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# Farmer responses to changing risk aversion, enterprise variability and resource endowments

Adam M. Komarek and T. Gordon MacAulay[†]

The focus of this article is on assessing how risk aversion, enterprise variability and resource endowments affect farm land-use decisions and economic returns. A theoretical model of a two-enterprise, two-constraint farm is developed, and then, an empirical illustration for an Australian farm is provided. The methodology used builds on previous expected mean-variance (EV) models by incorporating land and budget constraints. The Kuhn–Tucker conditions of the EV model are examined to highlight that changes in resource endowments have larger effects on economic returns, than do changes in risk aversion or enterprise gross margin variability. It was also found that combinations of enterprise mixes that do not use all available resources can produce higher economic returns, relative to some enterprise mixes that use all available resources.

**Key words:** economic returns, enterprise variability, farm enterprise choices, resource endowments, risk aversion.

## 1. Introduction

There is a long-standing debate on how risk and resource endowments can alter farming systems outcomes. One method to evaluate farm risk has been to assess how changes in relative risk aversion with respect to wealth alter economic returns (Freund 1956; Pannell *et al*. 2000; Hardaker *et al*. 2004; Gandorfer *et al*. 2011). Several studies have also examined how price and yield variability alter enterprise choices and income variability (Kingwell *et al*. 1992; Jacquet and Pluvinage 1997; Flaten and Lien 2007; Bell and Moore 2012; Sanfo and Gérard 2012). Underlying risk analyses are farmer resource endowments, and changes in these endowments will alter land usage and economic returns. In this article, the effects of changes in risk aversion, enterprise gross margin variability and resource endowments on acreage decisions and economic returns are examined by using the expected mean-variance (EV) model. The objective of the EV model is to maximise variance-weighted net income subject to a set of resource constraints.

Over the past decades, numerous stylised facts have emerged from studies on the economics of farm management under risk (Chavas 2008a; Chavas

[†] Adam M. Komarek (email: a.komarek@uws.edu.au) is at the School of Science and Health, University of Western Sydney, Locked Bag 1797, Penrith NSW 2751, Australia. T. Gordon MacAulay is at the Faculty of Agriculture and Environment, The University of Sydney, NSW 2006, Australia.


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*et al*. 2010). Key contributions include Coyle (1992), who formally derived the comparative statics of an unconstrained EV model and proposed that a farmer's dual indirect utility is increasing in output prices, decreasing in input prices and decreasing in output price variability. Coyle (1999) extended this to highlight that output supply is not only related to the dual's derivative but also to endogenous output variance. State-contingent modelling (Chambers and Quiggin 2000) has further advanced the field by providing a methodology to assess portfolio-choice behaviour independently of any specific risk attitudes. Despite this accumulation of knowledge, an important research agenda still remains in gauging how farmers respond to changes in risk preferences, enterprise gross margin variability and resource endowments.

Questions have been raised about whether risk really matters to agricultural producers (Just 2003). Pannell *et al*. (2000) point out that in many situations, risk has limited influence on farmer returns, and Chavas (2008b) suggests that the cost of facing production risk has recently declined. Furthermore, important studies, with their linkages to Rae (1971), highlight that divergent results exist in examining the value of information used to refine management decisions under conditions of uncertainty (Kingwell *et al*. 1993; Pannell *et al*. 2000; Pannell 2006). In this context, do changes in gross margin variability or risk preferences significantly alter farmer pay-offs? Or are there other factors that have larger influences on outcomes that need to be better understood and so should be a stronger focus for research? For example, do the gains associated with increasing resource endowments exceed the gains associated with reductions in gross margin variability?

In this article, a model is used to examine land allocation decisions on a farm in the Australian wheat–sheep zone, taking into account seasonal variability, two constraints and risk preferences. The model presented in Section two builds on Coyle (1999), who examined an unconstrained EV model, by incorporating land and budget constraints. The Kuhn–Tucker conditions (Kuhn and Tucker 1951) are derived for a two-enterprise and two-constraint case-study farm, both theoretically and then empirically using 10 years of on-farm data. In Section three, the case-study farm and empirical data used are described. Following this the empirical results relating to how changes in risk aversion, enterprise gross margin variability and resource endowments alter economic returns and enterprise mixes are presented.

## 2. Theoretical model

### 2.1. Theoretical EV model

Consider a farmer who has two enterprises that are expressed in hectares ($x_1$ and $x_2$). The farmer aims to maximise the certainty equivalent (CE) of income (pay-off), which unlike utility values, is expressed in monetary terms:

$$\mathrm{CE} = g_1 x_1 + g_2 x_2 - \frac{\alpha}{2} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \begin{bmatrix} q_1 & q_{12} \\ q_{21} & q_2 \end{bmatrix} [\, x_1 \quad x_2 \,]. \tag{1}$$

In Equation (1), $g_1$ and $g_2$ are the expected gross margins of $x_1$ and $x_2$, the gross margin variances of $x_1$ and $x_2$ are $q_1$ and $q_2$, and $q_{12}$ (=$q_{21}$) is the gross margin covariance between $x_1$ and $x_2$. The sum of the first two terms on the right-hand side of Equation (1) is the expected total gross margin. The difference between the expected income (expected total gross margin, $g_1x_1 + g_2x_2$) from the prospect and the CE of the prospect is the risk premium (Hardaker *et al.* 2004), that is, $\frac{\alpha}{2} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \begin{bmatrix} q_1 & q_{12} \\ q_{21} & q_2 \end{bmatrix} \begin{bmatrix} x_1 & x_2 \end{bmatrix}$. The term $\alpha$ is the farmer's absolute risk aversion coefficient, with larger values of $\alpha$ indicating greater aversion to risk (Freund 1956). More specifically $\alpha = r_r(w)/W$, where $r_r(w)$ is the farmer's relative risk aversion coefficient and $W$ is the farmer's wealth (net assets). The farm is constrained by a total budget ($B$) and total land area ($A$). The corresponding Lagrangian function is:

$$L = \mathrm{CE} + \lambda_b(B - c_1x_1 - c_2x_2) + \lambda_a(A - x_1 - x_2). \tag{2}$$

In Equation (2), $c_1$ and $c_2$ are the production costs per ha for $x_1$ and $x_2$. The Kuhn–Tucker conditions, which must be satisfied to yield candidates for a maximum of $L$ when the possibilities of inequality constraints are included, are given in Equations (4)–(7). These Kuhn–Tucker conditions are sufficient for a maximum, as the CE function is pseudo-concave, implying that:

$$\frac{\partial^2 \mathrm{CE}}{\partial x_1} = -q_1\alpha \le 0 \text{ and } \frac{\partial^2 \mathrm{CE}}{\partial x_2} = -q_2\alpha \le 0. \tag{3}$$

