Prize-linked savings mechanism in the portfolio selection framework

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Prize-linked savings (PLS) instruments implement the lottery-like component into the structure of traditional financial products. Following existing research based on both real and experimental data, such programs appeared highly successful in raising the overall savings rates within the given environments. PLS accounts seem to be treated by decision-makers as substitutes to ordinary lotteries, but this does not hold when comparing PLS with traditional interest-bearing savings products. This paper explains such empirical observations in a framework of portfolio selection problem. For that purpose, two models have been presented and used for deriving optimal portfolios in a presence of PLS, lottery and savings products. As shown in the analysis, the standard mean-variance model does not allow for a PLS instrument to be of optimum choice, whereas in the case of behavioural portfolio model allocating all disposable income to PLS can be in fact the best decision under certain individual conditions.

JEL Classifications: G11

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Introduction

The prize-linked savings mechanism combines the elements of a traditional savings account and a lottery (Kearney, Tufano, Guryan, and Hurst, 2010). The main idea is based on people willing to forgo the certain interest injections for the opportunity to gamble. The sacrificed interest payments are then collected and allocated to the common pool. These funds are eventually distributed to the account holders via a lottery mechanism, taking place in regular time intervals - usually at the end of the month.

The attractiveness of a PLS mechanism to potential buyers of such products has already been proved by several examples from around the world. One of the flagship examples here are the so-called Premium Bonds. The program was launched by the British government in 1956, and is running until now. Its leading aim was to increase the overall savings rate among British households following the end of World War 2. According to statistics, in addition to having achieved this objective, the program is actually successful enough to currently engage over 21 million bondholders. As a result, the overall value of liabilities arisen from the Premium Bond investments constitutes the highest among all the products offered by NS&I, and currently exceeds £54 bln (National Savings and Investments, 2015). At the beginning of 2015 the second jackpot of £1 mln in monthly drawings was introduced, and on 1 June the upper bound for deposits held in Premium Bonds was increased to £50,000.

Another well-known example of a PLS program is Million a Month Account (MaMA). It had been in operation by the First National Bank of South Africa during years 2005-2008, offering a 0.25 percent as a nominal interest rate along with monthly lottery draws of

financial prizes. The bank was forced to close the offer though after having been sued by the National Lottery Board. Despite the legal issues associated with the lottery component of MaMA program, its implementation appeared successful - the overall level of savings was raised by as much as 38% with respect to its mean value (Cole, Iverson, and Tufano, 2014). Furthermore, according to the research, this new banking product was purchased in a vast majority by both financially constrained individuals as well as those that had been holding no deposits before, what is consistent with results obtained from the experimental studies (Atalay, Bakhtiar, Cheung, and Slonim, 2014).

The existing research allows for thinking of the PLS mechanism as a potentially effective, but underestimated tool for encouraging people to save more. In addition, it appears to be the most cost-effective among the policies introduced thus far in the form of financial incentives (Kearney et al., 2010). As far as the previous research in the area is concerned though, the major work has been conducted very recently, still leaving some important questions unanswered.

The objective of the following paper is to state the problem of PLS investments in a portfolio selection framework. That is, with the use of two chosen models it is investigated whether and under what conditions investors will actually choose prize-linked savings products as their optimal decision. The models chosen fall within Modern Portfolio Theory (MPT) and Behavioural Portfolio Theory (BPT). The paper is organized as follows. Section 2 describes the data used for the analysis. Section 3 provides the theoretical backgrounds along with the investigation results for each of the portfolio selection models considered, and is followed by Section 4, which concludes and gives the directions for future research on the subject.

Data

For the purpose of the following analysis let us assume there exist three wealth allocation opportunities in the market - ordinary savings account, PLS-type account, and standard lottery tickets. The specific data considered here come from September 2007, from the South African market. The products available to an agent are then as follows:

1. Risk-free traditional savings account, offering a certain annual rate of interest, R_f. Its specific values for different thresholds of deposits paid in are given in Table 1.

TABLE 1. THRESHOLDS FOR INTEREST RATES ON SAVINGS ACCOUNT

DEPOSIT VALUE	INTEREST RATE
below R10,000	4%
[R10,000; R25,000)	4.25%
[R25,000; R250,000]	4.5% - 4.75%

Source: Cole, Iverson, and Tufano (2014).

2. Lottery tickets, where the prizes are drawn according to the distribution offered by the South African National Lottery main draw, that is, Lotto 6/49.

