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(In)efficiency of auctions with the asymmetry of rights

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The private entrepreneurs are not forced to limit themselves to the standard auction rules, and in case of the procurement auctions one can observe many hybrid or quasi-auction rules spontaneously introduced. The paper analyzes two of them, that are based on the common assumption: the second-best bidder is given an additional right to improve her initial bid, and therefore win the auction. The focus of the paper is on the analysis of price and allocative efficiency of such modifications, to determine whether they can serve as an alternative to the standard auction rules. Theoretical analyses, followed by the laboratory experiments, provide no proof for that conclusion though, as none of the rules under study beats the classical first-price sealed-bid auction.

JEL Classifications: C91, D44, D61

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Introduction

Auction mechanisms have been more and more widely applied in procurement, where buyers take advantage of their efficiency in bringing down the prices and in forcing the suppliers to provide a better quality product. The reverse auctions were not studied thoroughly in auction theory though, as they seemed to present no challenge to auction theory: all characteristics of standard auction designs are easily translated into the reversed auction case. But there are at least two reasons, why reverse auctions deserve a more careful attention. One is that, being used in procurement, the reverse auctions are quite often multi-criteria auctions. In standard auctions the seller typically cares only for the revenue, hence price is the only criterion of choosing the winning bidder. But in procurement the buyers offer different quality goods, and the non-price criteria might be evaluated in the auction process. The other reason is that in procurement auctions one observes a lot of innovations. A private procurer is by no means forced to apply a standard auction design, and in practice many modifications of the known rules are met. If such modifications turn out to be efficient, they might evolve into a new market institution.

The analysis of one type of such a modification is the subject of this paper. In particular, the paper deals with the auctions rules, in which the second-best bidder is given a special right: an opportunity to improve her bid. In practice it could look like this. The procurer starts an auction, and after collecting all the bids, he contacts the second-best bidder, and allows her to change her bid. If she does so, and if she outbids the bidder that initially made the best bid, she wins the auction. Before we proceed with a detailed analysis of this auction modification, one could ask a question of the potential benefits for the procurer, that come from this design. It is not hard to notice, that this modification would rather not work in a dynamic auction, like the English auction. As everyone would like to come second in the first stage of the auction, no one would be willing to bid the price, and it would stop on the level far from procurer's expectations. Moreover, the second-best bidder, knowing the final price from the second stage, would just bid it by the minimal increment, which provides no benefits for the buyer. But the described asymmetry of rights could potentially work in case of the sealed-bid auction. First, assuming the number of bidders is bigger than 2, one cannot offer a very high price, as he would most likely be outbid by more than one participant, and would have no chances to come second. Second, the strongest bidder (the one that expects to have the lowest costs), being afraid to be outbid by the second-best bidder, might decide to start with a very low price (close to his costs), so that the second-best bidder has no possibility to outbid him, without suffering a loss. This could actually be a reason for using this modification in the real life: by scaring the strongest bidders, that the second-best bidder might be given a chance to improve her bid, one makes the strongest bidders go down with their price, by the further extent than they initially planned. To the best knowledge of the author he is the first to analyze the properties of this auction innovation.

The rest of the paper is divided into 4 parts. The first one provides a theoretical background, with the definitions of auction rules under study, and hypotheses concerning two efficiency measures used. The second one gives details concerning the design of experiments, that were carried out in order to test the efficiency hypotheses. The third one presents the analysis of the results, divided into 3 groups: the efficiency tests, the impact of the participation costs on auction results, and the analysis of the bidders' behavior. The last part comprises the conclusions and suggestions for further research.

Theory and hypotheses

Auction literature is to a large extent devoted to the analysis of auctions' efficiency. An efficient auction is defined as the one that maximizes the total surplus from the transaction, and it is easy to demonstrate that in case of one criterion auctions (price auctions) this is reached, when the auction is won by the bidder with the highest valuation (in case of standard auctions), or with the lowest cost (in case of the reverse auctions)[1]. A degree to which a given auction design manages to maximize the total surplus will be referred to as its ***allocative efficiency***. Yet another property of auctions is optimality; an auction is called optimal when it maximizes the auctioneer's surplus (seller's surplus in case of standard auctions, and buyer's surplus in case of reverse auctions). The design of optimal auctions was first presented by (Myerson, 1981), and (Riley, Samuelson, 1981). But as the optimal auctions are often hardly implementable and most real life auctions are not optimal, an important topic in auction theory becomes a comparison of auctions' efficiency with respect to this aspect. This type of auction efficiency will be referred to in this paper as the ***price efficiency***.

Even though most auction literature is devoted to the standard auctions, the efficiency analyses can be extended over the reverse ones. When the assumptions of the Revenue Equivalence Principle (*REP*) hold[2], all standard auction designs have the same expected value of final price, and bidders' payments, and hence the same efficiency. But, contrary to *REP*'s assumptions, most bidders are in fact risk averse, which strongly affects their behavior and the auctions' efficiency. The theoretical analyses[3] predict that the highest price efficiency would be reached in case of the so called first-price auctions, e.g. the first-price sealed-bid auction[4] and the Dutch auction[5], whereas a lower price efficiency should

[1] In a reverse auction the total surplus is a sum of buyer's surplus, being a difference between the utility of the good u, and the price paid b, and the seller's surplus, which is a difference between the price and the cost c, and thus equals: $TS = (u-b)+(b-c) = u-c$. This is maximized for the minimal value of c.