The Kuhn–Tucker conditions require:

$$\frac{\partial L}{\partial x_1} = \frac{\partial \mathrm{CE}}{\partial x_1} - \lambda_a - c_1\lambda_b \le 0, \qquad \frac{\partial L}{\partial x_1} x_1 = 0 \tag{4}$$

$$\frac{\partial L}{\partial x_2} = \frac{\partial \mathrm{CE}}{\partial x_2} - \lambda_a - c_2\lambda_b \le 0, \qquad \frac{\partial L}{\partial x_2} x_2 = 0 \tag{5}$$

$$\frac{\partial L}{\partial \lambda_b} = B - c_1x_1 - c_2x_2 \le 0, \qquad \frac{\partial L}{\partial \lambda_b} \lambda_b = 0 \tag{6}$$

$$\frac{\partial L}{\partial \lambda_a} = A - x_1 - x_2 \le 0, \qquad \frac{\partial L}{\partial \lambda_a} \lambda_a = 0 \tag{7}$$

$$\lambda_b \ge 0, \lambda_a \ge 0, x_1 \ge 0, x_2 \ge 0$$

For this problem, there are 16 possible combinations of the conditions to be examined (Appendix, Table A1). For the current problem, it is reasonable to impose the requirement of strictly non-negative values for the enterprise levels giving four possible relevant cases that could be conditions for optimal solutions (Eqns 8–11). The optimal enterprise levels are indicated with a star (*).

$$\text{Case 1:} \frac{\partial L}{\partial x_1} = 0, \frac{\partial L}{\partial x_2} = 0, x_1^* + x_2^* = A \text{ and } \lambda_b = 0(c_1 x_1^* + c_2 x_2^* \leq B) \tag{8}$$

$$\text{Case 2:} \frac{\partial L}{\partial x_1} = 0, \frac{\partial L}{\partial x_2} = 0, c_1 x_1^* + c_2 x_2^* = B \text{ and } x_1^* + x_2^* = A \tag{9}$$

$$\text{Case 3:} \frac{\partial L}{\partial x_1} = 0, \frac{\partial L}{\partial x_2} = 0, c_1 x_1^* + c_2 x_2^* = B \text{ and } \lambda_a = 0(x_1^* + x_2^* \leq A) \tag{10}$$

$$\text{Case 4:} \frac{\partial L}{\partial x_1} = 0, \frac{\partial L}{\partial x_2} = 0(c_1 x_1^* + c_2 x_2^* \leq B \text{ and } x_1^* + x_2^* \leq A) \tag{11}$$

Case 1 includes a binding land constraint and a nonbinding budget constraint (shown in parenthesis). Case 2 includes a binding land constraint and a binding budget constraint. Case 3 includes a nonbinding land constraint (shown in parenthesis) and a binding budget constraint. Both constraints are nonbinding in case 4. Solutions for the constrained maximisation problems were established by simultaneously solving the equations for each of the four cases (Eqns 8–11) and also verifying that the complementary slackness conditions held. The mathematical solutions for the optimisation problem are available in the Appendix. Substituting the optimal solution values into Equation (1), i.e. replacing $(x_1, x_2)$ with $(x_1^*, x_2^*)$, generates a new function, CE*. This function is the indirect utility function (or equivalently, the value function) (Varian 1992) and is the maximum CE of income attainable given the constraint set.

The drivers of optimal enterprise levels and pay-offs generally follow standard intuition. When the budget constraint is nonbinding (case 1), land area (not enterprise costs and budget levels) affects $x_1^*$ and $x_2^*$. Conversely, when the land constraint is nonbinding (case 3), enterprise costs and budget levels (not land area) affect $x_1^*$ and $x_2^*$. When both constraints are nonbinding (case 4), $x_1^*$ and $x_2^*$ are independent of both land area and enterprise costs. In cases 1, 3 and 4, $x_1^*$ and $x_2^*$ are influenced by enterprise gross margins (and their variability) and risk aversion.

Simultaneously solving the land and budget constraints produced the optimal solution for case 2. In case 2, changes in enterprise costs, available land or available budget will shift the intersection of the land and budget constraints and thus produce new optimal enterprise combinations. A change in risk aversion or enterprise variability does not influence the enterprise combination when this combination is fully determined by simultaneously solving the land and budget constraints. However, the CE does change when risk aversion or enterprise variability change. If the farm has a nonbinding land constraint and a binding budget constraint or a nonbinding budget constraint and a binding land constraint, the optimal enterprise combination will always lie on the binding constraint and will be influenced by the shape of CE indifference curve. The CE indifference curve is linked to risk aversion and enterprise variability, and any change in these parameters will alter the optimal enterprise combination.

## 2.2. Theoretical implications of changes in enterprise gross margin variance–covariance

To investigate changes in the $CE^*, \lambda_a^*, \lambda_b^*, x_1^*$ and $x_2^*$ when variance and covariance change, the Envelope Theorem was used (Varian 1992). For most cases, the relationships are complex and signing the terms will ultimately depend on the numerical values of the parameters in the above equations. Notwithstanding this general intractability, some relationships were found.

Coyle (1992) proposed that when no constraints are considered, indirect utility is decreasing in covariance and own-price variance. This conclusion is extended here to the broader situation where there are resource constraints. It is found that when gross margin variance increases, CE* always decreases. In order to algebraically test the propositions of the unconstrained model in Coyle (1992), a comparative statics analysis is undertaken where the Envelope Theorem is used to determine how CE* changes when an exogenous variable changes (e.g. $q_{12}$). When there are no constraints considered, the change in CE* associated with a change in covariance is given in Equation (12).

$$\frac{\partial CE^*}{\partial q_{12}} = \frac{(g_2 q_1 - g_1 q_{12})(g_2 q_{12} - g_1 q_2)}{\left(q_{12}^2 - q_1 q_2\right)^2 \alpha} \tag{12}$$

Under almost all combinations of parameters, the sign of $\partial CE^*/\partial q_{12}$ in Equation (12) is negative. For example, if covariance, $q_{12}$, is zero or negative, a rise in covariance reduces CE*; however, if there are no constraints and if $\frac{g_1}{g_2} < \frac{q_{12}}{q_2}$, $q_1 q_2 > q_{12}^2$, and $q_{12} > 0$, an increase in covariance leads to an increase in CE*. The variance–covariance matrix is explicitly positive definite when $q_1 q_2 > q_{12}^2$.

