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Abstract

Since the early 1990s, researchers have routinely used count data models (such as the Poisson and negative binomial) to estimate the demand for recreational activities. Along with the success and popularity of count data models in recreational demand analysis during the last decade, a number of shortcomings of standard count data models became obvious to researchers. This had led to the development of new and more sophisticated model specifications. Furthermore, semi-parametric and non-parametric approaches have also made their way into count data models.

Despite these advances, however, one interesting issue has received little research attention in this area. This is related to the fast decay process of the dependent variable and the associated long tail. This phenomenon is observed quite frequently in recreational demand studies; most recreationists make one or two trips while a few of them make exceedingly large number of trips. This introduces an extreme form of overdispersion difficult to address in popular count data models. The major objective of this paper is to investigate the issues related to proper modelling of the fast decay process and the associated long tails in recreation demand analysis. For this purpose, we introduce two categories of alternative count data models. The first group includes four alternative count data models, each characterised by a single parameter while the second group includes one count data model characterised by two parameters. This paper demonstrates how these alternative models can be used to properly model the fast decay process and the associated long tail commonly observed in recreation demand analysis. The first four alternative count data models are based on an adaptation of the geometric, Borel, logarithmic and Yule probability distributions to count data models while the second group of models relied on the use of the generalised Poisson probability distribution.

All these alternative count data models are empirically implemented using the maximum likelihood estimation procedure and applied to study the demand for moose hunting in Northern Ontario. Econometric results indicate that most of the alternative count data models proposed in this paper are able to capture the fast decay process characterising the number of moose hunting trips. Overall they seem to perform as well as the conventional negative binomial model.and better than the Poisson specification. However further investigation of the econometric results reveal that the geometric and generalised Poisson model specifications fare better than the modified Borel and Yule regression models.

Keywords : fast decay process ; recreational demand; count data models ; Borel, Yule, logarithmic and generalised Poisson regression models.

1 - INTRODUCTION

Since the early 1990s, researchers have routinely used count data models (such as the Poisson and negative binomial) to estimate the demand for recreational activities. Along with the success and popularity of count data models in recreational demand analysis during the last decade, a number of shortcomings of standard count data models became obvious to researchers. For example, Habb and McConnell (1996) suggest augmenting commonly used count data models by introducing an individual specific weight parameter into the initial probability density function and by making it a function of only the variables that influence participation decisions to address the problem of zeros in recreation demand analysis. Other modifications have also been suggested to count data models to address certain institutional constraints such as the "bag limit" in big game hunting (Creel and Loomis 1992) and the possibility of visiting multiple recreation sites (Hausman et al., 1995).

In a typical recreation demand application of the benchmark Poisson model, the estimated model underpredicts the true frequency of zeros, overpredicts the true frequency of other small values and underpredicts the true frequency of large counts. This is well recognised in the count data literature (called overdispersion) and is caused by some form of unobserved heterogeneity in population parameter. Furthermore, a manifestation of this phenomenon is the existence of a variance larger than its mean. Three alternative approaches have been used to capture different forms of heterogeneity in the Poisson model by allowing the variance of the distribution to vary across counts. Following the parametric tradition, King (1989) and Winkelmann and Zimmermann (1991) proposed generalised count data models developed by exploiting the properties of the Katz family of probability distributions to tackle the problem of overdispersion. Note that these generalisations modify only the variance function but not the conditional mean. Recently, Cameron and Johansson (1997) proposed another parametric approach that simultaneously affects the specification of all conditional moments. In particular, they consider generalisations of the Poisson count data model based on a squared polynomial series expansion which permits flexible modelling of conditional moments and allows one to escape the restrictive framework of commonly used parametric count data models. Developments in nonparametric and semi-parametric econometrics during the last decade have also made their way into count data models. For example, Gurmu et al. (1996) proposed a specification in which the distribution of the variance is estimated non-parametrically using Laguerre series expansion estimators. Finally, Cooper (2000) proposed two nonparametric approaches, the pool adjacent violators approach and the kernel smoothing approach to travel cost analysis of recreation demand.

These developments in count data models are, indeed, exciting and have contributed to the growing popularity of count data models in recreation demand analysis. Despite these advances, however, one interesting issue has received little research attention in this area. This is related to the fast decay process of the dependent variable and the associated long tail. This phenomenon is observed quite frequently in recreational demand studies; most recreationists make one or two trips while a few of them make exceedingly a large number of trips. This introduces an extreme form of overdispersion difficult to address in popular count data models. The major objective of this paper is to investigate the issues related to proper modelling of the fast decay process and the associated long tails in recreation demand analysis. Although nonparametric approach makes no precise assumptions about functional form and allows the data to 'speak for themselves', good estimates of a nonparametric model can be obtained only with a very large amount of data (Delgado and Robinson 1992). While semi-parametric approach provides a compromise between parametric and nonparametric approaches and can reduce the potential for mispecification, it requires complex and delicate modelling efforts and careful fitting to the data. Even with careful modelling, the interpretation of the results remain open (Creel 1997). Moreover, Cooper's (2000) empirical analysis of an actual travel cost model (TCM) data set on waterfowl hunting shows that with proper econometric specification the parametric approach generates more reliable results than the semi-parametric or nonparametric approaches to recreation demand analysis.

In light of these observations and the fact that the data sets available for recreation demand analysis are often small, we employ the parametric approach to address the fast decay process and the associated long tail of the distribution. To this end, we introduce two categories of alternative count data models. The first group includes four alternative count data models each characterised by a single parameter while the second group includes one count data model characterised by two parameters. This paper demonstrates how these alternative models can be used to properly model the fast decay process and the associated long tail commonly observed in recreation demand analysis.

