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# Export-led growth in Tunisia: A wavelet filtering based analysis

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In this paper, we use a wavelet filtering based approach to study the econometric relationship between exports, imports, and economic growth for Tunisia, using quarterly data for the period 1961:1-2007:4. GDP is used as a proxy for economic growth. We explore the interactions between these primary macroeconomic inputs in a co-integrating framework. We also study the direction of causality between the three variables, based on the more robust Toda-Yamamoto modified Wald (MWALD) test. The much-studied relationship between these three primary indicators of the economy is explored with the help of the wavelet multi-resolution filtering technique. Instead of an analysis at the original series level, as is usually done, we first decompose the variables using wavelet decomposition technique at various scales of resolution and obtain relationship among components of the decomposed series matched to its scale. The analysis reveals interesting aspects of the interrelationship among the three fundamental macroeconomic variables.

**JEL Classifications:** C01, C02, C32

**Keywords:** Export, economic growth, cointegration, wavelet filtering, causality

## Introduction

The role of export to improve the growth potential of a country occupies the center stage in especially development literature where export promotion and increased openness gradually have replaced import substitution to enhance growth. This shift from import substitution to export promotion and increased openness implies as well a shift in the trade and industry policy from being highly import substituting and government controlled to become more liberalized and deregulated. This shift in policies has also been central in policy recommendations to developing countries concerning improvements of their growth potential. An increased openness to trade will enhance competition for firms producing for the international market. Such an environment generates incentives for an increased productivity and incentives for innovations as well as the possibility to pay higher wages in line with the increased productivity. Furthermore, an increased openness to trade is also central in international negotiations about trade and tariff barriers where trade theory suggests that all parties on aggregate will enhance their welfare position in relation to their autarky situation. An export-led growth (ELG) hypothesis which states that exports are keys to promoting economic growth provides one of the answers to this fundamental question. There is a considerable literature that investigates the link and causation between exports and economic growth, but the conclusions still remain a subject of debate.

A number of empirical studies have documented a strong and positive relationship between export and economic growth including Balassa (1978), Jung and Marshall (1985), Chow (1987), Ahmad and Kwan (1991), and Moosa (1999) among others. The results reveal evidence in support of the export-led growth hypothesis for various countries. Several studies have also shown that it is possible to have growth-led exports (GLE)

which has the reverse causal flow from economic growth to exports growth. In the GLE case, export expansion could be stimulated by productivity gains caused by increases in domestic levels of skilled-labour and technology (Bhagwati, 1988; Krugman, 1984). The third alternative is that of import-led growth (ILG) which suggests that economic growth could be driven primarily by growth in imports. Endogenous growth models show that imports can be a channel for long-run economic growth because it provides domestic firms with access to needed intermediate factors and foreign technology (Coe and Helpman, 1995). Growth in imports can serve as a medium for the transfer of growth-enhancing foreign R&D knowledge from developed to developing countries (Joy, 2001).

In this paper, we use a wavelet filtering based approach to study the econometric relationship between exports, imports and gross domestic product (GDP). We explore the interactions between these primary macroeconomic indicators in a co-integrating and vector autoregression framework and their dynamic causality under the augmented level VAR setup. The much studied relationship between these three primary indicators of the economy is explored with the help of the wavelet multi-resolution filtering technique. Instead of an analysis at the original series level, as is usually done, we first decompose the variables using wavelet decomposition technique at various scales of resolution and obtain relationship among components of the decomposed series matched to its scale. The analysis reveals interesting aspects of the interrelationship among the three fundamental macroeconomic variables.

This paper contributes to the literature on the export-out growth nexus in the following ways. First, previous studies on the dynamic linkages between exports and economic growth are extended through the application of wavelet transform and through the application of recent advance in time series statistical technique (augmented level VAR modeling with integrated and co-integrated process of arbitrary orders (Toda and Yamamoto, 1995; Dolado and Lutkepohl, 1996)). In addition to employing recently developed time series modeling techniques, this study also expands on the three variables to include exports, imports and economic growth.

The rest of the paper is organized as following. In the following section we discuss the analytical framework and methodological issues, while the subsequent sections present empirical findings and summarize the paper's findings.

## Analytical framework and methodological issues

### Background on Wavelets

In this section we give a brief exposition of the relevant aspects of wavelet theory, without going into deeper detail about the mathematical involved. For precise mathematical statement we refer the reader to Meyer (1990), Mallat (1989), Daubechies (1992), Chui (1992), Percival and Walden (2000), Genacy et al. (2002) and Nason (2000).

