Managing Food Industry Business and Financial Risks with Commodity-Linked Credit Instruments

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Abstract

This paper reviews the use and structure of commodity-linked credit instruments. It is argued that in the absence of contingent markets food firms face increasing financial risk reduced investment, and limited access to debt markets. One strategy is to issue commodity-linked credit whose payment structure is linked to the price of an underlying commodity. In some cases, a commodity-linked bond (CLB) can be structured to provide an incentive to investors by sharing in profit gains. If the goal is to hedge financial risks, CLB's can also be constructed that reduce the loan principle or coupons depending on price movements.

I. Introduction

One of the most important managerial roles facing the agriculture and food industry is the management of business risks arises from commodity price fluctuations. The prevalence of these risks affects not only capital structure decisions on the optimal balance between debt and equity but also the timing of commitments for capital investment. Unmanaged, business risks give rise to higher costs of capital and volatile share prices. To mitigate risks a number of financial instruments such as forward contracts, futures, swaps, and options have been developed and most companies have established risk management strategies. For example farmers, grain marketers and other sellers of commodities can hedge there risk by taking short positions in futures markets or purchasing put options, while buyers of commodities can hedge risk by taking long positions in futures contracts or buying call options.

The purpose of this paper is to examine a different class of structured financial products, collectively referred to as Commodity-Linked Credit Instruments, that links the payoff from a derivative such as a forward contract, futures contract, or option to the repayment covenants of a loan or bond. In section II we discuss the relationship between financing, hedging, and investment, Section III introduces the structure of a commodity-linked bond and discusses how they can be used to balance business and financial risk for the food industry. Section IV develops a general framework to support the main contentions, from a finance point of view, of sections II and III. Section V provides an overview of CLB pricing issues including convenience yield, interest rate risk, convenience yields and bankruptcy risk. Section V concludes with a general model framework based upon the work of Harrison and Kreps (1979). Section VI delves into structure issues in more depth. While the models of section V provide a framework for understanding CLBs in general, in practice firms that have issued commodity-linked bonds have used a variety of different structures. Therefore, section VI builds on the review in Attah Mensa (1992) by reconstructing the payoff or boundary conditions to a number of actual CLB products. The paper concludes in section VI.I

II. Hedging, Business Risk, and Financial Risk

Hedging strategies that affect financial decisions and capital structure have not been widely studied. On the one hand it is well known that there are agency costs associated with business risks, particularly in the pricing and access to credit and credit markets. In agriculture Collins (1985) and Featherstone, Moss, Preckel and Baker (1988) have argued that total risk is the sum of business risk plus financial risk, and financial risk is defend as the incremental increase in the variability of the return on equity due to increased debt use. From this emerges the risk-balancing hypothesis that in order to maintain constancy in total risk any increase in leverage will need to be offset by a decrease in business (operating) risk. The corollary is that any actions taken to reduce business risk, relaxes constraints that limit the use of debt. In turn agency costs associated with the imposition of price or non-price credit rationing will be reduced. Results by Turvey and Baker (1989, 1990), and Mello and Parsons (2000) show that a firm with no debt gains little from hedging its price risks because the agency costs of debt are reduced or zero. However, zero debt is often suboptimal from a shareholder value point of view.

Hedging involves a transaction that shifts risks from states in which the opportunity cost of liquidity are low to those in which the opportunity costs of liquidity are high (Mello and Parsons, 2000). In this sense the purpose of hedging is to improve liquidity, reduce financial distress and the costs of external financing, and make value maximizing investments affordable. In addition, maintained liquidity provides the flexibility to undertake and plan future investment opportunities (Mello and Parsons, 2000) and thus the value of growth options implicit in a hedging strategy can lead to higher firm value. It has been argued by Mayers and Smith (1987) and Morrelec and Smith (2003) that hedging policies also allows the firm to control the underinvestment incentives associated with debt financing by increasing the number of states of nature in which shareholders are residual claimants. This is part of the free cash flow hypothesis of Jensen (1986) that argues that share prices will generally rise as residual cash above precommitted capital expenditures rise. Hedging strategies that reduce or eliminate low or negative free cash flow states of nature can lead to increases in firm value.

The forgoing provides a strong argument that commodity-based firms need to look at their risk management, capital structure and investment decisions as a simultaneous strategy, rather than separate managerial tasks. This process of strategic risk management has started in part to the real options paradigm in which a positive NPV investment might be postponed until certain underlying uncertainties or ambiguities are resolved. If, as in the food industry, the major cause of uncertainty is commodity price risk, and commodity price risk can be managed through hedging strategies, then the option to wait can be reduced significantly by altering the project's risk profile and the smooth pasting condition. At this point one can play a thought experiment on how financing the project or firm fits into the paradigm. As argued by Turvey and Baker (1989) and Mello and Parsons (2000) the purpose of a hedge is to provide needed liquidity in those price states of nature that need them most. This in turn increases debt capacity and reduces the cost of debt. Using this debt capacity to finance the project increases free cash flow and reduces the weighted average cost of capital. Increasing free cash flow will generally increase the value of common stock (Smith 1986) and decreasing the cost of capital will increase firm value via a ceterus paribus increase in the project's net present value.

