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ALTRUISM AND MARKETS

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1. Introduction

Consider the problem facing an econometrician who wishes to evaluate the effect on allocations of, say, a government bond issue in an economy with overlapping generations. The econometrician has read Barro (1974), so he knows that in a world of certainty, if there are "operational intergenerational transfers" then issuing a bond will have precisely the same effects on allocations as a tax increase would.

How can one test for the existence of operational intergenerational transfers? In Barro's formulation, these intergenerational links are forged using parental altruism. So long as the parent's wealth is sufficiently large relative to the child's (initial) wealth, the parent can select some non-negative bequest which serves to equate intergenerational marginal rates of substitution, at least in a world without uncertainty.

If there *is* uncertainty in, say, the child's income (realized after the death of the parent), then the parent's fixed bequest, no matter what the size, cannot possibly serve to equate marginal rates of substitution, except in expectation. The bequest does not suffice to insure the child against income risk.

We know, however, that if markets are complete then the child *will* be able to insure away idiosyncratic income risk. This brings us to the first moral of this paper:

> *If transfers are possible and markets are complete then allocations will be efficient, regardless of whether or not agents are altruistic. Obversely, in the absence of selfish exchange, altruistic behavior is not sufficient for efficiency.*

Neither of these propositions should be surprising; the usual formulations of altruistic behavior do not violate the conditions of the first

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welfare theorem, and there is no obvious reason to suppose that altruism is a perfect substitute for markets. In fact, it will turn out that both the classical welfare theorems continue to hold when preferences are altruistic. Altruism does, however, raise a technical difficulty having to do with the efficiency of conventionally defined competitive equilibria: we modify the usual definition of competitive equilibria slightly so as to insure the efficiency of what we term *altruistic competitive equilibria.*

Although altruism will not affect the efficiency of allocations in an economy with complete markets, it will affect the distribution of consumption. If there is great disparity in wealth, wealthy altruists will be made better off if they transfer some of their wealth to the less fortunate. Such transfers will not alone produce efficient allocations; the wealthy altruist and his beneficiary may be able to realize gains from selfish trade subsequent to any transfers.

This brings us back to the problem facing our econometrician. The econometrician has data on consumption allocations, but no satisfactory measures of initial endowments. Suppose that the econometrician wishes to test the hypothesis that operative intergenerational transfers exist, and seeks to test this by estimating the altruism of parent toward child. Such an effort will prove to be fruitless. Why? Because parent and child can agree on an efficient set of transfers, even in the complete absence of altruism. Accordingly, testing for the existence of an efficient set of transfers may accomplish the econometrician's primary aim, but the very existence of such an efficient set of transfers implies that the altruism coefficient that the econometrician seeks to estimate will be unidentified.[1]

1.1. **Organization.** Section 2 is a standard exercise in developing the first two welfare theorems for a multi-period endowment economy with uncertainty when agents are selfish. The set of Pareto optimal allocation rules are derived for the planning problem, and a set of prices and initial endowments supporting any Pareto optimum are found.

What do we mean by altruism? Section 3 defines this precisely, in a manner similar to Becker (1974). We define two sorts of altruism

[1]Perhaps the first to note this fact were the authors of an early empirical investigation of altruism,Altonji, Hayashi, and Kotlikoff (1992). Those authors note that their main test must be viewed as a test of "the altruism/life-cycle models with risk-sharing against the Keynesian/life-cycle models with no risk sharing." (p. 1184) As a consequence, some other authors have tended to view the test of Altonji, Hayashi, and Kotlikoff as a test, not of altruism *per se*, but of the full insurance corollary of the complete markets hypothesis within the extended family (e.g. Cochrane (1991), Deaton (1992), Townsend (1994)).

which I term "naive" (agents value other agents' consumption) and "sophisticated" (agents value other agents' utility).[2] Altruism between two agents may be one- or two-sided, but we show that both one- and two-sided altruism—whether naive or sophisticated—map into the naive two-sided framework without loss of generality.

Section 4 describes the mapping between altruistic and selfish allocation rules in a complete markets setting. We show that for any optimal altruistic allocation, there is an identical optimal selfish allocation. As a consequence, the two classical welfare theorems are unaffected by how altruistic preferences are. The converse, however, does not hold, and the distribution of resources is shown to be affected by the degree of altruism: higher levels of altruism have the effect of shrinking the set of Pareto efficient allocations. Observed consumption inequality places an upper bound on the (unobserved) degree of altruism.

Although altruism does not affect the classical link between competitive equilibria and Pareto optima provided by the second welfare theorem, it does require us to reconsider the usual definition of competitive equilibrium, as this definition does not permit agents to make transfers. In the absence of transfers, altruism may lead to a failure of the first welfare theorem: competitive equilibria may not be efficient. In Section 4.2 we demonstrate by example that competitive equilibria are often *not* efficient when preferences are altruistic, and propose to remedy this defect by introducing a somewhat more general definition of competitive equilbria, which we term *altruistic competitive equilibria.* The relationship between competitive equilibria and altruistic competitive equilibria is pursued in a companion paper, "Altruistic competitive equilibria."

