Certification of 0rigin and Brands Competition

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Abstract

We analyse the competition in quality and quantity between a foreign firm and a domestic firm. The domestic firm can belong to a certification of origin, whereas its rival uses a pure brand strategy. We will show how the certification can allow the domestic firm to position itself as a high quality producer and improve the average quality of the products offered on the market. If, however, the certified firm offers the low quality good, the certification can permit it to guarantee a higher profit than that of its competitor and to improve the consumers' surplus by favouring product standardisation.

Key words: Certification of origin, Quality, international competition
JEL Classification : L13,F12,F14

1 Introduction

Most of the micro-economic models which take into account the quality of the products offered to consumers consider that a given quality level can be acquired through investments by producers or, in other words, by a cost of production proportional to this quality level. This is also the case of the analysis in terms of vertical differentiation of products, from the pioneering contributions of Shaked and Sutton (1983), Mussa and Rosen (1978), Maskin and Riley (1979), to the more recent works of Motta (1993)

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and Lehmann-Grube (1997). These models indeed, include a good number of examples (innovation technology, improvement of production procedures, hiring qualified workers, spending on marketing,) which at the accounting level and the analysis of firms' behaviour on the markets certainly justifies this type of approach.

Nevertheless, quality improvement remains especially complex in an industrial process, particularly when it concerns products which have been transformed. Of course, these cannot always be considered as additional production costs. In the agro-food sector in particular, the quality certification systems are often based on the respect of a certain number of production rules and not just on the ex post checking of the quality of the product. The certification of origins in this way defines territorial limits outside of which a producer can no longer benefit from official certification. This delimitation has an effect on the quantities produced. In the wine-growing sector it is not rare that these certifications of origin specify, in addition, a limitation of the production of grapes (as is the case of the French "Appellations d'Origine Contrôlées" (AOC), or more generally the European "Quality Wines Produced in a Determined Region", (VQPRD). This is also found for cheeses with the definition of the breed of animal designated to produce the milk and is the most often not optimal from the point of view of quantities produced. It is the same for the quality labels in the meat sector which very severely restrict the producers on the number of animals per hectare that they have and these restrictions are often much more constraining than the need to invest in fixed or variable costs to satisfy production specification requirements. Thus, the quality certification systems in agriculture are based more on production control, and therefore on quality on the market, than on the necessity of incurring expensive but reproducible procedures for the production system.

This paper proposes comparing the two competing systems to improve the quality of products: The investment in research and development or brand promotion system, which we will call "brand system", and the "certification system" of the A.O.C. type or label of origin. The latter, as in the examples quoted above, imposes a control of the offer as a condition of having a level of certified quality. However, this system does not rely exclusively on a control of the offer, and the certified producers can always carry out research and development and advertising to increase the quality of product.

The quality labels and the certifications of origin are the principle tools which are used by the State to correct market dysfunction associated with consumers' lack of infor-

mation about the quality of the products (Gozlan, D., Marette, S. (2002). Most of the theoretical works therefore concentrate on the compared effectiveness of these systems of revealing information to consumers from a point of view of official economy (Shapiro C. (1983), Marette, S. et al (1999)). Other works always within the framework of limited information, analyse the private interests of a producer to certify the quality of his product (Linnemer, L., Perrot, A. (2000), Ibanez (2001)). Following the example of these works, the model which follows considers that the firms decide privately to submit to the system of official certification. However, it is not the "revealing" of information of the system of certification side which is appreciated in this model. It does not concern evaluating the benefits of the certification system in relation to a reference situation in which consumers have no information about the quality of the goods which are offered to them. On the contrary, we assume that the consumers are perfectly informed about the quality of the product. Naturally, we also assume the existence of a system of official certification which guarantees a minimum level of quality which is acknowledged by consumers. We then consider that the producer must make a choice between two strategies. On the one hand, if he chooses to respect the production specification requirements he in return is granted minimum "certified" quality and can then improve his quality with the help of traditional investments. On the other hand, the producer can adopt a classical system of quality investment. In agreement with the illustrations borrowed from the agro-food sector, the production specification requirements are expressed by a restriction of production capacity imposed exogenously by the establishment which controls the certification system. In other words, any firm wishing to adopt the certification system must submit to this capacity restriction in order to benefit from the minimum quality guarantee in exchange.

In this paper, we evaluate the impact of a certification associated with such an offer-restriction policy. Our model is close to the works of Krishna (1990), Das and Donnenfeld (1987), Herguera Kujal and Petrakis (2000), which we shall note in the continuation of the HKP text and Boccard and Wauthy (2000), which we note BW, who analyse the impact of import quota imposition or voluntary export restrictions on choices of quality in the area of competition between several countries.

When there is competition between two countries, we will compare a mixed competition system, where only the home country is certified, to a situation of competition between two non-certified producers. We will show how the unilateral adoption of a

certification system does not necessarily motivate the producer in the home country to improve his quality. We clarify the foreign aspect created by the certification system on the quality of the product offered by the producer in a foreign country as well. There again, we show how this quality can become deteriorated after setting up the certification system in the home country.

