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Representing climatic uncertainty in agricultural models – an application of state-contingent theory*

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The state-contingent approach to production uncertainty presents a more general model than the conventional stochastic production approach. Here we investigate whether the state-contingent approach offers a tractable framework for representing climatic uncertainty at a farm level. We developed a discrete stochastic programming (DSP) model of a representative wheat–sheep (mixed) farm in the Central West of NSW. More explicit recognition of climatic states, and associated state-contingent responses, led to optimal farm plans that were more profitable on average and less prone to the effects of variations in climate than comparable farm plans based on the expected value framework. The solutions from the DSP model also appeared to more closely resemble farm land use than the equivalent expected value model using the same data. We conclude that there are benefits of adopting a state-contingent view of uncertainty, giving support to its more widespread application to other problems.

Key words: climatic uncertainty, expected value, risk, state-contingent theory, stochastic programming.

1. Introduction

Although farmers face many sources of uncertainty, it is agriculture's basic dependence on climatically dependent biological systems that often exert the most influence on the sector from one year to the next. Australian farmers contend with a high degree of inter-annual rainfall variability, creating uncertainty and production risk. Anderson (1979) estimated that climate variability was responsible for just under 40 per cent of the variation in Australia's gross value of agricultural production and farm income.

As is the case with risk more generally, climatic risk imposes costs. Farmers facing climate risk have to plan for a range of possible seasonal

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conditions other than the one that ultimately occurs. This means that farmers do not use resources as they would if climate conditions in the approaching season were known. Moreover, traditional approaches to incorporating risk in modelling decision-making by farmers limit the scope of risk responsiveness of farmers and generate farm plans that often bear little resemblance to decisions made by farmers.

Chambers and Quiggin (2000) proposed that state-contingent production theory, based on the work of Arrow (1953) and Debreu (1959), was the best way to think about all problems of uncertainty. State-contingent theory implies that the production relationship between inputs and outputs depends ultimately on the state of nature that arises rather than being fixed across states. This contrasts with the commonly used stochastic production function approach where uncertainty is represented by an error term and where the role of inputs used in production remains the same irrespective of what state ultimately occurs.

State-contingent theory recognises that farmers choose from a set of technologies, which canvas the range of seasonal outcomes (rather than just a single expected season. It allows consideration of a broader set of responses to climate risk than traditional expected value approaches, which are only optimal when the 'expected season' coincides with the 'actual season'.

In this paper, we investigate whether the state-contingent approach, applied through discrete stochastic programming (DSP) (Cocks 1968; Rae 1971a,b), offers a preferable framework for representing climatic uncertainty at a farm level compared to expected value approaches. A brief overview of state-contingent theory is given in the next section, followed by a review of how state-contingent theory can be applied to climate risk. A DSP model of a representative wheat–sheep (mixed) farm in Central West NSW is outlined in Section 4. Section 5 presents decision criteria to discriminate between the DSP and expected value approaches. Results are presented in the penultimate section followed by some conclusions.

2. State-contingent theory

The Expected Utility (EU) model (Anderson *et al.* 1977) has been the standard economic framework for analysing decision problems under uncertainty, although empirical studies dating back to the 1950s have revealed a variety of choice behaviour that is inconsistent with EU (Starmer 2000). In contrast to criticisms relating to whether expected utility preferences are actually held by decision-makers, Chambers and Quiggin (2000) broaden the attack to the way in which production technologies are conventionally represented within the EU model. More specifically, they point out the problems with the use of simplified stochastic production functions that they argue, do not adequately consider the interaction between controlled and uncontrolled inputs (i.e. the state of nature).

The notion of state-contingent commodities allows decision-making under uncertainty to be approached in the same way as decision-making under certainty. 'The crucial insight of Arrow and Debreu was that, if uncertainty is represented by a set of possible states of nature, and uncertain outputs by vectors of state-contingent commodities, production under uncertainty can be represented as a multi-output technology, formally identical to a nonstochastic technology' (Quiggin and Chambers 2006, p. 153). From this equivalence, Chambers and Quiggin demonstrated that the tools and marginal principles used by economists in nonstochastic production theory are also applicable to problems involving uncertainty, provided that the problem is cast in state-contingent production terms.

Chambers and Quiggin (2000, p. 41) summarise the general state-contingent model where there are M outputs, N inputs and S states of nature. A vector of inputs, $x \in \Re_{+}^{N}$, is committed by the producer prior to nature selecting from a set of possible states, $\Omega = (1,\ldots,s,\ldots,S)$. The producer's technology $T(x, z)$ converts inputs, x, into a matrix of state-contingent outputs $z \in \Re_{+}^{SxM}$. Inputs that are variable *ex post* may be regarded as negative state-contingent outputs. Here, z is a matrix of *ex ante* or potential outputs since *ex post* only one state of nature occurs. An element of the z matrix, z_{ms}, for example, denotes the amount of output m (row) produced if state s (column) occurs. Given that inputs are committed prior to the state of nature being known, the total bundle of inputs is the same in every state while outputs are specific to an individual state. Uncertainty in the state-contingent model may therefore be considered as a two-period game with nature (Quiggin and Chambers 2006). In the first period, producers commit a vector of inputs (x) under conditions of uncertainty. In the second period, a state of nature is revealed, and a vector of state-contingent outputs (z_s) is produced.

The selection of one input bundle over another is equivalent to looking forward over the season and selecting one output matrix over another, where the columns of the output matrix define the states and the rows the respective outputs. The choice among such state-contingent output matrices can be considered an expression of risk aversion. A highly risk-averse producer would select an output matrix, which portrays little variability across different states. In other words, they would select an input bundle that maps into a low variability output matrix. For example, a highly risk-averse livestock producer could select such a low stocking rate, or carry such a high level of fodder reserves, that production outcomes across different climatic states would be similar. Such a strategy would surely stabilise production outcomes, but the producer would also incur large opportunity costs in doing so. A less risk-averse livestock producer would choose a stocking rate that traded-off production stability across states with economic returns possible under each state. Consequently, the degree of variability in outputs is an economic choice made by producers rather than something beyond their control.

