Department of Agricultural & Resource Economics, UCB
CUDARE Working Papers
(University of California, Berkeley)

Year 1998 *Paper* 835

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August 15, 2002

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Key words: Ambient tax, nonpoint source pollution, moral hazard, asymmetric information, differential games.

JEL Classification numbers: D82, H20, H40, Q20

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*I would like to thank, without implicating, Katti Millock, Jason Shogren, Leo Simon, Anastasios Xepapadeas, and seminar participants at UC Berkeley for their comments on an earlier draft.

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1 Introduction

When regulators are unable to measure firms' individual levels of pollution, standard Pigouvian taxes cannot be used to achieve target levels of emissions. For these "nonpoint source" pollution problems, it is not feasible to charge a firm a tax on the basis of its pollution. Segerson [13], drawing on Holmstrom's [5] analysis of moral hazard in teams, noted that one method of regulating nonpoint source pollution is to charge each firm a unit tax based on the aggregate level of pollution, i.e., an "ambient tax". There are two well-understood limitations to this solution. First, it requires that firms be large enough to recognize that they affect the aggregate level of pollution. Second, the ambient tax may result in large transfers to or from the firm. Large transfers to or from a firm may be politically costly, when the tax or subsidy is not commensurate with the firm's actions. It may also be more difficult to achieve the socially optimal number of firms in the industry when large taxes or subsidies are needed to achieve a pollution target.

The ambient tax is effective only if firms recognize that their emissions affect aggregate levels. If firms believed that aggregate emissions were fixed, they would view their tax burden as a fixed cost, and the tax would not affect their decisions about pollution. For some nonpoint source pollution problems, e.g. those associated with automobiles, there are so many polluters that it is reasonable for each to treat aggregate pollution as fixed. For these cases, the ambient tax is unlikely to be effective, and the analysis of this paper is not applicable. In other cases, it is possible to measure the aggregate emissions of a small group of polluters, but not practical to measure each member's emissions. For example, monitoring

stations can measure the pollution caused by a group of farmers in the same watershed, even when it is too costly to measure the pollution caused by individual farmers[1] . In this situation, the relevant aggregate emissions measure is that of the group of farmers within the watershed, rather than total emissions within the state or county. When a small number of polluters contribute to the nonpoint source problem, they may behave strategically with respect to other firms and/or the regulator. In that case, the ambient tax can be effective, and the analysis of this paper is relevant.

Regulatory policy that depends on ambient measures rather than solely on individual actions is increasingly important. Segerson [14] describes several such policies, including: (i) the Everglades Forever Act, where failure to reduce aggregate phosphorus levels causes land tax to increase; (ii) a policy for Lake Okeechobee, Florida, which taxes dairies if water quality goals are not met; (iii) the Coastal Zone Management Reauthorization Amendments; and (iv) a threat to list salmon as an endangered species unless voluntary measures succeed in restoring habitats in Oregon.

Since ambient taxes can be effective only if the number of polluters is small enough that polluters understand that they effect the aggregate level of pollution, we restrict attention to this case. The second limitation of the ambient tax remains: it may cause each firm's tax burden to be large, since each firm pays a unit tax on aggregate rather than individual emissions. In this case, the tax collection exceeds the damage done by pollution and there

[1] Farzin and Kaplan [2] discuss environmental damage associated with logging roads in Redwood National Park, California. There, monitoring stations are used to assess the contribution to the damage caused by groups of roads (which may be associated with different logging operators), although the contribution of individual roads cannot be measured.

is an "imbalance" in the social budget, analogous to the imbalance noted by Holmstrom. If the regulator subsidizes abatement, rather than taxing emissions, the budget imbalance is negative, rather than positive. Whether a tax or subsidy is used, there is an imbalance, and the nature of the problem is essentially the same. In either case, the transfer may increase problems associated with equity ("unfair" taxes) or efficiency (the wrong number of firms in the industry). For our purposes, the important issue is that an effective ambient tax might result in large transfers; it does really matter whether there is actually a "budget imbalance", as described above

The problem of large transfers associated with a unit ambient tax can be corrected by giving firms a lump sum transfer, or by using a non-linear tax. Under some conditions these taxes can achieve any level of budget balance. Hyde *et al.* [6] study the case where firms are heterogenous and the regulator knows the distribution of firms; a cost of public finance causes a trade-off between pollution control and the size of the transfer. Garvie and Keeler [3] study the trade-off between data collection and enforcement under a budget constraint.

The complexity of non-linear schemes may make them unattractive. In addition, the taxes may not be credible if a non-equilibrium event occurs. For example, suppose that a two-part tax involves a lump sum transfer and a unit ambient tax. If the aggregate level of pollution is at its equilibrium level, the net transfer is at the desired level. However, if for some reason the aggregate level of pollution is lower than the equilibrium level, the net transfer to polluters exceeds the desired level. In this case, the lump sum transfer may not be credible.

The simple ambient tax (without lump sum transfers) is likely to cause the problem of large transfers when the tax rate is fixed. However, the assumption of a fixed tax rate may be inappropriate. An ambient tax makes sense only if firms behave strategically. If firms recognize that they affect aggregate pollution, then it is plausible that they also understand that they might affect the unit tax. In that case, each firm has an incentive to behave in a way that reduces the unit tax. This incentive is greater when they pay the tax on aggregate, rather than individual emissions.

For example, suppose that the unit tax is s, aggregate emissions are X, and each of the n firms' individual emissions are $x = X/n$ in a symmetric equilibrium. With point source pollution, the firm pays sx in tax, and under the ambient tax the firm pays sX: $sX > sx$. For a given level of aggregate emissions X, a unit reduction in the tax saves the firm X under the ambient tax, and it saves the firm X/n under the point source tax. Thus, the firm's incentive to behave in a way that causes a tax reduction is much greater with the ambient tax. Consequently, we might expect the ambient unit tax needed to support X under nonpoint source pollution to be less than the unit tax needed to support X under point source pollution. In that case, the tax burden under the ambient tax may be less than the tax burden under the point source pollution tax. Firms' strategic behavior may not merely mitigate the problem of large transfers, but may cause it to disappear.