The respective probability density function relevant to purchase of one lottery ticket is given in Table 2. Price per ticket is equal to R 3.50, but it is possible to get an additional lottery entry for R 1.50. Revenues obtained from lottery ticket sales go in 34% to the central charitable distribution fund, 6% is charged as a retail commission, 10% is retained as operational cost, and the remaining 50% accounts for the prize pool. The jackpot starts at around R 2 mln, but it rolls over to the following draw in case there is no highest winner.

TABLE 2. DISTRIBUTION OF PRIZES IN NATIONAL LOTTERY

TIER	PRIZE POOL FRACTION	PROBABILITY
Match 6	18.25%	1/13,983,816
Match 5 + Bonus Ball	4%	1/2,330,636
Match 5	9%	1/55,491
Match 4 + Bonus Ball	5%	1/22,197
Match 4	16.75%	1/1,083
Match 3 + Bonus Ball	11%	1/812
Match 3	36%	1/61
No Prize	N/A	approx. 98.14%

Source: South African National Lottery, www.nationallottery.co.za

3. Prize-linked savings account, for which the specific characteristics are based on the MaMA program run by First National Bank in South Africa.

TABLE 3. DISTRIBUTION OF PRIZES IN A SINGLE
DRAW IN PLS PRODUCT

Number of prizes drawn	Prize value
1	R 1,000,000
4	R 100,000
20	R 20,000
200	R 1,000

Source: Cole, Iverson, and Tufano (2014).

The account does not earn any certain interest, but guarantees the capital payback. The prizes are drawn at the end of each month, and their distribution is provided in Table 3. For each R 100 paid in as a deposit to the MaMA account, a holder is given one chance to gamble - that is, a "ticket" of participation in the random draw of prizes.

Models of portfolio selection

Traditionally, as pointed by the modern portfolio theory (Markowitz, 1952), investors are assumed to care about a trade-off between risk and return when creating a portfolio. Specifically, it implies considering each component's mean and variance along with correlations among every two. The core issues associated with MPT include the following:

1. The higher the risk of an investment, the higher premium in return willbe demanded by an investor in order to compensate for that risk.

2. The greater the number of assets included, the lower the overall level of risk of the portfolio. However, this is true only up to some threshold - exceeding it will result in overdiversification, when the impact of increasing the amount of assets on portfolio's risk becomes insignificant.

3. The lower the correlation between two assets, the higher return on portfolio containing both of them could be achieved.

As opposed to the MPT though, behavioural portfolio theory (Shefrin and Statman, 2000) assumes the motivations of investors could be different than their portfolio value maximization only. It considers expected wealth, investor's desire for security and potential, and levels of aspiration along with the probabilities of achieving these. In terms of investor's willingness to reach a specific level of wealth on the portfolio, the theory is similar to Roy's safety-first theory (Roy, 1952). The behavioural model is based on Lopes' SP/A theory (Lopes, 1987) and Kahneman and Tversky's prospect theory (Kahneman and Tversky, 1979). It provides an algorithm for an optimal individual wealth allocation among some given investment opportunities.

MPT uses assumptions of von Neumann and Morgenstern's expected utility theory (Von Neumann and Morgenstern, 1944). As a result, an investor is believed to have a uniform attitude towards risk among different classes of assets. Therefore, following Markowitz' model, agents with low levels of initial wealth should never purchase lottery tickets if acting rationally. On the other hand, BPT provides different argument. Specifically, the theory suggests people distinguish between particular sections of their portfolios by classifying them to different mental accounts. Each of these are then associated with different goals, aspiration levels etc., while the correlations between accounts tend to be neglected. As a consequence, there are agents purchasing both insurance policies and lottery tickets at the same time, as indicated by the Friedman and Savage' puzzle (Friedman and Savage, 1948).

A. Modern portfolio theory

First, let us consider possible portfolios constructed with lottery tickets and PLS account - that is, available risky assets (Figure 1, Appendix). The analysis is conducted in the risk-return space, where vertical axis shows a rate of return, and along the horizontal axis portfolio's risk is indicated, as measured by standard deviation of its returns. All the possible portfolios then lie along the curve between the two points indicating portfolios consisting fully of either lotteries or PLS. The point lying on the vertical axis indicates a case of investing all the disposable wealth to a standard savings account.

