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Production Economics Paper No. 6103
Purdue University
March 20, 1961

THE MARGINAL CONDITIONS ASSOCIATED WITH PARETIAN OPTIMALITY, THEIR CRUCIAL ASSUMPTIONS AND THE RELATIONSHIP WITH EQUILIBRIUM UNDER PERFECT COMPETITION AND MONOPOLY

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The origin of New Welfare Economics is attributed to the work of Vilfredo Pareto. Paretian concepts proposed to dispense with the necessity of adding utilities of individuals by substituting ordinal measurements for cardinal utility. From this approach, optimum conditions of production and exchange were defined without comparing the satisfactions of different individuals. The "Paretian optimum" is said to refer to a situation in which it is impossible to make one person better off without making some other person worse off.

The contents of this paper refer to three major topics. First, the marginal conditions which are commonly identified with Paretian optimum are illustrated, and the additional assumptions which must be met to avoid ambiguity in the Paretian concepts are outlined. Secondly, when the underlying assumptions are fulfilled, the relationship of the marginal equivalences to equilibrium under perfect competition and monopoly is discussed. A third section examines alterations in the results of the analysis when the underlying assumptions are relaxed.

I

The wide extent to which Paretian optimum is associated with the conditions of tangency between transformation curves and/or indifference curves

1/The writer gratefully acknowledges the assistance of Professors G. E. Schuh and L. M. Eisgruber of the Department of Agricultural Economics and Professor R. K. Davidson of the Department of Economics, Purdue University, whose suggestions were invaluable in selecting source material and in bringing the major considerations into focus. Having chosen to translate some of the topics of welfare economics from mathematical terms to narrative, the writer assumes responsibility for any over-simplifications or other errors.

stems from the fact that these marginal equivalences, subject to further constraints, do agree with the above definition. The marginal conditions for optimality may be generally stated as:[2]

Requiring that the marginal rates of substitution between any two variables be (subjectively) equal for all individuals and (technically) equal for all alternative processes, with the common technical and subjective ratios being equivalent; otherwise there exists a physically attainable position that makes everyone better off.

Achieving the various marginal equivalences simultaneously constitutes only the necessary conditions for an optimum. In order to consider the marginal equivalences synonomous with Paretian optimum as defined in words above, certain underlying assumptions are implied.[3]

Assumptions underlying the identification of the marginal equivalences with Paretian optimum:

1. The distribution of income is given and the optimum is defined only with respect to the given distribution.
2. Total conditions: that all products and all factors of production are considered in the various marginal equivalences.
3. Second order conditions relative to the curvature of transformation functions and indifference curves: The most convenient assumption is to consider all transformation curves as concave to the origin and all indiffer-

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2/Samuelson, Paul A., "Comment on Welfare Economics ", A Survey of Contemporary Economics, Vol. II, Edited by Bernard F. Haley (Homewood, Illinois: Richard D. Irwin, Inc.) p. 38.

3/The lack of clarity on this point in much of the literature may be envisioned as a result of these assumptions being made explicit only in post-Pareto writings. As shall be shown later, the marginal equivalences can conceivably negate an optimum position, unless sufficiency conditions are met. Therefore this paper adopts the convention of describing the marginal equivalences with all the underlying assumptions fulfilled as "Paretian optimum." Any situation in which some of these assumptions are violated and still one person can be bettered without making others worse off will be referred to as "neo-Paretian optimum."

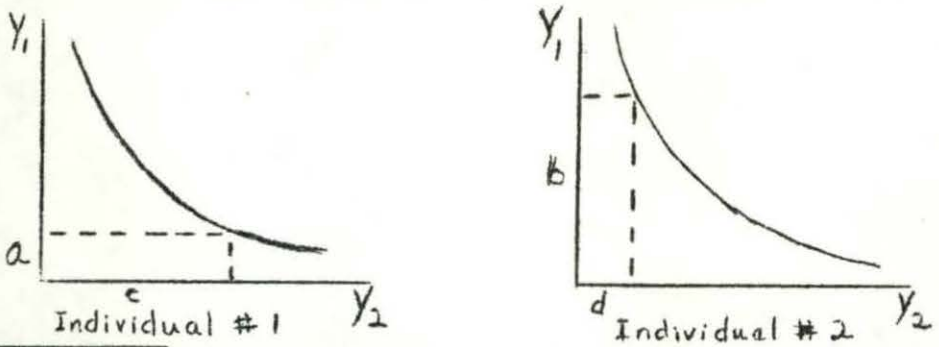
ence curves as convex to the origin.4/

4. The shape of community curves, at least around the point of equilibrium, is related to the shape of the curves of representative individuals or firms.

Marginal conditions: Preparatory to showing the divergence of equilibrium under perfect competition and monopoly, it is useful to discuss some of the variables which must be related simultaneously. Following Reder, these are known as the seven marginal conditions of maximum welfare.5/

1. Optimum allocation of products

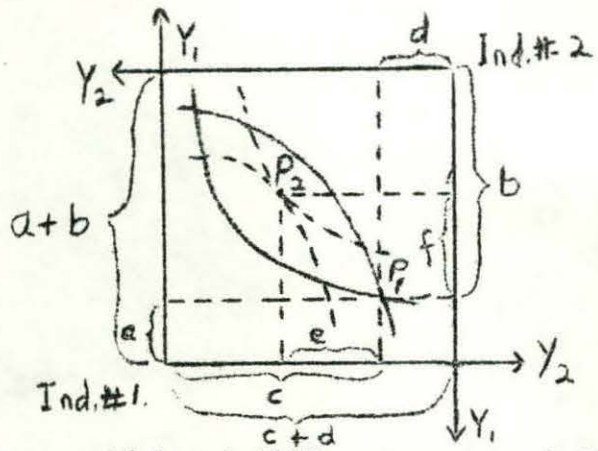
This condition requires that the marginal rate of substitution between any two products be the same for every individual who consumes both. To show the derivation of this condition, suppose the total stock of products, Y, and Y_2 are given and we consider a disequilibrium between two consumers.



