AgEcon SEARCH
RESEARCH IN AGRICULTURAL & APPLIED ECONOMICS

The World's Largest Open Access Agricultural & Applied Economics Digital Library

This document is discoverable and free to researchers across the globe due to the work of AgEcon Search.

Help ensure our sustainability.

Give to AgEcon Search

AgEcon Search
http://ageconsearch.umn.edu
aesearch@umn.edu

*Papers downloaded from **AgEcon Search** may be used for non-commercial purposes and personal study only. No other use, including posting to another Internet site, is permitted without permission from the copyright owner (not AgEcon Search), or as allowed under the provisions of Fair Use, U.S. Copyright Act, Title 17 U.S.C.*

Department of Agricultural & Resource Economics, UCB
CUDARE Working Papers
(University of California, Berkeley)

Year 2000 *Paper* 857

Learning-by-Doing and the Choice of Technology: the Role of Patience

Larry Karp * In Ho Lee †

*University of California, Berkeley and Giannini Foundation
†Seoul National University

This paper is posted at the eScholarship Repository, University of California.
http://repositories.cdlib.org/are_ucb/857
Copyright ©2000 by the authors.

Learning-by-Doing and the Choice of Technology: the Role of Patience

Abstract

If agents learn-by-doing and are myopic, less advanced firms might adopt new technologies while more advanced ¿rms stick with the old technology. This kind of overtaking can also occur if agents are forward looking and have high discount rates. However, overtaking never occurs if agents are sufficiently patient. A finite discount rate increases the set of states at which agents adopt new technologies, so more patient agents tend to upgrade their technology more frequently.

Learning-by-Doing and the Choice of Technology: The Role of Patience

Larry Karp and In Ho Lee*

University of California, Berkeley and University of Southampton

April 28, 2000

Abstract

If agents learn-by-doing and are myopic, less advanced firms might adopt new technologies while more advanced firms stick with the old technology. This kind of overtaking can also occur if agents are forward looking and have high discount rates. However, overtaking never occurs if agents are sufficiently patient. A finite discount rate increases the set of states at which agents adopt new technologies, so more patient agents tend to upgrade their technology more frequently.

keywords: learning-by-doing, overtaking, leapfrogging, technology adoption, economic growth
JEL Classification numbers D92, O14,O33

*We thank Boyan Jovanovic and an anonymous referee for helpful comments. Remaining errors are ours.

1 Introduction

Modern development economics emphasizes the role of technology in determining growth paths. Lucas (1993) identifies technology adoption as the most important explanation of the economic growth of several Asian countries. Recent textbooks such as Aghion and Howitt (1998) and Barro and Sala-i-Martin (1995) reflect the importance attributed to technology in explaining growth. Technological improvements can lead to divergence in growth paths when firms in the "advanced" country have a greater incentive to adopt new technology. In other circumstances, firms in less advanced countries may be more likely adopt the new technology, even when it is less profitable for them than for the advanced firms. The adoption decision depends on a comparison of profits under the new technology and under the next best alternative, i.e. on the opportunity cost of adoption. The opportunity cost of adoption may be higher for the more advanced firms, because of their proficiency in using the old technology. In this case, innovations in technology can contribute to the convergence of growth paths, or even to "overtaking" (or "leapfrogging") by the less advanced country.

There have been a number of historical examples where technology adoption has contributed to overtaking, both at the industry and country level. Industries in regions destroyed by war (such as in post-war Europe and Japan) sometimes rebuild using the latest technology, eventually overtaking established industries elsewhere. Start-up industries may begin with the latest technology which incumbents are slow to adopt. Brezis et al. (1993) cite cases where new technologies have contributed to overtaking by entire countries rather than individual sectors.

The incentives to adopt a new technology depend on the firm's ability to use the previous generation of technology. This ability may depend on the experience the firm has had with the technology, i.e., on the amount of learning-by-doing that has occurred. Chari and Hopenhayn (1991), Parente (1994) and Stokey (1988) study learning-by-doing as a force for sustained growth. Brezis et al. (1993), Krussell and Rios-Rull (1996), and Jovanovic and Nyarko (1996) (hereafter JN) show that learning-by-doing can give rise to the type of overtaking noted in the growth literature. An agent accustomed to an existing technology may be unwilling to adopt a newer technology which requires learning and leads to lower profits in the short run. An agent who is less

familiar with the existing technology has a lower opportunity cost of adopting the new technology. The second agent may adopt the new technology and eventually overtake the first, who was initially more advanced.[1]

If learning is a non-excludable public good (as in Brezis et al. (1993)) or if it is a private good but firms are myopic (as in JN), the adoption decision depends on a comparison of *current* profits under the old and new technology. However, forward-looking firms who internalize learning-by-doing would consider the future stream of payoffs in deciding whether to adopt the new technology. We show that overtaking may still occur, but it is less likely when firms are forward looking. Overtaking is less likely in markets with high discount factors.

Jovanovic and Nyarko's 1994 working paper had also studied the problem when the firm is patient. That working paper establishes that overtaking can occur with a positive but sufficiently small discount factor (our Theorems 1 and 2). In addition to addressing the problem in a formal way, our Proposition 3 and Theorem 3 highlight the important balancing acts behind the technology adoption decision. Our results show that a larger discount factor increases the set of parameter values at which upgrading occurs; the more patient the firms are, the greater the benefit of the new technology, which takes time to learn.

2 Model

We modify JN's model of learning-by-doing by including forward looking firms. The payoff in period t depends on a random parameter y_t. As the firm learns about the distribution of this parameter, its payoff increases. A firm working with a technology

[1]There is an industrial organization literature on leapfrogging which is closely related to the economic growth literature we cite in the text. The IO literature emphasizes firms' strategic incentives to change a decision, such as improving technology. Budd et al. (1993) review recent contributions to leapfrogging models in IO, and Brezis et al. (1993) discuss the relation between the two literatures. Motta et al. (1997) study a model in which trade changes a firm's strategic decision (quality, in their case), and overtaking can occur. Their model thus incorporates elements of both the IO and economic growth literatures.

of grade n chooses x in period t and receives the payoff:

$$q = \gamma^n \left[1 - (y_t - x)^2\right], \ \gamma > 1.$$

After observing the payoff, the firm can infer the value of y_t since it knows γ, n, x. This inferred value is $y_t = \theta_n + w_t$, the sum of two random variables; θ_n is a random variable that depends on the technology grade n_t, and w_t is an i.i.d. Normal random variable with zero mean and variance σ_w^2. The firm knows the distribution of w_t. It does not know the value of θ_n but has prior beliefs about it.

Before learning y_t the firm maximizes the expected payoff by setting x equal to the expected value of y_t, conditional on information available in period t:

$$x = E_t[y_t] = E_t[\theta_n]$$

where the second equality follows from the fact that w_t is white noise. This choice yields the expected payoff:

$$E_t[q] = \gamma^n[1 - \text{var}_t(\theta_n) - \sigma_w^2] \tag{1}$$

where $\text{var}_t(\theta_n)$ is the variance of θ_n conditional on information available in period t.