Armed with our new definition of altruistic competitive equilibria, and with the first welfare theorem to guide us, it is perfectly clear that any set of initial endowments will give rise to an efficient outcome, so long as markets are complete. This is true whether or not agents are altruistic, so that altruism is clearly not necessary for efficiency. However, much of the interest in altruism is motivated by situations in which the assumption of complete markets may be unrealistic. Section 5 shows with an an example that while altruism may improve the efficiency of allocations, it is not sufficient for full efficiency. The theme of market incompleteness is pursued in a second companion paper, "Altruism and Incomplete Markets."

Section 6 considers the particular case of intergenerational altruism, and shows by example that an absence of uncertainty is crucial to the

[2]A somewhat different distinction has been drawn elsewhere using the terms "paternalistic" and "non-paternalistic" altruism. See, e.g. Ray (1987).

sort of Ricardian equivalence results shown by Barro (1974) if no selfish trade is permitted within dynasties. Section 7 concludes.

2. Selfish Allocations

2.1. **Optimal Allocations.** Consider an economy with two selfish agents indexed by $i = 1, 2$. Each period agent i receives some endowment x_{it} which depends on the stochastic state of the economy, $\omega \in \{\omega_1, \omega_2, \ldots, \omega_m\} = \Omega$. The realization of ω should be taken to determine the state of the economy at every date.

The problem facing a social planner, then, is

$$\max_{c_{1t}, c_{2t}} \mathrm{E}_0 \sum_{t=1}^{T} \beta^{t-1} \left[\lambda_1 U^1(c_{1t}) + \lambda_2 U^2(c_{2t}) \right] \tag{1}$$

such that the the aggregate resource constraint is satisfied, or

$$c_{1t} + c_{2t} \leq x_{1t} + x_{2t} = \overline{x}_t \tag{2}$$

for all dates and states. The controls c_{it} $(i = 1, 2)$ map the endowment pair into a consumption pair, so that agents' consumption will generally vary across date-states. The utility functions $U^i : C \to \mathbb{R}_+$ are assumed to be strictly increasing, strictly concave, and continuously differentiable. The set of programming weights (λ_1, λ_2) is taken to be the interior of the unit simplex (denoted by Δ), so that $\lambda_1 + \lambda_2 = 1$, and $0 < \lambda_i < 1$ for $i = 1, 2$.

The first order conditions for the planner's problem are

$$\lambda_i U^{i\prime}(c_{it}) = \mu_t(\overline{x}_t) \qquad i = 1, 2;\ t = 1, \ldots, T,\ \forall \overline{x}_t \tag{3}$$

Where $\mu_t(\overline{x}_t)$ is the Lagrange multiplier on the resource constraint, scaled by β^{t-1}. We write the Lagrange multiplier as a function of the aggregate resource $\overline{x}_t$ to remind ourselves that this equation holds at every date-state. This equation captures the idea that the optimal allocation eliminates all idiosyncratic risk; consumption c_{it} depends only on the aggregate resource constraint and some non-varying programming weight. By varying the programming weights we can trace out the entire set of optimal allocations.

2.2. **Competitive Equilibria.** In order to solve the equilibrium problem, let us begin with the problem facing agent i at time zero:

$$\max_{c_{it}} \mathrm{E}_0 \sum_{t=1}^{T} \beta^{t-1} U^i(c_{it}) \tag{4}$$

such that his individual budget constraint is not violated:

$$\sum_{t=1}^{T} \sum_{\omega} p_t(\omega) c_{it}(\omega) \leq \sum_{t=1}^{T} \sum_{\omega} p_t(\omega) x_{it}(\omega) \tag{5}$$

Agent 1's first order conditions are

$$U^{1\prime}(c_{1t}(\omega)) = \xi_1 \frac{p_t(\omega)}{\beta^{t-1} \mathrm{Pr}(\omega)} \tag{6}$$

where ξ_1 is the Lagrangean multiplier on agent 1's time zero intertemporal budget constraint.

The second welfare theorem assures us that any Pareto optimal allocation (corresponding to a particular (λ_1, λ_2) pair) can be supported by a competitive equilibrium. For future reference, let us first define what we mean by a competitive equilibrium.