The inadequacies of the conventional count data models - the Poisson and negative binomial - to deal with the issue related to the fast decay process are highlighted in section 2. Alternative count data models capable of capturing the features of the fast decay process along with their basic properties are presented in sections 3 and 4. Section 5 describes data and empirical specifications of alternative count data models. The econometric results are discussed along with their policy implications in section 6. The final section of the paper summarises the major findings of this study and offers some concluding remarks.

2 - FAST DECAY PROCESS AND CONVENTIONAL COUNT DATA MODELS

A common observation in recreation demand studies is that a vast majority of the participants make at least one or two trips and the number of recreational trips higher than two falls rapidly. However, only a few overly enthusiastic recreationists make exceedingly large number of trips. Such idiosyncratic behaviour of recreationists generates trip data with special properties: the frequency of trips fall sharply after one or two trips but the distribution contains a long tail. This is called the fast decay process. As a result, its associated variance will be greater than its mean (overdispersion) and likely be an increasing (and possibly nonlinear) function of its mean. We revisit the popular count data models - the Poisson and the negative binomial models - in this section and comment on their ability to capture the fast decay process and the associated overdispersion.

The most widely used single parameter count data model in recreational demand analysis is the Poisson distribution. The basic Poisson model assumes that Y_i, the *ith* observation of the number of recreational trips for $i = 0, 1, 2,..,n$ follows a Poisson distribution and λ is the Poisson parameter to be estimated (see Table 1). A count data regression based on the Poisson distribution is specified by letting λ to vary over observations according to a specific function of a set of explanatory variables. The most commonly used specification for λ is:

$$\lambda_i = \exp(X_i\beta) \quad (1)$$

where X_i is a matrix of explanatory variables including a constant, β is a conformable vector of unknown parameters to be estimated and "exp" designates the exponential function.. Because of the exponential specification of λ, the basic Poisson model captures the discrete and nonnegative nature of the dependent variable and allows inference on the probability of trip occurrence. However, this specification also implies that the variance of the distribution is equal to its mean. This is a very restrictive property not often met in reality. In particular, when the dependent variable is characterised by a fast decay process, the so-called equidispersion property of the Poisson distribution is flagrantly violated. If this is not recognised and accounted for in modelling demand for recreation, the estimated parameters will be biased and inconsistent (Grogger and Carson 1991). The Poisson distribution admits the fast decay process only when the estimated value of the Poisson parameter (λ) is less than one and this seldom occurs in reality.

An alternative to the Poisson model has been proposed more than fifteen ago by Hausman *et al.* (1984) to deal with overdispersion in count data models. This alternative can be justified on the grounds that measurement errors and/or omission of explanatory variables could introduce additional heterogeneity and hence, overdispersion in the data. Under these conditions, it is assumed that the dependent variable is measured with a multiplicative error term ε_i which captures unobserved heterogeneity and this error term is uncorrelated with the explanatory variables. If the error term, ε_i, follows a Gamma distribution, a two parameter negative binomial model may be defined as (see also Table 1):

$$\Pr ob(Y_i = k; k = 0,1,2,...) = \frac{\Gamma(k+\nu)}{\Gamma(k+1)\bullet .\Gamma(\nu)} \bullet \left[\frac{\nu}{\nu+\lambda}\right]^{\nu} \bullet \left[\frac{\lambda}{\nu+\lambda}\right]^{k} \tag{2}$$

The expected value and the variance of this distribution are λ and $[\lambda+\lambda^2/\nu]$, respectively. The parameter ν is non-negative and called the precision parameter. It can also be noted that the variance is a quadratic function of its mean. To make sure that the mean λ is non-negative, the model is parameterised by assuming $\lambda_i = \exp(X_i^{'}\beta)$ where X_i is a vector of explanatory variables or covariates. A wide range of model specifications can be generated by setting the parameter ν as a function of the explanatory variables, X_i ,such that:

$$\nu_i = \frac{(\lambda_i)^m}{\alpha} = \left[\frac{1}{\alpha}\right]\left[\exp(X_i^{'}\beta)\right]^m \; ; \; \forall \;\; \alpha > 0 \tag{3}$$

where m is an arbitrary non-negative constant. By replacing ν_i in the variance by eq. (3) results in a generalised form of the variance such as:

$$Var(Y_i|X_i) = E[Y_i|X_i] + \alpha E[Y_i|X_i]^{2-m} = \lambda_i + \alpha \bullet (\lambda_i)^{2-m} \tag{4}$$

The associated probability distribution is now a "generalised" density function given by

$$\Pr ob(Y_i = k; k = 0,1,2,...) = \frac{\Gamma\left[k + \left[\frac{(\lambda_i)^m}{\alpha}\right]\right]}{\Gamma(k+1)\bullet .\Gamma\left[\frac{(\lambda_i)^m}{\alpha}\right]} \bullet \left\{\frac{(\alpha\lambda_i)^k (\lambda_i)^{\frac{m(\lambda_i)^m}{\alpha}}}{\left[(\lambda_i)^m + \alpha\lambda_i\right]^{(\frac{(\lambda_i)^m}{\alpha}+k)}}\right\} \tag{5}$$

A closer look at expressions (4) and (5) reveals that different forms of overdispersion can be captured in this model depending on the values taken by the parameters m and α. Moreover, it provides a convenient formulation for nesting popular count data models through linking the conditional mean and variance of the dependent variable as discussed below.

- A value of $\alpha = 0$ yields the Poisson model where variance and mean are equal.

- If m=1, $Var(Y_i | X_i)=E[Y_i | X_i]\bullet(1+\alpha)=\lambda_i\bullet(1+\alpha)$. This specification is called Negbin type I. It assumes a constant relationship between conditional mean and variance.
- When m=0, the precision parameter ν_i is constant and equal to $1/\alpha$. The variance of the distribution is equal to $\lambda\bullet(1+\alpha\lambda)$. This specification is known as Negbin Type II and has been extensively used in modelling the demand for recreational activities.
- a fast decay process is obtained when the parameter α assumes values greater than or equal to one.
- When $m \neq 1$ and/or $\neq 0$ and depending upon the values taken by this parameter, we have two types of specification to represent overdispersion. When $m < 1$, the conditional variance increases with the mean at a increasing rate. On the other hand, when $m > 1$ the variance increases with the mean at a decreasing rate.