When there are land and budget constraints, the change in CE* associated with a change in covariance is given in Equation 13. Situations arise when an increase in covariance can lead to an increase in CE* (Eqn 13). For example, when land and budgets are constrained (case 2), an increase in covariance increases CE* when $c_2 < c_1 < (B/A)$.

$$\frac{\partial CE^*}{\partial q_{12}} = \frac{(B - c_1 A)(B - c_2 A)\alpha}{(c_1 - c_2)^2}. \tag{13}$$

As the ratio of the budget to area increases, the probability of the cost of enterprise one being less than this ratio increases (assuming that per hectare costs are independent of area and budget). Thus, more capital-intensive farms (more total budget per hectare) may see a rise in the covariance increasing pay-offs; however, on more extensive farms (lower budget per hectare), the probability of observing $c_2 < c_1 < (B/A)$ would be expected to be lower.

Despite the above cases providing examples of conditions that result in an increase in covariance leading to an increased CE*, numerical values need to be used to validate these suggestions. To obtain insights into the role of variance-covariance, risk aversion attitudes and constraint values in influencing $\mathrm{CE}^*, \lambda_a^*, \lambda_b^*, x_1^*$ and $x_2^*$ the application of the Kuhn–Tucker conditions is illustrated using 10 years of data from an Australian farm.

## 3. Empirical illustration

### 3.1. Case-study farm and data

The four different cases implied by the Kuhn–Tucker conditions related to the EV model for a two-enterprise, two-constraint farm are examined for a farm located in the Australian wheat–sheep zone using Mathematica (Appendix). Increasing complexity by adding additional constraints and enterprises renders examining the Kuhn–Tucker conditions challenging. In this article, a simple model is used so that the Kuhn–Tucker conditions can be better understood. The illustration focusses on two significant constraints facing mixed farmers: cash and land. Although labour is an important constraint, cash and labour have a degree of substitutability, for example, hiring labour to assist with sheep husbandry activities reduces family labour requirements but increases cash requirements for the sheep enterprise. In addition, the two enterprises are aggregated enterprises with the crop enterprise covering all crops grown on the farm, and conversely, the livestock enterprise covering all livestock on the farm. Thus, the example provided covers all farm enterprises, at an aggregate scale, along with two major farm-level constraints.

The farm is located in the Coonamble shire on the central-western plains of New South Wales. This district is a predominately mixed farming district with an annual average rainfall of 502 mm. The 3990 ha family-owned farm was managed with a self-replacing merino flock and a small herd of beef cattle. The farmer grew winter crops, including wheat, barley and lupins and also grew forage crops, including forage oats and lucerne for livestock feed.

Interviews with the farmer provided 10 years of data from 1993 to 2002 on farm activities and management, and these data provided sufficient information to solve the Kuhn–Tucker conditions (Eqns 4–7) with numerical values. Enterprise gross margins were inflation- and trend-adjusted using the method described in Hardaker *et al.* (2004, Chapter 4) and applied in Lien and Hardaker (2001) and Lien *et al.* (2009). The adjustments included using agricultural price and cost indices (Australian Bureau of Agricultural and Resource Economics and Sciences 2011) to bring individual enterprise costs to 2002 money values. Following this, the inflation-corrected gross margin data were de-trended to attempt to accommodate any technology changes over time that would otherwise cause spurious positive correlations. Enterprise cost data were used in the budget constraint and were not de-trended, rather the average cost over the 10 years was used in the empirical model (values of $c_1$ and $c_2$) to correct for any possible time trend in costs. The budget was set at the average observed spending over the 10-year period (Table 1). The farm area was 3990 ha each year.

The model was comprised of two aggregate gross margin enterprises, one for livestock and one for cropping, and these were derived from the two livestock and five cropping enterprises that actually existed on the farm. The aggregate gross margin for crops and the aggregate gross margin for livestock were calculated as the weighted averages of individual crop or livestock enterprises, respectively. The weighting was based on the contribution of each individual enterprise's gross margin to each year's total crop or livestock gross margin. The variance–covariance matrix of enterprise gross margins was used to reflect risk and was based on 10 years of inflation and trend corrected aggregate enterprise gross margins.

The relative risk aversion coefficient, $r_r(w)$, was used to reflect the farmer's risk attitude, and in this study, $r_r(w)$ was initially set at 2, implying moderate risk aversion. The value appeared to be consistent with fieldwork interviews regarding the farmer's attitude towards risk and with previous studies suggesting that Australian farmers are slightly risk averse (Bond and Wonder 1980; Abadi Ghadim and Pannell 2003).

Using the data in Table 1, the resulting Lagrangian function is given in Equation (14). This equation was used to generate the empirical results.

$$L = 93\,x_1 + 52\,x_2 - \frac{6.57\times10^{-7}}{2}\begin{bmatrix}x_1\\x_2\end{bmatrix}\begin{bmatrix}6745 & 119\\119 & 103\end{bmatrix}\begin{bmatrix}x_1 & x_2\end{bmatrix} + \lambda_b(134{,}181 - 76\,x_1 - 24\,x_2) + \lambda_a(3990 - x_1 - x_2). \tag{14}$$

## 3.2. Empirical results

### *3.2.1. Pay-off function shape and maxima*

In order to better understand the pay-off function commonly used in EV studies, the shape of the CE of income indifference curve (Eqn 1) was plotted, and the global maxima for different data sets determined when no constraints were present (points P1 and P2 in Figure 1). For the case-study farm, the maximum CE occurred at 7358 ha of crops and 764,205 ha of livestock (Figure 1). If crop and livestock costs and returns (and their variability) were identical, then 253,583 ha of crops and 253,583 ha of livestock would maximise pay-offs. These maximums did not take into account farm-level constraints, and a feasible maximum will take into account available land and cash (amongst other resources). The actual farm size was 3990 ha and

**Table 1** Model parameters

| Variable | Unit | Value |
|---|---|---|
| Average crop gross margin (CV) | \$/ha | 93 (0.89) |
| Average livestock gross margin (CV) | \$/ha | 52 (0.19) |
| Average crop cost (CV) | \$/ha | 76 (0.36) |
| Average livestock cost (CV) | \$/ha | 24 (0.36) |
| Average budget (CV) | \$ | 134,181 (0.18) |
| Arable land area | ha | 3990 |
| Wealth in 2002 (net assets) | \$ | 3,043,000 |
| Relative risk aversion coefficient | Unitless | 2 |

Notes: Averages are based on 1993–2002 data. CV is the coefficient of variation (standard deviation divided by mean of 10 observed years). Using the Shapiro–Francia test, the hypothesis that the gross margin and costs of each enterprise and the total budget are normally distributed cannot be rejected at the 5 per cent level.