[2] See (Myerson, 1981), (Riley, Samuelson, 1981).

[3] Ibidem.

[4] Participants of the first-price sealed-bid auction make sealed bids, and the winner is the one who bids the best price. This price is used for the transaction.

[5] In a reverse Dutch auction, the price dynamically increases until someone accepts it.

be expected in case of the so called second-price auctions, e.g. the English auction[1], and the Vickrey auction[2]. As for the allocative efficiency this rank is reversed: because of the uncertainty resulting from the first-price auctions' rules, they're expected to have a lower allocative efficiency compared to the second-price auctions. The theoretical predictions have been confirmed in a number of experimental studies, see e.g. (Cox, Robertson, Smith, 1982), (Kagel, Levin, 1993), (Andreoni, Che, Kim, 2007). All those studies were conducted for the standard auctions, but their results can be extended over the reverse auctions case.

The first-price sealed-bid auction (henceforth denoted as *FPS*), and the English auction (henceforth denoted as *ENG*) are the two most popular auction designs, and would be used as the benchmark designs for the analysis of the efficiency of the asymmetric rules defined below.

In this paper the efficiency of two auction mechanisms with the asymmetry of rights will be put under the study. By the ***minimal overbid auction*** (*MOA*) the following two stage mechanism will be understood:

1) first all participants make sealed bids,
2) then, auctioneer contacts the second-best bidder, reveals to her the lowest price, and gives her an opportunity to outbid this price by the minimal increment.

If the second-best bidder in step 2) decides to outbid the price, she becomes the winner, otherwise the winner is the bidder who made the best bid in step 1).

By the ***second-best rebid auction*** (*SBR*) let us define the following two stage mechanism:

1) first all participants make sealed bids,
2) then, auctioneer contacts the second-best bidder, informs her that she made the second-best bid, and gives her an opportunity to rebid.

If the new bid made by the second-best bidder in step 2) is lower than best bid from step 1), she becomes the winner, otherwise the winner is the bidder who made the best bid in step 1).

The main difference between *MOA* and *SBR* lies in the information provided to the second-best bidder. In the former the second-best bidder learns the best price from the first stage, and so she will marginally outbid it if and only if her costs allow for that. In the latter the second-best bidder is not revealed the lowest price, and so it is more difficult for her to beat it.

For the further analysis standard assumptions concerning the bidding environment are made. The costs of the bidders are independently drawn from the common and publicly known uniform distribution. Bidder *i*'s profit is calculated as:

$$\pi_i = \begin{cases} b - c_i - w & \text{if she wins the auction,} \\ -w & \text{if she loses the auction} \end{cases}, \qquad (1)$$

where: b is the final price, c_i is the bidder *i*'s cost, and w is the non-refundable participation cost.

[1] The reverse English auction is a dynamic bidding mechanism, in which the bidders bid the price down, until no one is willing to decrease it further.

[2] Vickrey auction, also called the second-price sealed-bid auction, is a sealed-bid mechanism, in which a second-best price is used for the transaction.

In the experiment two treatments are introduced, one standard without the participation costs (thus in (1) $w = 0$), and one with a non-refundable participation cost, which is understood as the cost of preparing the bid, and henceforth is not a revenue for the auctioneer.

The consequence of the introduction of the entry cost for the auctions' efficiency has been studied in a number of theoretical and experimental papers, that differ by the way the entry costs are modelled, e.g. (Samuelson, 1985), (McAfee, McMillan, 1987), (Palfrey, Pevnitskaya, 2008). The entry cost, that is not the revenue of the auctioneer, decreases the profit of the winner, and thus the total surplus; hence the introduction of the non-refundable participation cost should lower the allocative efficiency, in terms of the percantage of the maximal total surplus reached. When the participation cost and the bidder's delivery cost is revealed to him *ex ante*, he decides whether to participate in the auction, basing his decision on the expected value of profit. Bidders with the high realizations of the delivery costs would refrain from joining the auction, and so the auction with the entry cost is expected to have a lower number of participants. As the optimal strategy of bidders in a sealed-bid auction is a function of the number of competitors, one should expect the participants to bid higher prices in the reverse auction, which would lead to the lower buyer's surplus and therefore the lower price efficiency. Higher prices in such auctions are also a consequence of the participation costs: bidders would most likely bid higher, to compansate for the higher total cost.

Because of the complexity od the auction rules under study, and the fact that both *MOA* and *SBR* do not meet the *REP*'s assumptions, it is difficult to derive the optimal bidders' strategies in their cases. But it is possible to prove one property of the minimal overbid auction.

Proposition 1. Let us assume that the costs of participants of the minimal overbid auction are independently drawn from the common uniform distribution on $\langle \alpha, \beta \rangle$, there are no participation costs, and bidders are risk neutral. Then an optimal bid in the first stage is higher than the optimal price in case of the first-price sealed-bid auction.

Proof. In appendix.

