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# Real Options and Competition: The Impact of Depreciation and Reinvestment

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## Real Options and Competition: The Impact of Depreciation and Reinvestment

### 1. Introduction

One of the most important developments in economics during the last decades was the recognition

that the Net Present Value (NPV) criterion in investment theory can be misleading under certain conditions. These conditions are: the returns of an investment are subject to an ongoing uncertainty, the investment is (at least partly) irreversible (i.e. the investment causes sunk costs), and the investor can suspend the investment decision for some time. If all these conditions are fulfilled, even in case of risk neutrality, it is not necessarily optimal to invest if the expected present value of the future returns covers the investment outlays. Rather, one should assign a positive value to the preservation of the flexibility whether to invest or not; in other words, waiting for new information has a value.

This insight led to the development of the real options approach to investment (Henry, 1974a, McDonald and Siegel, 1986, Pindyck, 1991).<sup>1</sup> It exploits the analogy between a financial option and a real investment. The opportunity to conduct an investment can be compared with a call option on financial markets: like the owner of a call, the investor has the right but not the obligation to pay a fixed sum  $I$  and to receive a stochastic cash flow with an expected discounted value  $V$ . While classical investment theory tells us this investment opportunity is worth  $V-I$ , i.e. the NPV, it is well known from the theory of financial derivatives that  $V-I$  measures only one part of the value of the option to invest, namely the intrinsic value. In addition,

the opportunity to invest has a continuation value, which is the discounted value of the expected appreciation of the option. The option should only be exercised if the intrinsic value exceeds the continuation value (cf. e.g. Dixit and Pindyck 1994).

Unfortunately, the practical application of the real options approach is not that easy. Analytical solutions of optimal investment triggers only exist for rather restricted situations, for example, if the expected returns of the investment follow a geometric Brownian motion (GBM) and the investment option never expires. Thus, for practical applications of the real options approach one either has to find evidence that the assumptions of a GBM and of an infinite lifetime of the option are fulfilled. Alternatively, one has to resort to approximation techniques to price them. Hull (2000), for instance, provides an overview of various methods.

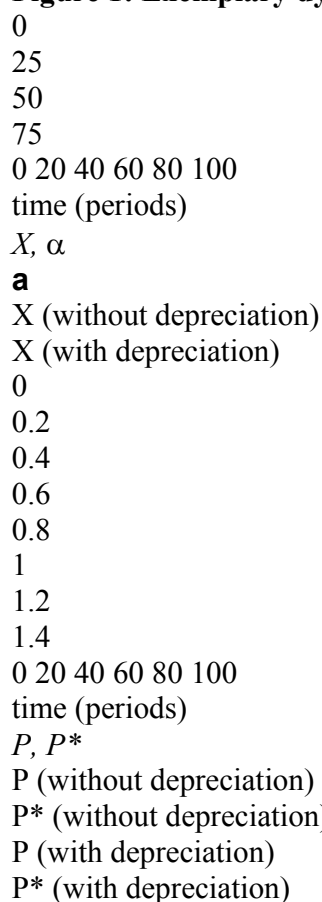
A look at the literature reveals that very often the first strategy is chosen: Authors take time series data on prices or returns for a given branch or market and apply unit root tests to find evidence for a random walk. Then the volatility of the returns is estimated and taken to compute the optimal investment trigger (e.g. Pietola and Wang, 2001; Bessen, 1999). However, for competitive industries this “standard procedure” seems to be problematic, because the evolution of the returns is hardly purely exogenous, as implicitly stated by the GBM assumption. Rather the evolution depends to some extent on the behavior of competitors. Accordingly, one could argue that deferring an investment until prices or returns are at least equal to

<sup>1</sup> The idea that the preservation of unique environmental goods and of historical buildings has an option value was first proposed by Arrow and Fischer (1974) and Henry (1974b).

the investment trigger may be inferior because competitors could enter the market at lower prices and prevent prices to rise. Dixit and Pindyck (1994), however, find that this argument does not hold. They show for certain settings that the optimal investment trigger  $P^*$  is not affected

by competition, i.e. the investment trigger is the same for exclusive investment options and for investment options under competition. Nevertheless, Dixit and Pindyck find that the price dynamics is somewhat different: The investment trigger forms a kind of reflecting barrier. As long as prices are lower than the trigger price, prices follow a geometric Brownian motion. If market conditions prosper, prices rise up to the trigger price and additional firms enter the market and prevent prices to rise above the trigger. If thereafter market conditions worsen, then those firms that have invested continue production and prices decline proportional to the market conditions. Figure 1 shows the dynamics for a price  $P_t$ , a demand parameter  $\alpha_t$ , output  $X_t$ , with  $P_t = \alpha_t / X_t$  and  $\alpha_t$  follows GBM.<sup>2</sup>

**Figure 1: Exemplary dynamics in competitive markets\***



\* Prices just above the trigger arise because of a time lag in production response.

