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by

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Key Words: Demand, Generalized Method of Moments, Rational Random Errors, Weak Separability

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Generalized Rational Random Errors

1. Introduction

More than a quarter of a century ago, Henri Theil developed an economic theory for the second moments of disturbances in behavioral models, which he called *rational random errors* (Theil 1971, 1975, 1976). In models of consumer choice, the main result of this theory is that the covariance matrix of the error terms is proportional to the Slutsky matrix. Although this result has received little attention over the past two decades, it is known to be a sufficient condition for total expenditure to be *strictly exogenous* (Engle, Hendry, and Richard, 1983) in demand models (Deaton 1975, 1986; Theil 1976). In this paper, I show that a generalization of the theory of rational random errors, which I call *generalized rational random errors*, is necessary and sufficient for strict exogeneity of group expenditure in separable demand models. I derive a simple, robust, and asymptotically normal t-test for this hypothesis based on the generalized methods of moments principle. An application to per capita annual U.S. food demand in the 20th century strongly rejects the exogeneity of food expenditure in a model that in all other respects appears to be highly compatible with the data set and with economic theory.

2. The Econometric Model

Consider the demand equations

$$(1) \qquad \boldsymbol{x}_t = \boldsymbol{h}^x(\boldsymbol{p}_{xt}, \boldsymbol{p}_{yt}, m_t, \boldsymbol{s}_t) + \boldsymbol{\varepsilon}_t, \;\; t = 1, ..., T,$$

where $\boldsymbol{x}_t \in \mathbb{R}_+^{n_x}$ is an n_x–vector of quantities demanded for the goods of interest, $\boldsymbol{y}_t \in \mathbb{R}_+^{n_y}$ is an n_y–vector of quantities demanded for all other goods, $\boldsymbol{p}_{xt} \in \mathbb{R}_+^{n_x}$ is an n_x–vector of market prices for the goods $\boldsymbol{x}_t$, $\boldsymbol{p}_{yt} \in \mathbb{R}_+^{n_y}$ is an n_y–vector of market prices for the other goods, $m_t \equiv \boldsymbol{p}'_{xt}\boldsymbol{x}_t + \boldsymbol{p}'_{yt}\boldsymbol{y}_t$ is total expenditure (income for brevity), $\boldsymbol{s}_t \in \mathbb{R}^k$ is a *k*–vector of demographic variables and other demand shifters, and $\boldsymbol{\varepsilon}_t$ is a vector of stochastic error terms independently distributed across *t* with $E\left(\boldsymbol{\varepsilon}_t\right) = \boldsymbol{0}$ and $E\left(\boldsymbol{\varepsilon}_t\boldsymbol{\varepsilon}'_t\right) = \boldsymbol{\Sigma}_t$. The mean demands for the goods $\boldsymbol{x}$ given market prices, income, and demographic variables are

$$E(\boldsymbol{x}_t \mid \boldsymbol{p}_{xt}, \boldsymbol{p}_{yt}, m_t, \boldsymbol{s}_t) = \boldsymbol{h}^x(\boldsymbol{p}_{xt}, \boldsymbol{p}_{yt}, m_t, \boldsymbol{s}_t), \tag{2}$$

Although $\boldsymbol{\Sigma}_t$ may well be time varying and dependent on prices, income, and/or the demographic variables, the budget identity does not imply that $\boldsymbol{\Sigma}_t\boldsymbol{p}_{xt} = 0$; that is, the error covariance matrix is not singular.

Weak separability of $\boldsymbol{x}$ from $\boldsymbol{y}$ in the consumer's preference function is equivalent to the demands for $\boldsymbol{x}$ having the structure

$$\boldsymbol{x}_t = \tilde{\boldsymbol{h}}^x(\boldsymbol{p}_{xt}, \mu_x(\boldsymbol{p}_{xt}, \boldsymbol{p}_{yt}, m_t, \boldsymbol{s}_t), \boldsymbol{s}_t) + \boldsymbol{\varepsilon}_t, \tag{3}$$

where $\mu_{\boldsymbol{x}}(\boldsymbol{p}_{xt}, \boldsymbol{p}_{yt}, m_t, \boldsymbol{s}_t) \equiv \boldsymbol{p}'_{xt}\boldsymbol{h}^x(\boldsymbol{p}_{xt}, \boldsymbol{p}_{yt}, m_t, \boldsymbol{s}_t) \equiv E(\boldsymbol{p}'_{xt}\boldsymbol{x}_t \mid \boldsymbol{p}_{xt}, \boldsymbol{p}_{yt}, m_t, \boldsymbol{s}_t)$ is the mean total expenditure on the separable goods given all prices, income, and demographic variables (Gorman, 1970; Blackorby, Primont, and Russell, 1978). Therefore, assuming $\boldsymbol{x}$ is weakly separable from $\boldsymbol{y}$, and defining total expenditures on *x* by $m_{xt} \equiv \boldsymbol{p}'_{xt}\boldsymbol{x}_t$, we have

$$m_{xt} = \mu_x(\boldsymbol{p}_{xt}, \boldsymbol{p}_{yt}, m_t, \boldsymbol{s}_t) + \upsilon_t, \tag{4}$$

where $\upsilon_t \equiv \boldsymbol{p}'_{xt}\boldsymbol{\varepsilon}_t \sim (0, \boldsymbol{p}'_{xt}\boldsymbol{\Sigma}_t\boldsymbol{p}_{xt})$, independent across *t*.

Standard empirical practice is estimate the complete system of *conditional demands* for the separable goods with *observed* group expenditure on the right-hand-side,

$$\boldsymbol{x}_t = \tilde{\boldsymbol{h}}^x(\boldsymbol{p}_{xt}, m_{xt}, \boldsymbol{s}_t) + \tilde{\boldsymbol{\varepsilon}}_t, \tag{5}$$

where $\tilde{\varepsilon}_t \equiv \varepsilon_t + \tilde{\boldsymbol{h}}^x(\boldsymbol{p}_{xt}, \mu_x(\boldsymbol{p}_{xt}, \boldsymbol{p}_{yt}, m_t, \boldsymbol{s}_t), \boldsymbol{s}_t) - \tilde{\boldsymbol{h}}^x(\boldsymbol{p}_{xt}, m_{xt}, \boldsymbol{s}_t)$ is the vector of *conditional demand residuals*. A first step in understanding the empirical implications of this practice is lemma 1, which follows directly from Jensen's inequality and the adding up condition.

Lemma 1. *$E\left(\upsilon_t \mid \boldsymbol{p}_{xt}, \boldsymbol{p}_{yt}, m_t, \boldsymbol{s}_t\right) = 0$ and $E\left(\tilde{\varepsilon}_t \mid \boldsymbol{p}_{xt}, \boldsymbol{p}_{yt}, m_t, \boldsymbol{s}_t\right) = \mathbf{0}$ if and only if*

$$\frac{\partial \tilde{\boldsymbol{h}}^x\left(\boldsymbol{p}_{xt}, m_{xt}, \boldsymbol{s}_t\right)}{\partial m_x} \equiv \beta(\boldsymbol{p}_{xt}, \boldsymbol{s}_t) \ \exists\ \beta : \mathbb{R}^{n_x} \times \mathbb{R}^k \to \mathbb{R}^{n_x} \ \textit{satisfying}\ \boldsymbol{p}'_{xt}\beta(\boldsymbol{p}_{xt}, \boldsymbol{s}_t) \equiv 1.$$

The upshot is that a consistent stochastic specification between the conditional demands and group expenditure – equivalently, between conditional and unconditional demands – restricts the structure of the conditional demand model for precisely the same reason that exact aggregation in income does (Gorman, 1953, 1961).[1]

A consequence of lemma 1 is that the relationship between conditional and unconditional demand residuals can be written as

$$\tilde{\varepsilon}_t \equiv \left[\mathbf{I} - \beta(\boldsymbol{p}_{xt}, \boldsymbol{s}_t)\boldsymbol{p}'_{xt}\right]\varepsilon_t. \tag{6}$$

It follows from this and the definition of the group expenditure residual that group expenditure and the conditional demand residuals are generically correlated,

$$E\left(\tilde{\varepsilon}_t \upsilon_t \mid \boldsymbol{p}_{xt}, \boldsymbol{p}_{yt}, m_t, \boldsymbol{s}_t\right) = \left[\mathbf{I} - \beta(\boldsymbol{p}_{xr}, \boldsymbol{s}_t)\boldsymbol{p}'_{xt}\right]\boldsymbol{\Sigma}_t \boldsymbol{p}_{xt}. \tag{7}$$

This leads us directly to the following result.