III. The Case for Commodity-Linked Bonds

In a perfect capital market hedging, financing and investment strategies can be viewed as separate transactions (Mello and Parsons 2000). For example a firm can borrow funds in one market while making hedging strategies in another. Keeping in mind the claim that hedging strategies are ineffectual for firms with no or little debt, the optimal strategy would be to link the hedging strategies with the financing strategies. In the alternative, if markets are not perfect, management may wish to issue a single credit instrument, a commodity-linked bond (CLB) for example, that combines both under one covenant. Broadly defined, these Commodity-linked bonds (CLBs) are debt instruments with a payment structure that is contingent on the outcomes of one or more underlying commodities. There are two types of commodity-linked bonds: the forward type and the option type (Attah Mensa 1992). With commodity bonds of the forward type, the coupon and/or principal payment to the bondholder are linearly related through a forward or futures contract to the price of a stated amount of the reference commodity. With commodity bonds of the option type, the coupon payments are similar to that of a conventional bond, but, upon maturity, the bondholder receives the face value plus an option to buy or sell a predetermined quantity of the commodity at a specified price. A food-based firm that faces increasing downside risks as prices fall can secure debt repayment by issuing bonds or otherwise acquiring debt by directly linking to that debt a put option on the underlying commodity. Likewise, a firm that faces the risk of rising commodity prices can mitigate financial risk by linking to its debt a call option on the underlying commodity. Should prices fall (or rise) below a trigger the bond principal would be reduced. These bonds would be issued to the market at a discount. The implicit transaction is that the issuing firm is buying a put or call option (or equivalent contingent claim) from the investor. A premium bond serves a different purpose. With these bonds the investor will share in the benefits of upside risk from the issuer. Implicitly the investor is purchasing a put (if the risk is that commodity prices fall) or call (if the risk is that commodity prices rise) option from the issuing firm. In either case the payoff from the option is used to secure the debt. On the supply side a firm would issue a CLB as part of its targeted risk management strategy, managing its balance sheet, or altering its capital structure with minimal impact on equity values. From the demand side, investors would purchase a CLB because they (usually) provide income through coupons, while allowing for diversification into commodity price risk, without being invested in commodities.

The idea of linking commodity price risk to the value of a bond is not new. During the U.S. civil war the confederate states issued war bonds with the payoff value linked to the price of cotton. If cotton prices rose the notional value of the bond also rose. The purpose of the cotton linkage was to encourage investment in the bonds. Since then a variety of structured products have been developed to either mitigate the financial risks associated with holding the bonds, or to incentivise the purchase of a bond. A recent example of a commodity-linked products can be emphasized by a new (2004) investment grade bond offered by Barclays Capital referred to as a collateralized commodity obligation. With over $800m committed, these bonds pay regular coupons but repay the principal based on the performance of a basket of commodities including gold, copper and Brent crude oil. Funds raised from the issue are used to buy floating-rate notes and

sell derivatives (known as commodity trigger swaps) on the commodities. Income from the derivatives generates income to pay for coupons on the note. The attraction of the collateralized commodity obligations is that the coupons are higher than yields on fixed income investments. The bonds are not of the option type. The bet is that the swap will yield a higher return on the commodity basket prior to maturity. If the swap generates a loss, money from the original bond sale is used to offset the loss. Losses are borne by different classes of investors. Investors can opt for more risk in principal repayment for a higher coupon, while others can risk less principal but at a lower coupon. From a risk perspective, such vehicles are attractive because commodity markets are reasonably uncorrelated with other financial markets and can therefore be used as a portfolio hedge. Furthermore, investors can buy commodity risk through an investment vehicle, which is very similar to fixed income securities [1].

The Barclay product illustrates the appetite for investment in commodity linked financial products, but these were not issued by a commodity-based firm, but rather an investment house. In contrast, Chidambaran, Fernando and Spindt (1999) describe how in 1993 and early 1994, Freeport-McMoRan Copper and Gold Inc. facing a substantial financing problem for the expansion of its Grasberg gold and copper mine in Indonesia. , the world's largest gold reserve and one of the world's largest copper reserves. Freeport-McMoRan needed to invest heavily in order to expand mine capacity and achieve the economies of scale required to be competitive. Despite a heavy debt burden and a stock that was trading below book value, Freeport-McMoRan raised $430 million through two series of gold-linked depositary shares at a cost below that of its existing debt. Chidambaran, Fernando and Spindt (1999) showed that this credit enhancement was achieved because the link to gold prices credibly reduced default risk and the associated distress cost.