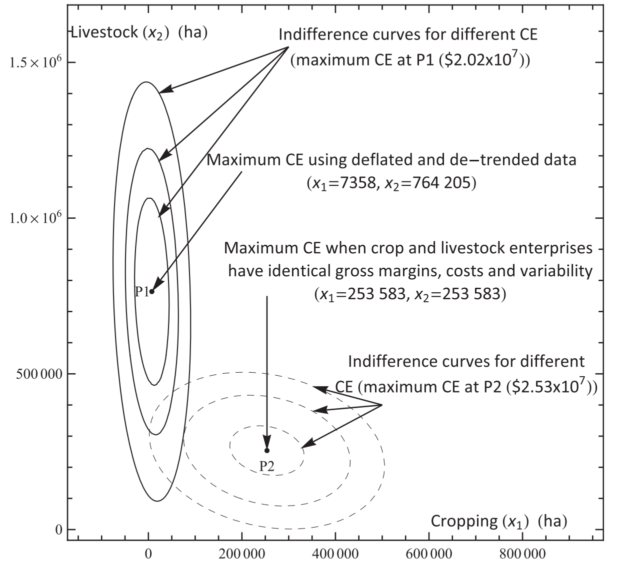


**Figure 1** Indifference curves associated with different *CEs* of income and the area of cropping and livestock that generates the maximum *CE* of income (points P1 and P2). Notes: *CE* is the certainty equivalent of income. Each indifference curve represents the same algebraic function (equation 1) with different data. Point P1 represents the maximum *CE* of income obtainable using deflated and de-trended data for the case study farm. Point P2 is an illustrative example to highlight the maximum obtainable *CE* of income when cropping and livestock costs and revenues (and both their variances) are identical.

available cash was \$134,181 (crop costs were \$76/ha and livestock costs were \$24/ha), thus a maximum of 1766 ha of crops could be grown (if no livestock were raised), and a maximum of 3990 ha of land could be allocated to livestock (if no crops were grown).

Because of the simplicity of these unconstrained models, the large farm sizes of 7358 ha of crops and 764,205 ha of livestock and 253,583 ha of crops and 253,583 ha of livestock when there are no resource constraints, are not likely to be realistic. The maximum unconstrained enterprise mixes (points

P1 and P2 in Figure 1) are currently outside the farmer's constraint set. Increases in available land and cash will move a farmer towards the unconstrained solution. In reality, other constraints (not in the model) will eventually become binding and limit land expansion, for example, labour availability or paying fixed costs. The result does however highlight the direction in which farm size may move over time and is in the general reflection of Australian farm sizes growing over time. The Productivity Commission (2005) report that the median farm size rose 23 per cent from 2720 ha in 1982 to 3370 ha in 2003.

These observations about the maximum obtainable CE also provide support for selecting the four Kuhn–Tucker cases (Eqns 8–11). As positive combinations of enterprises (nonzero levels of $x_1$ and $x_2$) maximise pay-offs, the theoretical Kuhn–Tucker solutions are focussed on those cases where $x_1$ and $x_2$ are positive (scenarios 7, 8, 15, 16 in the Appendix).

*3.2.2. Optimal solution*

Satisfaction of the Kuhn–Tucker conditions leads to three acceptable candidates for an optimal solution to Equations (4)–(7) (Figure 2). In cases 1, 2 and 3, enterprise mixes can lie between points A and B, be at point B and lie between points B and C, respectively. Using the case-study farm data, the solutions to each Kuhn–Tucker case were all at point B. At point B, the farmer grows 730 ha of crops and uses 3260 ha for livestock (Figure 2). This produces a CE of \$235,958. At point B, the shadow prices of land (ha) and cash (\$) were \$34.5 and \$0.72, respectively. Despite points A and C satisfying the Kuhn–Tucker conditions, the enterprise mixes at these points produced lower pay-offs compared to point B. For example, using one less hectare reduced the CE of income by \$34.5. This result was driven by the shape of the farmer's CE indifference curve, with the point of tangency between the CE indifference curve and the production possibilities frontier being at point B. The location of the optimal enterprise mix can also be determined using Equation (15).

$$\mathrm{CE}^* = \begin{cases} \mathrm{A-B} & \text{if} -1 \leq -\partial x_2^*/\partial x_1^* \\ \mathrm{B} & \text{if} -3.15 < -\partial x_2^*/\partial x_1^* < -1 \\ \mathrm{B-C} & \text{if} -3.15 \geq -\partial x_2^*/\partial x_1^* \end{cases} \tag{15}$$

At point B, the slope of the CE indifference curve $\left(-\partial x_2^*/\partial x_1^*\right)$ is $-1.77$, and this is steeper than the slope of the land constraint $(-1)$, but flatter than the slope of the budget constraint $(-3.15)$.

*3.2.3. Risk aversion, enterprise variance and different Kuhn–Tucker cases being optimal solutions*

In order to determine the effect of changes in risk aversion and enterprise variance on pay-offs, the level of risk was varied parametrically, with the outcomes of different levels of risk aversion and enterprise variance being assessed. Another reason for varying the level of the farmer's relative risk

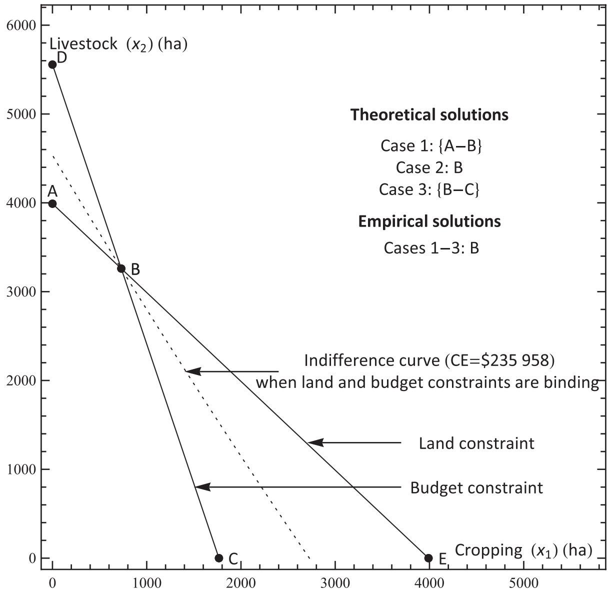


**Figure 2** Theoretical and empirical solutions for the three Kuhn-Tucker conditions using the baseline case-study farm data in Table 1 and Equation (14).

