Managing a Common Renewable Resource in Asymmetric Information

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Abstract

The clear definition of property rights is not a sufficient condition to prevent congestion effects in commons. In this paper we present how interesting can be the coordination among owners in the preservation of a common good. Our approach takes into account economic dynamics and incentive mechanisms in a hidden information context. We consider a natural resource which is being used up for a continuum of producers on a common property regime. We also consider that each producer has an individual performance index which is a hidden information for the rest of players. We introduce coordination in the sense of a global maximization of the joint profit. If there is no coordination among the producers, their behavior leads to complete rent dissipation. We focus our model in the case of the producers convinced to coordinate their actions in order to preserve their own economic sustainability. Under perfect information we find that the exclusion of a subset of producers can appear and how it is endogenously determined. Under asymmetric information we propose a quantity-transfer contract which lead us to the previous stationary disposal stock of the resource without exclusion.

KEY WORDS: Commons; Natural Resources; Dynamics; Asymmetric Information; Contracts.

1 Introduction

In the literature on natural resources and environmental policy we find many approaches concerning the burning problem of managing the commons. In most of them the key question is the concept of property. When rights of property are not clearly defined, the final issue of commons is often the complete rent dissipation (*open access regimes*). However the clear definition of property rights is not a sufficient condition to prevent congestion effects (see Ostrom (1990)).

In this paper we want to present how much interesting can be the coordination among owners in the preservation of a common good. Our approach takes into account economic dynamics of natural resources (see Shone (1997) and Chiang (1992)) and incentive mechanisms in a hidden information context (see Laffont and Tirole (1993) and Salanié (1994)).

We consider a natural resource which is being used up for a continuum of producers on a common property regime. We also consider that each producer has an individual performance index which is a hidden information for the rest of players (for a similar approach see Laffont and Tirole (1986)).

There are two cases in which we focus our model. The first case occurs when there is no coordination among the producers. In such a context at any time t each producer maximizes his own profit function following the myopic rule (Hardin 1968). The long run issue is the complete rent dissipation, as in a context of open access.

In the second case the producers are convinced to coordinate their actions in order to preserve their own economic sustainability. We assume a regulation authority who can impose compliance to the producers. The coordination appears in the sense of a global maximization of the joint profit.

In the first step we consider the perfect information context and the stationary equilibrium for which the disposal stock of the resource neither depends on the total amount of extraction nor on the number of producers extracting the resource. However the individual share of extraction and the shadow cost of the resource depend both on a parameter that we called the *frontier agent*.

The frontier agent is the last producer not allowed to extract nearest the first agent extracting the resource. We show how the frontier agent is endogenously determined by the renewability function of the resource and the profit function of the producers. Two extreme cases are founded. The first one appears when all producers are going on extracting the resource. In this case all of the producers extract and used up the resource, there is no exclusion and the disposal stock of the resource is always positive. The second one emerges when the frontier agent is "on the top" of the continuum of producers. This case is equivalent to the sole owner solution.

The key question is that the frontier agent implies exclusion. The problem happens when the regulator authority can not impose compliance to the producers and all of them access to the resource (Samuelson and Nordhaus 1990), even those who would be not allowed to extract the resource.

In the second step we consider the management in an asymmetric information context. The theoretical solution of this kind of problems is well known by the seminal approach of Baron and Myerson (1982) and many emphatic extensions (see Guesnerie and Laffont (1984) for a general approach; see Bourgeon, Jayet and Picard (1995) for application to agricultural and natural resources.). The cost of regulation is obviously greater than the case of coordination under perfect information. However we find the same stationary disposal stock of the resource and the same stationary global amount of extraction in any case considered in the paper (excepting the case of complete rent dissipation).

The main difference between the regulation by coordination in perfect information and the incentive mechanism proposed in asymmetric information is that the second one does not consider the exclusion of any subset of producers.

In section 2 we present the basic model. In section 3 we expose the case of coordination under perfect information with the possible appearance of the frontier agent problem. The case of asymmetric information is considered in section 4, where we propose the contract. We also compare the stationary equilibria. In section 5 we give some results assuming specific functions of the growth rate, the profits and the distribution of the producers' characteristics. We also make some particular assumptions about the profit function. Finally we add some concluding remarks in section 6.

2 The Model

Let us consider a renewable resource which is being extracted and used up for a continuum of producers. The production process is linked to an individual characteristics θ, which can be interpreted as a performance index. We assume that θ is distributed according to the cumulative function $\Gamma(\theta)$ and the density function $\gamma(\theta)$, positive and defined on the interval $[0,1]$. The stock of the resource is noted $x(t)$ as the *state variable* and $q(\theta,t)$ denotes the individual rate of extraction of the resource at time t as the *control variable*. The growth function of the resource is noted $F(x)$ and we assume that the initial stock $x(0) = x_0$ is given. The individual profit function is noted $\pi(q,\theta)$ with the properties $\pi'_q > 0$, $\pi''_{qq} < 0$ and $\pi''_{q\theta} > \theta$. The two first properties are general assumptions about the profit function as in many economic models. The last one means that the marginal productivity increases with the performance θ.