Lemma 2. *If $\boldsymbol{\Sigma}_t \boldsymbol{p}_{xt} \neq \mathbf{0}$ then $E\left(\tilde{\varepsilon}_t \upsilon_t \mid \boldsymbol{p}_{xt}, \boldsymbol{p}_{yt}, m_t, \boldsymbol{s}_t\right) = \mathbf{0}$ if and only if*

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[1] It is always possible to modify the stochastic specification to have a model with, say, budget shares on the left-hand-side and nonlinear functions of expenditure on the right-hand-side. However, a result analogous to Lemma 1 also applies to such cases, and a coherent statistical model restricts our attention to at most rank two demand systems that are linear in a single nonlinear function of expenditure (Edgerton, 1993). Our focus is on conditional demand models linear in group expenditure.

$$\frac{\partial \tilde{\boldsymbol{h}}^{x}\left(\boldsymbol{p}_{xt}, \mu_{x}(\boldsymbol{p}_{xt}, \boldsymbol{p}_{yt}, m_{t}, \boldsymbol{s}_{t}), \boldsymbol{s}_{t}\right)}{\partial m_{x}} = \beta(\boldsymbol{p}_{xt}, \boldsymbol{s}_{t}) = \left(\boldsymbol{p}_{xt}' \boldsymbol{\Sigma}_{t} \boldsymbol{p}_{xt}\right)^{-1} \boldsymbol{\Sigma}_{t} \boldsymbol{p}_{xt}.$$

We call this restriction the *generalized rational random errors* hypothesis.

Given weak separability and lemma 1, the *conditional* covariance matrix satisfies

$$\tilde{\Sigma}_{t} = \left[\mathbf{I} - \beta(\boldsymbol{p}_{xt}, \boldsymbol{s}_{t}) \boldsymbol{p}_{xt}'\right] \Sigma_{t} \left[\mathbf{I} - \boldsymbol{p}_{xt} \beta(\boldsymbol{p}_{xt}, \boldsymbol{s}_{t})'\right]. \tag{8}$$

The adding up condition for the group of separable goods implies,

$$\boldsymbol{p}_{xt}' \tilde{\boldsymbol{h}}^{x}(\boldsymbol{p}_{xt}, \mu_{x}(\boldsymbol{p}_{xt}, \boldsymbol{p}_{yt}, m_{t}, \boldsymbol{s}_{t}), \boldsymbol{s}_{t}) \equiv \mu_{x}(\boldsymbol{p}_{xt}, \boldsymbol{p}_{yt}, m_{t}, \boldsymbol{s}_{t}), \tag{9}$$

which when combined with (8) and lemma 1 implies that the conditional covariance matrix is singular,

$$\tilde{\Sigma}_{t} \boldsymbol{p}_{xt} \equiv \left[\mathbf{I} - \boldsymbol{p}_{xt} \beta(\boldsymbol{p}_{xt}, \boldsymbol{s}_{t})'\right] \boldsymbol{p}_{xt} \equiv \boldsymbol{p}_{xt} - \boldsymbol{p}_{xt} \equiv \mathbf{0}. \tag{10}$$

The *unconditional* Slutsky matrix for $\boldsymbol{x}$ is defined by

$$\boldsymbol{S}_{t} = \frac{\partial \boldsymbol{h}^{x}(\boldsymbol{p}_{xt}, \boldsymbol{p}_{yt}, m_{t}, \boldsymbol{s}_{t})}{\partial \boldsymbol{p}_{x}'} + \frac{\partial \boldsymbol{h}^{x}(\boldsymbol{p}_{xt}, \boldsymbol{p}_{yt}, m_{t}, \boldsymbol{s}_{t})}{\partial m} \boldsymbol{h}^{x}(\boldsymbol{p}_{xt}, \boldsymbol{p}_{yt}, m_{t}, \boldsymbol{s}_{t})', \tag{11}$$

while the *conditional* Slutsky matrix for $\boldsymbol{x}$ is defined by

$$\tilde{\boldsymbol{S}}_{t} = \frac{\partial \tilde{\boldsymbol{h}}^{x}(\boldsymbol{p}_{xt}, \mu_{x}(\boldsymbol{p}_{xt}, \boldsymbol{p}_{yt}, m_{t}, \boldsymbol{s}_{t}), \boldsymbol{s}_{t})}{\partial \boldsymbol{p}_{x}'} \tag{12}$$

$$+ \frac{\partial \tilde{\boldsymbol{h}}^{x}(\boldsymbol{p}_{xt}, \mu_{x}(\boldsymbol{p}_{xt}, \boldsymbol{p}_{yt}, m_{t}, \boldsymbol{s}_{t}), \boldsymbol{s}_{t})}{\partial m_{x}} \tilde{\boldsymbol{h}}^{x}(\boldsymbol{p}_{xt}, \mu_{x}(\boldsymbol{p}_{xt}, \boldsymbol{p}_{yt}, m_{t}, \boldsymbol{s}_{t}), \boldsymbol{s}_{t})'.$$

The connection between the conditional and unconditional Slutsky matrices for the separable goods can be established with the following identities:

$$\frac{\partial \boldsymbol{h}^{x}(\boldsymbol{p}_{xt}, \boldsymbol{p}_{yt}, m_{t}, \boldsymbol{s}_{t})}{\partial \boldsymbol{p}_{x}'} \equiv \frac{\partial \tilde{\boldsymbol{h}}^{x}(\boldsymbol{p}_{xt}, \mu_{xt}(\boldsymbol{p}_{xt}, \boldsymbol{p}_{yt}, m_{t}, \boldsymbol{s}_{t}), \boldsymbol{s}_{t})}{\partial \boldsymbol{p}_{x}'} \tag{13}$$

$$+\frac{\partial \tilde{\boldsymbol{h}}^x(\boldsymbol{p}_{xt}, \mu_x(\boldsymbol{p}_{xt}, \boldsymbol{p}_{yt}, m_t, \boldsymbol{s}_t), \boldsymbol{s}_t)}{\partial m_x} \frac{\partial \mu_x(\boldsymbol{p}_{xt}, \boldsymbol{p}_{yt}, m_t, \boldsymbol{s}_t)}{\partial \boldsymbol{p}'_x};$$

(14) $$\frac{\partial \boldsymbol{h}^x(\boldsymbol{p}_{xt}, \boldsymbol{p}_{yt}, m_t, \boldsymbol{s}_t)}{\partial m} \equiv \frac{\partial \tilde{\boldsymbol{h}}^x(\boldsymbol{p}_{xt}, \mu_x(\boldsymbol{p}_{xt}, \boldsymbol{p}_{yt}, m_t, \boldsymbol{s}_t), \boldsymbol{s}_t)}{\partial m_x} \frac{\partial \mu_x(\boldsymbol{p}_{xt}, \boldsymbol{p}_{yt}, m_t, \boldsymbol{s}_t)}{\partial m};$$

