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Department of Agricultural & Resource Economics, UCB
CUDARE Working Papers
(University of California, Berkeley)

Year 2002 *Paper* 956

Superior Forecasts of the U.S. Unemployment Rate Using a Nonparametric Method

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Key Words: Embedding dimension, Nonlinearity, Nonparametric, Unemployment rate.

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We are very grateful to Ruey Tsay for supplying data and for his comments. We also thank Alan Montgomery for his thoughtful comments.

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Superior Forecasts of the U.S. Unemployment Rate
Using a Nonparametric Method

I. Introduction

Predicting the unemployment rate is one of the most important applications of out-of-sampling forecasting for economists and policymakers. We demonstrate that a new data-driven method that is nonparametric in both functional form and distribution outperforms traditional data-driven, time-series models (which use only lagged observations of unemployment) and compares favorably with several economic-theory-based models (which also use other explanatory variables) in predicting unemployment rates. We compare the forecasts of various models using both quarterly and monthly data.

Recently, Montgomery, Zarnowitz, Tsay, and Tiao (1998, henceforth "MZTT") compared the forecasting abilities of a large number of linear and nonlinear models using unemployment rate data from 1948-1993. Using the data that they graciously supplied, we compare the forecasts from our nonparametric model to the models they studied and to the recent model developed by Carruth, Hooker and Oswald (1998, henceforth "CHO").

Both MZTT and CHO take to heart Ramsey's (1996) admonition: "If nonlinear models cannot forecast, what good are they?" Though their objective may be to enhance the understanding of the process governing the U.S. unemployment rates, they judge the "goodness" of the different models in terms of their forecasting abilities. We follow in this tradition by focusing on the relative forecasting abilities of the various models.

II. Nonlinear, Nonparametric Forecasting - A Short Summary

The nonlinear, nonparametric approach to forecasting that we use was introduced in Sugihara and May (1990) and Sugihara et al. (1996) and was modified by Mulhern and Caprara (1994) and Agnon, Golan, and Shearer (1999). In a recent paper, Fernandez-Rodriguez and Sosvilla-Rivero (1998) employ a closely related nearest neighbor approach to construct a test for nonlinearities in time-series data. In all of these studies, as well as in the one proposed here, forecasting is based on projecting historical patterns into the future without reliance on parametric assumptions on the distribution and functional form.

The method we use here is a variation of the nearest neighbor approach. For each point we wish to forecast, we find a simplex that contains this point, and use the points in the simplex to forecast the desired point. If such a simplex is not found, we use a combination of a noninclusive simplex and near points to forecast.

Let **y** be a T-dimensional vector of time-series observations of the unemployment rate. We start by breaking **y** into two mutually exclusive subsamples: the *history* and the *future*. Let the history include time periods 1,…, T_h and the future include points $T_{h+1},\ldots,T$. Next, we choose the embedding dimension E, which we discuss in the Appendix. We represent each lagged sequence of data points as a point in this E-dimensional space: $z_t^j = \left\{ y_t^j, y_{t-1}^j, y_{t-2}^j, \ldots, y_{t-(E-1)}^j \right\}$, where j = 1, 2,..., E+1, and t = E, E+1, ..., T. To simplify notation we rewrite z_t^j as $z = \left\{ y, y_{-1}, y_{-2}, \ldots, y_{-(E-1)} \right\}$.

The six basic steps of our algorithm are:[1]

Step 1. For each E-dimensional point z in the future subsample, we order the points in the history subsample from the closest to the furthest point using a distance measure.[2] The measure we use to determine the distance between point z and point z^* is $d = \sum_{i=1}^{E} \left| z_i - z_i^* \right|$.

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[1] In addition to these basic steps, our computer code includes a large number of additional controls that may be specified by the user (e.g., the number of close points for choosing the inclusive simplex, number of possible lags, etc.). Our C code is available upon request.

[2] As a practical matter to increase speed, we cam restrict the search to a limited number (here 40)

Step 2. We choose the E+1 closest points.

Step 3. We check whether these E+1 closest points form a *simplex* that contains the point z. If it contains z, we call it an "inclusive" simplex. Because it is based on the nearest points, it is likely to be the minimum volume simplex, where the volume is defined in terms of the determinant (Greene 1997).[3]

The inclusive simplex for point z is a collection of E+1 E-dimensional points: $S(z^1, z^2, \ldots, z^{E+1})$. These points are the vertices of an inclusive simplex iff the volume of the E+1 simplex *equals* the sum of the E+1 simplexes that are constructed by exchanging each one of the E+1 points with points in z.[4] Equivalently, S is an inclusive simplex iff the determinant of S equals the sum of the determinants of the E+1 other simplexes. The determinant for z is

$$D \equiv \begin{vmatrix} y^1 & y^1_{-1} & \cdots & y^1_{-(E-1)} & 1 \\ y^2 & y^2_{-1} & \cdots & y^2_{-(E-1)} & 1 \\ \vdots & \vdots & & \vdots & \vdots \\ y^{E+1} & y^{E+1}_{-1} & \cdots & y^{E+1}_{-(E-1)} & 1 \end{vmatrix}.$$

We substitute the values of z for each row in D to construct D_1, D_2, ..., D_E. Thus, S is inclusive if $D = D_1 + D_2 + \ldots + D_E$.