We also consider a public regulator searching for the maximization of the joint profit in the long run. The discount rate is noted δ. The regulator has to take account of the limited availability of the resource at any time. The maximization program of the regulation authority is the following one[1].

$$\max_{q(\theta,t)} \int_0^{+\infty} \int_0^1 \pi\left(q(\theta,t),\theta\right)\gamma(\theta)d\theta \; e^{-\delta t}dt$$

subject to

$$\dot{x} = F(x) - \int_0^1 q(\theta,t)\gamma(\theta)d\theta \tag{C1}$$

[1] The dotted symbol $\dot{x}$ denotes the time derivative $\frac{dx}{dt}$

$$\forall\theta \in [0,1]\ ;\ \forall t\ ;\ q(\theta,t) \geq 0 \tag{C2}$$

$$\forall t\ ;\ \int_0^1 q(\theta,t)\gamma(\theta)d\theta \leq x(t) \tag{C3}$$

In the renewability constraint (C1), $F(x)$ denotes the growth function of the resource and $\int_0^1 q(\theta,t)\gamma(\theta)d\theta$ denotes the total amount of consumption of the resource at any time t. We also introduce the constraint C2 in order to have non negative individual shares of extraction. Finally, the C3 constraint means that the disposal stock of the resource has to be at least equal to the total amount of consumption at any time t.

3 Coordination under Perfect Information

Let us recall that we focus on the case in which the resource is managed in a common property regime. In this section we also consider that the producers coordinate their actions to maintain the renewability of the resource. In such a context the disposal stock of the resource is not extracted at each time t. Consequently the constraint C3 is not binding and its associated Lagrange multiplier is nul.

Otherwise the producers are in the case of complete rent dissipation: at any time t each producer maximizes his own profit function following the myopic rule and damaging the renewability of the resource. That means

$$\pi'_q\left(q^*\left(\theta,t\right),\theta\right) = 0\ \forall\theta\ \forall t \Longleftrightarrow \dot{x} = F\left(x\right) - \int_0^1 q^*(\theta,t)\gamma(\theta)d\theta < 0 \tag{1}$$

where $q^*(t,\theta)$ denotes the optimal extraction rate for the agent θ. The stationary issue of this situation is a nul stock $x = 0$ and the whole cut of the production system.

Furthermore we suppose that the regulation authority can impose coordination to the agents in order to maximize the joint profit in the long run. In this way it is possible that only the more performer producers are allowed to extract the resource. Therefore the C2 constraint is binding for those θ who have no access to the resource. The Lagrange multiplier associated to the C2 constraint is noted $\varepsilon(\theta)$. The *costate variable* associated to $x(t)$ is noted $\mu(t)$.

In the case $\frac{\partial q}{\partial\theta} > 0$ (see proof in appendix A), we know that *if there exists a $\tilde{\theta}$ producer for who $q(\tilde{\theta},t) = 0$ and $\pi'_q(q(\tilde{\theta},t),\tilde{\theta}) = \mu$ then* from this $\tilde{\theta}$ agent to top ($\theta = 1$) all the producers extract; otherwise they are not allowed to use up the resource. We call $\tilde{\theta}$ the *frontier agent*. That means

$$q(\theta,t) = 0 \text{ and } \varepsilon(\theta) > 0\ ;\ \forall\theta \in \left[0,\tilde{\theta}\right] \tag{2}$$

$$q(\theta,t) > 0 \text{ and } \varepsilon(\theta) = 0\ ;\ \forall\theta \in \left(\tilde{\theta},1\right] \tag{3}$$

An interesting issue of the program described in section 2 is the stationary equilibrium $\frac{dx(t)}{dt} = 0$, $\frac{dq(\theta,t)}{dt} = 0$, $\frac{d\mu(t)}{dt} = 0$. The program of the regulator in this context of coordination leads to the following Hamiltonian function.

$$\max_{q(\theta,t)} H = \int_0^1 \pi\left(q(\theta,t),\theta\right)\gamma(\theta)d\theta + \mu\left[F\left(x\right) - \int_0^1 q(\theta,t)\gamma(\theta)d\theta\right] + \int_0^1 \varepsilon(\theta)q(\theta,t)\gamma(\theta)d\theta \quad (4)$$

From the maximization conditions of the program above (see appendix B) we find the stationary equilibrium as given in the next proposition[2].