Most applications of state-contingent theory to date have been policy related. These are reviewed in Quiggin and Chambers (2006) and include analyses of price stabilisation, crop area insurance, point source pollution and drought policy. There have been relatively few empirical applications of the state-contingent approach. O'Donnell and Griffiths (2006) applied the state-contingent model to econometrically estimate production frontiers of rice farmers in the Philippines. In contrast to conventional stochastic frontier studies, they found that most of the estimated output shortfalls were due to climatic variability rather than inefficiency. The state-contingent approach has also been applied to water and salinity issues in the Murray-Darling Basin (Adamson *et al.* 2007). The approach was found to better capture flexible responses to uncertainty about water availability, a key component of the environment confronting water managers.

3. Stochastic programming and state-contingent theory

Risk in agricultural problems is often of a sequential nature whereby famers have opportunities to respond to changes in their environment over time. In a two-period representation, there is a set of initial decisions (x_1), state-dependent outcomes (y_1) of those decisions, followed by a set of stage-2 decisions (x_2). The importance of the sequential nature of risk is increasingly recognised by agricultural economists (Kingwell *et al.* 1993; Pannell *et al.* 2000; Just 2003; Hardaker *et al.* 2004). Tactical adjustment is critical in uncertain environments because decisions made *ex ante* will rarely be optimal *ex post*. Broader recognition of the importance of time in decision-making is also reflected in the growth in the application of Real Options Theory (Dixit and Pindyck 1994) to longer-term investment decisions.

Discrete stochastic programming (DSP), originally developed by Dantzig (1955) and extended by Cocks (1968) and Rae (1971a,b), is the main modelling approach used to capture problems involving sequential risk. A key feature of DSP is that alternative states of nature are explicitly represented within the analysis. Adamson *et al.* (2007) noted that DSP was the closest approach to a state-contingent representation of uncertainty. A number of parallels can be drawn between DSP and a state-contingent view of uncertainty. Both make a clear distinction between the states of nature and the outcomes in those states. Both allow trade-offs over state-contingent outcomes. In other words, there is a possibility to substitute outcomes in one state of nature for outcomes in another state (*ex ante*). Each approach recognises that some decisions need to be taken prior to the resolution of uncertainty, and the probabilities of states are treated as being independent of the actions taken by decision-makers.

Based on Chambers and Quiggin (2000), net income in a given state within the state-contingent model can be defined as:

$$y_s = r_s - C(w,r,p) \tag{1}$$

where

$$r_s = \sum_{m=1}^{M} p_{ms} z_{ms} \tag{2}$$

$C(w,r,p)$ the revenue-cost function is the least cost way to produce a given vector of state-contingent revenues.

The problem can be thought of as maximising state-contingent income over all states and is written as:

$$\mathrm{Max} E[Y] = -C(w,r,p) + \sum_{s=1}^{S} \pi_s r_s \tag{3}$$

The $C(w,r,p)$ term in Equation (3) can be interpreted as the cost of inputs, capable of producing a set of state-contingent revenues, that must be committed prior to the state of nature being known. The $\sum_{s=1}^{S} \pi_s r_s$ term is the probability-weighted sum of state-contingent revenues made possible by that commitment of inputs. Here, π_s is the probability of state s occurring and r_s is the revenue received in state s. Interpreted this way, Equation (3) resembles the classic two-stage DSP problem described later in Equation (4). In both the DSP approach and in state-contingent theory, inputs are committed in the first stage under conditions of uncertainty and state-contingent outputs are only revealed in the second stage when the real state of nature is known. In either approach, once state s has been realised, state income is determined by the input vector committed in the first stage and the realised vector of outputs.

4. A DSP model of dryland farming in Central West NSW

4.1. The region

The Central West region of NSW is typical of mixed farming systems in south-eastern Australia. The Central West region is the same as that defined by Patton and Mullen (2001) and includes the local government areas of Lachlan, Bland, Forbes, Weddin, Gilgandra, Dubbo and Wellington (Figure 1). The town of Condobolin lies in the centre of the region.

Annual average rainfall varies from around 700 mm in the eastern part of the region to 400 mm in the west. Annual rainfall variability at Condobolin is shown in Figure 2a. Rainfall variability has an important influence on agricultural production. Rainfall is distributed relatively uniformly throughout the year (Figure 2b) although rainfall received during May to October is more effective and coincides with the main winter cropping period.

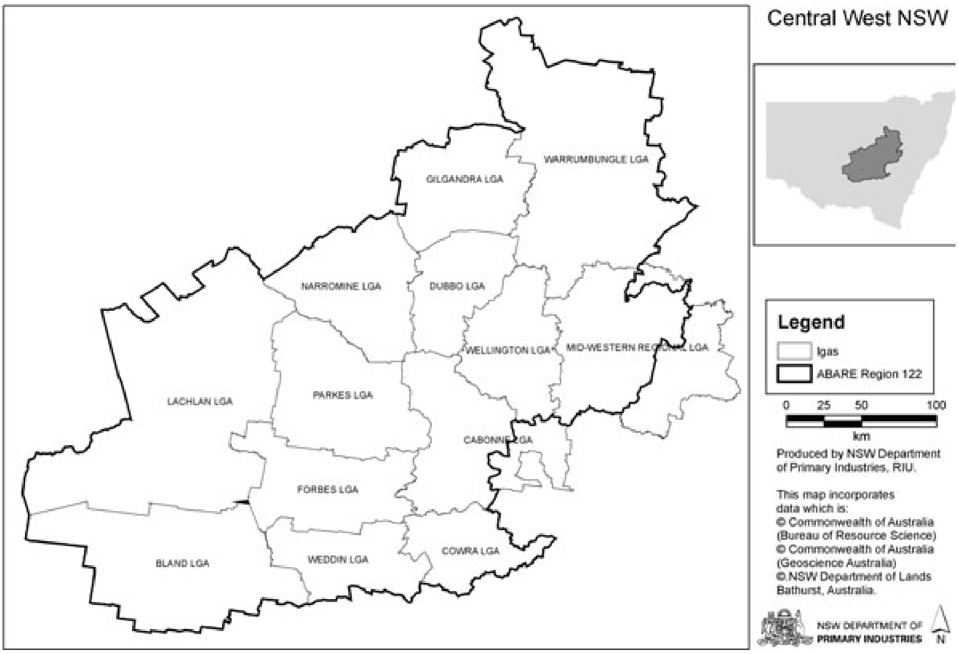

Figure 1 Central West NSW.