In Figure 1, we illustrate how an inclusive simplex is determined given that the embedding dimension is $E = 2$. We want to find the inclusive simplex for point z. The three closest two-dimensional neighboring points are z^1, z^2, and z^3. We start by defining the simplex consist-

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of the neighboring points.

[3] We experimented with additional searches to find the actual minimal volume simplex (i.e., checking all possible inclusive simplexes) and found that this simpler method usually finds the minimal volume simplex and, in any case, produces comparably good forecasts.

[4] Other nearest neighbor approaches are also reasonable. For example, Sugihara and May (1990) searched for the *minimal diameter* simplex or the "simple" nearest neighbor approach that chooses the k closest neighbors where k is determined by the researcher. In our approach, k always equals E+1.

ing of these three points as S = simplex(z^1, z^2, z^3). For this simplex to be inclusive, z must lie within the triangle formed by points z^1, z^2, and z^3. To determine if S is an inclusive simplex, we use the three neighboring points z^1, z^2, and z^3 together with z, to build the three simplexes S_1 = simplex(z^2, z^3, z), S_2 = simplex(z^1, z^3, z), and S_3 = simplex(z^1, z^2, z). Each of these three simplexes is constructed with two points of the three points that formed the original simplex S together with the two-dimensional point z. The simplex includes point z iff Det(S) = det(S_1) + det(S_2) + det(S_3). That is, the determinant of S equals the sum of the determinants of S_i (i = 1, 2, 3).[5] In the figure, the three simplexes (triangles) S_1, S_2, and S_3, combine to exactly form the simplex (triangle) S, which has vertices z^1, z^2, and z^3.

Step 4. If the three closest points do not constitute a simplex, we exchange one of the points with the next closest point. We check whether we have an inclusive simplex. If not, we continue to look at the next closest points.

We illustrate this approach in Figure 2, where we start with the three closest points z^1, z^{2*}, and z^3. That is, we have a point z^{2*} that is closer to z than is z^2. Unfortunately, these points do not constitute an inclusive simplex because z does not lie within the triangle these points form. Therefore, we need to exchange one of these points with the next closest point: point z_2.

The point to be exchanged is chosen as follows. First, we calculate the determinant of each one of the three simplexes[6]: Det(S_1) = -1, Det(S_2^*) = 1, and Det(S_3) = -2. Next, we compare the signs of the calculated determinants. The point, z_2^*, that is associated with the simplex

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[5] $Det(S_1) = \begin{vmatrix} 4.5 & 4 & 1 \\ 6 & 3 & 1 \\ 5 & 5 & 1 \end{vmatrix} = 2$, $Det(S_2) = \begin{vmatrix} 3 & 3 & 1 \\ 4.5 & 4 & 1 \\ 5 & 5 & 1 \end{vmatrix} = 1$, $Det(S_3) = \begin{vmatrix} 3 & 3 & 1 \\ 6 & 3 & 1 \\ 4.5 & 4 & 1 \end{vmatrix} = 3$,

$Det(S) = \begin{vmatrix} 3 & 3 & 1 \\ 6 & 3 & 1 \\ 5 & 5 & 1 \end{vmatrix} = 6$ and $Det(S) = Det(S_1) + Det(S_2) + Det(S_3)$.

[6] $Det(S_1) = \begin{vmatrix} 4.5 & 4 & 1 \\ 4 & 5 & 1 \\ 5 & 5 & 1 \end{vmatrix} = -1$, $Det(S_2^*) = \begin{vmatrix} 3 & 3 & 1 \\ 4.5 & 4 & 1 \\ 5 & 5 & 1 \end{vmatrix} = 1$ and $Det(S_3) = \begin{vmatrix} 3 & 3 & 1 \\ 4 & 5 & 1 \\ 4.5 & 4 & 1 \end{vmatrix} = -2$.

with the opposite sign to the other two is the one that is exchanged with the next closest point, z_2. Loosely speaking, the opposite sign of the determinant means that this point is "hidden" in the relevant space and therefore should be exchanged for another point. We note that in a higher dimension there could be more than a single hidden point. In that case, we exchange the point that farthest from z. This procedure continues until an inclusive simplex is found or there are no more substitutions possible. In Figure 2, after replacing z^{2*} with z^2, we have three points, z^1, z^2, and z^3, that constitute an inclusive simplex (as confirmed in our discussion of Figure 1).

Step 5. If no inclusive simplex exists (we cannot find three points that "surround" point z in the historical data set), we have two alternatives. Either, we choose the $E+1$ closest points, or we choose a simplex that is not inclusive. The first option of using the closest neighbors for forecasting is straightforward. The second option of using a non-perfect simplex involves several steps. We define the ratio $R = |Det(S)| / \sum_{i=1}^{E+1} |Det(S_i)|$. Because the determinant of an inclusive simplex equals the sum of the other determinants, if the target point z lies within the simplex, $R = 1$. If the point z to be forecast lies "outside" of the simplex, then $0 < R < 1$. The closer R is to one, the closer the proposed simplex comes to being inclusive. If we fail to find an inclusive simplex, we specify a minimal value that the ratio much equal or exceed in order to for a noninclusive simplex to be used. If we do not find such a noninclusive simplex with that large a ratio, we forecast using the E+1 closest E-dimensional points.