From the above, it can be seen that a fast decay process is better specified with a negative binomial model with $\alpha \geq 1$ and $m < 1$. This explains why the Negbin II specification provides a better representation of overdispersion (and hence fast decay process) than Negbin I. For this reason, the former has also been extensively used for modelling the demand for recreational activities. However, the "generalised" version of this model presented in eq. (5) is yet to receive wide application mainly because its probability distribution and the associated likelihood function are highly nonlinear with respect to the parameters. Such nonlinearities lead to difficult estimation problems and often it is difficult to obtain convergent estimates of the parameters from this model. Although Saha and Dong (1997) suggested a step by step estimation procedure to obtain a global optimum of the likelihood function associated with the "generalised" Negbin model, it still requires innovations in modelling and estimation to adequately model the fast decay process with the "generalised" negative binomial model. Compared with these challenges, the distributional features of other count data models appear attractive to model the fast decay process.

3 - ALTERNATIVE COUNT DATA MODELS AND ONE PARAMETER PROBABILITY DISTRIBUTIONS

A set of four alternative count data models and their basic features are presented in this section. They rely on a one-parameter probability distribution and are suitable for representing the fast decay process and the long tails. These models are presented in the same spirit as the original introduction of count data models in recreational demand analysis.

The geometric distribution

The geometric probability distribution suggested by Mullahy (1986) could be easily adapted to recreational demand for trips. This distribution is characterised by the parameter λ assumed to be positive. The mean and variance of this distribution are λ and $\lambda\bullet(1+\lambda)$, respectively (see Table 1). Since the variance is a quadratic function of the mean, the geometric distribution allows for over dispersion in data and can represent the fast decay process. The left-truncated geometric model has the following probability distribution:

$$\Pr ob(Y=k; k=0,1...n | k>0) = \frac{\Pr ob(Y=k)}{1-\Pr ob(Y=0)} = \frac{\lambda^k (1+\lambda)^{-(k+1)}}{(1-(1+\lambda)^{-1})} = \lambda^{k-1}(1+\lambda)^{-k} \qquad (7)$$

The geometric distribution is unimodal for k = 0 and can be obtained as a special case of the negative binomial model when the precision parameter v is equal to 1. The model is parameterised as $\lambda_i=\exp(X_i'\beta)$, where X is the matrix of explanatory variable.

Table 1: Characteristics of the probability distributions used in this study

Designation	Range of support values	Probability distribution	Expectation	Variance	Specific remarks
One-parameter distribution					
Poisson	0, 1, 2, 3, …n	$\Pr ob(Y=k)=\dfrac{e^{-\lambda}\lambda^{k}}{k!}$	λ	λ	V(Y) = E(Y)
Geometric	0, 1, 2, 3, …n	$\Pr ob(Y=k)=\lambda^{k}(1+\lambda)^{-(k+1)}$	λ	λ(1+λ)	V(Y) =E(Y)*[1+E(Y)]
Borel	1,2,3,…….n	$\Pr ob(Y=k)=\dfrac{k^{k-2}\lambda^{k-1}e^{-\lambda k}}{(k-1)!}$	1/(1-λ)	$\lambda/(1-\lambda)^3$	i) $0<\lambda<1$ ii) $V(Y)=[E(Y)]^2*[E(Y)-1]$
Logarithmic	1,2,3,…….n	$\Pr ob(Y=k)=\dfrac{-\lambda^{k}}{k\ln(1-\lambda)}$	$\dfrac{-\lambda}{(1-\lambda)\ln(1-\lambda)}$	$\dfrac{-\lambda(\ln(1-\lambda)+\lambda)}{(1-\lambda)^2(\ln(1-\lambda))^2}$	i) $0<\lambda<1$ ii) $V(Y)=E(Y)*\left[\dfrac{1}{1-\lambda}-E(Y)\right]$
Yule	1,2,3,…….n	$\Pr ob(Y=k)=\eta\mathrm{B}(k,\eta+1)=\dfrac{\eta\Gamma(k)\Gamma(\eta+1)}{\Gamma(k+\eta+1)}$	$\dfrac{\eta}{\eta-1}$	$\dfrac{\eta^2}{(\eta-1)^2(\eta-2)}$	i) $\eta>1$ ii) $\eta>2$ for the existence of the variance iii) $V(Y)=\dfrac{[E(Y)]^2[E(Y)-1]}{2-E(Y)}$
Two parameter distribution					
Negative binomial	0, 1, 2, 3, …n	$\Pr ob(Y=k)=\dfrac{\Gamma(k+\nu)}{\Gamma(1+k)\Gamma(\nu)}\left[\dfrac{\nu}{\nu+\lambda}\right]^{\nu}\left[\dfrac{\lambda}{\nu+\lambda}\right]^{k}$	λ	$\lambda+\lambda^2/\nu$	i)ν = 1, Negbin ≈ Geometric ii)ν → + ∝, Negbin ≈ Poisson iii)ν → 0, Left-truncated Negbin ≈ Logarithmic
Generalised Poisson	0, 1, 2, 3, …n	$Prob(Y=k)=\dfrac{\lambda(\lambda+\delta k)^{k-1}e^{-(\lambda+\delta k)}}{k!}$	$\dfrac{\lambda}{1-\delta}$	$\dfrac{\lambda}{(1-\delta)^3}$	i) λ > 0 and *δ* < 1 ii) δ= 0, Poisson iii) λ= δ < 1, Modifed Borel

