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by

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Abstract

This paper develops two-period analytical and numerical models to study the question: given a stock of greenhouse gases that poses a risk of future damages of unknown magnitude, and the possibility of learning about damages, how do sunk abatement capital and a nondegradable stock of greenhouse gases affect optimal first-period investment? We show that both affect investment, the former negatively and the latter positively. Additionally, endogenous risk— the risk of damages dependent on the stock of gases—results in an increase in optimal investment for any level of capital "sunkness" or greenhouse gas degradability. Quantitatively, though, the effect of sunk capital is much stronger than the effect of greenhouse gas irreversibility or that of endogenous risk.

1. Introduction

Climatologists report that, at current greenhouse gas emission levels, the stock of gases in the atmosphere may double the preindustrial level in the next few decades, which may, in turn, lead to an increase in global mean temperature by as much as 5.8^oC (IPCC 2001). This is a large and sudden increase in mean temperature considering that the world is only about 5^oC warmer now than in the last ice age. The increase in global mean temperature is expected to lead to disruptions in the world's climate. Whether these disruptions will cause economic damages and whether these damages will be catastrophic in nature is as yet uncertain. There are those who believe that global warming will lead to sudden and catastrophic economic damages. Others believe that damages will be modest, or even that the net impact of warming will be beneficial.

Given a threat of damages of an unknown magnitude, the question facing policymakers is whether they should change the rate at which greenhouse gases are being emitted today. In this paper we focus on four features of the natural and economic environments we believe bear on this decision and make the answer less than obvious: sunk or irreversible abatement capital; a non-degradable or irreversible stock of greenhouse gases; endogenous, and potentially catastrophic, damages; and future learning about the nature of damages.

Abatement capital is said to be sunk if resources once invested cannot be converted to consumption or other forms of capital. Investment in small on-site power generators that convert natural gas into electricity, natural gas that would otherwise be flared off or rented off because its sale is uneconomic, is one example of sunk abatement capital. If not used to generate electricity it would be difficult to put the capital invested in the power generators to any other use. On the other hand, forests that act as sinks by absorbing greenhouse gases from the atmosphere are a form of capital that can be converted into consumer products if the forests are not needed to absorb greenhouse gases. Forests are thus a form of capital that is not sunk. An obvious concern is whether the presence of sunk capital alters optimal emission control decisions today. Given the uncertainty, should less be invested if capital is sunk? Alternatively, should more be invested if capital is not sunk?[1]

A second important complicating factor is the non-degradability of the stock of greenhouse gases. The stock of greenhouse gases is said to be non-degradable if it cannot be reduced through abatement and if it does not decay naturally. Climatologists claim that some part of the stock of greenhouse gases will in fact be non-degradable. The atmospheric concentration of carbon is not expected to return to its original (pre-industrial) level but instead is expected to reach a new equilibrium where some fraction of total carbon dioxide emitted will remain in the atmosphere for several thousand years (Schultz and Kasting 1997, Joos, Muller-Furstenberger and Stephan 1999).[2] Should policymakers take steps to further reduce greenhouse gas emissions if, once emitted, gases remain in the atmosphere for thousands of years?

A third important concern for policy makers is the extent to which the risk of future damages is endogenous and whether or not damages will be catastrophic in nature. If the probability of damages occurring depends on the behavior of economic agents, then the risk should be considered to be endogenous. In the context of global warming, since the probability of damages depends on the stock of greenhouse gases, the risk of damages is in fact endogenous. The implications of endogenous risk are an important focus of our study. Furthermore, recent findings suggest that the possibility of damages being catastrophic in nature—one example being disintegration of the West Antarctic ice sheet—is more serious than economists (and others) have realized (Kerr 1998).[3] This suggestion is strengthened by the prospect that concentrations of greenhouse gases could, over the next couple of centuries, rise well beyond the conventionally assumed doubling of pre-industrial

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[1] Given that one possibly appropriate time scale for decisions on global warming is 50 years or more, one may object to the assumption of sunk capital, since some capital may decay completely over a 50 year period. This would be a legitimate objection if the agent was constrained to invest only in the first year, or first few years, of the period. As investment is however assumed to occur over the entire period, not all of the accumulated capital will decay. This implies that sunk capital will continue to matter even when a single period extends over 50 years.

[2] Peck and Teisberg (1995,1996) and Farzin and Tahvonen (1996) were to our knowledge the first to incorporate carbon cycle models in this spirit into the economic analysis of optimal emissions control.

[3] For a discussion of other catastrophic risks in the context of climate change, see IPCC (2001).

levels (Cline 1992). Again, given the possibility of catastrophic damages, should policymakers increase investment in abatement capital?

A final issue identified here that complicates policy decisions on global warming is how uncertainty is resolved over time. If uncertainty about the nature of damages due to global warming is resolved over time, then policymakers must decide whether they should wait to act until there is better information about the nature of damages. When time resolves uncertainty, Arrow and Fisher (1974) have shown that there is a premium or option value on policies that maintain flexibility. Irreversibility of capital and the stock of greenhouse gases are two potential sources of inflexibility. Investment in sunk capital today locks the economy into a particular use of resources which may turn out to be wasteful if tomorrow reveals that damages due to global warming are modest. Dixit and Pindyck (1994) and Kolstad (1996a,1996b) emphasize this possibility. One then expects that investment in sunk capital will be less than the investment that would be made if capital was convertible. With a non-degradable stock of greenhouse gases, on the other hand, emissions today affect the probability of future damages which may be revealed as catastrophic. To maintain the option of not having to bear large damages policymakers might increase investment in abatement today. Both Chichilnisky and Heal (1993) and Fisher and Hanemann (1993) emphasize this possibility.

