Term Structure of Volatility and Price Jumps in Agricultural Markets - Evidence from Option Data

Steen Koekebakker

Gudbrand Lien
E-mail: gudbrand.lien@nilf.no

Paper prepared for presentation at the Xth EAAE Congress
'Exploring Diversity in the European Agri-Food System',
Zaragoza (Spain), 28-31 August 2002

Term structure of volatility and price jumps in agricultural markets - evidence from option data*

Steen Koekebakker
Agder University College, Department of Economics and Business Administration
Sevicebox 422, 4604 Kristiansand, Norway

Gudbrand Lien (corresponding author)
Norwegian Agricultural Economics Research Institute
Box 8024 Dep, 0030 Oslo, Norway
Phone: +47 2236 7250, Fax: +47 2236 7299
E-mail: gudbrand.lien@nilf.no

Paper submitted to be presented at
X[th] congress, European Association of Agricultural Economics (EAAE)
28-31 august 2002, Zaragoza, Spain

Abstract

Empirical evidence suggests that agricultural futures price movements have fat-tailed distributions and exhibit sudden and unexpected price jumps. There is also evidence that the volatility of futures prices contains a term structure depending on both calendar-time and time to maturity. This paper extends Bates (1991) jump-diffusion option pricing model by including both seasonal and maturity effects in volatility. An in-sample fit to market option prices on wheat futures shows that our model outperforms previous models considered in the literature. A numerical example illustrates the economic significance of our results for option valuation.

Keywords: Option pricing; Futures; Term structure of volatility; Jump-diffusion; Agricultural markets

* An earlier version of this paper with the same title appeared as Discussion Paper 2001/19 at Norwegian School of Economics and Business Administration, Department of Finance and Management Science. We are grateful to Petter Bjerksund for helpful comments on an earlier draft.

Term structure of volatility and price jumps in agricultural markets - evidence from option data

Abstract

Empirical evidence suggests that agricultural futures price movements have fat-tailed distributions and exhibit sudden and unexpected price jumps. There is also evidence that the volatility of futures prices contains a term structure depending on both calendar-time and time to maturity. This paper extends Bates (1991) jump-diffusion option pricing model by including both seasonal and maturity effects in volatility. An in-sample fit to market option prices on wheat futures shows that our model outperforms previous models considered in the literature. A numerical example illustrates the economic significance of our results for option valuation.

Keywords: Option pricing; Futures; Term structure of volatility; Jump-diffusion; Agricultural markets

1 Introduction

Black (1976) derives a pricing model for European puts and calls on a commodity futures contract, assuming that the futures price follows a geometric Brownian motion (GBM). In the literature on agricultural futures markets (as in many other markets) however, several empirical regularities have been documented, indicating that the GBM assumption may be too simplistic. Research on futures prices has found distributions that are leptokurtic relative to the normal distributions (e.g. Hudson et al., 1987; Hall et al., 1989) and the prices often exhibit sudden, unexpected and discontinuous changes. Jump behaviour of this sort will typically occur due to abrupt changes in supply and demand conditions, and naturally it will affect option pricing. Hilliard and Reis (1999) used transactions data on soybean futures and futures options to test American versions of Black's (1976) diffusion and Bates' (1991) jump-diffusion option pricing models. Their results show that Bates' model performs considerably better than Black's model.

A number of studies have demonstrated the presence of a term structure of volatility in agricultural futures prices. Samuelson (1965) stated that the volatility of futures price changes per unit of time increases as the time to maturity decreases. This maturity effect is usually referred to as the "Samuelson hypothesis". Another view, the "state variable hypothesis" is that the variance of futures prices depends on the distribution of underlying state variables. For crop commodities with annual harvest, seasonality in the volatility of futures prices is typically expected. Empirical research on the former approach has produced mixed evidence on the maturity effect (Rutledge, 1976). Milonas (1986) found strong support for the maturity effect after controlling for the year effect, seasonality effect and the contract-month effect. Galloway and Kolb (1996) concluded that the maturity effect is an important source of volatility in futures prices for commodities that experience seasonal demand or supply, but not for commodities where the cost-of-carry model works well. Anderson (1985) found support for the maturity effect, but concluded it is secondary to the effect of seasonality. Anderson also concluded that the pricing of options on futures contracts should be made for the regular pattern to the volatility of futures. Bessembinder et al. (1996) have reconciled much of the early evidence on the "Samuelson hypothesis". They have shown that in markets where spot price changes include a temporary component so investors expect some portion of a typical price change to revert in the future, the "Samuelson hypothesis" will hold. Mean reversion is more likely to occur in agricultural commodity markets than in markets for precious metals or financial assets (Bessembinder et al., 1995), so we expect to see maturity effects in agricultural commodity markets.

Any regular pattern in the volatility is inconsistent with the underlying assumptions of the Black's (1976) and Bates' (1991) option pricing models. Choi and Longstaff (1985) applied the formula of Cox and Ross' (1976) for constant elasticity of variance option pricing in the presence of seasonal volatility. They found this superior to Black's model for pricing options on soybeans futures. Myers and Hanson (1993) present option-pricing models when time-varying volatility and excess kurtosis in the underlying futures price are modelled as a GARCH process. Empirical results suggest that the GARCH option-pricing model outperforms the standard Black model. Fackler and Tian (1999) proposed a simple one-factor spot price model with mean reversion (in the log price) and seasonal volatility. They show that futures prices consistent with this spot price model have a volatility term structure exhibiting both seasonality and maturity effects. Their empirical results indicate that both phenomena are present in the soybean futures and option markets.