aversion coefficient was to illustrate the bounds of the system and highlight at what levels of risk aversion does the budget constraint not become effective. At a moderate level of relative risk aversion ($r_r(w) = 2$), any reduction in land or cash usage reduced the CE. However, changes in risk aversion changed the slope of the CE indifference curve. As risk aversion increased, a point was reached when it was possible to have nonbinding constraints and obtain a CE that was equal to the CE obtained when all the land and budget were used. For example, when $r_r(w) < 25$ using all the land and budget maximised the CE of income. If $r_r(w)$ was increased from 2 to 4 the slope of the indifference curve was such that the optimal combination of enterprises was still at point B in Figure 2. However, if $r_r(w) > 25$, the CE was higher when the farmer did not use the entire budget (i.e. allocated the budget and land according to case 1). If $r_r(w) = 35$, the CE in case 1 was 0.8 per cent higher than in case 2 (point A1 compared to point B in Figure 3); however, budget outlays were 8.7 per cent higher in case 2, and in case 1, the cropping area was 29 per cent lower (521 ha relative to 730 ha; Figure 3). A farmer with a level of relative risk aversion of 4 is considered extremely risk averse and a farmer with a level of relative risk aversion of 0.5 is considered hardly risk averse at all (Hardaker *et al.* 2004); therefore, a level of relative risk aversion above 25 greatly exceeds normally expected values. Only if relative risk aversion levels take on values greatly in excess of those normally expected does the budget constraint become ineffective, thus this is unlikely to occur.

When $0 < r_r(w) < 25$, case 2 had the maximum CE. In the current case study, risk aversion has limited influence on CE*; for example, when

$r_r(w) = 0$, CE* = \$237,685 and when $r_r(w) = 2.4$, CE* = \$235,612. This is because livestock gross margin variability was low (coefficient of variation is 0.19, Table 1) and the resource constraints mean that in case 2 the solution contained a large area of livestock. Pannell *et al.* (2000) report results from whole-farm models representing Syrian farmers and find that CE* is reduced by approximately 17 per cent when $r_r(w)$ changes from 0 to 2.4. The farms in Syria had cropping as the main source of income, whereas in this study, livestock are the main source of income.

When the variance of an enterprise increases, the slope of the CE indifference curve changes, and eventually, a point is reached where the solution with a nonbinding budget constraint produces a higher CE, relative to using the entire budget. To assess how CE and enterprise mixes change when cropping variance changes, a Monte Carlo sampling method was used to update the variance–covariance matrix to reflect higher cropping gross margin variances. As the gross margin data were approximately normally distributed, the normal inverse function is used to generate a series of 10,000 gross margins from the normal distribution. The distribution maintained the observed average cropping gross margin (Table 1), but had a higher cropping gross margin variance relative to the observed data (and hence the covariance also changed).

The enterprise combinations that maximised CE were determined by using the updated variance–covariance matrix (as defined above). When the cropping gross margin standard deviation was increased by 3.6 times (relative to the observed standard deviation), cases 1 and 2 produced the same CE. However, in case 1, there was \$1718 of idle cash, and even though both cases

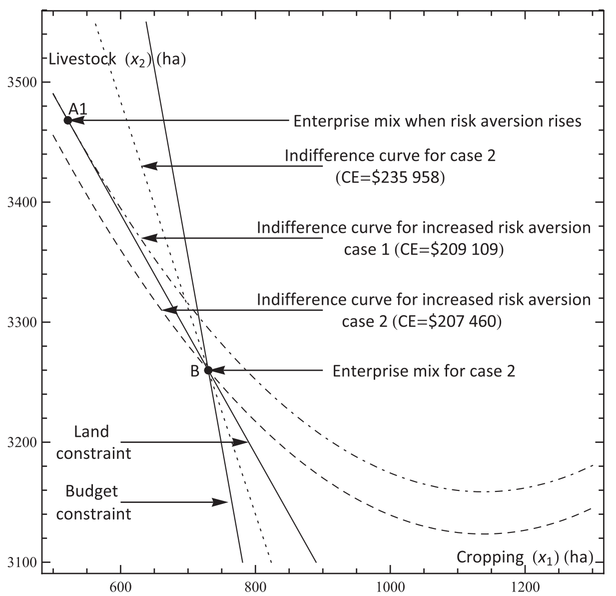


**Figure 3** Empirical solutions for different levels of risk aversion

used all available land, case 2 involved 64 ha more crops and 64 ha less livestock than in case 1. When the cropping gross margin variance increased by four times, the CE in case 1 was 0.4 per cent higher than in case 2; however, budget outlays were 6.3 per cent higher in case 2 ($134,181 compared with $126,099), and in case 1, the cropping area was 21 per cent lower (574 ha relative to 730 ha).

*3.2.4. A large range of different enterprise mixes produce the same pay-off*

Different enterprise mixes can achieve similar pay-offs, but with different variances. Pannell (2006) also highlights the observation that changes in enterprise mixes can have minimal influence on optimal economic pay-offs. Depending on a farmer's risk aversion preferences, a range of enterprise mixes will be possible that provide similar pay-offs. This range may change as the degree of risk aversion changes. For example, increased risk aversion may lead to a reduced willingness to specialise in one specific enterprise.

In this case study, relatively large changes in crop-livestock mixes produced the same CE of income, and as risk aversion increased, a smaller choice set of crop-livestock mixes existed for a specific CE of income. For example, the case-study farmer ($r_r(w) = 2$) could obtain a CE of income of $220,000 with a mix of 944 ha cropping and 2587 ha of livestock (tangency of the budget constraint and the CE indifference curve) or 311 ha cropping and 3678 ha of livestock (tangency of the land constraint and the CE indifference curve). These different enterprise mixes provided the same CE of income. When the level of risk aversion declined the range of land mixes to obtain a specific CE of income increased. For example, when the farmer was indifferent to risk ($r_r(w) = 0$), the combination of enterprises on the CE indifference curve was between 969 ha of cropping and 2509 ha of livestock (budget constrained) and 297 ha of cropping and 3693 ha of livestock (land constrained) and each produced a CE of income of $220,000.

