.Others	Excluding medicare & rent .

3.1. Estimates of Marginal Budget Shares, Cost of Subsistence, and Income Elasticities.

Estimates of Marginal Budget Shares, Income Elasticities, and other important parameters of the LES and The ELES are given for 1975, 1979, and 1980 budget sets for Addis Ababa, and for 1980 Rural Ethiopia[10] budget set in Tables 2,3,4, and 5, respectively. It is seen that nearly all are statistically significant[11] and comply with the restrictions stated for regularity purposes. The ELES was applied to the 1975 Addis Ababa Household Budget data and to the 1980 Rural Household Budget data, and the LES for the remaining budget sets. The main reason being that estimates of total cost of subsistence, which plays a crucial role in estimating the Frisch parameter, resulted in a negative value under ELES for the 1979, and 1980 budget studies for Addis Ababa creating a problem of interpretation (In fact negative intercept terms for Engel functions imply Luxury goods if the absolute value is within an acceptable range, see e.g., Deaton and Muellbauer [27]). Alternatively, the LES offered a better estimate particularly in the context of the Frisch parameter.

TABLE 2- Estimates Of Selected Parameters Of The ELES[*]
(1975 Budget Study For Addis Ababa)

Expenditure Group	$p^*\gamma$ (in Birr)	R	V_i	E	β[**]
1.Food	8.8668	.88265	.5387	.7895	.4253 (.03887)
2.HH. Items	1.55893	.76135	.1558	.984	.1558 (.02151)
3.Clothing	1.8409	.61135	.1066	.813	.08667 (.01733)
4.Transport	-5.91065	.77828	.0771	2.2951	.17695 (.02367)
5.Personal Care	-.13525	.73807	.0144	1.1958	.01722 (.002571)
6.Recreation	-1.8297	.68050	.0251	1.7532	.07907 (.01358)
7.Others	.0971	.64310	.0623	.9865	.06146 (.01147)

[*] The number of income group cells used are 15 for statistical reasons.
[**] figures in bracket are standard deviation estimates

TABLE 3 - Estimates of Selected Parameters of The LES*
(1979 Addis Ababa Budget Study)

Items	β_i**	$P_i \gamma_i$	V_i	E_i	R^2
1.Food	.48996 (.04081)	16.7109	.5322	.9206	.8472
2.HH Items	.13009 (.02939)	8.176	.1971	.66	.4298
3.Clothing	.090802 (.0227)	2.6113	.0914	.9935	.3811
4.Transport	.1432 (.0028)	1.2886	.1019	1.4053	.62245
5.Personal Care	.00978 (.00279)	.1205	.0077	1.2701	.4298
6.Recreation	.04133 (.00648)	-.6226	.0146	2.8308	.60998
7.Others	.0948 (.01137)	.0245	.0551	1.7205	.7279

* The number of income group cells used are 28.
** The Figures in the bracket are standard error estimates.

TABLE 4. Estimates of Selected Parameters of LES*
(1980 Budget Study For Addis Ababa)

Items	β_i**	$P_i \gamma_i$	V_i	E_i	R^2
1.Food	.3761 (.01777)	19.161	.4673	.8048	.9492
2.HH Items	.17298 (.02332)	7.294	.1957	.8839	.6963
3.Clothing	.1019 (.01244)	2.9101	.0979	1.0408	.7365
4.Transport	.08365 (.0287)	-.7882	.0405	2.0654	.2618
5.Personal Care	.0776 (.00829)	2.6252	.0797	.974	.785
6.Recreation	.07861 (.01081)	-.6759	.0389	2.0209	.688
7.Others	.10913 (.01259)	1.1258	.0799	1.3658	.7579

* The number of income group cells used are 26.
** The figures in parenthesis are standard error estimates.

TABLE 5: **Estimates Of Selected Parameters Of The ELES[*]**
(Rural Household Budget Study 1980)

Item	β_i [**]	$p_i\gamma_i$ [***]	V_i	E_i	R^2
1.Food	.6741 (.0178)	108.195	.7034	.9583	.98623
2.HH Items	.1647 (.0047)	33.1743	.1902	.8659	.98399
3.Clothing	.068 (.0075)	10.307	.0692	.9827	.803656
4.Transport	.0207 (.0045)	-1.2824	.0091	2.2747	.52455
5.Personal Care	.0704 (.015)	-5.6013	.0274	2.569	.52469
6.Recreation	.002 (.0021)	-.1978	.0008	2.5	.39654

* The number of income group cells used are 22.
** The figures in parenthesis are standard error estimates
*** The "cost of Subsistence " figures are for a year, whereas for Addis Ababa Budget studies it is for a month.

Between 1975, 1979, and 1980 no significant change is observed in the values of β for Addis Ababa. This more or less supports the LES. In nearly all cases, Food expenditure took the highest share, and was followed by Housing Items expenditure. Transport expenditure took second place for 1975 and 1979 budget study though its share fell substantially in 1980. Expenditure share on Clothing remained nearly unchanged.

There is a causal relationship cutting across marginal budget shares, average budget shares, cost of subsistence and thus income or expenditure elasticities under LES. To see this, differentiating average budget shares expressions for LES with respect to income (expenditure) establishes that average budget share increases (decreases) as expenditure increases if total cost of subsistence multiplied by marginal budget share of the ith commodity exceeds (is less than) the cost of subsistence of commodity i. Following this derivation, one observes from Table 2 that for 1975 data the average budget share declined as real income increased for Food, Housing Items, and Clothing. For the other expenditure groups this value showed a positive correlation with income (expenditure). This is more

or less supported by the average income elasticity values given in Table 2. For 1979, average budget share declined with respect to real income for all commodity groups excepting for Recreation, showing a moderate discord with mean income elasticities. The same result also holds true for the 1980 Addis Ababa budget set as is clear from Table 4. The one reason for the inconsistency between the rate of change in the average budget shares with respect to income ($\delta v / \delta y$) and income elasticity for 1979 and 1980 data sets could be the use of sample averages instead of weighted mean income to calculate the income elasticity values. It is important to mention the fact that for the sample data 1979 and 1980 income distribution was biased against the low income group[12].