Overall return rate (R_p) and risk (s_p^2) of each portfolio are evaluated respectively according to the following formulas:

$$R_p = w_{lottery} \cdot R_{lottery} + w_{PLS} \cdot R_{PLS} , \tag{1}$$

$$s_p^2 = w_{lottery}^2 \cdot s_{lottery}^2 + w_{PLS}^2 \cdot s_{PLS}^2 + 2w_{lottery} \cdot w_{PLS} \cdot s_{lottery} \cdot s_{PLS} \cdot \rho , \tag{2}$$

$w_{lottery}$ and w_{PLS} stand for weights of each asset in an investor's portfolio, $R_{lottery}$ and R_{PLS} are the mean rates of return achieved on each asset, while $s_{lottery}^2$ and s_{PLS}^2 provide the variances of return distributions for each asset, respectively. ρ is a Pearson correlation coefficient between the two return distributions.

According to the analysis of all available portfolios created by purchasing both lottery tickets and PLS products, a minimum-variance portfolio (the one carrying the lowest possible risk) is in our example the same as a market portfolio (the best portfolio possible to be constructed with the risky assets only) - in both cases it is a portfolio consisting fully of PLS products. In other words, because the pure lottery option carries higher risk than PLS along with a lower average return, according to the mean-variance methodology, it is never going to be chosen by a rational risk-averse investor. The argument continues with a risk-free option - since any portfolio constructed with the use of PLS products and/or lottery tickets bears both higher risk and lower return, an optimal solution for a rational risk-averse investor is to invest in a savings account only.

Statistical summary of distributions of returns on each of the options considered are provided by Table 4.

TABLE 4. SUMMARY STATISTICS FOR MPT MODEL

	RATE OF RETURN	RISK (STANDARD DEVIATION)
Lottery	-59.36%	278.64%
Prize-Linked Savings Account	1.72%	32.85%
Traditional Savings Account	4%	0
Market Portfolio	1.72%	32.85%
Optimal Portfolio	4%	0

Source: Own calculations.

Following the initial assumptions of modern portfolio theory, investors are essentially risk-averse. That is, the function of their utility over investment's return and risk structure is increasing and concave. This results in indifference curves in risk-return space being concave as well, and utility level increasing along the inflating rate of return.

According to another assumption of the model, the ultimate goal of investors is to maximize their utility over risk and return. Therefore, considering all the aforementioned utility function's features, the optimal choice for an investor is to allocate all the disposable money in a traditional savings account - this way achieving an indifference curve with a highest possible utility.

The assumption of risk aversion means that one can determine the general form of their utility function in a following form:

$$U(\bar{R},s) = \bar{R} - b \cdot A \cdot s^2, \qquad (3)$$

In formula (3), U stands for the utility level, $\bar{R}$ is a mean level of return on a portfolio, b is any real number such that $b \geq 0$, A is a risk aversion coefficient, where $A > 0$ indicates a risk-averse person, and s^2 measures risk of a portfolio with regard to the variance of its returns. Letting $\bar{R}_1, \bar{R}_2, \bar{R}_3$ be the mean returns on $\bar{R}_1$ - worth investments in lottery gamble, PLS product and savings account, respectively, as well as s_1, s_2 be the standard deviations of returns on lottery gamble and PLS product, we know that $\bar{R}_3 > \bar{R}_2 > \bar{R}_1$, $s_1 > s_2 > s_3$, and $s_3 - 0$. As a result, for any value of $b > 0$ and $A > 0$, it is true that $U(\bar{R}_3, s_3) > U(\bar{R}_1, s_1)$ as well as $U(\bar{R}_3, s_3) > U(\bar{R}_2, s_2)$.

Let us now relax the MPT assumption of risk aversion. Therefore, in the functional form of utility in risk-return space of (3), we allow for $A < 0$, in which case an investor is a risk-seeker. The condition under which investor's optimal choice would be to allocate all the available wealth in lottery tickets is:

$$\begin{cases} U(\bar{R}_1, s_1) > U(\bar{R}_2, s_2) \\ U(\bar{R}_1, s_1) > U(\bar{R}_3, s_3) \end{cases}, \qquad (4)$$

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After having substituted for the respective utility functions, the solution to the above system implies:

$$A < \frac{\bar{R}_1 - \bar{R}_3}{b \cdot s_1^2} , \tag{5}$$

On the other hand, for the traditional savings account to be an optimal choice for a risk-seeking investor, $U(\bar{R}_3, s_3)$ should be higher than each of the other two options. The solution then would be as follows:

$$A > \frac{\bar{R}_1 - \bar{R}_3}{b \cdot s_1^2} , \tag{6}$$

For a prize-linked savings product to be an optimal allocation for a risk-seeker, the following conditions should be met:

$$\begin{cases} U(\bar{R}_2, s_2) > U(\bar{R}_1, s_1) \\ U(\bar{R}_2, s_2) > U(\bar{R}_3, s_3) \end{cases}, \tag{7}$$