Definition 1. A competitive equilibrium is a set of positive prices $\{p\}$ and an allocation c_i for each agent i such that

1. Given $\{p\}$ and some initial endowment x_i, c_i solves agent i's problem given by (4,5)
2. Given the set of solutions to the agents' problems, prices $\{p\}$ clear markets:

$$\sum_i p_t(\omega) x_{it}(\omega) = \sum_i p_t(\omega) c_{it}(\omega) \qquad \text{for all } t = 1, 2, \ldots, T; \omega \in \Omega.$$

Now, having defined what we mean by a competitive equilibrium, let us show that the second welfare theorem holds in this environment by constructing a competitive equilibrium with which to decentralize any particular Pareto optimal allocation. In order to decentralize a particular optimal allocation, we need to choose a set of prices and endowments to implement the consumption allocation recommended by (3). Combining (3) with (6) gives us

$$\xi_1 \frac{p_t(\omega)}{\beta^{t-1} \mathrm{Pr}(\omega)} = \frac{\mu_t(\omega)}{\lambda_1}$$

so that we can decentralize a particular optimal allocation by choosing a set of time zero Arrow-Debreu prices

$$p_t(\omega) = \mu_t(\omega)\beta^{t-1}\Pr(\omega)$$

and an initial distribution of wealth such that

$$\xi_1 = \frac{1}{\lambda_1}$$

and similarly for agent 2.

2.3. **Core Allocations.** We've been talking about a model with only two agents, which stretches the logic of competitive markets to its limit. While our results to this point extend straightforwardly to the case of any arbitrary finite number of agents, it may make more sense to think about the set of core allocations, and assume that some bargaining process will determine precisely what point in the core final allocations will correspond to.

The set of core allocations is a subset of the set of Pareto optimal allocations. Core allocations are that set of allocations which are efficient and which satisfy a set of constraints dictating that each agent will be at least as well off ex ante as if he simply consumed his endowment at each date-state, or

$$\mathrm{E}_0 \sum_{t=1}^{T} \beta^{t-1} U^i(c_{it}) \geq \mathrm{E}_0 \sum_{t=1}^{T} \beta^{t-1} U^i(x_{it}) \qquad i = 1, 2.$$

Let w_i denote some time zero expected, discounted utility for agent i. Let $(\underline{w}_1, \underline{w}_2)$ denote the utility pair if each agent simply consumes his endowment. Finally, let λ_i be a function mapping from the set of feasible utility levels to the $(0, 1)$ interval, such that $\lambda_i(w_i)$ is the programming weight which delivers discounted expected utility w_i to agent i in the solution to the planner's problem.[3] Then the set of programming weights which delivers the core allocations can be written as

$$,\ (\underline{w}_1, \underline{w}_2) = \{(\lambda_1, \lambda_2) \in \Delta \,|\, \lambda_1(\underline{w}_1) \leq \lambda_1 \leq 1 - \lambda_2(\underline{w}_2)\}$$

Note that this set, and hence the core, is monotonically decreasing in each of $\underline{w}_1, \underline{w}_2$.

[3]Our assumption that the $U^i(\cdot)$ are strictly increasing functions is sufficient to guarantee that this mapping is one-to-one.

3. Different Forms of Altruism

3.1. **Naive Two-sided Altruism.** An altruistic individual has a preference ordering which is defined over own consumption and the consumption of others. In the case of two indiduals, the mapping $\tilde{U} : C \times C \to \mathbb{R}_+$ is a general way of writing that preferences are altruistic.

The form of altruistic preferences we're concerned with in this note are of the additive form:

$$\tilde{U}^1(c_1, c_2) = U^1(c_1) + \theta_1 U^2(c_2)$$

and

$$\tilde{U}^2(c_1, c_2) = U^2(c_2) + \theta_2 U^1(c_1)$$

for agents 1 and 2, respectively. We take $\theta_i \in [0, 1]$.[4] Note that if the altruism weights $\theta_i = 0$ for $i = 1, 2$ then this reduces to precisely the selfish case; if the altruism weights are equal to one, then each agent cares as much about the other as he does himself.[5]

3.2. **One-sided Altruism.** If the altruism weight of the first agent, θ_1, is greater than zero and the altruism weight of the second agent, θ_2 is equal to zero, then this is a case of one-sided altruism, as in Barro (1974), who identifies agent one in this instance as a parent household and agent two as a child household.

3.3. **Sophisticated Two-sided Altruism.** If one agent is genuinely concerned with the welfare (rather than just the consumption) of a second agent, and the second agent similarly cares about the welfare of the first agent, then we should redefine the preferences given above to depend on own consumption and other utility, or (abusing notation)

$$\hat{U}^1(c_1, \hat{U}^2) = U^1(c_1) + \theta_1 \hat{U}^2$$

and

$$\hat{U}^2(c_2, \hat{U}^1) = U^2(c_2) + \theta_2 \hat{U}^1$$

[4]Because allocations are invariant to a positive affine transformation of agents' preferences, this restriction involves no loss of generality.

[5]If we wish to capture the notion that each agent actually cares *more* about the other agent than he does himself, we can accomodate this by pretending that each in fact *is* the other agent. This trick, of course, will only work if both agents are selfless. See Kimball (1987) for a treatment of the case in which one agent's extreme concern for the other is unrequited.