Firms have an incentive to behave strategically if they believe that they can influence the level of the tax. That belief is reasonable if the tax actually does adjust to firms' decisions (as with the land tax under the Everglades Forever Act). When the regulator does not know

firms' abatement costs, as in Karp and Livernois ([8] hereafter KL) or is uncertain about the degree of intra-industry cooperation, as in Millock and Saline (1998), she may not be able to (or may not want to) commit to a fixed tax. It is sensible to change the tax if it does not support the pollution target. To describe the effect of strategic behavior under nonpoint source pollution in the simplest manner, I assume that the tax adjusts linearly when actual emissions differ from an exogenous level of target emissions (as in KL).

For a given tax rule, I compare the steady state tax burden under the ambient tax and under the tax that is levied on individual emissions. Both taxes achieve the same steady state level of aggregate emissions. The ambient tax is usually viewed as a response to the regulator's inability to observe the emissions of individual firms. However, firms might *prefer* an ambient tax. If the regulator actually observes individual emissions, she may be unable to credibly commit to using an ambient tax. In this case, contrary to appearances, firms benefit from the regulator's inability to observe individual emissions.

The possibility that apparently beneficial information is actually harmful occurs in many settings. When a regulator has imperfect information, she may need to act in a way which, under some circumstances, would be very costly to firms. In order to avoid bad outcomes, firms are induced to behave in a way which is in their collective self-interest, i.e., to cooperate. Cremer [1] discusses a similar possibility in a principal-agent setting. There, additional information may make it difficult for the principal to commit to threats, thus decreasing the principal's leverage over agents, and making the agents worse off.

In the setting here, "better information" means the ability to identify the level of emis-

sions of a subset of firms rather than simply the aggregate level of emissions for all firms. I examine the conditions under which better information actually does harm firms. For example, a group of farmers in a particular area might be able to install a monitoring device that measures the total emissions for that group: they form a kind of club. If the payoff of club members is lower than in the equilibrium without the club, they would not want to invest in the monitoring equipment.

The paper most closely related to this study is Xepapadeas [18]. He studies a model in which firms' tax payments depend on the difference between the actual and socially optimal levels of the pollutant.[2] For an asymptotically efficient tax rule, this difference is zero, so firms pay zero tax in the steady state. This tax is an example of the use of transfers (or equivalently, of a two-part tax) to correct the budget imbalance problem, at least in the steady state. Shortle *et al.* [15] review the theoretical and empirical literature on regulation of nonpoint pollution problems.[3]

The next section adapts the model in KL to the nonpoint source pollution setting. It compares the tax burdens at an open-loop steady state for point source and nonpoint source pollution, for a given aggregate target level of emissions.[4] The next two sections study

[2] Xepapadeas [18] considers the case of a stock pollutant, but he concentrates (in his analysis of the deterministic problem) on a tax rule that achieves the socially optimal steady state. The level of this steady state, and thus the level of emissions that supports it, can be treated as exogenous to the problem of determining the tax rule. Thus, the distinction between a stock pollutant and a flow pollutant is not essential (for the deterministic problem), although it does change some technical details.

[3] See also Herriges et al. [4] and Xepapadeas [17], two papers which follow Rasmusen [12] in showing that strategic behavior amongst firms can resolve the budget balance problem. In their model, a firm is randomly chosen to pay the tax when ambient emissions exceed a target. The randomness of the penalty and the asymmetry of outcomes might make the mechanism politically unattractive.

[4] The target should depend on damages and abatement costs, but does not necessarily depend on the regulator's ability to measure firms' emissions. Therefore, the target might be the same for both types of

the comparative statics of the open-loop equilibrium, and the set of Markov Perfect steady states. A conclusion summarizes the results.

2 The Open-Loop Equilibrium in a Dynamic Tax Game

This section studies the open-loop equilibrium in a dynamic tax game. In this setting, the open-loop equilibrium is time consistent, but it is not subgame perfect. This shortcoming is well-known, but by using the open-loop equilibrium it is possible to make some interesting points in a simple setting. I first describe the model and compare steady state tax payments under an ambient tax and one that is based on individual emissions. Subsequent sections discuss issues arising from the assumed symmetry of the equilibrium, the role of asymmetric information, and effect of firm heterogeneity. The final subsection presents the phase portrait.

2.1 A comparison of ambient and point-source taxes

The representative firm emits x, a flow. Its concave restricted profit function is $R(x)$.[5] There are n firms, and industry emissions are X. Except where I note otherwise, I assume that firms are identical and the equilibrium is symmetric (i.e., $x = X/n$ *in equilibrium*). The exogenous target level of emissions is X^*. If the regulator knew the firm's restricted profit function, could measure the firm's emissions, and could commit to a fixed tax, she could achieve the target X^* by charging the full information unit tax $s^* \equiv R'(X^*/n)$.

pollution, and I treat it as exogenous.

[5] Liski [9] allows firms to have fixed costs, which makes the profit function convex near the origin. In this situation, the tax adjustment rule may not support the target level of emissions even when the industry consists of a single firm.

The unit tax at time t is $s(t)$. Firms take as given the tax adjustment rule:

$$\begin{aligned} \dot{s}(t) &= \alpha\left(X(t) - X^*\right) \text{ if } s > 0, \text{or if } s = 0 \text{ and } X \geq X^* \\ \dot{s}(t) &= 0 \text{ if } s = 0 \text{ and } X < X^*. \end{aligned} \tag{1}$$

where $\alpha > 0$ is an exogenous speed-of-adjustment parameter. The emissions tax increases whenever aggregate emissions exceed the target, decreases whenever the tax is positive and aggregate emissions are less than the target, and is constant whenever aggregate emissions are equal to the target. The tax never becomes negative.