In this paper we assume that the futures price follows a jump-diffusion process. The diffusion term includes time dependent volatility that captures (possibly) both a seasonal and a maturity effect. We derive a futures option pricing model given our specified futures price dynamics, and we test our model empirically using eleven years of data on American futures option prices on wheat from Chicago Board of Trade (CBOT). We find that our model does a better job in explaining the option prices than the models previously suggested in the literature. The maturity effect is especially strong in this market. A numerical example illustrates the economic significance of our results. This paper is organised as follows: In the next section we present the model and derive the option pricing model. Thereafter the data are described and preliminary evidence on volatility term structure and jump effects is given, then the empirical results are presented. Finally, we illustrate the economic significance of volatility term structure and jump parameters and a numerical example is given. The paper ends with a summary and concluding comments.

2 The model

We shall present a jump-diffusion model for the futures price dynamics and derive an option pricing model for a European futures option. Fundamental to the pricing of contingent claims is the derivation from the real world distribution of the asset price, to the equivalent "risk-neutral" distribution, or the equivalent martingale measure (EMM) in modern terminology. The value of a contingent claim is the expected value under the EMM discounted by the risk free rate. In the paper by Merton (1976), jumps are assumed to be symmetric (zero mean) and nonsystematic. In a stock market model, this means that jumps are of no concern to an investor with a well-diversified portfolio, since jumps on average cancel out. Given such assumptions of firm specific jump risk, parameters concerning the jump part are equal under both the real world probability measure and the EMM. In our setting, focusing on wheat futures prices, the assumption of non-systematic jump risk may be inappropriate. If, for example, bad weather results in a poor harvest, futures prices may jump. However, the occurrence of such an event is likely to move all the commodity futures prices in the same direction, and so diversifying the jump risk is impossible. In other words, jump risk is systematic. To derive the EMM when jump risk is systematic, we have to make assumptions about the price of jump risk. In this paper we follow Bates (1991) closely.[1] Bates assumed frictionless markets, optimally invested wealth follows a jump-diffusion, and a representative consumer with time-separable power utility. He then derived the EMM from the real world probability measure. Under the assumptions on preferences and technology, he showed that jump parameters under the EMM need to be adjusted according to the preferences of the representative consumer. In case of risk neutrality, the jump parameters are equal under both measures. The only difference between our model and that of Bates is that we impose time

[1] A full derivation of the EMM in an equilibrium setting is given in the appendix in Bates (1991).

dependence in the diffusion term of the GBM. It is well known that the diffusion term is unchanged, going from one probability measure to an equivalent probability measure. Hence, the results in Bates apply to our model as well. We shall set up the model directly under the EMM. Denote the price of a futures contract as $F(t,T^*)$, where t is today's date and T^* is the maturity date of the contract. The futures price is assumed to follow the following dynamics under the EMM:

$$\frac{dF(t,T^*)}{F(t,T^*)} = -\lambda\bar{\kappa}dt + \sigma(t,T^*)dB(t) + \kappa dq \tag{1}$$

where $B(t)$ is standard Brownian motion under the EMM and κ is the random percentage jump conditional upon a Poisson distributed event, q, occurring. We assume that $(1+\kappa)$ is a lognormal random variable with mean $(\gamma - 1/2\,v^2)$ and variance v^2. Consequently, the expected percentage jump size is $E[\kappa] \equiv \bar{\kappa} = e^{\gamma} - 1$. The frequency of Poisson events is λ and q is the Poisson counter with intensity λ. Note that the jump parameters are independent of time to maturity. This means that if a jump occurs, a parallel shift in the term structure of futures prices will occur. If we observe several futures contracts with time to maturity spanning several years into the future, the jump structure described above may seem inadequate. If, for example, exceptional bad weather (such as a hurricane) partly destroys a harvest, then futures prices are likely to jump. But we would expect contracts with maturity before the next harvest to experience a greater price change than contracts with maturity preceding the next harvest, since the next harvest is likely to turn out better than the previous one. This behaviour can easily be incorporated in our model by imposing a term structure on the jump amplitude. Such an extension is ignored in this paper since the maturity of the futures contracts analysed in this paper never exceed one year. Hence, in our data set, imposing parallel jumps may be a satisfactory assumption. The function $\sigma(t,T^*)$ represents the instantaneous volatility of the futures price conditional on no jumps. We want to capture two possible effects in the specification of the volatility function; periodic seasonality and maturity effect. We shall concentrate on the following candidate

$$\sigma(t,T^*) = \sigma(t)\sum_{i=1}^{I}\sigma_i(T^* - t) \tag{2}$$

The first term represents the time t dependent seasonal volatility pattern. We model the periodic function as a truncated Fourier series

$$\sigma(t) = \bar{\sigma} + \sum_{j=1}^{p}(\alpha_j \sin 2\pi t + \beta_j \cos 2\pi t)$$

The maturity effect is modelled by negative exponentials

$$\sigma_i(T^* - t) = e^{-\delta_i(T^* - t)}$$

This model provides a fairly rich volatility term structure, and as we shall see below, a straightforward closed-form pricing formula for vanilla European options can be derived.

2.1 Relation to other models in the commodity literature

This model nests several models proposed for commodities in the literature. The seminal Black's (1976) model is given by $\lambda = \delta_i = \alpha_j = \beta_j = 0$. The one-factor model of Schwartz (1997), that captures the maturity effect, appears if we set $\lambda = \alpha_j = \beta_j = 0$. The jump-diffusion model of

Bates (1991) is $\delta_i = \alpha_j = \beta_j = 0$. Bates (1991) extended with maturity effect is $\alpha_j = \beta_j = 0$, and Bates (1991) extended with seasonal effects is given by $\delta_i = 0$.