During the last years several authors have taken the finding of Dixit and Pindyck (1994) as an argument to ignore competition and to apply the already mentioned “the remainder of this paper, we will demonstrate that for many investment decisions this procedure

is not appropriate. The reason behind is a central assumption in the Dixit and Pindyck

<sup>2</sup> A particular implication of this price dynamics is that in average competition does not allow for profits. Investment at the trigger price  $P^*$  just fulfils the zero profit assumption which is a central equilibrium condition for competitive markets. Lower investment triggers imply losses and are consequently inferior. Higher triggers do not allow for profits because they do not allow to exercise the investment option.

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framework: the assumption that assets have an infinite lifetime. Consequently, the aggregate

output on the market may increase over time but it cannot decline as it is shown in figure 1, i.e. if assets do not need to be replaced and if prices cannot become negative (which is implicit for GBM) then the asset will be used for an infinite time. However, if one assumes that assets are subject to decay or that they have a limited lifetime, the price dynamics changes: There still is a certain trigger price that forms a reflecting barrier for an increasing demand parameter  $\alpha_t$ . However - and in contrast to the Dixit and Pindyck model - a decrease of  $\alpha_t$  can at least partly be compensated by a subsequent output decrease if there are some “depreciated” production facilities that will not be replaced because expected prices are lower than the trigger price. Hence, downward price reactions are dampened as it is shown in figure 1. Consequently, under competition the equilibrium investment trigger for assets with finite lifetime is lower than for identical investment opportunities that are exclusive. In principal, one could argue that the damping effect of depreciation causes a lower price volatility. Consequently, the application of the “standard procedure” may lead to lower investment

triggers anyway, i.e. the “standard procedure” may be appropriate. Our analysis does not support this conclusion. On the contrary: We find that the estimated price volatility does not significantly differ from the volatility of the demand parameter  $\alpha_t$ . Moreover, under certain conditions unit root tests fail as well. Thus - as already mentioned - we conclude that the “standard procedure” is not appropriate!

Our results are obtained by a discrete time agent-based approach in which  $N$  agents represent  $N$  identical farms (or more generally: firms) which compete on a certain market. Each of these firms possesses its individual investment trigger which is derived by linking the agent-based model with a genetic algorithm (cf. Arifovic, 1994). In section 2 the firms’ investment problems, their interaction, as well as the link to the genetic algorithm (GA) are presented in detail.

In section 3 results are presented and analyzed. Moreover, we identify a direct rule of determining the price dynamics in competitive markets with depreciable assets. This rule allows us to validate our findings as well as to compute investment triggers for different parameter settings with less computational effort than the agent-based approach. In section 4 the approach and our findings are summarized and discussed.

## 2. The Model

### 2.1. The investment problem

Consider a number of  $N = 50$  firms, each having repeatedly the opportunity to invest in identical assets or a fraction thereof, i.e. the assets are divisible. Initially no firm has invested. The asset has a maximum size of 1 and can be used by firm  $n$  to produce up to  $1, = n_t x$  unit of output per production period. Size, investment outlay and production are proportional. If a firm invests for the first time, its maximum initial investment outlay  $\max$

$, n_t M$  is  $I$ . The investment

outlay  $M_{t,n}$  is considered to be totally sunk after the investment is carried out. For every period, we consider a geometrical decay of the asset. The asset's productivity declines to  $(1-l)$

<sup>3</sup> Note, we understand depreciation as the deterioration of assets with increasing age.

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of the previous period's output, i.e. we consider a depreciation rate  $l$  such that

$n_t n_{t+1} x, , ) 1 ( \cdot - = \Delta + l$  .4 However, in every period, each firm can invest or reinvest in order to increase production or to regain a production capacity of up to one unit of output. The outlay  $M_{t,n}$  then has a maximum amount  $\max$

$, n_t M$  depending on the missing production capacity, i.e.



$${}_t x X$$

(3)

and demand is

$${}_t D$$

$${}_t P$$

$$X$$

$$a$$

= (4)

For identity of demand and supply, we get

$$S$$

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$$D$$

$$t$$

$${}_t X X$$

$$P$$

$$a a$$

= (5)

Consider now that the demand parameter  $\alpha_t$  follows geometric Brownian motion (GBM).

Assuming

discrete time and assuming the absence of a drift rate this can be modeled as

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(6)

with a volatility  $\sigma$ , a normally distributed random number  $\varepsilon_t$  and a time step length  $\Delta t$ . Note

that  $\alpha_t$  is the expected future demand parameter  ${}_{t t \Delta +} \hat{a}$  for GBM.