Substituting the second into the first, and solving the recursion, we have:

$$\hat{U}^1(c_1, \hat{U}^2) = \frac{1}{1 - \theta_1\theta_2}\left[U^1(c_1) + \theta_1 U^2(c_2)\right]$$

for altruism weights less than one. A symmetric relation holds for agent two, of course, so that this 'sophisticated' altruism is simply an affine transformation of 'naive' altruism, and there is little loss of generality in confining our investigation to the naive variety of altruism.[6]

4. Altruistic Allocations

4.1. **Optimal Altruistic Allocations.** Substituting the two-sided altruistic preferences of Section 3 for the selfish preferences of Section 2 gives us a modified planner's problem which takes into account the utility that altruistic agents receive from vicarious consumption:

(7)
$$\max_{c_{1t}, c_{2t}} \mathrm{E}_0 \sum_{t=1}^{T} \beta^{t-1}\left[\lambda_1\left(U^1(c_{1t}) + \theta_1 U^2(c_{2t})\right) + \lambda_2\left(U^2(c_{2t}) + \theta_2 U^1(c_{1t})\right)\right]$$

such that the the aggregate resource constraint (2) is satisfied. However, the planner's objective function can be rewritten as

$$\mathrm{E}_0 \sum_{t=1}^{T} \beta^{t-1}\left[\tilde{\lambda}_1 U^1(c_{1t}) + \tilde{\lambda}_2 U^2(c_{2t})\right] \tag{8}$$

where

$$\tilde{\lambda}_1 = \frac{\lambda_1 + \lambda_2\theta_2}{1 + \lambda_1\theta_1 + \lambda_2\theta_2}$$

and

$$\tilde{\lambda}_2 = \frac{\lambda_2 + \lambda_1\theta_1}{1 + \lambda_1\theta_1 + \lambda_2\theta_2}$$

[6]The little loss of generality involves a restriction that agents not both be *too* altruistic. At least one agent must be at least slightly selfish, so that $\theta_1\theta_2 < 1$. If this condition is not satisfied, then preferences defined here will not be bounded, a fact noted in a similar context by Miles Kimball (1987), who termed this a "Hall of Mirrors" problem. I am indebted to Casey Mulligan for what I think is an even more memorable characterization of this case: "One guy sniffs a flower and the world explodes."

These redefined programming weights are again elements of the interior of the unit simplex. Note, however, that as altruism increases, the set of redefined programming weights grows smaller.

Formally, denote the interior of the unit simplex (the set of the original programming weights with selfish agents) by $\Delta(0,0)$. Then let the set of redefined programming weights be given by $\Delta(\theta_1,\theta_2)$. The definitions of $(\tilde{\lambda}_1,\tilde{\lambda}_2)$ imply that

$$\Delta(\theta_1,\theta_2) = \left\{ (\lambda_1,\lambda_2) \in \Delta(0,0) \,\middle|\, \frac{\theta_2}{1+\theta_2} < \lambda_1 < \frac{1}{1+\theta_1} \right\} \tag{9}$$

Clearly $\Delta(\theta_1,\theta_2)$ is monotonically decreasing in each of θ_1,θ_2. Hence, increases in altruism imply a smaller set of efficient allocations. In the limit, with $\theta_1 = \theta_2 = 1$, the set of efficient allocations shrinks to a singleton, corresponding to the set of programming weights $\{(1/2,1/2)\}$ if agents' preferences are identical.[7]

4.2. **Altruistic Competitive Equilibria.** The decentralization of any particular Pareto optimal allocation (corresponding to some element of $\Delta(\theta_1,\theta_2)$) proceeds just as it did in Section 2. However, the correspondence between the marginal utility of wealth and programming weights must now be reinterpreted, since 'wealth' must now be taken to include discounted expected utility from others' consumption.

For sufficiently high levels of altruism, there will be some set of endowments for which conventionally defined competitive equilibria will fail to achieve efficiency. We can imagine, for example, the limiting case in which $\theta_1 = \theta_2 = 1$, and in which preferences are identical. The only efficient allocation given this level of altruism is an equal division of all resources. Suppose that instead of an equal division of resources, the

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[7]It is straightforward to extend this redefinition of programming weights to the case of n agents, but requires a slight extension of notation. Let θ_i^j measure how altruistic agent i is toward agent j, and define $\theta_i^i = 1$. Then the appropriately redefined programming weights are given by

$$\tilde{\lambda}_i = \frac{\sum_{j=1}^n \lambda_j \theta_j^i}{\sum_{k=1}^n \sum_{j=1}^n \lambda_j \theta_j^k}. \tag{10}$$

The set of redefined programming weights will be somewhat less conveniently defined by

$$\left\{ (\lambda_1,\lambda_2,\ldots,\lambda_n) \in \Delta^{n-1} \,\middle|\, \frac{\sum_{j\neq i} \lambda_j \theta_j^i}{\sum_{k\neq i}^n \sum_{j\neq i}^n \lambda_j \theta_j^k} < \lambda_i < \frac{1}{\sum_{j=1}^n \theta_i^j}; i = 1,2,\ldots n \right\} \tag{11}$$

where Δ^{n-1} denotes the Cartesian product of the interior of $n-1$ unit simplices.

initial endowments are such that one agent owns nearly all of the resources. Both agents will be better off if some resources are transferred to the second agent, but this transfer will violate the second agent's budget constraint unless at least one price is non-positive. But our definition of competitive equilibrium requires that all prices be positive, so no competitive equilibrium achieves an efficient allocation.