2 The model

2.1 The offer

Let's consider two firms located respectively in the home country and in a foreign country and which are competing on the domestic market. The firms in the two countries offer goods differing in quality on the national market of a standard size as 1.

When a firm becomes certified, it commits itself to limiting its production to level z in exchange for certification. In return for the restriction in capacity z to which it assents, the certified firm benefits immediately from a minimal quality level s. This exogenous parameter s reflects here the consumer satisfaction with such a system. A firm which chooses brand strategy is, on the contrary, free to furnish all the demand, but has no advantage for quality at the start.

Whatever the chosen strategy be, the firms can make investments to improve their quality in relation to their initial quality (respectively, s for a certified firm or 0 for a brand firm). These investments can represent technical innovation costs allowing for the objective improvement of the quality of the goods, as well as costs of brand promotion which contribute to improving the image and the reputation of the product. In these two examples, the entailed expenses are most often independent from the quantities produced, we therefore assume that a firm having adopted the brand strategy attempting to reach a quality k has a fixed cost for quality $F(k) = \frac{1}{2}k^2$ The consideration of fixed costs allows us to measure the producers' interest in innovating once they have benefited from a certification system[1]. If a firm adheres to a certification system the investment cost which must be spent to reach a quality level $k > s$, is written : $F(k) = \frac{1}{2}(k-s)^2$. As is shown in figure 1 below, the sums invested to reach quality level k are less.

[1]Differing from the other form of considering quality costs and concern variable costs (see for example, Champsaur and Rochet (1989), Motta (1993).

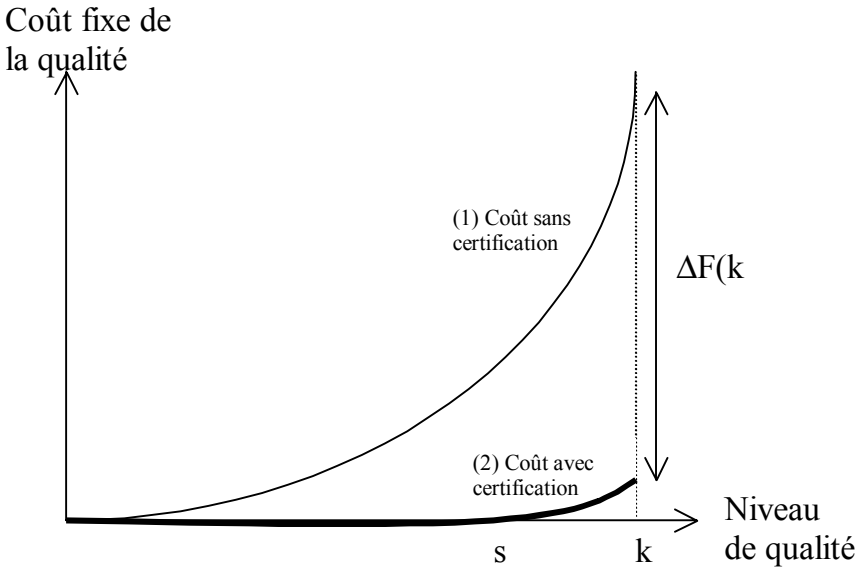

Figure 1: Cost structure according to strategies

The two curves (1) and (2) presented in figure 1 correspond respectively to the two strategies which a producer can adopt. For any quality level k, the brand strategy is more expensive than the certification strategy in terms of investment spending, the compensation being that the producer is limited in the quantity put on the market. The marginal cost of quality improvement is equally less when the firm is certified as well[2].

2.2 La demande

Whether the firms adopt a certification strategy or a brand strategy, we assume that the consumers uniformly perceive the more or less objective quality associated with these signals. The consumers on the domestic market are distinguished by a taste parameter θ which expresses the intensity of an individual's preferences for quality. The one dimensional parameter θ is uniformly distributed over an interval $[0, 1]$. We assume that each consumer can consume zero or one unity of goods.

The surplus $S_j(\theta)$ that an individual redeems from the taste parameter θ of the goods of quality k_j, purchased, is given as:

[2]If we admit that an inverse relation really exists between the quality potential of a product and the quantity produced, this hypothesis reflects the least cost of quality improvement of a modified product coming from the best quality raw material.

Now if we reason at the level of fixed capacity restriction z; the low quality firm is always encouraged to increase its quality whereas its competitor decreases it when certification level s increases; the difference in the products decreases. In fact, the more the certification level increases the more the certified firm can attain a high quality for a lower investment. On the other hand, s dissuades the high quality firm from investing in quality. This effect is paradoxical because when s increases, the certified firm increases its quality, with the result that the high quality firm could, in order to maintain a sufficient product differentiation, also be incited to raise its quality level. We will explain this result in section 5.