Technology grades are integer-valued and the firm can move up by at most one grade in a single period. There is no pecuniary cost of switching, but skills acquired in working with the old technology are only partially transferable, so there are learning costs. Different grades of technology are linked to each other according to the following relationship:

$$\theta_{n+1} = \sqrt{\alpha}\, \theta_n + \epsilon_{n+1} \tag{2}$$

where $\epsilon_{n+1} \sim N(0, \sigma_\epsilon^2)$ and θ_n and ϵ_{n+1} are independent.

The firm updates its prior on θ_n based on the signal y_t. Denote the precision of the unknown technological parameter θ_n in period t by η_t and the precision of w_t by ν : $\eta_t = \frac{1}{\text{var}_t(\theta_n)}$, $\nu = \frac{1}{\sigma_w^2}$. We assume that $\nu > 1$, which implies that firms earn positive profits for sufficiently large η. In period 1 the firm begins with a Normal prior on the current technology (the value of θ) with precision η_1.

We now describe how η changes over time. First suppose that the firm does not upgrade technology in period t. Using the assumptions that w_t is a Normal random

variable, and that the prior on θ_n is Normal, the precision in period $t+1$ is (see DeGroot, 1970):

$$\eta_{t+1} = \eta_t + \nu. \tag{3}$$

If the firm upgrades, the variance is updated through two steps. The first step is due to the technology switch and the second is due to the observation of the outcome from the new technology. The first step transforms the variance (prior to the switch) $\text{var}_t(\theta_n)$ to $\alpha \cdot \text{var}_t(\theta_n) + \sigma_\epsilon^2$ (the variance after the switch) due to the transformation of θ_n as in equation (2). The firm then chooses x, observes q_t, infers y_t, and updates its beliefs about the value of θ_{n+1}. The second step transforms the post-switch variance using equation (3). Combining the two, the precision in the period after a switch occurs is

$$\eta_{t+1} = \frac{1}{\alpha/\eta_t + \sigma_\epsilon^2} + \nu = \frac{\eta_t}{\alpha + \eta_t \sigma_\epsilon^2} + \nu \equiv h\left(\eta_t\right) + \nu. \tag{4}$$

The function $h(\eta)$ represents the first step of the updating procedure. Hereafter we restrict attention to state space where $\eta \geq h(\eta)$.[2]

A forward-looking firm maximizes the present value of the infinite stream of payoffs with a discount factor, $\beta > 0$. In period 1 it starts with an arbitrary grade of technology, which we normalize to be grade $n = 0$. Define $k_t = 0$ if the firm keeps the current technology in period t and $k_t = 1$ if it upgrades. The strategy profile is $(k_1, k_2, \ldots)$. Define $T_n = \min_t\{k_1 + k_2 + ... + k_t \geq n\}$, the period in which it switches to the nth grade of technology, with the convention that $T_0 = 0$. Given a strategy $(k_1, k_2, \ldots)$, we can use the single-period expected payoff in equation (1) to compute the discounted expected payoff. We use the definition $\mu \equiv 1 - \sigma_w^2 = \frac{\nu - 1}{\nu} > 0$ in the single-period payoff function, equation (1).

The sequence problem which maximizes the discounted expected payoff, given the

[2]This restriction is innocuous, since for any initial condition it must be satisfied in finite time, regardless of the agent's upgrade decisions. If the restriction is satisfied at any period, it holds in all subsequent periods. Moreover, given the interpretation of the function $h(\eta)$, the model is sensible only when the restriction is satisfied. (If $\eta < h(\eta)$, upgrading increases precision, which means that the agent knows more about the new technology than about the old technology.)

initial precision η and technology grade $n = 0$ is:

$$W^*(\eta, 0) = \max_{(k_1, k_2, \dots)} W(k_1, k_2, \dots : \eta, 0)$$
$$= \sum_{n=0}^{\infty} \gamma^n \beta^{T_n} \{ \sum_{t=T_n}^{T_{n+1}-1} \beta^{t-T_n} [\mu - \frac{1}{\eta_t}] \} \tag{SP}$$

where η_t is updated according to either equation (3) or equation (4). The first argument of W^* is the precision, η, and the second is the technology grade, n (here $n = 0$).

We use the sequence problem to formulate the dynamic programming equation (DPE). The payoff from the firm's choice depends on the grade of the technology and the precision. Hence the DPE has two state variables, n_t and η_t:

$$\widetilde{V}(\eta_t, n_t) = \max_{k_t \in \{0,1\}} \{F(k_t; \eta_t, n_t) + \beta \widetilde{V}(\eta_{t+1}, n_t + k_t)\} \tag{5}$$

where

$$F(k_t; \eta, n) = \begin{cases} \gamma^n [\mu - \frac{1}{\eta}] & \text{if } k_t = 0 \\ \gamma^{n+1} [\mu - \frac{1}{h(\eta)}] & \text{if } k_t = 1, \end{cases}$$

$$\eta_{t+1} = \begin{cases} \eta_t + \nu & \text{if } k_t = 0 \\ h(\eta_t) + \nu & \text{if } k_t = 1, \end{cases}$$

and

$$n_{t+1} = n_t + k_t.$$

An optimal policy, $k^*(\eta, n)$, solves the DPE (5).

3 Preliminaries

Since we use the DPE in later analysis, we begin by showing that its solution exists and that it solves the problem (SP) under the following

Assumption 1 $\beta\gamma < 1$.

Proposition 1 *1. There exists a solution to the DPE (5).*

2. Under Assumption 1, the solution to the DPE satisfying

$$\lim_{t\to\infty} \beta^t \widetilde{V}(\eta_t, n_t) = 0$$

is the unique solution to the sequence problem (SP).

Next we show that the optimal upgrade rule depends on the value of η, but not on the grade of technology n or on time, t.

Proposition 2 *The optimal upgrade rule depends only on η, i.e. $k = k^*(\eta)$.*

The proofs of these and subsequent results are in the Appendix.

4 Choice of Technology

4.1 Myopic Case

We first review JN's results for the case where firms base their current adoption decisions only on profits in the current period. Here, firms solve the problem $\max\{\mu - \frac{1}{\eta}, \gamma(\mu - \frac{1}{h(\eta)})\}$, which uses the definition of $h(\eta)$. The first term in the maximand equals profits if the firm sticks with the current technology, and the second equals profits if the firm upgrades to the next generation of technology. The factor γ^n affects profits under both alternatives, but not the adoption decision.

The firm sticks with the current technology if and only if the current precision satisfies the inequality

$$z(\eta) \equiv \frac{\alpha\gamma + \gamma\sigma_\epsilon^2\eta - 1}{\eta} - \mu(\gamma - 1) \geq 0.$$

The function $z(\eta)$ gives the increased profits, in the current period, resulting from not upgrading. In other words, $z(\eta)$ is the cost of adoption. The slope of $z(\eta)$ has the same sign as $1 - \alpha\gamma$. If there exists a positive root of $z(\eta) = 0$, it is unique. Denote this root (when it exists) as $\eta^c \equiv \frac{1-\alpha\gamma}{\gamma\sigma_\epsilon^2 - \mu(\gamma-1)}$. The firm is indifferent between upgrading and sticking if and only if $\eta = \eta^c$, i.e. when the opportunity cost of adoption is zero.