Although standard definitions of equilibrium often permit prices to be zero, this inefficiency is not simply due to our definition of competitive equilibrium. Modify the example by considering the case in which $\theta_2 < 1$. The first agent will still want to transfer resources to the second agent so as to achieve an equal division of resources. Again, the second agent's budget constraint given the proposed transfer will be violated unless at least one price is non-positive. Let us choose prices so that the second agent's budget constraint is satisfied given the proposed transfer. But at these prices, the proposed consumption allocation will no longer satisfy the second agent's problem: at any non-positive price, he will demand more than half of any resource so priced. Ligon (1995b) gives a set of necessary and sufficient conditions for the effiency of competitive equilibrium.

All that the second welfare theorem implies is that a decentralizing competitive equilibrium exists for *some* set of initial endowments: the decentralizing set of initial endowments in this example are those that imply an equal division of wealth.[8] In the second example given by Figure 1, the connection between competitive equilibria and Pareto optima is made clear by the contract curve illustrated in that figure: although competitive equilibria are inefficient when the altruist possesses a sufficiently large share of the total wealth of the economy, Pareto optima also do not exist in this region.

In any case, this inefficiency is surely more a technical problem than a conceptual one. In each of the examples above, the wealthier agent can act unilaterally in order to make himself better off simply by transferring resources to the poorer agent, who will surely not object to such charity. Accordingly, if one agent has more resources initially than the other, then it may be in both of their interests to transfer goods so that the optimum is achieved. If such transfers are permitted, then we can simply incorporate this into our definition of a competitive equilibrium

[8] Note here the careful distinction between an equal division of *resources* and an equal division of *wealth*. The former implies that each agent possesses an equal quantity of every consumption good; the latter only that each possesses a bundle of goods having equal value in equilibrium. In particular, there may be welfare improving trade from each agent's initial endowment position.

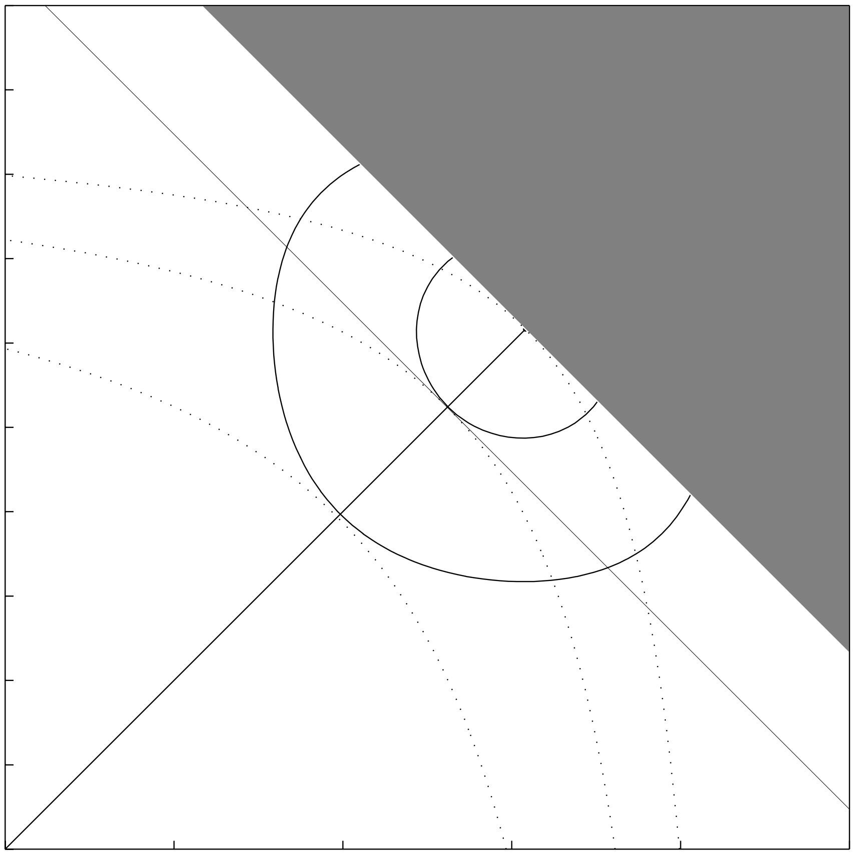

FIGURE 1. Inefficiency of Competitive Equilibrium. The solid indifference curves belong to an altruist, while the dotted lines belong to a selfish agent. The contract curve terminates at the altruist's preferred allocation, $\overline{c}$, since this point Pareto dominates any point beyond it. For any initial endowment in the non-shaded area of the Edgeworth box, such as $\vec{x_1}$, any ordinary competitive equilibrium will be efficient. For any initial endowment in the shaded part of the box, a competitive equilibrium cannot achieve efficiency, so no market clearing price can achieve an allocation in the non-shaded portion of the box..

in this setting, so that we might redefine what we mean by competitive equilibrium.

