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SOME NOTES ON THE DETERMINATION OF SUPPLY FUNCTIONS FROM COST AND PRODUCTION FUNCTIONS*

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Introduction

The purpose of this paper is to explore some of the possibilities for using firm level production and cost functions as a basis for estimating agricultural product supply functions. This paper is further confined to certain aspects of this problem when cross section farm records and surveys are used as basic data for estimation.

We assume that the supply response of an industry may be looked upon as the aggregation of individual firm experience and action. Further, cross section estimates of sector or industry relationships are based on an assumption that individual units of observation are relatively homogeneous in certain respects, e.g., with respect to production functions. As compared with time series analysis, the problem of estimation is one of accounting for spatial heterogeneity rather than changes or shifts in relations over time.

Since the production function is basic to both estimation of cost relationships and of supply functions some of the theoretical production relationships and their implications for estimation of supply response or functions are discussed first. The theory in this area is well developed for static situations, i.e., the analysis abstracts from time as a variable. No attempt will be made to exhaustively present this static theory, but a sketch of some pertinent relationships is made as a basis for further discussion. This theory is presented mostly in the context of certainty of knowledge with respect to prices, quantities of the productive services of factors of production and their relationships to the forthcoming products. There is no well known and well developed body of theory in this area beyond this static level. The lack of such theory is,

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of course, a bottle neck to empirical research. The gaps on the theory side as recently listed by Bachman and Nerlove[1/] are

"1. An adequate theory of aggregation of firm supply functions."

"2. An adequate theory of behavior under uncertainty."

"3. An adequate operational theory of investment for the firm, that is, an empirically useful theory of how so-called fixed factors are varied over time in response to economic forces."

"4. A theory of, or at least techniques of measuring, the diffusion of technological changes and their specific effects on the production possibilities open to the firm."

Theory

Single product situation

We suppose that a single product forthcoming in a given period is some function of the input of variable factor services given certain amounts of fixed factor services which may be considered as a group or technical unit. Variable and fixed designate services the use of which respectively affect a change in costs or do not affect costs as output is changed given the period in question.[2/] Relationships between the product and factor services indicate the maximum amount of product forthcoming from any combination of factors (technically efficient production). Accordingly, with appropriate secondary conditions we can assert the economic theorem[3/] that in order for total cost to be a minimum

$$\frac{\frac{\partial Y}{\partial X_1}}{P_1} = \frac{\frac{\partial Y}{\partial X_2}}{P_2} = \cdots = \frac{\frac{\partial Y}{\partial X_n}}{P_n} = \frac{1}{\lambda}$$

where $\frac{\partial Y}{\partial X_i}$ are the partial derivatives of the above mentioned production function and represent the marginal productivities of the factor services, X_i.
P_i represent the factor service prices.

1/ Kenneth L. Bachman and Marc Nerlove. Memorandum on the analysis of changes in agricultural supply. USDA A.R.S. Farm Economics Division. 1959. p. 3-4.
2/ Sune Carlson. A study of the pure theory of production. P. S. King and Son, Ltd. London. 1939. p. 12.
3/ Paul Anthony Samuelson. Foundations of economic analysis. Harvard University Press. 1948. p. 60.

for any given output, marginal productivity of the last dollar input ($1/\lambda$) must be equal in every use. The combination of points of minimum cost for different levels of output is termed the expansion path. A firm that increases production in the least costly manner remains on its expansion path which especially in the short run may not be linear. Marginal cost is the addition to total cost brought about by increasing output by one unit while remaining on the expansion path. There is no reason why all firms should have the same marginal cost curves. In fact, it is expected that firms will generally have different short run cost curves. Lacking perfect knowledge in times past and present the firms have been and are being organized in various ways involving different combinations of fixed assets.