2.2 Valuation of futures options

Valuation of both European and American futures options in this model are slight generalisations of the formula given in Bates (1991) and Merton (1976). Let *n* be the number of jumps occurring in the interval $[t,T]$. Then the solution to equation (1) is

$$F(T,T^*) = F(t,T)\exp\left(-\lambda\bar{\kappa}(T-t) - 1/2\int_t^T \sigma(s,t)^2 ds + \int_t^T \sigma(s,T)dB(s)\right)\prod_{j=0}^{n}(1+\kappa_j) \tag{3}$$

The value of a European futures call option written on the contract $F(t,T^*)$ where $T \le T^*$ with strike price *K* and maturity at time *T*, is given by

$$c(F(t,T^*),T) = e^{-r(T-t)}\sum_{n=0}^{\infty}(\Pr\_n\_jumps)\left(F(t,T^*)e^{b(n)(T-t)}N(d_{1n}) - KN(d_{2n})\right)$$

$$= e^{-r(T-t)}\sum_{n=0}^{\infty}\left(\frac{e^{-\lambda T}(\lambda T)^n}{n!}\right)\left(F(t,T^*)e^{b(n)(T-t)}N(d_{1n}) - KN(d_{2n})\right)$$

where

$$b(n) = -\lambda\bar{\kappa}(T-t) + \frac{n\gamma}{(T-t)} \qquad d_{1n} = \frac{\ln\left(\frac{F(t,T^*)}{K}\right) + b(n)(T-t) + \frac{1}{2}\omega^2 + nv^2}{\sqrt{\omega^2 + nv^2}}$$

$$d_{2n} = d_{1n} - \sqrt{\omega^2 + nv^2} \qquad \omega = \sqrt{\int_t^T \sigma(s,T^*)^2 ds}$$

Put options can be calculated explicitly, or they can be found via the futures option put-call parity. In the empirical part of this paper, we use data on American futures options, consequently some modification of the above model is required. Bates (1991) derives an approximation for an American option in the jump-diffusion framework. His approximation follows the work of Barone-Adesi and Whaley (1987) in the standard case where the underlying asset follows a GBM. We use the same approximation as described by Bates (1991), replacing the constant volatility in his setting with the time-dependent volatility given by ω above (we name this model Bates SM later in the paper).

3 Preliminary analysis and data description

Weekly data were obtained for call options on wheat futures and for the underlying futures contract traded on the CBOT from January 1989 until December 1999. Wheat futures contracts are available with expiration in March, May, July, September, and December. We first present a simple regression model to illustrate the term structure of volatility present in our eleven years sample of futures data.

3.1 Term structure effects in futures price volatility

We ran the following regression for each of the five contracts:

$$V_t = \eta_1 + \sum_{k=2}^{12} \eta_k D_{kt} + e_t \tag{4}$$

where V_t is estimated standard deviation of the log changes of wheat futures prices for month t based on daily data, D_{kt} are seasonal dummy variables for month t: $k=2$, February, …, $k=12$, December, and e_t is an error term assumed to follow an AR(1) process. The regression model was estimated by Hildreth and Lu (1960) grid search method.[2]

Table 1 Estimates of seasonality and maturity coefficients, March, May, July, September and December wheat futures contracts, 1989-1999. *t*-values are in parentheses

	March		May		July		September		December	
η_1	0.062	(7.32)	0.011	(1.25)	0.027	(1.86)	0.009	(0.95)	0.003	(0.18)
η_2	0.061	(0.20)	0.010	(0.05)	0.030	(0.40)	0.009	(0.01)	0.004	(0.11)
η_3	0.060	(0.24)	0.032	(2.11)	0.035	(0.71)	0.013	(0.39)	0.014	(0.93)
η_4	0.009	(4.99)	0.065	(5.04)	0.054	(2.15)	0.016	(0.63)	0.032	(2.11)
η_5	0.010	(4.61)	0.071	(5.33)	0.067	(2.93)	0.015	(0.54)	0.035	(2.17)
η_6	0.011	(4.44)	0.008	(0.24)	0.072	(3.17)	0.017	(0.65)	0.035	(2.08)
η_7	0.012	(4.32)	0.010	(0.10)	0.077	(3.47)	0.048	(3.30)	0.040	(2.37)
η_8	0.012	(4.34)	0.010	(0.02)	0.013	(0.91)	0.073	(5.38)	0.055	(3.31)
η_9	0.010	(4.65)	0.009	(0.16)	0.009	(1.24)	0.077	(5.89)	0.073	(4.65)
η_{10}	0.010	(4.83)	0.009	(0.12)	0.019	(0.56)	0.004	(0.41)	0.084	(5.79)
η_{11}	0.010	(5.31)	0.010	(0.12)	0.019	(0.67)	0.005	(0.36)	0.096	(7.58)
η_{12}	0.032	(3.93)	0.008	(0.30)	0.024	(0.33)	0.006	(0.43)	0.098	(10.03)
Adj R^2	0.58		0.56		0.68		0.65		0.73	

In Table 1 the results from the regression are reported in the following way; January is the constant term, η_1, February is $\eta_1+\eta_2$ etc. From the results in Table 1 we see a very pronounced maturity effect, and weak evidence of seasonality for each contract. Looking for example at the March contract we see that volatility starts to rise in December. The volatility in January, February and March is approximately six times the volatility in April.[3] We also see that the volatilities of the remaining months of the March contract are significantly different from volatility in January. Note also that the summer months have slightly higher volatilities than April and the autumn months. We find this pattern for the other contracts as well. In this paper we shall investigate whether this term structure effect is priced in the option market.

[2] OLS generally displayed autocorrelated residuals. The Hildreth and Lu grid search procedure was employed to yield consistent parameter estimates.

[3] The low *t*-statistics in February and March simply imply that the volatilities in those months are indistinguishable from the volatility in January.

