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Taxes versus Quotas for a Stock Pollutant

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Abstract

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We compare the effects of taxes and quotas for an environmental problem in which the regulator and polluter have asymmetric information about abatement costs, and the environmental damage depends on the stock of pollution. We thus extend, to a dynamic framework, previous studies in which environmental damages depend on the flow of pollution. As with the static analysis, an increase in the slope of the marginal abatement cost curve, or a decrease in the slope of the marginal damage curve, favors taxes. In addition, in the dynamic model, an increase in the discount rate or the stock decay rate favor the use of taxes. Taxes certainly dominate quotas if the length of a period during which decisions are constant is sufficiently small. An empirical illustration suggests that taxes dominate quotas for the control of greenhouse gasses.

Keywords: Pollution control, asymmetric information, taxes and quotas, stochastic control

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1. Introduction

Asymmetric information plays an important role in environmental regulation when the polluter knows more than the regulator about the abatement cost function. In this situation, the first-best optimum can seldom be reached by using emission taxes or quotas. The first-best optimum equates the marginal abatement costs of the pollutants and the marginal environmental damage.

Weitzman (1974) compared the expected payoff, under asymmetric information, for taxes and quotas. He assumed linear marginal costs, uncertainty only about the level of the marginal cost curves (not their slopes) and no correlation between the uncertainty of the abatement cost and the environmental cost. Under these assumptions, an emissions tax dominates a quota if and only if the marginal abatement cost curve is steeper than the marginal environmental cost curve.

Subsequent contributions to this topic fall into two categories: (a) modifying the assumptions in Weitzman's analysis[1], and (b) considering policy tools other than an emission tax and a direct specification of the emission level[2]. More complex policies can reduce the potential loss in social welfare associated with asymmetric information about abatement costs. However, in practice, policy-makers have not used these more sophisticated methods of environmental regulation.

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[1] See e.g. Yohe (1977), Watson and Ridker (1984),Malcomson (1978), and Stavins (1996). Tisato and Allen (1993) compare taxes and marketable permits.

[2] See e.g. Dasgupta et al. (1980), Kwerel (1977), McKitrick (1997), Roberts and Spence (1976).

This earlier literature assumes that the environmental damage comes from the *flow* of emissions. However, for several important environmental problems, damages depend on the *stock,* and not the flow, of the pollution. Examples of such problems include climate change (due to atmospheric concentration of greenhouse gases), depletion of the ozone layer (due to cumulative emissions of CFCs), deterioration of soil and water quality (due to acid rain), and the pollution of rivers, lakes and oceans from emissions of heavy metals.

We revisit the problem originally posed by Weitzman, replacing the flow pollutant with a stock pollutant. We assume that the cost shocks are serially uncorrelated. Section 2 presents the basic model. In sections 3 and 4 we analyze the case in which the entire trajectory of the emissions tax or quota must be determined at the initial time (the open-loop policy). Section 5 studies the opposite extreme, where the quota or tax can be adjusted in light of new information (the feedback policy). We obtain the comparative statics of the policy ranking with respect to the slopes of marginal abatement costs and marginal damages, the discount rate, and the decay rate of the stock. We also emphasize the effect, on the policy ranking, of the "length of a stage", defined as the amount of time during which decisions are constant. Section 6 provides an empirical illustration, which suggests that taxes dominate quotas for the control of greenhouse gasses.

Staring (1995) obtained the criterion for the policy ranking in the open-loop setting, without recognizing the importance of the length of a stage. The current paper was first distributed as a FEEM working paper in May 1998. In the intervening 3.5 years several related papers have been written.

Shortly after our paper was distributed (but working independently), Newell and

Pizer (1998) generalized the open-loop setting by considering serially correlated cost shocks, but ignoring the importance of length of a stage. Under the assumption that the regulator's priors equal their stationary values, they reproduce the comparative statics results that we obtain in Section 4, and they generalized these by showing that more positively correlated shocks favor quotas. A later version of their paper provides an empirical application, reproducing the conclusions that we report in Section 6.

The inclusion of serial correlation greatly complicates analysis of the feedback (but not the open-loop) setting, because it requires a two-dimensional state variable. Arguably, serial correlation is most interesting in the feedback setting, where the regulator has the opportunity to learn about the current cost shock by using previous observations of the firm's response to taxes, or equilibrium quota prices. In the open-loop setting, the regulator cannot update information and therefore cannot use the correlation to learn about the shock. In our view, serially correlated shocks are not central to the issue of stock regulation with asymmetric information. In the interest of obtaining clear results and maintaining focus, we decided not to include serial correlation.

A later paper, (Karp and Zhang (1999)) generalizes the feedback model here (by considering serially correlated shocks) and generalizes the open-loop model in Newell and Pizer (by allowing arbitrary priors). The policy comparisons under the feedback structure are less clear, because of the complexities caused by the additional state variable. However, the more general setting makes it possible to examine the manner in which different policies provide different information about the cost shocks.

All of these papers maintain the assumption of linear-quadratic functions with additive cost shocks. These functional assumptions make it possible to obtain analytic

comparisons of the taxes and quotas, but they imply that the expected trajectories are the same under the two policies and that the difference in payoffs (and the policy ranking) is independent of the stock of pollution. Neither of these properties holds when shocks are multiplicative, as in Hoel and Karp (1999). However, that model does not yield clear analytic results.

Karp and Zhang (2000) consider the case where firms with rational expectations make investment decisions in abatement capital. Those decisions depend on the firms' belief about future policies; the firms' decisions affect the future abatement costs. Karp and Zhang (2001) study a model in which the regulator does not know, but is able to learn about, the stochastic relation between pollution stock and environmental damages. In a model where damages are associated with pollution flow rather than stock, Costello and Karp (2001) examine the effect of relaxing the assumption that the quota is binding with probability 1.