Proposition 1 is helpful in forming the hypotheses concerning the relative efficiency of minimal overbid auction, compared to the first-price sealed-bid auction. As bidders in *MOA* are expected to bid higher than in *FPS*, *MOA* has a lower price efficiency than *FPS* (*hypothesis H1*). On the other hand, due to risk aversion *MOA* is more price efficient than *ENG* (*hypothesis H2*). As we know nothing of the theoretical properties of *SBR* it is harder to form null hypotheses concerning this auction mechanism. Assuming that it is similar to *MOA*, let us form the hypotheses similarly, i.e.: *SBR* has a lower price efficiency than *FPS* (*hypothesis H3*), and higher price efficiency than *ENG* (*hypothesis H4*).

As for the allocative efficiency both minimal overbid auction, and second-best rebid auction give additional rights to the second-best bidder, thus increasing his chances for winning the auction. This lowers the allocative efficiency, and so both *MOA* has a lower allocative efficiency than *FPS* (*hypothesis H6*), and *ENG* (*hypothesis H7*), and *SBR* has a lower allocative efficiency than *FPS* (*hypothesis H8*), and *ENG* (*hypothesis H9*).

Finally we can form some hypotheses concerning the relative efficiency of *MOA* compared to *SBR*. As it is harder for the second-best bidder to beat the winning price in *SBR* this auction should be characterized by the higher allocative efficiency than *MOA* (*hypothesis H5*). At the same time, potential reduction of the price in *SBR* goes deeper than in *MOA* (in which the price is outbid only marginally), and so this mechanism should also have higher price efficiency than *MOA* (*hypothesis H10*).

Theoretical analysis predict poor properties of the herein defined asymmetric rights auction rules, as they are expected to have both the lower price and allocative efficiency compared to first-price sealed-bid auction. But the theoretical predictions concerning the price efficiency are derived from Proposition 1, which was proven for the risk neutral bidders. It is not known ex ante, how risk aversion will affect the actual efficiency of those mechanisms. The answer to that can be reached through the experiments.

Experimental design

The experiments were carried out with the 1st year students of Wroclaw University of Economics. Students competed in the 4-person groups. Their costs were independently drawn from a uniform distribution on $\langle 200, 400 \rangle$. An abstract frame was used, with the information provided formed like this: "*You take part in the procurement auction. The participant who asks the lowest price wins the auction (and is paid the price asked)*. (…)". The term "*procurement*" was used to help the students better understand the idea of the auction, but apart from that no additional information concerning the market, or commodity was used[1].

Four auction mechanisms were used (*FPS*, *ENG*, *MOA*, *SBR*), but none of the students competed in more than two auction designs[2]. As has been mentioned, two environments were modeled: one without the participation costs, and one with the non-refundable entry cost, which equaled $w = 4$. In case of the auctions with the entry costs, the bidders learnt about their cost values before the beginning of the auction, and in case of high values of costs could decide not to enter, in order to avoid paying the participation costs, and a potential loss.

As the English auction course is straightforward in case of no entry cost treatment, it was decided not to run the experiment in this case, and instead to use the results from simulations. Each auction was first played in a trial mode (not included in the further analyses) and then repeated a number of times (8-10), so that the participants could learn the optimal strategies. The results were also controlled for the learning effects. In those cases in which the first round results were substantially different than the rest of the decisions, the first round was omitted in the analyses. Table 1 provide the aggregate information on the number of auctions run and the number of participating students[3].

TABLE 1. SUMMARY OF THE EXPERIMENTAL DESIGN

Auction design	Without participation costs		With participation costs	
	No. of auctions	No. of students	No. of auctions	No. of students
FPS	359	240	154	88
ENG	-	-	124	56
MOA	145	72	152	72
SBR	132	64	128	64

Source: Own studies.

As the Polish system of financing the scientific research makes it very difficult to introduce the monetary incentives, the students were motivated by the additional points for the exam, depending on their final profit reached in the auction games. Additional

[1] Detailed instructions available on request.

[2] The experiment was controlled for the possible effect of the sequence in which the auctions are played. No such effect was observed.

[3] The experimental results described in this paper are a part of a bigger study devoted to the analysis of the efficiency of various auction and quasi-auction designs. Because of that the number of students participating in the different auctions is so varied; the first-price sealed-bid auction was played the most often, as it was also used as a benchmark for the analysis of other auction mechanisms not mentioned in this paper. See (Kuśmierczyk, 2013).

analyses showed that it worked as a very strong incentive, probably even stronger that the standard monetary rewards[1].

All experiments were designed and carried out using zTree (Fischbacher, 2007).

Results

Efficiency measures

The most important goal of the experiments was the evaluation of the efficiency of the newly defined auction mechanisms, and comparison with the standard auction rules. Price efficiency of an auction was measured as an average value of μ, calculated for each auction in the following way:

$$\mu^j = b^j - c_{1:n}^j, \qquad (2)$$

where: j is the auction index, b^j is the final price reached in the auction, and $c_{1:n}^j$ is the lowest cost of n bidders participating in the j-th auction.

Average values of μ, calculated for all auction designs that are under the analysis, allow for the price efficiency comparison. The more price efficient is the mechanism, the closer is the final price to the lowest value of costs among the auction participants, and therefore the lower is the average value of μ.