According to estimated average income elasticities for Addis Ababa, except for Food and Housing all commodity groups were luxury items in 1975; and in 1979 and 1980 Clothing became a necessity, and the rest conformed with 1975 situation. The picture for the rural household is quite revealing in many ways. Average budget shares declined for Food, Clothing and Housing expenditures, and increased for Transportation, Personal Care and Recreation with income. This more or less accorded with the income elasticity estimates. Food and Clothing for the rural household in 1980 were near-neutral commodities implying a unitary income elasticity. Also, both marginal budget shares and average budget shares values for Food were very high compared with that of Addis Ababa households. Rural households generally spent a significant amount of their income on Food. This result is not surprising in the setting of a subsistence economy. In fact, a 67% marginal or average budget share is not high for a household that produces and consumes generally Food items. It rather questions the strength of the edifice upon which the assertion that Ethiopian rural household is subsistence is formed. Housing is another expenditure item that was a necessity according to the income elasticity values. Considering the adequate statistical significance of parameter estimates, this result could not be by accident. We may conjecture that Housing items in the rural community do not make much of a difference across various income groups. Other expenditure groups are all luxuries to the rural community; including transportation. It may be a bit hasty, but tempting to note the limited market potential available for manufacturing items and services in rural Ethiopia during that period. In general, the very high income elasticity values (even for necessities) for both urban and

rural households depicts what is expected of a typical poor developing country scenario and in the process of economic growth these values tend to fall (see, for instance, LLuch and Powell [48]).

3.2. Saving Parameters and Welfare Implications of LES

One of the uses of parameter estimates from system of demand equations is the inference one may make about important features of the household. Estimates of marginal propensity to save (MPS) from ELES model for the 1975, 1979, and 1980 budget studies for Addis Ababa and for the rural household budget study are given in Table 6. These values for Addis Ababa vindicate the common perception that since 1975 things in the economy have gone really bad and rising prices eroded the purchasing power of households income which affected significantly their saving behaviour. MPS declined from .0964 to .0295 in these periods.

Table 6:Estimates of MPS and Frisch Parameter

Year	Estimate of MPS*	Estimate of Frisch Parameter(ω)	Total Average cost of Subsistence (birr per month)
1975(A.A)	.0964	-1.1777(ELES)	8.51
1979(A.A)	.0231	-1.7325(LES)	28.3246
1980(A.A)	.0295	-1.6592(LES)	31.7269
1980(Rural)	.0678	-1.5461(ELES)	11.29

* ELES model.

In contrast to Addis Ababa the MPS of the rural households in 1980 was remarkably higher(about .07). Here again the common belief that rural households in Ethiopia seldom save, and when they do they spend it on investment items is brought to question. As explained in section 2.3.1 the saving data does not include investment expenditure or other forms of saving than cash. When we consider the fact that the process of saving determination by households works through the substitution effect (i.e., effect of alternative forms of savings on the decision of holding savings in a cash form) , and the income effect

(as is the case in here) then, the value of MPS estimated for the rural households here indicates existence of some potential for mobilization in that period. It was, in fact, pointed out (e.g. Brüne [11]) that Ethiopian agriculture did better in 1979 and 1980 than in the years immediately before or after. This result, combined with estimates of Marginal Budget share, therefore call for a rethinking about the degree of monetization assumed to exist in the rural community in that period. Even more so, effort has to be made to closely study household saving behaviour for planning purposes in view of the growing conviction that the assumptions underlying aggregate saving behaviour, which is frequently used, are in practice untenable (see, Schimdt-Hebbel et.al [64] for a detailed discussion of the inappropriateness of relying on aggregate saving behaviour which hinge upon the Ricardian Equivalence assertion).

The process of estimating ω under LES is closely related with the estimates of total cost of "subsistence" and the mean income. The former is crucial when it comes to the proper sign ω should take so that the LES demand system satisfies all regularities needed by the choice theory. For this reason, LES and ELES models were alternatively applied as explained in section 3.1, to the budget studies where they appropriately satisfied these restrictions. A similar study by Getachew [36] for the 1975 budget study estimated the Frisch parameter outside of the demand system. The absolute value of his estimate for the Lower income group households was 3.02, which can be seen to be largely at variance with our estimates. This is not a surprising result given the different methodologies employed and the sensitivity of the Frisch Parameter itself to various demand specifications. There can be no dispute, however, regarding the advantage of direct methods over indirect ones in retaining relative accuracy. In concluding his work (which was probably the first of its kind in the Ethiopian context), Getachew [36] observed that " Finally, to the knowledge of this writer, no study seems to have been done to estimate the money flexibility [i.e., the Frisch parameter] for Ethiopia. In this paper this value was interpolated from the results of other studies undertaken elsewhere. It would be a worthwhile research project to stipulate this parameter on the basis of some Ethiopian data for the urban and rural population". Though long due, his call has been more or less answered by the attempts made in this study.

Estimate of the Frisch Parameter for all data sets confirm again the general impression many held about the Ethiopian Economy at least for the periods under discussion: From Table 6 it can be seen that the values of ω increased sharply for Addis Ababa data set between 1975 and 1979, and showed a slight decline in 1980. In fact, if we took account of income distribution factors to measure mean income, then, ω for 1979 and 1980 budget studies would be to the tune of -2.56 and -2.5, respectively, which further pronounces the gravity of welfare deterioration. By many accounts real income of households through these periods indeed declined quite substantially, and thus their welfare. One could not fail to notice the per person estimated cost of subsistence given in Table 6. We may go a bit further to reflect on its implication to the incidence of poverty if we take account of the percentage of people earning an income below this subsistence level from the samples. In 1980, the percentage of households in Addis Ababa earning per regularly employed an income level less than the cost of subsistence per person estimated for that year was about 43. If we make the not so unreasonable assumption that the rate of cost of subsistence between 1979 and 1980 (12%) apply to the years until 1990, and that income distribution remained neutral to real income changes in these years, then in 1990 the same percentage would come to 75. It seems that this value is not much far from those cited frequently in recent works (e.g. Eshetu [1992]) regarding intensity of poverty. In fact, there is no dispute concerning the reliability of budget studies in providing better estimate of income distribution aspects and incidence of poverty.