4/For a maximum position to be attained at a point of tangency, the second order conditions only require th t the degree of concavity of indifference curves be less than the concavity of transformation curves, and vice-versa in event of convex transformation curves. However the equilibrium at such a point of tangency may be unstable even at competitive pricing. In case an indifference curve is concave in some ranges and convex in others, multiple points of tangency are possible. Other characteristics commonly assumed for indifference curves are (1) downward sloping to the right (2) non-intersecting and (3) smooth. If indifference curves sloped in a northeasterly direction, it is argued that the items considered in the indifference surface are not economic goods. Intersection of indifference curves is inconsistent with transitive logic. The question of indifference curves which are not smooth has both analytical and realistic problems attached to it. (a) If an indifference curve is kinked, there are many budget lines which may touch the same point. (b) Where there are indivisibilities, i.e. where a good can only be purchased or produced in discrete units, the marginal analysis cannot apply. The latter point is particularly relevant to choice of durable goods and jobs. [See: Little, I.M.D. A Critique of Welfare Economics (Oxford University Press, Second Edition, 1960) pp. 24-28 and pp. 166-167.]

5/Reder, M. W. Studies in the Theory of Welfare Economics (New York: Columbia University Press, 1951).

Initially individual #1 has a of Y_1 and c of Y_2 while individual #2 has b of Y_1 and d of Y_2. By superimposing the indifference map of individual #2 on that of individual #1, the original distribution of Y_1 and Y_2 may be shown as P_1. Trading can put one or both consumers on a higher indifference curve; e.g. to reach P_2 which is on a higher indifference curve, individual #1 might give up e of Y_2 for f of Y_1. Regarding the exchange situation, the Paretian optimality conditions relate to the contract curve. The contract curve is the locus of points of tangency of indifference curves. Any point on the contract curve is preferable to a point off it, but the Paretian optima cannot specify a preference for one position on the contract curve over another, since this would involve redistributions of income.

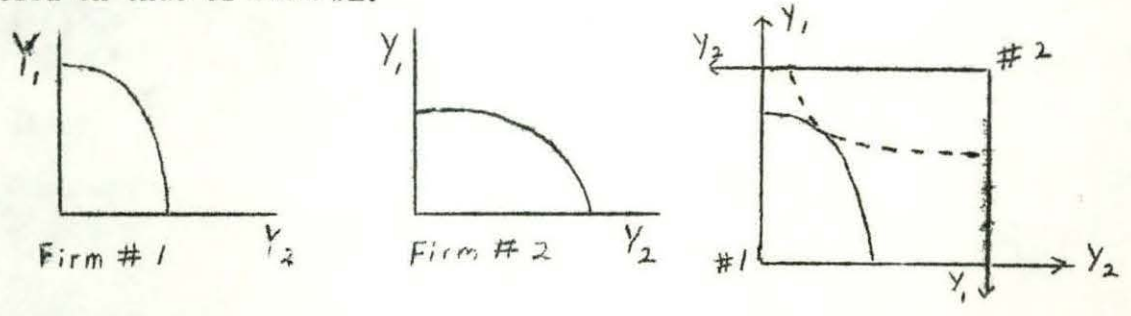


Thus the first marginal condition specifies that:

$$\left(\frac{MU_{y_2}}{MU_{y_1}}\right)_1 = \left(\frac{MU_{y_2}}{MU_{y_1}}\right)_2$$

2. Optimum degree of specialization among firms

This condition requires that the marginal rates of transformation between any two products be the same for any two firms that produce both. Therefore, given the transformation curve of two firms between products Y_1 and Y_2 with given quantities of factors available to each firm, this condition concerns a tangency between the transformation curve of Firm # 2 when superimposed on that of Firm #1.

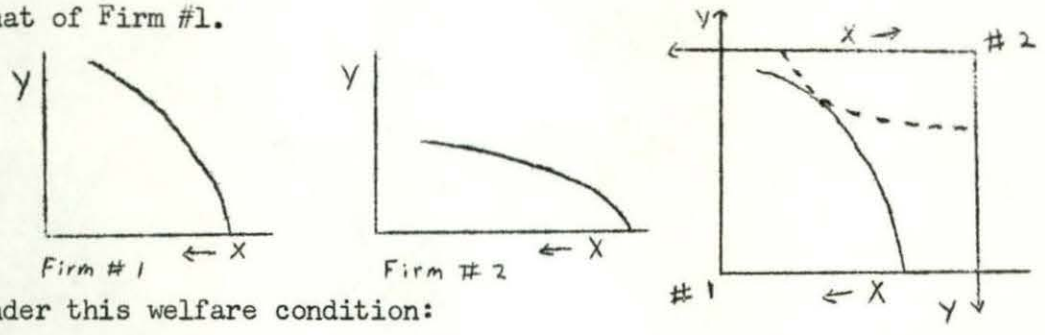


Under this welfare condition:

$$\left(\frac{MPP(x_1 \ldots x_n)y_1}{MPP(x_1 \ldots x_n)y_2}\right)_1 = \left(\frac{MPP(x_1 \ldots x_n)y_1}{(x_1 \ldots x_n)y_2}\right)_2$$

3. Optimum factor-product relationship among firms

This condition requires that the marginal rate of transformation between any factor and any product be the same between any two firms using the factor and producing the product. Given the production function of each firm when reading the level of input X from right to left, this condition concerns a tangency between the production function of Firm #2 when superimposed on that of Firm #1.