	$\alpha\gamma > 1$	$\alpha\gamma < 1$
$\sigma_\epsilon^2 > \frac{\mu(\gamma-1)}{\gamma}$	stagnation; never upgrade $\nexists \eta^c$	(possible) overtaking; upgrade if $\eta < \eta^c$
$\sigma_\epsilon^2 < \frac{\mu(\gamma-1)}{\gamma}$	standard case; upgrade if $\eta > \eta^c$	continual upgrading $\nexists \eta^c$

Table 1: The Myopic Model

Table 1 summarizes the relation between the parameter values and the optimal decision. In entries along the diagonal, it is optimal either never to upgrade or to upgrade in every period, regardless of the value of η. In these situations, η^c does not exist. In the lower left entry of Table 1, firms with low precision stick with the current technology until they learn to use it sufficiently well (until $\eta \geq \eta^c$), at which time they upgrade. We refer to this as the "standard" case.

In the upper right entry, it is optimal to upgrade only if the firm has low precision. A firm that is relatively unfamiliar with the current technology (i.e., has low precision $\eta < \eta^c$) upgrades, whereas the firm that knows how to use the current technology well (i.e. has high precision $\eta > \eta^c$) sticks with it. In this situation, the firm with lower initial precision (and thus, lower initial profits) may eventually obtain higher profits: it continues to upgrade its technology even though it never becomes expert at using it. In that sense, it overtakes the firm with high initial precision.

In order to guarantee that overtaking occurs, we need the following additional restriction. Define η_s as the (unique) positive steady state to equation (4).

Assumption 2 $\eta^c > \eta_s$.

The following lemma summarizes the overtaking result in JN.

Lemma 1 *(Overtaking) When $\alpha\gamma < 1$, $\sigma_\epsilon^2 > \frac{\mu(\gamma-1)}{\gamma}$, and Assumption 2 holds, a firm with initial precision $\eta < \eta^c$ eventually earns higher profits than a firm with initial precision $\eta > \eta^c$.*

If Assumption 2 did not hold, all firms would eventually cease to upgrade, and overtaking might not occur. Hereafter, when discussing the case of overtaking, we maintain Assumption 2.

4.2 General Case

This section generalizes the results from the myopic setting. All of the four possibilities described in Table 1 remain when β is positive. Thus, the possibility that overtaking occurs does not rely on the assumption that firms are myopic. However, if firms are sufficiently patient, overtaking cannot occur. We also show that a positive value of β never decreases, and typically increases the set of precision levels at which upgrading is optimal. In this sense, a forward looking firm upgrades more frequently than a myopic firm.

Overtaking requires that there is an interval of η over which the firm is willing to upgrade. Moreover, if the initial precision lies in this interval, the equilibrium technology sequence is unbounded: $\lim_{t\to\infty} n_t = \infty$. There is also a critical value of η, which we denote $\overline{\eta}$, above which the firm never upgrades. Thus, if one firm begins with precision in the interval for which upgrading continues, and a second firm begins with $\eta > \overline{\eta}$, the first firm eventually uses a higher grade technology and receives higher profits in every period, regardless of the initial technologies (the initial values of n).

The next two theorems analyze the first row of Table 1 when $\beta > 0$. Theorem 1 shows that overtaking is a generic possibility. Theorem 2 shows that a sufficiently large value of β eliminates the possibility of overtaking.[3]

Theorem 1 *If Assumptions 1 and 2 hold and $\alpha\gamma < 1$, $\sigma_\epsilon^2 > \frac{\mu(\gamma-1)}{\gamma}$ (so that overtaking occurs when $\beta = 0$), overtaking can occur for small positive values of β.*

Theorem 1 demonstrates the robustness of the overtaking result for a small positive discount factor. Although the possibility of overtaking is generic, it never occurs if firms are sufficiently patient.

Theorem 2 *Suppose Assumption 1 holds.*

1. *If $\alpha\gamma < 1$, $\sigma_\epsilon^2 > \frac{\mu(\gamma-1)}{\gamma}$, and Assumption 2 holds (so that overtaking occurs when $\beta = 0$), there exists $\beta^* < \frac{1}{\gamma}$ such that for all $\beta \geq \beta^*$, overtaking cannot occur.*

[3] We had completed our analysis of this problem before learning of Jovanovic and Nyarko's working paper (1994), which contains theorems 1 and 2. We include our proofs of these theorems in order to make this paper self-contained.

2. *If* $\alpha\gamma > 1$, $\sigma_\epsilon^2 > \frac{\mu(\gamma-1)}{\gamma}$ *(so that it is never optimal to upgrade when* $\beta = 0$*), and in addition* $\mu - \frac{1}{h(\eta_s)} > 0$,[4] *it is sometimes optimal to upgrade when* $\beta \geq \beta^*$.

We next show how forward-looking behavior changes the set of η at which upgrading is optimal. We define $\eta^{c\beta}$ as a value of η at which the firm with discount factor β is indifferent between sticking with the current technology and upgrading. That is, $\eta^{c\beta}$ satisfies

$$z(\eta) = \beta\left[\gamma V(h(\eta) + \nu) - V(\eta + \nu)\right] \tag{6}$$

(so $\eta^{c0} = \eta^c$). As with the static case, $\eta^{c\beta}$ may not exist, in which case the firm either upgrades in every period, or never upgrades. Unlike the static case, we have not shown that $\eta^{c\beta}$ is unique. When we refer to $\eta^{c\beta}$ we always mean any value of η that satisfies equation (6).

We show that $z(\eta^{c\beta}) > 0$ for $\beta > 0$. This inequality means that at a level of precision where the firm is indifferent between upgrading and sticking, upgrading reduces profits in the current period. We have

Proposition 3 *For* $\beta > 0$*, at a level of precision where the firm is indifferent between upgrading and sticking, upgrading causes losses in the current period:* $z(\eta^{c\beta}) > 0$.

Proposition 3 implies that forward-looking firms upgrade to the new technology because the future benefit from the new technology exceeds the short-term cost from discarding the old technology. In other words, forward looking firms upgrade when the current payoff from the new technology is strictly less than the current payoff from the old technology. In contrast, myopic firms upgrade only when the current payoff from the new technology is at least as great as the current payoff from the old technology.

Define the "upgrade set" $\Delta^\beta = \{\eta : k(\eta) = 1\}$, the set of η for which it is optimal to upgrade, given β. Table 1 implicitly defines Δ^0 (the upgrade set for $\beta = 0$) under different configurations of parameter values. The following theorem compares the upgrade sets for $\beta = 0$ and for $0 < \beta < \frac{1}{\gamma}$ under these four configurations of parameter values.