Let us begin this task by defining $\tau_t^{ij}(\omega)$ as a time t, state ω net transfer of the consumption good from agent i to agent j. Note that $\tau_t^{ij}(\omega)$ is necessarily equal to $-\tau_t^{ji}(\omega)$. Accordingly, the new problem facing agent i is

$$\max_{\tau_t^{ij}, c_{it}} \mathrm{E}_0 \sum_{t=1}^{T} \beta^{t-1} U^1(c_{it}) \tag{12}$$

such that his individual budget constraint is not violated:

$$\sum_{t=1}^{T} \sum_{\omega} p_t(\omega) c_{it}(\omega) \leq \sum_{t=1}^{T} \sum_{\omega} p_t(\omega)(x_{it}(\omega) + \sum_{j} \tau_t^{ji}(\omega)) \tag{13}$$

and such that, given transfers he expects to receive $\{\tau^{ij}\}$ he does not take more than is offered:

$$\tau_t^{ij}(\omega) \geq \min(0, \tau_t^{ij}(\omega)) \text{for all } t,\, \omega \in \Omega. \tag{14}$$

Having set up the agent's problem, we are in a position to define an altruistic competitive equilibrium:

Definition 2. A altruistic competitive equilibrium is a set of prices $\{p\}$, transfers $\{\tau\}$, and an allocation c_i for each agent i such that

1. Given $\{p\}$ and some initial endowment x_i, (τ^{ij}, c_i) solves agent i's problem given by (12-14)
2. Given the set of solutions to the agents' problems, prices $\{p\}$ clear markets:

$$\sum_{i} p_t(\omega)(x_{it}(\omega) + \sum_{j} \tau_t^{ji}(\omega)) = \sum_{i} p_t(\omega) c_{it}(\omega) \qquad \text{for all } t = 1, 2, \ldots, T; \omega \in \Omega.$$

Ligon (1995b) provides a proof of the existence and efficiency of this notion of equilibrium, along with a discussion of its relationship to the conventional competitive equilibrium of Definition 1.

4.3. **Core Allocations.** Recall that the set of core allocations is a subset of the set of Pareto optimal allocations. Since increasing altruism has the effect of reducing the size of the set of optima, increasing altruism will weakly reduce the size of core. If the set of initial endowments yields levels of utility $(\underline{w}_1, \underline{w}_2)$, then the core will shrink with small increases in altruism if and only if $(\lambda_1(w_1), \lambda_2(w_2)) \notin \Delta(\theta_1, \theta_2)$.

5. Altruistic Insurance

Whether or not agents are altruistic, there will remain a role for selfish trade except in the extreme case that $\theta_1 = \theta_2 = 1$.

Consider the following example,[9] in which there is a single consumption good but two possible states of nature, $\omega \in \{\omega_1, \omega_2\}$. There is a single period ($T = 1$), and agents are equally altruistic ($\theta_1 = \theta_2 = \theta < 1$). Each agent i receives some endowment realization $x_i(\omega)$, as follows:

$$x_i(\omega) = \begin{cases} \overline{x} & \text{if } \omega = \omega_i \\ 0 & \text{otherwise} \end{cases}$$

for $i = 1, 2$, so that in one state of the world, agent one receives the aggregate endowment, while in the other state of the world agent two receives it.

Now, with or without altruism the competitive allocation in this example is given by

$$U^{1\prime}(c_1(\omega)) = U^{2\prime}(c_2(\omega)) \qquad \text{for } \omega = \omega_1, \omega_2.$$

so that marginal utilities are equated in both states.

Suppose, however, that markets are incomplete and that the ex ante insurance contracts which are used to achieve the competitive allocations are not allowed, so that altruism provides the only means of smoothing consumption. When the state is ω_1, the first agent will be able to make transfers to the second agent, and conversely when the state is ω_2. Consumption allocations will be determined by

$$U^{1\prime}(c_1(\omega_1)) = \theta U^{2\prime}(c_2(\omega_1))$$

and

$$U^{1\prime}(c_1(\omega_2)) = \frac{1}{\theta} U^{2\prime}(c_2(\omega_2))$$

Because these altruistic transfers fail to equate agents' marginal utilities in different states, these purely altruistic allocations are dominated by the competitive solution.

The logic of this can be seen in the accompanying Edgeworth box, figure 2. Suppose that we normalize $\bar{x} = 1$. Then we can view the axes of the Edgeworth box as the quantity of good in each of the two possible states. Because agent one receives the entire endowment in the first state, and agent two the entire endowment in the second, the initial endowment is found at point A. If agents were purely selfish, the core of this economy would be the line segment from points b to g;

[9]The example of this section was suggested by Ned Prescott.