For the rural households ω was not much different from their urban counterparts; in fact better to some extent. A survey of the value of ω estimated for different countries can be found in Brown and Deaton [10], Goldberger und Gameltsos [37] and Lluch and Powell [48]. In almost all cases the empirical values confirmed Frisch's theoretical observation: as real income of consumers increase the marginal utility of income declines. This is what is exactly observed for Ethiopian data sets too. We may point out further here the opportunity of measuring the " True Cost of Living " index using the expenditure function. We may go about this by asking the question that how much income would an individual consumer need in order to attain the same level of utility under two different sets of prices. Suppose we define vector of prices of n commodities as $P(T)=[p_1(T),...,p_n(T)]$ and $P(0)=$

$[p_1(0),....,p_n(0)]$, respectively, for some terminal period T and initial period 0. Then, the "True" Cost of Living Index for period T and 0 is given by,

$$E(T,0) = y^*(T)/y(0) \qquad ......(18)$$

where, $y^*(T)$ is income required at the terminal period T to attain the same utility as under the initial period 0. Computation of the "True" Cost of Living thus requires existence of a utility function and its specification. In our case, since we have estimated the parameters of a demand system generated from the Stone-Geary utility function we will have no problem. Therefore, we illustrate here the application by taking 1980 as an initial period 0, and 1990 First Quarter as terminal period T. There is no special reason for choosing the periods. It can be shown quite easily that the "True" Cost of Living Index underlying the LES is given by (see Goldberger and Gamaletsos [37] for details) :

$$E(T,0) = (1 - |\Phi(0)|) \sum_{i=1}^{n} w_i^*(0) p_i(T,0) + |\Phi(0)| \prod_{i=1}^{n} p_i(T,0)^{\beta_i} \quad [19]$$

where, $w_i^*(0) = p_i(0)\gamma_i / \sum_k p_k(0)\gamma_k$ = the subsistence budget share of the ith commodity in period 0,

$-\phi(0) = y(0) - \sum_k p_k(0)\gamma_k / y(0)$ ≡ the supernumerary ratio in period 0,

$p_i(T,0) = p_i(T)/p_i(0)$ ≡ the price relative for good i, i.e., the ratio of its price in period T to that in period 0. The computations made by inserting the relevant information into equation [19] resulted in the following:

$E(T,0) = .3973 (1.6847) + .6027 (1.1516) = 1.583.$

If we note the Cost of Living Index provided by the Conventional method computed from the Retail Price Index published by the CSA we have $\pi(T)/\pi(0)$, where $\pi(T)$ and $\pi(0)$ stand, respectively, for General Price Indices for periods T and 0. In our illustration they are the years 1980 and 1990 First Quarter. Accordingly, $\pi(T) = 522.9$ and $\pi(0) = 353.5$ as given in Quarterly Bulletin [51]. The Conventional Cost of Living Index would thus be equal to 1.479. Since the "True" Cost of Living index omitted only one commodity group (medicare) out of the eight commodity group that had been used since 1963 to compute the

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General Price index, no doubt that the Conventional method understated the Cost of Living Index. As it is the omission of medicare could not have made much a difference since its share in the 85.4 total weight of the eight commodity group is a mere 1.8 (see Quarterly Bulletin [51]), and its individual Cost of Living Index alone was in the same neighbourhood (1.302) as the Conventional Cost of Living Index itself. Rather, it is interesting, more so encouraging to see that the two measures of welfare changes came up with a value quite close to each other given the distinct approaches used in their evaluation. This is further pronounced if we note the same Cost of Living Indices given by the 1979 estimated parameters of the LES and the Conventional Cost of Living Index for the years between 1979 and 1990 First Quarter. They were computed to be, respectively, 1.59 and 1.54, and so much closer as it could be. Therefore we may be at least rest assured to continue to use the price series published by CSA with the due reservation as far as the group of commodities under consideration are concerned. A word of caution has to be made however here. The fact that both measures of welfare conveniently left out the rent component of household expenditure, the above estimates severely understate changes in Cost of Living as a result of rising housing rent. If, however, there were a reasonable mean estimate of rent paid by households in 1980, then the LES could incorporate rent in the calculation of Cost of Living Index since all budget studies did report per person mean expenditure of households on rent for all commodity groups. Not only house rents, but also the inclusion of expenditure on children (including school fees, etc.) improve parameter estimates and specially offer a very useful insight into the formulation of optimal indirect tax structures as is discussed in Ray [60]. From our result above, we could only say that Cost of Living deteriorated the welfare of the fixed-income earning households in the concerned periods so much that they would have needed a nominal increase in income of about 60% in 1990 just to remain as well-off as they had been in 1980. Considering that salaries and wages remained unchanged for the large part of these periods to many households, one could only imagine the consequent human misery and plight following the inflationary process. As a final remark, it might be much more instructive to support results of the estimated "true" Cost of Living Index here by estimating alternative demand systems using the available budget studies.