Applications to agriculture and agribusiness have however been limited. Jin and Turvey (2002) illustrate how debt linked to commodities can be used by farmers, and Myers and Thompson, Myers, and Attah Mensa (1992) discuss the linking of commodity price risk to bond repayment in less developed countries. Using Costa Rica as a base case Myers and Thompson find that the country's bond portfolio should include as much as 30% commodity-linked debt. Innes (1993), offers a possible explanation of why CLB's might not have taken root in agriculture. He develops a principle-agent model to investigate the potential moral hazard when the bond repayment is contingent on firm output and firm output in turn depends on managerial effort. He argues (in proposition 6) that if prices and outputs are independent then the optimum contract is a pure commodity contract which pays the minimum of a promised payout or the actual production (a result which is generalizeable to a price contingent contract if prices are a deterministic function of supply). This would hold for most atomistic farms or firms that are price takers, but the notion of moral hazard in production of firms whose processing or marketing activities can influence market prices may require a moral hazard premium that markets are not willing to bear. On the other hand O'Hara (1984, 1990) demonstrates that conventional loans can be pareto dominated by financial contracts explicitly incorporating characteristics of the borrower's product market, identifies conditions under which commodity-linked debt is desirable, when more complicated revenue-linked loans

[1] By Dan Wilchins http://yahoo.reuters.com/financeQuoteCompanyNewsArticle.jhtml?duid=mtfh63325_2004-12-07_23-12-00_n07291675_newsml

are optimal, and how the type of lending contract can have real effects on production decisions.

Firm solvency is also an issue. With limited liability a firm cannot be compelled to pay more than its contingent profits. In this case, the optimum contract according to Innes (1993) is a pure bond with no price linkages. Schwartz (1982) examines a related issue that gives investors a claim to the assets of the firm in the case of bankruptcy, but the cost of the bond (i.e. its discount) will increase with the probability of bankruptcy.

The use of commodity linked bonds is inexplicably tied to firm value and capital structure decisions. In regards to Commodity-Linked Bonds Miralloc and Smith (2003) have examined how firms jointly determine financing, hedging and investment decisions. They argue that optimum leverage reflects the tradeoff between under and over investment and show that hybrid debt financing comprised of a commodity linked bond of the forward type (i.e. with a linked forward contract) reduces agency costs and incentives to over and under investment, and increases firm value relative to standard debt financing. This implies that one of the advantages to issuing a CLB is that it precommits the firm to a hedging strategy for the life of the bond in an environment where covenants to using futures, options or swaps are prohibitive. The intuitive logic behind this idea is that by precommiting to a hedging strategy within the covenants of a bond, ensures that the investors are compensated for whatever risks are implied by the hedge. Without such precommitments bondholder covenants are designed to ensure that speculation in uncorrelated risks are limited by restricting all types of derivatives strategies.

One of the criticisms of CLBs, at least for tradeable securities, is that the CLB can be constructed from issuing a bond and placing a separate hedge on the price risk. One might ask why CLBs need to be issued at all! Part of the answer, as discussed above, is that covenants on the bond issue may prohibit taking positions in forward or derivative markets. However, Morellec and Smith (2003) also provide an argument by looking at what happens to firm value if shareholders unwind the commodity part by taking a long position in the forward contract. They show that while such a strategy will transfer cash from bondholders to shareholders, it will fail to maximize shareholder value because of a loss of control of deviations from investment policy, and that unwinding increases the price at which the firm defaults (increasing the probability of default).

IV. The Conceptual Frameowrk

We define financial risk as the incremental increase in the variability of the return on equity due to financial leverage. We assume, for purposes of ease, that there is an identifiable mapping between the probability distribution of a commodity price (traded or not) and the return on assets, $\int_{l}^{P^*} f(P)dP \rightarrow \int^{ROA^*} g(ROA)dROA$ and this is the sole source of business risk. Furthermore, the interest rate charged on loans or bonds will generally increase as either leverage, business risk, or both increase. We further define the ROE according to

$$(1) \qquad R\tilde{O}E = R\tilde{O}A\left[\frac{A}{E}\right] - i\left(\sigma_{ROA}, \frac{D}{E}\right)\left[\frac{D}{E}\right]$$

with expectation

(2) $$E[ROE] = E[ROA]\left[\frac{A}{E}\right] - i\left[\frac{D}{E}\right],$$

variance,

(3) $$\sigma^2_{ROE} = \sigma^2_{ROA}\left[\frac{A}{E}\right]^2$$

and standard deviation

(4) $$\sigma_{ROE} = \sigma_{ROA}\left[1 + \frac{D}{E}\right].$$

Because commodity prices represent the sole source of variance in ROA, it is the sole source of business risk. For an unlevered firm, the variability in ROA will be the same as the variability in ROE. However as debt (D) rises relative to equity (E) variance in ROE rises. Thus total risk, the sum of business risk and financial risk, is captured by the variability in equity returns. To gain some insights into the relationship between the underlying business risk and debt, we totally modestly expand the model of Collins and Featherstone et al to examine the optimum debt structure. Assuming negative exponential utility and absolute risk aversion, λ, and defining the D/E ratio as δ, the expected utility of equity returns is given by

$$EU(ROE) = ROA(1+\delta) - i(\delta)\delta - \frac{\lambda}{2}\sigma^2_{ROA}\left[1+\delta\right]^2$$