Proposition 1 *The maximization of the program of the regulator authority in the case of common property under coordination leads to the following stationary equilibrium characterized by the equations*

$$F'(\bar{x}) = \delta \quad (5)$$

$$\int_{\tilde{\theta}}^1 \bar{q}(\theta)\gamma(\theta)d\theta = F\left(\bar{x}\right) \quad (6)$$

$$\bar{\mu} = \varepsilon(\theta) + \pi_q'\left(\bar{q}(\theta),\theta\right) \quad (7)$$

$$\theta > \tilde{\theta} \Rightarrow \varepsilon(\theta) = 0\ ;\ \theta \leq \tilde{\theta} \Rightarrow \varepsilon(\theta) > 0 \quad (8)$$

As we can see the stationary disposal stock depends only on the growth function and on the discount rate. So the total amount of consumption of the resource and the number of producers extracting it do not have influence on the long run issue of the disposal resource. All the disposal stock, the individual extraction rate, the frontier agent and the shadow cost of the resource are endogenously determined.

The Frontier Agent

As shown in section 3, the frontier agent $\tilde{\theta}$ emerges naturally from the growth function and the profit function. We find two extreme cases.

First, when there is no $\tilde{\theta}$. In such a case all of the producers are allowed to extract the resource. The coordination (in the sense of maximize the joint profit) is well accepted by all of the agents because there is no exclusion of any subset of producers.

The other extreme case, when $\tilde{\theta} = 1$, is equivalent to the sole owner solution. If there will be an only producer it will be the $\theta = 1$ producer.

Let us now compare the case in which there is no a frontier agent and the general case. We note $\left(\overline{\overline{x}}, \overline{\overline{q}}(\theta), \overline{\overline{\mu}}\right)$ the stationary equilibrium when $q\left(\theta,t\right) > 0$ for all of the producers (see the appendix F). We find that the stock equilibrium is equivalent in both cases ($\bar{x} = \overline{\overline{x}}$). By $\frac{\partial q}{\partial \theta} > 0$ and by the equations 6 and 25 we know that $\bar{q}\left(\theta\right) \geq \overline{\overline{q}}\left(\theta\right)$. Consequently and by $\pi_{qq}'' < 0$ and the equations 7

[2] Note that for writing simplification and for any stationary variable in the paper we are going to write $\bar{x}\left(\theta\right)$ instead $\lim_{t \to +\infty} \bar{x}\left(\theta,t\right)$ in the case of perfect information and $\hat{x}\left(\theta\right)$ instead $\lim_{t \to +\infty} \hat{x}\left(\theta,t\right)$ in the case of asymmetric information.

and 26 we are allowed to compare the shadow cost of the resource in the sense $\bar{\mu} \leq \overline{\overline{\mu}}$. Furthermore when $\tilde{\theta} = 0$ we find $\bar{q} = \overline{\overline{q}}$, so the equivalence between the first extreme case and the general one is done.

In all of the other cases, for any $\tilde{\theta} \in (0, 1)$ the exclusion of some producers is required. The problem of coordination with exclusion emerges.

4 Regulation under Asymmetric Information

Let us now think about a context of asymmetric information where the producers know their own θ characteristics which is unknown by the regulator. As the usual application of the revelation principle, we retain a direct revealing mechanism which combines an individual share of extraction $\hat{q}(\theta, t)$ and an individual transfer $\tau(\theta, t)$. This mechanism requires that the principal asks each producer for his θ characteristics. The regulator has to design the best mechanism leading any producer to reveal his own truthful characteristics.

The maximization program of any θ producer accepting the contract is

$$\max_{\hat{\theta}} \pi(\hat{q}(\hat{\theta}, t), \theta) + \tau(\hat{\theta}, t)$$

where $\hat{\theta}$ is the announce of the θ characteristics.

Proposition 2 *The incentive constraints are*

$$\pi_q' \left(\hat{q}\left(\theta, t\right), \theta\right) \dot{\hat{q}} + \dot{\tau} = 0 \tag{IC1}$$

$$\dot{\hat{q}} \cdot \pi_{q\theta}'' \left(\hat{q}\left(\theta, t\right), \theta\right) > 0 \tag{IC2}$$

See the proof in appendix C.

The participation constraint guarantees positive profits for all the producers accepting the contract.

$$\pi\left(\hat{q}\left(\theta, t\right), \theta\right) + \tau(\theta, t) \geq \pi\left(q^*\left(\theta, t\right), \theta\right) \tag{PC}$$

Note that each producer has two options: to accept the contract or to refuse it and choose the extraction rate corresponding to the open access issue (see equation 1).