After appropriate transformations, the solution is defined by the following system:

$$\begin{cases} A > \dfrac{\bar{R}_1 - \bar{R}_2}{(s_1^2 - s_2^2) \cdot b} \\ A < \dfrac{\bar{R}_2 - \bar{R}_3}{b \cdot s_2^2} \end{cases}, \tag{8}$$

However, according to the data used in the following analysis, we get that $\dfrac{\bar{R}_1 - \bar{R}_2}{b \cdot (s_1^2 - s_2^2) \cdot b} > \dfrac{\bar{R}_2 - \bar{R}_3}{b \cdot s_2^2}$, what results in an empty set of A. Therefore, despite investor's attitude towards risk, having the three options available, according to the modern portfolio theory, he will never choose a PLS product. Such conclusion is inconsistent with the real data, though, showing that the modern portfolio theory is unsuccessful in explaining why individuals find such instruments attractive.

B. Behavioural portfolio theory

Following the BPT model, agent's problem is expressed in the form of (9) under the condition given by (10):

$$max \, E_h(W) = \sum_{i=1}^{n} r_i \, W_i , \tag{9}$$

$$Pr(W \le A) \le \alpha, \qquad\qquad (10)$$

Where W is an investor's terminal wealth, maximized $E_h(W)$ is an expected value of W under the transformed decumulative function, A stands for individual aspiration level, and α determines the probability of ruin, defined as failing to reach an aspiration level. The space for decision making by agent is: $(E_h(W), Pr(W < A))$. In other words, we consider an expected wealth level of each portfolio available, estimated using individual probability weighting functions, as well as probabilities of ruin corresponding to each of such portfolios. Supposing the three wealth allocation opportunities are available, the corresponding numbers of future payoffs are:

- there are 8 states associated with security 1 - lottery (corresponding variables are subscribed with 'L');
- there are 5 states associated with security 2 - PLS account (subscription 'P');
- there is 1 state associated with security 3 - savings account (subscription 'S').

For the purpose of calculations, let us assume the initial wealth level (disposable to be allocated among three securities available), $(W_0) = R100$, so that one can either invest fully in a PLS product, or mix between lottery tickets and savings account. Let us also say that in the mixed portfolios, the maximum weight of portfolio value allocated to lottery tickets is 25%. The main reason for this assumption is the resulting number of contingent claims obtained - already for assumed $w_L = 25\%$, the number of iterations is $8^{10} = 1,073,741,824$. Hence, the limitation of lottery ticket spending is needed for technical purpose.

Apart from the aforementioned assumptions, it should be noted that throughout the paper no reinvestment is being considered. Otherwise, it would be possible, for example, for an individual to purchase lottery tickets with money won in PLS drawings. Last but not least, the time horizon is assumed to be of 1-year length. That is, with a full investment in PLS product, one gets a chance to participate in 12 drawings (one each month), plus 0.25% as a certain interest earned at the end of the year. All the assumptions put forward result in the following set of portfolios available to the agent:

- Portfolio 1: PLS only; 1 'entry' to a series of 12 financial prizes drawings with the same probability distributions along with the guarantee of capital return and 0.25% interest on the capital at the end of the period;
- Portfolios 2-11: lottery tickets and savings account deposits combined; money deposited to the savings account (W_S) for each quantity of lottery tickets purchased (n_L), $W_S = W_0 - n_L \cdot ticket_L$, where $ticket_L = R25$ is a price of lottery ticket. The total number of combinations (contingent claims) depends on n_L. Any amount deposited on savings account provides a capital return plus a 4% interest at the end of the period;
- Portfolio 12: savings account only; deposit of full endowment (W_0) at the traditional savings bank account, which gives a 100% capital return along with a certain 0.25% interest earned on it at the end of the period.

While deriving the aforementioned portfolios, the probability distributions of payoffs for each portfolio is derived. Then, every such payoff (W_i) corresponds to an i th contingent claim, such that if state i occurs in date one, the agent gets W, but zero otherwise, where $i = 1,2,...,n$. Since it is not possible to purchase some chosen claims only, for each portfolio including lottery tickets, the quantities purchased of each claim are the same, and equal to n_l.