(5) $$\delta^* = \frac{ROA - i(\delta) - \lambda\sigma^2_{ROA}}{\lambda\sigma^2_{ROA} + \frac{\partial i(\delta)}{\partial \delta}}$$

(6) $$EU(ROE) = ROA(1+\delta) - i(\delta)\delta - \frac{\lambda}{2}\sigma^2_{ROA}\left[1+\delta\right]^2$$

differentiate (6) and rearrange to get

(7) $$\delta^* = \frac{ROA - i(\delta) - \lambda\sigma^2_{ROA}}{\lambda\sigma^2_{ROA} + \frac{\partial i(\delta)}{\partial \delta}}$$

Clearly the optimal proportion of debt decreases with increases in business risk, and more so through increases in the interest rate. There is a simultaneity here that can be addressed through a structured CLB product, namely since the marginal change in interest rate is a function of business risk, then any actions to reduce business risk will allow for a risk-balanced increase in debt. Looked upon another way, totally differentiate (4) to find the direct relationship between debt and commodity price risk:

(8) $$\frac{dD}{d\sigma_P} = -\left[\frac{A\frac{\partial \sigma_{ROA}}{\partial \sigma_P}}{\sigma_{ROA}}\right] = -\left[\frac{A}{\sigma_{ROA}}\right]\frac{\partial \sigma_{ROA}}{\partial \sigma_P}.$$

This is not an uncommon results, and similar results can be found in the models of Collins (1985) and Featherstone, Moss, Preckel and Baker (1988). Essentially the result states that as underlying commodity price risks rise, there will be a decrease in debt in order to keep the change in total risk constant. This has been referred to as the risk balancing hypothesis. Essentially the economic argument is that as business risk increases there must be a concomitant reduction in debt. This can also be stated in elasticity form as

$$(9)\quad \varepsilon = -\left[\frac{A}{\sigma_{ROA}}\frac{\sigma_P}{D}\right]\frac{\partial \sigma_{ROA}}{\partial \sigma_P} = -\left[\left[\frac{\sigma_{ROA}}{\sigma_P}\right] \Big/ \left[\frac{D}{A}\right]\right]\frac{\partial \sigma_{ROA}}{\partial \sigma_P}.$$

For a 1% increase in price risk. The reduction in debt depends on the ratio of business risk to price risk, the debt to asset ratio and the marginal sensitivity of business risk to price risk.

We can also make an appeal to the Capital Asset Pricing Model to motivate the problem. It is well known that the estimated beta for stock of a leveraged firm will be higher than that of an unlevered firm with the same amount of systematic risk. Therefore, the security market line equation

$$(10)\quad E\left[ROE\right] = r_f + \beta_u\left(1+\frac{D}{E}\right)\left[r_m - r_f\right]$$

with r_f and r_m being the risk free and market rates of return respectively, and β_u the unlevered beta. The significance of this relationship is to indicate that as debt increases relative to equity the systematic risk increases as does the expected return on equity. This has two related impacts. First an increase in the market ROE increases the weighted average cost of capital which reduces the net present value of investments and second, an increase in equity returns and risk can lead to a reduction in share prices.

We can also examine the relationship between business risk, financial risk , cash flow and free cash flow. Net income is given by

$$(11)\quad E[ROE]^* E = E[ROA]A - iD$$

and by adding depreciation and deducting precommitments to investment I*, free cash flow can be defined as

$$(12)\quad FCF = \left(E[ROA]A - iD + x\right) - I^*$$

with the bracketed term representing operating cash flow. Under the Jenson (1986) free cash flow hypothesis firm value will increase with unexpected increases in the payout to corporate claimholders. Thus an increase in free cash flow suggests, perhaps, additional dividends to shareholders. This will result if operating income increases or debt decreases.