In this paper, then, we develop an analytical and a numerical model with learning to answer these questions. Specifically, we ask: given a stock of greenhouse gases that poses an endogenous threat of damages of an unknown magnitude and the possibility of learning about the nature of damages, how does the presence of sunk abatement capital and a non-degradable stock of greenhouse gases affect the optimal rate of investment in abatement capital? The opposing irreversibilities were, to our knowledge, first recognized and jointly analyzed by Kolstad (1996a), in a two-period model of irreversibilities in stock externalities. There Kolstad asks the question, how does the prospect of better second-period information about the consequences of the externality, in his example the damages from global warming, affect the desired level of first-period investment in abatement capital? The rate of learning is allowed to vary while the degree of capital "sunkness," and the decay rate of the stock of greenhouse gases, are fixed. He finds that, if learning is proceeding slowly enough, compared with the rates of pollution decay and capital depreciation, learning makes no difference. On the other hand, if learning is significant, either or both of the irreversibilities can affect the desired level of first-period emissions, in opposite directions. Which dominates depends on the relative magnitudes of the decay and depreciation rates and on expectations about damages.

In a second paper, a multi-period numerical simulation of optimal investment in control of greenhouse gases based on the DICE model (Nordhaus 1994b) and introducing, in addition to the capital stock irreversibility, a parametric representation of the rate of learning, Kolstad (1996b) finds a significant impact associated with the capital stock irreversibility and no impact with the emissions irreversibility. The reason, essentially, is that, in his parameterization, the non-negativity restriction on emissions, used in the model to define emissions irreversibility, is never binding. Too little investment in emission control in the early periods can be compensated by a bit more investment in later periods, but there is no scenario in which it would be optimal to emit negatively in the future to correct over-emission today.

This is consistent with the main analytical result in Ulph and Ulph (1997), a two-period model of global warming, irreversibility, and learning in which there is no explicit representation of investment in abatement, but, as in Kolstad, the decay rate of the stock of greenhouse gases is fixed, and rate of learning about damages is allowed to vary. A sufficient condition for there to be an irreversibility effect, that is, for first-period emissions with learning to be less than first-period emissions without learning, is that the non-negativity restriction on emissions, here too used to define emissions irreversibility, be binding in the no-learning case. Ulph and Ulph also provide a multi-period numerical simulation. For a variety of scenarios, they find very little difference between first-period emissions with learning and without, except for one case, characterized by a low discount rate and substantial uncertainty, in which emissions with learning are, surprisingly, significantly greater. Since there is no explicit capital stock irreversibility in the model or the simulation, it is not clear what is driving this result.

We take a somewhat different approach. In our model, learning is fixed, in the sense that the decision-maker is assumed to learn, by the start of the second period, whether a climate event – say a x^oC rise in global mean temperature – has occurred, and if it has, the nature of the impact, high damage or low. We then consider how the desired level of first-period investment in abatement capital varies with the degree of "sunkness" of the investment, and with the degree of non-degradability of the stock of gases. A second difference, with respect to Kolstad's model, is in the definition of sunk capital. Kolstad defines this in terms of capital durability: more durable capital is considered to be more sunk. We define it in a way we believe better reflects the irreversibility we are trying to capture: capital is sunk if it cannot be converted into consumption or into other forms of capital. This also seems to be consistent with the literature on irreversible investment (see (Pindyck 1991)). As we show in section 5, the results in this analysis are affected by the definition of sunk abatement capital. When sunkness is defined in terms of capital durability, then an increase in capital sunkness leads to an *increase* in investment in abatement capital. On the other hand, when it is defined in terms of capital convertibility, then an increase in sunkness leads to the exact opposite effect, namely to a *decrease* in investment. Of the two definitions, then, the latter leads to the more intuitive result.

Two other differences between our model and those of Kolstad and Ulph and Ulph are that (i) we treat the risk of high, or catastrophic, damage, as endogenous while they treat the risk as exogenous; and (ii) they define emissions irreversibility as a non-negativity constraint on emissions while we define it as a lower rate of decay of the stock of greenhouse gases in addition to the non-negativity constraint.

Presumably as a result of these differences, we find, contrary to prior results, that capital stock irreversibilities and emissions irreversibilities both affect the optimal rate of investment in abatement capital. While the former results in a decrease in investment, the latter results in an increase. Furthermore, endogenous risk has the same effect as emissions irreversibility on investment in that it too results in an increase in investment in abatement capital. Quantitatively, though, the effect of capital irreversibility is much stronger than either the effect of emissions irreversibility or of endogenous risk.

The rest of the paper is organized as follows. The next section describes the analytical model, the objective function, and the optimality conditions. Section 3 discusses the results of the analytical model. Section 4 describes the numerical model and its parameterization and section 5 the results of the numerical model. Section 6 concludes.

2. Analytical Model

Consider a two-period model where in each period a fixed endowment is allocated between consumption and investment in capital to reduce emissions. Emissions are a by-product of consumption and, if not controlled, add to the stock of greenhouse gases. Capital varies in its degree of convertibility, while the stock of greenhouse gases varies in its degree of degradability. In both periods utility is derived from consumption. In addition, in the second period there is a possibility of an event occurring that, in turn, leads to either high, possibly catastrophic, damages or low damages. A significant rise in global mean temperature—say 3.6^oC—would be such an event.[4] If this, in turn, leads to the disintegration of the West Antarctic Ice Sheet, with an attendant rise of $5-6$ meters in the sea level, for example, or to significant disruptions of thermohaline circulation, or one of another of the potentially serious impacts discussed in IPCC (2001), it seems safe to say damages would be high. If, on the other hand, warming does not lead to disaster, merely to scattered modest impacts,

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[4] Note that IPCC (2001) projects that under business-as-usual, globally averaged surface temperature will increase by $1.4-5.8^oC$ over the period 1990 to 2100. 3.6^oC is the mid point of this interval.

damages could be low. If warming occurs, then, in the second period the decision-maker needs to take account of damages, in addition to the utility from consumption. Finally, the probability of warming depends positively on the stock of greenhouse gases, that is, the risk of warming is endogenous.