The maximization program of the regulation authority is the following one.

$$\max_{\hat{q},\tau} \int_0^{+\infty} \int_0^1 \left[\pi\left(\hat{q}(\theta, t), \theta\right) - \lambda\tau(\theta, t)\right] \gamma(\theta) d\theta \ e^{-\delta t} dt$$

subject to

$$IC1, IC2, PC$$

$$\frac{dx}{dt} = F\left(x\right) - \int_0^1 \hat{q}(\theta, t)\gamma(\theta) d\theta \tag{C1'}$$

$$\forall\theta \in [0,1] \; ; \; \forall t \; ; \; \hat{q}(\theta,t) \geq 0 \tag{C2'}$$

$$\forall t \; ; \; \int_0^1 \hat{q}(\theta,t)\gamma(\theta)d\theta \leq x(t) \tag{C3'}$$

As in section 2, we consider that the C1' constraint is not binding, so its associated Lagrange multiplier is nul. The constraint IC2 will have to be verified ex-post, as usually practiced in theory of contracts. We note ν and ξ the Lagrange multipliers respectively associated to the PC and IC1 constraints. We know that the sign of the information rent related to the θ characteristics is negative[3] ($\frac{dR}{d\theta} < 0$), so the PC constraint is only saturated for $\theta = 1$ ($\xi = 0$ for $\theta \neq 1$ and $\xi > 0$ for $\theta = 1$). The opportunity cost of regulation is noted λ. The Hamiltonian function is writing such that

$$\begin{aligned} H &= \int_0^1 \left[\pi\left(\hat{q},\theta\right) - \lambda\tau\right]\gamma(\theta)d\theta + \mu\left[F\left(x\right) - \int_0^1 \hat{q}\gamma(\theta)d\theta\right] \\ &\quad + \int_0^1 \left[\varepsilon\left(\theta\right)\hat{q} + \nu\left(\pi_q'\left(\hat{q},\theta\right)\dot{\hat{q}} + \dot{\tau}\right) + \xi\left(\pi\left(\hat{q},\theta\right) + \tau - \pi(q^*,\theta)\right)\right]\gamma(\theta)d\theta \end{aligned}$$

which is equivalent (by IC1) to

$$\begin{aligned} H &= \int_0^1 \left[\pi\left(\hat{q},\theta\right) - \mu\hat{q} + \varepsilon\left(\theta\right)\hat{q}\right]\gamma(\theta)d\theta - \lambda\int_0^1 \tau\left(\theta\right)\gamma(\theta)d\theta + \mu F\left(x\right) \\ &\quad + \xi\left[\pi\left(\hat{q},1\right) + \tau\left(1\right) - \pi(q^*,1)\right] \end{aligned}$$

We can easily simplify this expression integrating by parts the $\int \tau\gamma d\theta$ term of the Hamiltonian[4]. The maximization program becomes

$$\max_{\hat{q}} H = \int_0^1 \left\{\left[\pi\left(\hat{q},\theta\right) - \mu\hat{q} + \varepsilon\left(\theta\right)\hat{q}\right]\gamma(\theta) - \lambda\dot{\hat{q}}\pi_q'\left(\hat{q},\theta\right)\Gamma\left(\theta\right)\right\}d\theta$$

We apply the Euler relation in order to find the optimal individual share of extraction $\hat{q}(\theta,t)$. As said above, the participation constraint is saturated for $\theta = 1$, so $\tau(1,t) = \pi\left(q^*\left(1\right),1\right) - \pi\left(\hat{q}\left(1\right),1\right)$. From PC, IC1 and $\frac{dR}{d\theta} < 0$ we obtain the optimal transfer of the contract.

Proposition 3 *The optimal contract proposed to the agents is* $\left[\hat{q}\left(\theta,t\right);\hat{\tau}\left(\theta,t\right)\right]$ *such that*

$$\hat{q}\left(\theta,t\right) : -\lambda\frac{\Gamma}{\gamma}\pi_{q\theta}''\left(\hat{q}(\theta,t),\theta\right) = \left(1+\lambda\right)\pi_q'\left(\hat{q}(\theta,t),\theta\right) + \varepsilon\left(\theta\right) - \mu\left(t\right) \tag{9}$$

$$\hat{\tau}\left(\theta,t\right) = \tau(1,t) - \int_0^1 \pi_q'\left(\hat{q}\left(\theta,t\right),\theta\right)\dot{\hat{q}}\gamma\left(\theta\right)d\theta \tag{10}$$

Let us consider the case in which all the producers are allowed to extract the resource. If $\hat{q}(\theta) > 0 \; \forall\theta$, then the Kuhn-Tucker condition is $\varepsilon\left(\theta\right) = 0 \; \forall\theta$. When the opportunity cost is such as $\lambda = 0$, we can easily find in equation 9 the Hotelling rule.