As with the standard ambient tax, each firm pays $s(t)X(t)$ at time t. Each firm is taxed on the basis of aggregate emissions. Each firm chooses a trajectory $\{x(t)\}_0^\infty$ to maximize the present discounted value of profits, $\int_0^\infty e^{-rt}[R(x) - s(t)X(t)]dt$, subject to (1) and $X \equiv x + \mathrm{X}_{-i}$. In an open-loop equilibrium (OLE), each firm takes as given the aggregate extraction trajectory of other firms, $\{X_{-i}\}_0^\infty$. The Hamiltonian and necessary conditions for a symmetric open-loop interior Nash equilibrium are:

$$H = R(x) - sX + \lambda\alpha(X - X^*) \tag{2}$$

$$R'(x) - s + \lambda\alpha = 0 \tag{3}$$

$$\dot{\lambda} - r\lambda = X. \tag{4}$$

The steady state open-loop ambient tax, s_∞^N ("N" for "Nonpoint"), is obtained by setting the time derivatives in (1) and (4) equal to 0 and using (3):

$$s_\infty^N = \max\left[0, s^* - \frac{\alpha X^*}{r}\right]. \tag{5}$$

In order to obtain the steady state point source pollution tax, s_∞^P, ("P" for "Point"), write the firm's flow of payoff as $R(x) - sx$ (instead of $R(x) - sX$) and make the corresponding changes in the Hamiltonian and necessary conditions. Evaluating these conditions at the steady state (see equation (5) in KL) gives the steady state tax under point source pollution:

$$s_\infty^P = \max\left[0, s^* - \frac{\alpha X^*}{rn}\right]. \tag{6}$$

Equations (5) and (6) imply that $s_\infty^N < s_\infty^P$ whenever s is positive (as I hereafter assume). Under both nonpoint and point source pollution, firms have an incentive to reduce current emissions in order to reduce the tax, and thereby reduce their future tax bill. This incentive is greater with the ambient tax than with the point source pollution tax. For a *given trajectory* of X, the absolute value of the shadow value of the tax (λ) is n times as large with nonpoint source pollution compared to point source pollution; this difference implies that the incentive to reduce pollution would be n times as large with nonpoint source pollution. The trajectories of X are different under the two taxes outside the steady state, but they are identical at the steady state (where $X(t) \equiv X^*$).

Define the steady state tax burden per firm under nonpoint source pollution as $NT \equiv s_\infty^N X^*$, and define the steady state tax burden per firm under point source pollution as $PT \equiv s_\infty^P X^*/n$. For $n > 1$, equations (5) and (6) imply[6]

$$NT < PT \Leftrightarrow n + 1 > \frac{rR'(X^*/n)}{\alpha X^*/n} \equiv z(n) \equiv \frac{rs^*}{\alpha x^*} \tag{7}$$

[6] The assumption that $s_\infty^P \geq 0$ implies (using equation (6) and the definition $s^* \equiv R'(X^*/n)$) that $z(n) \geq 1$. This restriction does not affect the results.

The function $z(n)$ defined in equation (7) is used at several different stages of our analysis; $z(n)$ depends on the profit function, the target level of emissions (X^*), the speed of adjustment parameter α and the discount rate r; we suppress those arguments for simplicity, but it is important to keep in mind that we can hold n constant and change $z(n)$ by changing any of these parameters. $z(n)$ is an increasing function of r/α and the full information tax s^*, and it is a decreasing function of the target level of emissions per firm. Equation (7) implies (for $s^P_\infty > 0$)

Proposition 1 *The steady state tax burden with the ambient tax is lower than the tax burden with point source pollution if and only if r/α and/or s^*/x^* are sufficiently small.*

Greater patience or more rapid adjustment of the tax both increase the difference (between point and nonpoint source pollution) in the strategic incentive of firms. When this difference is strong enough, the unit ambient tax is less than $1/n$′th of the unit tax under point source pollution, making the tax burden smaller under the former.

The equilibrium industry size under open access depends on the tax burden, and therefore depends on the tax rule. Proposition 1 shows that under some circumstances the equilibrium number of firms under open access is higher with the ambient tax.

2.2 The assumption of symmetry and homogeneity

It might seem that if the regulator knows that firms are homogenous and expects the equilibrium to be symmetric, then she has the same information as under point source pollution: she need merely divide the total amount of emissions by the number of firms to identify the equilibrium emissions of each firm. This conjecture is false, because it fails to distinguish

between what the regulator knows (or believes) to be true as a result of inferences based on the equilibrium outcome, and what she can directly observe. The distinction matter because it affects the firms' incentives.

In order to explain this point, suppose that the regulator uses a modified ambient tax in which each firm is charged for X/k units of emissions. A larger value of k means that each firm is held responsible for a smaller fraction of total pollution. For $k = n$, each firm is taxed on the fraction of total emissions it is responsible for *in a symmetric equilibrium.*

With this modified tax rule, equations (2) and (3) are changed by replacing s with s/k. The steady state tax is $(s^*\text{-}\ \alpha X/rk)k$, and each firm's steady state tax burden, denoted $NT(k)$, is

$$NT(k) \equiv \left(s^* - \frac{\alpha X}{rk}\right) X^*.$$

The tax burden is increasing in k: when firms pay for only $1/k$'th of total pollution, a larger tax is needed to reach the fixed target X^*. The percentage increase in the tax exceeds the percentage decrease in the tax base. For $k = n$, the steady state tax burden under this modified ambient tax always exceeds the tax burden under point source pollution: $NT(n) > PT$.

The explanation for this inequality is that if $k = n$ firms are responsible for the same amount of pollution in the steady state under both the modified tax rule and under the point source pollution rule. However, the tax rate is n times larger in the former. The reason for this difference is that with the point source pollution tax, the firm understands that it is responsible for a unit increase in its pollution, whereas under the modified tax, the

firm expects to be responsible for only $1/k$'th of the additional pollution. In the latter case the firm obviously has a smaller incentive to reduce pollution, thus requiring a larger tax to support a given target level of pollution.

2.3 The role of asymmetric information

In many situations, asymmetric information (between the regulator and firms) increases firms' profits because it enables them to extract rent or evade emissions control. In this model, asymmetric information with regard to costs always benefits firms. A further asymmetry of information, with regard to emissions, may also benefit firms.

The asymmetry of information with respect to costs enables the regulator to credibly commit to lower taxes in response to a reduction in emissions. The tax burden under *full information* about firms' abatement costs and their levels of emissions is $FT \equiv s^*X^*/n$. The full information tax burden is higher than the steady state tax burden when point source pollution is controlled by the tax adjustment rule, equation (1): $FT > PT$, since $s^P_\infty < s^*$

Asymmetric information about individual emissions levels (as opposed to costs) forces the regulator to be tough in order to meet a target. This increased toughness induces firms to behave more "cooperatively", thus reducing their *equilibrium tax*.

2.4 Heterogeneous firms

The previous discussion considers only homogenous firms. Firm heterogeneity does not alter the basic comparison between the tax burden under a standard and an ambient (adjustable) tax, but it leads to an important additional result: With heterogenous firms, the ambient

tax (unlike the point source tax based on own-emissions) leads to an efficient allocation of a given quantity of emissions.