Allocative efficiency was analyzed in the standard way, i.e. by the percentage of auctions ended up with the winning of the lowest-cost participant (denoted by *AL*), and by the average value of the percentage of the total surplus reached (denoted by *KH*[2]), i.e. the average value of:

$$KH = \frac{\beta - c^j - w \cdot n^j}{\beta - c_{1:n}^j - w} \cdot 100\%, \qquad (3)$$

where: j is the auction index, β is the price at which the buyer reaches zero surplus, set to be equal to maximal value of bidders' costs, i.e. 400, c^j is the cost of the winning bidder, w is the entry cost, n^j is the number of bidders who decided to enter the j-th auction, and $c_{1:n}^j$ is the lowest cost of n bidders participating in the j-th auction[3].

Table 2 presents the values of the efficiency measures in case of auctions with no entry cost, and Table 3 in case of auctions with the entry cost.

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[1] Practically all students, allowed to participate in the experiments, volunteered for it. When asked in a survey for the minimal price for which one would sell a point gained in an experiment, the median answer was 30 PLN. Students gained on the average 3.3 points (with the minimum being 2, and the maximum being 6), which can be evaluated as being worth about 100 PLN, i.e. almost 25 euro.

[2] Where *KH* stands for the Kaldor-Hicks efficiency.

[3] The total surplus was maximized when only the lowest cost bidder decided to enter the auction. If more bidders participated, the sunk participation costs lowered the total surplus.

TABLE 2. EFFICIENCY MEASURES, AUCTIONS WITH NO ENTRY COST

Auction design	Price efficiency		Allocative efficiency	
	average μ	median μ	*AL*	average *KH*
FPS	15.58	12	90%	99.1%
ENG	40 *a*	31.8 *b*	100% *a*	100% *a*
MOA	21.5	19	80.7%	98.3%
SBR	17.05	13	87.9%	99.1%

Source: Own studies.

Note: All auctions involved 4 participants, with the costs randomly drawn from the interval $\langle 200, 400\rangle$. The number of auctions and participants is given in Table 1. *a* - the expected value. *b* - the simulated value.

TABLE 3. EFFICIENCY MEASURES, AUCTIONS WITH THE ENTRY COST

Auction design	Price efficiency		Allocative efficiency	
	average μ	median μ	*AL*	average *KH*
FPS	25.51	18	80.5%	89.6%
ENG	58.2	41	91.1%	91.5%
MOA	32.52	23.5	65.8%	85%
SBR	26.19	20	78.1%	89.5%

Source: Own studies.

Note: All auctions involved 4 participants, with the costs randomly drawn from the interval $\langle 200, 400\rangle$ and a non-refundable entry cost $w = 4$. The number of auctions and participants is given in Table 1.

Let us analyze the price efficiency first. As can be observed in the tables the lowest values of both the average and the median μ were observed in case of first-price sealed-bid auction. The mechanism which comes second in terms of price efficiency is the second-best rebid auction, and minimal overbid auction is third. Those results are in line with hypotheses *H1-H5*. To test the statistical significance of the differences in μ values a one tail Wilcoxon-Mann-Whitney *U* test was used with the null hypothesis that the values of μ equal, and alternative hypothesis formed like in *H1-H5*[1]. The *p* values are presented in Table 4.

TABLE 4. RESULTS OF STATISTICAL TESTS CONCERNING THE PRICE EFFICIENCY

Hypotheses	*p* values	
	Auctions without participation costs	Auctions with participation costs
H1 ($MOA \prec FPS$)	2.7E-05 (**)	0.0011 (**)
H2 ($ENG \prec MOA$)	-	7.4E-06 (**)
H3 ($SBR \prec FPS$)	0.203	0.212
H4 ($ENG \prec SBR$)	-	5.8E-09 (**)
H5 ($MOA \prec SBR$)	0.0058 (**)	0.011 (*)

Source: Own studies.

Note: The *p* values from one tail Wilcoxon-Mann-Whitney *U* test, with the null hypotheses that values of μ are equal. The null hypothesis is rejected in favor of the alternative hypothesis *H1*, *H2*... for small values of *p*.

As one can see, the results of the tests are in accordance with the hypotheses *H1*, *H2*, *H4*, and *H5*. As predicted, minimal overbid auction turned out to be less price efficient than

[1] The distribution of profit margins offered by bidders in the experiment were not normal, but right asymmetric, so a rank test had to be used.

first-price sealed-bid auction, and second-best rebid auction, and both newly defined mechanisms were more price efficient than the English auction. Only in case of *H3* there are no statistical grounds to reject the null hypothesis of equality of μ measures, i.e. not enough data to support the hypothesis that *FPS* is more price efficient than *SBR*. This result is a consequence of small price differences in case of those two auctions, combined with a weak statistical power of the *U* test.

Let us now look into the results concerning the allocative efficiency. Again, most of the results are in line with hypotheses *H6-H10*. The English auction turned out to be the most efficient, and as the only one managed to reach allocative efficiency exceeding 90% in case of the experiments with the entry cost (Table 3). As expected, the second highest allocative efficiency was observed in case of the first-price sealed-bid auction, and the asymmetric rights auctions were the least efficient. To test the hypotheses in case of the *KH* measures again the *U* test was used, and to test the statistical significance of differences in *AL* measures a one tail two-proportion *z* test was used. The *p* values are presented in Table 5.