The marginal cost curve of the firm under conditions of perfect competition is looked upon as the supply curve of the firm. If marginal cost is above the average variable cost per unit of output the lowest cost at which the firm will offer a given quantity of product is the marginal cost of the corresponding output. Furthermore, if individual firms are ruled by the profit maximization motive the supply curve of the industry is the simple sum of the individual firm supply functions, other things being equal. The ceteris paribus conditions are (a) the firms do not affect their factor markets, i.e., the changes in quantities of factors demanded by the firms of the industry as a result of shifting levels of output do not affect the price of these factors or factor services to the firms, (b) the number of firms in the industry is given. If the conditions, (a) and (b), are not in effect the static marginal cost curves of the firms will not sum to the industry supply curve.

In the event that the total demand of the industry for factors of production affects the price of various factor services to the individual firm, the flexibility of factor prices must be incorporated into any method of aggregation

of firm level production response to product price change. If farmers have imperfect knowledge of price and production relationships or if farmers are under capital rationing pressures simple summation of marginal cost curves need not lead to industry supply functions. Similarly technological change may lead to unforeseen changes in firm behavior. If the profit maximization motives of the farmer are qualified by or in competition with household goals other elements than the marginal cost functions must also be considered in aggregating the individual firm actions to obtain supply response of the industry.

Multiple product situation

The theory of production and costs has been extended to include firms producing more than one product.[1/] If the products are independent in production, i.e., if the production of one product does not affect the costs of producing any other and vice versa[2/], the supply curves of the products at the firm level may be considered as specified in the same manner as for the single product firm. The production response of each product may be considered separately in the case of independent production. However, there would probably be no incentive for production of multiple products in such situations.

Also in the joint product situation in which the products are forthcoming in fixed proportion there is no difference between the single product and joint product situation at the production level. The products may be combined and considered as a single product for purposes of analyzing cost and production functions.

The multiple product situation differs from the single product production

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1/ Sune Carlson. A study on the pure theory of production. P. S. King & Son, Ltd. London, 1939.

2/ If service prices are assumed constant, technical independence is specified. It is possible for products to be interdependent in production because of (a) technical interdependence and/or (b) service prices varying with levels of product output.

case if products are interdependent because of technical and/or service price conditions. Technical interdependence occurs if the marginal productivity of one product is a function of the level of output of another product and/or the levels of service inputs of other products. If service prices are not constant the marginal cost of one product may be affected by the level of service input of other products when cost is assumed a function of levels of output. It may be noted that these interdependent situations describe products which while independent at certain levels of production of the products may be interdependent at others. Even with constant service prices this is true if certain factor services are fixed. Such a possibility may have implications of importance to short run and relatively short run analysis.

A more complete development of the foregoing points may be found in the references cited.1/

Estimation Problem

The estimation of supply functions from cross section data consists generally of (a) estimation of individual firm supply relationships and (b) aggregation of individual firm relationships into a supply function for the commodity in the industry or sector of the economy. Attempts to estimate firm supply relationships from cross section data may take either of two directions. In the first, underlying production functions may be estimated and then the firm's cost and supply functions are derived from the estimated production functions. In the second, supply relationships from the firm are obtained from the cost relationships which are estimated directly from financial data of the individual producing units. The first approach has the advantage of being more general. That is, production functions if sufficiently detailed and with inputs and

1/See also, Earl O Heady. Economics of agricultural production and resource use. New York. Prentice-Hall, Inc. 1952, Ch. 23 and James S. Plaxico. Aggregation of supply concepts and firm supply functions in farm size and output research, a study in research methods, p. 75-94. Cooperative among the southern agricultural experiment stations and USDA. Southern Cooperative series bulletin no. 56. June, 1958.

outputs in physical units may have alternative prices attached in order to analyze various cost and price situations. Theoretically the use of this approach permits one to arrive at a set of relations that will remain valid or change in known ways under a wider variety of circumstances than would an approach involving estimation of supply relationships directly from recorded cost data. The direct cost analysis approach has the advantage of requiring a type of data which is usually more easily acquired. Financial records are generally used for income tax filing and other purposes so the data can usually be had by survey or by simple record keeping of a type understood by farmers. However, cost relations estimated directly from such records reflect specific cost and price situations and are not easily modified to account for changes. The direct cost analysis approach also leads to problems in the case of multiple product firms because of the difficulty of allocating certain fixed costs among the various enterprises.