We now deduct the cost of a precommited hedging strategy $\omega(\Omega)$ where Ω represents the strategy. Substituting $\omega(\Omega)$ into free cash flow and solving gives the critical ROA for which free cash flow is zero and below which free cash flow is negative.

$$(13) \qquad ROA^* = \left[\frac{I^* + iD - \omega(\Omega) - x}{A} \right]$$

Using (13) the variance of cash flow, and free cash flow is given by

$$(14) \qquad \sigma^2_{FCF} = A^2 \sigma^2_{ROA}$$

which again is determined by the business risk. This along with E[ROA] defines the probability distribution for cash flow. The expected value of ROA conditional on the commodity price is then

$$(15) \qquad \int ROA(P) f(P \mid \Omega) dP = \int^{P^{-1}(ROA^*)} ROA(P) f(P \mid \Omega) dP + \int_{P^{-1}(ROA^*)} ROA(P) f(P \mid \Omega) dP$$

where $f(P \mid \Omega)$ is the probability distribution of prices received conditional on the hedging strategy and $P^{-1}(ROA^*)$ is the critical price that gives ROA^*. The first integrand on the right hand side describes states for which free cash flow is negative. Over this domain, cash flow can only be restored by increased borrowing or raising cash from new equity. In this range share value will diminish. The second integrand of the right hand side describes states of nature for which free cash flow is increasing. Over this range management can increase dividends or repurchase shares. Prices will increase.

The hedging variable Ω is a moderating variable. Its existence will reduce the variability and downside risk in ROA. If it is a hedge with a futures contract or a forward contract variance will be reduced on both the upper and lower tails. If agents are risk neutral, hedging might not affect share value at all, but if agents are risk adverse the marginal utility of a decrease in downside risk will more valuable than the disutility of lower upside potential and in this context share values may increase with the hedge. If Ω represents an option strategy then the lower probability tail will be truncated at some $P^{-1}(ROA^*)$=P*. Assuming in this case that

$$(16) \qquad \omega(\Omega) = e^{-rT} \int^{P^*} (P^* - P) f(P) dP$$

fairly prices the transference of downside risk, and ω(Ω) is a precommited expense, the truncation of downside risk increases the expected value of free cash flow and share prices will increase.

With these characteristics the problem faced by a commodity-based food firm becomes self evident when it wants to issue bonds to finance investments: In order to increase debt it must somehow take actions to reduce business risk. While the relationship between hedging price risks and financial leverage have been discussed previously (Turvey and Baker 1989) the idea of linking price risks as part of a structured debt product has not. A review of the academic literature is presented in the next section. This is followed by a detailed review of a number of structured products that have appeared as either an idea in the academic literature or implemented in practice.

V. The Theoretical Framework for Commodity-Linked Bonds

The academic literature has focused primarily on theoretical models of arbitrage priced to equilibrium. In most cases the standard model is provided with one or more of the following factors included: commodity price risk, interest rate risk, convenience yield risk, and firm value or bankruptcy risk. In practice, however, the structured models do not normally conform to what is described in theory and the theoretical models should more generally be considered as providing guiding principles. The standard model for pricing commodity-linked securities uses the option pricing framework as pioneered by Black and Scholes (1973) and extended by Merton (1973) and Cox and Ross (1976). Schwartz (1982) provides a general framework for valuing CLBs. He considers the underlying commodity price risk, default risk, and interest rate risk and provides a second–order partial differential equation in four variables that governs the value of CLBs at any point in time. Schwartz states that the solution to the general problem, subject to certain boundary conditions, is difficult even by numerical methods, and he only provides some closed form solutions in special cases. In his paper there is no discussion about convenience yields. Carr (1987) derives a closed form pricing formula for a commodity-linked bond where the bond prices follows a third order geometric Brownian motion (lognormal) without referring to the interest rate process. His formula encompasses Schwartz's solution as a special case. However he leaves out the convenience yield from his model.

Gibson and Schwartz (1990) consider stochastic convenience yields for the valuation of commodity derivatives. Assuming that the price of the underlying commodity has a lognormal stationary distribution and net marginal convenience yields follow the mean reverting pattern, they derive the partial differential equation for the price of commodity derivatives defined as functions of the underling commodity spot price and the net marginal convenience yields. They also estimate the parameters for the behavior of the net marginal convenience yield from market data, and then calculate numerically the futures price of the commodity. Schwartz (1997) extends this model by introducing a third stochastic factor, the instantaneous interest rates. Hilliard and Reis (1998) extend this three factors model by introducing jumps in the spot price of the commodity and by using the term structure of interest rate to eliminate the market price of interest rate risk in their fundamental price equation. Milterson and Schwartz (1998) develop a new arbitrage model that includes three factors: the spot price of the commodity, forward interest rates and convenience yields. They address issues about the forward interest rates and convenience yield based on the Heath-Jarrow-Morton (HJM 1992) framework and obtain closed form solutions for options on commodity futures as well as commodity forwards in the Gaussian case. None of the above models actually considers the default of the issuing firms as the commodity contingent claims mature.