For example, suppose in Figure 2 that point z^2 did not exist (nor any other point that could be used to form an inclusive simplex). The simplex for the available points are z^1, z^{2*}, and z^3 has an $R = 0.5$. If we specify the minimal acceptance value of $R \leq 0.5$, say $R = 0.3$, then these three points (a noninclusive simplex) are chosen. If we choose a criterion of $R > 0.5$, then these points are not chosen, and instead we forecast using the three two-dimensional closest neighbors.

Step 6. Finally, we forecast by projecting the domain of the simplex into its range. That is, the forecasts are based on the "observed" historical time-series data. We follow the change of

the E+1 surrounding points μ-steps into the future, where the forecasted point is an exponentially weighted average of these neighbors:[7]

$$\hat{y}_{t+\mu} = \frac{\sum_j y^j_{t+\mu}\, e^{-d_j}}{\sum_j e^{-d_j}},$$

where j = 1, 2, ..., E+1 and d_j is the distance between the points.

III. Comparison Models

We compare our results to those from various data-based and economic-theory-based models reported by MZTT and CHO (see those papers for details). MZTT estimated two univariate linear models: an ARIMA(1, 1, 0) model and a modified version that adjusts for seasonality using a multiplicative seasonal ARMA(4, 4). They re-estimated the threshold autoregressive (TAR) model of Tong (1983), which is a version of the familiar tent-map and takes account of the unemployment rate's asymmetric cyclical behavior, where the unemployment rate increases at a faster rate than it decreases. They also re-estimated Hamilton's (1989, 1990) Markov switching autoregressive (MSA) model that captures asymmetric behavior using a hidden Markov process that switches between two autoregressive models.

In addition, MZTT estimated an "economic-theory-based" model, a bivariate autoregressive (AR) model, which uses initial claims of unemployment to predict unemployment rates. Presumably initial claims lead changes in the actual unemployment rate. We also compare our nonparametric model forecasts to those of several other models reported in MZTT (but that are not re-estimated there).[8] CHO develop an efficiency-wage theoretical framework in which they

[7] See Sugihara and May (1990) and Satchell and Timmermann (1995). It would be possible to use other weights instead, such as the linear weights in Fernandez-Rodriguez et al., forthcoming.

[8] The BVAR is a Bayesian vector autoregressive model with six variables and six quarterly lags estimated sequentially with data available in 1993. The Sims (1989) model is a nine-variable, five-lag BVAR model that allows time variation in coefficients and forecast error variance and

use real prices and real interest rates (in addition to lagged unemployment rates) to predict unemployment levels.

IV. Results

Using both the MSE and RMSE criteria (as MZTT suggested), we compare forecasts of the various models using both quarterly and monthly data. Aggregating monthly data into quarterly observations may exclude important information. As MZTT noted, models based on quarterly data implicitly assume that the aggregation of monthly data is linear. If the data were generated by a nonlinear process, the aggregation from monthly to quarterly process may result in an essential loss in information that will adversely affect our forecasts. Further, as MZTT observed, the reformulation of the standard quarterly models as monthly ones is problematic and most recent work is done in terms of quarterly forecasts. Consequently, they estimated only the ARIMA and bivariate ARMA models using monthly data. For comparison purposes, we estimated the nonparametric model using both quarterly and monthly data.

We generate out-of-sample forecasts for the same period as MZTT do with their parametric models. We also use their rolling forecast approach: Each model is estimated using the observations from the historical period, and then forecasts are generated for each period (forecasting origin) using these estimated coefficients. In Table 1, the historical period is the 83 quarters from 1948:1-1968:2. The rolling forecasts are then calculated for each origin beginning with 1968:3 and ending with 1993:3. MZTT ended in 1993:3 because the Labor Department changed its survey technique in the next quarter, which resulted in an increase in the unemployment rate by 0.5%.

Table 1 shows the relative MSE of forecasts from the various models, where the benchmark is the nonparametric model with an embedding dimension of four estimated using quarterly

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nonnormality in disturbances. The Michigan Research Seminar in Quantitative Economics (RSQE) macroeconomic forecasting model is estimated over a slightly different time period than are the other models. MZTT include this model in their presentation because it dominates a consensus forecast for long-term (forth and fifth quarters) forecast horizons.

data. The MSE of this benchmark model appears in the last row.[9] The MSEs of the other quarterly models are many times larger that those of our nonparametric model.

MZTT presented additional monthly forecasts for the ARIMA and bivariate ARMA model using extra information in the form of the actual unemployment rate data for the first month of the first quarter to be forecast.[10] Where their forecasts use this extra information and the nonparametric models do not, these time-series models forecast better than the nonparametric model in the first quarter, but not in subsequent quarters.

Finally, Table 1 also compares the forecasts of the various estimation models to a well-known, consensus forecast, the group median forecast of the Survey of Professional Forecasters (SPF), which is currently collected by the Federal Reserve Bank of Philadelphia. To improve the consensus forecasts, MZTT used only those forecasters who had participated in at least 10 surveys. The forecasters may have access to many estimated models and additional information. In particular, some forecasters had access to between zero and two months of data for the first quarter they forecast. Nonetheless, the quarterly nonparametric model dominates the SPF in all but the fourth and fifth forecasted quarters.