Single product firm production function approach

If the output of a single product is considered as a function of certain resource inputs, the conventional procedure of predicting the total output curve or surface as a regression equation may be followed. On the product side the output may be measured in physical units or as a value product that is a constant multiple of the physical product. Except in controlled experimental situations the number of possible variables on the input side is too large to permit working with all variables. Hence, researchers have aggregated the factor services (or investments in factors) into categories on the basis of their being technical complements and near perfect substitutes.[1/] This categorization has led to the measurement of input categories such as machinery service in value terms. The specification of the input categories and the measurement of the appropriate

1/ Glenn L. Johnson. Classification and accounting problems in fitting production functions in farm record and survey data. In Earl O. Heady and others, eds. Resource productivity, returns to scale, and farm size. p. 90-96. Ames, Iowa. The Iowa State College Press. 1956.

variables raises the same problems of estimation considered in some detail in the literature concerned with the estimation of production function for farm management or intra-firm purposes.[1/] As has been pointed out elsewhere[2/] biases may result from failure to include important variables such as management service. As usual the problem of multicollinearity will continue to plague researchers trying to obtain production function estimates from cross section data. These problems may be no greater in estimating supply relationships than in the usual production function analysis where they have caused considerable concern. However, few attempts have been made to carry production function estimation at the firm level to supply functions on an industry basis and the implications of these problems for aggregate level analysis aren't spelled out yet.

As noted, discussion of production function estimation opens a Pandora's box. It is not the purpose of this paper to review in detail these problems. Most of the problems are well-known to researchers who have tried to estimate a production function. The articles cited offer possibilities for solutions to the problems at least in certain instances.

However, the use of cross section data gives rise to the suspicion that each observation may represent a point on a different production surface especially since firms are found using different technologies. A possible solution under relatively short run conditions lies in the selection of the cross section samples. If strata are delineated in a manner to represent firms with common technologies and/or other factors likely to affect technology, separate production function estimates may be made for these situations. For example, firms may be classified into strata by size, age of operator,

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1/A number of production function estimation problems were reviewed in Earl O. Heady and others, eds. Resource productivity, returns to scale, and farm size. Reference cited by a number of authors in this book may be used to further follow up the problem.

2/Z. Griliches. Specification bias in estimates of production functions. Journal of Farm Economics. 39:8-20. 1957.

production practices such as use of milking parlors, etc. This stratification procedure besides aiding in identification of the production functions may be useful in reducing the biases caused by failure to include variables such as management. One intuitively feels that in such situations management and other non-included variables are more likely to cause randomly distributed disturbances. That is, the correlation between management factors and included variables is apt to be relatively less given the extent of the variation within the homogeneous strata than among strata situation. Among strata, management might be expected to vary in proportion to capital inputs but when the size of the firm is specified and hence the corresponding capital inputs are specified within limits the correlation will ordinarily be lower.

If in the short run one can assume that the number of firms in each strata will remain relatively constant, the firm supply relations may be estimated and subtotaled for the strata and then added to the supply function of each other stratum. (It is assumed that appropriate sampling techniques are used within strata so that the resulting firm supply function for each stratum is representative for that stratum.) More specifically this procedure involves first estimating the outputs for various factor combinations within each stratum by the use of the production function. One generally uses the combinations of factors corresponding to points on the expansion path defined by the production function and expected factor service prices (values). The levels of certain factor service inputs would be fixed according to the length of run considerations. Since the relationship of various levels of output to the combined (minimum) value of the factor services required to produce these outputs is the variable cost curve of the firm, the strata marginal costs are obtained by estimating with this function the additional cost that accompanies each added unit of output. Multiplying the successive output levels by the number of

firms in each stratum and relating to the corresponding marginal costs leads to a stratum supply curve.[1/] To obtain the industry or sector supply curve the strata output levels corresponding to the specified marginal cost situations are simply added together.