(15) $$\frac{\partial \mu_{xt}(\boldsymbol{p}_{xt}, \boldsymbol{p}_{yt}, m_t, \boldsymbol{s}_t)}{\partial \boldsymbol{p}_{xt}} \equiv \boldsymbol{h}^x(\boldsymbol{p}_{xt}, \boldsymbol{p}_{yt}, m_t, \boldsymbol{s}_t) + \frac{\partial \boldsymbol{h}^x(\boldsymbol{p}_{xt}, \boldsymbol{p}_{yt}, m_t, \boldsymbol{s}_t)'}{\partial \boldsymbol{p}_{xt}} \boldsymbol{p}_{xt};$$

(16) $$\frac{\partial \mu_x(\boldsymbol{p}_{xt}, \boldsymbol{p}_{yt}, m_t, \boldsymbol{s}_t)}{\partial m} \equiv \boldsymbol{p}'_{xt} \frac{\partial \boldsymbol{h}^x(\boldsymbol{p}_{xt}, \boldsymbol{p}_{yt}, m_t, \boldsymbol{s}_t)}{\partial m};$$

(17) $$\boldsymbol{p}'_{xt} \frac{\partial \tilde{\boldsymbol{h}}^x(\boldsymbol{p}_{xt}, \mu_x(\boldsymbol{p}_{xt}, \boldsymbol{p}_{yt}, m_t, \boldsymbol{s}_t), \boldsymbol{s}_t)}{\partial m_x} \equiv 1;$$

and

(18) $$\boldsymbol{h}^x(\boldsymbol{p}_{xt}, \boldsymbol{p}_{yt}, m_t, \boldsymbol{s}_t) \equiv \boldsymbol{h}^x(\boldsymbol{p}_{xt}, \mu_x(\boldsymbol{p}_{xt}, \boldsymbol{p}_{yt}, m_t, \boldsymbol{s}_t), \boldsymbol{s}_t).$$

We make the following sequence of substitutions,

(19) $$\boldsymbol{S}_t \equiv \frac{\partial \boldsymbol{h}^x}{\partial \boldsymbol{p}'_x} + \frac{\partial \boldsymbol{h}^x}{\partial m}(\boldsymbol{h}^x)'$$

$$\equiv \frac{\partial \tilde{\boldsymbol{h}}^x}{\partial \boldsymbol{p}'_x} + \frac{\partial \tilde{\boldsymbol{h}}^x}{\partial m_x}\left(\frac{\partial \mu_x}{\partial \boldsymbol{p}'_x}\right) + \frac{\partial \tilde{\boldsymbol{h}}^x}{\partial m_x}\left(\frac{\partial \mu_x}{\partial m}\right)(\boldsymbol{h}^x)'$$

$$\equiv \frac{\partial \tilde{\boldsymbol{h}}^x}{\partial \boldsymbol{p}'_x} + \frac{\partial \tilde{\boldsymbol{h}}^x}{\partial m_x}\left((\tilde{\boldsymbol{h}}^x)' + \boldsymbol{p}'_{xt}\frac{\partial \boldsymbol{h}^x}{\partial \boldsymbol{p}'_x}\right) + \frac{\partial \tilde{\boldsymbol{h}}^x}{\partial m_x}\left(\boldsymbol{p}'_{xt}\frac{\partial \boldsymbol{h}^x}{\partial m}\right)(\boldsymbol{h}^x)'$$

$$\equiv \tilde{\boldsymbol{S}}_t + \left(\frac{\partial \tilde{\boldsymbol{h}}^x}{\partial m_x}\boldsymbol{p}'_{xt}\right)\boldsymbol{S}_t.$$

and then solve for $\tilde{\boldsymbol{S}}_t$ to obtain

(20) $$\tilde{\boldsymbol{S}}_t \equiv [\boldsymbol{I} - \beta(\boldsymbol{p}_{xt}, \boldsymbol{s}_t)\boldsymbol{p}'_{xt}]\boldsymbol{S}_t.$$

Since $\tilde{\boldsymbol{S}}_t \boldsymbol{p}_{xt} \equiv \boldsymbol{0}$, equivalently, $\boldsymbol{p}'_{xt}\beta(\boldsymbol{p}_{xt}, \boldsymbol{s}_t) \equiv 1$, this implies

$$\tilde{\boldsymbol{S}}_t \equiv \left[\boldsymbol{I} - \beta(\boldsymbol{p}_{xt}, \boldsymbol{s}_t)\boldsymbol{p}'_{xt}\right]\boldsymbol{S}_t\left[\boldsymbol{I} - \boldsymbol{p}_x\beta(\boldsymbol{p}_{xt}, \boldsymbol{s}_t)'\right]. \tag{21}$$

Note that $(\boldsymbol{\Sigma}_t, \tilde{\boldsymbol{\Sigma}}_t)$ and $(\boldsymbol{S}_t, \tilde{\boldsymbol{S}}_t)$ share precisely the same structural connection from the unconditional to the conditional level of the demand model. Moreover, note from (19) and $\tilde{\boldsymbol{S}}_t \boldsymbol{p}_{xt} \equiv \boldsymbol{0}$ that, as long as $\boldsymbol{S}_t \boldsymbol{p}_{xt} \neq \boldsymbol{0}$, we have the identity

$$\frac{\partial \tilde{\boldsymbol{h}}^x(\boldsymbol{p}_{xt}, \mu_x(\boldsymbol{p}_{xt}, \boldsymbol{p}_{yt}, m_t, \boldsymbol{s}_t), \boldsymbol{s}_t)}{\partial m_x} \equiv \left(\boldsymbol{p}'_{xt}\boldsymbol{S}_t\boldsymbol{p}_{xt}\right)^{-1}\boldsymbol{S}_t\boldsymbol{p}_{xt}. \tag{22}$$

Therefore, defining the positive-valued function,

$$\varphi(\boldsymbol{p}_{xt}, \boldsymbol{p}_{yt}, m_t, \boldsymbol{s}_t) \equiv -\left(\boldsymbol{p}'_{xt}\boldsymbol{\Sigma}_t\boldsymbol{p}_{xt}\right)^{-1}\left(\boldsymbol{p}'_{xt}\boldsymbol{S}_t\boldsymbol{p}_{xt}\right), \tag{23}$$

we can restate the GRREH in the following way.

Lemma 3. *If* $\boldsymbol{\Sigma}_t \boldsymbol{p}_{xt} \neq \boldsymbol{0}$, *then* $E\left(\tilde{\varepsilon}_t \upsilon_t \mid \boldsymbol{p}_{xt}, \boldsymbol{p}_{yt}, m_t, \boldsymbol{s}_t\right) = \boldsymbol{0}$ *if and only if*

$\boldsymbol{\Sigma}_t \boldsymbol{p}_{xt} \equiv -\varphi_t \boldsymbol{S}_t \boldsymbol{p}_{xt}$.

Lemma 3 establishes the relationship between *generalized rational random errors* and *rational random errors*. In particular, the *rational random errors* hypothesis implies that the unconditional covariance matrix and the Slutsky matrix are related by $\boldsymbol{\Sigma}_t \equiv -\varphi_t \boldsymbol{S}_t$ for some function $\varphi_t\colon \mathbb{R}_+^{n+k+1} \to \mathbb{R}_+$, where $n = n_x + n_y$. *Rational random errors* is sufficient, but not necessary for the *generalized rational random errors* hypothesis. As a counterexample to necessity, let $\boldsymbol{\Omega}_t$ be the Hessian matrix for any function $g(\boldsymbol{p}_{xt}, \boldsymbol{p}_{yt}, m_t, \boldsymbol{s}_t)$ that is 1° homogeneous and convex in $\boldsymbol{p}_{xt}$, and let $\boldsymbol{\Sigma}_t \equiv \boldsymbol{\Omega}_t - \varphi \boldsymbol{S}_t$.