2. The Model

We use a discrete time formulation, in which each stage lasts for h units of time. Units of time are arbitrary; in our subsequent discussion we choose years as the unit of time. A new realization of the random shock occurs at intervals of h units of time; agents are able to change their decision (e.g., the tax and the level of emissions) every h units of time. The decision variables (taxes or quotas, and the firm's response to the tax) and the realization of the random shock are constant within a stage. The firm observes the current cost shock and the current tax before deciding how to respond to an emission tax. A smaller value of h corresponds to the case where information arrives more often and

agents are able to change their decisions more frequently, i.e. there is more flexibility.[3]

When the unit of time is a year, $h = 1$ means that agents can change their decisions once a year, while $h=1/12$ means that agents can change their decisions once a month. We treat the value of h as a parameter in the model, an exogenous constant.

In most discrete stage control problems, the value of the payoff (the value function) depends on the length of a stage. For example, if a resource owner is able to adjust output twice in a year rather than once in a year, she is able to use a more flexible policy and (typically) obtains a higher payoff. The length of a stage is usually not an interesting parameter, and it is customary to normalize it to 1. In comparing taxes and quotas, we need to compare two value functions. A change in the length of a stage has different effects on these two value functions. We introduce the parameter h in order to make the relation between the length of a stage and the policy ranking easy to study. Section 4 discusses the parameter h in more detail.

Let $x(t)$ be the constant flow of pollutant during the stage beginning at time t, so $x(t)h$ is the contribution to the stock during that stage. We define Δ as fraction of the stock remaining in the next stage in the absence of additional pollution. With additional pollution $x(t)h$, the stock at time $t+h$ is

$$S(t+h) = \Delta S(t) + x(t)h. \tag{1}$$

We use a representative firm model, and for notational convenience we normalize the number of representative firms to 1. In each period the firm incurs abatement costs. These costs are increasing and convex in the amount of abatement. Thus, the amount of

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[3] A more general specification would allow the arrival of new information and changes in the firms' and the regulator's decisions to occur at different intervals.

abatement costs that the firm avoids – the benefit of emissions – is an increasing concave function of emissions. The *flow* of this benefit is given by the quadratic function $B(x,\theta)$:

$$B=B\big(x(t),\theta(t)\big)=f+\big(a+\theta(t)\big)x(t)-\frac{b}{2}x(t)^2 . \tag{2}$$

The parameters *f, a,* and *b* are positive, and $\theta(t)$ is the realization of a random variable. We can interpret $B(x,\theta)$ as a restricted profit function in which prices are suppressed. Since *B* is a rate, the total benefit from emissions obtained in the stage beginning at time t is $B\big(x(t),\theta(t)\big)h.$

As in Weitzman's model, uncertainty affects the level but not the slope of marginal benefits. We assume that the random variable $\theta(t)$ is independently and identically distributed (*iid*), with mean *0* and variance σ^2. This assumption enables us to obtain a clear comparison with Weitzman's result, regardless of whether the regulator uses an open-loop or a feedback control rule. That is, we are able to rank policies under persistent asymmetric information when damages depend on stocks rather than flows. The assumption of zero correlation means that observations of previous responses to a tax, or previous equilibrium quota prices under a quota system, provide no information to the regulator about the current cost shock.

We assume that if the regulator uses a quota, it is always binding. This assumption requires that the lower bound of the support of θ be sufficiently large. Thus, when the regulator chooses quantity restrictions, $dS \equiv S(t+h) - S(t)$ is nonstochastic, and the expectation of the flow of benefits is independent of σ: $E(B) = f + ax - bx^2/2$ (since $E\theta = 0$).

The representative firm understands that its emissions decisions have no (appreciable) effect on aggregate emissions. Since it takes aggregate emissions as exogenous, it understands that it cannot affect future regulations. It is therefore rational for the representative firm to behave non-strategically. Since the firm takes both the current and future policies as exogenous, the firm solves a sequence of static optimization problems.

The regulator maximizes expected social utility: the expected payoff of the firm, net of emissions taxes (if any) minus the expected damages. If the regulator uses a tax, $p(t)$, firms choose x to maximize $[B(\bullet) - p(t)x]$. The first order condition is $a + \theta(t) - bx(t) = p(t)$, which we rewrite as

$$x(t) = \frac{a - p(t)}{b} + \frac{\theta(t)}{b} \equiv z(t) + \frac{\theta(t)}{b}. \tag{3}$$

Choosing $p(t)$ is equivalent to choosing $z(t)$, the expectation of the flow of pollution. Hereafter, we treat a regulator who uses taxes as choosing $z(t)$. From (2) and (3), the *regulator's* expectation at time $s < t$ of the flow of benefits at stage t, conditional on $z(t)$, is

$$E\big(B(x(t), \theta(t))\big)h = E\left[B\left(z(t) + \frac{\theta(t)}{b}, \theta(t)\right)\right]h = \left(f + az(t) - \frac{b}{2}z(t)^2 + \frac{\sigma^2}{2b}\right)h \tag{4}$$

The *flow* of damages during the stage beginning at time t is

$$D = D(S(t)) = cS(t) + \frac{g}{2}S(t)^2 \tag{5}$$

where c and g are positive parameters. The total damage during the stage is $D(S(t))h$.