Wavelets are building block functions and localized in time or space. They are obtained from a simple function  $\psi(t)$ , called the mother wavelet, by translations and dilations. The wavelet  $\psi_{j,k}(t)$  is obtained from the mother wavelet by shrinking by a factor of  $2^j$  and translating by  $2^j k$  to obtain

$$\psi_{j,k}(t) = 2^{j/2} \psi(2^j t - k) \quad (1)$$

Except in some special cases, there is no analytic formula for computing wavelet functions. To evaluate a wavelet function, use the dilation equation

$$\varphi(t) = \sqrt{2} \sum_k l_k \varphi(2t - k) \quad (2)$$

where  $\varphi(t)$  is the so called scaling function (or father wavelet), satisfying  $\int_{\mathbb{R}} \varphi(t) dt = 1$ . We can obtain the mother wavelet  $\psi(t)$  from the father wavelet through

$$\psi(t) = \sqrt{2} \sum_k h_k \varphi(2t - k) \quad (3)$$

with  $h_k = (-1)^k l_{1-k}$ , called the quadrature mirror filter relation, where the coefficients  $l_k$  and  $h_k$  are the low-pass and high-pass filter coefficients defined as

$$l_k = \sqrt{2} \int_{\mathbb{R}} \varphi(t) \varphi(2t - k) dt \quad (4)$$

$$h_k = \sqrt{2} \int_{\mathbb{R}} \psi(t) \varphi(2t - k) dt \quad (5)$$

For the wavelet series representation of a function, we expand the function in terms of some orthonormal base different from the trigonometric base. The standard wavelet series representation in terms of basis functions is given by:

$$f(t) = \sum_k c_{j_0,k} \varphi_{j_0,k}(t) + \sum_{j \geq j_0} \sum_k d_{j,k} \psi_{j,k}(t) \quad (6)$$

where  $c_{j_0,k}$  and  $d_{j,k}$ 's are the coefficients of the wavelet series representation given by:

$c_{j_0,k} = \int_{\mathbb{R}} f(t) \varphi_{j_0,k}(t) dt$  and  $d_{j,k} = \int_{\mathbb{R}} f(t) \psi_{j,k}(t) dt$ ;  $j_0$  is called the *coarsest scale* of the wavelet representation,  $\varphi_{j_0,k}(t)$  is an orthonormal basis function given by

$$\varphi_{j_0,k}(t) = 2^{j_0/2} \varphi(2^{j_0/2} t - k) \quad (7)$$

Wavelet decomposition is usually obtained using an algorithm referred to as the Mallat's Pyramid algorithm (Mallat, 1989). This algorithm due to Mallat consists of a sequence of application of low-pass and high-pass filters. The procedure starts with the data  $c_0 = (c_{0,0}, \dots, c_{0,T-1})$ , where  $c_{0,i} = X_i$ ;  $i = 0, 1, \dots, T-1$ . In the  $j^{\text{th}}$  step, the algorithm computes  $c_{j,k}$  and  $d_{j,k}$  from the smooth coefficients of level  $j-1$ ,  $c_{j-1,k}$  through

$$c_{j,k} = \sum_n l_{2k-n} c_{j-1,n} \quad (8)$$

$$d_{j,k} = \sum_n h_{2k-n} c_{j-1,n} \quad (9)$$

### Unit root test

A unit root test tests whether a time series variable is non-stationary using an autoregressive model. The most famous test is the Augmented Dickey-Fuller test (Dickey and Fuller, 1979). Another test is the Phillips-Perron test (Phillips and Perron, 1988). The ADF and PP unit root tests are for the null hypothesis that a time series  $y_t$  is  $I(1)$ . Stationarity tests, on the other hand, are for the null that  $y_t$  is  $I(0)$ . The most commonly used stationarity test, the KPSS test, is due to Kwiatkowski, Phillips, Schmidt and Shin (Kwiatkowski et al., 1992).

### Cointegration analysis

The second stage involves testing for the existence of a long-run equilibrium relationship between real exports, real imports and GDP within a multivariate framework.

Further, we explore a co-integrating relationship among the variables. We first consider co-integration testing in a univariate time series setup. If a time series  $Y_t$ , with no deterministic component, can be represented by a stationary and invertible ARMA process after differencing  $d$  times, the series is said to be integrated of order  $d$ , i.e.,  $Y_t \sim I(d)$ .

Furthermore, if all elements of a vector time series  $Y_t$  are  $I(d)$  and there exists a vector  $\beta \neq 0$  such that  $\beta^T Y_t \sim I(d-b)$  for any  $b > 0$ , the vector process is said to be co-integrated  $CI(d, b)$ , with  $\beta$  as the co-integrating vector. A special case of  $b = d = 1$  is of importance in analysis of economic time series, as this implies long run equilibrium (stationary) relationship among the variables involved in co-integration. We can infer that in such a situation, in the long run, the  $I(1)$  variables are “tied together” even though they might drift apart in the short run.