3. The Hypothesis and Test Statistic

In principle, the *generalized rational random errors* hypothesis is testable. The remainder of this section presents a derivation of a simple, consistent *t*–test for this purpose. Suppose that the *generalized rational random errors* hypothesis is true. Then

$$\boldsymbol{\varepsilon}_t \boldsymbol{\varepsilon}_t' \boldsymbol{p}_{xt} = \boldsymbol{\varepsilon}_t \upsilon_t = -\varphi_t \boldsymbol{S}_t \boldsymbol{p}_{xt} + \boldsymbol{u}_t, \tag{24}$$

where $E(\boldsymbol{u}_t) = \boldsymbol{0}$, $E(\boldsymbol{u}_t \boldsymbol{u}_t') = \boldsymbol{\Phi}_t$, say, and the $\boldsymbol{u}_t$ are independently distributed across *t*. Therefore, let $\boldsymbol{z}_t = \boldsymbol{\varepsilon}_t \upsilon_t$, $w_{it} = \sum_{j=1}^{n_x} s_{ijt} p_{x_j t}$, and $\boldsymbol{w_t} = [w_{1t} \quad \cdots \quad w_{n_x t}]'$. An unbiased estimator for φ_t is obtained by ordinary least squares (OLS),

$$\hat{\varphi}_t = -\left(\boldsymbol{w}_t' \boldsymbol{w}_t\right)^{-1} \boldsymbol{w}_t' \boldsymbol{z}_t . \tag{25}$$

In practice, $\boldsymbol{z}_t$ and $\boldsymbol{w}_t$ are not observed, but consistent estimates can be readily obtained. This does not alter any of the asymptotic results that follow, and for notational brevity this minor issue is ignored. For each *t* the OLS errors are

$$\hat{\boldsymbol{u}}_t = \left[\boldsymbol{I} - \boldsymbol{w}_t \left(\boldsymbol{w}_t' \boldsymbol{w}_t\right)^{-1} \boldsymbol{w}_t'\right] \boldsymbol{u}_t = \mathbb{M}_t \boldsymbol{u}_t, \tag{26}$$

with $E\left(\hat{\boldsymbol{u}}_t \hat{\boldsymbol{u}}_t'\right) = \mathbb{M}_t \boldsymbol{\Phi}_t \mathbb{M}_t$. The average OLS residual within each time period is defined by $\bar{\hat{u}}_{\cdot t} = \frac{1}{n_x} \boldsymbol{\iota}_{n_x}' \hat{\boldsymbol{u}}_t$, where $\boldsymbol{\iota}_{n_x} = [1 \quad \cdots \quad 1]'$, with variance

$$E\left(\bar{\hat{u}}_{\cdot t}^2\right) = \tfrac{1}{n_x^2} \boldsymbol{\iota}_{n_x}' \mathbb{M}_t \boldsymbol{\Phi}_t \mathbb{M}_t \boldsymbol{\iota}_{n_x} . \tag{27}$$

The overall average residual is $\bar{\hat{u}} = \frac{1}{T}[\bar{\hat{u}}_{\cdot 1} \cdots \bar{\hat{u}}_{\cdot T}]\boldsymbol{\iota}_T$, with variance

$$E\left(\bar{\hat{u}}^2\right) = \left(\tfrac{1}{n_x T}\right)^2 \sum_{t=1}^{T} \boldsymbol{\iota}_{n_x}' \mathbb{M}_t \boldsymbol{\Phi}_t \mathbb{M}_t \boldsymbol{\iota}_{n_x} . \tag{28}$$

Under standard assumptions for the error terms u_{it}, a robust herteroscedasticity-consistent estimator for the variance term in (20) is (Hansen, 1982; White, 1980)

(29) $$\hat{\sigma}^2_{\bar{\hat{u}}} = \left(\tfrac{1}{n_x T}\right)^2 \sum_{t=1}^{T} \iota'_{n_x} \mathbb{M}_t \hat{\boldsymbol{u}}_t \hat{\boldsymbol{u}}'_t \mathbb{M}_t \iota_{n_x} .$$

However, since $\hat{\boldsymbol{u}}_t = \mathbb{M}_t \boldsymbol{u}_t$ and $\mathbb{M}_t \mathbb{M}_t = \mathbb{M}_t = \mathbb{M}'_t$, this estimator simplifies to

(30) $$\hat{\sigma}^2_{\bar{\hat{u}}} = \left(\tfrac{1}{n_x T}\right)^2 \sum_{t=1}^{T} \iota'_{n_x} \hat{\boldsymbol{u}}_t \hat{\boldsymbol{u}}'_t \iota_{n_x}$$

$$= \tfrac{1}{T^2} \sum_{t=1}^{T} \left(\tfrac{1}{n_x} \iota'_{n_x} \hat{\boldsymbol{u}}_t\right)^2$$

$$= \tfrac{1}{T^2} \sum_{t=1}^{T} (\bar{\hat{u}}_{\cdot t})^2$$

Therefore, restating the *generalized rational random errors* hypothesis as

$$H_{\text{o}}:\ E(u_{it}) = 0\ \forall\ i = 1,\ldots,n_x,\ \forall\ t = 1,\ldots,T\ ,$$

$$H_A:\ \exists\ i,t \text{ such that } E(u_{it}) \neq 0\, ,$$

a simple t–statistic that has a standard normal asymptotic distribution is

(31) $$t = \frac{T\bar{\hat{u}}}{\sqrt{\sum_{t=1}^{T}\left(\bar{\hat{u}}_{\cdot t}\right)^2}} .$$

3. An Application to U.S. Food Demand

Let y be a scalar representing nonfood expenditures, let $\pi(\boldsymbol{p}_y)$ be an increasing, 1° homogeneous, concave function of nonfood prices, and assume a quadratic (quasi-)utility function (LaFrance and Hanemann) for foods and nonfood expenditures,

(32) $$u(\boldsymbol{x}, y, \boldsymbol{s}) = (\boldsymbol{x} - \boldsymbol{\alpha}_{\boldsymbol{x}}(\boldsymbol{s}))' \boldsymbol{B}_{\boldsymbol{xx}} (\boldsymbol{x} - \boldsymbol{\alpha}_{\boldsymbol{x}}(\boldsymbol{s})) + \beta_{yy}(y - \alpha_y(\boldsymbol{s}))^2$$

$$+ 2(\boldsymbol{x} - \boldsymbol{\alpha}_{\boldsymbol{x}}(\boldsymbol{s}))' \boldsymbol{\beta}_{\boldsymbol{x}y}(y - \alpha_y(\boldsymbol{s})) .$$

Maximizing $u(\boldsymbol{x}, y, \boldsymbol{s})$ with respect to $(\boldsymbol{x}, y)$ subject to $\boldsymbol{p}'_{\boldsymbol{x}}\boldsymbol{x} + \pi(\boldsymbol{p}_y)y \leq m$, gives the unconditional demands for $\boldsymbol{x}$ as

$$\text{(33)} \qquad \boldsymbol{h}^x(\boldsymbol{p}_x, \pi(\boldsymbol{p}_y), m, \boldsymbol{s}) = \alpha_x(\boldsymbol{s})$$

$$+\left(\frac{m - \alpha_x(\boldsymbol{s})'\boldsymbol{p}_x - \alpha_y(\boldsymbol{s})\pi(\boldsymbol{p}_y)}{\boldsymbol{p}_x'\boldsymbol{C}_{xx}\boldsymbol{p}_x + 2\boldsymbol{p}_x'\gamma_{xy}\pi(\boldsymbol{p}_y) + \gamma_{yy}\pi(\boldsymbol{p}_y)^2}\right)\cdot\left(\boldsymbol{C}_{xx}\boldsymbol{p}_x + \gamma_{xy}\pi(\boldsymbol{p}_y)\right),$$

where $\boldsymbol{C} = \begin{bmatrix} \boldsymbol{C}_{xx} & \gamma_{xy} \\ \gamma_{xy}' & \gamma_{yy} \end{bmatrix} = \begin{bmatrix} \boldsymbol{B}_{xx} & \beta_{xy} \\ \beta_{xy}' & \beta_{yy} \end{bmatrix}^{-1}$.