It is, of course, likely that the firms as a group will influence factor prices by their combined action. Factor price flexibilities or the group output effect on prices may be taken into account in the single product model. As long as one assumes that no individual firm influences price perceptibly, it is only necessary that the price be specified for the industry or group of firms as a whole for each total (group) output level. Conceptually one has only to set these prices at their expected level for the output in total and then maneuver the individual strata models to a point on the respective expansion paths that leads to the desired total output for all strata but keeps the marginal cost the same for all firms among strata.

As output levels for the group of firms is changed marginal costs to the individual firm within strata shift, e.g., upward as suggested in Figure 1.

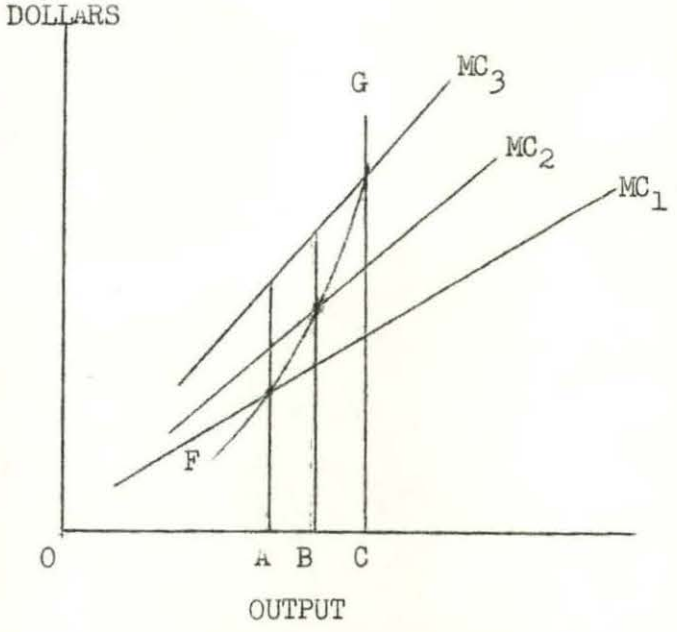

Fig. 1. Cost and supply relations for hypothetical average firm in a strata. (Linear relationship used for convenience in presentation.)

1/ For a mathematical derivation in the static case of the individual firm supply function from a Cobb-Douglas type production function see: Bachman and Nerlove. op. cit. p. 39-41.

OA represents the individual firm share of total output given the initial price and total output specification for the group of firms. OB specifies the individual firm share with a second specification and similarly for OC. It is the sum or aggregation of curves such as FG which becomes the supply curve of the industry or group of firms rather than the aggregation of marginal cost curves per se as was the case when the factor service prices did not change.

It is possible that even when firms know their production cost relationships and are able to adjust outputs to maximize short run profits, they may choose not to do so. Various reasons are hypothesized for such action, e.g., influence of holding other goals than profit maximization. If the suggestions for deriving a supply function are used various adjustment factors might be devised. Similarly the effects of risk and uncertainty modify any such analysis. Nevertheless, it is felt that until more general theories of firm behavior under risk and uncertainty and conflicting goals are devised, approaching supply analysis through static classic production functions may offer worthwhile insights to the researcher and policy maker. Differences between estimates made with the static model and the actual supply responses in reality may be analyzed or rationalized in the light of such factors as risk and uncertainty as a first step toward more adequate analysis of the effects of these factors. In the long run changes in the factors by which strata are specified may become of major interest. Technological change cannot be ignored as an important factor in supply analysis. Since there is no well-developed body of theory upon which to draw here the problems of estimation are going to be more difficult. One might suggest as does Klein[1] that cross section data might be taken over time to form a time series of cross section data. Such a procedure might permit the introduction of such techniques as distributed lags[2] or possibly analysis

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1/ Laurence R. Klein. A textbook of econometrics. Evanston, Ill. Row, Peterson and Co. 1953. p. 236-240.

2/ Zvi Griliches. The demand for fertilizer in an inter-state study. Journal of The American Statistical Association. 54:382-383.

of covariance[1] and other techniques. It must also be recognized that a related problem is the changing form of fixed assets within individual firm that takes place both with or without changing technology.