2.1. **Primitives.** In each period the decision-maker, or economic agent, derives utility from consumption, C_t, which is given by the function $U(C_t)$ where $U : R^+ \rightarrow R$ and $t = 1, 2$. We assume that the function U is increasing, concave and twice continuously differentiable in consumption. If warming occurs in the second period, then, in addition to the utility from consumption, the agent bears damages from the stock of greenhouse gases that in turn are revealed to be high or low. High damages are given by the function $\theta^h D(M_t)$ where $D : R^+ \rightarrow R$ and $t = 2$ while low damages are similarly given by the function $\theta^l D(M_t)$. θ^h is assumed to be greater than θ^l. Further we assume that the function D is increasing, convex and twice continuously differentiable in the stock of greenhouse gases.

The agent receives a fixed endowment R in each period that she allocates to either consumption or investment, I_t. In each period she also has the option of increasing consumption over and above R by disinvesting in abatement capital, K_{t-1}. The budget constraint with positive and negative investment is then given by

$$C_t = \begin{cases} R - I_t & \text{if } I_t \geq 0, \\ R + \Phi|I_t| & \text{if } I_t \leq 0, \end{cases} \tag{1}$$

for $t = 1, 2$. Φ is a parameter that reflects the cost of converting abatement capital into consumption. If abatement capital is prohibitively costly to convert then $\Phi = 0$ and if capital can be converted into consumption costlessly then $\Phi = 1$. For $\Phi \in (0, 1)$ capital can be converted into consumption, but at a cost. In each period, then, the constraint on consumption is given by $0 \leq C_t \leq R + \Phi(1-\delta_k)K_{t-1}$ where δ_k is the rate of capital depreciation.

The stock of capital changes from one period to the next due to depreciation and investment. Its equation of motion is

$$K_t = \begin{cases} (1-\delta_K)K_{t-1} + I_t & \text{if } I_t \geq 0, \\ (1-\delta_K)K_{t-1} - |I_t| & \text{if } I_t \leq 0. \end{cases} \tag{2}$$

Uncontrolled emissions in each period, E_t, are assumed proportional to consumption, $E_t = \sigma C_t$ where σ is a constant. Emissions can be controlled in each period with capital according to the function $H(K_t)$ where $H : R \rightarrow [0,1]$. The mapping is restricted to the zero-one interval to reflect the assumption that abatement capital cannot reduce the stock of greenhouse gases or that emissions are restricted to be non-negative. We assume that the function H is increasing, concave and twice continuously differentiable.

The stock of greenhouse gases changes from one period to the next as a result of natural decay, δ_M, and net emissions. The equation of motion is

$$M_t = (1-\delta_M)M_{t-1} + E_t(1 - H(K_t)). \tag{3}$$

As δ_M approaches zero the stock of greenhouse gases becomes more non-degradable.

In the first period the agent assigns probabilities to the occurrence of warming and to the magnitude of the damages caused by the stock of greenhouse gases should warming occur. Let the function $p(M_1)$ where $p : R \rightarrow [0,1]$ denote the probability of warming and the parameter q the probability that the damages associated with the corresponding stock of greenhouse gases will be high. If the risk of warming is exogenous then $p_1(M_1) = 0$ and if the risk is endogenous then

$p_1(M_1) > 0$, where the subscript denotes differentiation.[5] At the beginning of the second period the agent learns whether or not warming has occurred and, if it has, the magnitude of the damages.

2.2. **Objective Function.** The problem is to choose how much of the resource endowment to consume and how much to invest in the first and the second period to maximize the sum of utility over both periods. In symbols, the problem is

$$(4) \quad \max_{C_1, I_1} \left(U(C_1) + (1 - p(M_1)) \left(\max_{C_2, I_2} U(C_2) \right) \right. \\ \left. + p(M_1) q \left(\max_{C_2, I_2} U(C_2) - \theta^h D(M_2) \right) + p(M_1)(1-q) \left(\max_{C_2, I_2} U(C_2) - \theta^l D(M_2) \right) \right),$$

subject to the constraints given by equations (1)—(3).

2.3. **Optimality Conditions.** Since the agent learns about the nature of damages from the stock of greenhouse gases in the second period, the optimization problem is solved through backwards induction.

2.3.1. *Second Period.* There are three potential states of nature in the second period: first, where warming does not occur; second, where warming occurs and damages are high; and third, where warming occurs and damages are low. Let C_2^*, C_2^h and C_2^l be the optimal consumption level in the second period with no warming, warming and high damages, and warming and low damages, respectively. Then

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[5] In the introduction we spoke of the risk of catastrophic damages as being endogenous, in the sense that the probability of the damages occurring depends on the stock of greenhouse gases, which is in turn endogenous in the model (and in reality). In our more formal statement here this dependence is represented in the p variable. The q variable could also depend on M, but since (as indicated in equation (4) just below) the two probabilities are compounded, their product clearly depends on M.

$$C_2^* = \operatorname{argmax} U(C_2),$$

$$C_2^h = \operatorname{argmax} U(C_2) - \theta^h D(M_2),$$

$$C_2^l = \operatorname{argmax} U(C_2) - \theta^l D(M_2).$$

Given the structure of the utility functions, $C_2^* \geq C_2^l \geq C_2^h$. Further, if warming does not occur then there is no need for abatement and the agent will disinvest the entire stock of capital. Consumption in the second period with no warming, C_2^*, will thus be equal to $R + \Phi(1-\delta_k)K_1$ and $I_2^* = -(1-\delta_k)K_1$. If warming occurs, then whether the damages are high or low, for analytical simplicity we assume that the agent chooses to invest a part of the endowment so that $C_2^h < R$ and $C_2^l < R$. We feel that this assumption is reasonable as it amounts to assuming that the damage caused by the stock of greenhouse gases is sufficient, and the inherited capital stock is small enough, to warrant some investment. However, since $\theta^l < \theta^h$, $C_2^h < C_2^l$.