Atta-Mensah (1992) established a general model for pricing CLBs, which includes four factors: the spot price of the underlying commodity, firm value, interest rates, and convenience yields. He follows Schwartz (1982) to derive a second-order partial differential equation as well as the boundary conditions. Recently, Miura and Yamauchi (1998) assume that both the spot price of the commodity and firm value

follows geometric Brownian motions and that the net marginal convenience yield and interest rate follow mean-reverting processes and then price CLBs. They derive the partial differential equation, which must be satisfied by the bond and obtain the mathematical formula for the price of CLBs. Evnine (1983) extends the Cox, Ross, and Rubinstein (1979) option pricing model to incorporate an option on two or more stocks. Rajan (1991) applies Evnine's (1983) model to CLBs, prices CLBs in the presence of default risk and commodity price risk, and then compares his results to Schwartz's (1982) results.

In recent years the no-arbitrage model has been used to deal with the interest rate when pricing financial products (Hull, 2000). Ho and Lee (1986) present a no-arbitrage model and Hull and White (1990) extend the Ho and Lee (1986) model. Heath, Jarrow, and Morton (HJM) (1992) develop a general no-arbitrage model based on several factors and derive the relationship between the drift and standard deviation of an instantaneous forward rate.

Kaldor (1939) first gives the definition for the convenience yield of a commodity in the economic literature, and then offers an explanation of the relationship between the spot and future prices of a commodity. Later many scholars such as Working (1948), Brennan (1958), and Frechette (1997) reached the conclusion that the valuation equation for derivatives that are indexed to a commodity must take account of the convenience yield of the commodity linked to it. Fama and French (1987) provide evidence that the marginal convenience yield varies seasonally for most agricultural and animal products, and furthermore they find evidence of a mean-reverting process for metals' convenience yield using futures data from the London Metals Exchanges (LME). Gibson and Schwartz (1990) find that a constant convenience yield assumption did not work well for pricing oil indexed bonds and argued that convenience yield needs to be explicitly considered in modeling any meaningful valuation model. For pricing commodity–linked bonds, Ingersoll (1982) points out that the convenience yield should be considered when pricing CLBs. Atta-Mensah (1992) assumes that the convenience yields follow the Brownian motion in his model. Miura and Yamauchi (1998) postulate that the convenience yields follow a mean-reverting process to value CLBs. Miltersen and Schwartz (1998) use the HJM approach for interest rates and convenience yields to value the options on commodity futures.

The models developed to date are complex. Commodity-linked bonds as described above do not consider advanced features such as bankruptcy risk, stochastic interest rates or convenience yields. Bankruptcy risk was considered If there is a risk of bankruptcy (Schwartz (1982), Carr (1987) Miura and Yamauchi(1998)) then bondholders receive max of firm value or bond value, i.e.

$$(17) \qquad B(,T) = Min\left[V_T, F + Max(0, P_t - K)\right]$$

Also complicating pricing issues is the fact that the Value of bond = present value of coupons plus present value of payoff. In other words interest rates and convenience yields convenience yields are not constant over time (Milterson and Schwartz, 1998). Several approaches to including interest rates and convenience yields into a CLB problem have been presented in the literature based on either Brownian motion or HJM framework. Following the structure outlined in Milterson and Schwartz (1998), Harrison & Kreps

(1979) and Harrison & Pliska (1981) the idea is to calculate the discounted expected value of payment directly

$$(18) \qquad B(.,0) = E[\int_0^T e^{-\int_0^v f(s,s)ds} cdv] + E\left[e^{-\int_0^T f(s,s)ds} \min\left[V(T), F + \max(0, P_T - K)\right]\right]$$

where $\int_0^v f(s,s)ds$ v=t,T is the adjusted discount rate which can include convenience yield and interest rate risk under a variety of assumptions. In the simplest of cases with zero default risk (exclude V(T)), a nonstoreable commodity (no convenience yield) and no interest rate risk the value of a commodity-linked bond with constant coupon rate is given by

(19)

$$B(.,0) = \frac{c}{r}(1 - e^{-rT}) + Fe^{-rT} + S(0)N(\frac{\ln\frac{S(0)}{K} + rT + \frac{1}{2}\sigma_3^2}{\sigma_3}) - Ke^{-rT}N(\frac{\ln\frac{S(0)}{K} + rT - \frac{1}{2}\sigma_3^2}{\sigma_3})$$

This expression is exactly the same as the results obtained by Schwartz (1982) and Atta-Mensah (1992) and is similar to the one used in Jin and Turvey (2002). To reduce this further, note that if F = 0, c = 0 (no bond), and interest rate =r (constant), then P (0, T) = exp(-r*T) and (19) reduces to the standard Black (1976) model for pricing options on futures. In other words the simplest of structured products is simply the sum of the present value of the cash flow from the bond investment plus the option value of the commodity linkage.