TABLE 5. RESULTS OF STATISTICAL TESTS CONCERNING THE ALLOCATIVE EFFICIENCY

Hypotheses	*p* values			
	Auctions without participation costs		Auctions with participation costs	
	AL	*KH*	*AL*	*KH*
H6 ($MOA \prec FPS$)	0.0023 (**)	0.052	0.0018 (**)	-
H7 ($MOA \prec ENG$)	-	-	3E-07 (**)	-
H8 ($SBR \prec FPS$)	0.252	0.367	0.31	-
H9 ($SBR \prec ENG$)	-	-	0.002 (**)	-
H10 ($MOA \prec SBR$)	0.051	0.144	0.0114 (*)	-

Source: Own studies.

Note: The *p* values in case of *AL* measure from one tail two-proportion *z* test, and in case of *KH* measure from one tail Wilcoxon-Mann-Whitney *U* test. In all tests the null hypotheses of equality of measure values is rejected in favor of the alternative hypothesis *H6*, *H7*... for small values of *p*.

The weak statistical power of the *U* test and problems with its application in case of auctions with participation costs[1] make this test very inconclusive, so let us focus on the results of the *z* test applied to check the statistical significance of the differences in the values of the *AL* measure. The test results confirm that the highest allocative efficiency is reached in case of the English auction: the proportion of auctions finished with the winning of the lowest cost firm when this rule is applied is significantly higher than in case of *MOA* or *SBR* rules (thus confirming hypotheses *H7* and *H9*)[2]. There are no statistical grounds to differentiate between *FPS* and *SBR*; even though the allocative efficiency of first-price sealed-bid auction in 3 out of 4 cases reported in Tables 2-3 is higher than in case of second-best rebid auction (and equal in the last case), the data provide not enough statistical evidence to reject the null hypothesis of equality of the measures in favor of *H8*. But there is enough data to validate the hypothesis of a very weak allocative efficiency of

[1] There was a problem with the application of the *U* test in case of the *KH* measure in auctions with participation cost, because of which its results are not reported. To illustrate the problem compare the efficiency of *FPS* and *ENG*. As one can see from Table 3 $KH(ENG) > KH(FPS)$, but the *U* test provides evidence to reject the null hypothesis of equality of measures in favor of the hypothesis that *FPS* is… more efficient. This result is a consequence of applying a rank test to two distributions that differ significantly. In case of the English auction a bigger number of bidders decided to participate in the auction (see later analysis), which slightly lowered the total surplus left. The losses of surplus were not that big, but it weighted a lot in a U test, which just looks at the rank. On the other hand the test undervalued the huge losses of the surplus in case of *FPS*, resulting from the auctions without any participants. All of that radically affected the results of the *U* test.

[2] And also significantly higher than in case of *FPS*. The *p* value in case of tests of statistical significance of *AL* values between *ENG* and *FPS* in case of auctions with entry cost (not reported in Table 5) equals 0.0066.

the minimal overbid auction, which is clearly less efficient than *FPS* (in accordance with *H6*), and weakly less efficient than *SBR* (in accordance with *H10*).

None of the two newly defined auction rules turned out to have a particularly high efficiency, nevertheless obviously the worse of the two turned to be the *MOA* rule. Before we proceed to final conclusions, let us take a more careful look at the results concerning the role of the participation costs, and the individual bidders' strategies.

The effect of the participation costs

As predicted, the introduction of the non-refundable entry costs had a negative effect on bidders willingness to participate in the auction. Table 6 provides information on the distribution of number of bidders participating in the auction.

TABLE 6. DISTRIBUTION OF THE NUMBER OF BIDDERS IN AUCTIONS WITH ENTRY COST

Number of bidders	Auction design			
	FPS	*ENG*	*MOA*	*SBR*
0	0.065	0.040	0.099	0.047
1	0.312	0.143	0.237	0.281
2	0.338	0.270	0.362	0.352
3	0.240	0.373	0.243	0.258
4	0.045	0.174	0.059	0.062
average	1.89	2.5	1.93	2.01

Source: Own studies.

Note: All auctions involved 4 participants, with the costs randomly drawn from the interval $\langle 200, 400\rangle$ and a non-refundable entry cost $w = 4$. The number of auctions and participants is given in Table 1.

The entry costs decreased the number of participating bidders from 4 to about 2, thus substantially limiting the competition level. One has to remember though, that the actual number of competitors was known to the bidders only in case of the English auction; all the other designs were sealed-bid and did not provide such an information. So in most cases the bidders could just expect the number of bidders to be less than 4, but did not know their exact number.