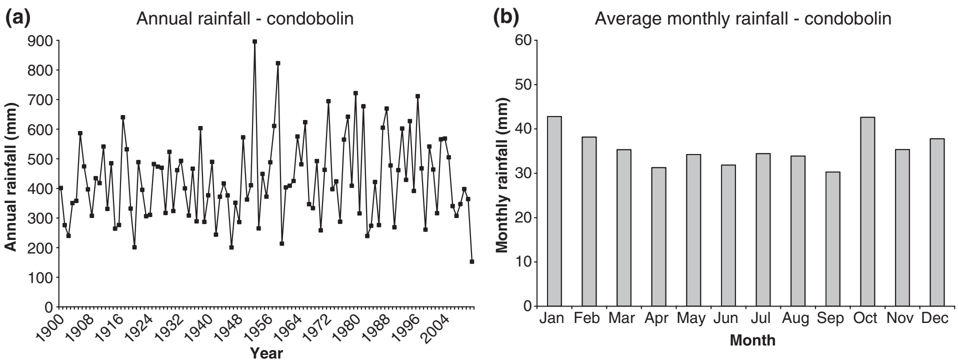

Figure 2 Condobolin rainfall (1900–2006).

Farms in the Central West have adopted mixed dryland farming systems. Over the 1990 to 2005 period, winter cereals accounted for just over 80 per cent of the total crop area (ABARE 2007). Wheat was by far the most important cereal comprising just over 70 per cent of the winter cereal area. Noncereal crops, including canola, field peas and lupins, accounted for around 20 per cent of the crop area. Livestock activities have been typically based around wool production supported by a mixture of native and improved pasture species. However, changes in the price relativities between wool, sheep and lamb prices have favoured some shift towards first- and second-cross lamb production. There has also been a trend towards larger crop areas at the expense of livestock. Typically about 40–50 per cent of total farm area is used for cropping activities. Farmers tend to employ pasture phases of 3–4 years in length, which are generally undersown with the last crop in the cropping phase. Opening crops are usually long-fallow wheat or canola and exploit the build-up of soil moisture and nutrients that occurs over the fallow period.

4.2. Overview of the DSP model

The DSP model representing the eastern farming system of the Central West is fully described in Crean *et al*. (2012) and is based on the representative farm characteristics identified by Patton and Mullen (2001). The farm has a total area of 1,500 ha, consisting of annual winter cropping (640 ha), annual pasture (640 ha) and fallow (220 ha). Livestock activities include first-cross lamb production (1,050 ewes) and merino wool production (450 wethers), giving an overall stocking rate of 1.95 dry sheep equivalents per hectare (dse's/ha).

Two simulation models, the Agricultural Production Systems Simulator (APSIM) and the Sustainable Grazing Systems (SGS) model, were used in this study to quantify the interactions between climatic conditions, production and management for crop and livestock activities, respectively. ASPIM simulates biophysical processes in cropping systems and has been extensively applied in Australia (Keating *et al*. 2002). The wheat module of APSIM was configured for use in the case study region and was run under different sets of planting conditions considered important in farmers' decisions. Nine sets of planting conditions were assessed reflecting different combinations of planting dates (early – 20 April; mid – 10 May; and late – 5 June) and starting soil moisture levels (low – 30 mm; average – 60 mm; and high – 100 mm).

The SGS Pasture Model simulates biophysical processes in a pasture system (Johnson *et al*. 2003). The SGS model, configured for the case study region, was used to simulate annual pasture and lucerne production.

Three discrete rainfall states[1] were defined based on growing season rainfall received at Condobolin (s = dry, average or wet). The growing season was defined as May to October, and the period assessed was 1902–2006 (105 years). The dry state contained the lowest third of years (growing season rainfall of 0–177 mm), the average state contained the middle third (178–249 mm) and the wet state contained the upper third (>249 mm). Each

[1] Specifying the number of states is an important consideration in the application of the state-contingent approach. While more states could be considered, one would expect that the marginal benefits of increasing the number of states to decrease quite quickly. At the same time, the marginal cost of including more states is unlikely to be trivial. The appropriate trade-off will depend on the particular problem under consideration.

year was categorised as belonging to one of these terciles. Crop and pasture production models were then simulated over the 1902–2006 period, and the outputs summarised by rainfall state.

A two-stage DSP model was developed for the case study region (Figure 3) where time was divided into the 'present' and the 'future'. A standard linear programming model can be developed into a DSP model by introducing a second period decision. The $x \rightarrow s$ format of static linear programming changes to $x_1 \rightarrow s \rightarrow x_2\ (s, x_1)$ in the DSP case. Here, x_1 is a vector of stage-1 decisions, s is the state of nature and $x_2\ (s, x_1)$ is a vector of stage-2 decisions, contingent upon earlier stage-1 decisions and the state of nature.