4.2 The certified firm offers the high quality good

We now turn to determine the *mc* equilibrium for which the certified firm offers the high quality good when the brand firm offers the low quality. The quantity produced by the high quality firm is limited by the capacity restriction, and the equilibrium quantities are:

$$\left| \begin{array}{l} q_l^{mc} = \frac{(1-z)}{2} \\ q_h^{mc} = z \end{array} \right. \tag{12}$$

We notice that the equlibrium in quantity is not dependant on the quality levels k_l and k_h chosen by the firms when the high quality is restricted by capacity. The first stage of the game is then resolved and leads to the following qualities:

$$\left| \begin{array}{l} k_l^{mc} = \frac{(1-z)^2}{4} \\ k_h^{mc} = s + (1-z)\, z \end{array} \right. \tag{13}$$

In this situation, we easily show that with s fixed, when z decreases, the difference in the products decreases. More precisely, the quality chosen by the certified firm is lower whereas the quality chosen by the brand firm increases. Once again these results are connected to a wealth-effect and agree with those of HKP. Furthermore, with a fixed restriction capacity, an increase of the certification level s allows the certified firm to improve its quality without the brand firm having to change its own. The differentiation of the products therefore is increased. There again, we would expect that, facing an

increase in quality, of the certified firm its brand competitor would reduce its investment in quality. This paradoxical effect directly ensues from the independence between the quantities and the qualities of the chosen equilibrium. Once again, it is necessary to verify the incentives of the firms playing leapfrog.

Lemma 3 : A $g(z)$ function exists, decreasing in z, in such a way the if $s < g(z)$, the brand firm which, in the framework of this equilibrium, offers the low quality good is moved to offer a higher quality than its competitor.

Thus, as soon as $s < g(z)$ an equilibrium no longer exists for which the certified firm offers the high quality good. We will now synthesise the results obtained in the two cases.

4.3 Synthesis of the results

We are now able to prove the existence and to characterise all the equilibrium of the game in which the firm from the home country becomes certified. The following proposition synthesises the results obtained on the existence of an equilibrium.

Lemma 4 *:* Whatever the values of the parameters $(z, s) \in \left[0, \frac{1}{4}\right]^2$, at least one perfect subgame equilibrium exists when a certified firm and a brand firm compete.

Proof : we will show that $g(z)$ is inferior to the function $f(z)$ for all of z. For the detail of the calculations refer to appendix 4.

Contrary to the cases studied in the preceding section, the two equilibrium no longer always appear simultaneously. Indeed, for certain parameter values, only one perfect equilibrium exists, whereas, for the intermediary values of z and s, the two equilibrium coexist. The following proposition reviews the situation on these results:

Proposition 1 *:* : Since a certification system is defined by a pair of parameters $(z, s) \in \left[0, \frac{1}{4}\right]^2$:

* when $s > f(z)$, the equilibrium for which the certified firm produces the high quality good is the only sub-game perfect equilibrium.

* when $s < g(z)$, the equilibrium for which the certified firm offers the low quality good is the only sub-game perfect equilibrium.

* when $g(z) < s < f(z)$, both sub-game perfect equilibrium exist.

Proof : is immediately concluded from lemmas 2 and 3.

The following figure illustrates proposition 1 in plan (z, s) :