Table 6 provides an interesting statistics. In turns out that the bidders' willingness to participate in the auction was the highest in case of the English auction, and the lowest in case of the first-price sealed-bid auction. It looks as though bidders preferred to take part in a dynamic setting, and avoided the sealed-bid one. The asymmetric rights auctions, i.e. *MOA* and *SBR* can be regarded as a mixed type auctions: they start as sealed-bid, but then give an option to the second-best bidder to react, thus introducing some dynamics in the second stage. Consequently, the bidders' willingness to participate in those auctions was lower than in *ENG* but higher than in *FPS*. But the decision, whether to enter the auction or not, should not depend on whether the auction is dynamic or sealed-bid, but on the estimated probability of winning. How does the auction design affect the probability of winning? It is a known fact that the expected number of participating bidders is the lowest in case of the auctions with the highest allocative efficiency. Weaker bidders (with higher realizations of cost parameters) are very unlikely to win the English auction, as the lowest cost bidder, always has an opportunity to outbid them. But their chances of winning are much higher in case of inefficient auctions, like for instance the minimal overbid auction, as it gives special rights to weaker bidders. As a consequence one should expect the lowest willingness to enter the auction in case of *ENG*, and the highest in case of *MOA*, contrary to what was observed. Apparently participants of the experiments based their decisions on different properties of the auction design. The reason why the dynamic setting attracted more bidders, could be its transparency. Bidders seemed to avoid designs which provide additional uncertainty (like *FPS*) in favor of the ones in which all is transparent and "controllable".

As has been mentioned, the actual number of bidders didn't affect directly the bidders' strategies and auctions' results in case of *FPS*, *MOA* and *SBR*, as this number was never revealed. The only exception is the case when no one entered the auction, when both buyer's surplus and total surplus was zero. As can be seen from Table 6, the percentage of auctions with no bidders was the lowest in case of *ENG* (about 4%), and the highest in case of *MOA* (almost 10%). But the buyer's surplus also equaled 0 in case of the English auction with one participant, as she did not have to bid the price down, and won with the maximal price. Taking it into consideration, one can see that the English auction was the one in which the buyer's surplus was most often equal 0 (18% of all cases), which is one of the reasons of low price efficiency of this auctions design.

Generally, the results of the efficiency measures presented in Tables 2 and 3 support the hypothesis that the introduction of a non-refundable entry cost have a negative effect on both price and allocative efficiency. In all the cases under study the value of μ is significantly higher when the entry cost is introduced, and the difference in measures is higher than the entry cost $w = 4$, and cannot be explained as a simple consequence of increasing the profit margin by the value of the additional cost. The values of allocative efficiency measures dropped by about 10 percantage points.

Individual bids

Previous analyses demonstrated the weak efficiency of the newly defined mechanisms. As a last step let us take a closer look at the individual bids, which could shed some light on the strategic situation of the bidders, and explain the observed failures of those mechanisms. The analyses will be limited to the results of auctions with no participation costs, as their introduction did not change substantially the strategic behavior of bidders.

Let us assume that all bidders maximize the value of the *CRRA* utility function[1], given by:

$$u(b,c) = (b-c)^r, \qquad (4)$$

where $1-r$ is the Arrow-Pratt constant relative risk aversion parameter. The parameter r takes values from $\langle 0,1 \rangle$ and the lower its value the more risk-averse is the bidder. Assuming there are no participation costs, and the bidders' costs are independently drawn from the uniform distribution on $\langle \alpha, \beta \rangle$, one can derive an optimal strategy in the first-price sealed-bid auction, which is given by[2]:

$$b(c) = c + \frac{r}{n-1+r} \cdot (\beta - c), \qquad (5)$$

Formula (5) has a very convenient interpretation: the risk-averse bidder in *FPS* should bid above his cost level, and the more risk-averse he is, the closer should be the bid to his cost value. Transforming (5), one gets:

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[1] The constant relative risk aversion (*CRRA*) utility function is quite often applied to model the decisions of the risk-averse bidders. See for instance (Cox, Robertson, Smith, 1982).

[2] The formula is derived as a Nash-Bayesian optimal strategy in the game. See (Cox, Robertson, Smith, 1982) for the analogous formula in case of the standard auctions.

$$b-c=\frac{r}{n-1+r}\cdot(\beta-c)\,, \qquad (6)$$

which is basis for the linear regression model:

$$y_i=a\cdot x_i+\xi_i\,, \qquad (7)$$

where: $y_i:=b_i-c_i$, $x_i:=\beta-c_i$, $a:=\frac{r}{n-1+r}$, and ξ_i is the random error.

Using the ordinary least squares method we can find the value of the estimator $\hat{a}$, and use it to estimate $\hat{r}$:

$$\hat{r}=\frac{\hat{a}\cdot(n-1)}{1-\hat{a}} \qquad (8)$$

Unfortunately little is known about the significance of the estimated parameter, and $\hat{r}$ is not even an unbiased estimator of r [1]. Nevertheless values of $\hat{r}$ have been calculated, basing on the data from experiments concerning FPS, and first stages of MOA, and SBR. The results are summarized in Table 7.

TABLE 7. ESTIMATION OF THE CRRA RISK AVERSION PARAMETERS IN AUCTIONS WITH NO ENTRY COST

Auction design	Linear regression model	
	$\hat{r}$	R^2
FPS	0.333	0.963
MOA	0.513	0.916
SBR	0.417	0.951

Source: Own studies.

Note: All auctions involved 4 participants, with the costs randomly drawn from the interval $\langle 200, 400\rangle$ and no entry cost. The number of auctions and participants is given in Table 1.