The farm-planning problem is to choose the mix of agricultural activities to maximise the expected level of net farm income across climatic states. The following two-stage DSP model was used:

$$\mathrm{Max}E[Y] = \sum_{s=1}^{S} \pi_s y_s = \left(-\sum_{j=1}^{J} c_{1j} x_{1j} + \sum_{s=1}^{S} \pi_s \sum_{n=1}^{N} c_{2ns} x_{2ns} \right) \tag{4}$$

subject to:

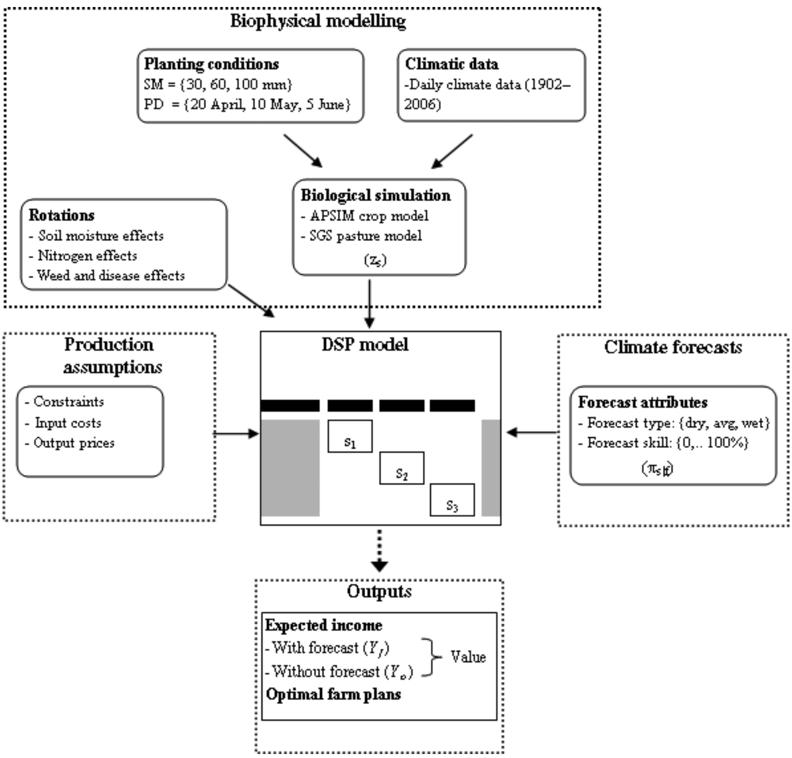

Figure 3 Overview of DSP model.

Land, labour and capital constraints

$$\sum_{j=1}^{J} a_{1ij}x_{1j} + \sum_{n=1}^{N} a_{2ins}x_{2ns} \leq b_i \text{ for all } i,\ s \tag{5}$$

Use of crop and pasture outputs

$$\sum_{j=1}^{J} a_{1mjs}x_{1j} + \sum_{n=1}^{N} a_{2mns}x_{2ns} \leq 0 \text{ for all } m, s \tag{6}$$

Non-negative activity levels

$$x_{1j}, x_{2ns} \geq 0 \text{ for all } j, n, s \tag{7}$$

Where model parameters are as follows:

π_s, probability of state s; c_{1j}, the cost of growing crop or pasture j in stage 1 ($/ha); a_{1ij}, the quantity of resource i required by crop or pasture j in stage 1 (units/ha); a_{1mjs}, the quantity of output m produced by crop or pasture j in state s (t/ha); c_{2ns}, the net revenue (+ve) or cost (−ve) from activity n in state s ($/unit); a_{2ins}, the quantity of resource i required by activity n in state s; a_{2mns} the quantity of output m required by activity n in state s (tonnes); b_i, the availability of resource i; and the model variables are as follows: y_s, farm income in state s; x_{1j}, the area of crop or pasture activity j grown in stage 1; x_{2ns}, the level of activity n chosen in state s in stage 2.

The objective function Equation (4) maximises the expected net farm income from crop, pasture and livestock production decisions across three climatic states. The outcomes of each state are weighted by their probabilities. Expected net farm income is maximised subject to constraints on availability of land, labour and capital, which must be satisfied in each state, see Equation (5). The DSP model was set up as a linear programming problem (LP) and solved using the What's Best!® R8.0 add-in to Microsoft Excel®.

The objective function reflects a two-stage decision process. The $\sum c_{1j}x_{1j}$ term of Equation (4) indicates a commitment of inputs (e.g. variable costs of growing wheat) based on the selection of stage-1 activities (x_{1j}), while the $\sum_{s=1}^{S} \pi_s \sum_{n=1}^{N} c_{2ns}x_{2ns}$ term reflects expected state-contingent revenue derived from stage-2 activities (x_{2ns}) (e.g. harvest and sale of wheat). The inputs committed in stage 1 are the same in every state of nature, whereas the inputs selected in stage 2 are specific to each state.

In stage 1 of the DSP approach, the farmer makes decisions about the areas of crop, pastures and fallow, taking into account the probabilities of dry, average and wet states. With only the probabilities known, Stage-1 decisions must trade-off returns across states in order to be optimal in the face of uncertainty. In stage 2, decisions are taken about the end use of crops (e.g. sell grain, store grain, graze crops and cut crops for hay) and pastures (e.g. graze and cut for hay), which are contingent upon both the state of nature and the decisions taken in stage 1. The representation of production in this way attempts to capture the flexibility that farmers have over the choice of production technologies when faced with climatic uncertainty. The DSP model contains 111 stage-1 activities, 456 stage-2 activities and 255 resource constraints that apply across either or both of the stages.

There are a number of important aspects of the above formulation. First, the DSP model has two clearly identifiable stages, one prior to and one postuncertainty from the first stage being resolved. Stage-1 decisions cannot anticipate one state over another, must be chosen *a priori* and must be feasible for each state (McCarl and Spreen 2007).

Second, there is a direct linkage between stage-1 and 2 decisions. The linkage is contained within constraint matrices (a_{1mjs} a_{2ins} a_{2mns} in Eqn 6). Stage-2 decisions (x_{2s}) depend on both the stage-1 decision and the state of nature. In a mixed farming example, a decision in stage 1 to run a large number of livestock may require in stage 2 either the retention of crop output on farm, grazing of failed crops or the purchase of additional fodder if a dry state occurs. An important characteristic of the DSP approach is that it requires explicit consideration of all adjustments that can be made to production in each state of nature when determining the optimal set of stage-1 decisions.

Third, uncertainty about which state of nature will occur is reflected in the probabilities that a decision-maker assigns to each state (π_s), which are unaffected by the action of the decision-maker. Changes in state probabilities thus alter the weighting given to state-contingent revenues in the objective function, which in turn influences the optimality of stage-1 decisions. The optimal solution to the DSP problem takes into account the outcomes across all states of nature. For this reason, DSP solutions are often thought of as solutions that are 'well hedged' for uncertainty.