Firm  $n$  invests in period  $t$  if the expected price  ${}^*$

$${}_{n t t} P P \geq {}_{\Delta +}$$
 with

<sup>4</sup> The use of the decay parameter  $l$  is analogous to the probabilistic approach presented in Dixit and Pindyck (1994, pp 200). To understand this, simply consider that any firm  $n$  actually considers of an infinite number of identical infinitely small firms.

<sup>5</sup> Note, that equation (15) implicitly assumes risk neutrality.

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$I D (8)$

The questions now are: Which firms invest? And how much do they invest? Therefore, let us assume that firms with lower trigger prices \*

$nP$  have a stronger tendency to invest. Consequently,



all firms can be sorted according to their trigger prices, starting with the lowest investment trigger, i.e. \*

1  
\*

$+ \leq_n P P$ . The following propositions are straightforward:

**Proposition 1:** If firm  $n$  does not invest in  $t$  then firm  $n+1$  will also not invest in  $t$ , i.e.

$0 \ 0 \ 1, = \Rightarrow = + M M_{n t n t}$

**Proposition 2:** If firm  $n$  does invest in  $t$  then firm  $n-1$  will invest  $\max$

$1, -_n t M$  in  $t$ , i.e.

$1 \ 0 \ 1, 1$

$\max$

$1, 1, = \Rightarrow = \Rightarrow > - + - - x M M M_{n t n t n t n t}$

**Proposition 3:** In every period  $t$ , a marginal (or last) firm  $o$

$_{t n}$  exists which invests  $\max$

$_{, o n t} M$ , and

$N n_o$

$_{t \leq} \leq 0 .6$

If one would assume that any firm  $o$

$_{t n} >$  does neither invest nor reinvest, then total production

in  $t + \Delta t$  would be

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Accordingly, the expected price in  $t + \Delta t$  would be

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$D(10)$

Now,  $\circ$

$t n$  can be identified by iteratively testing all firms for  $\circ$

$\circ$

$t$

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$t t n P P_{\Delta+} \leq \hat{*}$ . The last firm that

fulfils the investment condition (10) is  $\circ$

$t n$ .

According to proposition 3 and the subsequent considerations, we only consider firms which

either invest  $\max$

$, n t M$  or 0. However, we may find the situation that  $\circ$

$\circ$

$n$

$t t n P P_{\Delta++} \leq \hat{*}$

1. In this case we

can consider that firm  $1 + \circ n$  can invest  $1, + \circ n t M$ , with  $\max$

$1, 1, 0++ < \circ \circ n t n t M M$  and without violating

the condition that the trigger price is less or equal to the expected price. Based on equa-

tions (7), (8), and (10) we can derive the condition

$\circ$  Notice,  $n \circ$

$t$  is zero if there is no investor in period  $t$ .

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**a**

(12)

Equation (12) is an equilibrium condition: All firms which fully invest and hence produce at maximum capacity have trigger prices which are less or equal to the trigger price of firm

$1 + \alpha n$  which is also equal to the expected price for  $t + \Delta t$ . All firms which do not invest have

trigger prices which are higher than or equal to the expected price for  $t + \Delta t$ .

For a given set of trigger prices  $P^*$  and arbitrary initializations for  $\alpha_0$ , the expected profitability

of each strategy

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can be determined simultaneously by a sufficiently high number of repeated stochastic simulations

of the market. For our analysis, we consider 5000 repetitions to be sufficient. The remaining question is, how to determine appropriate sets of trigger prices \*

$P_n$ ? For this, the  $N$  firms

market model is combined with a genetic algorithm (GA).

## **2.2. The Genetic Algorithm and its implementation<sup>7</sup>**

GA are a heuristic optimization technique which has been developed in analogy to the concepts of natural evolution and the terminology used reflects this. Even though there is no “standard GA” but many variations of GA, there are some basic elements which are common to all GA (cf. Holland, 1975, Goldberg, 1989, Forrest, 1993, Mitchell, 1996).<sup>8</sup> The first task of an application of GA is to specify a way of representing each possible solution or strategy as a string of genes which is located on one or more chromosomes. Usually this is achieved by representing solutions (e.g. strategies, numbers, etc.) as binary bits, i.e. zeroes or ones, which form the genes. Since our problem is relatively simple, i.e. we just search for a single value (i.e. every strategy just consists of a certain trigger price), we take the investment trigger as a real value and apply the GA operators to the trigger price itself. The second task is to define a population of  $N$  genomes to which the genetic operators, i.e. selection, crossover and mutation, can be applied. The population size here is 50 genomes. This allows us to directly map the set of genomes to the firms' strategies, i.e. every firm's trigger price in our model is represented by one genome of the genome population. Vice versa every genome can be understood

as the strategy of a certain firm.

Each application of the genetic operators to the population of genomes creates a new, modified generation of genomes. The number of generations depends on the problem to be solved.