Our empirical model specifies deflated expenditures, rather than quantities, as dependent variables (Brown and Walker, 1989),

$$\text{(34)} \qquad \boldsymbol{e}_x \equiv \boldsymbol{P}_x\boldsymbol{x} = \boldsymbol{P}_x\alpha_x(\mathbf{s}) + \left(\frac{m - \alpha_x(\mathbf{s})'\boldsymbol{p}_x - \alpha_y(\boldsymbol{s})}{\boldsymbol{p}_x'\boldsymbol{C}_{xx}\boldsymbol{p}_x + 2\gamma_{xy}'\boldsymbol{p}_x + \gamma_{yy}}\right)\boldsymbol{P}_x\left(\boldsymbol{C}_{xx}\boldsymbol{p}_x + \gamma_{xy}\right) + \varepsilon_x,$$

where m and $\boldsymbol{p}_x$ are deflated by π and $\boldsymbol{P}_x \equiv \mathrm{diag}(p_{xi})$. Adding up implies $\iota'\varepsilon_x + \varepsilon_y \equiv 0$, where ι is an n_x-vector of ones and ε_y is the residual for nonfood expenditure. The estimation method is nonlinear seemingly unrelated regression equations (SURE) with one iteration on the residual covariance matrix. This produces consistent, efficient, and asymptotically normal parameter estimates under standard conditions (Malinvaud; Rothenberg and Leenders), while avoiding a spurious over fit of a subset of equations, which can result from iterative SURE methods.[2]

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[2] The reason for this can be seen by writing the estimated covariance matrix, say $\hat{\boldsymbol{\Sigma}}$, at a given iteration in factored form as $\hat{\boldsymbol{\Sigma}} = \boldsymbol{Q}\Delta\boldsymbol{Q}'$, where $\boldsymbol{QQ}' = \boldsymbol{Q}'\boldsymbol{Q} = \boldsymbol{I}$, and $\Delta = \mathrm{diag}(\delta_i)$ is the diagonal matrix of eigen values. If any of the δ_i is small relative to all others, then while $\hat{\boldsymbol{\Sigma}}^{-1}$ is held fixed during the next iteration on the structural parameters, the linear combination of the ε_t's associated with that eigen value carries a large relative weight in the sum of squares criterion. This linear combination of residuals can approach a perfect fit, leading to singularity of the estimated covariance matrix.

Weak separability of foods from nonfood expenditures, which is necessary and sufficient for separability of foods from all other goods (LaFrance and Hanemann), implies $\gamma_{xy} = 0$. The Slutsky matrix for food is

$$(35) \qquad \boldsymbol{S}_{xx} = \left(\frac{m - \alpha_x(\mathbf{s})'\boldsymbol{p}_x - \alpha_y(\boldsymbol{s})\pi}{\boldsymbol{p}_x'\boldsymbol{C}_{xx}\boldsymbol{p}_x + 2\gamma_{xy}'\boldsymbol{p}_x\pi_x + \pi^2} \right) \left[\boldsymbol{C}_{xx}' - \left(\frac{(\boldsymbol{C}_{xx}\boldsymbol{p}_x + \gamma_{xy}\pi)(\boldsymbol{p}_x'\boldsymbol{C}_{xx}' + \gamma_{xy}\pi)}{\boldsymbol{p}_x'\boldsymbol{C}_{xx}\boldsymbol{p}_x + 2\gamma_{xy}'\boldsymbol{p}_x\pi + \pi^2} \right) \right].$$

Symmetry is accommodated by $\frac{1}{2}n_x(n_x\text{-}1)$ linear parameter restrictions on $\boldsymbol{C}_{\boldsymbol{xx}}$. Quasi-concavity of preferences in $(\boldsymbol{x}, y)$ implies that at least n_x Eigen values of $\boldsymbol{C}$ must be positive (Lau, 1978). In this model, however, weak separability implies that preferences are additively separable in $\boldsymbol{x}$ and y. Quasi-concavity then requires concavity either in $\boldsymbol{x}$ or in y (Gorman 1995b).[3] Treating foods and other goods symmetrically implies that all Eigen values of $\boldsymbol{C}_{xx}$ must be positive. Hence, let $\boldsymbol{C}_{xx} = \boldsymbol{LL}'$, $\boldsymbol{L}$ lower triangular. These parameter restrictions ensure *global weak integrability* (LaFrance and Hanemann, 1989).

$\boldsymbol{L}$ can have reduced rank unless the symmetric estimate of $\boldsymbol{C}_{\boldsymbol{xx}}$ is positive definite. In such a case, curvature is not binding. In the alternative situation where $\boldsymbol{L}$ has a reduced rank, say $n_x\text{-}g$, $0 \le g \le n_x$, all elements on and below the last g diagonal elements will vanish. This gives the greatest number of independent parameters associated with a symmetric, positive semidefinite matrix $\boldsymbol{C}_{\boldsymbol{xx}}$ that has rank $n_x\text{-}g$ (Diewert and Wales,

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[3] This can be demonstrated as follows. Quasi-concavity requires

$$[d\boldsymbol{x}' \quad dy]\begin{bmatrix} u_{xx} & \boldsymbol{0} \\ \boldsymbol{0}' & u_{yy} \end{bmatrix}\begin{bmatrix} d\boldsymbol{x} \\ dy \end{bmatrix} \le 0 \;\; \forall \begin{bmatrix} d\boldsymbol{x} \\ dy \end{bmatrix} \ne \begin{bmatrix} \boldsymbol{0} \\ 0 \end{bmatrix} \ni [d\boldsymbol{x}' \quad dy]\begin{bmatrix} u_{\boldsymbol{x}} \\ u_y \end{bmatrix} = 0 .$$

Setting $dy = 0$ implies that $d\boldsymbol{x}'u_{\boldsymbol{xx}}d\boldsymbol{x} \le 0 \;\forall\; d\boldsymbol{x}'u_{\boldsymbol{x}} = 0$, so that the sectoral utility function for foods must be quasi-concave. But if $u_{\boldsymbol{xx}}$ is indefinite (has a positive eigen value) and $u_{yy} > 0$, the sign condition fails for joint quasi-concavity of u in $(\boldsymbol{x}, y)$.

1993), and ½$g(g$+1) restrictions for curvature and ½$n_x(n_x$–1) symmetry restrictions.

The food consumption data consist of 21 equations and 76 annual time series observations over the period 1918-1994. [4] The sample period is 1919-41 and 1947-94, with 1942-1946 excluded to account for World War II. The unrestricted model has 615 parameters; there are 210 parameter restrictions associated with symmetry of $\boldsymbol{C_{xx}}$; and 238 restrictions associated with symmetry and positive semidefiniteness of $\boldsymbol{C_{xx}}$.[5] Per capita consumption of twenty-one food items and corresponding average retail prices for those items were constructed several USDA and Bureau of Labor Statistics sources. The consumer price index for all nonfood items is used for the "price" of nonfood expenditures. Income is measured as per capita disposable personal income. Demographics include the first three moments (mean, variance, and skewness) of the age distribution for the U.S. population and proportions of the population that are Black and neither White nor Black. Habit formation is incorporated through including lagged food quantities elements of $\boldsymbol{s}$ (Pollak and Wales, 1981).

A summary of model specification tests is reported in Table 1. A test of separability in the unrestricted model falls right on the margin of rejection at a 5 percent significance level. Although this warrants further consideration, separability of foods from nonfoods is maintained. No model version shows significant evidence of misspecification using either systemwide or single equation stability tests. The unrestricted model fails to re-

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[4] Detailed descriptions of the data, its sources and construction methods, and data set are available from the author on request.