The standard procedure for testing co-integration is through Engel-Granger test (Granger, 1987). The test is a two-step procedure where if  $X_1, X_2, \dots, X_k$  are  $k$   $I(1)$  variables. Then first we find the OLS regression of say,  $X_1$  on  $(X_2, \dots, X_k)$ , i.e.,

$$X_{1t} = \alpha + \beta_1 X_{2t} + \dots + \beta_{k-1} X_{kt} + \varepsilon_t$$

Next we apply stationarity test, like the ADF test on the estimated residuals and infer that  $(X_1, X_2, \dots, X_k)$  is a set of co-integrated variables if and only if the estimated residuals are stationary. The co-integrating vector in such a situation is  $(1, -\beta_1, -\beta_2, \dots, -\beta_{k-1})$ . The long run equilibrium relationship between the variables being  $X_1 = \alpha + \beta_1 X_2 + \dots + \beta_{k-1} X_k$ .

Existence of con-integration can also be tested under VAR setup. The VAR based con-integration testing is known as the Johansen's tests. Under the VAR setup, we consider the vector autoregressive formulation with stationary errors

$$Y_t = \mu + \sum_{i=1}^p \phi_i Y_{t-i} + \varepsilon_t \quad (10)$$

The first difference formulation of the above model is

$$\Delta Y_t = \mu + \Gamma_1 \Delta Y_{t-1} + \Gamma_2 \Delta Y_{t-2} + \cdots + \Gamma_p \Delta Y_{t-p+1} + \Pi Y_{t-p} + \varepsilon_t$$

Where,  $\Gamma_i = (\phi_1 + \phi_2 + \cdots + \phi_i) - I_k$  for  $i=1, 2, \dots, p-1$  and  $\Pi = (\phi_1 + \phi_2 + \cdots + \phi_p) - I_k$ .

The matrix  $\Pi$  contains information on possible co-integrating relations between  $k$  elements of  $Y_t$ .  $\text{Rank}(\Pi)$  equals the number of independent co-integrating vectors of the system. Number of distinct co-integrating vectors can thus be obtained by checking the significance of the characteristic roots of  $\Pi$ . Johansen (1988) uses maximum likelihood based approach for testing the number of characteristic roots that are significantly different from zero.

Johansen's "Trace Test" procedure sequentially tests the following hypotheses:

$$\begin{aligned} H_0^0 : r=0 \text{ vs } H_A^0 : r \geq 0 \\ H_0^1 : r \leq 1 \text{ vs } H_A^1 : r \geq 2 \\ \vdots \\ H_0^{k-1} : r \leq k-1 \text{ vs } H_A^{k-1} : r \geq k \end{aligned}$$

Where  $r$  denotes the number of co-integrating vectors in the system. Johansen's "Trace Statistics", for testing the  $r^{\text{th}}$  hypothesis is given by

$$\lambda_{\text{trace}}(r_0) = -T \sum_{i=r_0+1}^k \log(1 - \hat{\lambda}_i) \quad (11)$$

Where  $T$  is the sample size and  $\hat{\lambda}_i$  are the estimated eigenvalues of the matrix  $\Pi$ . If  $\text{rank}(\Pi) = r_0$  then  $\hat{\lambda}_{r_0+1}, \dots, \hat{\lambda}_k$  should all be close to zero and  $\lambda_{\text{trace}}(r_0)$  should be small. In contrast, if  $\text{rank}(\Pi) > r_0$  then some of  $\hat{\lambda}_{r_0+1}, \dots, \hat{\lambda}_k$  will be nonzero (but less than 1) and  $\lambda_{\text{trace}}(r_0)$  should be large. The asymptotic null distribution of  $\lambda_{\text{trace}}(r_0)$  is not chi-square but instead is a multivariate version of the Dickey-Fuller unit root distribution which depends on the dimension  $n - r_0$  and the specification of the deterministic terms.

Then the "maximum eigenvalue" test statistic for testing the  $r^{\text{th}}$  hypothesis is given by

$$\lambda_{\text{max}}(r_0) = -T \log(1 - \hat{\lambda}_{r_0+1}) \quad (12)$$

As with the trace statistic, the asymptotic null distribution of  $\lambda_{\text{max}}(r_0)$  is not chi-square but instead is a complicated function of Brownian motion, which depends on the dimension  $n - r_0$  and the specification of the deterministic terms.