In Table 2, we compare the quarterly nonparametric model to other models in a different period using the RMSE criterion for the period 1968:3 through 1989:4.[11] The monthly ARIMA model in Table 2 reflects MZTT's assumption that the first month after the forecasting origin is known. MZTT concluded that "much of the improvement in the short-term forecasts of the SPF survey and the Michigan RSQE forecasting model over the simple univariate ARIMA model is

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[9] The mean value of the forecasted period is 6.5 with $\sigma = 1.53$. The forecast standard deviations for the first five periods in the future are .14, .31, .56, .74, and .74.

[10] We might expect that the parametric models based on monthly data to forecast better than those based on quarterly data because monthly data contain substantially more information than quarterly data for a given length of time. The results in Table 1 are not entirely consistent with this belief. MZTT noted that the use of monthly data results in substantial improvement in short-term forecasts in ARIMA and bivariate ARMA models, but not necessarily in longer-term forecasts.

[11] Except for the nonparametric model forecasts, these statistics are from MZTT, Table 7.

due to the incorporation of monthly information through forecasts of exogenous variables and constant adjustments of the forecasts." They further observed that the SPF dominates the forecasts of most of their other models in terms of RMSE and hence they suggested that one can use the SPF as "a proxy for a full-information (although not an optimal) forecast." The quarterly nonparametric model compares reasonably well using the RMSE criterion. The nonparametric models dominate the quarterly ARIMA, TAR, and SIMS in all quarters and the monthly ARIMA in all but the first quarter (where the monthly ARIMA uses extra information). The nonparametric model performs about as well as the SPF, the Bayesian vector autoregressive (BVAR) model (with six variables and six quarterly lags), and the Michigan Research Seminar in Quantitative Economics (RSQE) macroeconomic forecasting model (estimated over a slightly different time period), though, those models are slightly superior to the nonparametric model in several quarters. The nonparametric model dominates the Sims (1989) model (nine-variable, five-lag BVAR model that allows time variation in coefficients and forecast error variance and nonnormality in disturbances). Further, the nonparametric model has smaller RMSEs than do the monthly ARIMA and bivariate ARMA models based on the same amount of data.

The success of the nonlinear, nonparametric model may suggest that unemployment data are highly nonlinear and chaotic. To investigate this issue further, we compare the monthly nonparametric model (with an embedding dimension of eight, which outperform the nonparametric quarterly forecasts) to MZTT's monthly ARIMA(2, 0, 1)(12, 0, 12) in Figure 3. The forecasts of the nonparametric model have much lower MSEs than does the ARIMA model, and the nonparametric model's advantage grows the more months in the future is the forecast. For the first month, the MSE of the ARIMA model is 1.73 times larger than that of the nonparametric model. By the tenth month, the ARIMA's MSE is 2.53 times larger. However, as is expected with nonlinear data, the MSE's increase exponentially with the periods forecasted implying the existence of a positive Lyaponov exponent.

As one might expect, the monthly unemployment rate data have a much more complex structure than do the quarterly data. Monthly unemployment rates are extremely difficult to pre-

dict with parametric models due to the high dimensionality and highly nonlinear structure of the system generating these quantities. The nonparametric forecasts based on monthly data have relative RMSE's of 1.05, 0.92, and 0.72 for the first three quarters respectively (see Table 2). These results show the superiority of the monthly data forecasts when compared to the other models in Table 2.[12]

For the monthly data, we used Theil's U criterion to compare the forecast accuracy of the nonparametric and ARIMA models relative to a random walk. For the ARIMA model, Theil's U values are .88, .74, .77, .85, .87, and .86 for the first 5 periods of the forecast. The corresponding values are .63, .66, .69, .56, and .53 respectively for the nonparametric simplex model with E = 8.[13]

Finally, we compare forecasts based on our method to those from the Error Correction Model (ECM) in Carruth, Hooker and Oswald (1998). The ECM method uses information on real oil prices and real interest rates as well as information on lagged unemployment rates; whereas our nonparametric method uses only information on lagged unemployment rates.

CHO used the ECM to estimate quarterly data from 1954 through 1978 and then predicted unemployment rates through the end of their data set. When using the correct lagged un-

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[12] To make sure that there are no basic structural changes during the period investigated (the complete sample: 1948:1-1993:3), we re-estimated the forecasts using "in-sample" data. That is, the history and future groups are the same and include the whole sample. However, to avoid redundancy between forecast and correct points, we sequentially exclude points in the "history" library of points that are in the neighborhood of each predicted point. (This practice eliminates a large number of points and thereby reduces the quality of the forecasts.) As is predicted by the theory, this curve lies above the "out-of-sample" curve (since the in-sample curve uses less information) but it exhibits the same qualitative structure (Figure 3) and the same optimal embedding dimension of eight. This result implies that the same basic mechanism generates the monthly unemployment data over the entire period analyzed.