To verify this claim, replace $R(x)$ by $R^i(x_i)$ and replace λ by λ_i in equations (2) - (4). Using (the analogue of) equation (4) and the steady state condition, it is obvious that $\lambda_i = \lambda_j$ for all i, j at every point in time. Equation (3) then implies that $R^{i\prime}(x_i) = R^{j'}(x_j)$ for all i, j and at all times, so a given level of emissions is allocated efficiently across firms.

Under a point source tax, the allocation trajectory is inefficient (KS, Proposition 1). When firms pay the tax based only on their own emissions, firms that emit at different levels have different shadow values of the tax (λ). In that case, equality of marginal profits does not hold. In the steady state, where aggregate emissions are the same for both point source and nonpoint source pollution, the ambient tax is unambiguously more efficient[7] .

For the general case of heterogenous firms, a simple comparison of the average tax burden under point source and nonpoint source pollution is not possible. However, for the special case where $R^{i\prime}(x_j) = a_i$ - bx_i the criterion in Proposition 1 is unchanged, except that it now applies to the *average* tax burden.[8] Thus, for heterogenous firms (with the above specification of $R(x)$, the average tax burden is lower under the ambient tax when α/r is large.

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[7] Outside the steady state, aggregate emissions differ under the standard and the ambient tax, so we cannot compare the efficiency of the two. Outside the steady state we merely note the *conditional* (on aggregate emissions) efficiency of the ambient tax.

[8] In order to verify this claim, define $a = \Sigma_i a_i/n$, use $R'(x_i) = a - bx_i$, and repeat the steps leading to equation (7).

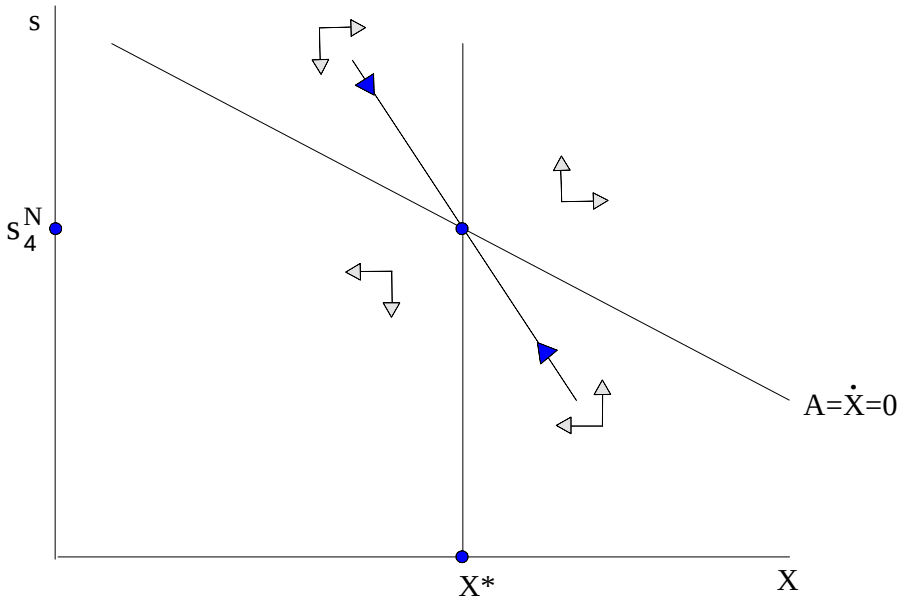

Figure 1: Open-loop Phase Portrait

2.5 Dynamics

For the purpose of comparing the open-loop equilibrium with the Markov Perfect equilibrium in Section 5, Figure 1 sketches the phase portrait of the open-loop trajectory. By differentiating equation (3) with respect to time and substituting (1), (3) and (4) into the result, we obtain the differential equation of the equilibrium level of X:

$$\dot{X} = \frac{-n[\alpha X^* - r(R'(X/n) - s)]}{R''(X/n)} \equiv A(X, s). \tag{8}$$

Inspection of the phase portrait shows that the steady state is a saddlepoint, and that the equilibrium trajectory involves monotonic adjustment of both X and s.

3 Strategic interactions

Three comparative statics questions shed light on the strategic interaction between firms and the regulator. First, I show that the inequality $NT < PT$ is more likely to hold when n is small. An increase in n decreases firms' strategic incentive under both the point source and the ambient tax, but the effect is stronger in the latter. Increasing n means that firms operate at a point where marginal profits are higher in equilibrium (because $R(x)$ is concave). Therefore, the full-information tax s^* also increases with n.

The second comparative statics experiment "fragments" (or replicates, or fissions) the market, in the manner described by Novshek and Sonnenschein [11], Hyde *et al.* [6] and Karp [7]. We can think of a single firm as fragmenting into m separate firms, without changing their aggregate technology. If behavior in the rest of the market were unchanged, these m new firms could obtain the same aggregate profit as the original firm. The change in their equilibrium profits is therefore due to the fact that the strategic behavior of m distinct firms differs from the strategic behavior of a single firm. Fragmentation of the market, unlike a simple increase in n, isolates the strategic effect of increasing the number of firms.

The third experiment allows a group of $m \leq n$ firms to form a "club", i.e. to invest in monitoring equipment that enables the regulator to distinguish the club's aggregate emissions from the aggregate emissions of non-members. Formation of a club is different from the inverse of fragmentation ("consolidation") in two respects. First, with consolidation, firms internalize some of the externality of their current action. This internalization does not occur with a club, since members consider only their own stream of profits. Second, formation of

a club narrows the definition of "ambient emissions", which now refers only to the aggregate emissions of the club members, not to the aggregate emissions of all firms. This change does not occur under consolidation.

3.1 A change in the number of firms

If the tax burden is higher under the ambient tax for some value of n, then it is also higher for all larger values of n. Formally, if $NT(n_o) > PT(n_o)$ for some n_o, then $NT(n) > PT(n)$ for all $n > n_o$.[9] In order to establish this relation, assume that $NT(n_o) > PT(n_o)$ and that $NT(n_o + v) < PT(n_o + v)$. These two inequalities, together with equation (**??**), imply that $v < 0$.

For example, for the constant elasticity profit function, $R(x) = x^\beta$, $0 < \beta < 1$, we can rewrite (7) as

$$NT < PT \Leftrightarrow y(n) \equiv \frac{\beta n^{2-\beta}}{n+1} < \frac{\alpha X^{*2-\beta}}{r} \equiv h. \tag{9}$$

The function $y(n)$, defined in (9), is increasing in n and unbounded as $n \to \infty$. Therefore the inequality $NT \leq PT$ requires that n is sufficiently small.