### Causality relationships

Causality denotes a necessary relationship between one event (called cause) and another event (called effect) which is the direct consequence of the first. In others words, whether one variable can help forecast another variable or not.

One way to address this question was proposed by (Granger, 1969) and popularized by Sims (Sims, 1972). Testing causality, in the Granger sense, involves using F-tests to test whether lagged information on a variable  $Y$  provides any statistically significant information about a variable  $X$  in the presence of lagged  $X$ . If not, then  $Y$  does not Granger-cause  $X$ . Here, assuming a particular autoregressive lag length  $p$ , we estimate the following unrestricted equation by ordinary least squares (OLS):

$$X_t = \mu + \sum_{i=1}^p \alpha_i X_{t-i} + \sum_{i=1}^p \beta_i Y_{t-i} + u_t \quad (13)$$

The null hypothesis under this setup of causality testing is framed as  $H_0 : Y$  does not Granger-cause  $X$ . In terms of model (13), we are interested in testing the following hypothesis

$$H_0 : \beta_1 = \beta_2 = \dots = \beta_p.$$

This is the simultaneous testing of a subset of regression parameters and can be tested using usual F-statistic. It is worth noting that with lagged dependent variables, as in Granger-causality regressions, the test is valid only asymptotically. The test procedure can be extended to cover the causality situation involving groups of variables.

The recent literature has shown that the conventional testing procedure for Granger non-causality using the F-statistic has size and power problems because of its dependence on the pre-testing for co-integration (Zapata and Rambaldi, 1997). A much more accurate and simpler procedure was proposed independently by Dolado and Lutkepohl (1996) and Toda and Yamamoto (1995) and is known as the “augmented VAR approach”. As shown by Zapata and Rambaldi (1997), the augmented VAR testing procedure is very simple to compute and is independent of the co-integration properties of the data.

The Toda and Yamamoto (1995) procedure uses a modified Wald (MWALD) test to test restrictions on the parameters of the VAR ( $k$ ) model.

This test has an asymptotic chi-squared distribution with  $k$  degrees of freedom in the limit when a VAR [ $k + d(\max)$ ] is estimated (where  $d(\max)$  is the maximal order of integration for the series in the system). Two steps are involved with implementing the procedure. The first step includes determination of the lag length ( $k$ ) and the maximum order of integration ( $d$ ) of the variables in the system. Measures such as the Akaike Information Criterion (AIC) and Hannan-Quinn (HQ) Information Criterion can be used to determine the appropriate lag structure of the VAR. Given the VAR ( $k$ ) selected, and that the order of integration  $d(\max)$  is determined, a level VAR can then be estimated with a total of  $p=[k + d(\max)]$  lags. The second step is to apply standard Wald tests to the first  $k$  VAR coefficient matrix (but not all lagged coefficients) to conduct inference on Granger causality.

## Empirical results

### Data description and descriptive statistics

For the present study, we have considered Tunisian macroeconomic time series data on exports, imports and GDP index. The data set, obtained from the National Statistics Institute of Tunisia, is quarterly and covers the periods 1961:1-2007:4. All the variables are in natural logarithms. The definitions of variables are the following:  $X = \log(\text{Real Exports})$ ,  $M = \log(\text{Real Imports})$  and  $GDP = \log(\text{Gross Domestic Product})$ .

The descriptive statistic is given in the Table 1. From this table, the mean is not significantly different from zero for either series. Normality is tested with the Jarque-Bera test, distributed as  $\chi^2(2)$  under the null hypothesis, so is strongly rejected for all series. Since rejection could be due to either excess kurtosis, or skewness. We report these statistics, separately in the Table 1.

The Figure 1 portrays the evolution of real exports, real imports and real GDP during the period of study.

### Wavelet decomposition

We first obtain a wavelet decomposition of the respective macroeconomic series. All the wavelet decompositions are done using a Daubechies extremal phase filters of length  $L = 4$ , that is  $D(4)$ , based on four non-zero coefficients, with periodic boundary conditions. The basic idea is thus to get a smooth component of the export, import and GDP series, without losing their underlying characteristics. Our understanding is that the resultant wavelet filtered series would provide us the true long-run econometric relationships.

We observe that the approximation series, corresponding to a level 3 decomposition of each of the macroeconomic indicators results in reasonable smoothing<sup>1</sup>. The application of the translation invariant wavelet transform with a number of scales  $J=3$  produce three vectors of details coefficients, that is  $d_1$ ,  $d_2$  and  $d_3$ , and one vector of wavelet smooth  $c_3$ . The vectors of details coefficients corresponding to this decomposition represent the short-term fluctuations and are stationary and hence do not contain any deterministic information about the respective series. Econometric analysis, as discussed in the following section, is carried out on the level 3 approximation series, which represent the smooth components of the respective macroeconomic indicators. In Figures 2-4 we report the wavelet decomposition of series.