We now have all of the elements of the model. The regulator's payoff is the present discounted value of the expectation of the stream of benefits minus costs, $[B(x,\theta)$

- *D(S)]h*, where the equation of motion is given by (1). With quantity restrictions, the evolution of the state is nonstochastic and the expected payoff within each period is also independent of σ^2. With a discount factor of β, the regulator wants to maximize the payoff

$$\sum_{i=0}^{\infty} \beta^i \left[f + ax(t+ih) - \frac{b}{2}x(t+ih)^2 - cS(t+ih) - \frac{g}{2}S(t+ih)^2 \right] h \tag{6}$$

subject to the equation of motion, equation (1). With taxes, the regulator wants to maximize

$$E\sum_{i=0}^{\infty} \beta^i \left[f + \frac{\sigma^2}{2b} + az(t+ih) - \frac{b}{2}z(t+ih)^2 - cS(t+ih) - \frac{g}{2}S(t+ih)^2 \right] h \tag{7}$$

subject to the constraint

$$S(t+h) = \Delta S(t) + z(t)h + \frac{\theta(t)h}{b}. \tag{8}$$

We obtain equation (8) by using equations (1) and (3). The expectation in (7) is taken with respect to the evolution of *S*. The random variable θ appears explicitly only in the constraint, equation (8). We have already replaced $E\{\theta^2/2b\}$ by $\sigma^2/2b$ in the single period payoff.

In this problem the payoffs are linear-quadratic functions of the state (pollution stock) and the control (the quota or tax), and uncertainty enters the equation of motion (8) additively. In this linear-quadratic control problem, the Principle of Certainty Equivalence holds. This principle states that the optimal policy rules in the stochastic model are identical to the rules obtained when the random variable in equation (8) is set equal to its expected value, 0.

There are three important consequences of this principle. The first is that the expected values of the stock and emissions trajectories are identical under optimal taxes and quotas, and under an open-loop and feedback policy. Only the higher moments of these trajectories differ. The payoff ranking therefore depends on these higher moments. The second consequence is that the payoff difference – and therefore the policy ranking – is independent of the value of the stock of pollution. Since the policy ranking depends only on the exogenous parameters but not on the stock, the regulator would never want to switch from one policy to another. Therefore, the assumption that the regulator always uses the same policy results in no loss in generality. The third consequence is that the payoff difference is proportional to the magnitude of uncertainty (the variance of the cost shock); the variance does not affect the sign of the payoff difference and therefore does not affect the policy ranking. None of these three properties hold in the model with multiplicative shocks, analyzed in Hoel and Karp (1999).

Before turning to the comparison of taxes and quantity restrictions, we note that if damages in the next period depend on the flow of pollution rather than the stock (e.g., $D(x)=\beta\left(cx+gx^2/2\right)$), our model is equivalent to Weitzman's. For that model, taxes are preferred to quotas if and only if $1 > \beta g/b$. With stock dependent damages, g and b have different units, so their ratio is not unit-free (unlike the number 1).

Table 1 gives the units of functions and parameters when we measure benefits and costs in dollars, stock in tons, and time in years. Since the ratio b/g is not a pure number, it cannot provide a criterion for ranking policies that is independent of other parameters in the model. However, the ranking is related to this ratio.

Table 1: Units of functions and parameters

x	$B(\bullet)$	$D(\bullet)$	b	g	$\dfrac{b}{g}$
$\dfrac{tons}{year}$	$\dfrac{dollars}{year}$	$\dfrac{dollars}{year}$	$\dfrac{dollars \times year}{tons^2}$	$\dfrac{dollars}{tons^2 \times year}$	$year^2$

3. Non-Flexible Tax or Quota

In this section we consider the case in which the trajectories of the tax and the quota must be determined at $t = 0$. The regulator uses an open-loop policy. Once he has chosen his policy trajectory, it is written in stone; in that sense, the regulator has no flexibility.[4]

We noted that when quotas are used, the expectation of the payoff is independent of σ and the evolution of S is nonstochastic. We define $Q(S(0))$ as the maximized value of the payoff in (6), subject to (1). We define $T(S(0);\sigma)$ as the maximized value of the payoff in (7), subject to (8) under an open-loop policy. When there is no uncertainty, the two policies obviously have the same payoff: $Q(S(0)) = T(S(0);0)$. The following result simplifies the comparison of the two policies.

Remark 1: The optimal quota trajectory is identical to the expected pollution

trajectory under optimal taxes, i.e. $x_i^* = z_i^*$ *for all i.*

This fact is a consequence of the linear-quadratic structure with additive errors.

We calculate expected damages under a tax by using equation (8) to obtain

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[4] The regulator is able to announce a time-varying policy, and in that sense does have flexibility. We define the inflexible regulator as one who has to choose a trajectory at time *0* and then follow it.

$$S(ih)=\Delta^i S(0)+h\left[z((i-1)h)+\Delta z((i-2)h)+...+\Delta^{i-1}z(0)\right]$$
$$+\frac{h}{b}\left[\theta((i-1)h)+\Delta\theta((i-2)h)+...+\Delta^{i-1}\theta(0)\right]. \tag{9}$$

In view of the quadratic form of damages, we can write the expectation of damages as

$$ED(S(ih))=D(ES(ih))+\frac{g}{2}\left[E(S(ih))^2-(ES(ih))^2\right]. \tag{10}$$

Equations (9) and (10) imply

$$ED(S(ih))=D(ES(ih))+\frac{gh^2\sigma^2}{2b^2}\frac{1-\Delta^{2i}}{1-\Delta^2}. \tag{11}$$