3.2 Indication of jump behaviour from option prices

If wheat futures prices are characterised solely by deterministic time-dependent volatility, they are lognormally distributed. Furthermore, the implied volatility from option prices will be constant across strike prices. However, if jumps are likely to occur, implied volatility will be skewed. In Figure 1 we have calculated implied volatility from call futures prices at January 18, 1995. When backing out implied volatilities, we used the formula derived by Black (1976) adjusting for the fact that the options are of American type using the approximation of Barone-Adesi and Whaley (1987). Figure 1 shows no horizontal pattern of implied volatility, but an implied "volatility smile". A jump diffusion model may produce such a pattern. When futures prices are allowed to jump upwards, out-of-the-money (OTM) call options have a higher probability of ending in-the-money (ITM) than otherwise would be the case, and they will trade at a higher price. This in turn creates an upward sloping volatility pattern for call options evident from Figure 1. For a call option ITM, the probability of a negative jump will cause the options to be worth more than would be the case in a lognormal world.

Figure 1 Implicit volatility patterns from CBOT wheat call options with maturity in May 19, 1995 at January 18, 1995. Implied volatility for American options are approximated as in Barone-Adesi and Whaley (1987)

3.3 Constructing the data set

From the preliminary analysis above we have seen evidence suggesting that our model, including both jumps and time dependent volatility, will capture important market characteristics. We have therefore tested our model on wheat futures option prices collected from CBOT. The eleven years of data consist of fifty-five futures contracts. The futures contracts matures in March, May, July, September, and December. At each point in time, there are five contracts traded, meaning that one year is the longest contract an investor can enter into. The options written on the contracts can be exercised prior to maturity, hence they are of American type. The last trading day for the options is the first Friday preceding the first notice day for the underlying wheat futures contract. The expiration day of a wheat futures option is on the first Saturday following the last day of trading.

We applied several exclusion filters to construct the data sample. First, our sample starts in 1989. We did not use prices prior to 1989 since market prices then were likely to be affected by government programs in the United States (price floor of market prices and government-held stocks). Second, only trades on Wednesdays were considered, yielding a panel data set with weekly frequency. Weekly sampling is simply a matter of convenience. Daily sampling would place extreme demands on computer memory and time. Third, only settlement (closing) prices were considered. Fourth, the last six trading days of each option contract were removed to avoid the expiration related price effects (these contracts may induce liquidity related biases). Fifth, to mitigate the impact of price discreteness on option valuation, price quotes lower than 2.5 cents/bu were deleted. Sixth, assuming that there is no arbitrage in this market, option prices lower or equal to their intrinsic values were removed. Three-month Treasury bill yields were used as a proxy for the risk free discount rate. The exogenous variables for each option in our data set are strike price, *K*, futures spot price, *F*, today's date, *t*, the maturity date of the option contract, *T*, the maturity date of the futures contract, T^*, the instantaneous risk-free interest rate, *r*, observed settlement option market price, C_{it}, where *i* is an index over transactions (calls of assorted strike prices and maturities), and *t* is an index over the Wednesdays in the sample.

4 Implicit parameter estimation and in-sample performance

4.1 Method

Besides the exogenous variables obtained from the data set, the option pricing formula requires some parameters as inputs. In the full model the following parameters need to be estimated: the season and maturity effect-related parameters $\bar{\sigma}, \alpha_j, \beta_j, \delta_i$ and the jump-related parameters $\bar{\kappa}, v, \lambda$. There are two main approaches to estimate these parameters; from time series analysis of the underlying asset price, or by inferring them from option prices (Bates, 1995). There are two main drawbacks of the former approach. First, very long time series are necessary to correctly estimate jump parameters, at least if prices jump rarely. Second, parameters obtained from this procedure correspond to the actual distribution, and hence the parameters cannot be used in an option pricing formula, since the parameters needed for option pricing are given under the EMM. The latter approach, to infer some or all of the distributional parameters from option prices conditional upon postulated models has been used in, e.g., Bates (1991, 1996, 2000); Bakshi et al. (1997); and Hilliard and Reis (1999). Implicit parameter estimation is based on the fact that options are forward looking assets and therefore contain information on future distributions. Implied estimation delivers the parameters under the EMM.

We infer model-specific parameters from option prices over an eleven years long time period. The model is separately estimated for March, May, July, September and December wheat futures contracts expiring in 1989 through 1999. In previous studies, implicit parameters have been inferred from option prices during a very short time interval, often daily (e.g., Bates (1991, 1996); Hilliard and Reis, 1999). However, this method can be applied to data spanning any interval that has sufficient number of trades (Hilliard and Reis, 1999). Daily recalibrations can fail to pick up longer horizon parameter instabilities (Bates, 2000). In this study, one of the aspects we focus on is the changing volatility during the year. Options written on a specific contract have only one maturity each year. If we were to use daily data, a model with time-dependent volatility would be indistinguishable from a model with constant volatility. Information of changing volatility will be revealed as the option prices change during the course of the year. In other words, we need a long time span, in order to be able to pick up volatility term structure effects in this market.

American option prices, C_{it}, are assumed to consist of model prices plus a random additive disturbance term:

$$C_{it} = C\left(F_{it}, K_i, t, T, T^*, r, \bar{\kappa}, v, \lambda, \bar{\sigma}, \alpha_j, \beta_j, \delta_i\right) + e_{it} \quad (5)$$

Equation (5) can be estimated using non-linear regression. The unknown implicit parameters $\bar{\kappa}, v, \lambda, \bar{\sigma}, \alpha_j, \beta_j, \delta_i$ are estimated by minimising the sum of squared errors (SSE) for all option in the sample given by

$$SSE = \sum_{t=1}^{T}\sum_{i=1}^{N}[C_{it} - C(\bullet)]^2 = \sum_{t=1}^{T}\sum_{i=1}^{N}[e_{it}]^2 \tag{6}$$

where i is an index over transactions (calls of assorted strike prices and maturities), and t is a time index. The parameters minimising (6) were found using the Quadratic-hill climbing algorithm in GAUSS.

