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Abstract:

We contribute to the literature on undesirable-output technology modeling by first discussing the limits of the recently proposed by-production approach of Murty *et al.* (2012) (hereafter we will refer to these authors as MRL) and second by proposing some possible extensions. We identify two theoretical limits and two practical drawbacks when using Data Envelopment Analysis (DEA) with this approach. Theoretically, MRL's by-production model is based on estimating two sub-technologies, one representing good outputs and the other one representing undesirable outputs. However, MRL assume independence of the sub-frontiers. In our paper, by contrast, we discuss the importance and implications of considering that all production processes are interconnected and should not be considered separately. Among the three extensions proposed, we argue that the introduction of some dependence constraints that link the two sub-technologies considered in this framework is very powerful. The two by-production approaches, MRL's and ours, are discussed under the restrictive assumption of fixed levels of inputs and under the flexible case of free choice of polluting input quantities. An application to a sample of 112 countries reveals that MRL model gives higher inefficiency scores compared to our extension with dependence constraints.

Keywords: by-production, cost disposability, factor bands, product couplings, dependence constraints, data envelopment analysis

JEL Classification: C61, D24, Q50

**Modélisation non-paramétrique de technologies polluantes :
une reformulation de l'approche coproduction**

Résumé :

Cet article contribue à la littérature sur la modélisation des technologies polluantes en proposant tout d'abord une analyse critique de l'approche coproduction récemment présentée par Murty *et al.* (2012) (MRL) et en discutant ensuite trois extensions possibles. Nous identifions deux limites théoriques et deux limites pratiques liées à la méthode d'enveloppement des données (data envelopment analysis). Théoriquement, le modèle de MRL est basé sur l'estimation de deux sous-technologies, une représentant la production des biens désirables et l'autre la génération des biens indésirables. Cependant MRL maintiennent une indépendance entre les sous-technologies. Dans cet article nous discutons de l'importance et des implications autour du fait que les processus de production sont interconnectés et ne devraient pas être considérés séparément. A partir des trois extensions discutées dans cet article, nous argumentons que celle basée sur l'introduction de contraintes de dépendance qui lient les deux sous-technologies en question est très puissante. Les deux approches, celle de MRL et la nôtre sont également discutées empiriquement en utilisant un échantillon de 112 pays. Les résultats montrent que l'approche de MRL donne des scores d'inefficience beaucoup plus élevés que notre extension avec l'introduction de contraintes de dépendance.

Mots-clefs : coproduction, cessibilité couteuse, couplage des facteurs, couplage des produits, contraintes de dépendance, méthode d'enveloppement des données

Classification JEL : C61, D24, Q50

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1. Introduction

Widespread societal environmental concerns and theory of externalities developments (Mishan, 1971; Cropper and Oates, 1992) have ushered in a new era in production economics with the emergence of multi-type output considerations in methodologies. In addition, the adoption of sustainable production behaviors has become key to many policy recommendations. The performance benchmarking literature (Tyteca, 1996; Allen, 1999; Zhou *et al.*, 2008; Song *et al.*, 2012) has therefore shown a keen interest in including the generation of undesirable outputs as by-products in production technology modeling. Following the seminal work produced by Pittman (1983), many non-parametric frontier estimation models have been developed (along the lines of Data Envelopment Analysis (DEA) where all firms are enveloped by a frontier made of the highest performing firms in the sample) to incorporate undesirable outputs into technology modeling. These models are based on the standard transformation function, and rely on specific disposability assumptions used to capture all the production technology's potential tradeoffs (substitution between inputs and outputs; marginal productivities). Most empirical applications treat undesirable outputs as additional inputs (Barbera and McConnell, 1990; Hailu and Veeman, 2000, 2001; Hailu, 2003; Considine and Larson, 2006) or work them into the technology as outputs, but under the weak disposability assumption (WDA) (Färe *et al.*, 1986; Färe *et al.*, 1989; Coggins and Swinton, 1996; Boyd and McClelland, 1999; Oude Lansink and Silva, 2003; Kuosmanen, 2005; Piot-Lepetit and Le Moing, 2007; Kuosmanen and Podinovski, 2009; Färe *et al.*, 2012). Such assumptions should make for a positive correlation between good and bad outputs. For example under the weak disposability assumption, it is costly for the firm to reduce its undesirable outputs since this implies a proportional reduction in good outputs due to the diversion of resources to the mitigation of undesirable outputs.

The limitations of these models, based as they are on a single functional relationship between inputs and outputs, are now well documented (Dakpo *et al.*, 2016). For instance, Førsund (2009), takes a profit function and a monetized pollutant damage function to show that, under the assumption that bads[1] are outputs (freely disposable), the maximal level of these detrimental

1 In this paper the terms bads, bad outputs, undesirable outputs, unintended outputs, detrimental outputs, pollutants and residuals are used interchangeably.