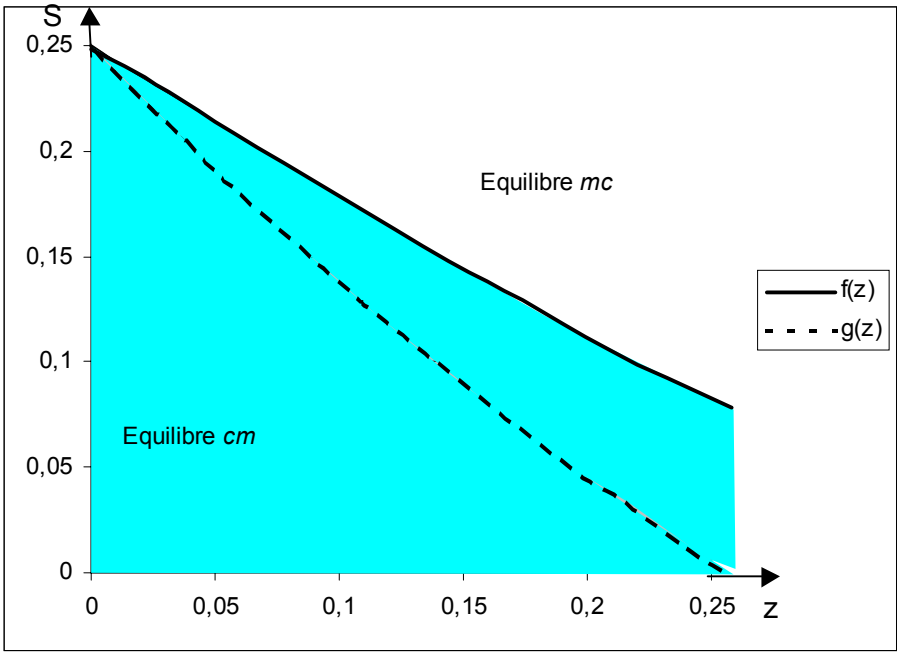

Figure 2 :Configuration of equilibrium *cm* and *mc*

In the above figure, as soon as $s > f(z)$, the only equilibrium which emerges is that for which the certified firm produces the high quality good. Indeed, when starting from the equilibrium situation where the certified firm produces the low quality good, and when $s > f(z)$, the level of certified quality s is high enough in relation to a less compelling capacity restriction, so that it becomes too expensive for the brand firm to keep its position of high quality. It is then in their interest to leapfrog with low quality, that is, to offer an inferior quality to its competitor. Inversely, when $s < g(z)$, the capacity restriction which the certified firm must respect is relatively strong in relation to the level of quality s guaranteed by this system, so that in starting from a equilibrium situation where the certified firm produces the high quality good, the low quality brand firm is moved to produce a higher quality than the certified firm. Thus, as soon as $s < g(z)$, the only perfect equilibrium which emerges is that for which the certified firm offers the low quality good.

In the framework of a sequential game with a strategic choice of quality and a competition of prices between two countries, BW shows how the choice of an import quota

permits the firm from the home country to position itself as leader in quality on the market and by the way, to obtain the higher profit (considering the result of Lehman-Grube). We obtain here a similar result since the choice of an appropriate certification system (z, s) can lead to a single equilibrium. Nonetheless, within the framework of our model becoming the producer of high quality does not permit obtaining the best profit systematically. Thus, we will demonstrate that because of existing asymmetry between the firms, the Lehmann-Grube (1997) result is no longer always verified.

Lemme 5 *:* Whatever the pairs (z, s) such as $s > g(z)$, the certified firm by offering the high quality product, makes a smaller profit than that of the brand firm which offers the lower quality. A function $h(z)$ exists decreasing in z, so that if $h(z) \leq s \leq f(z)$, the brand firm by offering the low quality product, makes a better profit.

Finally, when the certified firm offers the high quality good, they always make an lower profit than that of their competitor, whereas, when the certified firm offers the low quality good it can make a lower profit than its high quality competitor. Indeed, a capacity restriction z less than $\frac{1}{4}$ is much more restrictive for a high quality firm which, in the absence of a capacity restriction, would offer a greater quantity of the good than a firm which offers the low quality product. Therefore, contrary to BW, if the choice of a particular system (z, s) can steer the role of the certified firm, that is, the one to produce the high quality or the low quality of the good, it can never guarantee obtaining the best profit. On the contrary, the choice of a particular (z, s) can guarantee the best profit to the brand name competitor.

Undoubtedly, the two firms make the best profit when the certified firm offers the high quality good. The certified firm improves its profit because it offers the high quality, however it is then restricted even more by quantity, the competition finds itself even more diminished which favours the competitor. The two firms could come to a tacit agreement on such an equilibrium.

5 Competition certification-brand vs. brand-brand

We will study, firstly, the incentives of a low quality or high quality firm within the framework of the brand-brand competition to adopt the certification system which is

proposed to it unilaterally.

5.1 Profitability of the certification strategy

Even if a certification organism exists, the request for certification is always a matter of a private decision on the part of the producer. Under what conditions based on the (s, z) parameters would adherence to a certification system be profitable for a firm?

As it is represented in figure 3, we can distinguish three types of answers in function of the level of the parameters (s, z).