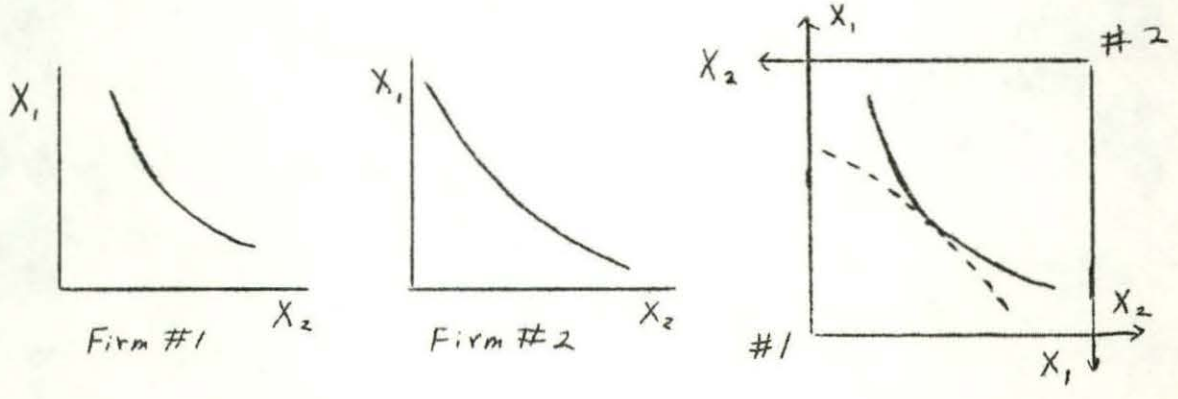


Under this welfare condition:

$$(MPP_{xy})_1 = (MPP_{xy})_2$$

4. Optimum allocation of factors among firms

This condition requires that the marginal technical rate of substitution between any pair of factors must be the same for any two firms using both to produce the same product. Given the isoproduct curves for two firms with the total quantity of factors given, this condition concerns the tangency of an isoproduct curve superimposed of Firm #2 with one for Firm #1.

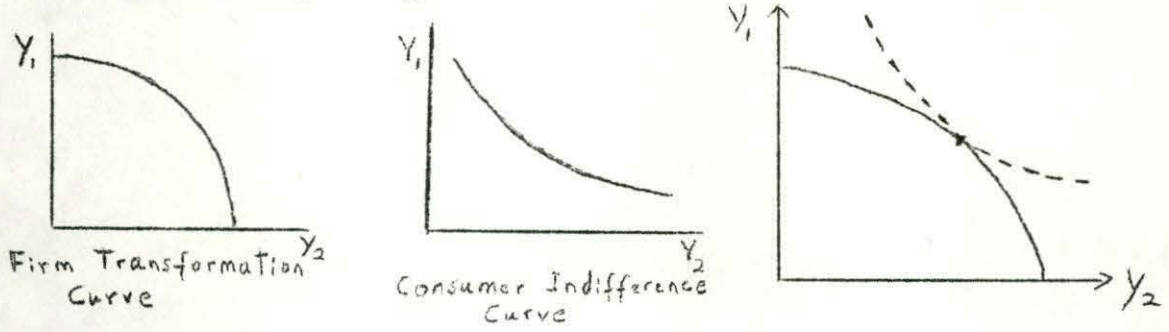


Under this welfare condition:

$$\left(\frac{MPP_{x_2y}}{MPP_{x_1y}}\right)_1 = \left(\frac{MPP_{x_2y}}{MPP_{x_1y}}\right)_2$$

5. Optimum direction of production

Concerned with the question of what to produce, this condition requires that the marginal rate of substitution between any pair of products for any person consuming both be the same as the marginal rate of transformation for any firm producing both. Therefore this condition points to a tangency of the indifference curve of a consumer with the transformation curve of a firm between products Y_1 and Y_2.

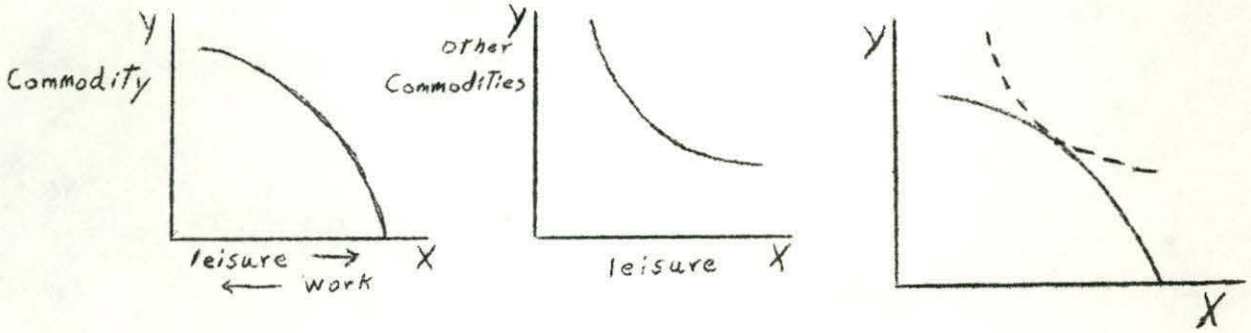


Under this condition:

$$\left(\frac{MU_{y_2}}{MU_{y_1}}\right) = \left(\frac{MPP(x_1 \ldots x_n)y_1}{MPP(x_1 \ldots x_n)y_2}\right)$$

6. Optimum amount of leisure and work for an individual

This condition requires a tangency between the transformation curve of work (negative leisure) into product and the indifference curve of the individual between leisure and other commodities.