[3] See the proof in appendix D.
[4] See appendix E.

4.1 The Stationary Equilibrium of the Contract

Our incentive mechanism leads to a long run equilibrium in which the total disposal stock of the resource is the same that in case of section 3, as well as the global amount of extraction of the common good.

Proposition 4 *The stationary equilibrium in asymmetric information leads to the stock $\hat{x}$, the individual share of extraction $\hat{q}(\theta)$ and the shadow cost of the resource $\hat{\mu}$ such as*

$$F'(\hat{x}) = \delta \tag{11}$$

$$\int_0^1 \hat{q}(\theta)\gamma(\theta)d\theta = F(\hat{x}) \tag{12}$$

$$\hat{\mu} = (1+\lambda)\pi'_q(\hat{q},\theta) + \lambda\frac{\Gamma}{\gamma}\pi''_{q\theta}(\hat{q},\theta) \tag{13}$$

The individual share of extraction and the shadow cost of the resource in the stationary equilibrium under asymmetric information generally differ both from the case of perfect information.

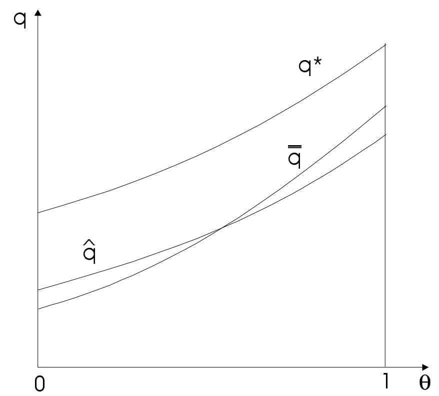

Figure 1: Distribution of quantities according to the characteristics θ (q^* : case of rent dissipation; $\bar{\bar{q}}$: perfect information without exclusion; $\hat{q}$: asymmetric information).

5 Illustration of the General Results

We show in this section a particular case of the general results of the preceding sections 3 and 4. Our first assumption concerns the individual profit function. We consider that there is a production system transforming a renewable resource into an output. The profit function is such that

$$\pi(q,\theta) = p\ln(1+q(\theta,t)) - (1-\theta)q(\theta,t)$$

where p is the exogenous market price of the output and $(1-\theta)$ denotes the marginal cost of extraction of the natural input (the renewable resource). The production function $\ln(1+q(\theta,t))$ is such that the gross margin verifies $\pi'_q > 0$, $\pi''_{qq} < 0$ and $\pi''_{q\theta} > \theta$. We also consider that the θ characteristics is distributed according to the uniform distribution on the interval $[0,1]$. Our last assumption concerns the growth function of the renewable resource. We consider the logistic law $rx\left(1-\frac{x}{k}\right)$, where r denotes the instantaneous growth rate at each time t and k the carrying capacity of the resource. As in the general case, the discount rate is noted δ. All of the other parameters take the same notation as in the section 2.

5.1 Coordination under Perfect Information

We first consider the case in which there is a frontier agent $\tilde{\theta} > 0$, so the equations 2 and 3 hold. The frontier agent verifies the equations $q(\tilde{\theta},t) = 0$ and $\pi'_q(q(\tilde{\theta},t),\tilde{\theta}) = \mu$, so that we find in this case

$$\tilde{\theta} = \mu + 1 - p \tag{14}$$

The program of the regulator in equation 4 becomes the following one for this particular case.

$$\begin{aligned} H &= \int_0^1 \{p\ln(1+q(\theta,t)) - (1-\theta)q(\theta,t)\}\,\gamma(\theta)d\theta + \qquad (15) \\ &\quad \mu\left[rx\left(1-\frac{x}{k}\right) - \int_0^1 q(\theta,t)\gamma(\theta)d\theta\right] + \int_0^1 \varepsilon(\theta)q(\theta,t)\gamma(\theta)d\theta \\ &\quad \text{subject to C1, C2 and C3.} \qquad (16) \end{aligned}$$

The maximization of the program above in the case of common property under coordination leads to the following stationary equilibrium (see the maximization conditions in appendix B)

$$\begin{aligned} \bar{x} &= \frac{(r-\delta)\,k}{2r} \\ \bar{q} &= \frac{p}{1-\theta+\bar{\mu}} - 1 \text{ if } \theta > \widetilde{\theta} \\ \bar{q} &= 0 \text{ if } \theta \leq \widetilde{\theta} \end{aligned}$$

The shadow cost of the resource $\bar{\mu}$ is implicitly shown by the equation

$$\frac{\bar{\mu}}{p} - \ln\frac{\bar{\mu}}{p} = 1 + \frac{k(r^2-\delta^2)}{4pr} \tag{17}$$