Single product firm cost analysis approach

Since elementary theory indicates that the supply curve for an industry is under certain conditions the sum of the individual firm marginal cost curves, estimation of the cost curves directly from the cross section firm records rather than starting with the underlying production function appears possible. Hence the differences between the cost-output positions of the cross section of firms as shown by appropriate financial records might be considered as data from which to estimate a total cost function. However, even when the firms operate with the same production functions it is not necessary that they have the same marginal cost curves. Marginal costs depend upon the level and distribution of fixed costs, or rather upon the nature of the fixed factors and their levels within a firm, i.e., upon length of run under consideration. Some way of grouping the firms according to the nature of the marginal cost function is needed. In other words homogeneous fixed plants must exist before a cross section would indicate points on the same variable cost structure.

If one aggregates all variable costs and estimates the function the costs are of output and fixed cost category levels, an assumption is made that farmers do combine resources so as to produce on the expansion path of the firm (at minimum cost per level of output). Otherwise some hybrid cost curve which would be difficult to interpret meaningfully is obtained. In practice firms vary rather widely in their fixed asset structure. In production function estimation the differences in services flowing from such fixed

1/ Irving Hoch. Estimation of agricultural resource productivities combining time series and cross section data. Unpublished Ph.D. thesis. Chicago, Ill. The University of Chicago Library. 1957.

factor situations were taken into account in the cross section estimate by considering such factors as variable from farm to farm and hence within the individual firm perhaps on a long run basis. Since marginal cost functions will vary depending upon the fixed asset structure, one could either (a) stratify the data to obtain comparable fixed asset structures and estimate individual cost functions for each or (b) include more than one fixed cost category in the model each with varying levels just as was done with factor services in the production function model. In a competitive model the production function may theoretically have per unit values attached to the services and any one of the categories made a function of output and the other categories. Elemental theory indicates the total cost function as the inverse of the production function which has undergone a value transformation on the input side. But this is the same as working with a production function in which value inputs are used. Hence, the detailed cost analysis approach comes back to the ordinary production function analysis approach.

Since most agricultural products are produced on multiple product farms, interest in the single product firm cases is somewhat limited. It has been presented in some detail for the purpose of giving an insight into the overall problem and because historically cross sectional estimation of production functions for multiple product agricultural firms has seldom proceeded beyond the use of single product variations.

Multiple product firm production function approach

The multiple product case, while of most general interest, is unfortunately the most involved and difficult to handle. Some sort of interdependence of the products in production or marketing exists or there would be little incentive of firms to produce more than one product. The nature of this interdependence affects the possibilities and methods of estimation that must be used to dis-

cover the production functions and related supply response estimates. Although few attempts have been made in agriculture to connect production functions to supply analysis directly, most firm level production function estimates have been based on multiple product firm data. However, the interdependence problem has not been satisfactorily handled in general. Researchers have often tried to avoid the issue by either selecting firms as nearly homogeneous in their output complex as could be obtained and then aggregating the product outputs in terms of gross income or they have divided the product outputs into several relatively homogeneous categories such as livestock and crops and then fitted separate production functions for these categories using various accounting procedures to allocate an appropriate share of the input services to each output category. Once output and input categories have been designated estimation has proceeded as in the single product case, usually by least squares regression fits of an equation. The usual choice has been a variation of the Cobb-Douglas production function.

The use of independent estimates of the separate production function by enterprise is theoretically feasible as pointed out by Beringer[1/] if the production functions are technically independent. In practice as noted before separating the products in the multi-product firm for the purpose of estimating individual production functions leads to accounting-allocation problems on the input side. It is difficult to determine how much of a feed floor service or building service to allocate to the production of each enterprise or product. On the other hand, treating the entire complex or portion of the products as a group leads to an index number problem.

1/ Christoph Beringer. Estimating enterprise production functions from input-output data on multiple enterprise farms. Journal of Farm Economics. 38:923-930. 1956.