[4] When η^c does not exist, we obviously cannot invoke Assumption 2. We therefore impose this inequality directly.

Theorem 3 *For* $0 < \beta < \frac{1}{\gamma}$, $\Delta^0 \subseteq \Delta^\beta$.

Theorem 3 means that forward-looking firms are "more likely" to upgrade than myopic firms. For example, if overtaking occurs in the myopic setting, the introduction of a positive discount factor reduces (and according to Theorem 2 may eliminate) the values of η above which further upgrading never occurs. In addition, if $\sigma_\epsilon^2 < \frac{\mu(\gamma-1)}{\gamma}$ (the second row in Table 1) so that overtaking does not occur when $\beta = 0$, then overtaking cannot occur when $\beta > 0$. Finally, if $\sigma_\epsilon^2 > \frac{\mu(\gamma-1)}{\gamma}$, $\alpha\gamma > 1$ (the upper left entry in Table 1) myopic firms would never upgrade. For these parameter values and $\beta > 0$, firms might upgrade when η is sufficiently large. In this case, the introduction of a positive discount factor transforms the "stagnation" scenario to the "standard" scenario, in which firms wait until they are sufficiently familiar with the current technology before upgrading.

Theorem 3 compares the upgrade sets for a myopic and a forward looking firm. Under the hypothesis that the value function is differentiable in β, we can show that for small values of β the upgrade set is monotone in β: $\Delta^\beta \subseteq \Delta^{\beta'}$ for $\beta' > \beta$. In this case, as the firm becomes more patient, the set of precision levels at which it upgrades increases. Unfortunately, this is a local result (at $\beta = 0$) and it assumes differentiability of the value function.

In order to check the robustness of this local result, we solved the dynamic programming problem numerically using the method of value function iteration.[5] In these simulations we treat the state variable as $var(\theta) \equiv \frac{1}{\eta} \geq 0$. The restriction $\eta \geq h(\eta)$ implies that $var(\theta) \leq \varphi$, where φ solves $\frac{1}{\varphi} = h(\frac{1}{\varphi})$. Thus, the state space is the interval $[0, \varphi]$. For all of our simulations, we find that $\eta^{c\beta}$ is unique, which implies that the upgrade set is connected; if the firm upgrades at two levels of $var(\theta)$, it upgrades at any convex combination of those values. Also, the upgrade set is monotonic in β: more patient firms are more likely to adopt new technologies.

Figure 1 graphs the critical value of $var(\theta)$ (which equals the inverse of $\eta^{c\beta}$) as

[5] We put a grid on the state space and used linear interpolation to calculate the value function at points off the grid. We iterated on the value function until the largest change in the value function (over all points on the grid, from one iteration to the next) was no greater than 10^{-10}. In most cases we achieved this tolerance in fewer than 75 iterations, although for values of β that approached $\frac{1}{\gamma}$ we needed nearly 400 iterations.

a function of β for three values of γ, 1.1, 1.3 and 1.5. The vertical intercepts of the graphs show the inverse of the critical precision for the myopic firm, η^c. The other parameter values in this simulation are: $\alpha = 0.5$, $\sigma_\epsilon^2 = 0.25$, and $\nu = 3.33$ (so $\mu = 0.7$). In the static model ($\beta = 0$) there is overtaking. For all values of β, firms upgrade if $var(\theta)$ exceeds the critical level (the graphs). In every case, the critical value is positive for small values of β (Theorem 1) and it equals 0 for values of $\beta < \frac{1}{\gamma}$ (Theorem 2). In addition to illustrating these theorems, the numerical results show that the upgrade set is connected and is monotonic in β. Figure 1 shows that an increase in γ decreases the critical variance, and thus increases the upgrade set. A larger value of γ increases the advantage of upgrading.

This model illustrates the manner in which technological inventions or changes in financial institutions can precipitate a process of continued innovation. Initially, an economy may be quite stagnant, as it was prior to the industrial revolution. During this period, improvement in productive methods are gradual and of decreasing marginal value. A technological invention increases γ, and the development of financial markets may lower the interest rate, increasing β. Either of these kinds of changes can move the economy into a region where innovation continues indefinitely, as in the post industrial revolution era.

The model assumes that there is no spillover of information across firms. There are a number of ways that we can think of informational spillovers. If all firms had exactly the same information, they would make the same decisions and there could be no possibility of overtaking. A more realistic view is that one firm's learning is more informative to firms with similar technologies, and that a firm learns only from firms with a more advanced technology; i.e. spillovers are asymmetric. (For example, the additional information acquired by someone who does wordprocessing is more useful to someone who currently uses a typewriter than it is to someone who uses only pen and paper, and the additional learning by the person who uses pen and paper does not help the other two agents.) Equation (2) is consistent with this assumption. A lower value of α implies that the information content of a signal decays rapidly as the distance between generations of technology increases. (See footnote 7 of JN.)

JN show that overtaking can occur with this model of spillovers when $\beta = 0$. This possibility also arises for small β, as a modification of their argument shows. When

α is sufficiently small, spillovers are not very important, and their presence cannot eliminate overtaking. JN also point out that in order for spillovers to change a firm's decision not to upgrade, the spillovers must have an effect "early on", while the leader is still using a technology similar to the laggard's. This qualitative result should also hold in the case where $\beta > 0$.

In other respects, a positive value of β may change the effect of spillovers among a group of firms that behave nonstrategically. Suppose that firms are small, and that each firm takes the investment trajectory of other firms as given. Each firm recognizes that by failing to keep up with firms that are using more advanced technologies, it lowers its ability to learn from them in the future. This recognition increases the value of upgrading in the current period rather than in the future. A larger value of β increases the importance of this effect, and thus tends to increase the role of spillovers in promoting innovation.

However, in an equilibrium where overtaking occurs, a firm recognizes that by delaying innovation it increases the number of firms who will be using the same or a higher level of technology in the future. This delay increases the firm's ability to learn in the future and provides an incentive to postpone adoption. A larger value of β magnifies this incentive.

Thus, for a specific firm, it is unclear whether spillovers increase or decrease the incentive to upgrade when $\beta > 0$. Consequently, the equilibrium effect of spillovers is uncertain. Note that the incentive to delay vanishes when $\beta = 0$ or if there is no overtaking in equilibrium. In either of those cases, spillovers have only the positive effect on innovation at both the individual and the aggregate levels.

4.3 Generalized Technology Choice

The firm's choice set is rather restricted, since it must either upgrade to the next level or continue with the current technology. In reality, firms can choose from a variety of technologies. To analyze the consequence of a wider choice, we consider the extreme opposite possibility in this subsection. Here we assume that the firm can choose any (possibly infinite) level of upgrade. We refer to the case where k can be any non-negative number as the unrestricted model, and the case where k must take the value 0 or 1 as the restricted model.