[5] The bottom 7 rows of $\boldsymbol{L}$ equal zero, the number of negative Eigen values of $\boldsymbol{C_{xx}}$ in the symmetric model, generating 28 parameter restrictions for curvature.

ject at the 5 percent significance level, while the symmetric and quasi-concave models fail to reject at the 10 percent level.[6]

Symmetry is not rejected at the 5 percent level of significance, while symmetry and quasi-concavity is not rejected at the 10 percent level. Neither the symmetric nor the quasi-concave model shows evidence of autocorrelation, while the unrestricted model suggests a low level of negative serial correlation (ρ = -.15). It is noteworthy that high levels of serial correlation are common in time series demand models, especially in those that are been restricted to satisfy Slutsky symmetry. This model shows little evidence of serial correlation and even less as economic restrictions are imposed. There also is little evidence of skewness in the residuals. All three versions of the model show evidence of leptokurtosis. Since the estimation and inference methods employed here are robust to thick tails as long as the fourth moments exist, this is not a serious concern. In addition, neither restricted model shows evidence of thicker tails than the unrestricted model.[7]

Tests of *generalized rational random errors*, equivalently, of *strict exogeneity* of food expenditure (Engle, Hendry, and Richard, 1983; Hendry, 1995), are rejected at any reasonable level of significance in all models. The implication is that food expenditure is correlated with the conditional error terms in all three versions of the model. The stan-

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[6] The systemwide test statistics denoted by $B_T(z)$ in table 1 are asymptotically distributed as Brownian bridges, and are constructed from partial sums of the transformed regression errors, $\hat{\boldsymbol{\Sigma}}^{-1/2}\hat{\boldsymbol{\varepsilon}}_t$. See LaFrance (1999) for derivation and a discussion. Also, see Ploberger and Krämer (1992) for the derivation of linear single equation versions of the conditional means tests.

[7] The point estimate for the coefficient of excess kurtosis in the unrestricted model falls well within a 95 percent confidence interval of the corresponding estimate for the quasi-concave model. In other words, the parameter restrictions associated with symmetry and jointly with symmetry and quasi-concavity do not appear to create spurious outliers in the data.

dard practice of using observed group expenditure on the right-hand-side of a system of conditional demands is clearly inappropriate for this model and data set. LaFrance (1991) has shown that group expenditure can *never* be *weak*, *strong*, or *super exogenous* (see Engle, Hendry and Richard (1983) for derivation and discussion). Thus, no concept of *exogeneity* can rationalize food expenditure as an exogenous right-hand-side variable in a conditional demand model of U.S. per capita food consumption.

Table 2 reports equation summary statistics for the *incomplete demand system* form of the symmetric, quasi-concave, separable model (see LaFrance (1985) or LaFrance and Hanemann (1989) for derivation and discussion). Table 3 presents the estimated parameters associated with the constant terms, demographics, and lagged quantities consumed, with estimated asymptotic standard errors in parentheses below the parameter estimates. Table 4 presents the parameter estimates for the food sector's negative inverse Hessian, with estimated asymptotic standard errors in parentheses below the coefficient estimates. It is worth emphasizing that this model *globally* satisfies the restrictions implied by economic theory.

4. Implications

The rational random errors hypothesis is sufficient for strict exogeneity of group expenditure in separable demand models. This paper derives the generalized rational random errors hypothesis as a necessary and sufficient condition for strict exogeneity of group expenditure in separable demand models. A simple, robust, asymptotically normal t-test of this hypothesis is derived based on the generalized method of moments.

An empirical application is made with per capita annual U.S. food demand in the 20^{th} century. An *incomplete demand systems* approach is used, which does not rely on any form of exogeneity of group expenditure to obtain consistent, asymptotically efficient parameter estimates. Extensive destructive testing a la Hendry (1995) suggests that, in all respects, the empirical model is compatible with the data and economic theory.

Tests of *generalized rational random errors*, however, are rejected at all reasonable levels of significance in all versions of the empirical model. The implication is that food expenditure is correlated with the error terms in a conditional demand model for U.S. food demand. Using group expenditure on the right-hand-side of a system of conditional demands, for this model and data set at least, would lead to biased and inconsistent parameter estimates and invalid statistical inferences. Food expenditure cannot be *weakly*, *strongly*, *super* or *strictly exogenous*, exhausting the logical possibilities. Treating group expenditure as exogenous in separable demand models is a flawed empirical practice.

References

Blackorby, C., D. Primont and R.R. Russell. *Duality, Separability and Functional Structure: Theory and Economic Applications*, New York: North-Holland, 1978.

Brown, B. W. and M. B. Walker. "The Random Utility Hypothesis and Inference in Demand Systems." *Econometrica* 57 (1989): 815-829.

Deaton, A. *Models and Projections of Demand in Post War Britain* London: Chapman and Hall, 1975.

_________."Demand Analysis." In *Handbook of Econometrics*, Vol. III, Eds. Z. Griliches and M. Intriligator. New York: North-Holland, 1986:1767-1839.

Diewert, W. E. and T. J. Wales. "Flexible Functional Forms and Global Curvature Conditions." *Econometrica* 55 (1987): 43-68.

Edgerton, D.L. "On the Estimation of Separable Demand Models." *Journal of Agricultural and Resource Economics* 18 (1993): 141-146.

Engle, R. F., D. F. Hendry, and J.F. Richard. "Exogeneity." *Econometrica* 51 (1983): 277-304.

Gorman, W.M. "Community Preference Fields." *Econometrica* 21 (1953): 63-80.

____________. "On a Class of Preference Fields." *Metroeconomica* 13 (1961): 53-56.

____________. "Two-Stage Budgeting." Unpublished typescript, University of North Carolina, 1970. Reprinted as Chapter 2 in C. Blackorby and T. Shorrocks, Eds. *Separability and Aggregation: Collected Works of W. M. Gorman*, Oxford: Clarendon Press, 1995a.

____________. "The Concavity of Additive Utility Functions." Unpublished typescript, University of North Carolina, October 1970. Reprinted as Chapter 9 in C. Blackorby and T. Shorrocks, Eds. *Separability and Aggregation: Collected Works of W. M. Gorman*, Oxford: Clarendon Press, 1995b.

Hansen, L. P. "Large Sample Properties of Generalized Methods of Moments Estimators." *Econometrica* 50 (1982): 1029-1054.

Hendry, D. F. *Dynamic Econometrics* Oxford: Oxford University Press, 1995.

LaFrance, J. T. "Linear Demand Functions in Theory and Practice." *Journal of Economic Theory* 37 (1985): 147-166.

____________. "When is Expenditure 'Exogenous' in Separable Demand Models?" *Western Journal of Agricultural Economics* 16 (1991):49-62.

____________. "The Structure of U.S. Food Demand." Working Paper No. 862, Department of Agricultural and Resource Economics, University of California, Berkeley, CA, 1999.

LaFrance, J.T. and W.M. Hanemann. "The Dual Structure of Incomplete Demand Systems." *American Journal of Agricultural Economics* 71 (1989):262-274.

Lau, L. J. "Testing and Imposing Monotonicity, Convexity and Quasi-convexity Constraints." Appendix A.4 in *Production Economics: A Dual Approach to Theory and Applications*, M. Fuss and D. McFadden (eds.), Amsterdam: North-Holland Publishing Co., 1978: 409-455.

Malinvaud, E. *Statistical Methods of Economics*, Second edition, Amsterdam: North-Holland, 1980.

Ploberger, W. and W. Krämer. "The CUSUM Test with OLS Residuals." *Econometrica*

60 (1992): 271-286.

Pollak, R.A. and T.J. Wales. “Demographic Variables in Demand Analysis.” *Econometrica* 49 (1981):1533-1551.

Rothenberg, T.J. and C.T. Leenders. “Efficient Estimation of Simultaneous Equation Systems.” *Econometrica* 32 (1964):57-76.

Theil, H. "An Economic Theory of the Second Moments of Disturbances of Behavioral Equations." *American Economic Review* 61 (1971): 190-194.

Theil, H. "The Theory of Rational Random Behavior and its Application to Demand Analysis." *European Economic Review* 6 (1975): 217-226.