*3.2.5. Responsiveness of pay-offs to changes in risk aversion, enterprise variability and resource endowments*

The Envelope Theorem was used to assess how changes in risk and resource endowments alter pay-offs. In the Envelope Theorem, the partial derivative of the value function (i.e. after inserting the optimal levels of $x_1$ and $x_2$ into the objective function) with respect to an exogenous variable (e.g. the cropping gross margin variance, $q_1$, or farm area, A) can be evaluated at the optimum. To make comparisons between variables, the elasticity of response was calculated (the formulae used are in Tables 2 and 3). The elasticities were a relative value that measured the percentage change in the certainty equivalent of income (CE*) when one parameter (independent of other parameters) changed by 1 per cent; for example, how a 1 per cent change in budget or a 1 per cent change in cropping gross margin variance changed CE*. The elasticities were unitless measures and were evaluated at a point and used the same parameter values as in Equation (14). The parameters used were based on 10 years of historical data from the case-study farm, and no speculation was made regarding how parameters might change in the future. If in the future the budget changed by 1 per cent and concurrently, the cropping gross margin variance changed by 10 per cent the consequences for comparing the absolute change in the CE* elasticity would be different.

In case 2, for a 1 per cent increase in the budget, the CE* increased by 0.40 per cent, and for a 1 per cent increase in land area, the CE* increased by 0.58 per cent. These elasticities relate to land (ha) and cash ($) shadow prices of $34.5 and $0.72, respectively. To gauge whether investing in additional resources is worthwhile, the costs of purchasing land and cash must be considered. In 1996, the price of agricultural land for a typical farm in the study location was $174/ha (Department of Financial Services 2011), thus it will take 7 years to generate a positive net present value on the purchase of an additional hectare (assuming a discount rate of 6 per cent and the shadow price of land not changing over the 7 years).

An increase in covariance increases CE* when the farmer was land and budget constrained and $c_2 < c_1 < (b/A)$. (Section 2.2). This condition was tested using the case-study farm data (Table 1), and it was found that $c_1 > c_2$; however, $Ac_1$ = $303,240, and this exceeded the budget. Thus, for the case-study farm when covariance increases, CE* will not increase, and this is consistent with the results in Table 2.

Marginal increases in the crop gross margin variance and risk aversion reduced the CE*. With a 1 per cent reduction in crop gross margin variance, the CE of income increased by 0.07 and 0.005 per cent in cases 1 and 2, respectively (Table 2). Although, reductions in cropping variance and risk aversion had positive effects on the CE of income, the elasticities were not as large as equivalent changes in area or budget (Table 2).

In order to understand why changes in resource endowments had relatively larger effects on the CE of income than changes in cropping gross margin variance and risk preferences, the components of the pay-off function were

**Table 2** Certainty equivalent (CE) of income elasticities of budget, area, crop enterprise variance and absolute risk aversion at the optimum in case 1 (binding land constraint and nonbinding budget constraint) and case 2 (binding land constraint and binding budget constraint)

| Change variable | Elasticity formulae | Binding land constraint and nonbinding budget constraint (case 1) | Binding land constraint and binding budget constraint (case 2) |
|---|---|---|---|
| Budget | $\frac{\partial \mathrm{CE}^*}{\partial b} \times \frac{b}{\mathrm{CE}^*}$ | 0 | 0.40 |
| Area | $\frac{\partial \mathrm{CE}^*}{\partial A} \times \frac{A}{\mathrm{CE}^*}$ | 0.89 | 0.58 |
| Cropping gross margin variance | $\frac{\partial \mathrm{CE}^*}{\partial q_1} \times \frac{q_1}{\mathrm{CE}^*}$ | −0.07 | −0.005 |
| Absolute risk aversion coefficient | $\frac{\partial \mathrm{CE}^*}{\alpha} \times \frac{\alpha}{\mathrm{CE}^*}$ | −0.10 | −0.007 |

**Table 3** Different elasticities of crop area ($x_1^*$), livestock area ($x_2^*$), total gross margin (GM) and risk premium (RP) at the optimum in case 1 (binding land constraint and nonbinding budget constraint) and case 2 (binding land constraint and binding budget constraint)

| Change variable | Response variable | Elasticity formulae | Binding land constraint and nonbinding budget constraint (case 1) | Binding land constraint and binding budget constraint (case 2) |
|---|---|---|---|---|
| Budget | $x_1^*, x_2^*$ | $\frac{\partial x_1^*}{\partial b} \times \frac{b}{x_1^*}, \frac{\partial x_2^*}{\partial b} \times \frac{b}{x_2^*}$ | 0, 0 | 3.54, −0.79 |
| | GM*, RP* | $\frac{\partial \mathrm{GM}^*}{\partial b} \times \frac{b}{\mathrm{GM}^*}, \frac{\partial \mathrm{RP}^*}{\partial b} \times \frac{b}{\mathrm{RP}^*}$ | 0, 0 | 0.44, 4.82 |
| Area | $x_1^*, x_2^*$ | $\frac{\partial x_1^*}{\partial A} \times \frac{A}{x_1^*}, \frac{\partial x_2^*}{\partial A} \times \frac{A}{x_2^*}$ | −0.01, 1.22 | −2.54, 1.79 |
| | GM*, RP* | $\frac{\partial \mathrm{GM}^*}{\partial A} \times \frac{A}{\mathrm{GM}^*}, \frac{\partial \mathrm{RP}^*}{\partial A} \times \frac{A}{\mathrm{RP}^*}$ | 0.87, 0.62 | 0.56, −2.82 |
| Cropping gross margin variance | $x_1^*, x_2^*$ | $\frac{\partial x_1^*}{\partial q_1} \times \frac{q_1}{x_1^*}, \frac{\partial x_2^*}{\partial q_1} \times \frac{q_1}{x_2^*}$ | −1.02, 0.22 | 0, 0 |
| | GM*, RP* | $\frac{\partial \mathrm{GM}^*}{\partial q_1} \times \frac{q_1}{\mathrm{GM}^*}, \frac{\partial \mathrm{RP}^*}{\partial q_1} \times \frac{q_1}{\mathrm{RP}^*}$ | −0.12, −0.70 | 0, 0.68 |
| Absolute risk aversion coefficient | $x_1^*, x_2^*$ | $\frac{\partial x_1^*}{\partial \alpha} \times \frac{\alpha}{x_1^*}, \frac{\partial x_2^*}{\partial \alpha} \times \frac{\alpha}{x_2^*}$ | −1.01, 0.22 | 0, 0 |
| | GM*, RP* | $\frac{\partial \mathrm{GM}^*}{\partial \alpha} \times \frac{\alpha}{\mathrm{GM}^*}, \frac{\partial \mathrm{RP}^*}{\partial \alpha} \times \frac{\alpha}{\mathrm{RP}^*}$ | −0.12, −0.38 | 0, 1 |