The first order condition for optimality in the second period given that warming occurs is then

$$\frac{dU(C_2)}{dC_2} - \theta^i \frac{dD(M_2)}{dM_2} \frac{\partial M_2}{\partial C_2} = 0.$$

2.3.2. *First Period.* The problem in the first period can now be written as

$$\max_{C_1, I_1} \Big(U(C_1) + (1 - p(M_1))U(C_2^*) + \\ p(M_1)q\big(U(C_2^h) - \theta^h D(M_2^h)\big) + p(M_1)(1-q)\big(U(C_2^l) - \theta^l D(M_2^l)\big)\Big), \tag{5}$$

where M_2^h is the stock of greenhouse gases in the second period given that warming occurs, damages are high, and consumption is optimally chosen, and M_2^l is similarly defined for low damages.

Differentiating equation (5) with respect to C_1 and setting the result to zero gives the optimal level of consumption in the first period. Let C_1^* be that optimal consumption level.

3. Analytical Results

3.1. **Endogenous Risk.**

Proposition 1. *The optimal level of investment (consumption) in the first period when the risk of warming is exogenous is never greater (less) than the optimal level when the risk is endogenous.*[6]

This result follows from two facts. The first is that, since utility with warming is strictly less than utility without warming, no warming is strictly preferred to warming. Second, when risk is endogenous, higher emissions in the first period increase the probability of warming in the second period. The optimizing agent thus does not increase emissions in the first period as that would increase the risk of warming.

3.2. **Sunk Capital.** In our model capital is considered to be perfectly sunk if resources once invested in abatement capital cannot be converted to consumption. The effect of a change in the degree of capital sunkness is captured by differentiating the first order condition with respect to Φ (the parameter that reflects the cost of converting capital). Note that a decrease in Φ is equivalent to a decrease in capital convertibility or to an increase in the degree of capital sunkness.

Proposition 2. $\frac{dC_1}{d\Phi} < 0$ *if* $\frac{\frac{-d^2U(C_2)}{dC_2^2}C_2^*}{\frac{dU(C_2)}{dC_2}} \leq 1,$

where $-\frac{\frac{d^2U(C_2)}{dC_2^2}C_2^*}{\frac{dU(C_2)}{dC_2}}$ is the coefficient of relative risk aversion or the inverse of the coefficient of intertemporal substitution. In words, the proposition states that if the coefficient of relative risk aversion is less than one (or the coefficient of intertemporal substitution is greater than one) then an increase in the degree of capital sunkness leads to a decrease in investment. An increase in capital sunkness (or a decrease in capital convertibility) leads to a decrease in the shadow value of capital which, in turn, should lead to a decrease in investment in the first period. However, when

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[6] *All proofs are given in the appendix.*

risk aversion is high (greater than one) or intertemporal substitutability is low (less than one), there is a counterbalancing effect that may in fact lead to an increase in investment even when the shadow value of capital is decreasing. In this case the agent cares a lot about smoothing consumption over time, so that a decrease in capital convertibility, which decreases expected consumption in the second period, leads the agent to decrease consumption in the first period in order to smooth consumption over the two periods. When risk aversion is low, or intertemporal substitutability is high, there is no such counterbalancing effect, so a decrease in capital convertibility leads unambiguously to a decrease in investment in the first period. Note that the proposition does not state that if the coefficient of risk aversion is greater than one then investment is always a decreasing function of capital convertibility, only that it may be.

The proposition holds irrespective of whether the risk of warming is endogenous or exogenous. That is, if the coefficient of relative risk aversion is less than one, then irrespective of whether the risk of warming is endogenous or exogenous, investment is a decreasing function of the degree of sunkness of capital. However, if the coefficient of relative risk aversion is greater than one, then whether the risk of warming is exogenous or endogenous does affect the relationship between investment and the degree of capital sunkness. Specifically, if the coefficient of relative risk aversion is greater than one, then investment is more likely to be an increasing function of capital convertibility if the risk of warming occurring is endogenous. Another way to state this is that if it is optimal to increase investment with an increase in capital convertibility when the risk of warming is exogenous, it will continue to be optimal to increase investment with an increase in capital convertibility when the risk is endogenous. However, if it is optimal to decrease investment with an increase in capital convertibility when the risk is exogenous, it may not be optimal to decrease investment when the risk is endogenous.

The foregoing discussion implies that what might be called the intuitive result, that investment in abatement capital optimally increases (decreases) with an increase (decrease) in capital convertibility, is more likely when the risk of warming is recognized as endogenous. The reason is that when the risk is endogenous and there is an increase in capital convertibility, there is a further benefit from increasing investment which counterbalances the desire to decrease investment in order to smooth consumption over time. When the risk is endogenous, an increase in investment, by decreasing the stock of greenhouse gases, decreases the probability of warming occurring and increases the probability of being in a world where it will be optimal to convert all the existing capital into consumption. An increase in convertibility implies more consumption per unit of capital converted or more consumption from the entire stock of capital. This increases the benefit of investing to decrease the risk of warming.

3.3. **Non-degradable Stocks.** Unfortunately, one can say little analytically about the effect of a change in the decay rate of the stock of greenhouse gases on the optimal level of investment in the first period. The derivative of optimal consumption in the first period with respect to the decay rate of the stock of greenhouse gases is ambiguous. The reason for this is that a change in the decay rate has two opposing effects on the shadow value of capital. On the one hand, a decrease in the decay rate of the stock of greenhouse gases, by increasing the stock of greenhouse gases that is passed on to the second period from the first, increases the shadow value of capital, capital that is needed to abate what is now a larger stock of greenhouse gases. On the other hand, though, a decrease in the decay rate also leads to a decrease in consumption in the second period, which by reducing the new emissions that get added to the stock of greenhouse gases in the second period, reduces the shadow value of capital. Consequently, the effect of a change in the decay rate of the stock of greenhouse gases on the shadow value of capital, and thus investment in the first period, is ambiguous.