Theil, H. *Theory and Measurement of Consumer Demand*, Volume II, Amsterdam: North-Holland, 1976.

White, H. “A Heteroskedasticity-Consistent Covariance Matrix Estimator and a Direct Test for Heteroskedasticity.” *Econometrica* 48 (1980): 817-38.

Table 1. Model Diagnostics; Sample Period 1919-41 and 1947-94.

	Unrestricted	Symmetric	Quasiconcave
Trace	1415.7	1228.1	1249.3
ρ	-.135	-.039	-.027
σ_ρ	.027	.029	.029
t_ρ	5.02	1.35	0.94
η_3	.147	.045	.066
σ_{η_3}	.063	.063	.063
t_{η_3}	2.31	0.71	1.05
η_4	.451	.675	.628
σ_{η_4}	.127	.127	.127
t_{η_4}	3.55	5.32	4.94
Jarque-Bera	17.96	28.80	25.51
P-value	1.3×10^{-4}	5.6×10^{-7}	2.9×10^{-6}
GRREH Tests			
$\bar{\hat{u}}$	1.624	5.150	5.061
$\sigma_{\bar{\hat{u}}}$	.343	1.364	1.332
$t_{\bar{\hat{u}}}$	4.739	3.776	3.800
P-value	1.1×10^{-5}	8.0×10^{-5}	7.2×10^{-5}
F-Tests			
Separability	1.56		
P-value	.05		
Theory	-	1.18	1.12
P-value	-	.06	.12
Systemwide Specification Tests			
1st Moment			
$\max\lvert B_T(z)\rvert$	.41	.42	.47
P-value	.996	.995	.98
2nd Moment			
$\max\lvert B_T(z)\rvert$	1.36	1.22	1.06
P-value	.05	.10	.22

ρ is the common first order autocorrelation coefficient; η_3 is the coefficient of skewness; η_4 is the coefficient of excess kurtosis; and Jarque-Bera the $\chi^2(2)$ test for normality.

Table 2. Single Equation Statistics; Symmetric, Quasi-Concave Model.

	$\mathbf{R^2}$	$\sqrt{T}\,\bar{\hat{\varepsilon}}_i / \hat{\boldsymbol{\sigma}}_i$	$\max\limits_{0 \le z \le 1} \lvert B_{iT}(z) \rvert$
Milk & Cream	.9973	.122	.326
Butter	.9965	-.164	.463
Cheese	.9983	-.026	.529
Frozen Dairy Products	.9877	-.190	.402
Other Dairy Products	.9867	.073	.507
Beef & Veal	.9951	-.058	.438
Pork	.9747	.043	.561
Other Meat	.9590	.032	.380
Fish	.9949	.148	.447
Poultry	.9893	.171	.607
Fresh Citrus Fruit	.6717	.301	.728
Fresh Noncitrus Fruit	.9487	-.297	.560
Fresh Vegetables	.9882	-.137	.346
Potatoes	.9648	.240	.807
Processed Fruit	.9882	-.020	.518
Processed Vegetables	.9891	-.124	.426
Fats & Oils	.9737	-.124	.394
Eggs	.9989	-.240	.473
Cereal Products	.9889	-.082	.413
Sugar	.9878	-.243	.478
Coffee, Tea, & Cocoa	.9803	-.242	.493

Table 3. Demographics and Habits; Symmetric, Quasi-Concave Model.

		Age Distribution			Ethnicity		Habits
	Constant	Mean	Variance	Skewness	Black	Others	x_{t-1}
Milk & Cream	374.1 (79.38)	-2.277 (2.419)	3.334 (0.673)	-.7492 (.7256)	-20.42 (13.43)	-3.503 (9.101)	.3680 (.0577)
Butter	4.975 (13.61)	.0268 (.2576)	-.2941 (.0917)	-.0263 (.0785)	1.222 (1.922)	-2.2417 (1.160)	.7394 (.0840)
Cheese	-16.21 (11.65)	.6015 (.3331)	-.1178 (.0798)	.0795 (.0846)	.2766 (1.883)	3.090 (1.322)	.5023 (.1090)
Frozen Dairy	-39.11 (27.78)	.0238 (.7482)	.8168 (.2791)	.0291 (.1956)	1.036 (4.214)	.7565 (2.674)	.3924 (.1204)
Other Dairy	34.47 (24.15)	-.2323 (.7751)	1.097 (.2870)	-.4906 (.1816)	-3.843 (4.511)	.8026 (2.492)	.3123 (.1345)
Beef & Veal	-377.7 (29.42)	1.801 (.8655)	1.859 (.2144)	-.0224 (.2424)	31.75 (5.089)	-21.30 (3.395)	.0206 (.0471)
Pork	151.0 (27.13)	.9419 (.8654)	.9444 (.2288)	.1261 (.2402)	-16.34 (4.947)	5.227 (3.285)	.0758 (.0396)
Other Meat	27.33 (13.13)	.1009 (.4196)	-.0149 (.1134)	.0907 (.1117)	-1.812 (2.419)	-.0203 (1.563)	.0727 (.1251)
Fish	42.94 (12.23)	.2894 (.3341)	-.1963 (.0812)	.1513 (.0904)	-4.272 (1.988)	5.653 (1.348)	.2578 (.0856)
Poultry	31.00 (20.66)	.0496 (.5240)	.2493 (.1646)	.0662 (.1441)	-3.797 (3.321)	12.92 (2.863)	.5027 (.0753)
Fresh Citrus	69.05 (40.34)	6.657 (1.444)	-.3189 (.3063)	.1289 (.3339)	-22.90 (7.486)	6.247 (4.749)	-.0509 (.0933)
Fresh Non-citrus	1060.5 (97.33)	-4.393 (2.474)	-4.086 (.6862)	.5838 (.6709)	-67.74 (15.08)	59.81 (10.52)	-.4825 (.0756)
Fresh Veges	221.1 (50.57)	7.054 (1.554)	.3274 (.3485)	1.508 (.3852)	-45.84 (8.965)	34.11 (5.937)	.1745 (.0929)
Potatoes	575.1 (99.17)	-9.599 (2.806)	-2.288 (.6742)	.0084 (.7207)	-5.856 (15.78)	18.17 (9.994)	-.0283 (.0943)

Numbers in parentheses are estimated asymptotic standard errors.

Table 3. Continued.

		Age Distribution			Ethnicity		Habits
	Constant	**Mean**	**Variance**	**Skewness**	**Black**	**Others**	x_{t-1}
Processed Fruit	-210.4 (41.89)	3.128 (1.062)	1.248 (.2697)	.2809 (.2948)	7.200 (6.499)	2.189 (4.287)	.2783 (.0753)
Processed Veges	41.94 (44.17)	7.004 (1.455)	-.3176 (.3406)	1.803 (.3598)	-28.89 (7.324)	20.99 (4.813)	.3156 (.0677)
Fats & Oils	22.08 (23.39)	3.297 (.7035)	-.2674 (.1852)	.9019 (.1940)	-12.80 (4.065)	15.66 (2.999)	.2192 (.0790)
Eggs	54.11 (16.58)	-.7929 (.4156)	.3898 (.1562)	-.1679 (.1098)	-2.399 (2.652)	-.4565 (1.757)	.7207 (.0631)
Flour & Cereals	1074.9 (125.8)	-9.290 (2.631)	-4.503 (.7121)	.1861 (.6382)	-47.94 (13.74)	51.32 (9.412)	.2835 (.0881)
Sugar & Sweeteners	186.8 (53.46)	6.610 (1.738)	-2.381 (.3701)	1.791 (.4986)	-26.13 (10.19)	24.17 (6.890)	.0388 (.0589)
Coffee, Tea & Cocoa	22.33 (9.056)	.7490 (.3007)	.2174 (.0716)	-.0055 (.0781)	-4.127 (1.662)	1.595 (1.109)	.2142 (.0600)
Nonfood	-4017.5 (1238.0)	317.5 (38.21)	14.67 (11.42)	88.94 (9.828)	-907.7 (185.4)	1273.5 (139.8)	--- ---

Numbers in parentheses are estimated asymptotic standard errors.