VI. Commodity-Linked Bonds in Practice

As indicated above, not all commodity –linked bonds follow the structure as described above. In practice, the types of CLBs that have been considered or issued have been varied. To gain a sense of the structure of these financial products, this section provides a review of various products described in the literature. For the most part he products in this section were described in Attah Mensah (1992) without explicit solutions or statements of the boundary conditions. Therefore, what follows is based on reconstructing these conditions based upon his written descriptions. In all cases it is assumed that interest rates are deterministic, bankruptcy risk is negligible, and convenience yields are zero (or at least deterministic).

In 1981, Inco issued a $CDN90m bond that was linked to the price of copper and nickel. The bond had a fixed coupon rate of 10% per year but at maturity the face value or monetary value of the bond was tied to a specified amount of copper or nickel. The basic structure of this bond's terminal value is given by

$$(20) \qquad B(,T) = c + F + \frac{F}{P_0} Max\left[P_T - P_0, 0\right]$$

Where F is the face value of the bonds, c, is the coupon, P_o is the initial price of the underlying commodity and P_T is the price at expiration. The value of the bond is linked to a call option which allowed investors to share in any gains in the market value of the

commodity between t=0 and t=T. However, if prices fell, investors were shielded from losses in principle.

An example of a CLB of the forward type was proposed by Myers and Thompson who examined the issuance of commodity linked bonds for LDC debt. The bond they propose rises or falls depending on the value of an underlying commodity. In general the terminal value of a zero-coupon bond is given by

$$(21) \qquad B_{forward}(,T) = F + \frac{F}{P_0}(P_T - P_0)$$

which is equivalent to a hedged position with a long forward contract. If prices rise in the LDC then the terminal value of the bond increases, but when prices fall the amount to be repaid falls.

In 1980 the Sunshine Mining Company issued 15-year silver-linked bonds with each $1,000 of par linked to 50oz of silver and a coupon rate of 8.5%. The bonds had a special feature in that they were redeemable only if the price of silver was in excess of $U.S. 40/oz in 1995. In other words, the redemption value was zero if prices fell below $40 with probability ρ. The expected value of this bond at T was

$$(22) \qquad B(,T) = c + \rho(P_T < \hat{P})0 + (1 - \rho(P_T < \hat{P}))Max\left[F, x[\frac{F}{x} + Max[0, P_T - \frac{F}{x}]\right]$$

where x (x=50) is a quantity reference. For x=50 the initial reference price was $20/oz ($1,000/ 50 oz). The option gain Max(0, P_T-20) would only occur if P_T>40. At expiration its expected value, including the $85 coupon was given by

$$(23) \qquad B(,T) = 85 + \rho(P_T > 40)Max\left[1,000, 50\left[\frac{1,000}{50} + Max[0, P_T - \frac{1,000}{50}\right]\right].$$

In other words, if the price was less than $40/oz the expiration payoff would include only the coupon.

A similar type of protected bond was a $U.S.20m petroleum bond issued in 1981 by the Petro-Lewis corporation. These were 5-year bonds with a 5% coupon. At maturity the notes offered a capped share in price gains. Based on 18.5 barrels of crude oil, a shared gain occurred if prices increased between $38 and $68/barrel. The general form is given by

$$(24) \qquad B(,T) = c + F + Max\left[0, Min\left[x_{oil}(P_T - P_0), x_{oil}(P_{\max} - P_0)\right]\right]$$

or after substituting for known values

$$(25) \qquad B(,T) = c + F + Max\left[0, 18.5Min\left[(P_T - 36.16), (68 - 36.16\right]\right]$$

A different type of 3-year petroBond was issued by the Government of Mexico in the late 1970's. Each 1,000 peso bond was linked to 1.95354 barrels of oil, and each bond had a coupon of 12.65823%. At maturity the holder received either the face value of the

bonds (F=1,000 pesos) or the value of the reference plus all coupons. The general form is given by

$$(26) \qquad B(,T) = c + Max\left[F, x_{oil} P_T + \sum_1^3 c_t \right]$$

or upon substitution of known values the terminal payment was

$$(27) \qquad V(,T) = 126.5823 + Max\left[1{,}000, 1.95354 P_T + \sum_1^3 126.5823 \right].$$

Funds have also been raised using commodity-linked preferred shares. Cominco Ltd raised $US $54M in preferred shares. Each preferred share had a warrant convertible into common equity based upon share prices and the price of metals. The warrant is given by

$$(28) \qquad B(,t) = \eta \frac{V_{S,t}}{V_{P,t}}$$

And the value of the preferred shares would be described by

$$(29) \qquad V(,t) = V_{pref,t} + V_{S,t}\left[\eta \frac{V_{S,t}}{V_{P,t}} \right]$$

Similarly, Echo Bay Mines issued gold indexed securities in the form of $1,550,000 preferred voting shares. Each share was comprised of a $U.S. 3.00 coupon plus four gold warrants. Each warrant was linked to 0.0706 ounces of gold at $US 595/0z.