Lastly, the objective function assumes risk neutrality. However, DSP can also accommodate a nonlinear utility function to reflect risk aversion and would therefore be solved as a nonlinear program (McCarl and Spreen 2007). Risk preferences were not pursued here in the light of what appears to be a growing consensus, within the agricultural economics literature, that the importance of capturing risk aversion within normative studies may have been overemphasised relative to the need to better represent production possibilities (Pannell *et al.* 2000; Just and Pope 2003; Hardaker *et al.* 2004).

The question remains whether the application of a DSP model will result in substantially different optimal solutions to expected value models based around the occurrence of a single state. Moreover, does the additional value obtained from a DSP approach justify its larger costs and added complexity?

5. Measuring the gains from modelling risk in a state-contingent way

Uncertainty imposes costs because decisions made *ex ante* are rarely optimal *ex post*. By measuring the cost of uncertainty, an estimate can be obtained of the pay-offs from better representing uncertainty in models of agricultural decision-making. Two concepts from the stochastic programming literature (Birge and Louveaux 1997), the Expected Value of Perfect Information (EVPI) and the Value of the Stochastic Solution (VSS), provide a basis on which these judgments can be made. EVPI and VSS are calculated from alternative approaches to representing uncertainty. The EVPI measures the maximum amount that a decision-maker would be prepared to pay for perfect information about the future. The VSS measures the gains from more realistically representing uncertainty and is calculated by comparing solutions from alternative approaches to representing uncertainty. There are four modelling approaches relevant to these calculations, and each of these is described below.

The 'wait and see' (WS) model involves solving a conventional deterministic model for each climatic state. It assumes that the decision-maker can wait and see what the state of nature is prior to making any commitment. WS problems are a generalisation of conventional sensitivity analysis and parametric linear programming (Birge and Louveaux 1997). The maximum amount of net farm income obtained for a single state is a standard maximisation problem of the following form:

$$y_s = \mathrm{Max}\, y(x, s) \tag{8}$$

where y_s is maximum level of net farm income obtained under state s, and x is a vector of decisions chosen on the basis that state s was known to be the true state at the time the model was solved (i.e. the case of perfect information). The model is solved separately for each state so that there are optimal farm plans for the dry, average and wet states. The optimal WS solution is represented by the decision set (x^*_{1s}, x^*_{2s}). The overall value of the WS solution across all states requires y_s to be weighted by the probability of each state:

$$Y_{\mathrm{WS}} = \sum_{s=1}^{S} \pi_s y_s \tag{9}$$

The expected value $(\mathrm{EV}^{\mathrm{A}})$[2] approach requires that the random parameters in the model be replaced by their expected values. Like the WS approach, in the EV^{A} approach, all uncertainty about random parameters is dealt with prior to the model being solved. The EV^{A} approach remains a simple deterministic maximisation problem:

$$Y_{\mathrm{EV^A}} = \mathrm{Max}\, Y(x, \bar{s}) \tag{10}$$

where $Y_{\mathrm{EV^A}}$ is the maximum level of expected net farm income achieved in the expected state $\bar{s}$ and x is a vector of decisions chosen on the basis of $\bar{s}$. The

[2] EV^{A} is used here to avoid confusion with mean-variance (E-V) notation used in the agricultural economics literature.

optimal EV^{A} solution is represented by the decision set $(x^*_{1\bar{s}}, x^*_{2\bar{s}})$. The optimal stage-1 decisions $x^*_{1\bar{s}}$ are chosen based on a single expected state, as indicated by subscript $\bar{s}$. In this solution, crop and pasture yields take their values from the single expected state rather being state-contingent as indicated previously by the term a_{1mjs}.

The actual result of implementing the stage-1 decisions of the EV^{A} solution ultimately depends on which state of nature actually occurs. A further measure referred to in the literature is the expected result of using the expected value solution[3] ($Y_{\mathrm{EV}}{}^{\mathrm{B}}$). The optimal EV^{B} solution is represented by the decision set $(x^*_{1\bar{s}}, x^*_{2s})$. Stage-1 decisions are fixed at their values from the deterministic EV^{A} solution $x^*_{1\bar{s}}$ (e.g. areas of crop and pasture). The model is then solved separately for each state with flexibility to determine optimal stage-2 decisions x^*_{2s} (e.g. the best use of crop and pasture output in this instance). The level of income from each EV^{B} solution is weighted by state probabilities to form an expected value ($Y_{\mathrm{EV}}{}^{\mathrm{B}}$) as shown in Equation (11).

$$Y_{\mathrm{EV}^{\mathrm{B}}} = \sum_{s=1}^{S} \pi_s y_{s|\mathrm{EV}^{\mathrm{A}}} \tag{11}$$

The optimal DSP solution is represented by the decision set (x^*_1, x^*_{2s}). Stage-1 decisions x^*_1 do not have a state subscript, implying that the decisions must be chosen prior to the state of nature being revealed. As a consequence, stage-1 decisions must sacrifice returns in some states in order to be optimal across states. This is the principal difference between the DSP model and the WS and expected value approaches. It differs from the WS model that presumes uncertainty can be resolved prior to a choice being made and is also different from the expected value model that presumes a single expected state is the best guide to selection of inputs and outputs. The approach here requires that the decision-maker is aware of the possible states and can assign a probability to each of those states. This is not a strong assumption in the context of annual climate risk that can be readily informed by historical observations, but might pose challenges in other contexts.

Net farm income derived from the solutions of the above models provides the basis for the calculation of the VSS. Madansky (1960) established a series of inequalities that govern the relationship between the solutions including:

$$Y_{\mathrm{EV}^{\mathrm{B}}} \leq Y_{\mathrm{DSP}} \leq Y_{\mathrm{WS}} \tag{12}$$

From Equation (12), the value of $Y_{\mathrm{EV}}{}^{\mathrm{B}}$ will always be less than or equal to Y_{DSP}. Any feasible solution considered by the EV^{B} model would have already been considered in finding the optimal solution to the DSP model (Di

[3] EV^{B} is used here to more clearly denote the clear linkage between the two solutions.