Many alternative criteria could be used to evaluate performance of option pricing models. The overall sum of squared errors (SSE) is used as a broad summary measure to determine how well each alternative option pricing model fits actual market prices. Assuming normality of the error term, nested models can be tested using F-test statistic.[4] Bates (1996, 2000) points out that the option pricing model is poorly identified. This means that when we minimise the non-linear function (5), quite different parameter values can yield virtually identical results. As a result of this, parameter estimates should be interpreted with care.

4.2 Implied parameters and in-sample pricing fit

The following models were estimated (abbreviations used later in the paper are in parentheses): Black's (1976) diffusion (Black76), Bates's (1991) jump-diffusion (Bates91), Black's model with season and maturity effect (Black SM) and Bates with season and maturity effect (Bates SM). Table 2 shows implicit parameter estimates for March, May, July[5], September and December wheat options. For the Black SM and Bates SM estimation was done with the maturity effects of order 1, i.e., only one parameter for α, β and δ, respectively.[6] As a result of forcing eleven years of data into one option pricing model with constant parameters, the SSE is quite large. However, R^2 values are high and vary between 0.967 and 0.988 between contracts and models.

---

[4] The F statistic is computed as $F[J, n-K] = \frac{(SSE_R - SSE_U)/J}{SSE_U/n-K}$ where SSE_U and SSE_R are sum squared errors for unrestricted and restricted models respectively, J is number of restrictions, n is number of observations in the sample, and K is number of parameters in the unrestricted model. In the nonlinear setting, the F distribution is only approximate (Greene, 1993, p. 336).

[5] For July contracts with the Bates SM model we had a problem in minimising function (6) in one step, so the parameters for this model were estimated in two steps. In step one all parameters except α_1 and β_1 were estimated. The parameters $\bar{\sigma}$ and δ_1 from step one were then used as constants in step two.

[6] We have also done some estimation of order 2 for both seasonal parameters and maturity parameters. Generally, using SSE as the performance criterion there is little improvement from including seasonal and maturity effects of order 2 compared to the more restrictive order 1 seasonal and maturity effects. Estimations of order 2 for only the seasonal parameters gave almost the same results as estimation of order 2 for both maturity and seasonal parameters, and are not reported here. However, the results are available from the authors upon request.

Table 2 Implicit parameter estimates for various models on March, May, July, September and December contracts on wheat in the period 1989-1999. 4264, 3859, 5074, 3971 and 5231 observations, respectively. *t*-values are in parentheses

	Black76		Black SM		Bates91		Bates SM	
March contracts								
σ	0.21	(514.7)	0.85	(1072)	0.15	(132.1)	1.18	(955.0)
γ					0.04	(51.5)	0.04	(47.9)
$\bar{\kappa}$					0.04		0.04	
ν					0.19	(542.8)	0.19	(215.4)
λ					0.57	(61.3)	0.59	(45.2)
δ_1			2.85	(247.3)			3.98	(812.6)
α_1			-0.11	-(22.6)			-0.11	-(10.2)
β_1			-0.57	-(223.4)			-1.00	-(151.8)
SSE	2 300 600		2 035 600		2 016 600		1 822 600	
May contracts								
σ	0.20	(1388)	0.25	(2897)	0.18	(2146)	0.23	(11.4)
γ					0.08	(6.4)	0.05	(5.9)
$\bar{\kappa}$					0.09		0.06	
ν					0.26	(673.8)	0.17	(466.9)
λ					0.14	(21.4)	0.60	(8.4)
δ_1			0.36	(3935)			0.71	(3.3)
α_1			-0.02	-(74.0)			-0.03	-(1.9)
β_1			-0.02	-(121.3)			-0.05	-(7.0)
SSE	1 514 000		1 458 200		1 399 100		1 299 000	
July contracts								
σ	0.21	(1102)	0.22	(889.7)	0.13	(598.0)	0.39	(183.2)
γ					0.04	(89.4)	0.02	(71.5)
$\bar{\kappa}$					0.04		0.02	
ν					0.05	(206.5)	0.15	(225.2)
λ					6.49	(578.8)	1.52	(93.8)
δ_1			0.01	(0.9)			4.49	(177.0)
α_1			-0.03	-(26.0)			-0.15	-(5.8)
β_1			-0.08	-(76.7)			-0.10	-(6.1)
SSE	4 793 100		3 848 100		4 609 900		3 840 900	
September contracts								
σ	0.24	(330.8)	4.00	(1027)	0.18	(1290)	0.34	(706.9)
γ					0.11	(158.1)	0.14	(21.3)
$\bar{\kappa}$					0.12		0.16	
ν					0.17	(60.8)	0.46	(636.3)
λ					0.56	(60.7)	0.14	(23.7)
δ_1			7.86	(533.8)			1.20	(173.2)
α_1			2.41	(444.3)			-0.15	(421.4)
β_1			2.46	(502.3)			-0.03	(169.8)
SSE	5 591 300		4 664 100		5 335 900		4 242 600	
December contracts								
σ	0.23	(805.3)	0.29	(477.0)	0.15	(156.5)	0.30	(24.5)
γ					0.01	(78.0)	0.05	(271.1)
$\bar{\kappa}$					0.01		0.05	
ν					0.24	(61.3)	0.35	(402.1)
λ					0.65	(442.1)	0.22	(24.4)
δ_1			1.03	(268.1)			1.56	(21.7)
α_1			0.01	(4.7)			0.05	(5.7)
β_1			-0.12	-(144.8)			-0.12	-(11.3)
SSE	4 734 500		4 548 000		4 360 800		4 173 200	

The results provide clear evidence of the importance of the seasonal and maturity effects; Bates SM performed best for all contracts. Furthermore, the inclusion of seasonal and maturity effects in Black76 sometimes gave approximately the same and sometimes better fit than Bates91 jump diffusion model. This indicates that the volatility term structure may be more important, in terms of option pricing, than the possibility of jumps. As Hilliard and Reis (1999) found this analysis also shows that Bates91 performed better than Black76. We have formally tested the models against each other using *F*-tests. The results given in Table 3, indicate that we can reject the other models proposed in the literature in favour of our model with both jump and time dependent volatility.