We already noted that uncertainty increases the single-period expected benefits (under taxes) by the amount $h\sigma^2/2b$ (see equation (7)). Using equations (7) and (11), and Remark 1, we can write $T(S(0);\sigma)$ - $Q(S(0))$ as

$$T(S(0);\sigma)-Q(S(0))=\sum_{i=0}^{\infty}\beta^i\left[\frac{\sigma^2 h}{2b}-\frac{gh^2\sigma^2}{2b^2}\frac{1-\Delta^{2i}}{1-\Delta^2}h\right], \tag{12}$$

which we simplify to obtain

$$T(S(0);\sigma)-Q(S(0))=\frac{\sigma^2 h}{(1-\beta)2b}\left(1-\frac{gh^2\beta}{b(1-\beta\Delta^2)}\right). \tag{13}$$

Equation (13) implies that an emission tax is superior to a quota (i.e., *T[S(0)] > Q[S(0)]*) if and only if $g/b < \varphi_1$, where the critical value φ_1 is

$$\varphi_1\equiv\frac{1-\beta\Delta^2}{\beta h^2}. \tag{14}$$

We use the expression for φ_1 to show how the policy ranking depends on the decay rate and the discount rate and on the length of a stage. For a given length of a stage, an increase in either β or Δ is equivalent to a lower discount or decay rate. Since

the function φ_1 is decreasing in β and Δ, an increase in the decay rate or the discount rate favors the use of taxes.

The future stock is a random variable under taxes; under quotas the regulator can control the future stock exactly. Since damages are convex in the stock, an increase in the randomness of the stock (holding fixed its expected trajectory) increases future expected damages. This difference favors the use of quotas. As the stock decays more rapidly (smaller Δ) or as the future becomes less important (smaller β), this advantage is less significant, making it "less likely" that the regulator wants to use quotas.

4. The Parameter h

This Section discusses the parameter h in detail. As noted in Section 2, we write benefits and damages as flows, and define the single stage benefit and damage as the number of dollars per year, multiplied by the length of a stage. This formulation enables us to change the length of a stage without changing the parameters of the benefit and damage function. This simplification makes it easy to see the relation between the policy ranking on the length of a period.

The parameters β and Δ depend on h: $\beta \equiv e^{-rh}$ and $\Delta \equiv e^{-\delta h}$, where r is the continuous yearly discount rate and δ is the continuous yearly decay rate. With these definitions, when we change the length of a stage we hold fixed the rate of discount and decay.

We could have avoided introducing the parameter h by maintaining the normalization $h = 1$, i.e. by using a suitable definition of a unit of time. In that case, when changing the length of a stage, we would need to change the definition of a unit of time. That change would require changing parameters in the benefit and damage

function. Our decision to have the model depend explicitly on *h* makes those kinds of changes unnecessary. It also makes the importance of the length of a stage obvious in formulae below.

Before discussing how the length of a period affects the policy ranking, we need to explain the role of a particular set of assumptions. Our model assumes that the flow of costs and benefits within a period are not discounted; that current emissions do not contribute to the stock level during the same period; and that there is no stock decay within a period. These types of assumptions are standard for discrete stage control problems, and they are not related to our use of the parameter *h*. That is, we would have adopted these assumptions even if we had maintained the normalization *h=1* as described in the previous paragraph. However, the plausibility of these assumptions does depend on the length of a period.

In order to understand this dependence, it helps to consider an alternative to our assumptions. To this end, suppose that discounting occurs continuously at rate *r* within a period and that the stock also changes continuously, decaying at rate δ and increasing at rate *x*. We do not change any other assumptions: the tax *p(t)* is announced and the shock $\theta(t)$ occurs at the beginning of a period, and the firm chooses a constant rate of emissions for that period, $x(t) = z(t) + \theta(t)/b$. Under the alternative assumption of a continuous decay rate *within a period*, the evolution of *S* within a period beginning at time *t* is

$$\frac{dS(\tau)}{dt} = z(t) + \frac{\theta(t)}{b} - \delta S(\tau); \quad t < \tau < t + h.$$

With this model of continuous decay and continuous discounting, we can calculate the expected payoff for a period of length *h*, conditional on *z(t)* and *S(t)*. This payoff is a linear-quadratic function of on *z(t)* and *S(t)*. To a first order approximation in *h*

(evaluated at $h=0$), this expression equals the summand in equation (7). (The formulae for the general case when $h>>0$ are available upon request.)

Consequently, our model (in which variables are constant and there is no discounting or decay within a period) is a good approximation of a more general model (with continuous discounting and decay) provided that the length of a period is small. If a stage lasts for a long time, it is obviously unreasonable to hold the stock constant and to ignore discounting during the stage.[5] Thus, when we consider the effect of h on the policy ranking, we are interested in the situation where h is small.

We can now provide an intuitive explanation for the effect of h on the policy ranking. We noted above that the expected value of the regulator's payoff under quotas does not depend on the cost shock. Under taxes, on the other hand, the variance of the cost shock appears in the single period payoff, and the cost shock makes the evolution of the state random – see equation (8).

The variance of the cost shock increases the expectation of the single period payoff by $\sigma^2 h/2b$, an amount proportional to $\sigma^2 h$. Under taxes (unlike quotas) the firm can adjust emissions in response to the cost shock, leading to a higher expected payoff, for a given level of expected emissions. However, conditional on information at time t, $dS \equiv S(t+h) - S(t)$ is a random variable under taxes, but is deterministic under quotas; $\text{var}(dS) = (\sigma h/b)^2$ under taxes. Since damages are convex in the stock, expected damages are higher (for a given level of emissions) the higher is the variance of dS. Since the damage function is quadratic, the increase in expected damages in a single

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[5] For example, when h is large and there is no decay within a period, it makes a big difference whether we assume that current emissions contribute to the stock in the next period (as in equation (1)) or in the current period. When h is small, these two

period is proportional to the variance of the stock. This increase in expected damages is discounted, because of our assumption that current emissions cause future (not current) damages.