Notes: B(η, k) and Γ(k) designates the Beta and Gamma functions, respectively. They are linked to each other by the following relationship: B(η, k) = Γ(η)Γ(k)/Γ(η+k)

Borel distribution

Another count data model capable of capturing the fast decay process is the Borel distribution (see Table 1). It was originally developed in the context of queuing theory[1]. The Borel distribution cannot be used in its original form to model the demand for recreational trips because it admits only positive values for the random variable (i.e., the number of trips) and thus excludes zeroes. This could be overcome by shifting the Borel distribution to the left so it has support *0*, *1*, *2*…(i.e. by obtaining $Z = Y-1$). The resulting probability distribution can be called a modified Borel distribution and defined as follows:

$$\Pr ob(Z = k; k = 0,1,....n) = \frac{(k+1)^{k-1} \lambda^k \exp[-\lambda(k+1)]}{k!} \tag{8}$$

where the parameter λ is positive and smaller than one. The modified Borel probability distribution is unimodal for $k = 0$. The left-truncated modified Borel distribution has the following form:

$$\Pr ob(Z = k; k = 0,1,....n | k > 0) = \frac{(k+1)^{k-1} \lambda^k \exp[-\lambda(k+1)]}{k![1-\exp(-\lambda)]} \tag{9}$$

The mean of this distribution is $\lambda/(1-\lambda)$, while the variance is equal to $\lambda/(1-\lambda)^{3.}$ Since the variance of the modified Borel distribution is a polynomial of degree three of its mean, it allows for a richer kind of overdispersion and better representation of the fast decay process than the geometric model.

To model the recreational demand for trips, a modified Borel regression can be parameterised such that $\lambda=1/(1+\exp(-X'\beta))$ where **X** is the matrix of explanatory variables.

The logarithmic distribution

The third alternative is based on the logarithmic distribution developed by Fisher *et al.* (1943). The density function and the basic characteristics of this distribution are presented in Table 1. The logarithmic distribution is unimodal for $Y = 1$ and it is the limiting distribution of a left truncated Negbin II distribution when the precision parameter v (or α) approaches 0 (tends to $+\infty$). In its original form, this distribution also excludes zero values. Consequently, it cannot be used for modelling the demand for recreational activities. For the purpose of this paper, we use a random variable, $Z = Y - 1$, and develop a modified logarithmic distribution which can be written as:

$$\Pr ob(Z = k; k = 0,1,..n) = \frac{-\lambda^{k+1}}{(k+1)\ln(1-\lambda)} \tag{10}$$

where λ is the parameter of this distribution that is positive and smaller than one. The mean of the modified logarithmic distribution, *E(Z)* is given by

$$E(Z) = E(Y) - 1 = \frac{\lambda[\ln(1-\lambda)-1] - \ln(1-\lambda)}{(1-\lambda)\ln(1-\lambda)} \tag{11}$$

while the variance is linked to its mean through the following non-linear relationship:

[1] Conceptualised first by Borel (1942), this probability distribution described the distribution of the total number of customers served before a queue vanishes given a simple queue with random arrival times of customers (at constant rate) and a constant time occupied in serving each customer (Johnson *et al*, 1992). According to our knowledge, there has been no application of the Borel distribution in economics.

$$E(Z)= [E(Z)+1]^2*[-ln(1-\lambda)-1] \tag{12}$$

Unlike the original logarithmic distribution, the modified version is characterised by a variance which is always greater than the mean regardless of the values taken by λ over the range [0, 1].The left truncated probability density for the modified logarithmic distribution can be defined as:

$$\Pr ob(Z = k; k = 0,1,....n \big| k > 0) = \frac{\Pr ob(Z = k)}{1 - \Pr ob(Z = 0)} = \frac{-\lambda^{k+1}}{(k+1)[\ln(1-\lambda)+\lambda]} \tag{13}$$

While the logarithmic distribution can handle both under- and over-dispersed data generating processes, the over-dispersion is satisfied only when the variance to mean ratio is greater than unity. The later condition is readily satisfied by the modified logarithmic distribution.

Assuming the parameter of the distribution λ is a logistic function of the explanatory variables, the mean of the modified logarithmic distribution can be derived as

$$E(Z) = \frac{\exp(X'\beta)}{\log\left[1+\exp(X'\beta)\right]} - 1 \tag{14}$$

the Yule distribution

The last one-parameter count data model suggested in this paper is based on the Yule distribution (see Table 1). Like the Borel and logarithmic probability distributions, the Yule distribution in its original form does not accommodate zero values and hence cannot be employed to analyse the demand for recreational trips. To overcome this problem, we shift the Yule distribution to the left so that it has support *0*, *1*, *2*,..*n* (i.e. by obtaining *Z* = *Y*-1). The resulting probability distribution is a modified Yule distribution defined as (Johnson *et al*,1992)

$$\Pr ob(Z = k; k = 0,1,....n) = \eta \mathrm{B}(k+1, \eta+1) = \frac{\eta\Gamma(k+1)\Gamma(\eta+2)}{\Gamma(k+\eta+2)} \tag{15}$$

where *B(.)* and *Γ(.)* are the Beta and Gamma functions, respectively and the parameter η is greater than one. The modified Yule distribution is unimodal for *k*=0. The mean is now equal to E(Z)=1/(η-1) while its variance is linked to its mean through the following relationship :

$$V(Z) = \sigma^2 = \frac{\left[E(Z)+1\right]^2}{1-E(Z)} \tag{16}$$

It can be seen from eq. (16) that the variance exists only if the mean is smaller than one. This property of the modified Yule distribution can be viewed as a weakness in case of recreational demand analysis because the average number of trips is usually greater than 1.

To model the recreational demand for trips, a modified Yule regression can be parameterised such that the distribution parameter[2] η is equal to exp(-**X**'β)+1 where **X** is the matrix of explanatory variables.