[13] The corresponding values are .97, .86, .95, .99, and 1.09 for the nonparametric simplex model with E = 7, and 4.07, 2.92, 2.55, 2.35, and 2.18 for the best nearest-neighbor model (e.g., Fernandez-Rodriguez and Sosvilla-Rivero forthcoming) with E = 2. As in Figure 3, the results show the relative superiority of the nonparametric-simplex model with E = 8 and demonstrate that the simplex approach dominates the "simple" nearest neighbor approach for these data.

employment rates, their method predicts well (CHO's Figure 2) except for a brief period of overshooting from late 1980 through early 1981. Using the same quarterly unemployment data (but ignoring the extra information that the ECM method employs), we forecasted unemployment rates over the same period (one step ahead forecasts) and compare our forecasts with the actual unemployment rates in Figure 4. Our nonparametric method performs well in general but undershoots during the 1982-1984 period of very high unemployment rates.

We also repeated CHO's experiment of using data from 1954:4-1990:4 to predict the recession of 1991-1992 (cf. CHO's Figure 3). In Figure 5, we compare our nonparametric estimates based on both quarterly and monthly data to those from CHO's ECM approach (where the averages of the real oil prices and interest rates over the 1991-1992 period were used). Our forecasts based on quarterly data (RMSE = 0.55 over the eight quarters) are comparable to those from the ECM (RMSE = 0.54). Our forecasts based on monthly data (RMSE = 0.18) substantially outperform those from the ECM method that is based on only quarterly data but that includes extra information about the real oil price and real interest rate.

V. Conclusions

Our results suggest that a relatively simple, nonlinear, nonparametric estimation method provides superior short-term and moderate-term forecasts of unemployment rates. Additional research is required to explain why the nonparametric model equals or dominates structural and other economic-theory models that use more information.

One possible explanation for the forecasting superiority of our highly nonlinear, nonparametric approach is that traditional, relatively simple time-series models as well as the more complex econometric models cannot capture the high dimensionality and very nonlinear structure of the true system. Economists do not know the exact dynamical structure generating the unemployment levels. Consequently, it is difficult to build reasonable structural models. The nonparametric approach does not require that we understand the structure exactly.

Further, due to the structure of the data, parametric studies usually work with the first difference of the data (to avoid unit root problems). For a highly nonlinear data set, taking differences may reduce the informational content of the data and thus affect the quality of the forecasts. This restriction does not apply to nonparametric models.

Appendix: Choosing the Embedding Dimension

The best embedding dimension for the quarterly nonparametric model is four, in the sense that we get the lowest MSE in the first quarter with $E = 4$. For example, the MSEs of forecasting one quarter into the future for the simplex approach are 0.27, 0.24, 0.08, 0.02, and 0.05 for $E = 1, 2, ..., 5$ respectively. [In contrast, the MSEs for the nearest neighbor approach are 1.01, 1.41, 1.87, 2.47, and 2.80, respectively.] Thus, using a lower or a higher value of E yields forecasts that are inferior to that of $E = 4$ even for one-period-into-the-future forecasts. Because E reflects the number of real variables or the autoregressive order, this result is consistent with the various estimators used by MZTT. Their ARIMA model is characterized by three parameters, while both the TAR and MSA models are characterized by six parameters each. Thus, they all are roughly similar in dimension to the nonparametric model. An embedding dimension of four means that the underlying attractor (where the dynamical system will "end up" eventually) has a dimension of about two because the (approximate) dimension of the attractor is smaller than $(E-1)/2$ (Sugihara and May, 1990; Agnon *et al.*, 1999).

For the monthly model, the optimal embedding dimension is seven or eight, which corresponds to a chaotic attractor of about three to four. Using the simplex method, the MSEs for one period forecast into the future are 0.12, 0.07, 0.06, 0.08, 0.07, 0.017, 0.018, and 0.02 for $E = 1, 2, ..., 10$, respectively. For the nearest-neighbor approach, the MSEs are 0.93, 0.63, 0.68, 0.79, 0.90, 1.04, 1.18, 1.36, 1.51, and 1.66, respectively. Again, the nearest neighbor approach has a much lower "optimal" embedding dimension, but is inferior to the simplex approach.

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Table 1
Relative MSEs of Forecasts from Various Models

	Steps Ahead				
	1	2	3	4	5
Quarterly Data					
Nonparametric	1	1	1	1	1
Seasonal ARIMA	3.81	3.20	1.91	1.55	1.41
ARIMA(1,1,0)	3.97	3.59	2.14	1.77	1.69
TAR	3.81	3.33	1.89	1.52	1.45
MSA	4.54	4.45	2.67	2.25	2.26
Bivariate AR	4.58	3.87	2.06	1.62	1.45
Monthly Data					
No Extra Data					
ARIMA	3.05	2.95	1.96	1.74	1.70
Bivariate ARMA	2.21	2.34	1.49	1.29	1.25
First Month Known					
ARIMA	.92	1.86	1.47	1.40	1.49
Bivariate ARMA	.88	2.02	1.43	1.26	1.22
Consensus Forecasts *(First or Second Month Known to Some)*					
SPF Median Forecast	1.24	1.64	1.11	.93	.91
MSE of Quarterly Nonparametric Model	.02	.10	.35	.73	1.10

Source: Forecasts for all but the nonparametric models are from MZTT, Tables 1 and 4. The nonparametric estimates were calculated for this paper.