In concluding this sub-section what can be said for the moment is that demand systems generated as such are very useful indicators of welfare changes of households to policy makers, planners, the government and others concerned in the issue. The existence of poverty alone and its implicit acknowledgement may not suffice in the realm of plan formulation. Its intensity and magnitude have to be established to justify quantitative plan targets and to determine direction of policy actions. We may cite the concern Indian Planners showed to the incidence of poverty and its implication for the planning process, specially with regard to the setting of growth objectives (see,e.g., Perspective Planning Division [55], and Mahalanobis [49]). It need not be argued that at initial stage, economic growth worsens the degree of income inequality[13], and therefore in a setting of abject poverty, no serious plan would rather dispense with income distribution matters when fixing the GNP growth rate. In the preparation of the Ethiopian 10th Year Perspective Plan [52], there was no an explicit attempt to treat issues of income inequality at all. Ironically, the domestic resource required to realize the GNP growth rate stipulated that households would cover a substantial percentage of total domestic saving the manner of whose determination had never been studied. In his observation of the growth constraint to the Ethiopian economy Eshetu [33, 32] emphasized the lack of surplus to be mobilized for investment and of market for manufacturing goods due to low level of income of households. It may be added here that even the available savings may not be transferred quite easily into investment the way the plan assumed, since households saving behaviour is not the same as the government's. It is therefore essential that concerned government authorities start in earnest to generate reliable data for the purpose of studying household consumption pattern and saving behaviour in order to get a better picture of the growth options open to the country.

3.3. Estimates of Own-and Cross Price Elasticities.

Price responses from LES models may be computed using the relations given in equation 17. Accordingly estimates of uncompensated own-and-cross price elasticities evaluated at sample mean are given in Tables 7,8,9, and 10 for the periods under investigation. It is apparent from the Tables that all uncompensated own price elasticities

are negative in absolute conformity with the utility basis of LES model. In fact the problem of finding a price response that are consistent with theoretical parametric restrictions have been discussed in Goldeberger and Gamaletsos [37], Lluch and Powell [48] and many others(see e.g. Brown and Deaton [10], and Blundell [9]). Therefore, in this regard LES and ELES model did not violate the theoretical restrictions imposed. If we look at the absolute values of estimated own price elasticities we notice the relatively higher magnitude for the 1975 Addis Ababa budget study than for 1979 and 1980. This difference has been no doubt because of the varying information used to identify the parameters of LES. The price information utilized for 1979, and 1980 gave a better result as is expected from a seven-group commodity classification. That is, own price elasticities could be well understood if they are inelastic for a commodity which is highly aggregated for the within substitution effect has been suppressed by the additive assumption in LES. On the other hand LES parameters for 1975 budget study were identified using saving information which has lesser role in measuring price responses than income responses. According to Tables 7,8,9 Food, Clothing and Housing Items were price inelastic in all cases, and Recreation price elastic. For the rural community the picture is that only Personal Care and Recreation were price elastic, the remaining being price inelastic, and one can explain the reason easily too.

Table 7: Estimate of Uncompensated Own-and-cross Price Elasticities
Addis Ababa Budget Study (ELES), 1975

	1	2	3	4	5	6	7
1	-0.8106	-0.0202	-0.0261	-0.0002	0.0578	0.0174	-0.0082
2	-0.1748	-0.8394	-0.0325	0.0002	0.0720	0.0216	-0.0099
3	-0.1444	-0.0208	-0.7171	0.0002	0.0595	0.0179	-0.0080
4	-0.2124	-0.0306	-0.0395	-1.0152	0.0875	0.0263	-0.0121
5	-0.4076	-0.0588	-0.0757	0.0005	-1.7808	0.0505	-0.0232
6	-0.3114	-0.0449	-0.0579	0.0004	0.1283	-1.4500	-0.0177
7	-0.1752	-0.0253	-0.0326	0.0002	0.0722	0.0217	-0.8382

Table 8: Estimate of Uncompensated Own-and-cross Price Elasticities
Addis Ababa Budget Study(LES) 1979

	1	2	3	4	5	6	7
1	-0.761	-0.11253	-0.0359	-0.01777	-0.00189	0.00850	-0.0004
2	-0.1646	-0.46140	-0.0257	-0.01270	-0.00140	0.00610	-0.0003
3	-0.2478	-0.12120	-0.6122	-0.01910	-0.00200	0.00920	-0.0070
4	-0.3505	-0.17150	-0.0548	-0.83820	-0.00290	0.01300	-0.0005
5	-0.3167	-0.15498	-0.0495	-0.02190	-0.73570	0.01176	-0.0005
6	-0.7059	-0.34540	-0.1104	-0.05450	-0.00580	-1.60780	-0.0011
7	-0.4291	-0.20995	-0.0669	-0.03310	-0.00360	0.01600	-0.9934

Table 9: Estimate of Uncompensated Own-and-cross Price Elasticities
Addis Ababa Budget Study (LES) 1980

	1	2	3	4	5	6	7
1	-0.6787	-0.07360	-0.0294	0.00800	-0.0265	0.0068	-0.01135
2	-0.2127	-0.61400	-0.0323	0.00875	-0.0291	0.0075	-0.01250
3	-0.2505	-0.08080	-0.6653	0.00880	-0.0291	0.0075	-0.01250
4	-0.4971	-0.18888	-0.0754	-1.22244	-0.0679	0.0176	-0.02910
5	-0.2340	-0.08900	-0.0356	0.00960	-0.6193	0.0083	-0.01370
6	-0.4863	-0.18470	-0.0738	0.02000	-0.0665	-1.2006	-0.02850
7	-0.3287	-0.12480	-0.0499	0.01350	-0.0449	0.0116	-0.84250

Table 10: Estimate of Uncompensated Own-and-cross Price Elasticities
Rural Household Budget Study (ELES) 1980

	1	2	3	4	5	6
1	-0.8761	-0.0241	-0.0802	0.00410	0.01740	0.00050
2	-0.2628	-0.6604	-0.0740	0.00422	0.01780	0.00050
3	-0.2315	-0.0218	-0.6326	0.00371	0.01574	0.00070
4	-0.6083	-0.0573	-0.1904	-0.14615	0.04130	0.00114
5	-0.6870	-0.0647	-0.2150	0.01102	-1.61540	0.00130
6	-0.6690	-0.0630	-0.1885	0.01070	0.04540	-1.61560

By construction, all cross uncompensated price elasticities under LES and ELES have to be negative in view of the regularity conditions delineated in section II. This means that all goods are gross complements, implying income effects completely swamp off substitution effects. From our empirical estimate we observe that Food, Clothing and Housing were gross complements in all cases, and Entertainment a gross substitute with all other commodities. Personal Care expenditure and Transport were gross substitutes to the rural household. The issue of complementarity and substitutability may not have to be taken literally, more so in case of an additive preference, as in LES. In fact, we may reflect for a moment on the intuitive interpretation of our result by observing the fact that goods are gross complements if substitution in consumption is less important than the purchasing power implication of a change in price. This is mainly true of goods that are either independent or complements in consumption. By construction, our estimated demand system rules out complementarity. Thus, Food, clothing and Housing may be independents in consumption but complements in income effect because of their relative importance in total budget and on the average they are necessities to the individual consumer. The LES also has a clear implication to the issue of compensated and equivalent variation regarding consumer's surplus discussed in detail in Seade [63] and the distinguishing feature between Marshallian demand function and compensated demand function as in Hicks [39].