This holds true irrespective of whether the risk of warming is exogenous or endogenous. However, when the risk is endogenous, there are two additional, and again opposing, effects on the shadow value of capital. On the one hand, an increase in the decay rate decreases the stock of greenhouse gases in the first period which in turn reduces the probability that warming will occur. This then implies that the agent is less likely to bear damages and so less likely to use abatement capital to reduce the stock of greenhouse gases. This reduces the shadow value of capital and leads to a decrease in investment. On the other hand, a decrease in the probability of warming also means that the agent is more likely to disinvest the entire stock of capital. This in turn increases the shadow value of capital. The net effect of a change in the rate of decay of greenhouse gases on the optimal level of investment is thus ambiguous.

We now turn to a numerical model to resolve the ambiguities with respect to the decay rate of the stock of greenhouse gases, as well as to indicate the relative importance of the various effects for a plausible parameterization.

4. Numerical Model

Our numerical model mostly follows the analytical model described in section 2. It does, however, differ in a few respects to allow us to draw on the functional forms and parameter values given in the most recent version of the well known and widely used DICE model (Nordhaus and Boyer 2000).

4.1. Model Structure.

A fixed endowment received in each of two periods is allocated between consumption and investment. Capital increases from one period to the next due to investment and decreases due to depreciation and disinvestment. Furthermore, capital abates only the flow of emissions and not the stock of greenhouse gases. Emissions are a by-product of fixed gross output, and if not controlled, add to the stock of greenhouse gases. The stock increases from one period to the next due to net emissions and decreases due to natural decay. Also, the stock affects radiative forcing in the atmosphere which, in turn, affects temperature.

In both periods utility is derived from consumption. In addition, in the second period there is a possibility of an event—warming—occurring which, in turn, leads to either high, possibly catastrophic, damages or low damages. Damages are a function of atmospheric temperature. At the beginning of the second period, the agent learns whether warming has occurred and, if so, the nature of the damages. The probability of warming is determined either exogenously or endogenously, in the latter case as a function of the stock of greenhouse gases. Finally, capital varies in its degree of convertibility, while the stock of greenhouse gases varies in its degree of degradability.

There are three differences between the analytical model described in section 2 and the numerical model described in this section. These are: (i) in the analytical model, emissions are a by-product of consumption while in the numerical model they are a by-product of gross output; (ii) in the analytical model there is only one variable that represents the climate sector, namely the stock of greenhouse gases, while in the numerical model the climate sector is represented by four variables—the stock of greenhouse gases, atmospheric radiative forcing, atmospheric temperature and ocean temperature; and (iii) in the analytical model damages are a function of the stock of greenhouse gases while in the numerical model they are a function of the atmospheric temperature. None of these differences are significant enough to affect the results but facilitate use of the functional forms and parameter values from the DICE model.

We need to say something about the interpretation of time scales in the numerical model. Although the model has only two periods, each represents more than a single calendar year. As in DICE, the time steps are decades. In addition to being consistent with DICE, this seems like a realistic policy formulation. Further, given the parameterization, at least a decade is needed to see significant change in the stock of greenhouse gases which, in turn, is needed to see the effects of a change in the decay rate of the stock and of endogenous risk on optimal first period investment. The agent receives a fixed endowment in each calendar year which she then invests or consumes, but investment is assumed not to change within a period but only between periods. This assumption is needed to allow for learning in the numerical model and to obtain a closed loop solution. These in turn require the model to be solved through backwards induction, which we have thus far found difficult to introduce into a numerical model with more than two periods, as for example a 10-period model in which each period represents a decade.[7] We hope to solve the computational problem in future work, but the current version is consistent with the main question posed in the paper: What happens to optimal decisions in the first period when there is a change in some underlying system parameter?

4.2. **Functional Forms.**

All the functional forms used in the simulations, other than those used for the abatement function, evolution of the stock of greenhouse gases, the damage function, and the endogenous risk function, are taken from the DICE model. We now describe the four equations that differ.

4.2.1. *Abatement Function.* Although the DICE model includes the cost of controlling emissions, it does not allow for investment in abatement capital so that it cannot be used to study the implications of greater or lesser sunkness of capital. Further, the DICE model expresses the cost as a function of the fraction of emissions controlled, while in our model this fraction is expressed as a function of the cost of abatement. Thus rather than using the total cost of abatement function specified in DICE, we use its inverse and furthermore we express the proportion abated as a function of abatement capital, and not abatement costs. In symbols,

$$\mu_t = \left(\frac{a_1 K_t}{y_t}\right)^{a_2},$$

where μ_t is the proportion of emissions abated, K_t is the amount of abatement capital, y_t is gross output, and a_1 and a_2 are parameters.

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[7] Note that even though Kolstad (1996b) and Ulph and Ulph (1997) solve multi-period numerical models, all learning takes place after either the first period (Ulph and Ulph) or the second (Kolstad). Once learning has taken place the models, for the remaining periods, are solved forwards. Effectively only a two- or three-period model is solved through backwards induction.

Note that in order to bound μ_t between 0 and 1 we limit the amount of resources that the agent can devote to abatement capital. This in turn is achieved by limiting the fixed endowment to be a fraction of total gross output.