For any finite n, firms make positive after-tax profits in the steady state under the point source pollution tax: $R(X^*/n) - PT(n) > 0$, in view of the concavity of R. Under the ambient tax, as $n \to \infty$, $R(X^*/n)$ approaches 0 but $NT(n)$ remains strictly positive (perhaps infinite). Therefore, with open entry, the maximum feasible n under the ambient

[9] An equivalent statement of this result is that there is at most one crossing of the graphs of $NT(n)$ and $PT(n)$. If this crossing occurs at (the unique) n^*, then $d[NT(n^*) - PT(n*)]/dn > 0$.

tax satisfies $R(X^*/n) - NT(n) = 0$.

For the constant elasticity example, the non-negativity of steady state profits requires

$$(n\beta - 1)n^{-\beta} - \frac{\alpha X^{*2-\beta}}{r} < 0.$$

Since the first term on the right side is increasing in n and becomes unbounded as $n \to \infty$, non-negativity of profits requires that n be sufficiently small.

3.2 Fragmenting the market

We can think of a single firm with restricted profit function $R(x)$ fragmenting into m firms, each with restricted profit functions $R^m(x/m) \equiv R(x)/m$. If aggregate production of the original firm were distributed evenly across the m firms, aggregate profit would remain constant.

Under this type of fragmentation, $R^{m'}\left(\frac{x}{m}\right) \equiv R'(x)$, so the full information tax is independent of m; it equals $s^* = R'\left(\frac{X^*}{n}\right)$ as previously defined. (Recall that when n increases, s^* increases.) From equation (5) the number of firms has no direct effect on the steady state ambient tax; the number of firms affects the steady state ambient tax rate only via the effect of this number on s^*. Since s^* is independent of m, the steady state ambient tax is independent of m.

Suppose that there are originally n firms, and each of these fragments into m smaller firms, so that there are now mn firms. Define $NT^m(m,n)$ as the steady state tax burden of a group of m firms under the ambient tax. As the previous paragraph explains, the steady state ambient tax rate is unchanged, and since each firm pays this tax applied to the total

amount of pollution, we have $NT^m = m \cdot NT$.

Define $PT^m(m,n)$ as the steady state tax payments of a group of m firms under the point source tax. The steady state point source pollution tax is $max[0, s^* - \alpha X^*/rmn]$. (See equation (6), which shows that the tax depends on the number of firms.). For a positive tax, we have $PT^m(m,n) = (s^* - \alpha X^*/rmn)\, X^*/n$. These expressions imply

$$NT^m < PT^m \Leftrightarrow \frac{mn+1}{m} > z(n). \tag{10}$$

Comparison of equations (7) and (10) shows that fragmentation of the market makes it less likely that the tax burden is lower under the ambient tax (since $(mn+1)/m$ is decreasing in m).

3.3 Formation of clubs

Millock *et al.* [10] consider the possibility that individual firms are able to invest in monitoring equipment.[10] Firms that make this investment can be charged on the basis of individual, rather than aggregate emissions. More generally, firms may be able to form *clubs*. Members of a club purchase monitoring equipment that makes it possible to measure the aggregate emissions of that club. If the aggregate emissions of the club is X^c, and aggregate emissions of all firms is X, each member pays sX^c rather than sX. Since monitoring produces benefits (a lower base on which the tax is calculated) and costs (a higher equilibrium unit tax), it might seem that under some circumstances the industry would prefer partial monitoring,

[10] They note that most pollution problems exist somewhere on a continuum between point and nonpoint source pollution. The location on this continuum depends on the amount of monitoring, which might be chosen by either the regulator or by firms.

rather than either extreme. However, providing that the monitoring equipment is sufficiently cheap, firms prefer the outcome under either point source pollution or the ambient tax to an intermediate outcome.

To verify this claim, suppose that κ symmetric clubs form; each club has n/κ members. Larger values of κ imply a larger number of clubs, i.e. more investment in monitoring. For the special case of $\kappa = n$ each firm has installed equipment that makes it possible for the regulator to measure that firm's emissions. Treating κ as a continuous variable (ignoring the integer constraint) is unimportant. The restriction to symmetry is important, because it disguises the fact that the formation of a club by some firms affects the incentives of non-members to form their own clubs. (We return to this issue below.) However, analysis of the symmetric case provides a basis for understanding those incentives.

In a symmetric steady state, aggregate equilibrium emissions for each club are X^*/κ and (as before) each firm emits $x^* = X^*/n$. Using equations (2) and (4) (replacing sX with sX^c), we can calculate the steady state tax burden for each club-member, denoted $NCT(\kappa,n)$, and its derivative $\partial NCT/\partial\kappa$

$$NCT\left(\kappa,n\right) \equiv \left[s^* - \frac{\alpha X^*}{r\kappa}\right]\frac{X^*}{k}; \qquad \frac{\partial NCT}{\partial \kappa} = \frac{\alpha X^{*2}}{r\kappa^2 n}\left[\frac{2n}{\kappa} - z\left(n\right)\right]. \tag{11}$$

"N" denotes nonpoint source pollution and "C" denotes club. (I assume the tax is positive.)

For $\kappa = 1$ we have the original nonpoint source pollution $[NCT(1,n) = NT]$, and for $\kappa = n$ the problem has been converted to point source pollution $[NCT(n,n) = PT]$. The derivative $\partial NCT/\partial\kappa$ has a single turning point, at $\hat{\kappa} = 2n/\mathrm{z}(n)$. There are four possibilities,