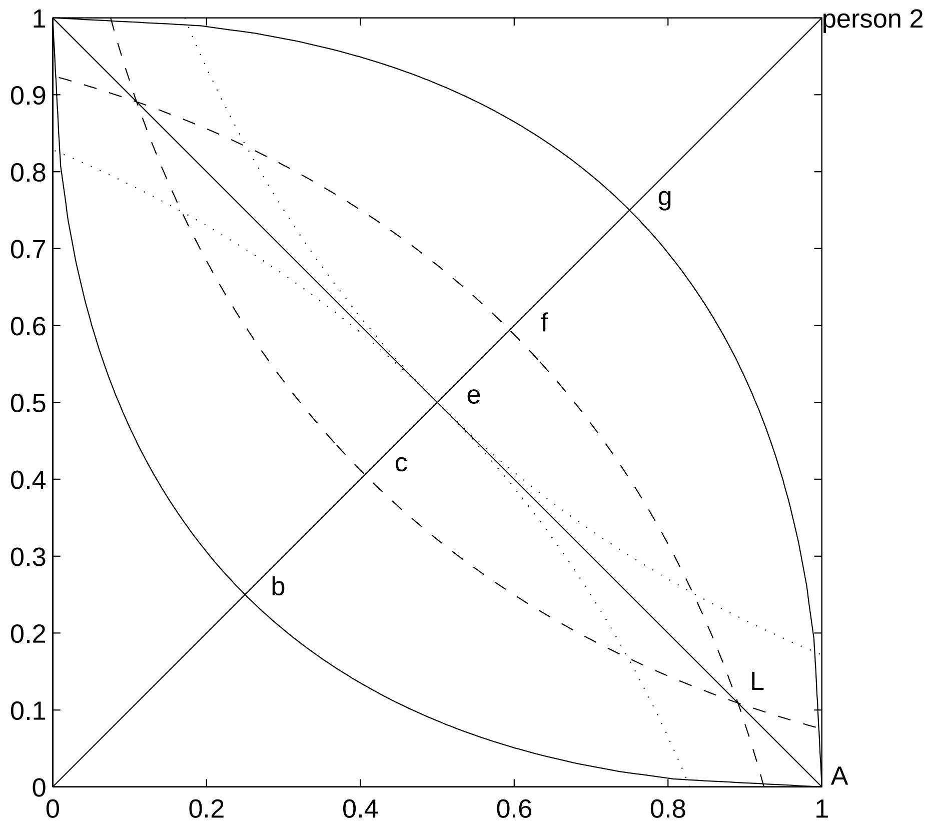

FIGURE 2. Effects of Altruism on the Core

the unique competitive equilibrium given the endowment point A lies at e.

But now suppose that agents are altruistic. If the first agent happens to receive the endowment, she can make herself better off if she makes a unilateral transfer to the second agent; the second agent will behave similarly if he happens to receive the endowment. Because each agent knows that the other agent is altruistic, an initial endowment point at A becomes an endowment point at L given the unilateral transfers that each agent would choose to make. Accordingly, the core of this economy at this level of altruism is a subset of the selfish core, and extends only from point c to point f.

In the limiting case that $\theta_1 = \theta_2 = 1$, the core would shrink to a singleton at e, but if altruism is less extreme, how can the two agents move from the altruistic endowment point L to an efficient allocation on the line segment cf? Neither will be willing to make a larger unilateral transfer, so the only alternative is to trade.

6. Intergenerational Altruism

Appeals to altruism are perhaps most commonly found in the literature on intergenerational transfers, in which parents harbor altruistic motives toward their children. The analysis of the previous section is easily modified for this case; if we simply label agent one the parent and agent two the child, setting $\theta_2 = 0$ yields preferences similar to those considered by many authors, beginning with Barro (1974).

In order to fully recover the case considered by Barro, however, we need to cast our model into an overlapping generations framework. Imagine that our economy begins with a parent and child in the first period. The parent dies at the end of the first period, while the child lives through the first and second periods.[10] Accordingly, let the utility of the parent be defined over own consumption in the first period and over the child's expected utility evaluated in the first period:

$$\hat{U}^1(c_1^o, \hat{U}^2) = U^1(c_1^o) + \theta \hat{U}^2$$

where

$$\hat{U}^2(c_2^y, c_w^o) = U(c_2^y) + \beta_2 U(c_2^o),$$

and where c_i^y denotes consumption of the ith generation in the first period of its life, c_i^o denotes consumption in the second period of i's life, and β_2 is the second generation's discount factor.