It can range from some 50 to a couple of thousand. In most GA applications the first genera-

<sup>7</sup> The following representation of GA draws on Balmann and Happe (2001).

<sup>8</sup> For other GA-applications to real options cf. Balmann, Mußhoff and Odening (2001) and Diaz (2000).

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tion of genomes is initialized by random values or it is set arbitrarily. During the following generations, the genome population passes through the following steps:

### *a) Fitness Evaluation*

Each time before the GA operators b) to d) are applied, the goodness of every genome is evaluated by applying a fitness function. This function assigns a score to each genome in the current population according to the capability of the genome strategy to solve the problem at hand. The better the strategy performs, the higher its fitness value. For our applications, the fitness value is directly derived from the strategy's average profitability  $P_n(P_n^*)$  or payoff in 5000 stochastic simulations of the market model.

### *b) Selection and Replication*

Selection determines the genetic material to be reproduced in the next generation. The fitter the genome (i.e. the better adapted it is to the problem) the more likely it is to be selected for reproduction. Selection can be implemented in many different ways. In this model the 20 most successful genomes always survive. The next 15 genomes are replaced with a certain likelihood by the 15 most successful genomes of the last simulation series. The next 10 genomes are replaced by the 10 fittest genomes with a higher likelihood. And the least 5 successful genomes are always replaced by the 5 most successful genomes. Summarizing, the 5 most successful genomes can quadruplicate, the next 5 can triplicate, and the next 5 most

successful  
strategies can double.

#### c) Crossover

Figure 2 shows the simplest case of a 1-point-crossover, where the coded strings of two parent genomes are split at a randomly chosen locus and the sub-strings before and after the locus are exchanged between the two parent genomes resulting in two offspring. This technique is also used for our GA implementation. With a certain likelihood, for every genome  $a$  a partner  $b$  is randomly chosen from the selected genomes. The values are cut at a randomly chosen digit. If e.g., the numbers are cut after the third digit, offspring  $a'$  gets the first three digits of parent  $a$  and all further digits of parent  $b$  and vice versa. Thus the triggers  $a=1.2345678$  and  $b=1.1111111$  become  $a'=1.2311111$  and  $b'=1.1145678$ .

#### Figure 2: Example of a 1-point-crossover after the 3<sup>rd</sup> digit

parent genomes    offspring genomes

$a \dots 1\ 2\ 3\ 4\ 5\ 6\ 7\ 8 \dots$      $a' \dots 1\ 2\ 3\ 1\ 1\ 1\ 1 \dots$

$b \dots 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1 \dots$

→

$b' \dots 1\ 1\ 1\ 4\ 5\ 6\ 7\ 8 \dots$

#### d) Mutation

Mutation also brings new genetic varieties into the population of genomes. Furthermore, mutation

serves as a reminder or insurance operator because it is able to recover genetic material into the population which was lost in previous generations (Mitchell 1996). This insures the population against an early and permanent fixation on an inferior genotype. Mutation is implemented

here by multiplying every solution with a certain, but small likelihood with a random number between 0.95 and 1.05. The mutation likelihood as well as the range of the ran-

dom number may be chosen according to experience as well as according to the already obtained results. A flow diagram can be found in the appendix.

In one particular point our GA application deviates from conventional applications. Here, the GA is not just used to solve a more or less complex optimization problem in which the goodness of the solution and the problem at hand are directly related. In our case, the goodness of a solution rather depends on the alternative solutions generated by the GA. In other words: in conventional GA applications the fitness of a genome can be obtained directly from a comparison of payoffs of the different solutions because the payoffs are independent of the competing solutions. Here, a solution's payoff depends on the other solutions. Thus, we are applying the GA to a game theoretic setting and we are not searching for an optimal solution, but for an equilibrium solution, i.e. the Nash-equilibrium strategy.<sup>9</sup>

### 2.3. The scenarios

The model as it is presented above can be used for many different scenarios. However, our motivation is to demonstrate that the "standard procedure" of the real options approach (cf. section 1) leads to wrong results for reasonable assumptions, i.e. we argue that the standard approach overestimates the investment trigger. Hence, in order to falsify this approach, it is sufficient to demonstrate the principal impact of depreciation for one specific scenario. This specific scenario is based on an interest rate of  $r = 6\%$ ,  $I_{t=5\%} = 8.36364$ , and no further production costs. This implies total production costs of 1 per unit of output. The volatility  $\sigma$  is

assumed to be 0.2. For the case without depreciation, i.e.  $\lambda = 0$ , and average production costs of 1 the investment costs are adjusted to  $I_{I=0\%} = 16.66667$ . In order to consider that our model is based on discrete time steps while the theoretical literature usually is based on continuous time, we vary the time step length  $\Delta t$  from 1 to 0.1 and we will show that smaller time steps do not offer any evidence against our basic message.<sup>10</sup> The total time span  $T$  simulated in every stochastic simulation is determined as 100 years. For later periods the expected returns are set equal to the returns in year 100. The possible error can be assumed to be negligible since later returns are discounted by more than 99.7%.