Let us now consider the extreme case in which there is no frontier agent. In this extreme case $\varepsilon(\theta) = 0\ \forall\theta$ and all of the producers are allowed to extract the resource. The stationary equilibrium in such a case leads us to

$$\overline{\overline{\mu}} = \frac{1}{1-e^{-\frac{1}{p}\left[r\bar{x}\left(1-\frac{\bar{x}}{k}\right)+1\right]}} - 1 \text{ and } \overline{\overline{q}} = \frac{p}{1-\theta+\overline{\overline{\mu}}} - 1$$

The only difference founded between the stationary equilibrium in the general case $\tilde{\theta} > 0$ and the stationary equilibrium in the extreme case where $\tilde{\theta}$ does not exist is the value of the shadow cost of the resource. As shown is section 3, the intuitive solution suggests that $\bar{\mu} < \overline{\overline{\mu}}$ (and consequently $\bar{q}(\theta) > \overline{\overline{q}}(\theta)$ if $\theta > \tilde{\theta}$ and $0 = \bar{q}(\theta) < \overline{\overline{q}}(\theta)$ if $\theta \leq \tilde{\theta}$).

5.2 Regulation under Asymmetric Information

Let us now consider the particular case in the context of asymmetric information. From the proposition 3 we obtain the following optimal contract

$$\hat{q}(\theta, t) = \frac{p(1+\lambda)}{1+\lambda-\varepsilon(\theta)+\mu(t)-\theta(1+2\lambda)} - 1 \tag{18}$$

$$\hat{\tau}(\theta, t) = \tau(1,t) - \int_0^1 \left[\frac{p}{1+\hat{q}(\theta,t)} - (1-\theta)\right] \dot{\hat{q}} d\theta \tag{19}$$

The stationary equilibrium of the particular case in asymmetric information leads to the stock $\hat{x}$, the individual share of extraction $\hat{q}(\theta)$ and the shadow cost of the resource $\hat{\mu}$, such that

$$\hat{x} = \frac{(r-\delta)k}{2r} \tag{20}$$

$$\int_0^1 \hat{q}(\theta)\gamma(\theta)d\theta = \frac{k(r^2-\delta^2)}{4r} \tag{21}$$

$$\hat{\mu} = (1+\lambda)\left[\frac{p}{1+\hat{q}(\theta)} - (1-\theta)\right] + \lambda\theta \tag{22}$$

The following proposition give us the comparison between the stationary equilibrium in the case of perfect information without exclusion of any subset of producers $\left(\overline{\overline{x}}, \overline{\overline{q}}(\theta), \overline{\overline{\mu}}\right)$ and in the case of asymmetric information $(\hat{x}, \hat{q}(\theta), \hat{\mu})$.

Proposition 5 *From the equations defining the stationary equilibrium in the cases of perfect information and asymmetric information, we compare the corresponding stationary shadow prices in the following way:*

$$\bar{\mu} \leq \hat{\mu} \text{ and } \bar{q}(\theta) \geq \overline{\overline{q}}(\theta)$$

$$\bar{\mu} = \hat{\mu} \text{ if } \lambda = 0$$

Corollary 6 *When $\lambda > 0$ and for little values of θ and the opportunity cost, we can expect probably $\hat{q}(\theta) > \overline{\overline{q}}(\theta)$.*

See the proof in appendix G.

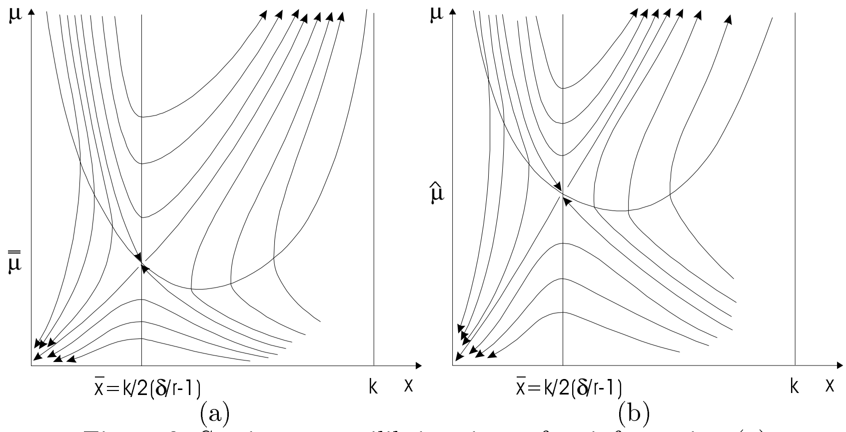

Figure 2. Stationary equilibrium in perfect information (a) and in asymmetric information (b).