We first modify the link between different grades of technology as in JN (equation (3) on page 1301) for the unrestricted model. That is, we replace our equation (2) with

$$\theta_{n+k} = \alpha^{k/2}\theta_n + \epsilon_k \tag{7}$$

where $\epsilon_k \sim N(0, \rho_k \sigma_\epsilon^2)$ and

$$\rho_k = \begin{cases} (1-\alpha^k)/(1-\alpha) & \text{if } \alpha \neq 1, \\ k & \text{if } \alpha = 1 \end{cases}$$

and θ_n and ϵ_k are independent. The variance after a k-stage upgrade is

$$\frac{\alpha^k}{\eta_t} + \frac{1-\alpha^k}{1-\alpha}\sigma_\epsilon^2. \tag{8}$$

The single period profit of a k-stage upgrade is[6]

$$P(\eta, k) = \gamma^k \left[\left(\mu - \frac{\sigma_\epsilon^2}{1-\alpha} \right) + \alpha^k \left(\frac{\sigma_\epsilon^2}{1-\alpha} - \frac{1}{\eta_t} \right) \right]. \tag{9}$$

For comparison we consider the myopic firm first.

Proposition 4 *Suppose the firm is myopic.*

1. (a) *If $\alpha < 1$ and $\mu < \frac{\sigma_\epsilon^2}{1-\alpha}$, then it is optimal to set k finite (possibly 0) and the payoff is bounded.*

   (b) *If $\alpha < 1$ and $\mu > \frac{\sigma_\epsilon^2}{1-\alpha}$, then it is optimal to set $k = \infty$, and the payoff is unbounded.*

2. *If $\alpha > 1$, a finite k (possibly 0) is optimal.*

The conditions under Part 1(a) of Proposition 4 are consistent with parameters in the upper right corner of Table 1, so the over-taking result is not overturned by the generalized technology choice. The parameter restrictions in the lower right corner of Table 1 imply that the conditions in Part 1(b) are met; therefore, when continual upgrading is optimal under the restricted technology, the firm always prefers an immediate infinite upgrade. The condition under Part 2 of the proposition is consistent with

[6] Hereafter we ignore the special case $\alpha = 1$ since it has the same characterization as the case $\alpha > 1$.

the left column of Table 1; here, as in JN (Proposition 4.1 on page 1305), a finite k is optimal.

Next we consider the case where $\beta > 0$. If it is optimal to set $k = \infty$ when $\beta = 0$ in the unrestricted model, a patient firm ($\beta > 0$) finds it optimal to set $k = \infty$ in the unrestricted model, *a fortiori*. Hence the optimal choice under the condition of Part 1(b) of Proposition 4 is unbounded.

When the optimal choice of the myopic firm is bounded, a comprehensive analysis is fairly difficult. Therefore we ask the following limited question: If the patient firm chooses $k = 1$ in the restricted model, will it necessarily choose $k = \infty$ in the unrestricted model? That is, if the optimal solution in the restricted model is at the upper boundary, does the optimal solution in the unrestricted model equal infinity?

Proposition 5 *If $\alpha\gamma > 1, \sigma_\epsilon^2 < \frac{\mu(\gamma-1)}{\gamma}$ and $\eta > \eta^c$ then it is optimal to set $k = 1$ in the restricted model while $k < \infty$ in the unrestricted model.*

We conclude that the conditions that insure that the firm is on the boundary $k = 1$ in the restricted model are not sufficient to insure that the firm would be at the upper boundary when it has a larger (possibly infinite) choice set.

5 Conclusion

When skills are only partly transferable across generations of technology, greater familiarity with an existing technology may make it easier to upgrade. However, greater skill at using the existing technology also leads to a higher opportunity cost of upgrading. A firm that is less skilled has a lower opportunity cost and may upgrade, even though it cannot use the new technology as profitably as a more skilled firm that chooses not to upgrade. The less skilled firm may continue to upgrade to increasingly sophisticated technologies, even though it never becomes expert at using any of them. It eventually achieves higher profits than the more skilled firm.

This kind of overtaking can occur even when firms are forward looking, as in Parente's (1994) model. However, overtaking never occurs if firms are sufficiently patient.

When the myopic firm's upgrade decision depends non-trivially on its skill level, a forward looking firm decides to upgrade for a larger set of skill levels. In this sense,

forward looking firms are more likely to upgrade, and they upgrade more frequently.

Low levels of economic development are often associated with inefficient financial markets and a high discount rate. The high cost of capital discourages adoption of a new technology, and thus impedes development. Somewhat paradoxically, a high discount rate may also make overtaking more likely. Thus, a situation where technologically backward firms overtake their relatively advanced rivals is more likely to occur in markets where discount rates are high.

6 Appendix: Proofs

Proof. (*Proposition 1.*) The proof for part 1 of the proposition is standard; define the operator

$$\mathcal{T}\widetilde{V} = \max_{k\in\{0,1\}}\{F(k:\eta_t, n_t) + \beta\widetilde{V}(\eta_{t+1}, n_t + k)\}.$$

Since $\mathcal{T}$ is a contraction mapping with modulus β, the solution to (5) exists.

The second part follows from the result that if the solution to the sequence problem (SP) is bounded, the solution to the DPE satisfying

$$\lim_{t\to\infty}\beta^t\widetilde{V}(\eta_t, n_t) = 0$$

is the unique solution to the sequence problem (SP). (Theorem 4.3 on p.72 of Stokey and Lucas (1989)) Hence it suffices to prove that Assumption 1 implies that the solution to the sequence problem (SP) is bounded.

If $\beta\gamma < 1$, then

$$\begin{aligned} W^*(\eta, 0) &= \max_{(k_1,k_2,\dots)} W(k_1, k_2, \dots : \eta_{1,1}) \\ &= \sum_{n=1}^{\infty}\gamma^{n-1}\beta^{T_{n-1}}\{\sum_{t=T_{n-1}}^{T_n-1}\beta^{t-T_{n-1}}[\mu - \frac{1}{\eta_t}]\} \\ &\leq \sum_{t=1}^{\infty}\beta^t\gamma^t\mu = \frac{\mu}{1-\beta\gamma} < \infty. \end{aligned}$$

Therefore the solution to the sequence problem (SP) is bounded. ■

Proof. *(Proposition 2.)* We use the fact that $F(k;\eta,n) = \gamma^n F(k;\eta,0)$ to "guess" the trial solution: $\widetilde{V}(\eta,n) = \gamma^n V(\eta)$ for some function V Given the uniqueness of

$\widetilde{V}(\eta, n)$, this trial solution must be correct if it solves the DPE. Since the equation of motion of η is independent of n, we can substitute the trial solution into equation (5) to obtain an equivalent DPE

$$\gamma^n V(\eta_t) = \max_{k_t \in \{0,1\}} \gamma^n \{F(k; \eta_t, 0) + \beta \gamma^k V(\eta_{t+1})\}. \tag{10}$$

Dividing both sides by γ^n results in a DPE – and thus an optimal decision rule – which is independent of both n and t. ■