_____. Problems in finding a method to estimate marginal value productivities for input and investment categories on multiple enterprise farms. In Earl O. Heady and others, eds. Resource productivity, returns to scale, and farm size. p. 106-113. Ames, Iowa, the Iowa State College Press. 1956.

An example of this problem can be illustrated. If two products, A and B, are aggregated into a single value product category, one can easily obtain biased estimates of the production coefficients, especially labor. For, if the cross section sample is one in which considerable substitution of the two products has occurred one (B) may be a larger user of labor than the other (A). The use of labor may be correlated with the substitution of this product (B) for the other (A). Now if the prices of the products are such that product B leads to lower total value from the same resources, it is possible as B increases in the cross section data, other resources constant, that gross income decreases, i.e., those farms with other resources comparable but having more labor may tend to substitute B for A. A cross section estimate of the production function will often show negative labor coefficients in this case although the addition of a unit of labor in the production of either product may be positive and the marginal value product of a unit of labor used in B higher than the corresponding marginal value product in A.

Intra firm analysis information of considerable interest can often be obtained when this problem is avoided by limiting the analysis to farms with relatively the same proportions of A and B in production. In supply analysis it is the substitution effects among the products that may be of major concern. The supply function of B cannot be considered separately from possible substitution possibilities between A and B.

A possible solution to such problems is suggested by Klein.[1/] He uses a production function model in which he includes the various output categories. Some are designated as independent variables for purposes of regression estimation. In the example he gives, a Cobb-Douglas variation is used. As he points out a theoretically unacceptable relation between the products under

1/ Klein. op. cit. p 226-236.

pure competition, i.e., diminishing marginal rates of substitution, occurs with this model. He suggests the possibility of other equations indicating that he has tried one which at least permits a constant marginal rate of substitution among products.

Multiple product firm direct cost analysis approach

Again, a direct cost analysis approach might be tried. Total cost functions may be derived in the multiple product case. Average cost functions are not meaningful, but marginal cost function derivation is possible. However, such costs are subject to the conditions implied by the output levels of the other products. One might, therefore, set out to estimate the total costs of a production process as a function of the output levels of the various products. Suppose a cross section of accounting records supplies the basic data. Presuming that an appropriate mathematical model has been chosen to represent the relationships involved and that it has been successfully fitted, marginal costs of any product given specific levels of others may be calculated as a first derivative of cost with respect to output.

Assuming that the firms have the same cost structure given the same factor prices, cross section estimates of the total cost function are based upon inter-firm differences in cost and output. If all firms were operating efficiently they should all be at the same point in their cost relationships. Only a point estimate could be achieved. In order to identify the cost structure it must further happen that the firms differ with respect to output positions for some reason that does not prevent them from having the same cost structure. If firms had and have different expectations, such a situation might exist. Some firms might overproduce and others might underproduce relative to the "optimum level". Under these conditions inter-firm differences would reflect differences in the

total cost structure of a nature found within individual firms. However, when estimating such cost functions from cross sectional data one may not even then be approximating the desired cost function because many firms may have had ex ante expectations which led to factor service combinations other than on the expansion path.

Cost curves developed in this manner would reflect a given set of cost conditions (factor service prices and production function). Difficulties in meeting the assumptions are obvious. Short run firm supply curves may be obtained by examining marginal cost of a particular product. Difficulties arise when analysis is to be made for a length of run which permits the factor service prices to vary as a function of the output levels.

The marginal cost curves for individual products can be specified only if the interrelationships of costs among the products are specified. It is difficult to assess the impact of changes in service prices upon these curves, since one assumes that the firms tend to operate upon the expansion paths designating service combinations for each product and also upon the expansion path indicating an optimum combination of products. Any cost complex is a shifting function of the underlying physical production function relations of substitution and the changing factor service prices. Hence, this approach has very definite limitations especially beyond the short run.

The foregoing notes are not intended as a complete coverage of the problem under discussion, but it is hoped that they will serve as basis for productive discussion in this workshop.