[2] We implicitly assume that the parameter η is equal to the inverse of the logistic function of the explanatory variables.

Each of the four count data models presented above captures overdispersion through a variance which is an increasing function of the mean. Note, however, the relationship between the mean values and the variance differ across the models. A simple numerical simulation was performed to obtain the probability distributions associated with each of the four count data models presented above. A number of interesting features emerge from a graphical representation of these simulation results (not reported here due to lack of space). First, irrespective of the mean values, the modified Yule distribution has the highest mode, followed by the modified Borel, modified logarithmic and geometric distributions. Second the nature of the decay process is slightly different across the four distributions. The decay process is more pronounced in case of modified Borel, logarithmic and Yule distributions than in the case of a geometric distribution. The modified Borel distribution also accommodates a longer tail than the geometric distribution.

The four alternative count data models presented in this section are not difficult to implement for studying recreational demand for trips. It is also relatively easy to generate estimates of consumers' surplus per trip for the geometric, modified Borel and modified Yule models. The way that these three regression models are parameterised yields a demand for recreational activities that is a semi-logarithmic function of explanatory trips and the associated consumer surplus per trip is equal to -1/price coefficient. However, the derivation of the consumer surplus for the modified logarithmic count data model involves a more complicated procedure[3].

4 - ALTERNATIVE COUNT DATA MODELS AND THE GENERALISED POISSON DISTRIBUTION

An assumption implicit in most count data analysis is that the occurrence of one count is independent of that of another count. While this may be a reasonable assumption for modelling many physical processes, it may not be so in social sciences. For example, the independent occurrence assumption may not be plausible when one is dealing with the number of visits to a doctor (Pohlmeier and Ulrich 1995) or the number of trips to a recreation site (Creel and Loomis 1990; Grogger and Carson 1991). Since the generalised Poisson distribution allows for the probability of an event to depend on the number of events already occurred (Consul and Shoukri 1985), this distribution may be particularly useful in recreational demand analysis. Introduced by Consul and Jain (1973), the generalised Poisson distribution has recently be used in a regression context by Consul and Famoye (1992), Famoye (1993) and Santos Silva (1997). Here, we concentrate on the aspects of the generalised Poisson distribution relevant for recreational demand analysis.

Following Consul (1989), the generalised Poisson distribution can be defined as (see Table1)

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[3] For this model, the expression (14) defining the demand for recreational activities is quite complex at first sight because its involves a ratio of an exponential function over a logarithmic function. The computation of the consumer surplus is going to require some special treatment using the *LogIntegral* function. For this purpose, we derive the following expression of consumer surplus for this distribution

$$CS_i = \int_P^{Pch}\left[\frac{\exp(a+bp)}{Log\left[1+\exp(a+bp)\right]}-1\right]dP_i = \left[-P+\frac{LogIntegral(1+\exp(a+bo))}{b}\right]_P^{Pch}$$

where Pch is the choke price defined as the price at which the quantity demanded tends to zero. *LogIntegral* is defined as follows:

$$LogIntegral(z) = \int_0^z \frac{dt}{Log(t)}$$

The consumer surplus per trip is then obtained by dividing $CS(Z_i)$ by the expected number of trips.

$$Prob(Y = k;\ k = 0,1,2..,\ n) = \begin{cases} \dfrac{\lambda(\lambda + \delta k)^{k-1} \exp\left[-(\lambda + \delta k)\right]}{k!} \\ 0 \ for \ \ y > m \ \ when \ \ \delta < 0 \end{cases} \tag{17}$$

where > 0, max (-1, -λ/m) < δ < 1 and m ≥ 4 is the largest positive integer for which λ + δm > 0 when δ is negative. The mean and variance of the generalised Poisson distribution are μ= E(Y) = λ/(1-δ) and σ^2 = V(Y) = λ/$(1-\delta)^3$ = μ/$(1-\delta)^2$, respectively. Note that the variance is greater than, equal to, or less than the mean if δ is positive, zero or negative. Moreover, when δ is positive, both the mean and variance increase as the value of δ increases. However, the variance increases faster than the mean. This property is very useful in recreational demand studies where the dependent variable is characterised by overdispersion. The generalised Poisson distribution also admits underdispersion and equidispersion.

Using the fact that μ = λ/(1-δ) = λρ, it is also possible to express the generalised Poisson distribution as a function if its mean. The resulting distribution can be expressed as follows:

$$Prob(Y = k;\ k = 0,1,2...,\ n) = \begin{cases} \dfrac{\mu\left[\mu + (\rho - 1)k\right]^{k-1} \rho^{-k}\ e^{-\left[(\mu + (\rho - 1)k / \rho\right]}}{k!} \\ 0 \ for \ \ y > m \ \ when \ \ \rho < 1 \end{cases} \tag{18}$$

where, ρ ≥ max (1/2, 1-μ/4) and *m* is the largest positive integer for which $\mu + m\ (\rho - 1) > 0$ when ρ is less than one. The variance is given by $\sigma^2 = V(Y) = \rho^2 \bullet \mu$. When $\rho = 1$, the generalised Poisson is equivalent to the Poisson model while the modified Borel probability distribution is obtained if ρ=1/(1-λ) or if ρ=(1+μ). Any values of ρ > 1 represents count data process with overdispersion and and 0,5 ≤ ρ < 1 characterises count data with underdispersion when μ > 2. Note that, the variance is proportional to its mean, thus implying a constant variance to mean ratio (like in the case of Negbin I). This property may not be the most suitable for capturing the type of overdispersion associated with the fast decay process.