Suppose for a moment that that there is no uncertainty in this economy; the parent household receives an endowment of x_1 in the first period with certainty, while the child household will receive an endowment of x_2 in the second period, again with certainty. The altruistic parent, in choosing how much of the consumption good to give to the child in the first period, and how large a bequest to leave, will choose these to satisfy

$$\max U^1(c_1^o) + \theta[U^2(c_2^y) + \beta_2 U^2(c_2^o)]$$

such that the parent's budget constraint is satisfied:

$$c_1^o + c_2^y + b \leq x_1$$

[10]The second generation might give birth to a third generation in the second period, and the third to a fourth in the third period, and so on. Each generation may have different initial endowments and different levels of altruism. However, we can assume that the second generation is fruitless without loss of generality because the first generation cannot control allocations in periods beyond the second.

as well as the child's budget constraint,

$$c_2^o \leq x_2 + bR$$

where b denotes the (non-negative) bequest, and R the return on the bequest left to the second generation. Altruistic transfers to the child household will be chosen to satisfy

$$U^{1\prime}(c_1^o) = \theta U^{2\prime}(c_2^y) = \theta\ \beta_2\ R\ U^{2\prime}(c_2^o) \tag{15}$$

when the non-negativity constraint on b is non-binding (or when there are "operational intergenerational transfers," in Barro's (1974) terminology.

The sharing rule in this case bears a close resemblance to the sharing rule of the previous section in the case without trade. In that case, there was a sharp contrast between allocations determined purely on the basis of altruistic transfers, and allocations involving trade.

How do the competitive equilibrium allocations with altruism compare with the purely altruistic allocations? Equation (15) shows consumption is allocated to the child in such a way that the child cannot gain from additional trade (the child's intertemporal marginal rate of substitution is equated to the intertemporal marginal rate of transformation), so there are no gains to be had by adding markets in this case.

Could we observe these same allocations if the parent household was selfish and there were complete markets? From equation (6), a complete markets equilibrium would be characterized by

$$U^{1\prime}(c_1^o) = \frac{\xi_1}{\xi_2} U^{2\prime}(c_2^y)$$

so that the answer is yes if there exists some set of endowments such that $\xi_1/\xi_2 = \theta$.[11] An econometrician who observes consumption allocations and who does not know the value of θ or initial endowments would not be able to distinguish between selfish and altruistic behavior when there is no uncertainty.

What about an economy in which there is uncertainty? Adapting our model of the previous section, we might imagine that the random variable ω takes on values of ω_1, ω_2 indicating whether the endowment realization $\bar{x}$ is realized during the first or second periods; that is,

[11] And if preferences are 'nice,' we would expect that at least one such set of endowments does exist for $\theta \in (0, 1)$.

$$x_i(\omega) = \begin{cases} \overline{x} & \text{if } \omega = \omega_i \\ 0 & \text{otherwise} \end{cases}$$

How does this uncertainty affect allocations? If we permit only altruistic transfers, and no trade, then the parent will choose transfers in the first period to satisfy

$$U^{1\prime}(c_1^o(\omega_1)) = \theta U^{2\prime}(c_2^y(\omega_1)) = \theta\beta_2 U^{2\prime}(c_2^o(\omega_1)).$$

After the beginning of the first period, the parent either does or does not receive an endowment of $\bar{x}$, and all uncertainty is resolved. If the parent receives the endowment, then she will make a period one transfer of the consumption good and leave a bequest so that the marginal conditions above are satisfied. However, if the parent does not receive the endowment, then the non-negativity constraint on the bequest will be binding, and the parent's consumption will be zero. If the child cannot borrow against his income next period, he will consume the entire endowment in period two; otherwise, he will borrow so as to have some consumption in period one.

What if there are trading opportunities for the two agents at the beginning of period one, before ω is realized? There is scope for mutual insurance, and with complete markets all idiosyncratic risk will be eliminated: (15) will hold, just as it did in the case of complete certainty.

7. Conclusion

The theme of this paper has been that altruism does not improve the efficiency of allocation. Altruism *does* generally affect the distribution of resources, and from a normative standpoint, one may view such redistribution favorably. Moving slightly beyond the Pareto criterion, if (other things being equal) one prefers greater consumption equality to less, one might wish that people were more altruistic, but such wishful thinking cannot be justified purely on grounds of efficiency.

Researchers who wish to appeal to altruism to explain allocations in an environment with utility-maximizing agents should take care to distinguish the effects of altruism from those of possibly selfish exchange. Observations that agents smooth their consumption over time and states most certainly does not constitute evidence for altruistic behavior, though it is not inconsistent with it.

Much of the motivation for researchers interested in altruism stems from Barro (1974); it is not difficult to show using the arguments given in this paper that if children are born with some endowment (say some small amount of inalienable human capital), then it is possible to rationalize the sort of Ricardian equivalence observed in that paper without resorting to altruism as an explanation.

The bottom line of this paper is that if markets are complete and endowments are unobserved, then altruism of the sort modelled here will not help to explain observed allocations. A corollary of this point is that, in the absence of knowledge regarding initial endowments, observed allocations cannot be used to infer the existence of altruism.

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