Proof. *(Lemma 1.)* The firm with initial precision $\eta > \eta^c$ never upgrades, so $\eta_t \to \infty$ and its profits converge to $\gamma^{n_0}\mu$, where n_0 is the initial grade of technology. The firm with initial precision $\eta < \eta^c$continues to upgrade in every period so $n_t \to \infty$ and $\eta_t \to \eta_s$. Thus, its profits approach $+\infty$ provided that $\mu - \frac{1}{h(\eta_s)} > 0$. Suppose to the contrary that $\mu - \frac{1}{h(\eta_s)} \leq 0$. In that case, $\gamma(\mu - \frac{1}{h(\eta_s)}) \leq \mu - \frac{1}{h(\eta_s)} < \mu - \frac{1}{\eta_s}$ (since $h(\eta_s) < \eta_s$), so it is not optimal to upgrade at η_s, contradicting the assumptions of the lemma. ■

Proof. *(Theorem 1.)* We show that for sufficiently small but positive values of β, it is optimal to upgrade in every period when η is small, and it is optimal never to upgrade when η is large. Using equation (10) and the definition of $z(\eta)$, it is optimal not to upgrade if

$$z(\eta) > \beta \left[\gamma V(h(\eta) + \nu) - V(\eta + \nu)\right]. \tag{11}$$

$V(\eta)$ is nondecreasing and $V(\frac{1}{\mu}) > 0$, since the strategy of never upgrading in the future gives a stream of positive payoffs when $\eta > \frac{1}{\mu}$. Thus, for $\eta \geq \frac{1}{\mu}$, the right side of equation (11) is bounded above by $\beta\gamma V(h(\eta) + \nu)$. For all η, $\beta\gamma V(h(\eta) + \nu)$ is bounded above by $\frac{\beta\gamma\mu}{1-\beta\gamma}$ (which equals the present value of the payoff if a new technology is adopted in every period and the precision instantly becomes infinite). Define η^* as the unique positive solution to $z(\eta) = \frac{\beta\gamma\mu}{1-\beta\gamma}$. Given the assumed parameter restrictions, η^* exists for β sufficiently small but positive. Thus, equation (11) is satisfied, and it is optimal not to upgrade for $\eta \geq \overline{\eta} \equiv \max\{\eta^*, \frac{1}{\mu}\}$.

It is optimal to upgrade if the inequality in equation (11) is reversed. The right side of equation (11) is approximately 0 for small β; the left side is independent of β and is strictly negative for η in the neighborhood of η_s (since $\eta_s < \eta^c$). Therefore,

for sufficiently small β there exists a critical value of η greater than η_s, below which it is optimal to upgrade. If the initial value of η is below this critical value, the firm upgrades in every period. Since $\mu - \frac{1}{h(\eta_s)} > 0$ by lemma 1, overtaking occurs ■

Proof. (*Theorem 2.)* 1. Overtaking requires that firms with sufficiently high precision never upgrade. We show that never upgrading in the future cannot be an optimal policy when β is large. Define $\pi_s \equiv \mu - \frac{1}{h(\eta_s)}$, which is positive by lemma 1. Therefore the value of the optimal program at η_s is $V(\eta_s) \geq \frac{\pi_s}{1-\beta\gamma}$. The payoff from never upgrading is bounded above by $\frac{\mu}{1-\beta}$. Monotonicity of $V(\eta)$ implies that it is not optimal to stick with the current technology forever if $\frac{\pi_s}{1-\beta\gamma} > \frac{\mu}{1-\beta}$, i.e. if $\beta > \frac{\mu-\pi_s}{\gamma\mu-\pi_s} \equiv \beta^*$. Since $\gamma > 1$, $\beta^* < \frac{1}{\gamma}$. Thus there exists a range of parameter values that satisfy Assumptions 1 and 2 and $\alpha\gamma < 1$, $\sigma_\epsilon^2 > \frac{\mu(\gamma-1)}{\gamma}$, for which overtaking cannot occur.

2. The proof of part 2 uses the same argument to show that never upgrading is not optimal when β is sufficiently large. ■

The proof of Proposition 3 uses the following two lemmas:

Lemma 2 *Define the function* $\chi(\eta, w) \equiv (\gamma - 1)\mu + \frac{1}{\eta+w} - \frac{\gamma}{h(\eta)+w}$. $\chi(\eta, w)$ *is an increasing function of* w.

Proof. Differentiate the function χ and use the restriction that $\eta > h(\eta)$. ■

Lemma 3 $h(\eta) + \nu > h(\eta + \nu)$.

Proof. $h(\eta + \nu) < h(\eta) + h'(\eta)\nu < h(\eta) + \nu$ where the first inequality follows from concavity and the second from the restriction $\eta > h(\eta)$ which implies $h'(\eta) < 1$. ■

Proof. *(Proposition 3.)* Suppose to the contrary that

$$z(\eta^{c\beta}) \leq 0 \tag{12}$$

We derive a contradiction for the two interesting cases.[7] Case 1: it is optimal to upgrade at $\eta^{c\beta} - \epsilon$ for small positive ϵ and it is optimal to stick with the current technology

[7] We ignore the unlikely possibility that the firm prefers to upgrade (or prefers to stick) for both $\eta^{c\beta} \pm \epsilon$, ϵ small. Even if this situation could arise, it is plausible that a perturbation of parameters would eliminate it.

for T periods at $\eta^{c\beta}+\epsilon$. (We allow the possibility that $T=\infty$, a necessary condition for overtaking.) Case 2: It is optimal to stick at $\eta^{c\beta}-\epsilon$ for small positive ϵ and it is optimal to upgrade at $\eta^{c\beta}+\epsilon$. (Case 2 corresponds to the second row of Table 1.)

Case 1. Choose $\eta=\eta^{c\beta}+\epsilon$, so that the optimal policy yields the payoff

$$V(\eta)=\sum_{t=0}^{T-1}\beta^t\left(\mu-\frac{1}{\eta+t\nu}\right)+\beta^T\gamma V(h(\eta+T\nu))$$

where T (possibly infinite) is the optimal time of the next upgrade. Consider the deviation of moving forward the time of the next upgrade, e.g. upgrading at time 0 rather than time T. The payoff corresponding to this deviation is $D(\eta)$

$$D(\eta)=\sum_{t=0}^{T-1}\beta^t\gamma\left(\mu-\frac{1}{h(\eta)+t\nu}\right)+\beta^T\gamma V(h(\eta)+T\nu).$$

Using these expressions, we have

$$D(\eta)-V(\eta)=\left[\sum_{t=0}^{T-1}\beta^t\chi(\eta,t\nu)\right]+\beta^T\gamma\left\{V(h(\eta)+T\nu)-V(h(\eta+T\nu)\right\}.$$

Evaluate this difference at $\eta=\eta^{c\beta}$, where $\chi(\eta,0)=-z(\eta)\geq 0$ by equation (12). By lemma 2, $\chi(\eta,t\nu)>0$ for $t>0$, so the term in the square brackets is positive. By lemma 3 and monotonicity of V, the term in the curly brackets is positive. Therefore $D(\eta^{c\beta})-V(\eta^{c\beta})>0$, which contradicts optimality.