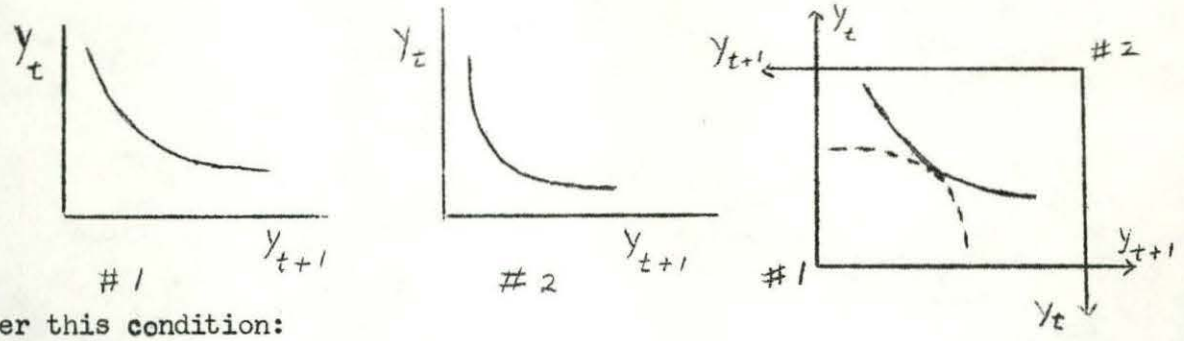


Under this condition:

$$MPP_{xy} = \frac{MU_x}{MU_y}$$

7. Intertemporal optimum

Concerned with lending and borrowing this condition requires that the marginal rate of substitution between control of items at different points in time must be the same for any two individuals, firms, or a pair consisting of a firm and an individual. Therefore the indifference curve of #2 between Y_t and Y_{t+1} must be tangent to that of #1 when superimposed.



Under this condition:

$$\left(\frac{MU_{y_{t+1}}}{MU_{y_t}}\right)_1 = \left(\frac{MU_{y_{t+1}}}{MU_{y_t}}\right)_2$$

In summary, the respective marginal conditions associated with "Paretian optimum" specify:

Condition 1. That whatever is produced is distributed among the various consumers in a manner compatible with maximum welfare (where it is assumed that people should get what they want.)

Conditions 2, 3, and 4. That whatever is produced must be produced in the most efficient way possible.

Condition 5. The relation between technical conditions of production and state of consumer preferences, which must be satisfied in order that welfare be maximized.

Condition 6. The allocation of a factor between rendering direct services to the factor owner (e.g. leisure) and working for reward.

Condition 7. The optimum amount of lending and borrowing in a given period of time.

II

Attention is now directed toward relating the marginal equivalences, when all underlying assumptions are fulfilled, to equilibrium in markets of perfect competition and monopoly. The terms commonly found in the theory of the firm, which are analogous to the marginal rates of substitution referred to in the various marginal equivalences, are marginal utility (MU) and marginal product (MP) or ratios thereof. In this section, the notation will be preserved to consider Y as product and X as factor.

Under perfect competition, each consumer or firm faces the same price for its product or factor. Therefore any relationship between price and other variables which specifies an equilibrium for one consumer or firm under perfect competition does so for others. The competitive relation to the seven marginal conditions is as follows:

1. $$\left(\frac{MU_{y_2}}{MU_{y_1}}\right)_1 = \left(\frac{MU_{y_2}}{MU_{y_1}}\right)_2 = \frac{P_{y_2}}{P_{y_1}}$$

since under pure competition, each consumer allocates his income in such a manner that the utility added by the last dollar spent on Y_1 and Y_2 respectively are equal, i.e. $\frac{MU_{y_1}}{P_{y_1}} = \frac{MU_{y_2}}{P_{y_2}}$.

2. $$\left(\frac{MPP_{(x_1 \ldots x_n)y_1}}{MPP_{(x_1 \ldots x_n)y_2}}\right)_1 = \left(\frac{MPP_{(x_1 \ldots x_n)y_1}}{MPP_{(x_1 \ldots x_n)y_2}}\right)_2 = \frac{P_{y_2}}{P_{y_1}}$$

since under pure competition, the firm allocates its resources in such a manner that the value added in producing alternative products is equal, i.e., $P_{y_1} \cdot MPP_{y_1} = P_{y_2} \cdot MPP_{y_2}$

$$3. \quad (MPP_{xy})_1 = (MPP_{xy})_2 = \frac{P_x}{P_y}$$

since the firm under pure competition faces constant prices of factors and products and in equilibrium marginal cost must equal marginal revenue, equilibrium is also specified when $P_y \cdot MPP_{xy} = P_x$

$$4. \left(\frac{MPP_{x_2y}}{MPP_{x_1y}}\right)_1 = \left(\frac{MPP_{x_2y}}{MPP_{x_1y}}\right)_2 = \frac{P_{x_2}}{P_{x_1}}$$

since the firm under pure competion allocates factors in such a manner that the cost of increasing output via use of one input is equal to the cost when using another input, i.e., $\frac{MPP_{x_1y}}{P_{x_1}} = \frac{MPP_{x_2y}}{P_{x_2}}$

$$5. \left(\frac{MU_{y_2}}{MU_{y_1}}\right) = \left(\frac{MPP(x_1 \cdots x_n)y_1}{MPP(x_1 \cdots x_n)y_2}\right) = \frac{P_{y_2}}{P_{y_1}}$$