Table 3 Model specification tests for March, May, July, September and December contracts

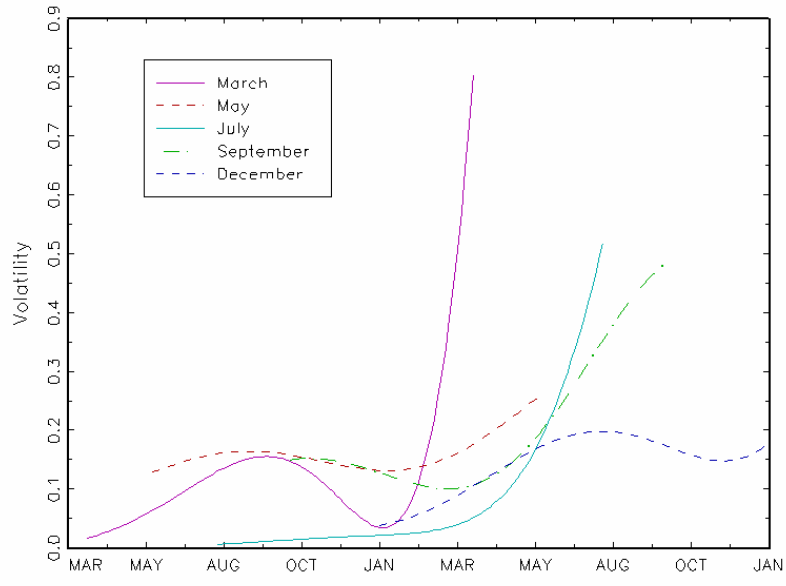
Null hypothesis	Restrictions	F-value	$F_{0.95}$-critical	Decision
March contracts				
Bates SM = Bates91	$\delta_1 = \alpha_1 = \beta_1 = 0)$	151.0	8.5	Reject H0
Bates91 = Black76	$\bar{\kappa} = \nu = \lambda = 0$	202.1	8.5	Reject H0
Black SM = Black76	$\delta_1 = \alpha_1 = \beta_1 = 0)$	187.0	8.5	Reject H0
May contracts				
Bates SM = Bates91	$\delta_1 = \alpha_1 = \beta_1 = 0)$	98.9	8.5	Reject H0
Bates91 = Black76	$\bar{\kappa} = \nu = \lambda = 0$	105.5	8.5	Reject H0
Black SM = Black76	$\delta_1 = \alpha_1 = \beta_1 = 0)$	49.2	8.5	Reject H0
July contracts				
Bates SM = Bates91	$\delta_1 = \alpha_1 = \beta_1 = 0)$	338.2	8.5	Reject H0
Bates91 = Black76	$\bar{\kappa} = \nu = \lambda = 0$	67.2	8.5	Reject H0
Black SM = Black76	$\delta_1 = \alpha_1 = \beta_1 = 0)$	415.0	8.5	Reject H0
September contracts				
Bates SM = Bates91	$\delta_1 = \alpha_1 = \beta_1 = 0)$	340.5	8.5	Reject H0
Bates91 = Black76	$\bar{\kappa} = \nu = \lambda = 0$	63.3	8.5	Reject H0
Black SM = Black76	$\delta_1 = \alpha_1 = \beta_1 = 0)$	262.9	8.5	Reject H0
December contracts				
Bates SM = Bates91	$\delta_1 = \alpha_1 = \beta_1 = 0)$	78.3	8.5	Reject H0
Bates91 = Black76	$\bar{\kappa} = \nu = \lambda = 0$	149.3	8.5	Reject H0
Black SM = Black76	$\delta_1 = \alpha_1 = \beta_1 = 0)$	71.4	8.5	Reject H0

4.3 A closer look at the volatility term structure

From Table 2 we also see that parameters governing the volatility dynamics differ somewhat across contracts. This may be explained partly by the fact that different parameter values may cause quite similar option prices, as mentioned above. We have plotted the volatility term structure for each contract in Figure 2, using the estimated parameters in Table 2. For each contract, the volatility term structure spans one year, and ends as the futures contract expires.

We see that March, July and September contracts reveal the most profound maturity effect. The December contract combines high summer volatility and a maturity effect during autumn. In sum, the December contract seems to be more volatile during the second half of the year. The July contract shows few signs of seasonality at all, but from Table 2 we see that the seasonal parameters are significantly different. Again, this illustrates that the maturity effect has a far bigger impact on the term structure of volatility than the seasonal effect.

Figure 2 Estimated term-structure of the volatility from option data for March, May, July, September and December futures contracts



4.4 A closer look at the jump parameters

As argued elsewhere, implied volatility curves reveal the effects of jumps on option prices. As an illustration of the effect of jumps on implied volatility, we computed theoretical option prices on American calls for different strikes using parameters from the full model (Bates SM) of the May contract in Table 2. The futures price is set to $F(t,T^*) = 3000$, the maturity of the contract $T^* = 7$ months, and the risk free rate $r = 0.05$. We backed out implied volatility curves using 5 strikes ($K = 2400$, 2700, 3000, 3300 and 3600) for three different option maturities ($T = 2$, 4 and 6 months). The results are given in Figure 3.