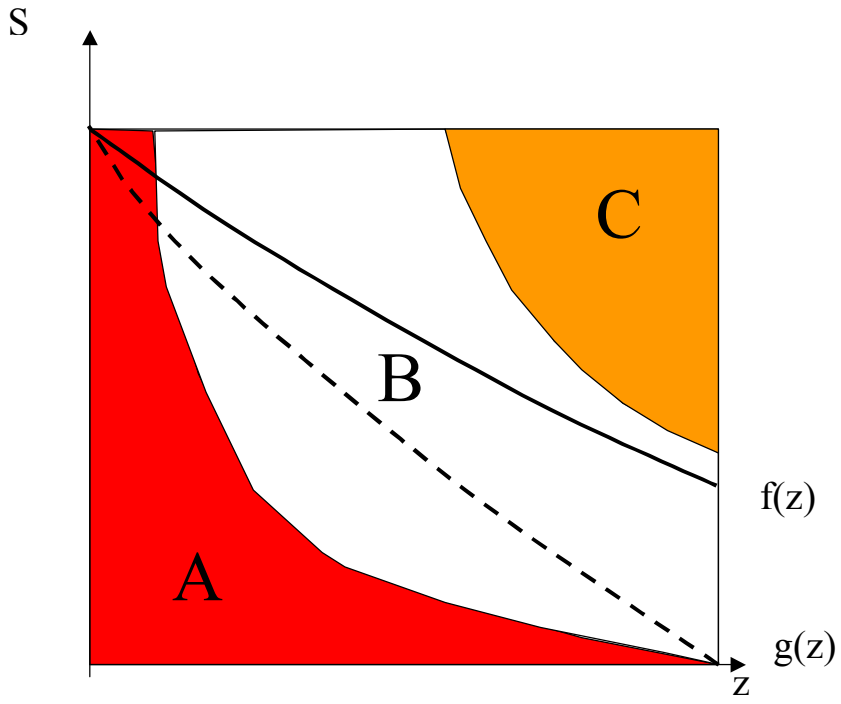

Figure 3 : Profitability of the adoption of a certification strategy

The equations for the limits of these three regions are given in the annex. In zone **A**, the certification system is too unfavourable for a firm to choose to adopt it. Particularly, if s is nil, taking on a capacity restriction can never be advantageous for a firm whether it be placed initially with high quality or with low quality. In zone **B**, the certification system becomes more advantageous and this time a brand firm with low quality will systematically choose to be certified. If (z, s) is sufficiently high, the certification system will even be able to allow it to automatically place itself with high quality. Finally,

in zone **C**, whether or not the firm be initially of low or high quality, it will choose to become certified and systematically offer the better quality good. Therefore, a firm initially offering the high quality in the brand-brand context, will never accept being certified if it risks, finding itself with low quality by doing so. Finally, as the certification system becomes both less restricting and more estimable, it is more quickly favourable to the producer that was with low quality initially.

5.2 Consequences of certification on average quality

We will first analyse the theoretical case for which the certification level is offered gratuitously to one of the firms, that is, without imposing the capacity restriction on it in return (s can then be interpreted as simply a brand image advantage acquired by the firm). After that, we will analyse the combined effect of s and of z on the chosen qualities.

■ **If the certified firm offers low quality**

In the case where s is offered gratuitously to the low quality firm, the firm should naturally increase its quality. The competing brand name firm should also raise its quality, but this time for inciting differentiation regarding its competitor. However, as soon as we add a capacity restriction to a fixed s, the competing brand name firm should reduce its investments in quality from now on. When s increases this restraint effect is reinforced. This investment restraint can be explained with the help of the following graph:

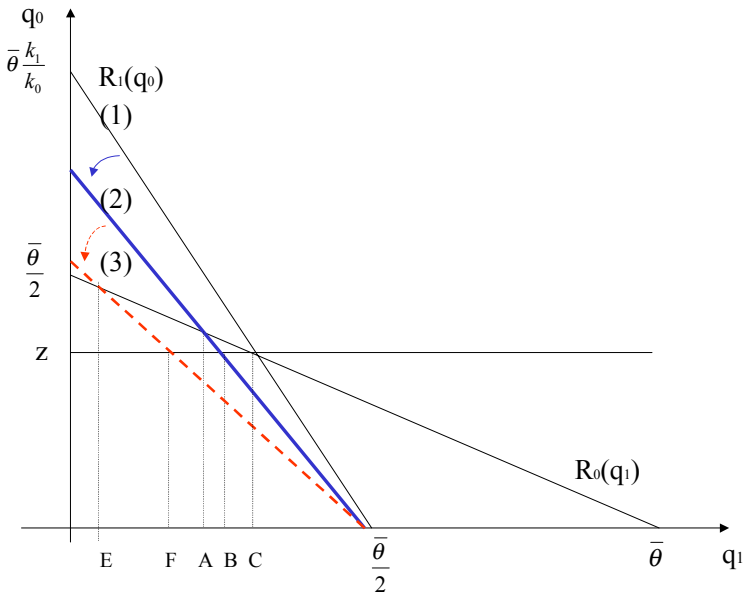

Figure 4: Effect of an increase of s on the incentive to invest in quality

First, let's consider the function of the best reaction (1) of the brand name firm for a fixed level of the parameter s. The impact of a reduction of quality k_1 on the loss of quantity q_1 of the brand name firm can be seen by going to the reaction function (2). For a same reduction in quality, the brand name firm under goes a loss of demand BC if its competitor has a capacity restriction and AC if it is not subject to the restriction. The ratio $\frac{BC}{AC}$ is inferior to 1, which explains the effect of the restraint in quality investment felt by the brand name firm when its competitor has a capacity restriction. This curve (2) which corresponds to a reduction of k_1, can also well correspond to an increase of s at fixed initial quality level (indeed, s makes k_0 increase). If we now reproduce a new equivalent reduction of k_1, we move towards curve (3). This time the relative loss of demand incited in the presence of a capacity restriction in relation to a situation without a restriction, is written $\frac{FB}{EA}$. We proove that $\frac{FB}{EA} < \frac{BC}{AC}$, which permits us to conclude that when s increases, the restraint to invest by the high quality firm linked to the imposition of a capacity restriction on its competitor, is strengthened. This effect is principally due to the capacity restriction which engenders a rigidity in terms of loss of demand and leads to a lessening of quality on the part of the competitor. Indeed, when even the brand name firm's quality lessens, the consumers who would like to turn to the low quality product cannot do so because the quantity offered is restricted. It is paradoxical that this effect is even greater as s is increased. Undoubtedly, we could

have thought that when s increases engendering low quality, the incentive to differentiate would alleviate this effect of restraint.