In (2) above, the profit maximizing output of the competitive firm was shown for y_1 and y_2 and in (1) it was shown that consumers maximize satisfaction when: $\frac{MU_{y_1}}{P_{y_1}} = \frac{MU_{y_2}}{P_{y_2}}$. Under pure competition, the relative prices are the same since neither the individual consumer or producer can affect prices.

The adaptability of pricing under pure competition to the remainder of the marginal conditions is summarized as follows:

$$6. \quad MPP_{xy} = \frac{MU_x}{MU_y} = \frac{P_x}{P_y}$$

$$7. \left(\frac{MU_{y_{t+1}}}{MU_{y_t}}\right)_1 = \left(\frac{MU_{y_{t+1}}}{MU_{y_t}}\right)_2 = \frac{P_{t+1}}{P_t}$$

Thus equilibrium conditions in a market of pure competition are consistent with all the marginal conditions of Paretian optimality when the additional assumptions are met.

In a market of monopoly, the criteria of Paretian optima are not simultaneously fulfilled. The divergence arises in connection with condition #5, i.e. the relative prices of y_1 and y_2 which would maximize consumer satisfaction would be different from the relative prices which maximize profits for the monopolistic and competitive firm respectively.

Condition 5 may be restated as:

$$MRS_{y_2\ y_1} = MRT_{y_2\ y_1}$$

The principle for maximizing satisfaction of the consumer remains:

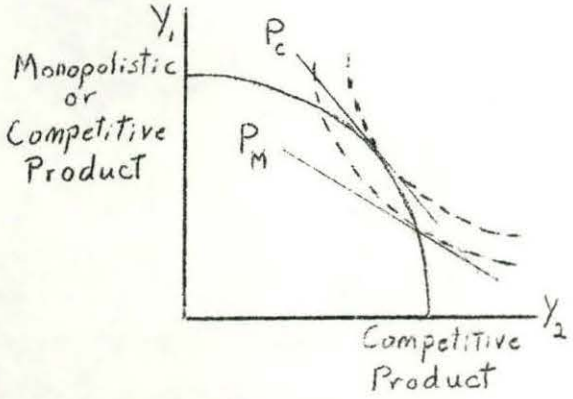
$$MRS_{y_2\ y_1} = \frac{P_{y_2}}{P_{y_1}}$$

Where $MRT_{y_2\ y_1}$ represents the technical possibilities in production, if y_1 is produced by a monopoly and y_2 is produced by firms under pure competition, the output desisions are based on:

$$MRT_{y_2\ y_1} = \frac{MR_{y_2}}{MR_{y_1}}$$

With y_2 produced under pure competition, $MR_{y_2} = P_{y_2}$, but where y_1 is produced by a monopoly, the producer faces a downward sloping demand curve for the product so that $MR_{y_1} < P_{y_1}$. The particular relationship of marginal revenue to price under monopoly is: $MR = P\ (1 - \frac{1}{\epsilon})$ where ϵ is the price elasticity of demand for the monopolist's product. Thus if two products are produced under alternative competitive structures, the relative prices which satisfy the profit maximizing principles in production are different from the price ratios which provide consumer equilibrium. This may be graphically presented if we consider the technologically determined transformation curve and the community

indifference curves for two products along with the alternative price lines when one product may be produced under either monopolistic or competitive conditions.[6/] P_c indicates the relative prices when Y_1 as well as Y_2 is produced under pure competition. P_m shows the change in relative prices when Y_1 is produced by a monopoly. Note: P_m is tangent to an indifference curve but not the higher indifference curve tangent to the transformation curve.



The so-called "misallocation of factors" by monopoly does not refer to a lower price paid for factors by monopoly. Instead reference is made to the fact that a monopoly restricts the amount of factors employed to the point where marginal physical productivity times marginal revenue is equal to the price of the factor, and that under monopoly, MR $<$ P of product whereas MR = P for the competitive firm. Where the imperfections in the market are on the product side, the monopoly must pay the going price for factors. Thus the "misallocation of factors" arises from a violation of marginal condition #5 by the monopoly, i.e. factors are not allocated in a manner consistent with maximizing consumer satisfaction.

III

The marginal equivalences are only first order conditions for maximum welfare. The additional assumptions to insure optimality have been cited. It is only in the sense that the marginal equivalences are supplemented by the

6/At this point, we abstract from the problems of deriving community curves by making the simplifying assumption that community curves are symmetrical to those of a "representative" firm or individual. In the absence of external effects and wrongly shaped individual curves, this procedure follows the traditional logic of welfare economists who adopt a criterion of an increase in welfare of actual individuals to construct the welfare of the community.

additional assumptions that a stable equilibrium is obtained under perfect competion which provides for maximum welfare. If some of the assumptions are violated, using the marginal equivalences to define Paretian optimality appears self-negating. In such a case, welfare comparisons of perfect competition and monopoly on the basis of their correspondence to Paretian optimum are impossible. Let us examine the effect of relaxing the usual assumptions underlying the marginal equivalences.