### Test results for unit roots

The tests are performed on all the component series obtained through wavelet decompositions of the original data. First we present the unit root test results and stationarity properties of the respective approximation series by performing the Augmented Dickey Fuller, Phillips-Perron and KPSS tests in top half of Table 2.

We observe from this table that the respective approximation series are non stationary time series processes with respect to three test's statistics. In order to find out their order of integration, we further investigate the unit root test properties of their first difference series. From this test (bottom half of Table 2), we observe that the first difference series of Exports and Imports are all stationary. Hence we conclude that the 4 level

<sup>1</sup> In general,  $J$  is the highest resolution level such that  $2^J \leq \sqrt{T} \leq 2^{J+1}$ .



approximation series of Export, Import and GDP are all order one integrated, i.e.  $I(1)$  time series processes (Table 3).

### **Cointegration test results**

The cointegration tests were performed utilizing the Johansen (1991 and 1995) methodology. The Johansen methodology is a generalization of the Dickey-Fuller test. Two likelihood ratio tests,  $\lambda_{\max}$  and  $\lambda_{\text{trace}}$ , were used to test the hypotheses regarding the number of cointegrating vectors.

Before implementing the Johansen procedure for co-integration analysis, the autoregression order of the VAR in equation (10) has to be correctly specified. Therefore, to select the correct specification, we based our decision on the Schwarz' Bayesian information criterion (BIC) and selected  $p = 3$ .

We next proceed to testing for co-integration using the maximum likelihood approach developed by Johansen (1991 and 1995). The results of testing for the number of co-integrating vectors are reported in Table 4, which presents the maximum eigenvalue and the trace statistics, the p-value at 5% of significance level as well as the corresponding  $\lambda$  values. The maximum eigenvalue tests for at most  $r$  co-integrating vectors against the alternative of exactly  $r + 1$ , and the trace tests for at most  $r$  cointegrating vectors against an alternative of at least  $r + 1$  vectors.

(Johansen, 1991) discussed the likelihood based co-integration theory for such a model without constant terms. It turns out, however, that the presence of a constant in the non-stationary part of the data generating process plays a crucial role in the statistical analysis and for the interpretation of the model. In particular, (Johansen, 1991) proved that the asymptotic distribution of the test statistics and estimators in an error-correction model is not invariant to the assumption made about the constant term. Osterwald-Lenum (1992) also proved that the distribution of the trace statistic could be affected by the presence of a constant term. Thus, we used the Johansen (1995) likelihood ratio test to test whether the model should contain a constant term. The results of this test indicate that a model with an unrestricted constant should be adopted for the analysis. Both the maximum eigenvalue and trace statistics indicate the existence of a unique long-run relationship among the three endogenous variables included in the analysis.

### **Test results for Granger Non-causality**

Results from a VAR estimated using the procedure developed by Toda and Yamamoto (1995) are presented in Table 5.

For the purposes of this paper, the procedure is applied as follows:

1. Since the VAR model contains three lags and since the highest order of integration in the data is one, we first estimate a VAR in levels with four lags, then
2. We test jointly that the first three lags of the relevant variable are zero using a Wald test, which has a  $\chi^2$  distribution.

Table 5 reports the results of testing for Granger non-causality between GDP, Exports expansions and Imports expansions. The results suggest that there are clear indications of existence of a bi-directional causality between Exports and GDP (both the ELG and GLE hypotheses are supported by the data at the 5% level of significance.). Nevertheless, there is no relevant causality between import and export growths at 10% level of significance.

Consequently, we can conclude that, in Tunisia, there is a simultaneous cause and effect between economic growth and export growth. This simultaneity arises from the fact that, at the initial stages of development, economic growth promotes exports, but along the way exports start generating the capital needed for further economic growth.

## Conclusion

In this paper, we use the wavelet based filtering technique for establishing relationships among macroeconomic indicators of exports, imports and economic growth. An accurate analysis of co-integrated long-run equilibrium or causal relationships among these macroeconomic indicators is important. We consider the case of the Tunisian economy and explore and extract interesting relationships using wavelet technique. Using quarterly data over the time period 1961:1-2007:4, after filtering the series using wavelet technique, we have analyzed the time series properties of the exports, imports and economic growth variables in order to determine the appropriate functional form for testing the ELG hypothesis.