Case 2. Choose $\eta\leq\eta^{c\beta}$ with $\eta+\nu>\eta^{c\beta}$. The optimal policy at such a value of η is to wait until the next period to upgrade, which leads to the payoff

$$V(\eta)=\left(\mu-\frac{1}{\eta}\right)+\beta\gamma\left(\mu-\frac{1}{h(\eta+\nu)}\right)+\beta^2\gamma V(h(\eta+\nu)+\nu).$$

Consider the deviation of upgrading in the current period rather than in the next one. We again denote the value of this deviation as $D(\eta)$:

$$D(\eta)=\gamma\left(\mu-\frac{1}{h(\eta)}\right)+\beta\gamma\left(\mu-\frac{1}{h(\eta)+\nu}\right)+\beta^2\gamma V(h(\eta)+2\nu).$$

The difference in the payoff is

$$\begin{aligned}D(\eta)-V(\eta) &= -z(\eta)+\beta\gamma\left[\frac{1}{h(\eta+\nu)}-\frac{1}{h(\eta)+\nu}\right]\\ &\quad+\beta^2\gamma\left\{V(h(\eta)+2\nu)-V(h(\eta+\nu)+\nu)\right\}.\end{aligned}$$

Evaluate this difference at $\eta = \eta^{c\beta}$. The first term on the right side is non-negative by equation (12), the second term (square brackets) is positive by lemma 3, and the third term (curly brackets) is positive by lemma 3 and the monotonicity of the value function. Consequently, $D(\eta) - V(\eta) > 0$, which contradicts optimality. ■

We use Proposition 3 to compare the critical values $\eta^{c\beta}$ and η^c, and thus to obtain an intermediate result needed for Theorem 3. In order to allow for the possibility that $\eta^{c\beta}$ is not unique, we define $\overline{\eta}^{c\beta} = \max\{\eta^{c\beta}\}$ and $\underline{\eta}^{c\beta} = \min\{\eta^{c\beta}\}$. We have

Corollary 1 *Suppose $\eta^{c\beta}$ and η^c exist. Then $\overline{\eta}^{c\beta} < \eta^c$ for $\alpha\gamma > 1$, and $\underline{\eta}^{c\beta} > \eta^c$ for $\alpha\gamma < 1$.*

Proof. By inspection, $z(\eta)$ is monotonic, and the derivative $\frac{dz}{d\eta}$ has the same sign as $1 - \alpha\gamma$. From Proposition 3, $z(\eta^{c\beta}) > 0 = z(\eta^c)$. Hence, when $\frac{dz}{d\eta} > 0, \eta^{c\beta} > \eta^c$, implying $\underline{\eta}^{c\beta} > \eta^c$ for $\alpha\gamma < 1$. When $\frac{dz}{d\eta} < 0, \eta^{c\beta} < \eta^c$, implying $\overline{\eta}^{c\beta} < \eta^c$ for $\alpha\gamma > 1$. ■

Proof. (*Theorem 3.)* We prove the claim for the three separate cases in Table 1.

(i) If, for $\beta = 0$, there is either stagnation ($\alpha\gamma > 1$ and $\sigma_\epsilon^2 > \frac{\mu(\gamma-1)}{\gamma}$) or continual upgrading ($\alpha\gamma < 1$ and $\sigma_\epsilon^2 < \frac{\mu(\gamma-1)}{\gamma}$), then $\Delta^0 \subseteq \Delta^\beta$.

(ii) If the "standard case" occurs when $\beta = 0$ ($\alpha\gamma > 1$ and $\sigma_\epsilon^2 < \frac{\mu(\gamma-1)}{\gamma}$), then $\Delta^0 \subset \Delta^\beta$.

(iii) If overtaking is possible when $\beta = 0$ ($\alpha\gamma < 1$, and $\sigma_\epsilon^2 > \frac{\mu(\gamma-1)}{\gamma}$), then $\Delta^0 \subset \Delta^\beta$.

We take these cases in turn.

(i) Under stagnation, $\Delta^0 = \emptyset \subseteq \Delta^\beta$. (From Theorem 2, Δ^β may be nonempty, in which case $\Delta^0 \subset \Delta^\beta$.) Under continual overtaking, $\Delta^0 = \Re^+$, and it is straightforward to show that $\Delta^\beta = \Re^+$.

(ii) In this case, $\Delta^0 = \{\eta : \eta > \eta^c\}$. If $\eta^{c\beta}$ exists, then it must be the case that $\Delta^\beta \supseteq \{\eta : \eta > \overline{\eta}^{c\beta}\}$. If this relation did not hold, then for sufficiently large η, it is optimal never to upgrade. However, using the inequality $\sigma_\epsilon^2 < \frac{\mu(\gamma-1)}{\gamma}$ we can show that for sufficiently large η the payoff of upgrading once and then never subsequently upgrading is greater than the payoff of never upgrading. Since $\overline{\eta}^{c\beta} < \eta^c$

from Corollary 1, we obtain $\Delta^\beta \supseteq \{\eta : \eta > \overline{\eta}^{c\beta}\} \supset \Delta^0$. If $\eta^{c\beta}$ does not exist, it is optimal to upgrade for all η, so $\Delta^\beta = \Re^+$.

(iii) In this case, $\Delta^0 = \{\eta : \eta \leq \eta^c\}$. If $\eta^{c\beta}$ exists, then from Corollary 1, $\underline{\eta}^{c\beta} > \eta^c$. We need to show that $\Delta^\beta \supseteq \{\eta : \eta < \underline{\eta}^{c\beta}\}$. (This relationship implies that for $\beta > 0$ it is strictly better to upgrade at $\eta = \eta^c$.) Suppose, to the contrary, that for $\eta < \underline{\eta}^{c\beta}$ it is optimal not to upgrade. Then at $\eta = \eta^c$ it is optimal to stick with the current technology for $T \geq 1$ periods, where T is the smallest integer that satisfies $\eta = \eta^c + T\nu \geq \eta^{c\beta}$. At time T time it is optimal to upgrade. Consider the deviation of upgrading in the current period (when $\eta = \eta^c$) rather than waiting T periods. The additional profits resulting from this deviation, rather than following the optimal program, are

$$\left[\sum_{t=0}^{T-1} \beta^t \chi(\eta^c, t\nu)\right] + \beta^T \left\{V(h(\eta^c) + T\nu) - V(h(\eta^c + T\nu))\right\}.$$

The first term (square brackets) is positive using the definition of η^c and lemma 2, and the second term (curly brackets) is positive by lemma 3 and monotonicity of $V(\eta)$. Consequently, it must be optimal to upgrade when $\eta = \eta^c$ and $\beta > 0$. Therefore $\Delta^\beta \supseteq \{\eta : \eta < \underline{\eta}^{c\beta}\} \supset \{\eta : \eta < \eta^c\} = \Delta^0$.