4.2.2. *Greenhouse Gas Stock Dynamics.* The DICE model contains three equations to capture the dynamics of the stock of greenhouse gases. Specifically, it has an equation for the stock in the atmosphere, one for the stock in the upper ocean and another for the stock in the deep ocean. An earlier version of DICE (Nordhaus 1994b), on the other hand, had only one equation to capture the dynamics of the stock of greenhouse gases. It is this equation that we use in our simulations as it better fits our analytical model which also represents stock dynamics with one equation. However, we have modified the equation to better reflect the major change in climate dynamics between the 1994 and 1999 DICE models, namely that once in the atmosphere emissions of greenhouse gases stay longer than specified in the 1994 model. Thus rather than the 1994 equation

$$M_t = 596.4 + \beta E_t + (1 - \delta_m)(M_{t-1} - 596.4),$$

we use the equation

$$M_t = 596.4 + E_t + (1 - \delta_m)(M_{t-1} - 596.4),$$

where M_t is the stock of greenhouse gases, E_t are the net emissions, $1 - \beta$ is the decay in the net emissions and δ_m is the decay in the stock of greenhouse gases. Note that the difference between the two equations is that in the 1994 equation emissions decay within a period.

4.2.3. *Damage Function.* To allow for both high and low damages, a feature not included in DICE, we draw on Nordhaus (1994a), which reports the results of a survey of experts in the field of climate change who were asked to provide their best estimate of the economic impact of global warming.[8] The survey asked the experts to estimate the loss, as a percent of gross world product, that would result from a doubling in the atmospheric concentration of carbon dioxide by the mid-21^{st} century, leading to a 3^o C rise in temperature by 2090. According to the survey, the experts estimated the most likely loss from a 3^o C rise in temperature to be 3.6% of gross world output, while the greatest loss was predicted to be 21% of gross world output. We have used these responses to estimate the damage functions for the numerical model. Specifically, we have fitted a convex function between an increase in atmospheric temperature and the percent of gross world output lost through the points $[0.43, 0]$ and $[3, 3.6]$ for the low damage function and the points $[0.43, 0]$ and $[3, 21]$ for the high damage function with the additional assumption that the intercept is zero. The basic functional form, again drawn from the 1999 DICE model, is given by:

$$D_t = \theta_1 T_t + \theta_2 T_t^2,$$

where D_t are the damages, T_t is the increase in atmospheric temperature, and θ_1 and θ_2 are parameters. Finally, rather then damages affecting the total output available for consumption, as in DICE, we assume, in keeping with our analytical model, that damages enter the utility function directly.

4.2.4. *Endogenous Risk Function.* To allow for the possibility of endogenous risk of warming, again a feature not included in DICE, we use the following functional form:

$$p_t = \frac{2}{1 + \exp(-bM_t)} - 1,$$

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[8] We recognize that estimates from the 1994 survey of experts may be somewhat dated, especially since newer versions of the IPCC reports have upped their estimates for surface temperature increases under the business-as-usual scenario. However, these were the only estimates of the range of economic damages that may be caused by global warming that we could find in the literature.

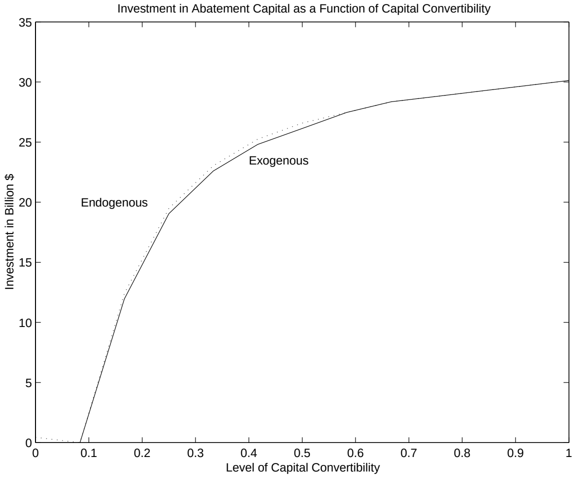

Figure 1. Investment in Abatement Capital as a Function of Capital Convertibility

where p_t is the probability of warming occurring, M_t is the stock of greenhouse gases and b is a parameter. Note that this functional form constrains p_t to lie between 0 and 1 and makes it an increasing function of the stock.

5. Numerical Results

The numerical model is solved using MATLAB and by searching over a finite, though large, control space. Sensitivity analysis was conducted for the parameters p and q, as we were unable to find point estimates of these in the literature. The results presented hold for the range of parameter values used in the sensitivity analysis. The computer code is available from the authors upon request.

Figure 1 shows the effect of a change in the degree of capital sunkness on the optimal level of investment in the first period. For both types of risk, an increase in capital convertibility, or a decrease in capital sunkness, leads to an increase in investment. The effect is dramatic. When the risk of warming is exogenous, optimal investment in the first period goes from zero when capital is perfectly sunk to about \$30 billion when capital is perfectly convertible. Investment under endogenous risk is somewhat higher for lower levels of capital convertibity and the same for higher levels of convertibility. Since, in keeping with DICE, utility from consumption is represented by a