If alternative distributions of income are considered: The uniqueness of Paretian optimum to the given distribution of income admits to the possibility of more than one optimum. Thus when alternative distributions of income are to be considered from the standpoint of welfare maximization, the question is comparable to the problem of ascertaining the highest of several peaks on a mountain. For this purpose, some judgement of an essentially ethical nature must be introduced to allow interpersonal comparisons of utility. Such a function (of the Bergson type) is in sharp constrast to the Paretian insistence on the impossibility of making interpersonal comparisons of well-being. The form of the Bergson function is: $W(u^1,\ldots,u^V)$, where the u's are the utility functions, or choice indicators, of the V men comprising the group.[7] The function either summarizes or implies a detailed set of ethical judgements regarding the way in which one man's welfare is to be added to another's.

I. M. B. Little apparently attempts to blend the notion of a system of "bribes" with a Bergson function.[8] A sufficient criterion is adopted from the Kaldor-Hicks-Scitovsky school which expresses a change as economically desirable (i.e. an improvement) if it results in a good redistribution of

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7/Boulding, Kenneth A. "Welfare Economics", A Survey of Contemporary Economics, Vol. II. op. cit. pp. 16-17.

8/Little, op. cit. pp. 117-123.

welfare and if a policy of redistributing money by lump-sum transfers could not make everyone as well off as they would be if the change were made. Another consideration is that such a change does not prejudice the chances of still better changes. Thus emphasis is placed on a final condition for achieving the "optimum" (among all the Paretian optima), namely that there should be an ideal distribution of welfare among individuals.

If not all products were produced: The maximum welfare position is not uniquely defined if a product is omitted from consideration in the various marginal equivalences cited earlier. This phase of Hicks' total conditions presents special problems in the case of products which have not been invented. The analysis might either (1) proceed on the basis of an optimum defined only for products currently known to producers and consumers or (2) be conducted theoretically by an all-seeing economist who takes into account all present and future products with regard to technology and preferences.

The possibility exists, based on decreasing costs, that a product might be produced under the equilibrium conditions for monopoly whereas it would not be produced by firms under perfect competition, i.e. competitive firms cannot obtain the economies of scale justifying production whereas a monopoly can do so. Under these circumstances, when the level of output for decreasing long-run unit costs represents large fractions of the total demand for the product, production by a monopoly violates the first order conditions of Paretian optimum. The case involves finite decisions outside the realm of Paretian criteria by asking consumers whether a given abundance of fewer commodities is preferred to an alternative scarcity of a greater range of commodities.9/

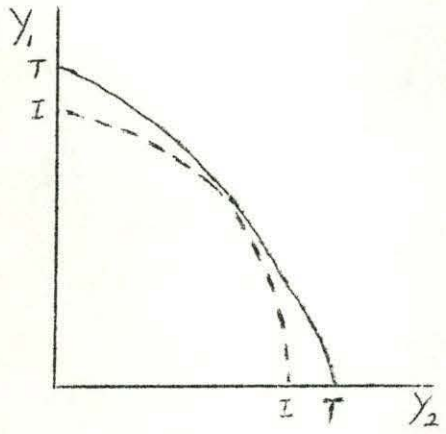
9/Samuelson, P. A. Foundations of Economic Analysis (Cambridge: Harvard University Press, 1958) p. 241.

Corner solutions, to be discussed shortly, allow for omission of certain products from production or consumption. While inconsistent with Paretian optimum strictly associated with marginal equivalences, corner solutions do not appear unrealistic. When an analysis is considered on a multi-commodity basis, a neo-Paretian optimum can easily exist due to a corner solution. It is suggested by the large number of commodities, especially indivisible goods, which one does not consume at all.

If not all factors were used: According to Hicks' total conditions, if welfare is to be a maximum, it must be impossible to increase welfare by using a factor not otherwise used. If a factor is omitted from consideration in the various marginal equivalences, the lack of a uniquely defined maximum makes it impossible to compare, on the basis of Paretian criteria, the relative conformance of monopoly and perfectly competitive equilibrium to maximum welfare. Again the possibility of corner solutions permits non-use of a factor. The case is theoretically conceivable when the marginal utility of the factor in direct consumption exceeds in all ranges the marginal utility of products which could be produced with the factor. Secondly, non-use would occur for a factor which is specific to a product which would not be produced due to a corner solution.

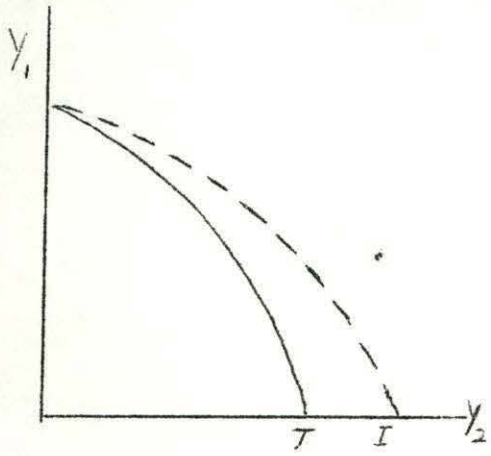
If not all indifference curves were convex to the origin: The usual assumption of convex indifference curves implies that the amount of Y_1 which one is willing to give up to get additional units of Y_2 becomes progressively smaller as more Y_2 is acquired; i.e. the marginal rate of substitution between commodities is decreasing. Three conceivable cases will be cited of "wrongly shaped" indifference curves. Indifference curves (I) between commodities, Y_1 and Y_2, will be considered in relation to transformation curves (T) for the two commodities.