The study find that, the exports, imports and GDP are co-integrated. From the causality tests we have seen that there exist a bi-directional relationship between the Exports and GDP, no relevant causality between import and export growths at 10 % level of significance and a bi-directional relationship between import and economic growths.

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## Appendix

TABLE 1. SUMMARY STATISTICS

	mean	std.dev	skewness	kurtosis	JB	p. value
Y	3.1171	0.0512	-0.5492	-1.0741	18.2965	0.0001
X	3.0664	0.0688	-0.5950	-1.0281	19.2133	0.0001
M	3.0659	0.0773	-0.7715	-0.8498	24.3055	0.0000

FIGURE 1. GDP, EXPORT AND IMPORT SERIES

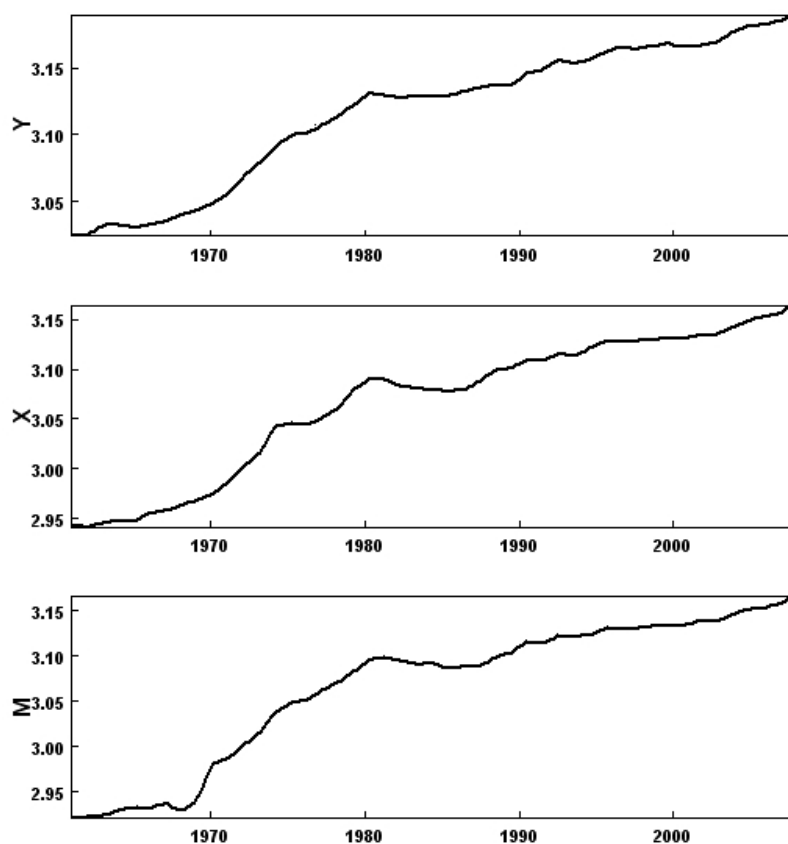


FIGURE 2. WAVELET DECOMPOSITION OF EXPORT SERIES

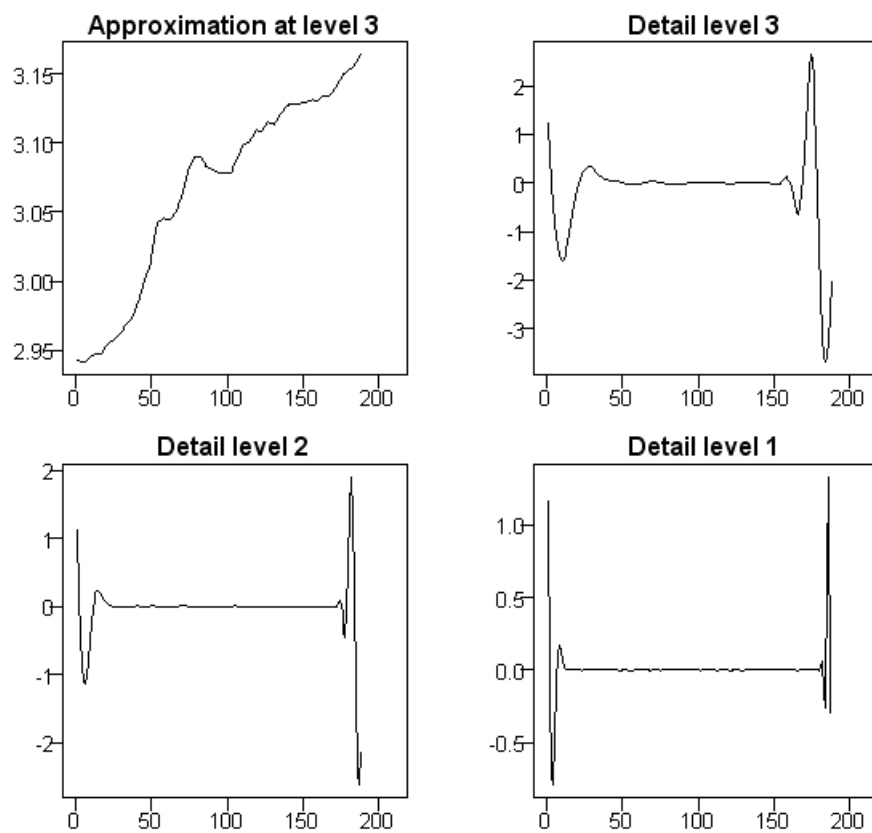


FIGURE 3. WAVELET DECOMPOSITION OF GDP SERIES

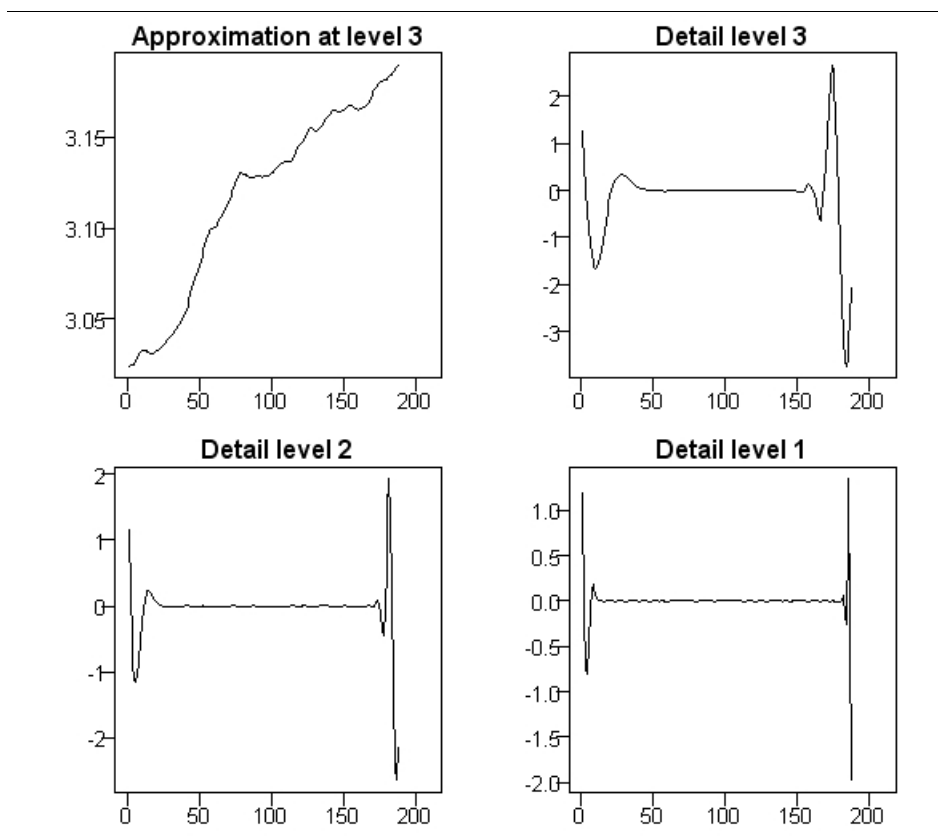


FIGURE 4. WAVELET DECOMPOSITION OF IMPORTS SERIES

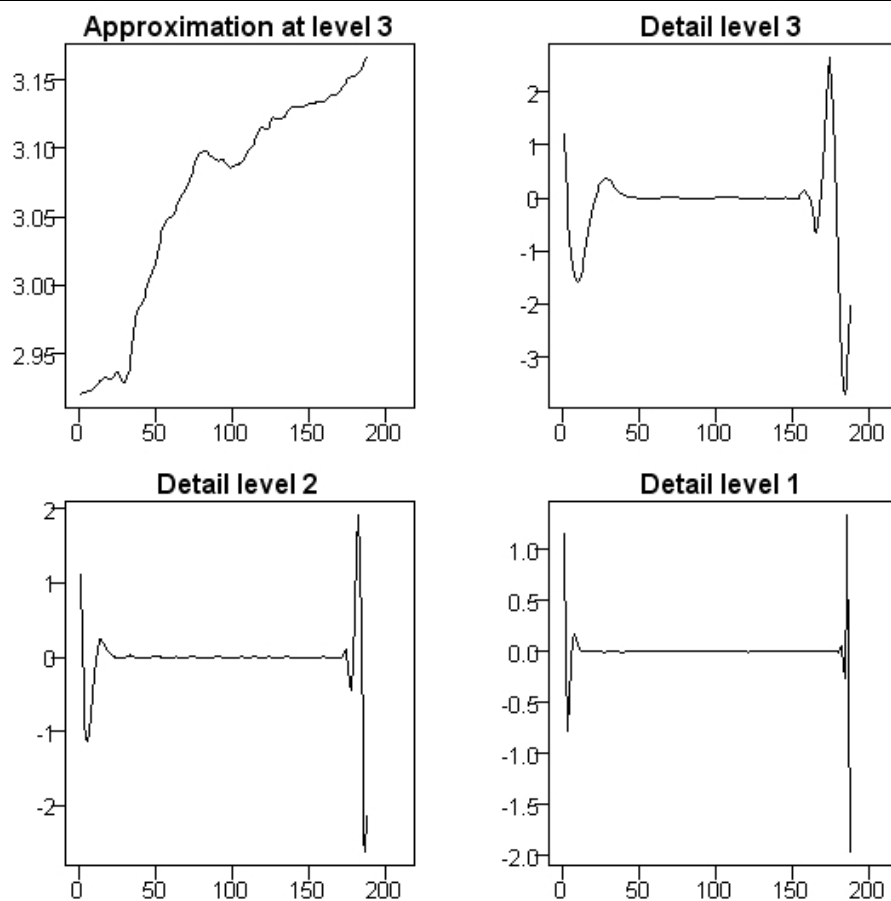