6 Concluding Remarks

Common property regimes suffer often congestion effects even when rights of property are clearly defined. In this paper we have considered the case in which a renewable resource is overexploited in absence of any kind of coordination among owners. The final issue is the complete rent dissipation.

However when a coordination appears in the management of the common, we can expect a different issue. In our model we introduce coordination in the way of there is a regulator maximizing the joint profit of a continuum of owners.

Under perfect information we show how a subset of producers can be efficiently excluded of the access to the common. The frontier between those agents who are allowed to used up the resource and those who do not extract it is endogenously determined. The characteristics of the renewability function and the profit function determines both where the frontier agent is placed with respect to the continuum of producers. Nevertheless it is possible that all the producers are allowed to exploit the resource. This problem of partial exclusion of producers arises in the perfect information context.

In the case of asymmetric information, we consider that the problem of commons can be solved by a mechanism design based on contracts. We focus on change of the stationary equilibrium when information context changes. The management of the commons in the case of asymmetric information leads to a higher shadow price of the disposal stock, instead the stationary disposal stock remains unchanged. Some added results are exhibited when we consider some particular functions defining the renewability of the resources, the profitability of producers and the density of the performance characteristics. Individual extraction quota changes when the information context changes and when the opportunity cost of public funds is positive. When the opportunity cost of public funds is positive, we can expect that the less efficient agents are supplied by increasing quota instead the more efficient are asked for decreasing their extraction rate.

Appendices

A The Variation of the Individual Extraction Quantity Related to the Variation of θ

Let consider the total derivative of the constraint 7 related to θ. In the case $q(\theta) > 0$, so $\varepsilon(\theta) = 0$, we obtain

$$\pi''_{qq}(\bar{q}, \theta) \frac{\partial \bar{q}}{\partial \theta} + \pi''_{q\theta}(\bar{q}, \theta) = 0$$

which implies that $\frac{\partial \bar{q}}{\partial \theta} = -\frac{\pi''_{q\theta}(\bar{q},\theta)}{\pi''_{qq}(\bar{q},\theta)}$ has a positive sign under $\hat{q}$ the assumptions $\pi''_{q\theta} > 0$ and $\pi''_{qq} < 0$. So that

$$\frac{\partial \bar{q}}{\partial \theta} > 0$$

B The maximization conditions in the case of common property under coordination

$$\begin{aligned}
\pi'_q(q(\theta, t), \theta) &= \mu - \varepsilon(\theta) \text{ ; } \varepsilon(\theta) = 0 \text{ if } \theta > \tilde{\theta} \text{ and } \varepsilon(\theta) > 0 \text{ if } \theta \leq \tilde{\theta} \\
\frac{\dot{\mu}}{\mu} &= \delta - F'(x) \\
\dot{x} &= F(x) - \int_{\tilde{\theta}}^{1} q(\theta, t)\gamma(\theta) d\theta \\
0 &= \lim_{t \to +\infty} e^{-\delta t} \mu(t)\ x(t)
\end{aligned}$$

C The Second Order Incentive Constraint

Let us consider the derivative of the IC1 constraint related to $\tilde{\theta}$. The second order condition is such that

$$\pi''_{qq}(\hat{q}, \theta) \dot{\hat{q}}^2 + \pi'_q(\hat{q}, \theta) \ddot{\hat{q}} + \ddot{\tau} < 0 \tag{23}$$

We proceed now to differentiate the IC1 constraint for any θ. In any case the optimal announce $\tilde{\theta}$ has to be the truth (θ) following the revelation principle.

$$\frac{dIC1}{d\theta} \Rightarrow \pi''_{qq}(\hat{q}, \theta) \dot{\hat{q}}^2 + \pi''_{q\theta}(\hat{q}, \theta) \dot{\hat{q}} + \pi'_q(\hat{q}, \theta) \ddot{\hat{q}} + \ddot{\tau} = 0$$

So that, by the equation 23, we find

$$\pi''_{qq}(\hat{q}, \theta) \dot{\hat{q}}^2 + \pi'_q(\hat{q}, \theta) \ddot{\hat{q}} + \ddot{\tau} = -\pi''_{q\theta}(\hat{q}, \theta) \dot{\hat{q}} < 0$$

The IC2 constraint is such that

$$\pi''_{q\theta}(\hat{q},\theta)\,\dot{\hat{q}} > 0$$

D The Sign of the Information Rent

Let be the information rent

$$R(\theta) = \pi(\hat{q},\theta) - \pi(q^*,\theta) + \tau$$

Let compute the total derivative of the information rent related to θ.

$$\frac{dR}{d\theta} = \pi'_q(\hat{q},\theta)\,\dot{\hat{q}} + \pi'_\theta(\hat{q},\theta) - \pi'_q(q^*,\theta)\,\dot{q^*} - \pi'_\theta(q^*,\theta) + \dot{\tau}$$

By the equation 1 and IC1 we obtain

$$\frac{dR}{d\theta} = \pi'_\theta(\hat{q},\theta) - \pi'_\theta(q^*,\theta)$$

which has a negative sign because $\pi''_{q\theta} > 0$ and $q^* > \hat{q}$. So that

$$\frac{dR}{d\theta} < 0$$

E Integration by Parts

Let consider the $\int \tau\gamma d\theta$ term of the Hamiltonian function in section 4.