Domenica *et al*. 2009). The difference between the first two terms is defined as the VSS, a measure of the benefits of using a DSP approach relative to an expected value approach.

$$\mathrm{VSS} = Y_{\mathrm{DSP}} - Y_{\mathrm{EV^B}} \quad (13)$$

Given that both the DSP and $\mathrm{EV^B}$ models provide the flexibility to respond to state conditions in stage 2, the source of VSS can be attributed to differences between stage-1 decisions. The key distinction is that the DSP model anticipates the future in making stage-1 decisions by explicitly considering state-contingent responses. In contrast, the $\mathrm{EV^B}$ model only responds to the current state, given that stage-1 decisions have previously been already set in place by a naive $\mathrm{EV^A}$ model.

Equation (12) also stipulates that the value of Y_{WS} should be always greater than or equal to that of Y_{DSP}. Any solution found to be optimal for the DSP model, under conditions of imperfect information, would have also been considered by the WS model under perfect information. The difference between the second two terms in Equation (12) is known as the EVPI:

$$\mathrm{EVPI} = Y_{\mathrm{WS}} - Y_{\mathrm{DSP}} \quad (14)$$

The EVPI measures the maximum amount that a decision-maker would be prepared to pay for perfect information about the future. It is the difference in expected net farm income obtained from implementing the optimal farm plan, given perfect knowledge of each state of nature, and that obtained from implementing the best hedged farm plan from the DSP model.

It is also apparent from Equation (12) that using an expected value model to represent uncertainty rather than the DSP model will generally increase the estimate of the EVPI because Y_{DSP} will generally exceed $Y_{\mathrm{EV}}{}^{\mathrm{B}}$. Modelling approaches that give solutions not well hedged for uncertainty will overstate the value derived from information that reduces uncertainty and thereby bias the valuation of technologies like climate forecasts.

6. Results

6.1. Comparisons of returns

The returns from DSP and expected value approaches were assessed over three planting dates. The average level of net farm income reported for each solution is contained in Figure 4a, and a breakdown of the results for each planting date is contained in Figure 4b–d.

The DSP farm plan was found to have a higher level of expected net farm income (across states) than the $\mathrm{EV^B}$ farm plan for all planting dates (Figure 4a). The VSS ranged from \$12,848 (30 per cent increase) under the 20 April planting date, to \$15,512 (44 per cent increase) under the 10 May

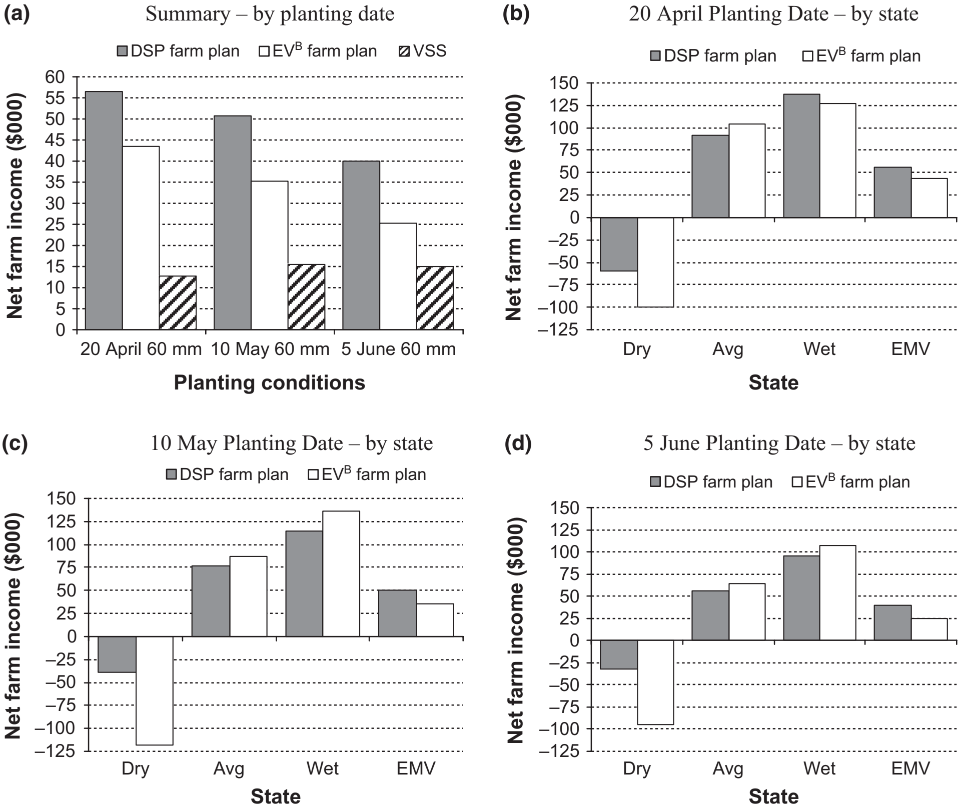

Figure 4 The value of the various farm plans by planting date. EMV is the expected monetary value. It weights net farm income in each state by the respective probabilities.

planting date, to $14,918 (59 per cent increase) under the 5 June planting date. The extent of gains from the DSP farm plan over that of the EV^B farm plan suggests that a more explicit representation of uncertainty is valuable.

Under the 20 April planting date, the DSP farm plan was more profitable than the EV^B farm plan in both dry and wet states, but was inferior in the average state. Under the 10 May and 5 June planting dates, the DSP farm plan was more profitable in the dry state (farm losses were reduced). Some of the gain in the dry state was, however, offset by slightly lower levels of income in average and wet states. A consistent finding across all planting conditions is that the DSP farm plan limits the losses in net farm income that occur in a dry state.