TABLE 2. UNIT ROOT TEST RESULTS OF LEVEL 4 APPROXIMATION SERIES AND THEIR FIRST DIFFERENCE SERIES

	ADF Test		Philips Perron Test		KPSS Test	
	statistics	p-value	statistics	p-value	statistics	p-value
X	-2.08588	0.48641	-1.96583	0.95592	4.47958	0.010
M	-1.91339	0.56832	-1.50448	0.97650	4.31119	0.010
Y	-1.85945	0.59555	-1.43931	0.97571	4.53364	0.010
$\Delta X$	-3.42928	0.05154	-28.65515	0.010	0.37662	0.08723
$\Delta M$	-3.89190	0.01589	-31.51380	0.010	0.56147	0.02782
$\Delta Y$	-3.20722	0.08855	-28.38377	0.010	0.51762	0.03770

TABLE 3. UNIT ROOT TEST STATISTICS OF THE DETAILS TIME SERIES

		ADF Test		Philips-Perron Test		KPSS Test	
		statistics	p-value	statistics	p-value	statistics	p-value
Detail 1	X	-7.24741	0.010	-106.1960	0.010	0.02307	0.10
	M	-7.22903	0.010	-105.57160	0.010	0.02310	0.10
	Y	-7.21588	0.010	-107.69627	0.010	0.02301	0.10
Detail 2	X	-9.03592	0.010	-52.18433	0.010	0.05973	0.10
	M	-9.05793	0.010	-52.12218	0.010	0.05949	0.10