The essence of behavioural portfolio model lies in the subjective assessment of probability distribution of prizes. In principle, the model distinguishes between agents that tend to be either over-optimistic or over-pessimistic about the future potential outcomes of their decisions. These attitudes are expressed by individual transformed probability weighting function, $h(D)$. First, let us define D as a decumulative probability distribution function, that meets the following criterion:

$$D(x) = Pr(W \geq x), \tag{11}$$

such that $W_1 \leq W_2 \leq ... \leq W_{40}$ are the appropriate amounts of contingent claims related to the ith event. Next, the individual assessment parameters, q_S and q_p, are required. For $q_S > 0$, an agent is fearful about failing to reach particular financial goals (security/aspiration levels), therefore more weight is applied to lower values of future wealth, as shown by (12).

$$h_S(D) = D^{1+q_S}, \tag{12}$$

On the other hand, for $q_p > 0$, an agent is hopeful for the highest possible gains on his portfolio, therefore probabilities associated with higher wealth levels are greater than in real distribution, as expressed by (13).

$$h_p(D) = 1 - (1-D)^{1+q_p}, \tag{13}$$

The final form of individual weighting function is then given by the convex combination of its two components, for which δ measures the relative fear-hope strength for an individual:

$$h(D) = \delta h_S(D) + (1-\delta)h_p(D), \tag{14}$$

Figure 2 (Appendix) below presents the individual probability weighting functions for a pessimistic, non-biased, and optimistic agents, respectively.

As resulting from the particular features of each function, preference towards potential (excessive optimism of future outcomes) implies higher single probabilities for each level of terminal wealth. This means that the distribution of returns as assessed individually by an optimistic agent exhibits fatter right tail. An investor then hopes for higher-than-true chances to win the highest possible prizes. Basing on the individual weighting function

one would like to derive the single probabilities r_i associated with the i th event. Specifically, it is done as described by (15).

$$r_i = h(D_{i+1}) - h(D_i),$$ (15)

Naturally, the total spending cannot exceed investor's initial level of wealth, that is:

$$\sum_{i=1}^{n} v_i W_i \leq W_0,$$ (16)

Where W_0 is an investor's wealth at date zero, and v_i - price of claim related to the i th event.

Let us consider the results while controlling for an agent's preference (assessment of probability distribution). The other factor we would like to control for is individual aspiration level. Let us then assume two potential levels - R100 (which is equal to agent's initial wealth W_0) and R130. The appropriate results are thus as follows:

- For a non-biased agent (Figure 3, Appendix) we get: $q_s = q_p = 0$, what results in: $h_s(D) = h_p(D) = D$, and $h(D) = D$;
- For an agent afraid of failing to achieve security level of wealth (Figure 4), let us take $q_s = 9$, $q_p = 0$, and $\delta = 0.9$;
- For an agent hopeful for high rewards (Figure 5, Appendix), we have: $q_s = 0$, $q_p = 9$, and $\delta = 0.1$.

As the graphical representation of portfolio opportunity sets suggests, for all the types of investors considered - that is, non-biased, excessively optimistic and pessimistic agents - the probability of ruin minimizing solution with the best expected payoff is a portfolio consisting fully of PLS products. The situation tends to change along the change in aspiration level that an agent aims at achieving with his portfolio. Since $A = R130$ means in fact $130\% W_0$, or in other words, a 30% rate of return on the initial capital, it becomes highly difficult for an agent to earn such, while impossible in case of 100% funds deposited on a savings account. Therefore, the sets of portfolios in high-aspiration frameworks are concentrated within appropriate ranges of high ruin probabilities.

Conclusion

To sum up, the modern portfolio model based on the mean-variance analysis provides no clear explanation to the phenomenon of people getting interested in the prize-linked savings products. On the contrary, according to its concept, a rational agent would never allocate funds to this option, neither would he use it as a way for diversification of his portfolio. On the other hand, the behavioural model of portfolio selection allows for the choice of a PLS product as an optimal solution to the investor's problem. In case of mental accounts aimed at achieving high aspiration levels, the option of allocating all the available funds to a savings account becomes strictly dominated by a fully-PLS products portfolio. Based on the data under investigation of this paper, an investor willing to achieve a $130\% W_0$ aspiration level would most probably like to mix between purchasing

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lottery tickets and depositing money to a savings account (the analysis did not allow for mixing between any other two options). The future research may extend the funds allocation opportunities available to an agent in order to control for more factors affecting people's decisions, e.g. initial wealth level. Moreover, an important limitation of the analysis conducted in the paper is that it does not account for the time value of money. Therefore, further work on the subject could aim at building a multi-period model that would predict more realistic behaviour over time.