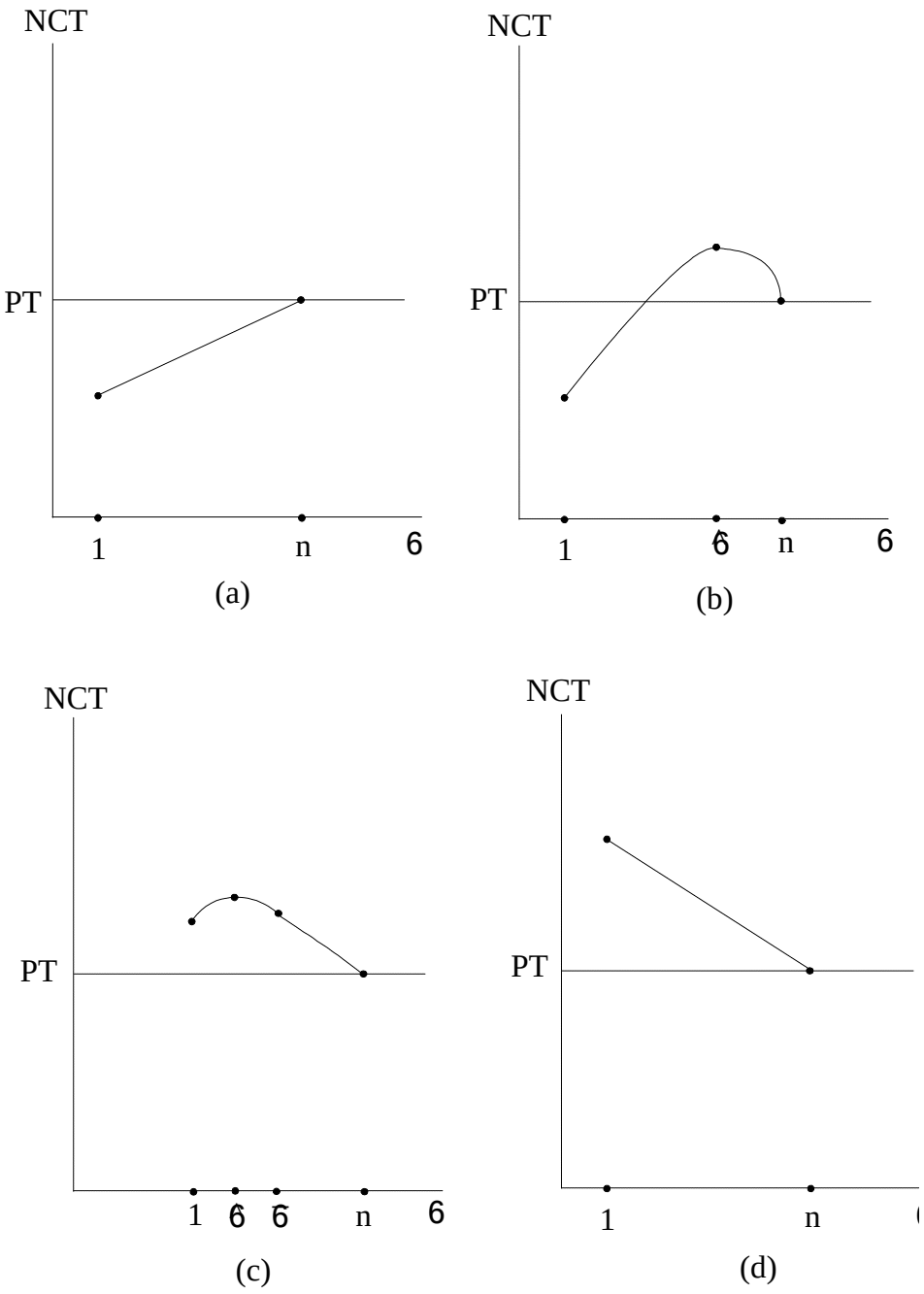

Figure 2: The Effects of Clubs

depending on the magnitude of $z(n)$ relative to n. Figure 2 graphs these four possible cases.

Case a: z < 2. $z(n) < 2n + 1$ In this case $\hat{\kappa} > n$ and $NCT(1, n) = NT < PT$ (from equation (7)). The industry tax burden is monotonically increasing in the number of clubs (figure 1a). **Case b:** $2 < z < n + 1$. Again, the tax burden is lower at $\kappa = 1$ than at $\kappa = n$, but NCT reaches its extreme value at $\hat{\kappa} < n$ (figure 1b, where there is a local maximum at $\hat{\kappa}$). **Case c:** $n + 1 < z < 2n$. Here $\partial NCT(n, n)/\partial k = 2 - z < 2 - n - 1 < 0$; $NCT(1,n) = NT > PT = NCT(n, n)$; and $n > \hat{\kappa} > 1$. A small amount of monitoring (small κ) increases the industry tax burden, but a sufficiently large amount of monitoring decreases the tax burden (figure 1c). **Case d:**$2n < z$. Here $NT > PT$ and $\hat{\kappa} > n$, so the industry tax burden is monotonically decreasing in κ (figure 1d).

For cases a and b (small z, e.g. a small r/α) the industry tax burden is minimized under the ambient tax (no monitoring). Even if monitoring equipment were free, the industry would not want to install it. Monitoring decreases each firm's incentives to keep the tax rate low and thus leads to a higher equilibrium tax burden. These incentives are particularly valuable, and therefore their loss is costly, if firms are patient (small r) and/or taxes adjust quickly (large α). For cases c and d (large z, e.g. large r/α) the industry tax burden is minimized under complete monitoring: if the monitoring equipment were sufficiently cheap, the industry would encourage each firm to install it.

The discussion above examines the effect of changing the number of clubs in a symmetric equilibrium. However, ithe focus on the aggregate payoff of firms in a symmetric outcome obscures the incentives of individual firms. Each firm's payoff might be higher when they

all install monitoring devices ($\kappa = n$) compared to when none of them do so ($\kappa = 1$) but that does not mean that it is an equilibrium for all firms to install the device. Individual and collective interests are unlikely to coincide in an "installation game", i.e. at an initial stage when each firm decides whether to join a club which installs monitoring equipment.

Suppose, for example, that z is small and the monitoring equipment is cheap. Collective rationality dictates that no monitoring equipment be installed. If each firm believes that no other firm will install the equipment, then it is in the interests of each firm to install it. Installation by that firm has a small effect on the equilibrium value of the tax rate, but causes a large decrease in the basis upon which its tax is levied, and thus reduces its own tax burden. In this circumstance, "non-installation" is not a noncooperative Nash equilibrium in the installation game. The possibility that individual and collective rationality do not coincide is, of course, a familiar result in industrial organization.

Alternatively, suppose that z is large and that the cost of monitoring is small but not negligible. Industry profits are maximized if every firm installs the monitoring device. However, if a firm believes that all other firms will install the equipment, that firm has no incentive to install it. By not buying the equipment, the firms saves the (small) installation costs, but the regulator is still able to calculate its emissions (since the regulator measures aggregate emissions and the emissions of all other firms). In this case,"universal installation" is not a noncooperative Nash equilibrium in the installation game.