■ **If the certified firm offers a high quality**

If the certified firm offers the high quality good, and if it is offered the certification level s gratuitously, it improves its quality whereas its competitor naturally reduces its investments. Then in studying the impact of an increase of s to z fixed, we verify that if the certified firm offers high quality, it continues to increase its investments whereas the certified firm keeps its investments unchanged.

We can then set foth the following proposition :

Proposition 2 : When the certified firm offers the low quality good, the average quality which is made available on the market to consumers decreases in relation to the equilibrium of brand competition. On the other hand, if the certified firm offers the high quality good, the average quality of the products offered to consumers decreases when $s < t(z)$, with $t(z) < f(z)$.

Indeed, when the certified firm offers low quality, it increases its quality but the quantity which it offers on the market diminishes because of the capacity restriction which it must respect. In addition, the high quality firm decreases its quality and what is more, offers a superior quality to that offered within the structure of brand-brand equilibrium. In all, the average quality decreases. If the certified firm offers the high quality, it increases its quality as soon as the certification level is sufficiently high, but its quantity is restricted (even more) because of its position of high quality. The low quality firm's quality remains unchanged so that in the end, it is the decrease in quantity produced of the high quality good that prevails and average quality decreases.

5.3 Consequences on the consumers' surplus

We will now study the influence of the unilateral adoption of a certification system on the surplus of consumers on the home market in relation to the reference situation of brand competition.

The expression of consumers' surplus in each of the two cases is written :

$$\left| \begin{array}{l} SC^{cm} = \frac{1}{8}\left(1-z^2\mu^2\right)\left(\frac{1}{4}+\frac{1}{2}\mu z+\mu z^2-\frac{3}{4}\mu^2 z^2\right) \\ SC^{mc} = \frac{1}{8}\left(1-z\right)^2\left[\left(1-z\right)z+\frac{z^2}{\mu}+\frac{(1-z)^2}{4}\right] \end{array} \right. \tag{14}$$

The functions SC^{cm} et SC^{mc} are increasing in μ. In other words, the consumers' surplus increases when the differentiation of the products diminishes, the effect of increased competition gaining over the beneficial effect related to the segmentation of the market. Both the expressions are naturally increasing in z. The less restrictive the certification system is in terms of quantity, the larger the global quantity put on the market is, which, of course, benefits consumers.

Proposition 3: : If the certified firm offers the high quality product, then the consumers' surplus is always weakened in relation to the competitive brand situation. On the other hand, if the certified firm offers the low quality product, a function $r(z) \leq f(z)$ exists so that if, $r(z) \leq s \leq f(z)$, the consumers' surplus increases in relation to the equilibrium of brand competition.

The consumers' surplus is at its maximum when the entire quantity is placed on high quality. Finally, when it is the quantity placed on the high quality good that is restricted by the certification system, the capacity restriction is relatively more prejudicial to consumers than when the certified firm offers the low quality good. The negative effect related to quantities always prevails even over the moderate improvement of the quality which can have a positive influence on the surplus.

When $r(z) \leq s \leq f(z)$, the product differentiation is very small compared to the brand competition situation. In addition the capacity restriction is relaxed, in such a way that the effect of the intensification of competition prevails over the damaging effect of the capacity restriction. The consumers' surplus is therefore increased. This result according to which the consumers' surplus can be improved in relation to a competitive brand situation, when the firm from the home country chooses to become certified and to offer the low quality good, is surprising. Indeed, we could naturally think that if s is strong enough to compensate for the capacity restriction, then this effect on the surplus was to be expected, however, the increase in the consumers' surplus would bring about then an improvement in the quality of the products offered. On the contrary, we have seen that when s increases, the average quality diminishes, and it is, after all, the

phenomenon of product standardisation which, leading to a tougher competition, allows consumers to benefit from a better quality-price ratio.

6 Conclusion

This paper has proposed the formalization of a certification system through control of the offer. In our model, we assume that the certification system is an exogenous given from the producer's point of view, and we analyse its implications in terms of profit for the firm that adopts it with average quality products and consumer surplus. We show how a certification system of the AOC or label of origin type, initially designed in order to promote the quality of the products and protect consumers, can be used as an arm in the competition game when it confronts a brand name production system. Inversely, we show how the adoption of this type of system for the reasons mentioned above or simply for individual profit improvement prospects, can, on the economic level, be in contradiction with its initial objectives. In reality, it appears through out model that the only way to effectively improve the consumers' surplus is to promote an equilibrium which is not too restrictive on the quantitative level and such that the certified firm positions itself on the low quality segment. This equilibrium corresponds paradoxically to a situation where the products are rather standardised and where the average quality is lower in relation to that which would prevail with a brand competitor. Moreover, the only means of achieving the best profit in the industry and therefore win the competition game, so to speak, is, once again, for the certified firm to offer the low quality good while continuing to respect the not too strenuous capacity restriction. This result is paradoxical and in contradiction, for example, with the French policy adopted by INAO[4] in the wine growing sector which, wanting to strengthen the reputation of the image and the virtuous quality of the AOC wines is on the way to adopting more restrictive production constraints. Such a policy would automatically risk driving the certified firm to position itself in the high quality area which, on the one hand would be damaging to consumers and on the other, would play the competition game by allowing it to make a better profit.