1. Indifference curves which are more concave than the transformation curve.



This case violates the second-order conditions for maximum welfare. The tangency denotes a minimum because a movement in either direction along a curve permits someone to become better off without making anyone else worse off. If each indifference curve in a set is more concave than the transformation curve, a higher indifference curve might intersect the transformation curve and this intersection denotes a physically attainable point which is preferable to consumers to the tangency of a lower indifference curve and the transformation curve.

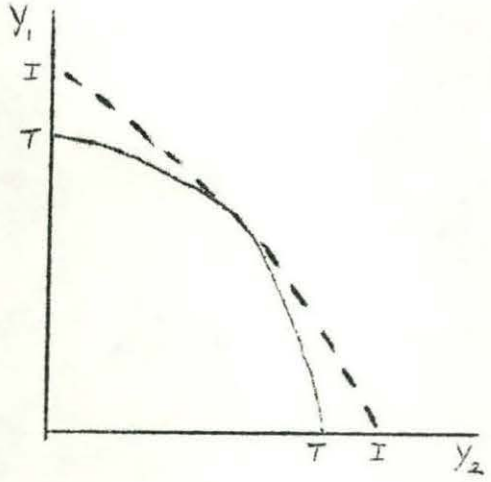
2. Corner solution with a concave indifference curve meeting a more concave transformation curve at one commodity axis.



Since the highest indifference curve touched by the transformation curve is at one commodity axis, this case constitutes neo-Paretian optimum, and the marginal equivalences are replaced mathematically by inequaltities. With a corner solution, no common price ratio, as under perfect competition, would simultaneously satisfy producers and consumers.10/ Therefore it has been suggested from the standpoint of social planning that this case be altered to the competitive solution by a corrective tax.

10/Graaff, J. de V. Theoretical Welfare Economics (Cambridge at the University Press, 1957) pp, 66-70.

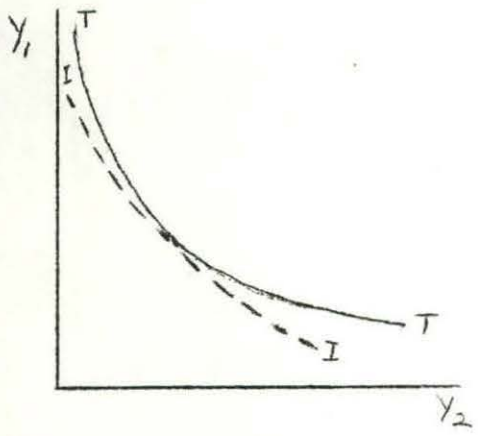
3. Tangency of a concave indifference curve with a more concave transformation curve.



The tangency defines a maximum. Producers would be willing to produce the quantities of the two products shown at the tangency if the price line were tangent to the same point. But at fixed relative prices, consumers could reach a higher indifference curve by choosing all of one product and none of the other. Therefore this case would result in an unstable equilibrium under perfect competition. Summarily, it is sufficient for a maximum that indifference curves only be less concave than transformation curves, but indifference curves must be convex to the origin to guarantee a stable equilibrium that is Pareto optimal.

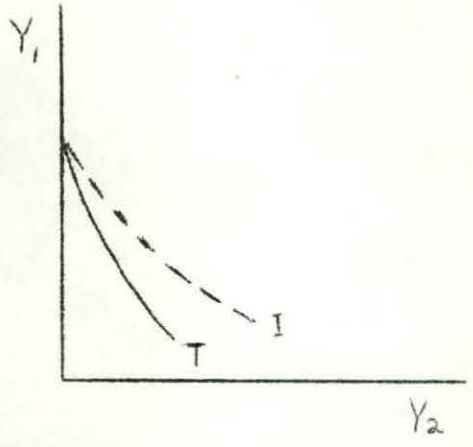
<u>If not all transformation curves were concave to the origin:</u> Again, there are three conceivable cases of "wrongly-shaped" curves.

1. Transformation curve which is more convex than indifference curves.



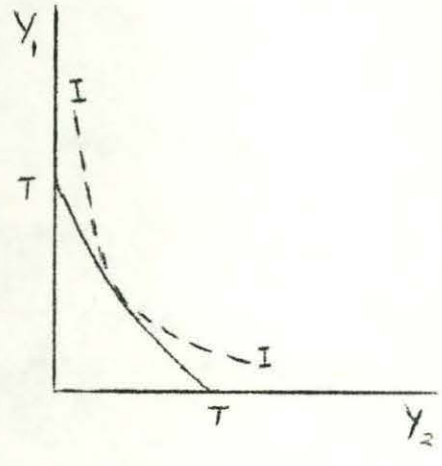
The tangency of such curves defines a minimum because any movement along the transformation curve would be preferable to consumers; i.e. a transformation curve which is more convex than the indifference curves would intersect a higher indifference curve at a point preferable to the tangency of the transformation curve with a lower indifference curve. Therefore the marginal equivalence in the absence of a specific second-order condition can define a minimum as well as a maximum.

2. Corner solution with a convex transformation curve meeting a more convex indifference curve at one commodity axis.



This case represents a neo-Paretian optimum, since the transformation curve reaches the highest possible indifference curve at one commodity axis. However, no common price ratio, as under perfect competition, will simultaneously satisfy producers and consumers.