To gain additional insights about the ability of the GP model to capture overdispersion associated with the fast decay process, a simulation exercise was performed for the one parameter GP probability distribution using the same values for the mean, μ but allowing different values of the parameter $\rho \geq 1$. A graphical examination of these simulation results indicates that the unimodality of the GPD is preserved for a value of ρ equal to one and a mean μ smaller than 0.5. Secondly, these results show that the L-shaped distribution is well represented for μ =0.5 and ρ greater than 1. It appears from the simulation exercise that the generalised Poisson distribution admits a fast decay process only under some restrictive conditions.

To overcome this problem, a restricted version of the generalised Poisson distribution (denoted RGPD hereafter) can be defined by making the parameter δ proportional to such that δ = α λ. Substituting this expression for δ in expression (18) yields the following RGP distribution:

$$\Pr ob(Y = 0; k = 0,1,....n) = \begin{Bmatrix} \dfrac{\lambda^k (1+\alpha k)^{k-1} \exp\left[-\lambda(1+\alpha k)\right]}{k!} \\ 0 \ \ for \ \ x > m \ \ when \ \ \alpha < 0 \end{Bmatrix} \text{ for } k = 0,\ 1,\ 2\ldots n \tag{19}$$

The domain of the parameter α is given by max ($-\lambda^{-1}$, -1/4) ≤ α ≤ λ^{-1} (Famoye, 1993). If $\alpha = 0$, the RGPD reduces to the Poisson distribution while for $\alpha = 1$ and λ< 1, we get a modified Borel distribution. The mean and variance associated with the RGPD are, μ = E (Y) = λ/(1-αλ) and σ^2= V (Y) = λ/$(1-\alpha\lambda)^3$ = μ*$[1 + \alpha\ \mu]^2$ respectively.

An alternative specification of the RGPD can also be obtained if the parameter λ is expressed as a function of the mean (μ). This yields a one-parameter probability distribution expressed as

$$\Pr ob(Y = 0; k = 0,1,....n) = A^k \frac{(1+\alpha k)^{k-1} \exp[-A(1+\alpha k)]}{k!} \tag{20}$$

where $A = \dfrac{\mu}{1+\alpha\mu}$

Overdispersion is obtained when $\alpha > 0$. It is interesting to note that the variance of this model is a third degree polynomial function of its mean. This allows for a richer type of overdispersion and is likely to model the fast decay process efficiently. A graphical representation of the one-parameter RGPD for a wide range of values of the mean, μ and the parameter, α clearly shows that well-defined L-shaped distributions are obtained regardless of the values of the mean, μ and the parameter, α. Therefore, the restricted generalised Poisson distribution appears to be well suited to represent the fast decay process. If we assume that the mean μ is an exponential function of explanatory variables so that $\mu_i = \exp(X_i'\beta)$, then it is possible to define generalised Poisson regression models which can be estimated either in restricted or unrestricted forms (Consul and Famoye 1990; Famoye 1993). Santos Silva (1997) has shown that the two (restricted and unrestricted) forms of the generalised Poisson regression model can be nested through a hybrid generalised Poisson model. To do so, the parameter α is linked to the covariates X_i, so that $\alpha_i = \alpha_i(X_i, \theta, \beta) = \theta_0 \exp[\theta_1 (X'\beta)]$ and that $\mu_i = \exp(X_i'\beta)$. Incorporating these expressions in (20) a hybrid generalised Poisson (denoted HGPI) regression model can be defined as:

$$\Pr ob(Y = k; k = 0,1,...n) = [A(X,\beta,\theta)]^k \bullet \frac{[1+\alpha(X,\theta,\beta)k]^{k-1} \exp[-A(X,\theta,\delta) \bullet (1+\alpha(X,\theta,\beta)k)]}{k!} \tag{21}$$

where $A(X,\beta,\delta)$ is now equal to $\dfrac{\exp(X'\beta)}{1+\alpha(X,\theta,\beta)\exp(X'\beta)}$ or to $\dfrac{\exp(X'\beta)}{1+\theta_0 \exp[(1+\theta_1)(X'\beta)]}$

A closer look at expression (21) reveals that, depending upon the form taken by the function $\alpha(X,\theta,\beta)$, the following model specifications can be obtained as special cases of this specification.

- when $\alpha_i(X_i, \theta, \beta)$ is a constant (when $\theta_1 = 0$), one obtains the restricted GP model(denoted HGDPII)[4]. In addition, if $\theta_0 = 1$, the model is reduced to the modified Borel regression model.

- If $\alpha_i(X_i, \theta, \beta)$ is proportional to $\exp(-X_i'\beta)$ (which is obtained when $\theta_1 = -1$), we obtain the GP regression model(denoted HGPDIII)[5].

- Finally, if $\alpha_i(X_i, \theta, \beta)$ is equal to zero (which is obtained when $\theta_0 = 0$), we obtain the Poisson regression model.

A left truncated HGP model could also be defined by adjusting expression (21) with the prob($Y_i > 0$) which is equal to $1-\exp[-A(X,\beta,\theta)]$. This leads to the following left truncated HGP model:

$$\Pr ob(Y = k; k > 0) = [A(X,\beta,\theta)]^k \bullet \frac{[1+\alpha(X,\beta,\theta)k]^{k-1} \exp[-A(X,\beta,\theta) \bullet (1+\alpha(X,\theta,\beta)k)]}{(k!) \bullet [1-\exp[-A(X,\beta,\theta)]]} \tag{22}$$

[4] θ_0 is then equal to the parameter α in expression (20).

[5] In this case, θ_0 is equal to ρ-1 and replacing it in (21) yields expression (18).

HGP demand models for recreational activities are obtained through $\mu=e^{(X\beta)}$ with the covariates X including price (i.e., travel costs). It is also interesting to note that the consumer surplus per trip obtained from this HGP model is equal to -1/(price coefficient). This is a major advantage of the HGP models.