Let note $u = \tau(\theta)$ and $dv = \gamma(\theta)\,d\theta$. Consequently $du = \dot{\tau}d\theta$ and $v = \Gamma(\theta)$. We compute the integration by parts $\left(\int udv = uv - \int vdu\right)$ and we find

$$\int_0^1 \tau(\theta)\,\gamma(\theta)d\theta = [\tau(\theta)\,\Gamma(\theta)]_0^1 - \int_0^1 \Gamma(\theta)\,\dot{\tau}(\theta)\,d\theta$$

which is equivalent (by IC1) to

$$\int_0^1 \tau(\theta)\,\gamma(\theta)d\theta = \tau(1) + \int_0^1 \Gamma\pi'_q(\hat{q},\theta)\,\dot{\hat{q}}d\theta$$

This last equation can be introduced in the Hamiltonian function.

F The First Extreme Case $q(\theta) > 0; \forall\theta$

As said in section 3, the constraints C2 and C3 are both not binding so its associated Lagrange multipliers are nul. The Hamiltonian function is

$$H = \int_0^1 \pi(q(\theta,t),\theta)\,\gamma(\theta)d\theta + \mu\left[F(x) - \int_0^1 q(\theta,t)\gamma(\theta)d\theta\right]$$

The maximization conditions are

$$\begin{aligned}
\pi_q'\left(q(\theta,t),\theta\right) &= \mu \; ; \forall\theta \in [0,1] \\
\frac{\dot{\mu}}{\mu} &= \delta - F'(x) \\
\dot{x} &= F\left(x\right) - \int_0^1 q(\theta,t)\gamma(\theta)d\theta \\
0 &= \lim_{t\to+\infty} e^{-\delta t}\mu(t)\; x(t)
\end{aligned}$$

The stationary equilibrium leads to the stock $\overline{\overline{x}}$, the individual share of extraction $\overline{\overline{q}}(\theta)$ and the shadow cost of the resource $\overline{\overline{\mu}}$ such as

$$F'(\overline{\overline{x}}) = \delta \tag{24}$$

$$\int_0^1 \overline{\overline{q}}(\theta)\gamma(\theta)d\theta = F\left(\overline{\overline{x}}\right) \tag{25}$$

$$\overline{\overline{\mu}} = \pi_q'\left(\overline{\overline{q}}(\theta),\theta\right) \tag{26}$$

If we assume $\pi_{qq}'' < 0$ and $\pi_{q\theta}'' > 0$, by equation 26 and appendix A we obtain

$$\frac{\partial\overline{\overline{q}}(\theta)}{\partial\theta} > 0 \tag{27}$$

G The Shadow Cost: Comparison between Perfect and Asymmetric Information

Let us consider the equations defining the stationary equilibrium in the case of perfect information (see the proposition 1) and the equations defining the stationary equilibrium in the case of asymmetric information (see the proposition 4). By $\pi_q' > 0$ and $\pi_{q\theta}'' > 0$, we know that $\hat{\mu}$ and $\overline{\overline{\mu}}$ have both positive sign.

If we consider $\hat{\mu} < \overline{\overline{\mu}}$, then by $\pi_{qq}'' < 0$ and the equations 7 and 13 we have

$$\forall\theta \; ; \; \pi_q'\left(\hat{q},\theta\right) < \pi_q'\left(\overline{\overline{q}},\theta\right)$$

which in terms of individual extraction quantities implies that

$$\forall\theta \; ; \; \hat{q}\left(\theta\right) < \overline{\overline{q}}\left(\theta\right)$$

However this last result is not possible because the total amount of extraction is equivalent in both cases (see equations 6 and 12). So that we find $\hat{\mu} \geq \overline{\overline{\mu}}$. Furthermore, when $\lambda = 0$, by the equations 7 and 13 we have $\hat{\mu} = \overline{\overline{\mu}}$.

When $\theta = 0$ the equation 13 becomes

$$(1+\lambda)\pi_q'(\hat{q}\left(\theta\right),\theta) > \pi_q'(\overline{\overline{q}}\left(\theta\right),\theta)$$

so that we can expect $\hat{q}\left(\theta\right) > \overline{\overline{q}}\left(\theta\right)$ if λ and θ approach to zero.

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