As a reference system for our market model, we determine the investment triggers also for the case that output prices directly follow GBM. Investment triggers for this problem are determined in two alternative ways. Firstly, we also apply GA in combination with stochastic simulations. Secondly, we apply stochastic simulations to alternative, arbitrary trigger prices and search for the solution with the highest average profit. Though this is double work, the latter approach offers additional evidence that the GA-technique is appropriate.

<sup>9</sup> A number of publications during the past 10 years show that this approach functions quite well. Examples and discussions are given for instance in Arifovic (1994), Axelrod (1997), Balmann and Happe (2000), Dawid (1996), Dawid and Kopel (1998) and Chattoe (1998).

<sup>10</sup> For  $\Delta t < 1$ , the parameters  $\lambda$ ,  $r$ , and  $\sigma$  are adjusted, i.e.  $(1-\lambda_{\Delta t}) = (1-\lambda)_{\Delta t}$ ,  $(1-r_{\Delta t}) = (1-r)_{\Delta t}$ ,  $\sigma_{\Delta t} = \sigma \Delta t^{0.5}$ .

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### 3. Results

Table 1 shows the results of the presented procedure and allows to compare results with the analytical solution. Accordingly, the trigger prices in the case of exclusive options (i.e. without considering market effects) generated by the GA approach are quite similar to the arbitrary simulation results. For the scenarios with and without depreciation, these values differ by about 1%. We have carried out the procedures repeatedly with varying random number seeds and we have obtained very similar results. The differences could be reduced further by increasing the number of repetitions of the stochastic simulations. Unfortunately, this would increase the computational efforts substantially and 1% is an insignificant error regarding the purpose to show the effects of competition and depreciation simulations.

**Table 1: Trigger prices depending on depreciation and competition ( $s = 0.2$ )**

Monopoly / exclusive option Competition

$\Delta t$  Infinite lifetime

( $\lambda=0$ )

Depreciation

( $\lambda=5\%$ )

Infinite lifetime

( $\lambda=0$ )

Depreciation

( $\lambda=5\%$ )

GA\*\* arbitrary GA\*\* arbitrary GA\*\* GA\*\*

0 1.7676\* 1.5194\* 1.7676\* n.a.

0.1 1.715 1.713 1.484 1.478 1.710 1.263

0.25 1.675 1.677 1.436 1.432 1.680 1.237

0.5 1.643 1.645 1.404 1.400 1.638 1.211

1 1.587 1.590 1.367 1.360 1.584 1.180

\* Analytical solution (cf. Dixit and Pindyck, 1994).

\*\* Average trigger prices of the genome population.

Reducing the time step length  $\Delta t$  leads to a convergence of trigger prices towards the analytical solution computed for continuous time, i.e.  $0 \rightarrow t$ .<sup>11</sup> This effect is not surprising, because reducing the time step length implies more frequent investment opportunities, i.e. one does not need to wait as long before one can decide again and can respond more quickly to revealed new information. In summary these results demonstrate that the GA approach leads to plausible results for the scenarios without competition.

Let us now consider competition. According to table 1, the GA model with competition leads for the scenarios with infinite lifetime of the assets ( $\lambda = 0$ ) to trigger prices which do not differ significantly from those of the scenarios without competition. Thus, these simulations are in accordance with the finding presented in Dixit and Pindyck (1994) according to which investment

triggers are not affected by competition. However, if we compare the investment triggers with depreciation, then competition leads to investment triggers which are some 13% to 15% lower than without competition. Hence, competition reduces the difference between trigger price and production costs by some 50%. Since the absolute as well as the relative differences

increase with reducing  $\Delta t$ , one can conclude that this phenomenon has also to be expected for a continuous time scenario. Accordingly, one has to state that competition matters if assets are depreciated and are subject to a reinvestment option!

<sup>11</sup> According to Dixit and Pindyck (1994), trigger prices for continuous time can be calculated as

$$I(rP) \left( \frac{1}{1 - \frac{1}{1 + \frac{1}{b}}} \right)$$

$$1 + \frac{1}{b}$$

$$1 + \frac{1}{b}$$

$$1 + \frac{1}{b}$$

$$1 + \frac{1}{b}$$

$$1 + \frac{1}{b}$$

$$1 + \frac{1}{b}$$

$$1 + \frac{1}{b}$$

$$1 + \frac{1}{b}$$

$$= \text{with } \frac{1}{b}$$

$$1 + \frac{1}{b}$$
 is the positive root of  $\left( \frac{1}{b} \right) \left( \frac{1}{b} \right) 0 1 2$

$$1 2 = + - - / b b s r .$$

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As already mentioned in the introduction, there is a simple explanation for this result:

Depreciation

allows for a certain market response to declining demand, i.e. to a declining  $\alpha$ . Consider that from period  $t$  to  $t+1$   $\alpha$  decreases by 5%. Accordingly, in  $t+1$  prices are 5% lower than expected in  $t$ , i.e. if the price starts at the trigger price  $P^*$ , in  $t+1$  the price is 5% lower than the trigger price  $P^*$ . Without depreciation the expected price for  $t+2$  would be equal to the actual price in  $t+1$ . However, if one considers 5% depreciation per period, i.e. a 5% reduction of the production capacities, then this reduction compensates the market deterioration and the expected price for  $t+2$  is equal to  $P^*$ . Consequently, as it is also shown in figure 1, depreciation

reduces the downside market risk and dampens price fluctuations. Hence, one can invest at lower trigger prices than in a scenario without depreciation. Figure 3 shows that the firms obtain profits which are not significantly different from zero. Hence, in accordance with Dixit and Pindyck (1994) the zero-profit assumption is fulfilled for all our market simulations, i.e. the results satisfy an essential equilibrium condition for competitive markets.

**Figure 3: NPV of the strategies of the genomes (50 generations, 5 000 simulations)\***

-0.01  
-0.005  
0  
0.005  
0.01  
0.015  
0.02  
0.025  
0 5 10 15 20 25 30 35 40 45 50  
number of genom

average  
standard deviation

\* The slight losses of the first 15 genomes are caused by too low trigger prices which arise through the GA. These reflections on the impact of depreciation on the market dynamics provoke further interesting

questions. We will concentrate on two. Firstly: Which insights gives the model regarding the price dynamics in relation to the dynamics of the demand parameter  $\alpha$ ? Second: Can the competitive price dynamics probably be simulated directly?

Let us start with the first question. Consider an equilibrium trigger  $P^*$  and assume that in period  $t-1$  firms have invested according to  $\hat{P}_t = P_t$ . From equations (5) and (6) we know that after the investment decisions are made,  $P_t$  purely depends on the relation of  $\alpha_t$  and  $\alpha_{t-D_t}$ . Hence, the price in  $t$  will be

$$\Delta \cdot \cdot + \Delta \cdot - \cdot = t t P P_{tt} e s$$

**s**  
2  
exp  
2  
\*(14)

Consider now that the actual price in period  $t$  is  $\hat{P}_t \geq P_t$ . Then the firms will respond and invest such that  $\hat{P}_t = P_t + \Delta$ . Now consider  $\hat{P}_t \geq P_t$ . Then, two cases have to be differentiated. If  $\hat{P}_t - P_t \geq \Delta$  then some firms will reinvest, such that  $\hat{P}_t = P_t + \Delta$ . Otherwise, if  $\hat{P}_t - P_t < \Delta$

no firm will reinvest and  $\hat{P}_t = P_t + \Delta$ . With this knowledge and in accordance

with equations (1) to (12) the price dynamics can be described as

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trigger. If  $P^*$  is higher, the dynamics should allow for profits. If  $P^*$  is smaller, this should imply losses. Accordingly, the equilibrium trigger price  $P^*$  can be determined by minimizing the square of the expected profits, i.e.

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**Table 2: Equilibrium Trigger and Volatility\*** $\Delta t$   $P^*$   $\hat{\sigma}$  (annualized)GA simulation  $\alpha_{t+\Delta t}$ ,  $\alpha_t$   $P_{t+\Delta t}$ ,  $P_t$   $\alpha_{t+4\Delta t}$ ,  $\alpha_t$   $P_{t+4\Delta t}$ ,  $P_t$   $\alpha_{t+10\Delta t}$ ,  $\alpha_t$   $P_{t+10\Delta t}$ ,  $P_t$ 

0.1 1.263 1.261 0.2000 0.2066 0.1996 0.1945 0.1989 0.1850

0.25 1.237 1.237 0.1998 0.2100 0.1987 0.1912 0.1967 0.1760

0.5 1.211 1.210 0.1995 0.2137 0.1975 0.1876 0.1934 0.1665

1 1.180 1.180 0.1993 0.2189 0.1950 0.1826 0.1865 0.1536

\* For  $\lambda = 5\%$ ,  $\delta = 0.2$ ,  $r = 6\%$ . The estimated volatility is based on 5 000 repeated stochastic simulations.