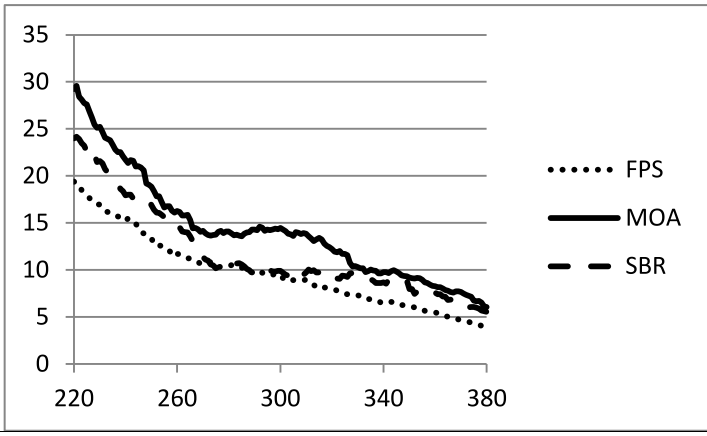
Even though one should be very careful when interpreting the results from Table 7[2], the results of the estimations allow us for some conclusions. The values of $\hat{r}$ in case of *MOA* and *SBR* turn out to be higher than in case of *FPS*. This suggests that participants of the minimal overbid auction and second-best rebid auction in the first stage are much less risk averse than bidders in the first-price sealed-bid auction. If introducing the asymmetric right auctions was supposed to persuade the bidders to bid closer to their cost levels, than the results are just the opposite: they bid at the higher profit margin. The potential reason for that might be a desire to come second in the auction, as this gives you additional bidding options.

[1] See discussion in (Kuśmierczyk, 2013, 153).

[2] First, the optimal strategies in first stage of *MOA* and *SBR* are not known, and there are no proofs that they are given by formula (5). The formula is just used to get a basic comparison between the bidders' strategies in different sealed-bid auctions. Second, (Cox, Smith, Walker, 1988) demonstrate that the *CRRA* function is useful in describing the strategic behavior of bidders in *FPS*, but one has to assume different values of risk aversion parameters among the bidders. Here we assume that the risk aversion of all bidders is the same, which is necessary once one wants to make comparisons between various auction mechanisms.

Yet more insight into the bidding strategies can be reached once we analyze, how the bidding strategies change with the value of costs. To see that, a relationship between the value of costs and the profit margin[1] bid by the auction participants has been analyzed. Figure 1 presents the relationships between the cost value c and moving averages[2] of the profit margins for the interval $\langle c-20, c+20 \rangle$.

FIGURE 1. WEIGHTED AVERAGE PROFIT MARGIN BID BY AUCTIONS' PARTICIPANTS AS A FUNCTION OF COSTS



Source: Own studies

Note: Data from the first stages of auctions without the participation costs. Weighted averages of the profit margins ($b-c$) bid for the costs from the interval $\langle c-20, c+20 \rangle$.

As Figure 1 demonstrates, the participants of *MOA* and *SBR* consistently bid the prices higher than bidders in *FPS*. Moreover the difference is significant where it matters the most, i.e. in case of the bidders with the lowest costs, who were most likely the winners of the auction. The second stage of the minimal overbid auction only marginally changes the final price, as the second-best bidder would never cut the price by more than 1, hence a huge difference in price efficiency of this auction compared to the first-price sealed-bid auction. The second-best bidders of *SBR* would generally cut the price by more than 1, which potentially could bring the price down, lower than in case of *FPS*. As the analysis presented in Tables 2 and 3 demonstrate though, this did not happen.

Conclusion

In case of privately held procurement auctions the auctioneers are free to arrange the auction rules in whatever manner they want, adding additional stages to the standard auction mechanisms, combining few of them into one, new mechanism, or introducing nontransparent rules. The paper analyzed two new mechanisms of this type, based on the assumption that the second-best bidder is given an extra right to change her bid, after the initial bids have been collected. Such innovations are observed in real life auctions, but usually introduced as a surprise. If this is the case, the additional stage cannot hurt the auctioneer: the new price by definition would be either equal or lower to the initial bid. But the goal of this paper was to determine, whether a modification of this type can be a ground for a new market institution, i.e. whether an asymmetric-rights auction would be

[1] Understood as a difference between the bidder's cost and the price bid, i.e. $b-c$.

[2] E.g. a cost value of 300 corresponds with the average value of profit margins bid by the participants with costs coming from the interval $\langle 280, 320 \rangle$.

characterized by a high efficiency, once it becomes an established auction mechanism, the rules of which are anticipated and incorporated into the bidders' strategies.

The theoretical analyses and the results of the conducted experiments suggest that the answer to the above stated question is negative. The minimal overbid auction (*MOA*) is both less price efficient and less allocatively efficient than the standard first-price sealed-bid auction, and so there is no point in concerning it a serious alternative to *FPS*. Moreover, even if bidders believed they participated in *FPS*, and were surprised by an additional stage of *MOA*, such a modification would be of no avail for the buyer, as a marginal reduction of the price, would be accompanied by an inefficient allocation. Of the two modifications analyzed, the second-best rebid auction (*SBR*) turned out to be much more efficient, nevertheless not efficient enough to replace the standard first-price sealed-bid auction. Even though in all cases there are no statistical grounds to reject the null hypotheses of equal efficiency of the abovementioned mechanisms, the actual numbers show that *SBR* never beats *FPS*. And there is no reason to implement a complex mechanism, when a simpler one is at least as good as its alternative. Of course, *SBR* would improve the price efficiency, if introduced as a surprise for the bidders, but the results of the analyses provide no grounds for the belief, that it could do so in the longer run, once it became a recognized and popular auction rule.