Thus, under taxes, the cost shock increases expected payoffs by an amount proportional to $\sigma^2 h$ and it decreases the expected payoff (i.e., it increases damages) by an amount proportional to $(\sigma h/b)^2$. The ratio of these two magnitudes (the functions associated with the benefits and the costs of using taxes rather than quotas) is $1/2h$, a decreasing function h. Therefore the policy ranking depends on h.

To provide a formal statement of this dependence, we use the definitions $\beta \equiv e^{-rh}$ and $\Delta \equiv e^{-\delta h}$ to rewrite equation (14) as

$$\varphi_1 \equiv \frac{1-\beta\Delta^2}{\beta h^2} = \frac{1-e^{-(r+2\delta)h}}{e^{-rh}h^2}. \tag{14'}$$

The function φ_1 is non-monotonic in h. A straightforward calculation shows that $\partial\varphi_1/\partial h$ is decreasing for small values of h and is increasing for large values of h. Also, it is straightforward to show that $\varphi_1 \to \infty$ as $h \to 0$ or as $h \to \infty$. Neither of these two limiting values of h is intrinsically interesting; however, these limiting cases and the fact that φ_1 is continuous in h means that if h is either very small or very large, taxes certainly dominate quotas.

We explained that our assumption that there is no decay or discounting within a stage is not reasonable when h is large. When h is large, discounting is important, since (by assumption) current emissions affect damages only in subsequent periods. For large h, *var(dS)* becomes large, but discounting causes it to have a negligible effect on the

formulations are approximately the same.

present value of damages. The benefit associated with taxes ($\sigma^2 h / 2b$) is received in the current period. Thus, when h is very large, taxes certainly dominate quotas, as equation (14') implies. However, when h is large, our model is not appropriate.

The meaning of "large" and "small" depends on the particular pollutant. If the pollutant decays very slowly (i.e., has a long half-life) a period of several years is "small". For example, suppose we choose units of time to be a year and we let $r = 0.03$ and $\delta = 0.0077$, corresponding to a yearly discount rate of 3% and a half-life of the stock of 90 years. For these values of r and δ, φ_1 is decreasing for $h < 60.4$. This inequality means that as long as $h < 60.4$, a further decrease in h increases the set of values of g/b for which taxes are preferred. It is reasonable to assume that the length of a period is much shorter than *60* years, so for a very persistent pollutant, an increase in h favors the use of quotas. The minimum value of φ_1 in this example is *0.0016*. Here, a sufficient condition for taxes to dominate quotas, regardless of the value of h is that $g/b < 0.0016$.

Our conclusion that for sufficiently small h, taxes dominate quotas implies nothing about the *magnitude* of the preference for taxes. Thus far we have treated σ^2 as a parameter. However, since σ^2 measures the amount of uncertainty in a period of length h, it is reasonable to view σ^2 as an increasing function of h. Our analysis does not depend on the precise form of $\sigma^2(h)$, but in order to consider limiting cases when $h \to 0$, we adopt

Assumption 1. σ^2 is of the same order of magnitude as h or smaller: $\sigma^2(h) \sim 0(h)$.
Under Assumption 1, the amount by which taxes are preferred to quotas approaches 0 as $h \to 0$. In the limit as the length of a period becomes very small, taxes and quotas are equivalent.

To verify this claim, we multiply and divide the right side of equation (13) by h to rewrite this equation as

$$T\left(S(0);\sigma\right)-Q\left(S(0)\right)=\left(\frac{\sigma^2(h)}{h}\right)\left[\frac{h^2}{\left(1-e^{-rh}\right)2b}\left(1-\frac{gh^2 e^{-rh}}{b\left(1-e^{-(r+2\delta)h}\right)}\right)\right].\tag{13$'$}$$

Under Assumption 1, the first term converges to a finite constant and (using *L'Hopital's Rule*) the term in square brackets converges to *0* as $h \to 0$. In the limit, taxes and quotas are equivalent, although for sufficiently small but positive h taxes are preferred to quotas. It is worth emphasizing that we do not view this limiting case as a continuous time model. It is simply a device for studying the discrete stage model when h is small.

If $\delta \to \infty$, $\varDelta \to 0$ and φ_1 approaches *1/β*. This limiting result reproduces Weitzman's result. We have *1/β* rather than 1 because the damages caused by this period's emissions are felt in the following period.

5. A Flexible Tax or Quota

This section studies the more interesting case where the regulator has the same degree of flexibility as firms. At the beginning of each stage, the regulator observes the current value of the stock, but not the current realization of θ, and chooses the current policy level. Here the regulator uses a feedback rule. The previous section considered the case where the regulator had to commit to a tax or quota trajectory at the initial time, i.e., he used an open-loop policy. The two cases thus represent two extreme assumptions about the regulator's ability to use new information.

 The regulator's increased flexibility has no value if he uses a quota. Equation (1) shows that under a quota the development of the stock is nonstochastic, so the regulator learns nothing from observing it. Therefore, nothing is gained by postponing the decision

of the level of the period t quota until t, rather than choosing the entire emission path at the initial time: the open-loop and feedback policies are identical and give the same payoff. In addition, the random variable θ does not affect the expectation of the current payoff. (The payoff in equation (6) is independent of σ^2.) The value of the regulator's program under a flexible quota is $Q(S)$, defined in the previous section.