Table 2
Relative RMSEs of Various Time Series and Econometrics Models

	Steps Ahead				
Model	1	2	3	4	5
Nonparametric (Quarterly data)	1.00	1.00	1.00	1.00	1.00
ARIMA (Quarterly data)	1.63	1.64	1.24	1.19	1.38
ARIMA (Monthly data, first month known)	.79	1.22	1.09	1.12	1.31
TAR (Quarterly data)	1.63	1.64	1.22	1.16	1.36
University of Michigan RSQE	.89	1.22	.87	.84	.98
BVAR	1.47	1.39	.96	.84	1.47
Sims	2.89	2.19	1.49	1.32	1.40
SPF median forecasts	.89	1.17	.93	.90	1.08
RMSE of the Quarterly Nonparametric Model	.19	.36	.69	.93	.95

Source: Forecasts for all but the nonparametric models are from MZTT, Table 7. The nonparametric forecasts were calculated for this paper.

Figure 1
An Example of a Minimum Volume Simplex

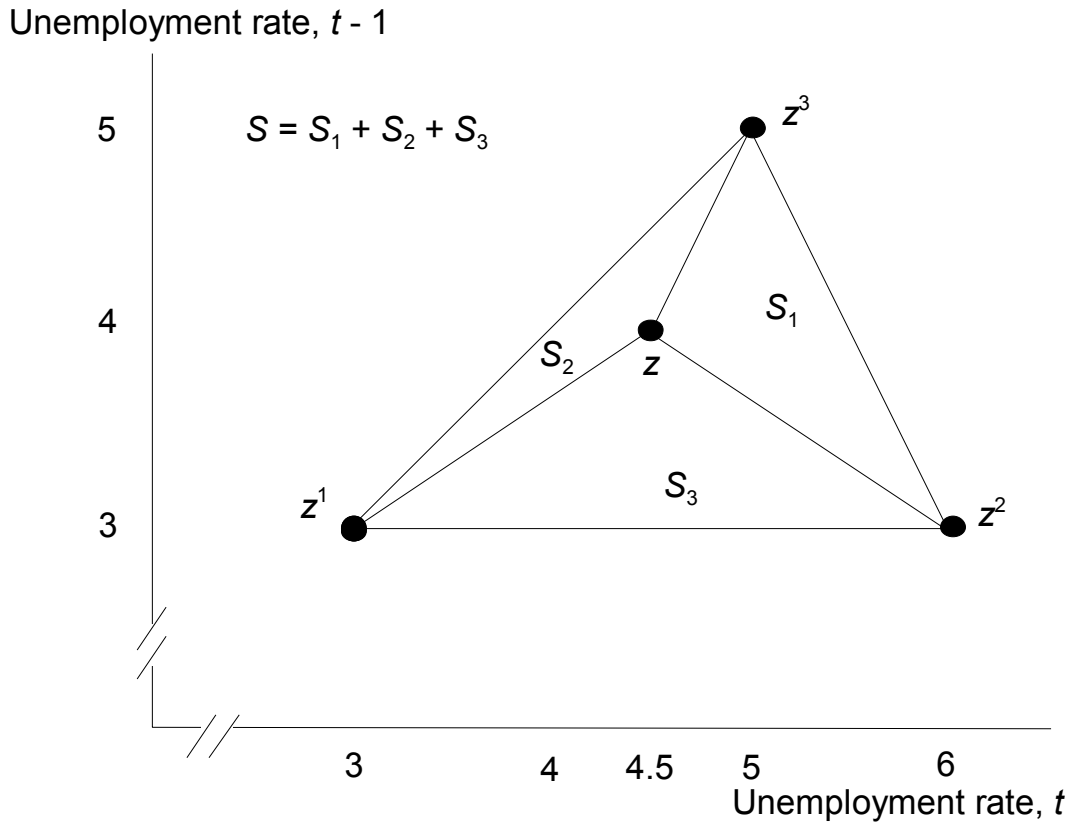

Figure 2
An Example of an Imperfect Simplex

Figure 3
MSEs for Monthly Forecasts

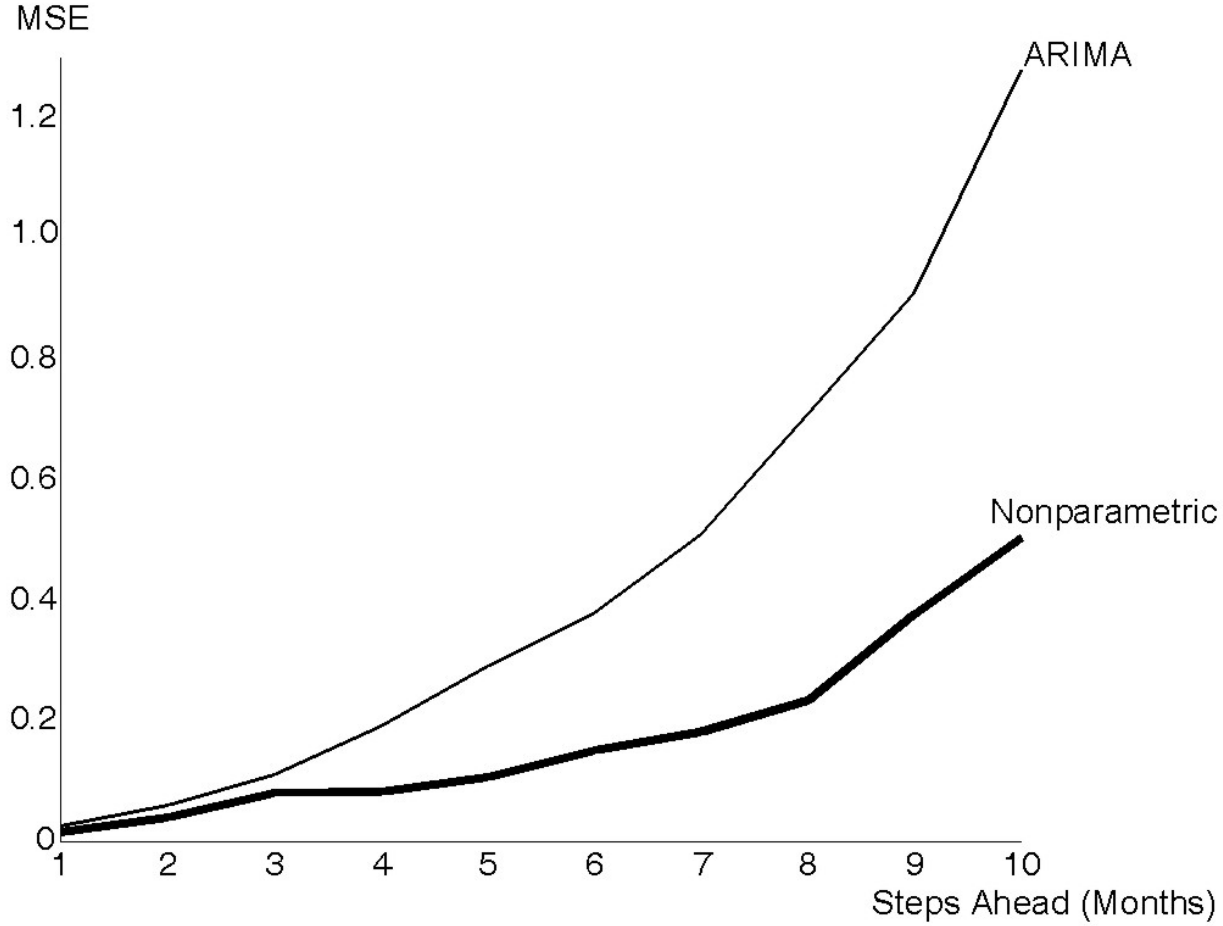

Figure 4
Comparison of Actual Unemployment Levels to Quarterly Nonparametric Forecasts 1979-1993