As mentioned in the introduction, one could raise the question whether the price volatility measured in the market is significantly lower than the volatility of  $\alpha$ . If this proved to be true, competition could probably be ignored, because the smoothing effect of depreciation is already implicit in the price volatility. However, this argument does not hold in general. According to table 2 the determined volatilities of  $\alpha_t$  and  $P_t$  are very similar and do not lead to meaningful differences in the short run. Only for longer periods (i.e. a multiple of  $\Delta t$ ), the price volatility is lower. This can be explained by the fact that to some extent demand reductions are always compensated over the next periods. Nevertheless, these slightly lower volatilities do not explain the reduction of the trigger price of considering competition.<sup>13</sup>

A second critical question is whether competitive prices can be considered as a random walk. Usually, the random walk hypothesis is tested by unit root tests, like a Dickey-Fuller (DF) and Augmented Dickey-Fuller (ADF) tests (cf. Pietola and Wang 2001). Table 3 shows the test results for our simulations. Accordingly, for many simulations the hypothesis that prices follow a random walk is rejected. However, in most cases the hypothesis is not rejected - this particularly holds for the ADF tests. Transferring this result to real markets means that unit root tests give no reliable justification to ignore competition to determine investment triggers.

**Table 3: Percentile rejection of the random walk hypothesis for demand parameter and price (Dickey-Fuller (DF) and Augmented Dickey-Fuller (ADF) test)\***

DF-test ADF-test

(first difference)

ADF-test

(first three differences)

 $\Delta t$  $\alpha_t$   $P_t$   $\alpha_t$   $P_t$   $\alpha_t$   $P_t$ 

0.1 3.1 43.9 0.5 23.9 0.0 6.3

0.25 3.5 42.8 0.4 21.7 0.0 5.6

0.5 3.2 42.0 0.8 17.1 0.0 3.2

1 3.7 39.3 0.6 14.6 0.0 1.3

\*  $\lambda = 5\%$ ,  $\delta = 0.2$ ,  $r = 6\%$ . The DF and ADF test are based on 1 000 repeated stochastic simulations. The null hypothesis of a unit root is tested at a 5% level.

Summarizing, one can conclude that the "standard procedure" of applying the real options approach to investments in competitive markets is highly problematic. The procedure of (i) testing market prices for a random walk, then (ii) estimating price volatilities, and finally (iii) calculating investment triggers by treating the investment as an exclusive option leads to an overestimation of the investment trigger. Instead, one should explicitly consider competition. In order to give an idea about the differences, tables 4 and 5 illustrate the impact of several parameter settings. The finding is that the depreciation rate has a decisive impact. As figure 5

<sup>13</sup> Considering  $\Delta t=1$ ,  $\left( \frac{1}{1.1950} \right)^4 = 0.711$  and  $\left( \frac{1}{1.1826} \right)^4 = 0.75$ . Hence the error is relatively

small compared to neglecting competition which implies  $P^*(\sigma = 0.2) = 1.36$ .

13

shows, the impact of depreciation on the relation of investment triggers with and without competition is most relevant for a depreciation rate between 5% and 50%, i.e. an asset's average lifetime of 2 to 20 years. This range covers most real investments.

**Table 4: Trigger prices for a monopolistic producer (*italic*) and under competition (**fat**) for various constellations of  $l$  and  $s$  ( $Dt = 0.25$ ,  $r = 6\%$ )**

$\sigma \quad l \quad 0\% \quad 5\% \quad 10\% \quad 20\% \quad 25\% \quad 50\% \quad 66\%$

0.1 *1.315*

**1.323**

*1.197*

**1.058**

*1.158*

**1.022**

*1.108*

**1.002**

*1.100*

**1.000**

*1.055*

**1.000**

*1.040*

**1.000**

0.2 *1.677*

**1.680**

*1.432*

**1.237**

*1.333*

**1.110**

*1.228*

**1.038**

*1.200*

**1.023**

*1.110*

**1.002**

*1.081*

**1.000**

0.3 *2.148*

**2.175**

*1.730*

**1.443**

*1.526*

**1.240**

*1.358*

**1.100**

*1.314*

**1.070**

*1.174*

**1.015**

*1.116*

**1.003**

**Table 5: Trigger prices for a monopolistic producer (*italic*) and under competition (fat) for various constellations of  $\lambda$  and  $r$  ( $Dt = 0.25$ ,  $s = 0.2$ )**

$r \quad \lambda$  0% 5% 10% 20% 25% 50% 66%

4% *1.917*

**1.909**

*1.491*

**1.258**

*1.366*

**1.118**

*1.246*

**1.040**

*1.209*

**1.025**

*1.115*

**1.002**

*1.084*

**1.000**

6% *1.677*

**1.680**

*1.432*

**1.237**

*1.333*

**1.110**

*1.228*

**1.038**

*1.200*

**1.023**

*1.110*

**1.002**

*1.081*

**1.000**

8% *1.548*

**1.556**

*1.385*

**1.186**

*1.320*

**1.110**

*1.220*

**1.038**

*1.200*

**1.023**

*1.109*

1.000

1.078

1.000

**Figure 5: Trigger prices for monopolistic producer vs. competition dependent on the depreciation rate ( $Dt = 0.25$ ,  $s = 0.2$ ,  $r = 6\%$ )**