The findings are consistent with the notion that DSP approaches result in strategies that are 'well hedged' compared with typical deterministic expected value approaches. By explicitly considering the consequences of alternative states, and their probability of occurrence, the farm plans proposed by the DSP approach were better hedged. By contrast, the final EV^B model simply responds to the conditions of each state, which were not planned for in the EV^A model.

6.2. Comparison of farm plans

Table 1 provides a summary of the results for the mid-planting date (10 May) including the optimal set of stage-1 and 2 decisions for the DSP, EV^A and EV^B farm plans. Note that the EV^A and EV^B plans share the same stage-1 decisions, but differ in respect to stage-2 decisions. EV^B can be thought of as the real consequences of implementing EV^A under climate variability. We also note the similarity between the DSP farm plan and the farm plan assembled for the study area from judgements by farmers and scientists about what farmers typically do as reported in Patton and Mullen (2001). The major difference between the farm plans concerns the number of livestock and crop areas. The EV^A farm plan was found to have a much higher overall stocking rate of 4.61 dse's/ha relative to the DSP farm plan of 2.66 dse's/ha. This is because the adverse consequences of high stock numbers in dry states are ignored in this formulation. Patton and Mullen's representative farm had a stocking rate of 1.95 dse's/ha.

Higher livestock numbers contained in the EV^A farm plan trigger substantially different stage-2 decisions in the EV^B farm plan for each planting date relative to the DSP farm plan. Much more grain was consumed on farm, and grain and hay purchases were much higher. Although these results are partial, they do suggest that model solutions that take into account more than one climatic state tend to be better hedged. In this case, the solution also better reflected aspects of the real farm system.

6.3. Robustness of DSP and expected value farm plans

Following Di Domenica *et al.* (2009), we used *ex post* simulation to further test both the robustness of the DSP farm plan relative to the expected value farm plans by incorporating simulated crop and pasture yields for each year of the analysis.

As shown in Figure 5, for each planting date, stage-1 decisions (i.e. area of pasture, fallow and crop, and number of livestock) of the previously solved DSP and EV^A models were inserted into an annual simulation model and fixed at their optimal values. The simulation models projected pasture and crop yields based on the inputs applied in Stage 1 and actual rainfall and temperature data from 1902 to 2006. The pasture and crop yields relating to the first year of the simulation period were then inserted into the model. The model was then solved, allowing stage-2 decisions to vary in response to the combined effects of the fixed stage-1 decisions and the simulated yields of the current year. The maximum level of farm income was recorded, and the analysis continued to the next year of the simulation. This process was undertaken for the three planting dates (20 April, 10 May and 5 June all at 60 mm of soil moisture) for each year of the simulation period (1902 to 2006).

Some changes were required in order to run the *ex post* simulation. Infeasibilities occurred in implementing decisions of expected value models in

Table 1 Ten May planting date – DSP, EV^A and EV^B farm plans

		Unit	DSP farm plan	EV^A farm plan	EV^B farm plan
Net farm income					
Dry	—	$	-$39,345	—	-$117,891
Avg	—	$	$76,868	—	$87,164
Wet	—	$	$114,926	—	$136,640
EMV	—	$	$50,816	$80,087	$35,304
VSS	—	$	$15,512	—	—
	—	%	44%	—	—
Stage-1 decisions					
Pasture & Fallow	Total pasture	ha	699	642	642
	Total fallow	ha	203	184	184
Livestock	Mer. ewes – 23u	hd	0	0	0
	Mer. wethers – 23u	hd	1023	0	0
	1st × Lambs	hd	1249	2938	2938
	2nd × lambs	hd	0	0	0
	Subtotal	dse's	3985	6919	6919
	Average	dse's/ha	2.66	4.61	4.61
Cropping	LF cereal	ha	203	184	184
	LF break	ha	0	0	0
	SF cereal	ha	203	184	184
	SF break	ha	0	0	0
	CC cereal	ha	95	153	153
	CC break	ha	95	153	153
	Total crop	ha	598	674	674
	Nitrogen	kg	9195	4257	4257
Stage-2 decisions					
DRY					
Crop disposal options	H & Sell crops	t	672	—	337
	H & Store crops	t	14	—	294
	Cut crops	t	0	—	0
	Graze crops	t	45	—	184
Fodder options	Buy grain	t	0	—	206
	Make pasture hay	t	59	—	0
	Buy pasture hay	t	0	—	69
AVG					
Crop disposal options	H & Sell crops	t	1284	1159	1261
	H & Store crops	t	0	158	140
	Cut crops	t	0	0	0
	Graze crops	t	0	0	0
Fodder options	Buy grain	t	0	24	32
	Make pasture hay	t	16	48	167
	Buy pasture hay	t	0	0	0
WET					
Crop disposal options	H & Sell crops	t	1620	—	1640
	H & Store crops	t	0	—	93
	Cut crops	t	0	—	0
	Graze crops	t	0	—	0
Fodder options	Buy grain	t	0	—	19
	Make pasture hay	t	0	—	143
	Buy pasture hay	t	0	—	0

Note: CC, continuos cropping; H, harvest; LF, long fallow; SF, short fallow.

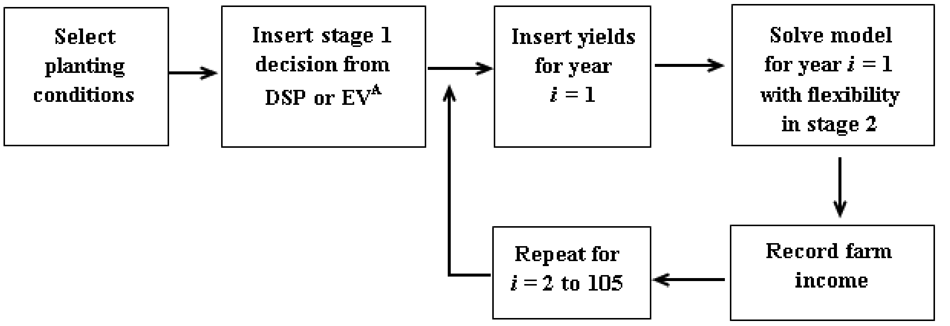

Figure 5 *Ex post* simulation of DSP and expected value farm plans.

very dry years, a further indication of the optimistic nature of EV^A. Dry years resulted in reduced crop and pasture production, creating additional demand for the purchase of grain and fodder, as well as higher demand for labour to undertake necessary supplementary feeding of livestock. To ensure a feasible solution in each year, constraints on the purchase of fodder and grain and the availability of casual labour were relaxed prior to the start of the simulation. We refer to this feasible solution again as the EV^B farm plan. In this case, the EV^B model is responding to annual climatic conditions for each of the 105 years given stage-1 decisions of EV^A, rather than the three more general states of dry, average and wet. The results of the analysis are shown in Figure 6 in the form of cumulative distribution functions of net farm income.