If the policy is an emission tax, the flow of pollution and thus the evolution of the stock is stochastic (equation 8). In this case, flexibility in setting the tax increases the regulator's payoff. The optimal emission tax at any time depends on the stock of the pollutant at that time. When the tax path must be chosen at time $t = 0$, the regulator chooses future taxes without knowing the future value of the stock. Denoting the optimal value of regulator's program under a flexible tax as $T^*(S;\ \sigma)$, it is clear that $T^*(S;\sigma) \geq T(S;\sigma)$. In general, the inequality is strict. When there is no ambiguity, we suppress the second argument of T^* and T.

The regulator who conditions taxes on current information is "more likely" to prefer taxes than quotas, compared to the regulator who must choose the tax trajectory at time 0. More precisely, there exist parameters such that the regulator would prefer a quota rather than a tax under an open-loop policy, but allowing the regulator to use a feedback policy reverses the ranking. This conclusion holds because we know that when $g/b > \varphi_1$, the "inflexible regulator" prefers the quota [i.e. $Q(S) > T(S)$] and that in general $T^*(S) > T(S)$. Therefore, (as we show below) there exist parameter values such that $T^*(S) > Q(S) > T(S)$.

Appendix A derives the criterion for a quota to be preferred to a tax when the regulator conditions the policy at time t on the stock at time t. There we show that quotas

are preferred to taxes if and only if g/b exceeds a critical value, denoted φ_2. This function

is defined as

$$\varphi_2 \equiv \left(\frac{2-\beta\Delta^2}{2\left(1-\beta\Delta^2\right)}\right)\left(\frac{1-\beta\Delta^2}{\beta h^2}\right) \equiv \gamma\varphi_1, \tag{15}$$

$$\gamma \equiv \frac{2-\beta\Delta^2}{2\left(1-\beta\Delta^2\right)} = \frac{2-e^{-(r+2\delta)h}}{2\left(1-e^{-(r+2\delta)h}\right)} > 1.$$

The function φ_1 is defined in equation (14). Since $\gamma > 1$, the critical value of g/b is higher

under the feedback policy, relative to the open-loop policy. That is, taxes are more likely

to be preferred under the feedback policy. In addition, γ is a decreasing function of h, so

the difference between the critical levels of g/b (under the open-loop and feedback

policies) is greatest when h is small.

All of the qualitative comparative statics of the critical levels under open-loop

and feedback policies are the same. The intuition is also the same. A simple calculation

shows that φ_2 is decreasing in β and Δ; increases in the discount and decay rates r and δ

makes taxes more attractive. In addition, φ_2 is non-monotonic in h, decreasing for small

h and increasing for large h. Since $\varphi_2 \to \infty$ as $h \to 0$, taxes are certainly preferred to

quotas for sufficiently small h. For the limiting value of $h = 0$, taxes and quotas are

equivalent. Finally, since $\gamma \to 1$ *as* $\Delta \to \infty$, we again reproduce the chief result from the

static model: when damages are associated with flows rather than stocks, the policy

ranking depends only on the relative slopes of marginal damages and abatement costs.

 To summarize, we have the following Proposition.

Proposition: For the quadratic model with additive uncertainty about abatement

costs, there are critical values of the ratio of the slope of marginal damages to the slope

of marginal abatement costs, g/b. These critical values are given by equation (14') when the regulator chooses the trajectory of policies at time 0, and by equation (15) when the regulator conditions the current policy on the current stock. The critical ratios are increasing in the decay and the discount rates, and non-monotonic in the parameter that measures the length of a period. If the length of a period is sufficiently small, the regulator wants to use taxes, but the payoff difference under the two policies is approximately 0.

6. An Empirical Illustration

We use the results of the previous sections to rank taxes and quotas in controlling CO_2, the major "greenhouse" gas. The control of greenhouse gasses is among the most important, or at least most hotly debated current environmental issues.

Even our parsimonious model stretches the limits of available data. We use two estimates for the slope of marginal damages, g: Falk and Mendelsohn's (1993; hereafter, FM) "high" estimate, and Reilly's (1992) estimate which we adapt by converting units from parts per million to tons. We use data from Nordhaus (1991) to estimate the parameter b, under a variety of specifications. All of these specifications lead to estimates with similar orders of magnitude. We choose the specification that produces the smallest estimate of b [= 5.94E(-8)], thus biasing the results in favor of the use of quotas. Table 2 summarizes the estimates of g and the ratio g/b. The appendix discusses the data more fully.

Table 2: Estimates of g/b (Using the estimate $b = 5.94E(-8)$.)

	FM	Reilly
Estimate of g	8.12E(-13)	1.19E(-12)
Estimate of g/b	1.37E(-5)	2E(-5)

We choose one unit of time equal to one year and set the discount rate r = .03 and the decay rate δ = .005. A review of the literature suggests that δ = .005 (implying a half-life of 139 years) is widely accepted as a point estimate for the decay rate for greenhouse gasses (FM and Nordhaus). Reilly uses δ = .0083 (implying a half-life of 83 years) in a study which focuses on CO_2. The values we choose for both the discount and decay rate are therefore plausible but conservative (i.e. small), thus tending to bias the results in favor of quotas.

We have no way of estimating the parameter h, but for units of time equal to one year, *h = 1* and *h = 10* are reasonable bounds. Table 2 presents the critical values φ_i for h ranging from .1 to 25.

Table 3: Critical Values of g/b (r = .03, δ = .005)

h	(open-loop) φ_1	(feedback) φ_2
.1	.4004	50.350
1	.0404	.535
10	.0044	.00897
25	.0021	.0028

Even though we chose parameter values from the plausible range in such a way to make *g/b* large and φ_i small (thus making it more likely that quotas dominate taxes) our calculations indicate that taxes lead to higher welfare (g/b << φ_i). Even if the larger (based on Reilly) estimate of *g* is too small by a factor of 1000, so that the actual value of *g/b* is approximately .02, taxes would still dominate quotas if the firm and the regulator were "reasonably flexible" (*h = 1*). If the estimate of *g* is too small by a factor of 100, taxes would still dominate quotas even if the firm and the regulator are inflexible (*h =*

10). Consequently, in spite of the data limitations, our results support the use of taxes rather than quantity restrictions to control greenhouse gasses.