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7 Appendix

7.1 $z < 0.25$

If one is certified : when $q_h^{cm} = \frac{k_h - k_l K}{2k_h}$, $q_l^{cm} = R_l(q_h) = \frac{1-\left(\frac{k_h - k_l q_l}{2k_h}\right)}{2} > z$, for every μ, if $z < \frac{1}{4}$.

When $q_l^{mc} = \frac{1-z}{2}$, $q_h^{mc} = \frac{k_1 - k_0\left(\frac{M-z}{2}\right)}{2k_1} > z$, for all μ, if $z < \frac{1}{3}$. So if $z < 0.25$, Then a certified firm is always constrained in capacity.

7.2 Lemme 1: $s(z,\mu)$ / $\frac{\partial s(z,\mu)}{\partial z} < 0$, and $\frac{\partial s(z,\mu)}{\partial \mu} > 0$.

$\frac{\partial s^{cm}(z,\mu)}{\partial \mu} = (1-4z^2) - 3z^2\mu^2$ is decreasing in z so, for $z = \frac{1}{4}$, $\frac{\partial s(z,\mu)}{\partial \mu} = \frac{3}{4}\left(1 - \frac{1}{4}\mu\right) > 0$, whatever $\mu \in [0,1]$.

$\frac{\partial s(z,\mu)}{\partial z} = z\left(4(1-\mu) - \mu^3\right) - 1 < 0$; This formula is strictly descreasing in μ, so a sufficient condition to be respected is that $\frac{\partial s(z,\mu)}{\partial z} < 0$ in $\mu = 0$. Because $z < \frac{1}{4}$, this condition is true when $\mu = 0$.

7.3 Certification-Brand Equilibria

7.3.1 When the certified firm offers the highest quality

$q_l^{mc} = \frac{M(1-z)}{2}$, $q_h^{mc} = zM$. In equilibrium, qualities k_l^{mc} et k_h^{mc} are as follows : $k_l^{mc} = \frac{(1-z)^2}{4}$ and $k_h^{mc} = (1-z)z + s$. Then profits are the following : $\pi_h^{mc} = \frac{(1-z)z}{8}\left[6z - 5z^2 + 8s - 1\right]$ and $\pi_l^{mc} = \frac{(1-z)^4}{32}$.

7.3.2 When the certified firm offers the lowest quality

$q_l^{cm} = zM$, $q_h^{cm} = \frac{M(1-\mu z)}{2}$, with $s = \frac{1}{4}\left[(1-4z^2)\mu - z^2\mu^3 - 2z(1-2z)\right]$. This formula defines μ for all value of (z,s). $k_h^{cm} = \frac{\overline{\theta}M}{4}(1 - z^2\mu^2)$ combined to the preceding equation allow us to find the equilibrium qualities. Similarly, profits are : $\pi_h^{cm} = \frac{k_h}{4}(1-\mu z)^2 - \frac{k_h^2}{2}$ et $\pi_l^{cm} = \frac{k_l^{cm}}{2}\left(1 + z(\mu - 2)\right)z - \frac{\left(k_l^{cm} - s\right)^2}{2}$.

7.4 Lemme 2 : $f(z)$ is decreasing in z, $f(z) < s^{cm}(z,1)$

■$f(z)$ / $\Pi_1^{leap}(k_1^{leap}) = \Pi_1(\mu, z)$ with $k_1^{leap} = \frac{(1-z)^2}{4}$ et $\Pi_1^{leap}(k_1^{leap}) = \frac{(1-z)^4}{32}$.

Then $f(z)$ is defined by both following equations :

$$f(\mu(z), z) = \frac{1}{4}\left[\left(1-4z^2\right)\mu - z^2\mu^3 - 2z(1-2z)\right]$$

with $\mu(z)$ the implicit function defined by :

$$\eta(\mu, z) = (1-z)^4 - \left(1-z^2\mu^2\right)\left(1+3\mu^2z^2-4\mu z\right) = 0 \tag{15}$$

We easily prove that $\frac{\partial\eta(\mu,z)}{\partial z} = 6-2\mu^2-8(1+\mu^3)z+3(1+3\mu^4)z^2$ is always decreasing in z and in μ. $\frac{\partial\eta(1,\frac{1}{4})}{\partial z} > 0$. So $\frac{\partial\eta(\mu,z)}{\partial z} > 0$.