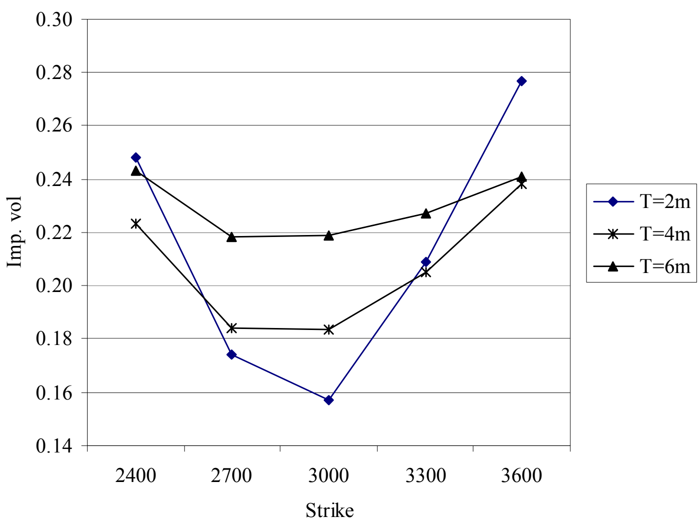


Figure 3 Implicit volatility patterns from CBOT wheat call options where options contracts have 2, 4 and 6 months to maturity, respectively and the underlying futures contract has 7 months to maturity. Implied volatility for American options are approximated as in Barone-Adesi and Whaley (1987)

We recognise the clear "smile" effect from Figure 1, caused by the possibility of both upward and downward jumps. It is also evident that this "smile" gets more pronounced as option expiration gets closer. If there is only a short time to maturity, far OTM options in a lognormal model will be worth relatively little, since an extreme upward price swings is very unlikely. In a jump-diffusion model, these options may end up ITM if a jump occurs, and consequently, these options will be relatively more valuable in a jump-diffusion than in a lognormal world. When there is long time to option maturity, the jump component plays a less prominent part when it comes to moving futures prices upwards or downwards. In the case of OTM options say, the diffusion term alone will be able to move the futures price so that the option will end up ITM.[7] We also note from Figure 3 that the volatility curve shifts upwards when option maturity increases. This fact is mainly caused by the maturity effect captured by the volatility term structure.

5 A numerical example

Finally, we provide a numerical example showing the economic significance of our findings. Assume that our model specification is correct; that both the volatility term structure and jumps are present in futures prices, and hence our option pricing formula calculates the true option price. What kind of mispricing will take place if we use the model of Black (1976) or Bates (1991) previously suggested in the literature? We stick to the example above and compute American call option prices based on parameters from the May contract for different option maturities. These prices are compared to Black76 and Bates91 model prices, again picking parameters from Table 2. The results are given in Table 4.

Table 4 Comparison of American wheat futures option prices using Black76, Bates91 and Bates SM for different strikes when the underlying futures contract has 7 months to maturity and the futures price is set to $F(t,T^*) = 3000$, and the risk free rate $r = 0.05$. Parameter estimates for the May contract in Table 2 is used

					%Diff.	
	K	Black76	Bates91	Bates SM	Black76 - Bates SM	Bates91 - Bates SM
T = 2m	2600	401.65	402.89	401.38	0.1 %	0.4 %
T = 7m*	3000	96.81	94.07	75.93	27.5 %	23.9 %
	3400	6.97	11.23	13.45	-48.1 %	-16.5 %
T = 4m	2600	414.12	414.64	409.74	1.1 %	1.2 %
T = 7m*	3000	136.02	134.96	124.30	9.4 %	8.6 %
	3400	25.45	30.95	31.89	-20.2 %	-2.9 %
T = 6m	2600	430.10	432.01	436.09	-1.4 %	-0.9 %
T = 7m*	3000	167.14	168.58	181.44	-7.9 %	-7.1 %
	3400	46.06	53.11	65.47	-29.6 %	-18.9 %

Concentrating on the last two columns, we see that Bates SM produce very different option prices than Black76 and Bates91. We note that the difference between Bates SM and Black76 is as much as 48% for the nearest OTM call. The general results are as follows: The prices from all

[7] In our special case, there is roughly equal chance for the jump to be either positive or negative under the EMM ($\bar{\kappa} \approx 0$). This means that as time to option expiration increases, multiple jumps will have a tendency to cancel each other out. This will enforce the flattening effect on the volatility smile as time to expiration increases. However, jump effects will in general be more visible in terms of implied volatility as time to expiration shortens (see Das and Sundaram (1999) for an investigation of term structure effects in a jump-diffusion model).

three models are more or less the same for ITM calls. This is due to the fact that the intrinsic value dominates the value of an option when deep ITM, and hence most models would produce quite similar results. The at-the-money (ATM) price differences are basically influenced by the term structure effect. Both Black76 and Bates91 use an average volatility for the whole period as input. The fact that the volatility of futures contract increases as maturity approaches, means that using an average value for the volatility will produce too high option prices for short maturity options and too low prices for long maturity options. We note that the prices from Black76 and Blates91 are in quite good agreement with each other; however, they differ quite severely from the Bates SM model. Last, the two alternative models produce significantly lower price for OTM calls than Bates SM. For the Black76 model, this fact is not surprising since OTM calls will be more valuable in a jump-diffusion world. The results from the Bates91 model deserve some explanation. We see that the parameters estimated for Bates91 give a less pronounced smile effect than Bates SM. This is because, as the volatility term structure is restricted to be flat, the jump parameters will influence both the prices across strikes, and the overall price level. From the discussion on implied volatility, the jump parameters influence both the "smile" and the level of the implied volatility curve.[8] In Bates SM, the term structure of volatility can take care of the level, and the jump parameters can "concentrate" on "smile" effects. Hence the parameters in Bates91, through the estimation method, emerge as a compromise of the two effects.