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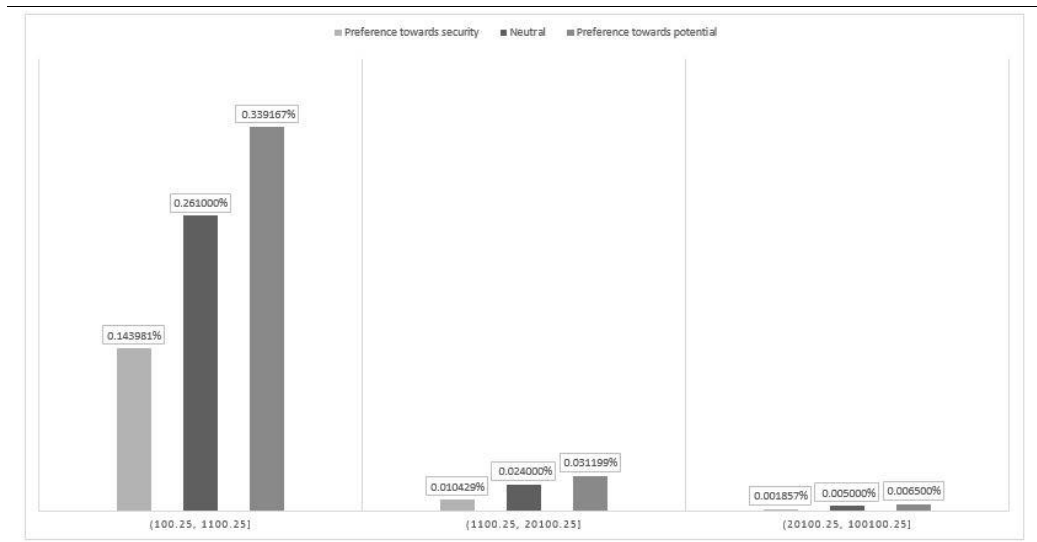
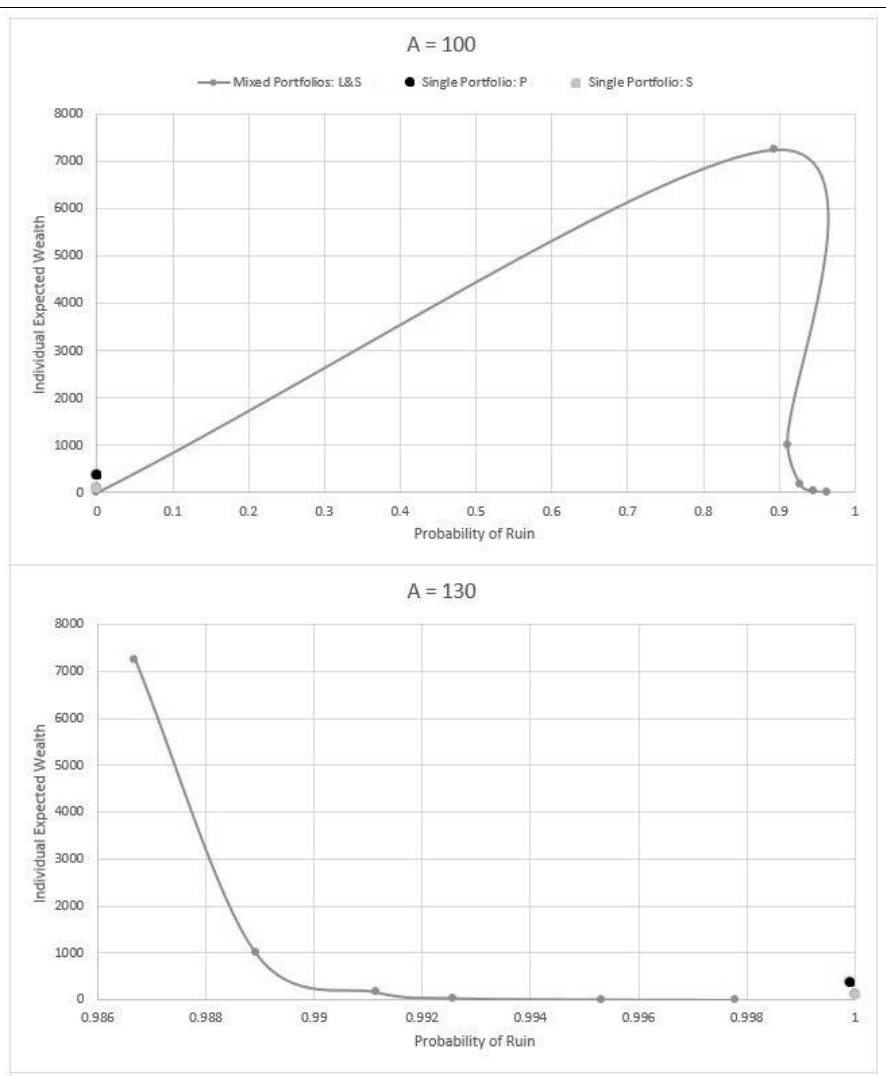## Appendix