The cumulative distribution function of net farm income from the DSP farm plan most often lies to the right of that for the EV^B farm plan. Formal tests of the simulation results were completed using Generalised Stochastic Dominance with Respect to a Function using Simetar software (Richardson *et al*. 2008). The lower and upper risk aversion coefficients were set to zero to reflect risk neutrality. The resulting tests confirmed that the DSP farm plan stochastically dominated the EV^B farm plan for all planting dates. Dry years led to negative farm income levels and to particularly large losses in some years (e.g. see the 0.10 probability) but the DSP farm plan provides a notable improvement over the EV^B farm plan. The objective function in the DSP model reflects risk neutrality. If this assumption was relaxed to reflect risk aversion, the DSP farm plan would be even more preferred. This can be gleaned from a comparison of the cumulative distribution functions of farm profit, which show that that the DSP farm plan consistently outperforms the EV^B farm plan in dry years when either low profits or losses are incurred.

7. Conclusions

Our objective was to assess whether applying state-contingent production theory through a DSP model accounts for climate risk in a better way than expected value, stochastic production function approaches. The relative

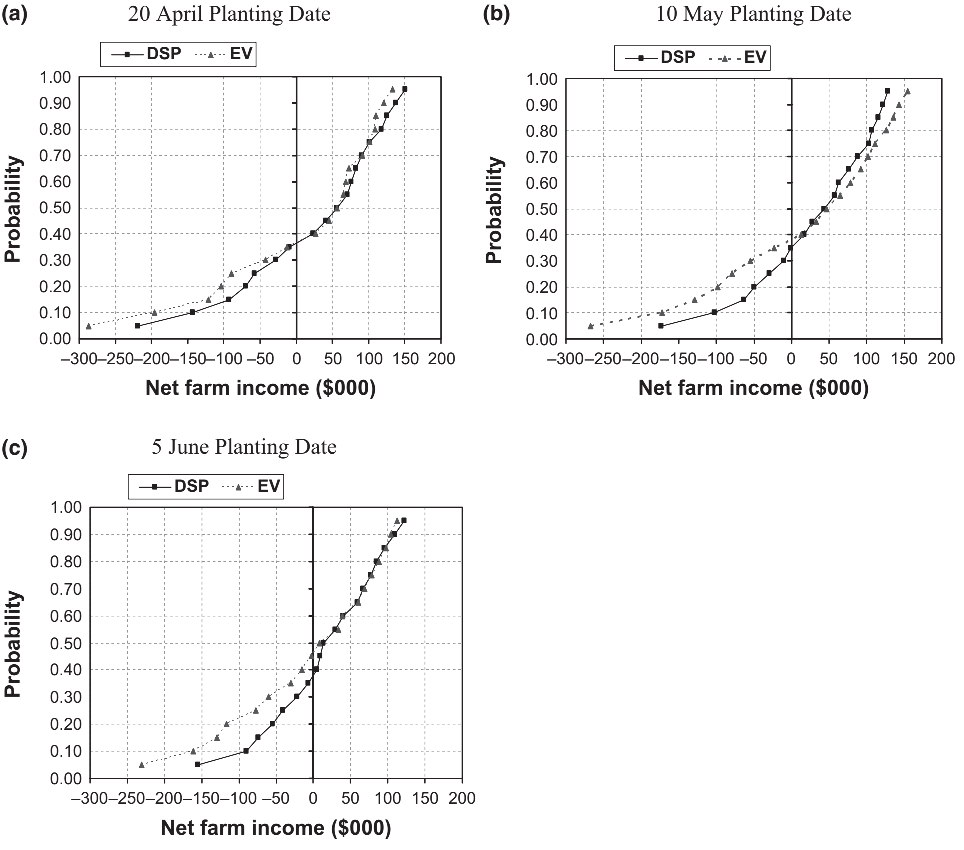

Figure 6 *Ex post* simulation results – comparison of DSP and EV^B farm plans (1902 to 2006 simulation period).

performance of the approaches was assessed in terms of differences in net farm income, and whether farm plans were consistent with both our expectations about how farmers adapt to climate risk and actual farmer behaviour.

The VSS is a measure of the benefits of adopting a DSP model and, by association, the gains from adopting a state-contingent view of uncertainty. The VSS was found to be positive over all planting dates and represented a sizeable improvement in net farm income over expected value approaches. The value of the DSP approach was further tested using *ex post* simulation. Optimal farm plans from the DSP model were found to be superior to the corresponding expected value farm plans in terms of second-degree stochastic dominance for all planting dates considered. The DSP farm plan resulted in notable reduction in downside risk and improvement in overall farm returns. The DSP farm plan was found to be better 'hedged' for uncertainty compared to the expected value approaches, with a lower overall stocking rate and generally a more diversified set of cropping and livestock activities. In this case, it also more closely resembled farm behaviour as reflected in representative farm plans for the case study area.

Many farm-level models aimed at representing agricultural production systems have incorporated risk using an expected value, stochastic production function approach. The approach provides an optimal farm plan for a single expected state, but not one that is optimal across states. An alternative approach is one that specifically incorporates future contingencies as a reasonable and realistic method of combating uncertainty, a central element of state-contingent theory. In this paper, we have shown that there are benefits of adopting a state-contingent view of uncertainty, giving support to its more widespread application to other problems.

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