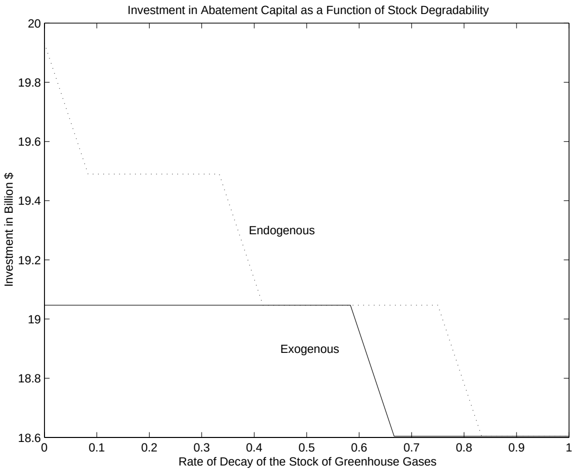

Figure 2. Investment in Abatement Capital as a Function of Stock Degradability

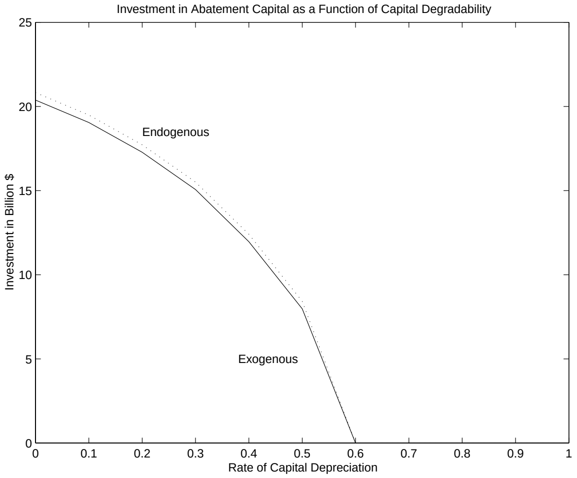

Figure 3. Investment in Abatement Capital as a Function of Capital Durability

log function, and since the coefficient of relative risk aversion for this functional form is equal to one, these results are consistent with the analytical results.

The analytical ambiguity surrounding the effect of a change in the decay rate of the stock of greenhouse gases on the optimal level of investment in the first period goes away in the numerical model. This is shown in figure 2 where investment unambiguously decreases with an increase in the degradability of the stock of greenhouse gases. The result holds for both exogenous and endogenous risk, though more strongly for the latter. As with capital sunkness, investment is greater under endogenous risk for any given level of greenhouse gas stock degradability.

If capital sunkness is defined in terms of durability instead of convertibility, with more durable capital being considered more sunk, then the effect of a change in the level of sunkness on optimal consumption and investment in the first period is quite different. As shown in figure 3, the optimal level of investment is an increasing function of the level of sunkness when sunkness is defined in terms of capital durability. Interestingly, Kolstad's definition does not produce his result, namely that sunk capital leads to a decrease in investment. The result is however obtained under our definition of sunk capital (of course there are some differences in the respective models). The reason for the perverse result using the durability definition is that, though a decrease in the rate of depreciation means capital is more sunk, it also increases the shadow value of capital. This, in turn, leads to greater investment. Note, finally, that investment is always greater under endogenous risk.

An interesting feature of the numerical results is that the effect on optimal first period investment of moving from reversible to irreversible greenhouse gas accumulation in the atmosphere is much smaller than the effect of moving from reversible to irreversible investment ($18.6 billion to $20 billion, versus $30 billion to zero). Presumably this is because even with very different decay rates, the concentration of greenhouse gases in the atmosphere does not change very much over a 10-year period, so temperature is not much affected, and neither are second period damages, which are a function of temperature.

The relatively small difference in investment for endogenous and exogenous risk, for any level of capital convertibility or durability, or greenhouse gas decay rate, is also explained by the small change in concentration of greenhouse gases over a period. Since atmospheric concentration does not change much, the probability of warming (and consequent damage) is not much affected, and investment under endogenous risk is only a little greater than under exogenous risk. That said, it should also be noted that the effects of climate irreversibility and endogenous risk though small are nonetheless significant. Moving from reversible to irreversible stocks of greenhouse gases increases investment by \$ 1.4 billion per year, and where the decay rate is low, as it is in reality, moving from exogenous to endogenous risk increases investment by a similar amount.

We should also note that the effects of climate irreversibility and endogenous risk would be strengthened if we allowed q, the probability of high damages, to depend on M, the concentration of greenhouse gases in the atmosphere, just as p depends on M. Since the probabilities are compounded, doing so would not affect the results of the analytical model, but clearly would affect the quantitative results of the numerical exercise, as the (compound) probability of high damages would increase more dramatically with an increase in M. However, even dramatic increases in the size of the resulting effects would leave them smaller than the irreversible investment effect, due to the small change in greenhouse gas concentrations over a period.

One way of interpreting the numerical results is that policy should be more concerned about the investment irreversibility than the climate irreversibility. Alternatively, one can think of a model in which there is more than one type of investment in reducing greenhouse gas emissions, with investments differing in degree of sunkness. Then the analog to our result would be that relatively convertible investments would be favored over relatively sunk ones—with endogenous risk and slow decay rates continuing to favor investment of any sort.

6. Concluding Remarks

This paper sets out to answer the question: given a stock of greenhouse gases that poses a risk of future damages of an unknown magnitude and the possibility of learning about the nature of damages, how does the presence of sunk abatement capital and a non-degradable stock of greenhouse gases affect the optimal rate of investment in abatement capital? We develop a two-period model and show analytically that investment in the first period will be greater under endogenous risk, in which the probability of warming and the resulting damages depend on the stock of greenhouse gases. We also show that, for both endogenous and exogenous risk, first-period investment is negatively related to the degree of sunkness of capital, if the coefficient of relative risk aversion is less than one, or the coefficient of intertemporal substitution is greater than one – a plausible condition. If the condition does not hold, investment may still be a decreasing function of sunkness, and this is more likely under endogenous risk. Finally, we argue that it is not possible to sign the relationship between the degradability of greenhouse gases and first-period investment.

We then develop a numerical model, based on the analytical model and taking relevant functional forms and parameters from the DICE model, and establish that, for a realistic parameterization, the relationship between degradability of greenhouse gases and investment is negative. The less degradable the stock of gases, the greater is optimal first-period investment. Quantitatively, the investment irreversibility effect is substantially larger than the climate irreversibility effects, essentially because the concentration of greenhouse gases in the atmosphere does not change much over a (10-year) period, and thus neither do the probability of warming or the level of next-period damages. One way of interpreting these results is that the investment mix should emphasize relatively less sunk forms of capital.