For the empirical application of price responses a recent study by Cragg [19] to issues of indirect tax reforms pointed out the differences between those obtained through additive preferences and those from semi-flexible functional forms like the normalized quadratic expenditure system in terms of their implication to aggregation. We shall nevertheless draw the reader's attention to the remarkable result of Deaton [22, p. 360] that says " Provided preferences are weakly separable between goods and leisure, and provided all consumers have parallel Engel curves for goods in terms of income, then, the differential commodity taxation is unnecessary given an optimal linear tax. Further, if there exists a separable group of goods for which the within group Engel curves are linear, then, all goods within-group should be taxed at the same rate." While this is a subject that indeed needs a separate treatment of its own, we may just reflect here on what the linearity of Engel curves implied by the LES, whose parameters we estimated, might mean to empirical

issues such as optimal tax rate determination. We have come quite close in supporting uniform commodity indirect tax rates for all commodity groups in question! If some more generalized systems like the quadratic Expenditure system and translog were estimated using the already available data sets there might be a broader room for making comparisons to the direction of reform the Ethiopian indirect tax system should take. One more constraint, however, that can be anticipated is how to find reliable actual tax rate for the groups of commodities considered in this study in the absence of published input-output data for Ethiopia. Nevertheless future investigation in this area is worth a try if at all we are supposed to know the way various economic variables are linked to each other in our economy under alternative settings.

Table 11: **Percentage Share of Own-Price Elasticities to Expenditure Elasticities for Addis Ababa Budget Studies**

Commodity group	1975	1979	1980
1	103	83	84
2	85	70	69
3	85	62	64
4	88	60	59
5	78	58	64
6	83	57	59
7	85	58	62

Further more, based on the Homogeneity property of demand systems we may note from Table 11 the relative importance of own price elasticities in the overall elasticity measures. Moreover, the importance of Food price increased between 1975 and 1980 in total expenditure elasticities, being in the order of 18% and 25% to total income elasticity accentuating further the sensitivity of overall price-elasticity to changes in the price of Food.

Finally, making use of the Slutsky decomposition we can compute the compensated price elasticities. Again as can be verified from Appendix C all compensated own price

elasticities were negative and compensated cross price elasticities positive, i.e., goods were net substitutes. All budget studies satisfied this condition in our empirical estimate. The issue of symmetry of the Slutsky substitution matrix implies unitary price elasticity and thus equality between marginal budget shares and average budget shares; this can occur if only Engel curves pass through the origin. Nevertheless concavity of the Substitution matrix is satisfied if marginal budget shares sum up to one, and all lie within specified range. In our case this has been satisfied in all cases.

CONCLUSION

The LES and ELES have been estimated using budget studies for Addis Ababa and Rural households for the years 1975,1979, 1980, and 1980-81, respectively. Parameter estimates to a larger extent satisfied the theoretical restrictions inherent in the models. An important insight was gained from the empirical results regarding the saving behaviour and consumption pattern for groups of aggregated commodities by urban as well as rural households. Particularly, the saving behaviour implied by ELES for Ethiopian rural households ran against the dominant perception ordinarily held by most of us: prevalence of subsistence consumption. The discussions brought out into light the welfare implication of changes in real income through the estimation of the Frisch Parameter which confirmed its theoretical edifice.

Further more, while in the literature much effort have gone into the task of empirically testing parametric restrictions implied by the choice theory, the simplicity of the LES does not require much of a concern in this regard. The Homogeneity and Aggregation restrictions are immediate results of the model. Concavity (negative semi-definiteness) of the Slutsky Substitution Matrix is guaranteed if marginal budget shares sum up, as they must under OLS, to unity. Therefore, one cannot claim to have found certain restrictions satisfied empirically that are already built in the model. However, important restrictions embodied in the utility function that generated LES need to be tested empirically. Regarding this, restrictions on marginal budget shares have all been satisfied

by our estimates; estimated cost of subsistence obeyed the restrictions implied by the model. As a result, we can say the LES and ELES more or less fitted well into budget studies for Ethiopia for the period under investigation.

Estimates of own-and-cross price elasticities gave results that are normally expected from the models except in some cases. In fact, goods were found to be gross complements if they were necessities and gross complements if they were luxuries. Of particular interest is the continuum observed regarding price responses between urban and rural households for selected commodities, especially in regard to gross complementarity.

Finally, the possibility of estimating generalized system like the Quadratic Expenditure System and the Translog Expenditure System for available Ethiopian budget studies using the Recoverability Theorem discussed in Dybrig [31] is an issue of interest for future endeavour. In this regard one may proceed to study the Ethiopian indirect system and incidence of poverty due to rising cost of living among different income groups using estimated demand systems.

APPENDIX A

Derivation of the Reduced Forms of LES and ELES

The LES demand system may be written in stochastic form :

$$p_{it}x_{it}^h = p_{it}\gamma_i + \beta i (y_t^h - \sum_{k-1}^n p_{kt}\gamma_k) + e_{it}^h \qquad [1]$$

h= 1.....H

t= 1,....T

where, N is for the total number of commodities; H is for the total number of households and T is for time in years.