5 - DATA DESCRIPTION AND EMPIRICAL SPECIFICATION

The data used in the illustrative applications of various count data models discussed earlier in this paper relate to the 1992 moose hunting season at the Wildlife management Unit #21A located in northern Ontario. It is a popular WMU for moose hunting because of its remoteness and moose population density. During the 1992 season some 1286 hunters received moose validation tags to hunt an adult moose at this hunting site and about 99% of these hunters were from Ontario. Most of the data came from the Ontario Ministry of Natural Resources. Data include the number of moose hunting trips made by each hunter to the WMU #21A and the travel cost per hunter (which includes vehicle related costs, a licence fee of CDN$26.50 per hunter per season, equipment costs, costs of food and lodging and time cost). The income variable consists of 1991 average employment income and other income at the Enumeration Area (EA) level. This information is based on 1991 census data and was adjusted to 1992 level using consumer price index (CPI). Further details on this dataset can be found in Sarker and Surry (1998).

The general specification of the travel cost model used is:

$$Y_i = f(\text{Prices (i.e., Travel Costs), Income}, \beta, \varepsilon_i) \qquad (23)$$

Where β is the vector of parameters and ε_i is the vector of residuals. The dependent variable is the number of moose hunting trips taken to WMU21A and is truncated at zero.

During the 1992 hunting season, hunters in Ontario made 2.35 moose hunting trips on an average to the WMU21A. Note that the data exhibit a quick decay process; about 78% of the sample hunters made only one trip during this season and the number of trips higher than one falls rapidly. However, a few hunters made more than 10 trips to this hunting site. Note also that the variance of the dependent variable is 12.89. Clearly, the equidispersion property of the Poisson distribution is not satisfied. We used this data set to illustrate the empirical applications of alternative count data models. The results are discussed in the following section.

6 -ECONOMETRIC RESULTS AND DISCUSSION:

We used an exponential form for all count data models and the maximum likelihood (ML) procedure to generate econometric results for all count data models (see appendix for a brief summary on estimation issues). The results for recreational for hunting trips in Ontario for alternative left-truncated count data models are presented in Table 2. In addition, left-truncated Poisson and negative binomial (Negbin II) models have been estimated for comparative purposes. The standard errors of the coefficients were estimated using the Eicker-White procedure.This procedure generates a heteroskedasticity-consistent covariance estimate which is asymptotically valid when the heteroskedasticity is of unknown form (White).

Leaving aside the Poisson specification, a first hand examination of the econometric results indicated that all the alternative count data model specifications proposed in this paper perform as well as the Negbin II model in terms of explanatory power and/or goodness of fit. Hence, based on a general R-squared measure (R^2_{LRT}) of goodness-of-fit which is related to the likelihood ratio test statistic for the joint significance of slope parameters, the modified Borel model gives the best specification followed by the geometric and negative binomial models. The other alternative count data models specifications have R^2_{LRT} ranging from 0.64 for the modified Yule model to 0.77 for HGPI Model. If we evaluate the overall performance of each model in terms of the Akaike information criterion (AIC), the lowest value (155.7) is reached by the modified logarithmic and geometric models. It is then followed by the Negbin II. All the other alternative count data models but one (HGPIII) have computed AIC values slightly smaller than the one obtained the negative binomial model. As far as the predictive power is concerned, the various values of the R^2_{COR} indicator clearly shows that HGP, geometric and modified logarithmic model specifications perform as well, if not better in some cases, as the conventional count data models. Two models – the modified Borel and Yule – do not fit very well this pattern. This finding is not surprising in light of the observations made earlier about the behaviour of these two one-parameter probability distribution which are characterised by high(est) modal values and longer tails.

Turning to the statistical performance of all the models, it can be noted that the estimated price coefficients (β_1) have the expected signs and are statistically significant in all cases. On the other hand, there is no uniformity in terms of magnitudes. Hence, if geometric, modified Borel, and HGPI and HGPII models have an estimated price coefficient similar in value to the estimated one for the negative binomial specification, such is not the case for the three remaining alternative count data models (Modified Borel, modified Yule and HGPIII). Another interesting point worth to make concerns the fact that the estimated income coefficients (β_2) are not statistically significant regardless of the model specifications. In addition, some of them have the wrong signs (Poisson and HGPIII models). An attractive feature of these alternative count data models deals with the fact that some of them are nested among each other, hence enabling us to test them using a likelihood ratio or Wald test. Hence, a Wald test applied to the Negbin II model shows that the estimated precision parameter (α) is not significantly different form one, indicating that the left-truncated geometric and Negbin II specifications yield similar results for this sample. Likelihood ratio tests applied to the Borel, HGP and Poisson models indicate that we can safely reject the Poisson and HGPIII specifications. On the other hand, we cannot reject the null hypothesis of an admissible restricted HGPII model (corresponding to the case where $\theta_1 = -1$). Similarly, we cannot accept the null hypothesis that the restrictive HGPII specification is a modified Borel model. To sum up, the statistical test results lead us to conclude that the geometric and the restricted generalised Poisson regression (HGPII) models can be viable alternative specifications to capture the fast decay process characterising the demand for moose hunting in Northern Ontario

Finally, the reliability of alternative count data models can also be judged by looking at the estimated benefits they generate. For this purpose, estimated consumer surplus (CS) per moose hunting trip along with their standard errors and 95% confidence intervals are reported in Table 3. An examination of these results reveals that the values of the consumer surplus are consistently close to each other for four out of seven alternative count data models. (the three HPD specifications and the geometric models). In these four cases, the estimated consumer surplus fall within the range of values [\$CDN 160 \$CDN 200] obtained for the Negbin II model. On the other hand, the estimated consumer surplus is much smaller for the modified logarithmic, modified Borel and modified Yule specifications. For these three models, the estimated values of the CS are less reliable.