FIGURE 1. OPPORTUNITY SET UNDER MPT
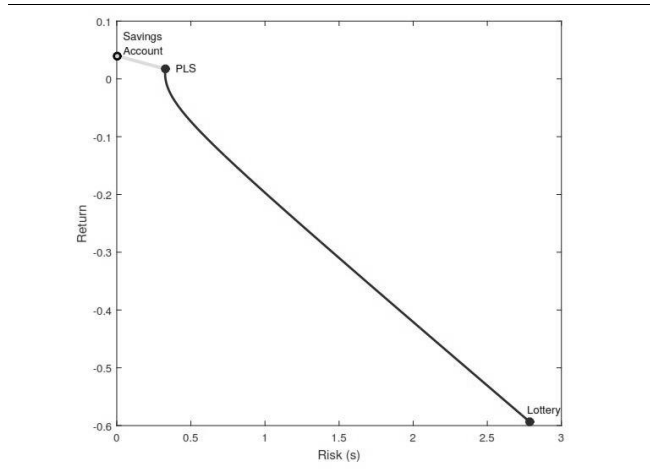
Source: Own calculations.

FIGURE 2. INDIVIDUAL PROBABILITY WEIGHTING FUNCTIONS FOR DIFFERENT TYPES OF PREFERENCES (BASED ON MONTE CARLO SIMULATION OF AN EXEMPLARY PORTFOLIO)
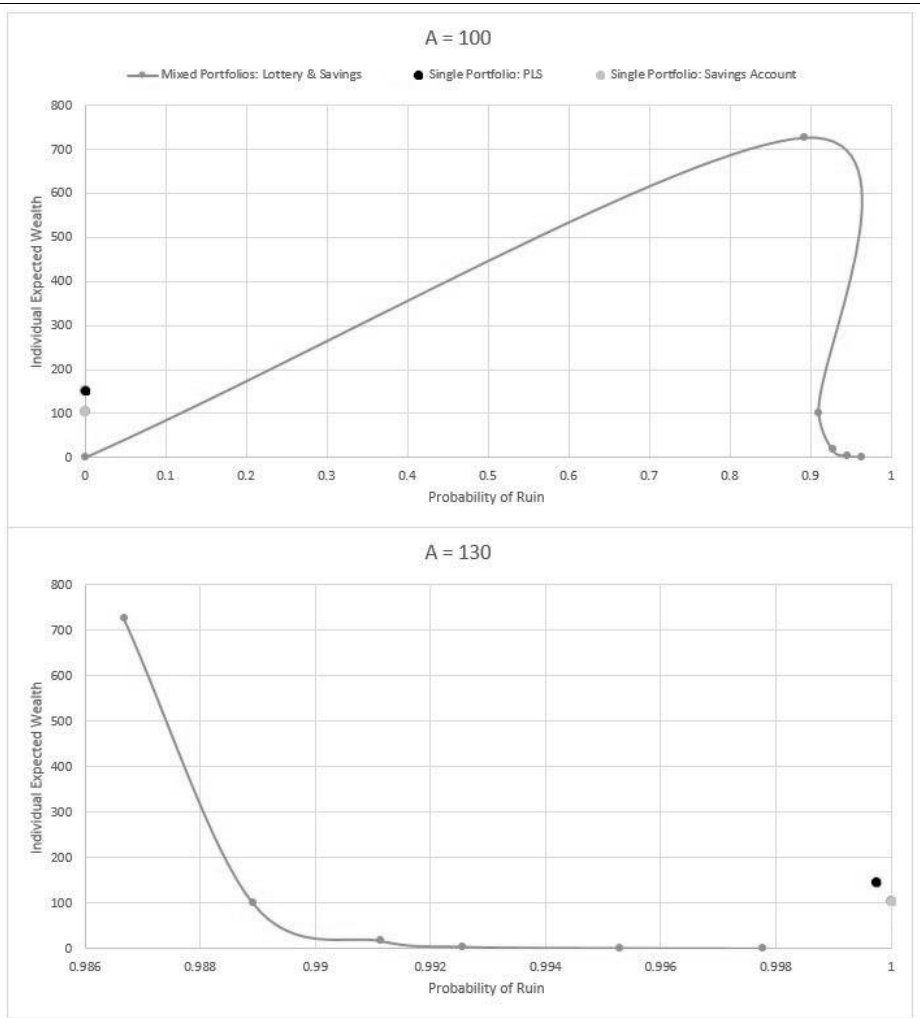