Econometric estimation of the parameters in [1] involves the pooling of cross-section and time series data, and standard Feasible Generalized Least Square methods are applied. The non-linearity within the parameters are taken care of following the algorithms developed by Parks [54]. The essential point to mark in [1] is the singularity of the Covariance matrix if its dimension is NxN. Barten[6] showed in the context of the Rotterdam model that dropping any one of the equations solves the singularity. Thus one can proceed with the estimation process along these lines. However, when the price information is limited to just a pair, the reduced form lends as well a convenient procedure to estimate all parameters of [1] through Indirect Least Squares. In fact, two Engel curves provide estimates of each periods β_i. Their intersection uniquely determines the n γ_i s (Pollak and Wales [57; 350]). The reasoning that there is a one to one relationship between the number of equations, number of parameters to be estimated and types of demand systems run deep int the Recoverability and Identification problem discussed in Dybring[31]. Thus we may write [1] for each period,0 and 1 , respectively as : where e_{i0} and e_{i1} are stochastic error terms, respectively for the ith commodity of the hth household in period 0.and 1. The rest of the notations take the definitions used in the text.

$$(p_{i0} \ x_{i0})^h = p_{i0}\gamma_i + \beta_{i0}(\ y_0^h - \sum_{k-1}^{n} p_{k0}\gamma_k \ ) - e_{i0}^h$$

$$(p_{i1}x_{i1})^h = p_{i1}\gamma k + \beta_{i1}(y_1^h - \sum_{k-1}^{n} p_{k1}\gamma_k) + e_{i1}^h$$

$$....[2]$$

And let.

$$\theta_{0i} = p_{i0}\gamma_i - \beta_{i0}\sum_{k-1}^{n} p_{k0}\gamma_k$$

$$\theta_{1i} = p_{i1}\gamma_i - \beta_{i1}\sum_{k-1}^{n} p_{k0}\gamma_k \qquad\qquad [3]$$

so that, we may rewrite [2] as :

$$(p_{i0}x_{i0}^h) = \theta_{0i} + \beta_{i0}y_0^h + e_{i0}^h$$

$$(p_{i1}x_{i1}^h) = \theta_{1i} + \beta_{i1}y_1^h + e_{i1}^h$$

$$.....[4]$$

Equation by equation OLS regression of [4] on the n-1 commodities give 2n-2 independent estimates for each period. Their intersection determines one more unknown so that the two equations in [4] can individually and jointly determine the estimate of 2n-1 independent parameters which is the exact number required to identify all parameters of the LES. We may demonstrate this as follows:

Intersection of two Engel curves is given by the relation :

$$\frac{\theta_{0i}}{p_{i0}} + \beta_{i0}\frac{y_0^h}{p_{i0}} = \frac{\theta_{1i}}{p_{i1}} + \beta_{i1}\frac{y_1^h}{p_{i1}}$$

$$.....[5]$$

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set

$$\frac{y_0^h}{p_{i0}} = \frac{y_1^h}{p_{il}} = y_i^*$$

so that,

$$\frac{\dfrac{\theta_{0l}}{p_{i0}} - \dfrac{\theta_{1l}}{p_{il}}}{\beta_{il} - \beta_{i0}} = y_i^*$$

.....[6]

Using [3] and [6], and the following identity at point of intersection:

$$\sum_{k=1}^{n} p_{k0} \equiv y_0^h \wedge \sum_{k=1}^{n} p_{kl} \gamma_k \equiv y_1^h$$

we have:

$$\sum_{i=1}^{n} p_{i0} \gamma_i = \sum_{i=1}^{n} \beta_{il} p_{i0} y_i^*$$

$$\sum_{i=1}^{n} p_{il} \gamma_i = \sum_{i=1}^{n} \beta_{il} p_{il} y_i^*$$

......[7]

that completely identify all the independent parameters of LES for each period.

QED.

The ELES stochastic form is given by :

where e_i is a stochastic error term for the i_{th} commodity.

Letting $\mu = \sum \beta_i$ we have $\beta_{n-1} = 1 - \mu$, therefore.

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$$p_i x_i = p_i \gamma_i + \beta_i (Z - \sum_{k=1}^{n} p_k \gamma_k) + e_i$$

$$S = \beta_{n+1} (Z - \sum_{k=1}^{n} p_k \gamma_k) + e_{n+1}$$

$$where, \gamma_{n+1} = 0$$

[8]

$$\sum_{i=1}^{n} p_i x_i = \sum_{i=1}^{n} p_i \gamma_i + \mu (Z - \sum_{k=1}^{n} p_k \gamma_k)$$

[9]

Combining [8] and [9] and setting $p_i \gamma_i = \gamma_i^*$ and $\beta_i^* = \sum \beta, \mu$, we have:

$$p_i x_i = \gamma_i^* - \beta_i^* \sum_{k=1}^{n} \gamma_k^* - \beta_i^* Z + e_i$$

$$S = -(1-\mu) \sum_{k=1}^{n} \gamma_k^* + (1-\mu) Z + e_{n+1}$$

[10]

The reduced form in [10] has $n-1$ independent equations with $2n$ independent parameters to be estimated (n γs and $n-1$ βs and μ). By rearranging [10], it is possible to estimate $2n$ independent parameters using Z and 1 as regressands from a cross section budget study. Since [10] is exactly identified if we drop one equation and each contains identical explanatory variables the Indirect Least Squares Method is efficient. For details see Howe [41].

APPENDIX B

Functional Forms of a Few Noted Demand Systems

1. The Linear Expenditure System [LES]

$$p_i x_i = p_i \gamma_i + \beta_i (y - \sum_k p_k \gamma_k)$$
$$\sum_i \beta_i = 1$$
$$i, k = 1, \dots, n.$$

where, x_i, y and p_i are, respectively, quantity demanded of the i_{th} commodity, y is total income of an individual consumer, and price of the ith commodity; β_i is a constant parameter, interpreted as marginal budget share $(= \delta(p_i x_i)/\delta y)$; and γ_i is a constant parameter interpreted as minimum quantity of the i_{th} commodity necessary for subsistence.