7- CONCLUDING REMARKS

The objective of this paper is to address the issue of fast decay process in recreational demand activities and how to represent it through appropriate count data models. Although recent semi- and

Table 2: Econometric results with truncated sample

Coefficients	**Conventional count data models**		**Alternative count data models**						
	Poisson	Negative binomial II	Geometric	Modified Logarithmic	Modified Borel	Modified Yule	Hybrid Generalised Poisson (HPD)		
							HPDI	HPDII	HPDIII
β_0	3.50676 (10.300)	2.71041 (3.409)	2.84549 (4.058)	3.61849 (3.783)	2.98393 (1.951)	5.10327 (1.690)	3.01089 (3.359)	2.71972 (3.515)	3.79399 (7.456)
β_1	-0.415995 (9.608)	-0.554112 (11.609)	-0.533041 (13.205)	-0.673957 (14.038)	-0.861240 (2.693)	-1.34123 (3.661)	-0.595716 (6.068)	-0.551482 (5.568)	-0.93545 (4.259)
β_2	-0.126691 (0.905)	0.136552 (0.657)	0.101115 (0.630)	0.315647 (1.633)	0.646148 (1.490)	1.14923 (1.548)	0.129268 (0.591)	0.204016 (0.635)	-0.174669 (0.618)
ρ									1.18404 (1.732)
θ_0							0.498998 (2.375)	0.344110 (1.535)	
θ_1							-0.223383 (1.717)		
α		1.36612 (1.425)							
Log L	-191.515	-152.510	-152.677	-152.765	-155.558	-154.611	-153.132	-153.753	-185.264
AIC	194.515	156.510	155.677	155.765	158.558	157.611	158.132	157.753	189.264
R^2_{LRT}	0.9555	0.7817	0.8033	0.7400	0.8889	0.6402	0.7676	0.7662	0.6764
R^2_{COR}	0.5472	0.4990	0.5065	0.4642	0.3109	0.0489	0.4922	0.4765	0.5436

Notes: The figures in parentheses are "t" values.
HGPI, HGPII and HGPIII designate three alternative specifications of the HGD regression model. HGPI is the unrestricted version (expression (22) while HPDII and HGPIII are restricted HGP model specifications. The former corresponds to the RHGP model (when θ_1 =-1) while the latter is the GPD specification (when θ_1 =0).
Log L designates the value of the logarithm of the likelihood function.
AIC is the Aikake information criterion. It is equal to –LogL+number of parameters.
R^2_{LRT} = 1-exp(-LRT/n) where n is the number of observations and LRT is the likelihood ratio test statistic for the joint significance of the slope parameters.
R^2_{COR} = square of the correlation coefficient between the observed and predicted numbers of trips. Note that the predicted number of trips is equal to the truncated expected number trips

Table 3: Estimated consumer surplus ($CDN) per moose hunting trip

Model specification	Consumer surplus per trip (CDN$)	Standard error	95% confidence interval
Conventional count data models			
Poisson	240.4	27.0	[199.3 301.4]
Negative binomial II	180.4	15.8	[156.0 215.9]
Alternative count data models			
Geometric	187.6	14.8	163.4 221.8]
Modified logarithmic	145.5	11.8	[128.0 175.1]
Modified Borel	116.1	172.7	[67.7 383.7]
Modified Yule	78.0	48.3	[48.9 163.3]
HPDI	167.9	17.1	[142.2 207.1]
HPDII	181.3	38.1	[134.3 271.9]
HPDIII	202.6	22.2	[169.1 254.0]

Note: Estimated standard errors and 95% confidence intervals are based on the results of a Monte Carlo simulation involving 1000 replications (see for example, Krinsky and Robb, 1986).

nonparametric approaches could have been used to capture this phenomenon, we propose instead seven alternative count data models. Four of them are based on the one parameter geometric, Borel, logarithmic and Yule probability distributions while the three remaining models rely on various versions of the generalised Poisson distribution. The empirical application to the demand for moose hunting in Northern Ontario indicates a satisfactory performance of four (including the geometric and the three generalised Poisson specifications) out of seven alternative count data models The implied estimated benefits (measured by CS per hunting trip) obtained with these four specifications are realistic and comparable with those obtained with the conventional negative binomial model. From this first empirical application of all these alternative count data models, what research directions should be suggested ? First, it would be fruitful to apply these alternative count data models to other situations dealing with recreational activities. Second, a more exhaustive analysis on the performance of these alternative models by comparing them to other more sophisticated approaches such as semi-parametric and nonparametric procedures would also worth pursuing. Finally, research is needed to develop a generalised regression model framework that could allow nesting of all alternative count data models along with the negative binomial model.

APPENDIX: ESTIMATION ISSUES OF THE ALTERNATIVE COUNT DATA MODELS

Lack of space prevented us from fully addressing estimation issues associated with the empirical implementation of these alternative count data models. The main question which comes to mind has to do with the existence of a global optimum (and the respect of regularity conditions) of the (Log) likelihood functions associated with each alternative count data model. Consul and Famoye (1992) and Famoye (1993) addressed these questions for the various versions of the GPD regression model and found that the ML estimation procedure yields efficient parameter estimates. These findings hold true for the modified Borel regression specification that is nested within the GDP model. The ML estimation of the geometric model parameters is straightforward since it is nested within Negative binomial regression model for which parameter ML estimation has been extensively studied in the econometric literature. As far as the modified Yule and logarithmic regression models are concerned, Johnson et al. (1992) indicated that estimate of the parameter (λ and η) of these two probability distributions could be obtained using the ML estimation procedure. We rely on this result but also on our own experience who showed that it was possible to obtain global convergence of the Log likelihood function of the modified Yule and logarithmic regression models. The development of all expressions defining the ML parameters estimates of all these alternative count data models and their underlying likelihood functions are available upon request. All the count data models have been estimated using the TSP computer program.

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