Table 3 also illustrates the magnitude of error that arises from ignoring *h,* and the difference between the criteria for ranking taxes and quotas under open-loop and feedback policies. For example, an increase in *h* from one to ten years deceases the critical ratio *g/b* by a factor of nearly 10 under open-loop policies, and by a factor of nearly 60 under feedback policies. When *h=1* the critical ratio under feedback policies is more than 10 times a large as the critical ratio under open-loop policies.

7. Conclusion

Previous literature ranked a tax and quota policy when abatement costs and environmental damages both depend on the flow of pollution, and the polluter has better information than the regulator concerning abatement costs. In that case, for linear-quadratic functions with additive uncertainty, the quota dominates the tax if and only if the slope of the marginal damage function is greater than the slope of abatement costs. We studied the situation where environmental damages depend on the pollution stock rather than the flow. In this circumstance, a direct comparison of the two slopes is not meaningful, since the units of the two are not the same.

 The intuition provided by the static model continues to hold, insofar as greater convexity of the damage function, or less concavity of the benefits function, make it more likely that a quota is preferred. However, when environmental damages depend on pollution stocks, the ranking of the two policies also depends on the discount and stock decay rates. Higher discount and/or higher decay rates increase the importance of current flows relative to future stock effects. The chief advantage of the quota is that it makes it

possible to control exactly the evolution of the stock. Since a higher discount rate and a higher decay rate both decrease the importance of future stock effects, they also decrease the value to the regulator of exact control of the evolution of the stock. Consequently, higher discount and decay rates make it more likely that a tax is preferred.

The ranking of the two policies also depends on the length of the period for which agents keep their decision variables unchanged. We showed that under both feedback and open-loop policies, the regulator always prefers taxes if the length of this period is very small. However, the difference between the payoffs vanishes as the length of the period becomes small. We also explained why the use of feedback rather than open-loop rules favors the use of taxes.

We used the theoretical results, together with estimates of marginal benefits and damages, to compare taxes and quotas in the control of greenhouse gasses. The point estimates suggest that taxes dominate quotas. In order to overturn this ranking, we would need to adjust key parameters by a factor of more than 1000.

Appendix A: Details of Feedback Solution and Derivation of Equation (15)

We note that for $\sigma = 0$, $T^*(S;\sigma) \equiv T(S;\sigma) \equiv Q(S)$. We use dynamic programming to determine the function $T^*(S;\sigma)$, and then show how this function is related to $Q(S)$ for $\sigma > 0$. The single period expected payoff in equation (7) is λh, with $\lambda \equiv [f + az - bz^2/2 + \sigma^2/2b - cS - gS^2/2]$. Using this definition of λ and equation (8), we write the Dynamic Programming Equation (DPE) under the flexible tax as

$$T^*(S) = \max_z \; \lambda(z,s)h + \beta E T^* \left(\Delta S + zh + \frac{\theta h}{b} \right). \tag{16}$$

The method of solving the DPE (16) is standard, so we merely sketch the steps. We know the value function is quadratic: $T^*(S) = \rho_0 + \rho_1 S + \rho_2 S^2/2$. We substitute this "trial solution" into equation (16), and use the first order condition to find the optimal control rule as a function of the state S, the known parameters, and the unknown parameters ρ_i. Substituting this control rule into (16) gives the maximized DPE, a quadratic equation in S. Equating coefficients of I, S and S^2 (in the maximized DPE) gives expressions for the values of ρ_i. The values of ρ_2 and ρ_0 satisfy

$$\rho_2 = \left(\frac{(\Delta\beta\rho_2)^2}{b - \beta h \rho_2} - g \right) h + \beta\Delta^2\rho_2 \tag{17}$$

and

$$\rho_0 = \left[f + \frac{(a + \beta\rho_1)^2}{2(b - \beta\rho_2 h)} \right] h + \beta\rho_0 + \frac{\sigma^2 h}{2b} \left(1 + \frac{\beta\rho_2 h}{b} \right). \tag{18}$$

The parameter σ affects the constant term $(f + \sigma^2/2b)h$ in the expected payoff flow and the variance of the additive error $(h\sigma/b)^2$ in the stock evolution. Therefore, in view of

well-known properties of the linear-quadratic control problem with additive errors, σ affects only the value of ρ_0; the values of ρ_1 and ρ_2 are independent of σ. In addition, from equation (18), the effect of σ on the value of ρ_0 depends on the value of ρ_2 but not on the value of ρ_1. Consequently, in order to determine how σ affects the value function, we do not need to know the value of ρ_1; we therefore do not include the equation that determines that parameter.

Since the values of ρ_1 and ρ_2 do not depend on σ, the control rule that determines the optimal tax as a function of the stock is also independent of σ. Again, this fact is a reflection of the Principle of Certainty Equivalence in the linear-quadratic control problem with additive errors. The *expected* flow of pollution is the same under taxes and quotas, in both the open-loop and in the feedback models.

The slope of the shadow value of the stock is the unique negative root of equation (17): $\hat{\rho}_2 = T*''$. From (18) we see that quotas are preferred to taxes (in the feedback setting) if and only if $\hat{\rho}_2 < -b / \beta h$. In the static problem, the ranking of taxes and quotas depends on the curvature of the benefits function (b) relative to the curvature of the damage function (g). In the dynamic problem with stock pollution, the ranking depends on the curvature of the benefits function relative to the curvature of the value function (ρ_2).