Likewise $\frac{\partial\eta(\mu,z)}{\partial\mu} = 4(1-\mu z)(1-3\mu^2z^2) > 0$ iff $\mu z < \frac{1}{\sqrt{3}}$ and this condition is always verified.

So $\mu'(z) < 0$. Finally, a sufficient condition for that $\frac{df(\mu(z),z)}{dz} < 0$ is that $z < \frac{1}{\sqrt{7}} \simeq 0,378$.

So $f(z)$ is monotonic and decreasing in z.

■ $f''(z) = 8\left(1-\mu-\frac{1}{4}\mu^3\right) - 16z\mu'(z) - 12z\mu^2\mu'(z) - 6\mu z^2\mu'(z) + \mu'(z)(1-4z^2-3z^2\mu^2)$.

A sufficient condition for that $f''(z)$ be positive is that $\mu''(z) > 0$,which is always verified. So the function $f(z)$ is convex. we now compare $f(z)$ with $s^{cm}(1,z) = \frac{1}{4}[1-z^2-2z]$ which is strictly decreasing and concave in z. $s^{cm}(1,0) = \frac{1}{4}$, et $s^{cm}\left(1,\frac{1}{4}\right) = 0,109$. However when $z = 0$, $\mu = 1$ and $f(\mu,z) = \frac{1}{4}$ and when $z = \frac{1}{4}, \mu = 0,787$ and $f(\mu,z) = 0,0774$. So $f(z) < s^{cm}(1,z)$ for all z.

7.5 Profit frontier

■ If a brand firm who offers the highest quality chooses to certify, then

→ When does it improve its profit?

* If it remains the highest quality producer:

$\pi_1^{mc} > \pi_1^m <=> s > \frac{0.01952}{z(1-z)} + \frac{1}{8}(5z^2-6z+1)$.

* If it offers the lowest quality product :

$\pi_0^{cm} > \pi_1^m ? <=> \frac{d\pi_0^{cm}(\mu)}{d\mu} = 1-z^2-3\mu^2z^2-3\mu^2z^4 > 0$ decreasing in z and in μ. When $z = 0.25$ and $\mu = 1$, $\frac{d\pi_0^{cm}(\mu)}{d\mu} = 0,727 > 0$.

So, $\frac{d\pi_0^{cm}(\mu)}{d\mu} > 0$,and for $\pi_0^{cm}(\mu=1) = \frac{1}{8}(1-z^2-2z+z^3)$

$Max_z\ \pi_0^{cm}(\mu=1) = 0.014355 < 0.0195$.

So, $\pi_0^{cm} < \pi_1^m$

$\rightrightarrows$ Does its competitor increase its profit?

* If the certified firm remains the highest quality producer

$\pi_0^m < \pi_0^{mc} <=> z < 0.457 =>$ always verified.

* If it offers the lowest quality product:

$\pi_0^m < \pi_1^{cm} <=> z < 0.289 =>$ always verified.

So, when the highest quality producer chooses to certify, it is always profitable for its competitor.

■ If the brand firm who offers the lowest quality product chooses to certify

$\rightarrow$ When does it improve its profit?

* If it remains the lowest quality producer :

$\pi_0^m < \pi_0^{cm} <=> s > s_1(z).s_1(z)$ is decreasing in z.

* If it offers the best quality

$\pi_0^m < \pi_1^{mc} <=> s > \frac{0.0027}{z(1-z)} + \frac{1}{8}(5z^2 - 6z + 1)$.

$\rightrightarrows$ Does its competitor increase its profit?

* If the certified firm remains the low quality producer :

$\pi_1^m > \pi_1^{cm} <=> s < s_2(z)$.

* If the certified firm offers the high quality

$\pi_1^m > \pi_0^{mc} => z < 0.111$. So if $z < 0.111$ then the competitor benefits from the certification of the domestic firm.

7.6 Consumers' surplus

The general expression of surplus with two qualities is : $SC(k_l, k_h, q_l, q_h) = \frac{1}{2}[k_l q_l^2 + k_h q_h^2 + 2k_l q_l q_h]$.

$SC^{cm} = \frac{1}{8}(1 - z^2\mu^2)\left(\frac{1}{4} + \frac{1}{2}\mu z + \mu z^2 - \frac{3}{4}\mu^2 z^2\right)$

$SC^{mc} = \frac{1}{2}\left(\frac{(1-z)^2}{16} + \frac{z(1-z)^3}{4} + z^3(1-z) + sz^2\right) = \frac{1}{8}(1-z)^2\left[(1-z)z + \frac{z^2}{\mu} + \frac{(1-z)^2}{4}\right]$

7.7 Average quality

$\overline{k^{cm}} = \frac{\left(1-\mu^2 z^2\right)(1+\mu z)}{(1+2z-\mu z)}$, knowing that $\frac{\partial \mu}{\partial s} > 0$, $\frac{\partial \overline{k^{cm}}}{\partial s} > 0$ if $1 - 2\mu z - 3(\mu z)^2 > 0$ that is iff $\mu z < \frac{1}{3}$. Average quality is increasing in s.