Source: Own calculations.

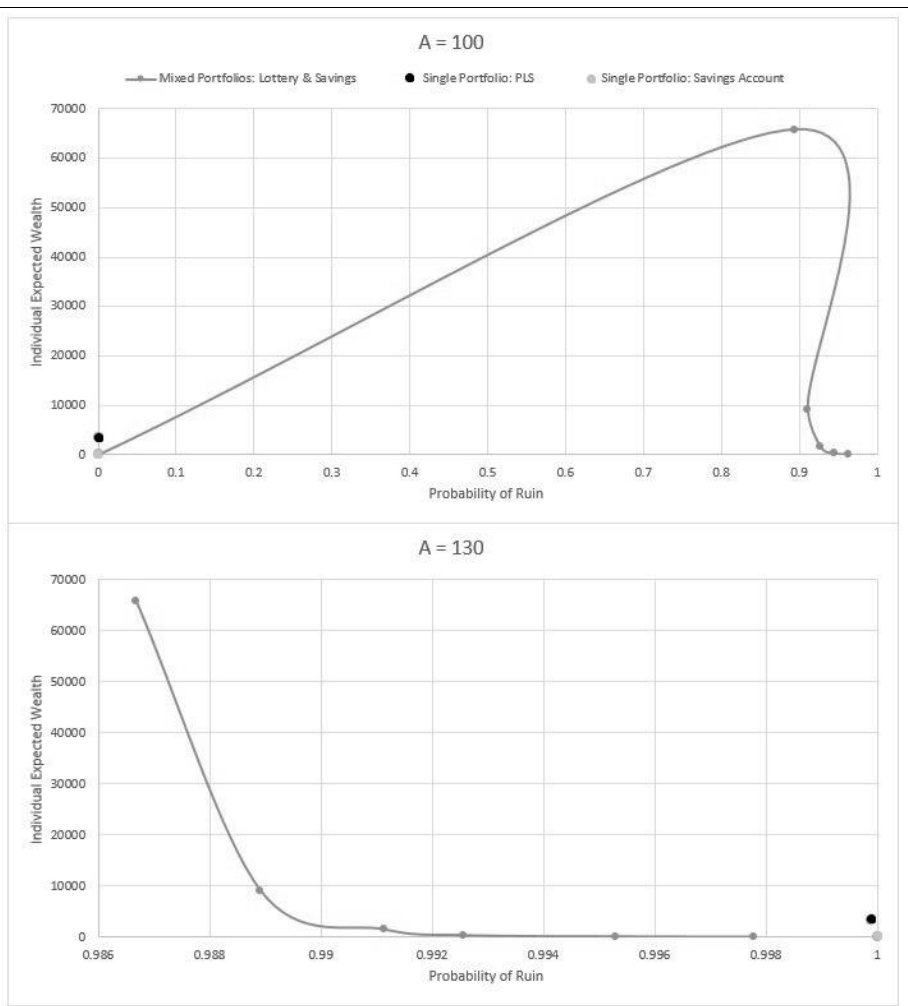FIGURE 3. DECISION SET FOR A NON-BIASED INVESTOR

Source: Own calculations.

FIGURE 4. DECISION SET FOR AN INVESTOR WITH
PREFERENCE FOR SECURITY LEVEL

Source: Own calculations.

FIGURE 5. DECISION SET FOR AN INVESTOR WITH PREFERENCE
FOR POTENTIAL HIGHEST GAINS

Source: Own calculations.