$$(30) \qquad B(,T_1,T_2,T_3,T_4) = x_{oz\_gold}(Max(P_{T1} - P_0, 0) + Max(P_{T2} - P_0, 0) + Max(P_{T3} - P_0, 0) + Max(P_{T4} - P_0, 0))$$

An alternative means of linkage is through the coupon rate. In 1988 Magna Corp issued 10-year copper indexed notes with the coupon rates linked to the price of copper. In this case the coupons were a random variable based upon the underlying value of the index I;

$$(31) \qquad \tilde{c} = cf(I,t)$$

Assuming a Brownian motion in I,

$$(32) \qquad dI = I\alpha dt + I\sigma dw$$

then by Ito's lemma

$$(33) \qquad \tilde{c} = cI[\alpha dt + \sigma dw]$$

with expected value

$$(34) \qquad E[c_T] = cE[\alpha]T$$

and variance

$$(35) \qquad Var[c_T] = c^2\sigma^2 T$$

The expected value of the bonds for any t was then given by

$$(36) \qquad V(,t) = E[c] + Fe^{-rT}$$

In another example the French government in 1973 issued 7% gold linked bonds with the redemption value indexed to the price of 1kg of gold. Each bond had a safeguard clause that both the coupon and the face value would be indexed if the Franc lost parity with gold. In this instance the payoff was of the forward type. Let $f(I,t)$ define the index-based safeguard clause. Then the terminal payoff value can be described by

$$(37) \qquad B(,T) = c + F + Ff(I,T)$$

which can be written more generally as

$$(38) \qquad B(,T) = c + F + \phi(P_T - P_0).$$

The payoff is even specific and is based on the probability of a specific event occurring in order to trigger the safety clause.

$$(39) \qquad B(,T) = c + \rho(t \le T)F + (1 - \rho(t \le T))Max(F, Ff(I,T))$$

But the safety clause also affected the coupon rate in the following way.

$$(40) \qquad \tilde{c} = \rho(t \le T)c + (1 - \rho(t \le T))cf(I,t))$$

so that the coupon payment was a random variable expressed as

$$(41) \qquad \tilde{c} = \rho(t \le T)c + (1 - \rho(t \le T))Max(c, cf(I,t))$$

Jin and Turvey (2002) provide one of the first examples dealing with agriculture. In their model they consider the value of an option linked to the value of a farm loan or mortgage. Defining c as the amortized, periodic, loan payment the payment schedule in each period is given by

$$(42) \qquad \begin{aligned} B_{call}(,t) &= c - \frac{c}{P_{t-1}} Max(P_t - P_{t-1}, 0) \\ B_{put}(,t) &= c - \frac{c}{P_{t-1}} Max(P_{t-1} - P_t, 0) \end{aligned}$$

In this formulation the option value has a random strike price based upon the price received in the preceding period, so the option value changes from year to year even if the amortized payment does not.

VII. Discussion

There are several considerations described in the preceding theory and review. First, all food firms face risks in the procurement or sales of their products. Theory suggests that in the absence of contingent markets these firms would face increasing financial risk and reduce investment in capital. Furthermore, in the absence of contingent markets, the volatility in commodity prices may be sufficiently high to reduce access to debt markets. Under these conditions an alternative strategy is to issue commodity-linked

bonds whose payment structure is linked to the price of an underlying commodity (or basket of commodities).

This paper has argued that commodity-linked credit can achieve these goals for food firms ranging from primary agriculture to marketing and warehousing and food processing. The structure of the proposed products can be varied. In some cases, a commodity linked bond can be structured to provide an incentive to investors by sharing in any profit gains from price movements. These bonds would normally be sold at a premium with the firm in essence selling an option to the investors. But if the goal is to hedge financial risks, CLB's can also be constructed that reduce the loan principle or coupons depending on price movements. Depending on the underlying commodity risks, these would normally be sold at a discount. Essentially the firm is purchasing an option from the investors.

For simple CLB's the payoff function can be constructed by using market based instruments, as long as any bond or loan covenants do not restrict derivatives investment. But these markets are sometimes not perfect and bundling the bond and commodity risk management can be an optimal strategy. In addition, CLBs can offer significant flexibility in risk management. Conceptually, firms trading in many commodities may want to construct a CLB with the payout based on a bundle of commodities to better match inventories and product lines, or if the main risk is volumetric rather than market, firms can issue weather-linked bonds.

This paper was developed at the conceptual level and did not provide any estimates of bond prices. Jin and Turvey (2002) provide some estimates for a simple structure. Nonetheless, for most CLB's the mechanics of pricing can be quite difficult and would require using Monte Carlo approaches. Closed form solutions are very difficult to obtain. In addition, this paper did not explore linking to commodities that are not traded. In that case the commodity part would require explicit consideration of the market price of risk

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