	Y	-9.06295	0.010	-52.53184	0.010	0.06004	0.10
Detail 3	X	-3.82256	0.0193	-23.60265	0.0288	0.12859	0.10
	M	-3.80145	0.0203	-23.54744	0.0293	0.12840	0.10
	Y	-3.83209	0.0188	-23.66561	0.0283	0.12948	0.10

TABLE 4. JOHANSEN'S TEST FOR MULTIPLE COINTEGRATION VECTORS<sup>1</sup>

$H_0$	$\lambda$	Trace Test		Max Test	
		Statistic	p-value	Statistic	p-value
$r = 0$	0.12389	34.855	0.0112	24.469	0.0142
$r \leq 1$	0.038899	10.387	0.2566	7.3401	0.4585
$r \leq 2$	0.016333	3.0466	0.0809	3.0466	0.0809

<sup>1</sup> Note that the p-values are computed via the approximations by (Doornik, 1998).



TABLE 5. TEST RESULTS FOR GRANGER NON-CAUSALITY

Null hypothesis	Statistics	p-value
Exports does not Granger-cause GDP	4.2963	0.2312
Imports does not Granger-cause GDP	2.8741	0.4114
GDP does not Granger-cause Exports	3.9581	0.2660
GDP does not Granger-cause Imports	0.7346	0.8650
Imports does not Granger-cause Exports	7.3567	0.0613
Exports does not Granger-cause Imports	6.2653	0.0994