If $\eta^{c\beta}$ does not exist, it is optimal to upgrade for all η, so $\Delta^\beta = \Re^+$. ■

Proof. (Proposition 4) The proof for the first part is straightforward from the single period profit function of k-stage upgrade, (9), since the second part of P vanishes for a large k. For the second part, notice that the second term in P, which is negative, dominates for a large k while the first term is positive, so a finite k (possibly 0) is optimal. ■

Proof. (Proposition 5) Let $\alpha > 1$, $\sigma_\epsilon^2 < \frac{\mu(\gamma-1)}{\gamma}$ and $\eta > \eta^c$ so that the conditions for the proposition hold. Since these conditions imply the parameter restrictions in the lower left corner of Table 1, the optimal technology choice in the restricted model is to set $k = 1$ (using Theorem 3). If the firm were to set $k = \infty$ in the unrestricted model, then (from equation (8)) the variance becomes unbounded, so all future payoffs are negative. Thus, the present discounted value of future payoffs is negative. From equation (9) the current payoff is infinitely negative. Thus, the payoff when $k = \infty$ is infinitely negative, so $k = \infty$ is not optimal. ■

References

[1] Aghion, P., and P. Howitt, *Endogenous Growth Theory*, (1998), MIT Press, Cambridge.

[2] Barro, R. J., and X. Sala-i-Martin, *Economic Growth*, (1995), McGraw-Hill, New York.

[3] Brezis, E. S., P.R. Krugman, and D. Tsiddon, Leapfrogging in International Competition: A Theory of Cycles in National Technological Leadership, *American Economic Review*, 83, (1993), 1211-1219.

[4] Budd, C, C Harris and J Vickers, A Model of the Evolution of Duopoly: Does the Asymmetry Between Firms Tend to Increase of Decrease? *Review of Economic Studies*, 60 (1993) 543 - 73.

[5] Chari, V. V., and H. Hopenhayn, Vintage Human Capital, Growth, and the Diffusion of New Technology, *Journal of Political Economy*, 99, (1991), 1142-1165.

[6] DeGroot, M. H., *Optimal Statistical Decision*, (1970), McGraw-Hill, New York.

[7] Jovanovic, B., and Y. Nyarko, The Bayesian Foundations of Learning-by-doing, NBER Working Paper No 4739 (1994).

[8] Jovanovic, B., and Y. Nyarko, Learning-by-doing and the Choice of Technology, 64, *Econometrica*, (1996), 1299-1310.

[9] Krusell, P., and J.-V. Rios-Rull, Vested Interests in a Positive Theory of Stagnation and Growth, *Review of Economic Studies*, 63, (1996), 301-329.

[10] Lucas, R. E., Making a Miracle, *Econometrica*, 61, (1993), 251-272.

[11] Parente, S., Technology Adoption, Learning-by-doing and Economic Growth, *Journal of Economic Theory*, 63, (1994), 349-369.

[12] Motta, M., J.F. Thisse, and A. Cabrales, On the Persistence of Leadership or Leapfrogging in International Trade, *International Economic Review*, 38, (1997), 809 - 824.

[13] Stokey, N. L., Learning-by-doing and the Introduction of New Goods, *Journal of Political Economy*, 96, (1988), 701-717.

[14] Stokey, N. L., and R. E. Lucas, Jr., *Recursive Methods in Economic Dynamics*, (1989), Harvard University Press, Cambridge and London.

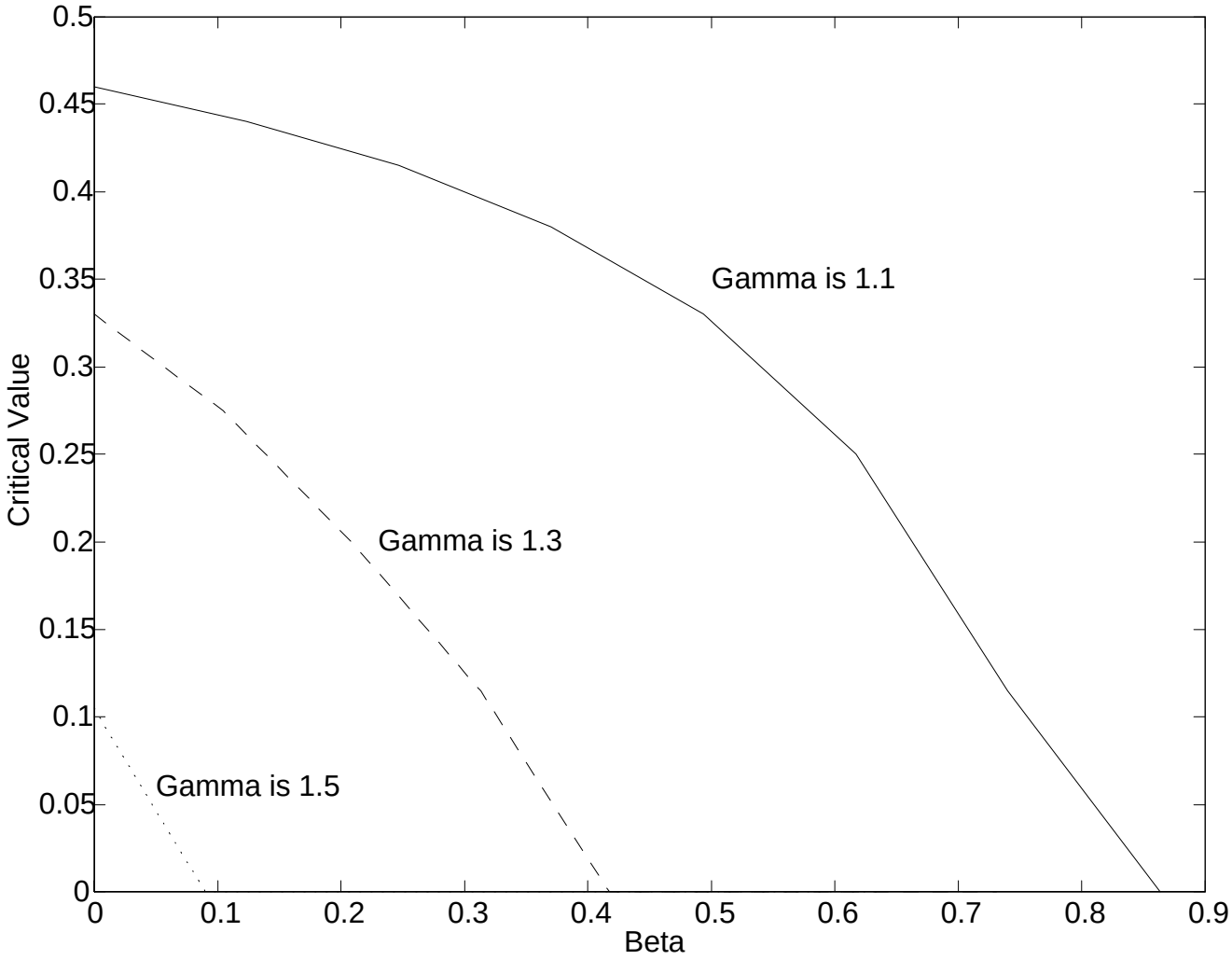

Figure 1: Numerical Results of Critical Values