See Goldberger and Gamaletsos [37] for the theoretical properties of the LES, and Howe [41], and Pollak and Wales [57] for their identification in empirical works and Lluch and Powell [48] for empirical application.

2. Indirect Addilog demand Model.

$$x_i(p,y) = \frac{a_i b_i y^{b_i} p_i^{-b_i - 1}}{\sum_j a_j b_j y^{b_j - 1}}$$
$$j = 1, \dots, n.$$

The above demand system is generated from an Indirect Utility Function (IUF) of the type:

$$v(p,y) = \sum_{i=1} a_i \left(\frac{y}{p_i}\right)^{b_i}$$
$$i = 1, \dots, n.$$

where, a_i and b_i are constant parameters(for the rest of the notations see the definitions

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given above or in the text). The econometric procedure to estimate the addilog demand system is discussed in Johnston [42], Varian [70].

3. The Rotterdam Demand System.

$$\bar{v}_{it}\Delta(\ln x_{it})=a_i+b_i\sum_k \bar{v}_{kt}\Delta(\ln x_{kt})+\sum_k S_{ik}\Delta(\ln p_{kt})$$
$$i,k=1,...,n.$$

where, $\Delta(\ln x_{it}) = \ln x_{it} - \ln x_{i,t-1}$

$\Delta(\ln p_{it}) = \ln p_{it} - \ln p_{i,t-1}$

$$\bar{v}_{it}=v_{it}+v_{i,t-1}=\frac{p_{it}x_{it}}{\sum_k p_{it}x_{it}}+\frac{p_{i,t-1}x_{i,t-1}}{\sum_k p_{k,t-1}x_{k,t-1}}$$
$$i,k=1,...,n.$$

$S_{ik}= v_i (E_{ik} + E_i v_k)$

where, t and t-1 refer to current and preceding period, respectively; v_i, E_{ik} and E_i are as defined in the text. a_i and b_i are constant parameters.

The theoretical basis and estimation procedure after the imposition of restrictions implied by the choice theory for the Rotterdam model is discussed in Barten [6], Lluch [47], Barten and Geyskens [8], and Keifer [43].

4. The Translog Reciprocal Indirect Utility Functions.

Define: $h(x_1,...,x_n) = 1/ v(p_1,...,p_n,y)$, i.e., $h(\mathbf{X})$ is a reciprocal of an IUF, and $q_i = p_i/y$.

$$\ln h(x_1,....,x_n) = a_0 + \sum_i a_i \ln x_i + \frac{1}{2} \sum_{i-1} b_{ij} \ln q_i \ln q_j$$
$$where, b_{ij} = b_{ji}, \forall i,j.$$

a_0, a_i, b_{ij} are constant parameters; and consumer demand functions are given by :

$$x_i(q) = \frac{q_i^{-1}(a_i + \sum_{i-1} b_{ij} \ln q_j)}{\sum_k a_k + \sum_{k-1} \sum_{m-1} b_{km} \ln q_m}$$
$$i,k,m = 1,...,n.$$

Varian observes [70, p. 132] that, " If the $b_{ij} \neq 0$ for any i and j, then the translog function will not satisfy the requirements for an indirect utility function globally, but may be regarded as approximation to any IUF. The translog function will be homogeneous of degree 1 if $\sum a_i = 1$, $b_{ij} = b_{ji}$, and $\sum b_{ij} = 0$ $\forall i$; since the demand equations are homogeneous of degree 0 in the parameters a_i and b_{ij}, it is necessary to impose normalization. The most convenient normalization is $\sum_i \sum_j b_{ij} = 0$."

5 Almost Ideal Demand System

The model is given by,

$$\frac{p_i x_i}{y} = v_i = \alpha_i + \sum_j \gamma_{ij} \log p_j + \beta_i \log(\frac{y}{P})$$
$$i,j = 1,...,n.$$
$$where,$$
$$\log P = \alpha_0 + \sum_k \alpha_k \log p_k + \frac{1}{2} \sum_j \sum_k \gamma_{kj} \log p_k \log p_j$$

where, $\alpha_0, \alpha_i, \beta_i$, and γ_{ij} are the constant parameters to be estimated under the

following restrictions implied by the choice theory:

$\sum \alpha_i = 1; \quad \sum \gamma_{ij} = 1, \quad \sum \beta_i = 0$ (additivity),

$\sum \gamma_{ij} = 0$ (homogeneity),

$\gamma_{ij} = \gamma_{ji}$ (symmetry).

P is generally taken as an approximation to the general consumer price index.

Details of the properties of this model and their estimation procedures are given in Deaton and Muellbauer [27], and Coyte and Landon [18].

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APPENDIX C

Estimates of Compensated Own-and-Cross Price Elasticities

Addis Ababa Household Budget Study

1975

	1	2	3	4	5	6	7
1	-0.3853	0.1028	0.0581	0.1186	0.0115	0.0530	0.0412
2	0.3553	-0.7074	0.0724	0.1478	0.0144	0.0660	0.0513
3	0.2936	0.1058	-0.6305	0.1221	0.0189	0.0546	0.0424
4	0.3448	0.2988	0.1689	-1.6039	0.0336	0.1541	0.1198
5	0.4318	0.2988	0.0880	0.1797	-0.9979	0.0803	0.0624
6	0.6331	0.2282	0.1290	0.2634	0.0256	-1.3709	0.0915
7	0.3562	0.1284	0.0726	0.1482	0.0144	0.0662	-0.7862