Table 4. Food Sector Negative Inverse Hessian; Symmetric, Quasi-concave Specification.

	Milk & Cream	Butter	Cheese	Frozen Dairy	Other Dairy	Beef & Veal	Pork	Other Meat	Fish	Poultry
Milk & Cream	.721									
	(.129)									
Butter	.00716	.00511								
	(.00646)	(.00103)								
Cheese	.00105	-.00102	.00447							
	(.0101)	(.00087)	(.00160)							
Frozen Dairy	-.0659	-.00235	-.00699	.142						
	(.0391)	(.00281)	(.00469)	(.0302)						
Other Dairy	-.119	-.00456	-.00697	.0125	.0704					
	(.0403)	(.00290)	(.00415)	(.0189)	(.0241)					
Beef & Veal	-.0279	.00250	-.00369	.00694	.0107	.0617				
	(.0104)	(.00096)	(.00135)	(.00402)	(.00371)	(.00564)				
Pork	.0100	-.00167	-.00499	.00742	.00764	-.0195	.0904			
	(.0127)	(.00140)	(.00185)	(.00506)	(.00472)	(.00312)	(.00834)			
Other Meat	.0305	-.00131	-.00071	-.0103	-.0137	-.0147	-.0106	.0376		
	(.0162)	(.00110)	(.00185)	(.00700)	(.00654)	(.00237)	(.00252)	(.00512)		
Fish	.0211	-.00271	.00418	-.00179	-.00536	-.00310	-.00680	-.00051	.00610	
	(.0086)	(.00081)	(.00111)	(.00428)	(.00383)	(.00124)	(.00176)	(.00176)	(.00139)	
Poultry	-.0664	.00033	.00258	-.0133	.00167	-.00550	-.00449	.00220	.00197	.0209
	(.0138)	(.00140)	(.00189)	(.00542)	(.00567)	(.00210)	(.00310)	(.00267)	(.00172)	(.00347)
Fresh Citrus	.00608	-.00022	.00063	-.00152	-.00075	-.00152	-.00837	.00103	.00086	-.00972
	(.0211)	(.00251)	(.00255)	(.00751)	(.00698)	(.00466)	(.00605)	(.00328)	(.00207)	(.00556)

Numbers in parentheses are estimated asymptotic standard errors.

Table 4. Continued.

	Milk & Cream	Butter	Cheese	Frozen Dairy	Other Dairy	Beef & Veal	Pork	Other Meat	Fish	Poultry
Fresh Non-Citrus	-.0934 (.0502)	-.0113 (.00492)	.00422 (.00692)	-.0704 (.0241)	.0192 (.0192)	-.0197 (.00988)	-.0248 (.0131)	-.0230 (.00968)	.00820 (.00651)	.0207 (.00951)
Fresh Veges	.0445 (.0367)	-.0109 (.00340)	.00033 (.00436)	.0256 (.0163)	.0155 (.0153)	-.00150 (.00576)	.0127 (.00783)	-.00893 (.00796)	.00771 (.00407)	-.00440 (.00653)
Potatoes	-.0212 (.0487)	.00887 (.00533)	-.00313 (.00704)	-.0178 (.0162)	.0130 (.0169)	-.00574 (.00945)	-.00284 (.0125)	.00138 (.00794)	-.00754 (.00647)	-.0110 (.0119)
Processed Fruit	-.00756 (.0160)	-.00406 (.00163)	.00048 (.00223)	-.0196 (.00701)	-.00406 (.00612)	.00118 (.00344)	.00687 (.00436)	-.00545 (.00277)	-.00045 (.00202)	-.00567 (.00335)
Processed Veges	.0130 (.0406)	.00394 (.00300)	.00169 (.00464)	.0445 (.0187)	-.00252 (.0176)	-.0236 (.00507)	-.0217 (.00650)	.0117 (.00748)	.0101 (.00415)	.0209 (.00641)
Fats & Oils	.0177 (.0171)	-.00329 (.00156)	.00930 (.00212)	-.0209 (.00894)	-.0177 (.00735)	-.0145 (.00283)	-.0109 (.00378)	.00763 (.00330)	.00867 (.00204)	.00758 (.00302)
Eggs	.0290 (.0146)	.00056 (.00129)	-.00172 (.00188)	-.00687 (.00598)	.00433 (.00611)	-.00116 (.00175)	.00478 (.00278)	-.00685 (.00270)	-.00321 (.00164)	-.00959 (.00260)
Flour & Cereals	-.256 (.0991)	.00083 (.00663)	-.00320 (.00918)	.0245 (.0404)	.0221 (.0360)	-.0293 (.0108)	-.0226 (.0135)	.00462 (.0172)	-.00690 (.00838)	.0363 (.0134)
Sugar & Sweeteners	-.0334 (.0270)	.00740 (.00266)	.00339 (.00352)	.00452 (.00777)	.0186 (.00819)	-.0179 (.00542)	-.00985 (.00662)	.00534 (.00479)	-.00156 (.00347)	.0153 (.00534)
Coffee, Tea & Cocoa	-.00406 (.00288)	-.00024 (.00035)	-.00075 (.00039)	.00231 (.00112)	.00185 (.00102)	.00016 (.00074)	.00179 (.00092)	.00031 (.00048)	-.00051 (.00035)	.00022 (.00095)

Numbers in parentheses are estimated asymptotic standard errors.

Table 4. Continued.

	Fresh Citrus	Fresh Noncitrus	Fresh Veges	Potatoes	Processed Fruit	Processed Veges	Fats & Oils	Eggs	Flour & Cereals	Sugar & Sweeteners	Coffee, Tea & Cocoa
Fresh Citrus	.0448 (.0106)										
Fresh Non-citrus	.00325 (.0210)	.260 (.0586)									
Fresh Veges	-.0176 (.0108)	.0199 (.0288)	.0636 (.0211)								
Potatoes	.0276 (.0218)	.0485 (.0495)	-.0534 (.0325)	.372 (.0747)							
Processed Fruit	.0200 (.00786)	.0198 (.0157)	-.0166 (.00954)	.0132 (.0171)	.0419 (.00811)						
Processed Veges	-.0111 (.00945)	-.00378 (.0241)	.00816 (.0169)	-.0132 (.0232)	-.0288 (.00900)	.124 (.0270)					
Fats & Oils	.00411 (.00570)	.0181 (.0125)	-.00275 (.00835)	.00822 (.0126)	.00650 (.00390)	.00454 (.00864)	.0244 (.00565)				
Eggs	-.00071 (.00477)	.0225 (.00898)	.0105 (.00689)	-.0296 (.00952)	.00128 (.00302)	-.0208 (.00654)	-.00563 (.00304)	.0244 (.00397)			
Flour & Cereals	-.0167 (.0186)	.161 (.0502)	-.00179 (.0332)	.00087 (.0501)	-.0218 (.0169)	.0419 (.0377)	.00410 (.0166)	.0206 (.0147)	.276 (.110)		
Sugar & Sweeteners	-.0152 (.0109)	-.0543 (.0219)	-.00448 (.0161)	-.0521 (.0265)	-.0128 (.00890)	.0393 (.0121)	.00670 (.00538)	.00812 (.00419)	.0247 (.0273)	.119 (.0194)	
Coffee, Tea & Cocoa	.00534 (.00189)	-.00465 (.00363)	.00370 (.00194)	.00534 (.00350)	-.00076 (.00141)	-.00287 (.00133)	-.00087 (.00084)	-.00129 (.000637)	-.00416 (.00281)	.00048 (.00179)	.00415 (.00051)

Numbers in parentheses are estimated asymptotic standard errors.