By rearranging equation (17) and dividing by h, we can write $\hat{\rho}_2$ as the unique negative root of $m(\rho_2) = 0$, where

$$m(\rho_2) \equiv \beta \rho_2^2 + \left(g\beta h \ - \ \frac{b(1 - \beta \Delta^2)}{h} \right) \rho_2 - gb. \tag{19}$$

We want to know whether $\hat{p}_2$ is greater or less then -b/βh. Since $m(0) < 0$, $m(\hat{p}_2) = 0$,

and $m'(\hat{p}_2) < 0$ we know that $\hat{p}_2 < -b/\beta h$ if and only if $m(-b/\beta h) < 0$. (See Figure 1.)

Using equation (19) to evaluate $m(-b/\beta h)$ gives

$$m\left(-\frac{b}{\beta h}\right) = 2b^2\left[\frac{2-\beta\Delta^2}{2\beta h^2} - \frac{g}{b}\right]. \tag{20}$$

Equation (20) and the previous remarks imply that quotas are preferred to taxes if and

only if g/b exceeds a critical value, denoted φ_2, defined in equation (15).

In the text we noted that the difference in payoffs under taxes and quotas vanishes

as h approaches 0. This conclusion follows because ρ_i approach limiting (finite) values

as $h \rightarrow 0$, and from equation (18) we see that in the limit ρ_0 is independent of σ^2.
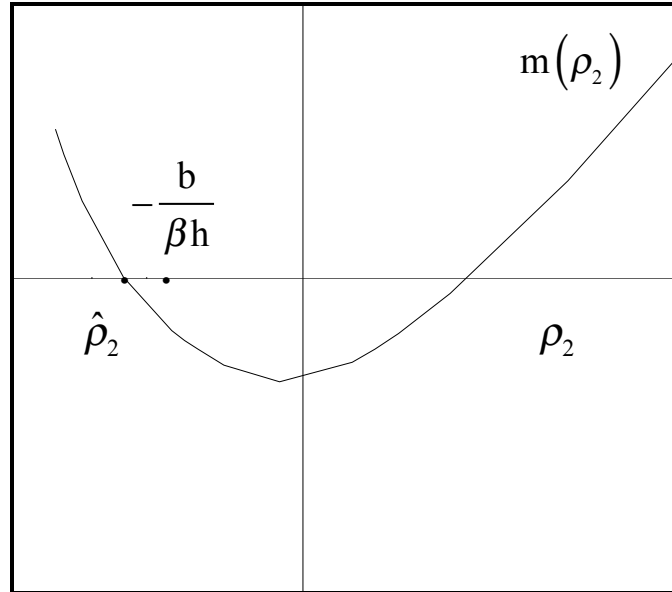

Figure 1: graph of $m(\rho_2)$ when $\hat{\rho}_2 < -b/\beta h$

Appendix B: Background for Empirical Illustration

We surveyed the literature on damage and abatement costs associated with greenhouse gasses. The volumes by Bruce et al. (1996), Cline (1992) and OECD (1992) and the papers by Barnes et al. (1993) and Manne (1993) provide background material and summarize previous estimates.

Falk and Mendelsohn (FM, 1993) use data from Nordhaus (1991) to estimate a linear marginal damage function, which provided our first estimate of the parameter g. Reilly (1992) estimates damages as a function of the concentration of greenhouse gasses (ppm). In 1990 the concentration of greenhouse gasses was 441 ppm and the stock of CO_2 was 800 billion tons. We used these quantities and the assumption of a linear relation between concentration rate and stock to convert Reilly's estimate, obtaining a second estimate of g. The two estimates differ by a factor of approximately two, which we regard as small, given the imprecision of all these numbers.

To get an idea of the range of plausible estimates of the slope of marginal damages, Table 3 reports estimates of the cost to the world (in billions of 1990 dollars) resulting from a doubling of the atmospheric stock of CO_2. Where the original study estimates the cost of damages for the US economy only, we assumed for the rest of the world the same ratio between damages and GDP as in the US. Using this ratio and data on world GDP we can then estimate the economic cost of damages for the world Thus, we can compare estimates across the studies.

Table 4 Damage Estimates In Billions of 1990 Dollars

Resulting from a Doubling of Tons of Carbon in Atmosphere

Cline (1992)	Fankhauser (1995)	FM (1993)	Maddison (1995)	Nordhaus (1991)	Nordhaus (1993)
220	260	50 (low) 400 (high)	300	50	266

These estimates vary by a factor of 8. Thus, it seems unlikely that estimates of g based on these studies (if such estimates were possible to construct) would vary by a factor of more than 1000. We noted in the text that the ranking of taxes over quotas would survive a thousand-fold increase in *g*.

Nordhaus (1991) reports estimates of total and marginal costs associated with different percentage reductions in greenhouse gasses. We converted these percentages to tons of greenhouse gas at 1990 levels, thus obtaining 15 "observations" of abatement and associated marginal and total costs. We used these data to estimate marginal abatement costs under a variety of specification. For example, we regressed total costs against abatement and $(abatement)^2$ with and without an intercept, and we regressed marginal costs against abatement with and without an intercept). Our estimates of *b* ranged from 5.94E(-8) to 8.2E(-8). We used the smallest value of *b* in our calculations, in order to make it more likely that quotas would be preferred.

Maddison (1995) estimates a cubic abatement cost function, using percentages rather than absolute level. We converted his estimates to levels and fit a quadratic function through the resulting curve. The resulting estimate of *b* was of the same order of magnitude as the estimates we obtained using Nordhaus's data.

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