Estimates of Compensated Own-and-Cross Price Elasticities

Addis Ababa Household Budget Study

1979

	1	2	3	4	5	6	7
1	-0.2710	0.0691	0.0482	0.0761	0.0052	0.0220	0.0504
2	0.1866	-0.3314	0.0346	0.0546	0.0037	0.0158	0.0361
3	0.2809	0.0746	-0.5214	0.0822	0.0056	0.0237	0.0543
4	0.1162	0.1055 *	0.0736	-0.6950	0.0079	0.0336	0.0769
5	0.3592	0.0954	0.0665	0.1650	-0.7259	0.0304	0.0695
6	0.8006	0.2126	0.1483	0.2341	0.0159	-1.5660	0.1548
7	0.4866	0.1292	0.0902	0.1423	0.0096	0.0411	-0.8990

Estimates of Compensated Own-and-Cross Price Elasticities
Addis Ababa Household Budget Study
1980

	1	2	3	4	5	6	7
1	-0.3026	0.0839	0.0494	0.0406	0.0376	0.0381	0.0529
2	0.2004	-0.4406	0.0543	0.0446	0.0414	0.0419	0.0582
3	0.2359	0.1085	-0.5634	0.0525	0.0487	0.0493	0.0685
4	0.4682	0.2153	0.1268	-0.6478	0.0966	0.0979	0.1358
5	0.2210	0.1015	0.0598	0.0491	-0.5410	0.0462	0.0641
6	0.4581	0.2107	0.1241	0.1019	0.0946	-1.1222	0.1329
7	0.3096	0.1424	0.0839	0.0688	0.0639	0.0647	-0.7330

Estimates of Compensated Own-and-Cross Price Elasticities
Rural Household Budget Study
1980

	1	2	3	4	5	6
1	-0.2020	0.0421	0.1021	0.0128	0.0436	0.0012
2	0.4285	-0.5924	0.1047	0.0305	0.0447	0.0013
3	0.3776	0.0381	-0.0922	0.0116	0.0394	0.0010
4	0.9920	0.1000	0.0305	-1.4408	0.1031	0.0029
5	0.7861	0.1129	0.2737	0.0344	-1.5446	0.0033
6	1.0900	0.1100	0.2663	0.0335	0.1138	-1.6138

Notes

1. Eventhough the original work of Slutsky appeared in French as early as 1915, it was only in 1936 that it attracted the attention of English speaking world and became a famous analytical tool in subsequent works (see for example, Brown and Deaton [10]).

2. monotonocity refers to invariability of preferences to appropriate transformations; for the proof of the theorem see Varian [70].

3. The most important identities of note are the following: (All notations below take the definitions given in the text)

a) $e(P, v(P, y)) \equiv y$, the minimal expenditure to reach utility $v(P, y)$ is y;

b) $v(P, e(P, u)) \equiv y$, the maximal utility from income $e(P, u)$ is u;

c) $h_i(P, u) \equiv x_i(P, e(P, u))$, the Hicksian demand at utility u is the same as the Marshallian demand at income $e(P, u)$.

d) $x_i(P, y) = h_i(P, v(P, y))$, the Marshallian demand at income y is the same as the Hicksian demand at utility $v(P, y)$.

Following these we have Roy's Identity, and Shephard's Lemma , respectively, given as:

$$x_i(P, y) = -\frac{\frac{\delta v(P, y)}{\delta p_i}}{\frac{\delta v(P, y)}{\delta y}}$$
$$i = 1 .. n$$

$$h_i(P, u) = \frac{\delta e(P, u)}{\delta p_i}$$

The proofs for both of the above results is given in Varian [70].

4. the derivation of the Slutsky Equation and a very simple proof of it is given in Varian [70] , and for intuitive interpretation see Green [38]. The original presentation is given in Slutsky

[65].

5. See Note 3 above.

6. Let the government's problem be stated as follows:

Maximize,

$$\psi = \psi\{v^1(p_1,\ldots,p_n,y^1),\ldots,v^H(p_1,\ldots,p_n,y^H)\}$$

Subject to: $\sum t_j X_j = R$ $\qquad\qquad$ $j=1,\ldots,n$

where, ψ is the SWF of the government,

$\quad$ t_j is the indirect tax rate on commodity j,

$\quad$ X_j is $\sum_i x_{ij}$,i= 1,...H, the sum of quantity of commodity x, consumed by the i_{th} household.

$\quad$ R is a lump sum fiscal revenue constraint. The solution to the above problem and its empirical investigation leads to the issue of tax reform. For the theoretical exposition see Deaton [21], Diamond and McFadden [1974] and for a recent application Cragg [19].

7. The demand systems estimated by Asmerom [4, 5] were in the Cobb-Douglas family which do not have a direct utility function (See,e.g., Green [38]). Therefore, all the regularities outlined by the choice theory are not exhibited in these systems.

8. For LES and ELES, " supernumerary" ratio is given by, respectively,

$$-\phi = 1 - \frac{\sum P_k \gamma_k}{\bar{y}}$$

$$-\phi = 1 - \frac{\sum p_k \gamma_k}{\bar{z}}.$$

9. Such a sampling unit is not typical of Ethiopia, rather common elsewhere. For example data employed by Darrough et.al [20] were generated in like manner in the context of Japanese households.

10. Estimation of demand systems for rural households here assumes that their production and consumption decisions are made independent of each other. The procedure is fairly justified on various grounds (e.g. see Deaton and Muellbauer [27]). An alternative is a model that incorporates both decisions simultaneously, see Alemayehu [2] for the theoretical exposition.

11. Adjustments for possible heteroscedastic error terms failed in all cases; thus OLS estimates were taken. For 1975 budget set out of the 29 income groups, the lower 16 income groups were taken for estimation on the grounds that Engel curves tended to be non-linear when the entire observation was taken. A semi-log specification, however, did not do well either.

12. The percentage of households below the income bracket Birr 449 were respectively 54.3, 87.8, 87.2, for 1975, 1979, and 1980 Addis Ababa budget sets. The sample size of households in the budget studies for 1975, 1979, and 1980 were respectively 200, 500, and 500.

13. This is a stylized fact generally accepted to hold and there is no known evidence to the contrary , see Kuznets [1962]. In this connection an attempt was made to estimate Cost of Living Index for high and low income groups. Because it turned out statistically insignificant it is not reported here.

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