decomposed. Changes in enterprise levels, and hence total gross margins, and risk premiums were examined for the different examples using two cases (Table 3). When land and budget were binding, changes in risk aversion or crop gross margin variance did not change the area of each enterprise at the optimum (Section 2), the changes only altered the risk premium component of the CE (Table 3). On the other hand, when land expanded by 1 per cent, the area of livestock increased and the area of cropping declined. This land reallocation had a net effect of increasing the total gross margin by 0.56 per cent and decreasing the risk premium by 2.82 per cent. As a result the CE of income increased by 0.58 per cent, and this exceeded the increase in the CE of income when cropping gross margin variability declined (0.005 per cent). The case when land was binding and budget was not binding was also assessed as farmers do not always have all resource constraints binding (case 1 in Tables 2 and 3). When land was binding and budget was not binding, a 1 per cent increase in land expanded livestock acreage by 1.22 per cent, and saw a 0.01 per cent reduction in cropping land, this increased gross margins by 0.87 per cent, but also increased the risk premium by 0.62 per cent, the net result was a 0.89 per cent rise in the CE. A decline in the cropping gross margin variance increased crop acreage by 1.02 per cent and reduced livestock acreage by 0.22 per cent. This resulted in a rise in gross margin of 0.12 per cent and an increased risk premium of 0.7 per cent, with a net result of a 0.07 per cent rise in the CE.

When the land and budget contraints were binding, the intersection of these two constraints will determine the optimal enterprise mix (point B in Figure 2). If the land constraint shifts outwards and the budget does not change, the new enterprise mix will be located on the budget constraint at a point with lower levels of cropping and higher levels of livestock (moving away from point B and closer to point D in Figure 2). If the cropping gross margin variance changed the CE of income changed, but the optimal enterprise mix was still at the intersection of the land and budget constraints. The situation differs when one constraint was binding and one was nonbinding. When land was binding, a shift in cropping gross margin variance will mean that the new optimal solution will be located at the intersection of the land constraint and the CE indifference curve. This point of tangency must be on the land constraint, but the change in cropping gross margin variance alters the shape of the CE indifference curve and this influences the new point of tangency. For the case-study farm, this point of tangency was a point that incorporated increased cropping acreage (moving away from point B towards point E in Figure 2).

## 4. Conclusion

In this article, a conceptual framework for examining changes in risk aversion, enterprise gross margin variability and resource endowments has been developed and applied to a case-study farm. Major findings of this study are that, for the case-study farm, (i) changes in resource endowments have a larger impact on pay-offs than changes in risk aversion or enterprise gross margin variability (Section 3.2.5); (ii) different enterprise mixes can provide similar financial pay-offs, but change farm structure and this will have implications for the overall variability of risk-weighted net incomes (Section 3.2.4); and (iii) nonbinding constraints can result in enterprise mixes that produce higher pay-offs, relative to solutions at full resource usage. It is only when large changes in risk aversion or enterprise gross margin variability occur does the budget constraint become nonbinding (Section 3.2.3). This indicates that in this case study, budget is an important constraint relative to the farmer's risk attitude.

Farm management economics research could have a greater focus on investigating how increases in resources, new technologies and management strategies can improve expected returns without excess concern for minimising income variability. Although managing seasonal variability is crucial to many farmers, a better understanding of underlying resource constraints should not be ignored when considering how farmers can improve their economic returns.

Testing the results across a wider range of farms could strengthen the implications for farm management. As a range of enterprise mixes produce equivalent pay-offs, other factors like labour resources, fixed costs and personal preferences for operating specific farming systems will likely explain land-use and resource allocation decisions. A logical step would be to extend the current analysis to incorporate labour constraints, fixed costs and carry-over effects of farm management decisions in a whole-farm modelling framework. Finally, the current analysis does not incorporate how the farmer responds to information that unfolds dynamically nor are learning processes explicitly addressed. Fine and Freund (1990) provide an excellent starting point to assess how production flexibility might be incorporated into risk modelling.

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## Appendix

Supplementary material associated with this article can be found alongside the online version. This includes the Mathematica code and data files associated with generating all the theoretical and empirical results.

The Kuhn–Tucker conditions expressed in Equations (4)–(7) can be more fully stated as Equations (A1)–(A8).

$$\frac{\partial L}{\partial x_1} = \frac{\partial \mathrm{CE}}{\partial x_1} - \lambda_a - c_1\lambda_b = 0 \text{ for } x_1 > 0 \tag{A1}$$

$$\frac{\partial L}{\partial x_2} = \frac{\partial \mathrm{CE}}{\partial x_2} - \lambda_a - c_2\lambda_b = 0 \text{ for } x_2 > 0 \tag{A2}$$

$$\frac{\partial L}{\partial \lambda_b} = B - c_1x_1 - c_2x_2 = 0 \text{ for } \lambda_b > 0 \tag{A3}$$

$$\frac{\partial L}{\partial \lambda_a} = A - x_1 - x_2 = 0 \text{ for } \lambda_a > 0 \tag{A4}$$

$$\frac{\partial L}{\partial x_1} = \frac{\partial \mathrm{CE}}{\partial x_1} - \lambda_a - c_1\lambda_b \leq 0 \text{ for } x_1 = 0 \tag{A5}$$

$$\frac{\partial L}{\partial x_2} = \frac{\partial \mathrm{CE}}{\partial x_2} - \lambda_a - c_2\lambda_b \leq 0 \text{ for } x_2 = 0 \tag{A6}$$

$$\frac{\partial L}{\partial \lambda_b} = B - c_1x_1 - c_2x_2 \leq 0 \text{ for } \lambda_b = 0 \tag{A7}$$

$$\frac{\partial L}{\partial \lambda_a} = A - x_1 - x_2 \leq 0 \text{ for } \lambda_a = 0 \tag{A8}$$

A Lagrangian formulation of a nonlinear programming problem with $n$ decision variables and $m$ constraints can yield $2^{m+n}$ possible combinations of solutions (i.e. the $n$ decision variables may be either zero or nonzero, and the $m$ Lagrangian multipliers may be either zero or nonzero (Lee *et al.* 1985). Thus, for the model under consideration, there are $2^{(2+2)} = 16$ combinations of solution values for $x_1$, $x_2$, $\lambda_b$ and $\lambda_a$ that must be considered when searching for the optimal solution. In principle, any of the 16 combinations could be an optimal solution; however, the focus for this study is on the four cases that are related to strictly positive enterprise levels. In order for a solution to qualify as a candidate for the optimal solution, it must satisfy the Kuhn–Tucker conditions noted in the right-hand column of Table A1. Scenarios 7 (case 4), 8 (case 3), 15 (case 1) and 16 (case 2) are the potential optimal solutions examined.