But that is not to say, that spontaneously created modifications of the standard auction rules cannot turn out to be efficient. In (Kuśmierczyk, 2013) there is an analysis of a longer list of hybrid auctions, and quasi-auction mechanisms, the design of which is based on the observations from the real life practices. Few of them turned out to be a potential alternative to the standard auction rules. One example is a two stage mechanism, that starts as a first-price sealed-bid auction but has an option of an additional English auction stage for the two bidders, who made the best bids in the first stage. The mechanism has an element of non-transparency, as it is up to the auctioneer to decide, whether he wants to start an additional stage, or is satisfied with the results of the first stage. Initial experiments have demonstrated, that this quasi-auction rule has a price efficiency comparable to the one of the first-price sealed-bid auction, but is much more efficient in terms of allocation[1].

In the market, where it is up to the buyers to design any auction rules that suit them, we will keep observing new rules being created. In a way it is an auction market for auction rules, in which the most efficient ones would win.

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Appendix

Proof of Proposition 1

The optimal strategy in (reverse) first-price sealed-bid auction with *n* risk neutral bidders, and the uniform distribution of costs on $\langle \alpha, \beta \rangle$, is given by[1]:

$$b(c) = c + \frac{\beta - c}{n}. \qquad (9)$$

Let us denote by $b(c)$ an optimal initial bid in the *SBR* auction for the bidder with a cost c, and let us assume that instead of using her optimal strategy, she decided to bid $\tilde{b} = b(\tilde{c})$. Denoting by $\pi(\tilde{c} \mid c)$ her profit, we get:

$$\mathrm{E}\pi(\tilde{c} \mid c) = \underbrace{\left(b(\tilde{c}) - c\right) \cdot P\left(b(\tilde{c}) < \min\{c_{-i}\}\right)}_{(1)} + \underbrace{\int_{b^{-1}(c)}^{\tilde{c}} \left(b(t) - c\right) \cdot P\left(\tilde{c} = c_{2:n}\right) \cdot f_{\min\{c_{-i}\}}\left(t \mid t < \tilde{c}\right) \mathrm{d}t}_{(2)},$$

where:

(1) describes the expected profit of the bidder, when the price offered by her in the first stage is lower than the minimal cost of the competition $\min\{c_{-i}\}$, and so no one would outbid her price in the second stage,

(2) describes the expected profit of the bidder, when she makes the second-lowest bid in the first stage ($\tilde{c} = c_{2:n}$), but her cost is lower than the lowest price in the first stage, and hence she can outbid it,

$b^{-1}(c)$ is the inverse function of the bid function $b(c)$, i.e. $b^{-1}(b(c)) = c$.

Substituting for $b(c)$ the optimal strategy in *FPS* given by (9), we get:

$$\mathrm{E}\pi(\tilde{c} \mid c) = \left(\tilde{c} + \frac{\beta - \tilde{c}}{n} - c\right) \cdot \left(\frac{\beta - \tilde{c} - \frac{\beta - \tilde{c}}{n}}{\beta - \alpha}\right)^{n-1} + \int_{\frac{nc - \beta}{n-1}}^{\tilde{c}} \left(t + \frac{\beta - t}{n} - c\right) \cdot \frac{1}{\tilde{c} - \alpha} \cdot (n-1) \cdot \frac{\tilde{c} - \alpha}{\beta - \alpha} \cdot \left(\frac{\beta - \tilde{c}}{\beta - \alpha}\right)^{n-2} \mathrm{d}t,$$

where: $\frac{nc - \beta}{n-1} = b^{-1}(c)$.

To check whether (9) is the optimal strategy, let us analyze the sign of $\frac{\mathrm{d}}{\mathrm{d}\tilde{c}} \mathrm{E}\pi(\tilde{c} \mid c)_{\tilde{c} = c}$.

$$\frac{\mathrm{d}}{\mathrm{d}\tilde{c}} \mathrm{E}\pi(\tilde{c} \mid c)_{\tilde{c} = c} = \left(1 - \frac{1}{n}\right)\left(\frac{n-1}{n} \cdot \frac{\beta - c}{\beta - \alpha}\right)^{n-1} - \left(\frac{\beta - c}{n}\right)(n-1)\left(\frac{n-1}{n}\right)^{n-1} \frac{(\beta - c)^{n-2}}{(\beta - \alpha)^{n-1}}$$

$$+ \left(c + \frac{\beta - c}{n} - c\right)(n-1)\frac{(\beta - c)^{n-2}}{(\beta - \alpha)^{n-1}} - \frac{(\beta - c)^2}{2n(n-1)}(n-1)(n-2)\frac{(\beta - c)^{n-3}}{(\beta - \alpha)^{n-1}} = \frac{1}{2} \cdot \left(\frac{\beta - c}{\beta - \alpha}\right)^{n-1} > 0$$

Positive sign of the derivative means that it pays to depart from the strategy (9), by mimicking an optimal strategy for the bidder with higher costs, which - due to the monotonic form of (9) - means bidding a higher price. *Q.E.D.*

[1] The first derivation of the optimal strategy for the case of standard FPS comes from (Vickrey, 1961).