The results provided here may be of great importance in other valuation contexts. For example, Hilliard and Reis (1999) argue that average based Asian options are popular in commodity over-the-counter (OTC) markets. They show that Asian option prices in the Black76 versus Bates91 differ even more than is the case for European/American options prices. Our results indicate, in addition to the jump effect, that Asian option prices will differ quite substantially depending on where in the life of the option the average is calculated. Especially, the relative strong maturity effect will give very different prices on Asian options depending on both the length of averaging period and how close the averaging period is to the maturity of the futures contract.

6 Summary and concluding comments

In this paper we have developed an option pricing model that incorporates several stylised facts reported in the literature on commodity futures price dynamics. The volatility may depend on both calendar-time and time to maturity. Furthermore, futures prices are allowed to make sudden discontinuous jumps. We estimated the parameters of the futures price dynamics by fitting our model to eleven years of wheat options data using non-linear least squares. Several models suggested in the literature are nested within our model, and they all gave significantly poorer fit compared with our more complete model formulation. In a numerical example we showed that ignoring term structure and jump effects in futures prices may lead to severe mis-pricing of options.

References

Anderson, R.W., 1985. Some determinants of the volatility of futures prices. *Journal of Futures Markets* 5: 331-348.

[8] This fact may partly explain the observation reported in Hilliard and Reis (1999) that parameter values are not stable over time. In their estimation procedure, they calibrate the model each day. Using their procedure, Bates91 will be able to replicate Bates SM on one given maturity. When either the option or futures maturity changes, the parameters in Bates91 must change to capture the volatility term structure effect. Hence we would expect unstable parameters in the analysis of Hilliard and Reis (1999) if, in fact, there exists volatility term structure effects in the underlying futures data.

Bakshi, G., Cao, C., Chen, Z., 1997. Empirical performance of alternative option pricing models. *Journal of Finance* 52: 2003-2049.

Barone-Adesi, G., Whaley, R.E., 1987. Efficient analytic approximation of American option values. *Journal of Finance* 42: 301-320.

Bates, D.S., 1991. The crash of '87: was it expected? the evidence from options markets. *Journal of Finance* 46: 1009-1044.

Bates, D.S., 1995. Testing option pricing models. Working Paper 5129, National Bureau of Economic Research.

Bates, D.S., 1996. Jumps and stochastic volatility: exchange rate processes implicit in Deutsche mark options. *Review of Financial Studies* 9: 69-107.

Bates, D.S., 2000. Post-'87 crash fears in the SP 500 futures option markets. *Journal of Econometrics* 94: 181-238.

Bessembinder, H., Coughenour, J.F., Seguin, P.J., Smoller, M.M., 1995. Mean reversion in equilibrium asset prices: evidence from the futures term structure. *Journal of Finance* 50: 361-375.

Bessembinder, H., Coughenour, J.F., Seguin, P.J., Smoller, M.M., 1996. Is there a term structure of futures volatilities? reevaluating the Samuelson hypothesis. *Journal of Derivatives* Winter: 45-58.

Black, F., 1976. The pricing of commodity contracts. *Journal of Financial Economics* 3: 167-179.

Choi, J.W., Longstaff, F.A., 1985. Pricing options on agricultural futures: an application of the constant elasticity of variance option pricing model. *Journal of Futures Markets* 5: 247-258.

Cox, J.C., Ross, S.A., 1976. The valuation of options for alternative stochastic processes. *Journal of Financial Economics* 3: 145-166.

Das, S.R., Sundaram, R.K., 1999. Of smiles and smirks: a term structure perspective. *Journal of Financial and Quantitative Analysis* 34: 211-239.

Fackler, P.L., Tian, Y., 1999. Volatility models for commodity markets. Working Paper, North Carolina Sate University.

Galloway, T.M., Kolb, R.W., 1996. Futures prices and the maturity effect. *Journal of Futures Markets* 16: 809-828.

Greene, W., 1993. *Econometric Analysis*, 2nd. ed. NJ, Prentice Hall.

Hall, J.A., Brorsen, B.W., Irwin, S.H., 1989. The distribution of futures prices: a test of the stable paretian and mixture of normal hypothesis. *Journal of Financial and Quantitative Analysis* 24: 105-116.

Hildreth, C., Lu, J., 1960. Demand relations with autocorrelated disturbances. *Technical Bulletin* 276, Michigan State University Agricultural Experiment Station.

Hilliard, J.E., Reis, J.A., 1999. Jump processes in commodity futures prices and option pricing. *American Journal of Agricultural Economics* 81: 273-286.

Hudson, M.A., Leuthold, R.M., Sarassoro, G.F., 1987. Commodity futures price changes: recent evidence on wheat, soybeans, and live cattle. *Journal of Futures Markets* 7: 287-301.

Merton, R.C., 1976. Option pricing when underlying stock returns are discontinuous. *Journal of Financial Economics* 3: 125-144.

Milonas, N.T., 1986. Price variability and the maturity effect in future markets. *Journal of Futures Markets* 6: 443-460.

Myers, R.J., Hanson, S.D., 1993. Pricing commodity options when underlying futures price exhibits time-varying volatility. *American Journal of Agricultural Economics* 75: 121-130.

Rutledge, D.J.S., 1976. A note on the variability of futures prices. *Review of Economics and Statistics* 58: 118-120.

Samuelson, P.A., 1965. Proof that properly anticipated prices fluctuate randomly. *Industrial Management Review* 6: 41-49.

Schwartz, E.S., 1997. The stochastic behavior of commodity prices: implications for valuation and hedging. *Journal of Finance* 52: 923-973.