**Table A1** Possible combinations of enterprise variables and shadow prices based on two enterprises and two constraints

| Scenario | $x_1$ | $x_2$ | $\lambda_b$ | $\lambda_a$ | Must satisfy Kuhn–Tucker conditions |
|---|---|---|---|---|---|
| 1 | =0 | =0 | =0 | =0 | A5, A6, A7, A8 |
| 2 | =0 | =0 | >0 | =0 | A5,A6, A3, A8 |
| 3 | =0 | >0 | =0 | =0 | A5, A2, A7, A8 |
| 4 | >0 | =0 | =0 | =0 | A1, A6, A7, A8 |
| 5 | =0 | >0 | >0 | =0 | A5, A2, A3, A8 |
| 6 | >0 | =0 | >0 | =0 | A1, A6, A3, A8 |
| 7 | >0 | >0 | =0 | =0 | A1, A2, A7, A8 |
| 8 | >0 | >0 | >0 | =0 | A1, A2, A3, A8 |
| 9 | =0 | =0 | =0 | >0 | A5, A6, A7, A4 |
| 10 | =0 | =0 | >0 | >0 | A5, A6, A3, A4 |
| 11 | =0 | >0 | =0 | >0 | A5, A2, A7, A4 |
| 12 | >0 | =0 | =0 | >0 | A1, A6, A7, A4 |
| 13 | =0 | >0 | >0 | >0 | A5, A2, A3, A4 |
| 14 | >0 | =0 | >0 | >0 | A1, A6, A3, A4 |
| 15 | >0 | >0 | =0 | >0 | A1, A2, A7, A4 |
| 16 | >0 | >0 | >0 | >0 | A1, A2, A3, A4 |

Simultaneously, solving the four cases in Mathematica (Eqns 8–11) generates the optimal enterprise mixes (Eqns A9–A12). In this article, a nonlinear programming problem was solved, with the corresponding Lagrangian function being given in Equation (2). The solutions to this nonlinear programming problem were generated by simultaneously solving the first-order conditions of the problem. An alternative approach would have been to solve the problem using a gradient algorithm. This alternative approach would involve setting up a model with an objective function and the two constraints and then using an optimisation search method to find the values of the choice variables that maximised CE, subject to the constraints. Either approach will give the same results; the approached used in this article also permitted the computation of the comparative statics.

The solutions in A9–A12 are listed in the format $\left(x_1^*, x_2^*\right)$.

$$\begin{aligned}&\text{case 1:}\left(\frac{g_1 - g_2 + A(q_2 - q_{12})\alpha}{(q_1 - 2q_{12} + q_2)\alpha}, \frac{g_2 - g_1 + A(q_1 - q_{12})\alpha}{(q_1 - 2q_{12} + q_2)\alpha}\right)\\ &\quad \text{if } (c_1 - c_2)(g_1 - g_2) + A(c_2(q_{11} - q_{12}) + (c_1(-q_{12} + q_{22})\alpha\\ &\quad \leq B(q_{11} - 2q_{12} + q_{22})\alpha\end{aligned} \tag{A9}$$

$$\text{case 2:}\left(\frac{B - c_2 A}{c_1 - c_2}, \frac{B - c_1 A}{c_2 - c_1}\right) \tag{A10}$$

$$\text{case 3:} \left( \frac{c_2^2 g_1 - c_1 c_2 g_2 - B c_2 q_{12} \alpha + B c_1 q_2 \alpha}{(c_2^2 q_1 - 2 c_1 c_2 q_{12} + c_1^2 q_2)\alpha}, \frac{c_1^2 g_2 - c_1 c_2 g_1 - B c_1 q_{12} \alpha + B c_2 q_1 \alpha}{(c_2^2 q_1 - 2 c_1 c_2 q_{12} + c_1^2 q_2)\alpha} \right)$$

$$\text{if } \frac{(c_1 - c_2)(-c_2 g_1 + c_1 g_2) + B(c_2(q_{11} - q_{12}) + c_1(-q_{12} + q_{22})\alpha}{(c_2^2 q_1 - 2 c_1 c_2 q_{12} + c_1^2 q_2)\alpha} \leq L \tag{A11}$$

$$\text{case 4:} \left( \frac{g_2 q_{12} - g_1 q_2}{(q_{12}^2 - q_1 q_2)\alpha}, \frac{g_1 q_{12} - g_2 q_1}{(q_{12}^2 - q_1 q_2)\alpha} \right) \tag{A12}$$

Substituting the solutions in Equations (A9)–(A12) into Equation (1) generates the indirect utility function (CE*; Equations A13–A16).

$$\text{case 1:} \frac{\left( (g_1 - g_2)^2 + 2A\alpha(g_2(q_1 - q_{12}) + g_1(q_2 - q_{12})) + A^2(q_{12}^2 - q_1 q_2)\alpha^2 \right)}{2\alpha(q_1 - 2q_{12} + q_2)} \tag{A13}$$

$$\text{case 2:} \frac{1}{2(c_1 - c_2)^2} \times \left( \begin{array}{l} 2(c_1 - c_2)(B(g_1 - g_2) - c_2 g_1 A + c_1 g_2 A) - \\ \left( (B - c_2 A)(-c_2 A q_1 + B(q_1 - 2q_{12} + 2c_1 A q_{12}) + (b - c_1 A)^2 q_2 \right) \alpha \end{array} \right) \tag{A14}$$

$$\text{case 3:} \frac{\left( \begin{array}{c} (c_2 g_1 - c_1 g_2)^2 + 2B(c_2 g_2 q_1 - c_2 g_1 q_{12} - c_1 g_2 q_{12} + c_1 g_1 q_2)\alpha \\ + B^2(q_{12}^2 - q_1 q_2)\alpha^2 \end{array} \right)}{\left( 2\left( c_2^2 q_1 - 2 c_1 c_2 q_{12} + c_1^2 q_{22} \right)\alpha \right)} \tag{A15}$$

$$\text{case 4:} - \frac{g_2^2 q_1 + g_1^2 q_2 - 2 g_1 g_2 q_{12}}{2\alpha\left( q_{12}^2 - q_1 q_2 \right)} \tag{A16}$$

## Supporting Information

Additional Supporting Information may be found in the online version of this article:

**Data S1.** Supporting information.