3. Tangency of a convex transformation curve with a more convex indifference curve.



The tangency defines a maximum. However at the fixed price ratio denoted at the tangency, the competitive firm would choose to specialize, i.e. produce at the corner. The maximum position of this case would result in an unstable competitive equilibrium. The equilibrium output of the monopolist might be in the stage of increasing marginal physical productivity since output is chosen at the point where marginal cost equals marginal revenue. However whether the output chosen by the monopolist is preferable to the competitive equilibrium cannot be determined on the basis of Paretian criteria. Also note that the convexity of a firm's transformation function may represent the advantages of specialization as well as economies of scale.

Non-symmetric considerations in the aggregation of individual curves to community curves: The simplifying assumption is often made that community curves have the image of a "representative" firm or individual. This analogy is invalid, or at best superficial, when "wrongly-shaped" individual curves aggregate to convex community indifference curves and concave community transformation functions or when there are external economies or diseconomies in production or consumption.

1. Rothenberg's case[11]: The most restrictive case of concave individual indifference curves (i.e. with identical tastes and incomes) is shown to aggregate to a linear (non-concave) aggregate constraint in the absence of external effects. For nonidentical tastes and unequal incomes, concave individual indifference curves aggregate to truly convex community indifference curves. The proof is as follows:

a. Start with each individual at one commodity axis (say Y_1).

b. Begin aggregation with the individual who is willing to make the largest possible sacrifice of Y_1 for a marginal unit of Y_2. With a concave indifference curve, this individual would exchange all his Y_1 for Y_2 before exchange by a second individual commences.

c. Move down a hierarchy based on willingness to sacrifice Y_1, thereby assuring that the marginal social rate of substitution is decreasing as more and more Y_2 is added.

Since persons left with only Y_1 when the supply of Y_2 is exhausted remain on the same indifference curve, a neo-Paretian optimum is obtained. Corner solutions, rather than marginal equalities, exist between pairs of consumers (and firms) but marginal equivalences exist with respect to the community curves.

11/Rothenberg, Jerome, "Non-Convexity, Aggregation, and Pareto Optimality", The Journal of Political Economy, October, 1960.

Rothenberg constructs a proof in an analogous manner for aggregating convex firm transformation curves to concave community transformation curves. Firms switch from production of Y_1 to Y_2 in accordance with the order in which the smallest sacrifice of Y_1 is involved.

The tangency of the derived community indifference and transformation curves meets the marginal and second-order conditions for a maximum. Equilibrium under perfect competition will lead to this neo-Paretian optimum since at all points other than the above tangency, a price line tangent to the community transformation curve constitutes a divergance from the marginal community rate of substitution in consumption and consumer demands will remove the disparity.

2. External economies and diseconomies in production and consumption: External effects in production occur when a firm's transformation function depends in some way on the amounts of the inputs or outputs of other firms. If we consider a product transformation function for a community (or total society), the marginal rate of transformation between commodities would differ between the community function and the individual firm's function where external effects occur. For example, in the presence of external economies, the reduction in product Y_1 for a marginal increment of product Y_2 would be less for society as a whole than for an individual firm. This conclusion is based on the possibility of society reshuffling inputs and outputs among firms when external effects exist. Therefore, a community transformation curve must take into account external effects when constructed from the properties of transformation curves of individual firms.

External effects in consumption occur when the shape or position of an individual's indifference curves depend upon the consumption of others. With a marginal decrement in consumption of Y_2, an individual's indifference map

shows the amount of Y_1 which would be required to obtain the same personal satisfaction. In the presence of external effects, one's preference map will be affected by a change in the consumption of others. For example, the marginal rate of substitution of an individual between telephones and other commodities might vary if there is a change in the number of his friends who have telephones. Snobbishness also illustrates a type of external effect. Therefore, in considering a community indifference map, not only the amount of Y_1 needed by the individual for his marginal loss of Y_2 must be accounted for but also the amount of Y_1 which must be given to (or taken away from) others must be included.

Perfect competition does not represent maximum social welfare where external effects exist since the tangency of the community transformation and community indifference curves does not coincide with the respective marginal rates of substitution for firms and individuals as determined by the Paretian optimality conditions. This circumstance leads to a divergence of the social marginal value of commodities from their private marginal values as expressed by price determined under perfect competition.

In summary, equilibrium under perfect competition represents the "Paretian optimum" defined by the necessary conditions described early in this paper. Perfect competition does not guarantee that the second-order conditions of maximum welfare are fulfilled, nor does it provide for maximum social welfare when there are external effects in production or consumption. The Paretian marginal conditions take the distribution of income as given without any connotations regarding whether the distribution is optimal. However monopoly equilibrium is not even consistent with all the first-order conditions identified with "Paretian optima." The divergence of equilibrium under monopoly from the Paretian conditions lies in the fact that product prices under monopoly

exceed the marginal cost.[12] Intuitively marginal cost may be considered as a measure of the cost to society of using resources to produce an additional unit of a given commodity. Price is a measure of the benefit to society from producing and additional unit. At equilibrium under perfect competition, price equals marginal cost but since price is greater than marginal cost under monopoly equilibrium, the benefit to society could be increased by producing larger quantities of the product currently under monopoly.

12/Henderson and Quandt, Microeconomic Theory: A Mathematical Approach (New York: McGraw-Hill, 1958).