Appendix A. Proofs of the propositions

A.1. **Proof of proposition 1.** The first order condition is given by the following equation

(6)

$$\frac{dU(C_1)}{dC_1}-\frac{dU(C_2)}{dC_2}\Phi(1-\delta_k)+p(M_1)\Big(\frac{dU(C_2^*)}{dC_2}\Phi(1-\delta_k)-q\theta^h\frac{dD(M_2^h)}{dM_2}\frac{\partial M_2^h}{\partial C_1}-(1-q)\theta^l\frac{dD(M_2^l)}{dM_2}\frac{\partial M_2^l}{\partial C_1}\Big)$$
$$-\frac{dp(M_1)}{dM_1}\frac{\partial M_1}{\partial C_1}\Big(U(C_2^*)-q(U(C_2^h)-\theta^h D(M_2^h))-(1-q)(U(C_2^l)-\theta^l D(M_2^l))\Big)=0,$$

where (assuming that the agent invests in abatement capital in the first period)

$$\frac{\partial M_1}{\partial C_1}=\sigma(1-H(K_1))+\sigma C_1\frac{dH(K_1)}{dK_1}>0.$$

The optimal level of consumption in the first period given that the risk is exogenous is obtained by setting $\frac{dp(M_1)}{dM_1}=0$. Let this level be denoted by C_1^{exo}. Now the first order condition, given that the risk is endogenous, but evaluated at C_1^{exo} is negative. Since the second order condition is non-positive, this implies that $C_1^{exo}>C_1^{endo}$ where C_1^{endo} is the optimal level of consumption in the first period given that the risk is endogenous.

A.2. **Proof of proposition 2.** First consider the case where the risk is exogenous. Set $\frac{\partial p(M_1)}{\partial M_1}=0$ and totally differentiate equation (6) with respect to C_1 and Φ. The denominator of $\frac{dC_1}{d\Phi}$ is the second order condition for optimality and is thus non-positive. The sign of $\frac{dC_1}{d\Phi}$ is then the opposite of the sign of the numerator. The numerator in turn is given by the expression

$$\big(1-p(M_1)\big)(1-\delta_k)\Big(\frac{dU(C_2)}{dC_2}+\frac{d^2U(C_2)}{dC_2^2}(C_2^*-R)\Big).$$

Dividing this by $\big(1-p(M_1)\big)(1-\delta_k)\frac{dU(C_2)}{dC_2}$ reduces the numerator to

$$1+\frac{\frac{d^2U(C_2)}{dC_2^2}}{\frac{dU(C_2)}{dC_2}}(C_2^*-R).$$

Consequently,

$$\frac{dC_1}{d\Phi} \gtreqless 0 \iff -\frac{\frac{d^2U(C_2)}{dC_2^2}}{\frac{dU(C_2)}{dC_2}}(C_2^*-R) \gtreqless 1.$$

Let α denote the coefficient of relative risk aversion. Then,

$$\frac{dC_1}{d\Phi} \gtreqless 0 \iff \alpha\frac{(C_2^*-R)}{C_2^*} \gtreqless 1,$$

or

$$\frac{dC_1}{d\Phi} \gtreqless 0 \iff \alpha \gtreqless \frac{C_2^*}{(C_2^*-R)}.$$

Since $C_2^* = R+\Phi(1-\delta_k)K_1$, $\frac{C_2^*}{C_2^*-R} > 1$. This in turn implies that when $\alpha \leq 1$, $\alpha < \frac{C_2^*}{(C_2^*-R)}$ and $\frac{dC_1}{d\Phi} < 0$. However, when $\alpha > 1$, $\alpha \gtreqless \frac{C_2^*}{(C_2^*-R)}$ and hence $\frac{dC_1}{d\Phi} \gtreqless 0$. Consequently,

$$\frac{dC_1}{d\Phi} < 0 if -\frac{\frac{d^2U(C_2)}{dC_2^2}C_2^*}{\frac{dU(C_2)}{dC_2}} \leq 1.$$

Next we consider the case where the risk is endogenous. Totally differentiate equation (6) with respect to C_1 and Φ. Once again, because of the second order condition, the sign of the derivative of consumption with respect to Φ is the opposite of the sign of the numerator of $\frac{dC_1^*}{d\Phi}$. In addition to the expression for exogenous risk, the numerator has the following additional term

$$\frac{dp(M_1)}{dM_1}\frac{\partial M_1}{\partial C_1}\frac{dU(C_2)}{dC_2}(1-\delta_k)K_1,$$

where $\frac{\partial M_1}{\partial C_1} = \sigma\big((1-H(K_1)) + C_1\frac{dH(K_1)}{K_1}\big) > 0$. Hence this additional equation is positive as well. Following the steps outlined in the proof for exogenous risk, when risk is endogenous

$$\frac{dC_1}{d\Phi} \gtreqless 0 \iff -\frac{\frac{d^2U(C_2)}{dC_2^2}}{\frac{dU(C_2)}{dC_2}}(C_2^* - R) \gtreqless 1 + \frac{\frac{dp(M_1)}{dM_1}}{1 - p(M_1)} \frac{\partial M_1}{\partial C_1} K_1.$$

Again,

$$\frac{dC_1}{d\Phi} < 0 if - \frac{\frac{d^2U(C_2)}{dC_2^2} C_2^*}{\frac{dU(C_2)}{dC_2}} \leq 1.$$

Parameter	Value
Fixed Endowment	\$ 67 Billion
Gross Output	\$ 22.5804 Trillion
Initial Abatement Capital	\$ 0
Initial Stock of Greenhouse Gas	785.3 Billion Tons of $C0_2$ Equivalent
Initial Atmospheric Temperature	0.58^o C
Initial Deep Ocean Temperature	0.07^o C
Rate of Capital Depreciation	0.1
Emissions Output Ratio	0.274
Abatement Parameter a_1	0.03
Abatement Parameter a_2	0.4651
Low Damage Parameter θ_1	-0.1673
Low Damage Parameter θ_2	0.4669
High Damage Parameter θ_1	-1.1712
High Damage Parameter θ_2	2.7237
Range of Φ	$[0, 1]$
Range of δ_m	$[0, 1]$

Appendix B. Parameter Values Used in the Numerical Simulations

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