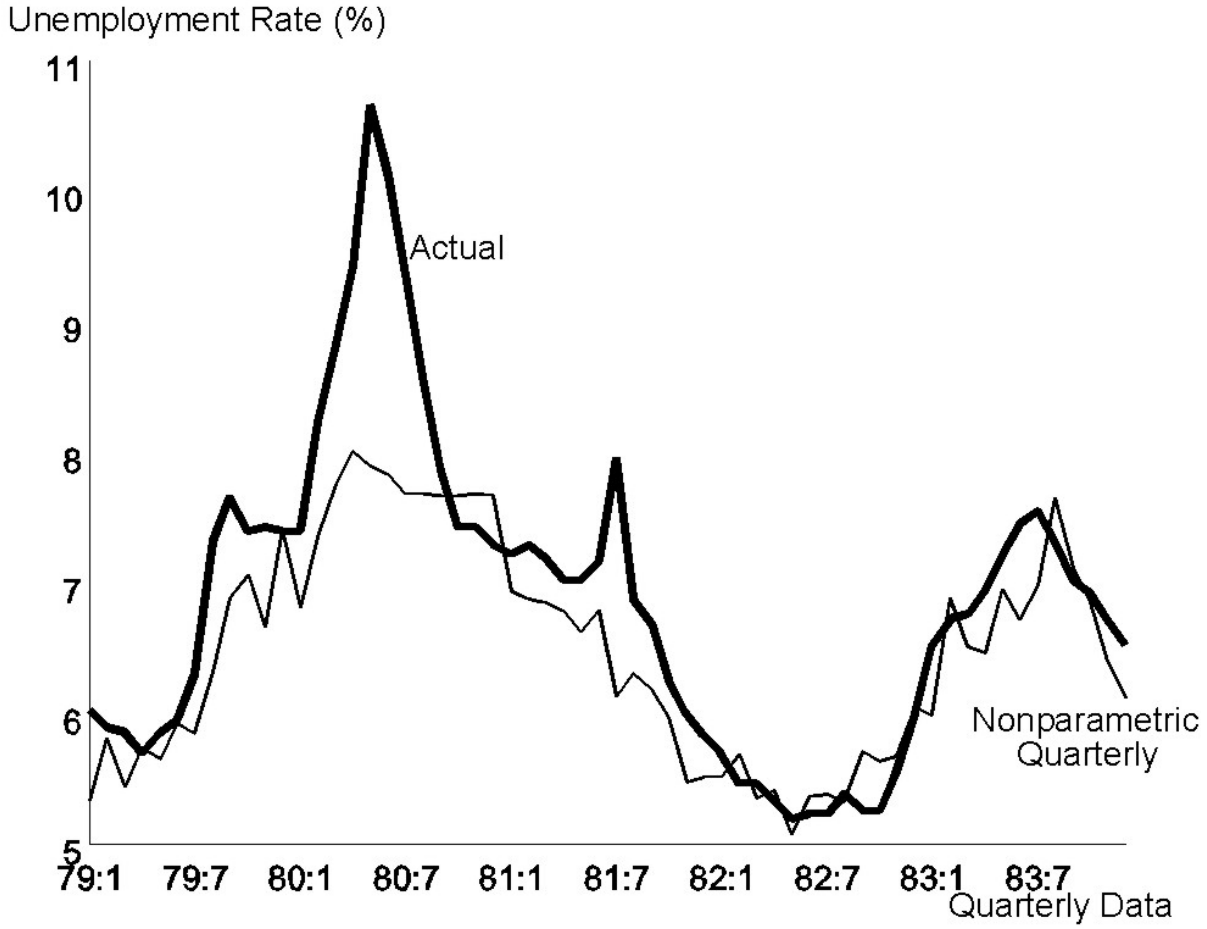
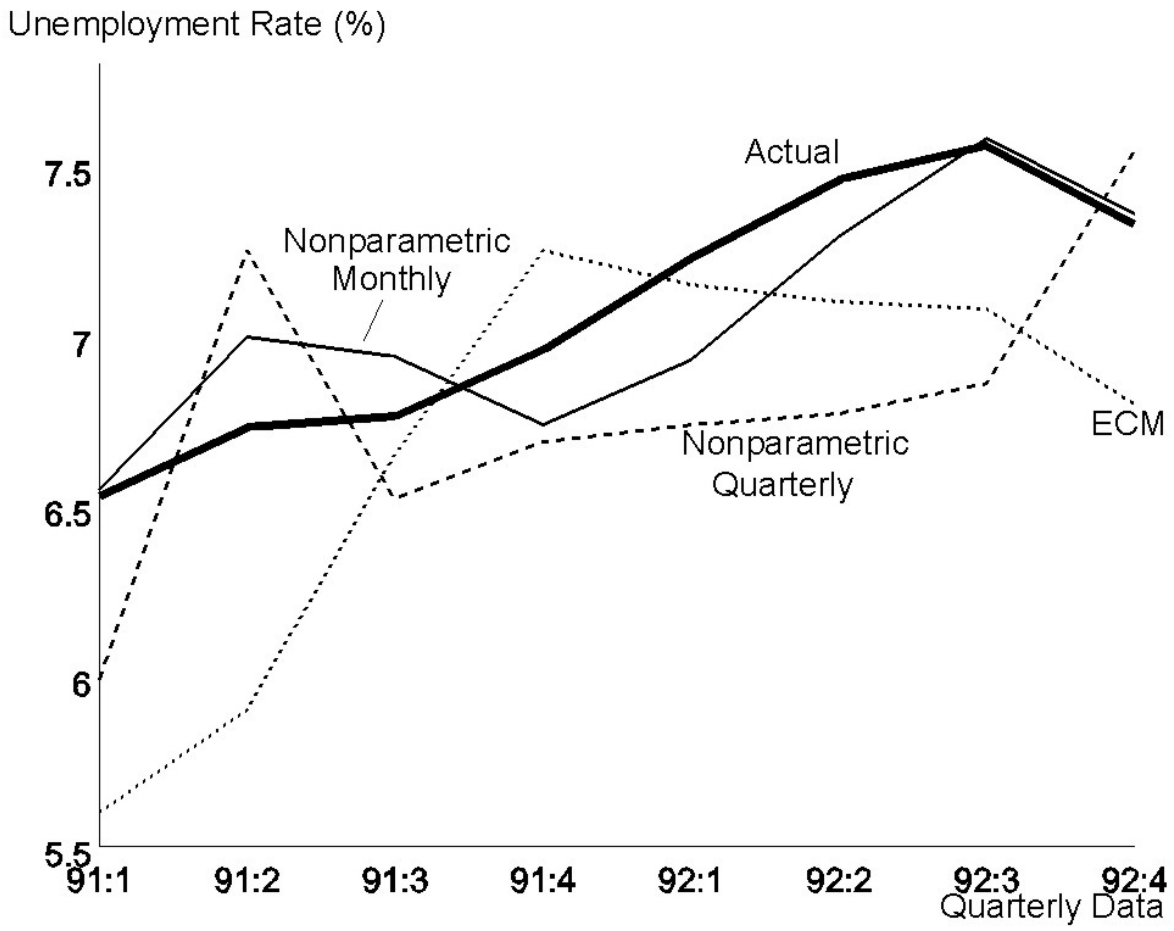
Figure 5
Comparison of Actual Unemployment Levels to Various Forecasts 1991-1992