We summarize the three comparative statics experiments described above in the following

Proposition 2 *An increase in the number of firms, or a fragmentation of the market (holding aggregate technology fixed) decreases the set of parameter values for which the steady*

state tax burden is lower under the ambient tax, relative to the point source pollution tax. In a symmetric equilibrium where firms can form clubs, and the regulator can monitor the emissions of each club, the industry tax burden is minimized when no clubs are formed (for small z) or when each firm forms a club consisting only of itself (for large z). The outcome that minimize collective taxes (non-installation or universal installation) is not, in general a noncooperative Nash equilibria in the installation game.

4 The Markov Perfect Tax

The OLE assumes that firms make binding decisions about their future emissions at time 0. In equilibrium no firm wants to change its plan, so the equilibrium is dynamically consistent. However, if any firm were to deviate from equilibrium, the remaining plan of all firms would not constitute an equilibrium for the subgame that begins after the time of the deviation. Thus, the OLE is not subgame perfect. The Markov Perfect equilibrium (MPE), in which all agents condition their decisions on the state variable (here, the current tax $s(t)$) is subgame perfect. The MPE is therefore arguably more plausible than the OLE. The greater complexity and the lack of uniqueness of the MPE make it more difficult to use that setting. Nevertheless, it is worth understanding how the OLE and MPE differ.

In general, the MPE is not unique, even with the restriction of continuous differentiability. Nonuniqueness occurs because of the "incomplete transversality condition", or the lack of a natural boundary condition in the problem. (See Tsutsui and Mino [16] and other papers cited in KL.) There exists a set of equilibrium stable steady states that corresponds to the set of continuously differentiable MPE. The smallest stable steady state MPE tax equals the open-loop steady state, s_∞^N.[11]

[11] KL obtain an analogous result for the point source pollution problem, although the phase portraits for

In a MPE, each firm regards the aggregate emissions of other firms as a function of the state: $X_{-i} = q(s)$. The function $q(s)$ is endogenous. If we replace $X_{-i}$by $q(s)$ in the representative firm's optimization problem, the first order condition (3) is unchanged, but (4) is replaced by[12]

$$\dot{\lambda} - r\lambda = (s - \alpha\lambda)q'(s) + x + q(s). \tag{12}$$

Equation (12) differs from (4) by the inclusion of the term $(s - \alpha\lambda)q'(s)$, which incorporates the firm's beliefs about how a change in the level of s will affect the aggregate future emissions of other firms. The assumption of symmetry implies $q(s) = (n-1)X(s)/n$, which implies $q'(s)\dot{s} = (n-1)\dot{X}/n$. These relations, together with equations (1), (3), and (12) can be substituted into the derivative with respect to time of equation (3). These manipulations produce the differential equation that describes the evolution of X:

$$\dot{X} = \frac{A\left(X, s\right)\left(X - X^*\right)}{B\left(X\right)}; \quad B\left(X\right) \equiv \left(X - X^*\right) + \frac{\left(n-1\right)R'\left(\frac{X}{n}\right)}{R''\left(\frac{X}{n}\right)} \tag{13}$$

where the function $A()$ was defined in equation (8).

Equation (13) illustrates the lack of a natural boundary condition. For the MPE, $\dot{X} = 0$ whenever $A(X, s) = 0$ or *whenever* $\dot{s} = 0$.

Figure 3 shows the phase portrait. In the neighborhood of X^*, $B < 0$, and for X

the two types of pollution are significantly different. In general, for one-state differential games, the set of MPE stable steady states need not be bounded by the open-loop steady state.

[12] The analysis relies exclusively on the necessary conditions to the representative firm's control problem, and assumes existence.

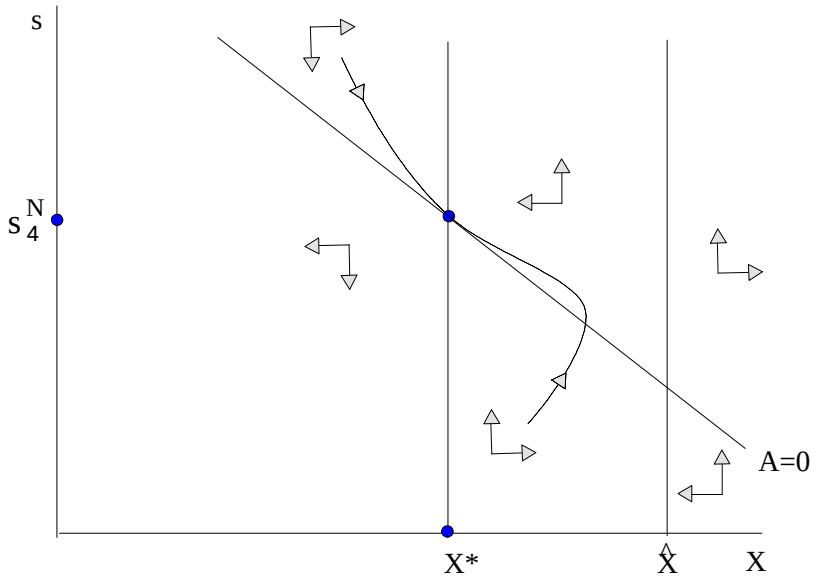

Figure 3: Markov Perfect Phase Portrait

sufficiently large, $B > 0$; $\hat{X}$ is defined as the smallest value of X that solves $B(X) = 0$. All trajectories that converge to a steady state satisfy $X \leq \hat{X}$ at every point. From figure 3 it is apparent that the smallest stable MPE steady state tax is the open-loop steady state, s_∞^N. Higher steady state taxes also satisfy the necessary conditions for the representative firm's optimization problem in a symmetric MPE.[13] Figure 3 shows the trajectory that converges to s_∞^N, drawn under the assumption that the trajectory is defined for all positive s.

Given the indeterminacy of the MPE steady state, it is natural to concentrate on the equilibrium most favorable to the firms, i.e. the equilibrium that reaches the steady state s_∞^N. For $s < s_\infty^N$, the open-loop trajectory approaches the steady state below the graph of $A = 0$ (figure 1), whereas the MPE approaches the steady state from above that graph

[13] For nonpoint source pollution (unlike for point source pollution), the necessary conditions do not imply an upper bound on the steady state MPE. Of course, the non-negativity of steady state profits puts an upper bound on the tax.