0.8

0.9

1

1.1

1.2

1.3

1.4

1.5

1.6

1.7

0 0.2 0.4 0.6 0.8 1

$\lambda$

$P^*$

monopoly

competition

Moreover, tables 4 and 5 show that the trigger prices differ only slightly from the production costs if the depreciation rate is larger than the volatility. This can be explained by the fact that with high depreciation rates almost every demand reduction can be compensated by a supply reduction within one period. In these cases the real options approach is practically irrelevant. Practically every market signal has a value for one period only. The resulting test statistics are quite interesting. For  $\lambda = 0.2$  and  $\sigma = 0.2$  the annualized average volatilities  $\hat{s}(\alpha_{t+4Dt})$  and  $\hat{s}(P_{t+4Dt})$  are 0.1987 and 0.1695, respectively. The hypothesis of a unit root is rejected by a Dickey-Fuller test at a 5% level for  $\alpha_t$  in 3.6% and for  $P_t$  in 98.9% of 1000 simulations. The 14

ADF test (first three differences) rejects the hypothesis for  $\alpha_t$  in 0.1% and for  $P_t$  in 92.4% of the simulations. A depreciation of 20% allows to compensate almost every demand shock in the following period. This has an interesting consequence. If on a certain market the assets are depreciated at a rate higher than the price volatility, prices cannot follow GBM even if demand is driven by GBM. If unit root tests suggest that they would, then further effects influence prices which have to be analyzed carefully.

#### 4. Summary and conclusions

This paper explicitly includes competition into a real options framework by using an agentbased approach of competing firms. The firms derive their investment triggers from a genetic algorithm which exploits the results of repeated stochastic simulations of the market. The results contradict the widespread opinion that optimal investment triggers are not affected by competition. The investment triggers under competition are substantially lower than those which we find and expected for exclusive options. Finally, our search for an explanation directed us to the impact of depreciation, i.e. to the fact that the lifetime of assets used for production is limited. The presence of depreciation, i.e. the necessity of reinvestments in order to maintain a certain production level, reduces the downside market risk of demand shocks.

Our main findings can be summarized as follows:

 The real options approach leads to investment triggers which are substantially higher than

the classical NPV criterion. However, if one considers depreciation and competition, the increase of the investment trigger is substantially reduced. If the depreciation rate is higher than the demand volatility, the real options approach is even practically irrelevant.

☞ The "standard approach" to determine investment triggers by ignoring competition is not healed by deriving the volatility from market prices. In general, the volatility of market prices and the demand parameter do not differ significantly. Moreover, unit root tests are no reliable instrument for testing the hypothesis that prices follow a random walk. Accordingly, the application of the real options approach to investments in competitive markets should explicitly consider the effects of competition.

☞ The results obtained by the agent-based model can be validated by an alternative model which has been identified by analyzing the simulation results. Instead of an agent-based approach, this second model is based on direct repeated stochastic simulations of the price dynamics. These price dynamics consider that the investment trigger delivers a kind of attractor for the prices. If a demand shock causes prices to be slightly higher or lower than the trigger price, the expected future price is the investment trigger. If prices are much lower, the movement towards the equilibrium depends on the demand volatility and the depreciation rate. For identical investment triggers and identical random numbers, the price dynamics of both modeling types are identical.

These results show several directions for further research. For instance, the experiments in this paper are based on geometric Brownian motion, geometric depreciation of assets, and isoelastic

demand functions only. However, at least the agent-based approach allows to modify these and many other assumptions in a straightforward way.

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## **Stochastic Simulation**

### **Genetic algorithm (GA)**

a) Evaluation of Fitness

b) Selection and Replication

c) Crossover

d) Mutation

**Stop**  $G \leq$

no yes

$g+1$

$S$

$S$

$s$

$s$

$\sum$

$=$

$\Pi$

$1$

$N$  = number of genomes (farms)

$n = 1, \dots, N$

$G$  = number of generations

$g = 1, \dots, G$

$S$  = number of stochastic simulations

$s = 1, \dots, S$



$T$  = Investment period

$t = 0, \dots, T$

$r$  = interest rate

$P$  = price

Calculation of  $P_0$

$n=1, t=0$

Calculation of  $P_t$

$s=1$

?  $N \ n \leq n+1$

?  $T \ t \leq$

no

yes  $t+1$

no

yes

?  $S \ s \leq s+1$

**Initialization**

\*

$n \ P$  set randomly

$g=1$

\*

1

^

$n \ t \ P \ P \geq +$

$t$

$t \ n \ t$

$s$

$n$

$s$

$n \ r \ p \ x \ - \ + \ + \ = \ ) \ 1 \ ( \ , \ P \ P$

no

yes

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**Appendix: Flow diagram of the agent-based simulation approach.**

yes

no