(figure 3). Therefore, in the neighborhood of the steady state for $s < s_\infty^N$, the MPE level of emissions is greater than the open-loop level of emissions.[14]

For all $s > s_\infty^N$ (not just in the neighborhood of the steady state), the MPE level of emissions exceeds the open-loop level. For $s > s_\infty^N$ both trajectories approach the steady state from above the graph of $A = 0$, so in both cases $dX/ds < 0$. We can rewrite equation (13) as

$$\dot{X} = \frac{A(X,s)}{C(X)}, \quad C(X) \equiv 1 + \frac{(n-1)R'}{(X - X^*)R''} > 1 \quad \text{for} \quad X < X^*.$$

The slopes of the MPE and OLE trajectories are

$$\begin{aligned} \text{MPE} \quad &: \quad \frac{ds}{dX} = \frac{\dot{s}}{\dot{X}} = \frac{\alpha(X - X^*)C(X)}{A(X,s)} \\ \text{OLE} \quad &: \quad \frac{ds}{dX} = \frac{\dot{s}}{\dot{X}} = \frac{\alpha(X - X^*)}{A(X,s)} \end{aligned}$$

If we evaluate these two slopes at any point in the region of phase space where $A > 0$ and $X < X^*$, we conclude that the trajectory in the MPE is steeper (since $C > 1$). Consequently, the MPE approaches the steady state s_∞^N on a trajectory above the open-loop trajectory. This comparison near the steady state, and the relative steepness of the two trajectories, imply that the trajectories can never cross in the region where $A > 0$ and $X < X^*$.

Thus, whenever $s > s_\infty^N$ and also for $s < s_\infty^N$ but not "too small", the level of emissions is greater in the MPE. This conclusion holds a fortiori for MPE that approach steady states larger than s_∞^N.

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[14] This comparison of emissions levels does not necessarily hold at small s, e.g. for s less than the intersection of the graphs of $A = 0$ and $X = \hat{X}$. There, the MPE trajectory (if it exists) must lie below the graph of $A = 0$. In this region, where $dX/ds > 0$, emissions may be lower in the MPE.

For all MPE trajectories above the graph $A = 0, dX/ds < 0$, and thus $dq(s)/ds < 0$: in response to an increase in firm i's current level of emissions, which causes an increase in the future tax, other firms reduce their future emissions. In this sense, firms' strategies are strategic substitutes.[15] In the MPE each firm has a greater incentive to pollute (relative to the OLE) because the resulting increase in the tax discourages other firms from polluting in the future. However, the policies are strategic complements for sufficiently small values of the tax. When the tax is very low, firms restrain themselves to some extent in order to keep the tax low. As the tax increases, this self-restraint diminishes, until eventually policies become strategic substitutes.

The OLE captures the firms' strategic behavior *vis a vis* the regulator; this strategic incentive always encourages firms to reduce current emissions in order to reduce the future tax. The MPE also captures firms' strategic incentive *vis a vis* other firms. That incentive encourages the firm to increase (respectively, reduce) emissions when strategies are strategic substitutes (respectively, complements). Thus, in a MPE, higher steady state taxes can occur, relative to the OLE. Even if the two equilibria result in the same steady state, emissions are higher in the MPE near the steady state.

The following proposition summarizes the results of this section.

Proposition 3 *There exists a continuum of candidate symmetric MPE satisfying the necessary conditions to firms' optimization problems. The smallest MPE steady state tax is the OLE steady state s_∞^N. Firms' strategies are strategic substitutes for any $s \geq s_\infty^N$, and are*

[15] Xepapadeas imposes a number of assumptions (in addition to differentiability) on firms' feedback strategies, including the assumption of strategic substitutability. Here I show (in a similar, but not identical model) that the Markov strategies must be strategic substitutes in the neighborhood of any stable steady state, and also for sufficiently large values of the tax.

strategic complements for s sufficiently small. For $s \neq s_\infty^N$, aggregate emissions are higher in any MPE (relative to the OLE), provided that the tax is sufficiently large.

5 Conclusion

If firms recognize that their decisions affect the aggregate level of emissions, then a policy that charges each firm an ambient tax can support the social optimum. In a static setting, such a tax is likely to result in large payments by firms. One solution to this problem is to use a two-part tax, or some other non-linear tax, which transfers revenue to firms. This paper explains why an adjustable ambient tax can provide an alternative that achieves a target level emissions without large tax payments.

If firms recognize that their decisions affect aggregate emissions, it is reasonable for them to recognize that their decisions might affect the tax rate. When we take into account strategic behavior, firms have an incentive to lower emissions in order to lower the tax. The steady state tax burden under the adjustable ambient tax is less than the burden where firms' payment is based on their own emissions, provided that: the tax adjusts quickly, firms are patient, and the number of firms is small.

An increase in the number of firms, or a fragmentation of the market, makes it less likely that firms prefer the ambient tax. If firms can join clubs that install monitoring equipment, the industry tax burden is minimized if no firm installs the equipment, or if every firm installs it. Intermediate solutions are never optimal. If firms make strategic decisions regarding installation, the non-cooperative equilibrium may lead to either too much or too little installation of monitoring equipment, relative to the industry optimum.

The usual motivation for an ambient tax is asymmetric information: the regulator's inability to measure and tax the individual's emissions. However, even with perfect information, firms *might* be better off if the regulator behaved as if she were unable to monitor individual emissions. By charging an ambient tax, the regulator causes each firm to behave in a manner more in the industry's collective self-interest, even though firms behave non-cooperatively. This result echoes a conclusion in the point source pollution model, where the regulator's ignorance of firms' abatement cost function may cause her to use an adjustable tax. There, firms' steady state equilibrium tax burden is always lower than under the full information tax. When the regulator is ignorant of not only the cost functions but also of individual emissions levels, firms may be still better off. These observations are special cases of a general point. When a regulator has imperfect information, she may need to act in a way which, under some circumstances, would be very costly to firms. In order to avoid those circumstances, firms' strategic incentives induce them to behave more cooperatively.

There exists a unique open-loop equilibrium, but there exists a continuum of policies satisfying the necessary conditions for (Markov) subgame perfection. In the most favorable of these (for the industry), the steady state tax equals the open-loop steady state. However, near the steady state, the level of emissions is higher in the Markov equilibrium. Here, subgame perfection, absent reputational effects and trigger strategies, has a tendency to erode firms' ability to cooperate.

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