outputs is zero. Actually, under this situation where the degree of assortment (or freedom of assortment) – defined by Frisch (1965) as the degree of freedom with which inputs can be directed to the production of any of the outputs – is maximal, all resources can be diverted at no cost to the production of the good outputs and thus generate zero levels of bads. This result appears to be unrealistic in the light of Ayres and Kneese (1969)'s materials balance idea, whereby the generation of pollutants is inevitable because of the use of pollution-generating inputs. Similarly, considering residuals as inputs is awkward. As argued by Førsund (2009), keeping all other inputs constant, an increase in the level of pollution cannot technically explain why a good output increases. Besides, there is no explicit relationship between the common production inputs and the residuals, and only some kind of tradeoff between these residuals and good outputs is captured. Moreover, no 'purification possibility' (pollution control) is accounted for. Similarly, Pethig (2003, 2006) takes the materials balance principle to demonstrate that bads cannot be treated as inputs since this is a violation of the first law of thermodynamics implying mass or energy conservation. In accordance with Frisch (1965), Førsund (2009) recommends using 'product couplings' and 'factor bands' to overcome the above-mentioned drawbacks. (Pure) product couplings refer to the introduction of additional constraints that depict the link between some outputs (here between the good output and the bad output) irrespective of the inputs.[2] (Pure) factor bands relate to the relationship between inputs regardless of the outputs. Although Førsund (2009) specifies the connection of the WDA with the idea of product couplings, this assumption falls down in that some parts of the technology boundary exhibit no opportunity costs in the abatement of unintended outputs. This is a significant limitation since it means that it is not costly to reduce bads (this situation is also investigated by Chen (2014)). Coelli *et al.* (2007) and Hoang and Coelli (2011) also prove the inconsistency of this assumption with respect to the materials balance principle.

MRL and Murty and Russell (2002) expand on these criticisms by demonstrating the irregularities that occur when using a single functional relationship to define a pollution-generating technology. It is easy to say that these approaches, based on a single feature of the production technology, work like black boxes in which the 'magic' is misused and hence fails to produce an explicit representation of the production processes involved. MRL and Murty and

2 A parametric application of product couplings can be seen in Bokusheva and Kumbhakar (2014) where the authors use a translog hedonic specification to link the good outputs to the bads. Previously, Fernández *et al.* (2002) and Fernández *et al.* (2005) use an output aggregator function based on the constant elasticity of transformation defined in Powell and Gruen (1968).

Russell (2002) then propose a better alternative, namely the by-production approach, which is based on a full description of the production processes and has sound theoretical grounds (Murty, 2012). To be more precise, the by-production approach estimates two sub-technologies: one for good outputs and the other for undesirable outputs. Theoretically, the overall technology should lie at the intersection of the two sub-technologies. However, the practical implementation in the case of the non-parametric DEA analysis proposed by MRL is simply an estimation of two independent sub-technologies. This independence between the two sub-technologies is criticized in Dakpo *et al.* (2016). However these latter authors do not provide theoretical and numerical evidence to support their criticism. We believe that the most serious drawback of the empirical model proposed by MRL is that no conditions referring to the product couplings or the factor band concepts are present in the DEA representation of the technologies.

In this paper we re-examine the by-production approach by discussing theoretically and empirically some solutions to the above-mentioned issues; solutions related to the introduction of an interconnection between the two technologies involved into the activity analysis model. This connection is set up by means of, first, the mass balance equation, second, a number of dependence constraints between the sub-technologies, and third, a direct estimation of a product coupling relation. About the dependence constraints solution attempts can be found in Dakpo *et al.* (2014) and Lozano (2015). In both papers, the authors failed to provide some theoretical discussion on the dependence constraints. In the former paper the authors have simply displayed the dependence constraints and in the latter the author has related to the network DEA literature based on the modeling of interconnected sub-processes to justify the incorporation of these constraints. In this paper we go a step further in the sense that we provide in a more elaborate way the relevance of these constraints using the materials balance theory. We discuss the different solutions under the restrictive assumption of fixed levels of inputs and under the flexible scenario of free choice of polluting input quantities. In addition, we define how overall efficiency can be computed based on our extensions of the by-production approach, using non-radial distance function estimation. After describing our new model's theoretical foundation, we apply it empirically to a sample of countries using the Enhanced Russell-Based Directional Distance Measure (ERBDDM) discussed in Chen *et al.* (2014). We present the results of the two models applied to this data set: (i) the classic by-production model proposed by MRL, and (ii) our extension introducing dependence constraints.

The paper is organized as follows. **Section 1** reviews the by-production modeling as developed by MRL and discusses some theoretical and empirical limits associated to it. **Section 2** discusses a theoretical solution to the limit of the MRL approach and presents the three novel practical extensions that we propose. **Section 3** empirically compares the two by-production approaches (the classic one and our extension based on the dependence constraints) using a data sample of 112 countries which generate carbon dioxide emissions. **Section 4** concludes.

2. The classic by-production modeling of MRL: concepts, model and limits

2.1. Concepts

'In this feverish world of ours, where one wants the economic analyses to produce easily understandable results quickly and at the least possible cost, some of us have fallen into the habit of assuming for simplicity that the hundreds and sometimes thousands of variables that enter into the analyses are linked together by very simple relationships' (Frisch, 1965 p.v). However, 'single production is a comparatively rare occurrence…' (Frisch, 1965 p.10). The concept of *multi-ware* production was thus introduced by Frisch (1965) to expound the ideas on modeling several connected products. Typically, the multi-ware production can also be viewed as a *multiproduct* case or a *multi-technology* firm description. By extension, from the organizational management literature, firms with several divisions each one associated to a specific output can be referred to as the *M-form* (multi-dimensional form) (Cherchye *et al.*, 2014). The technology that represents a M-form firm is thereby deemed to verify the *'almost non-jointness'* property since each specific output is associated to a different technology, and there are some technology-specific inputs and joint inputs common to all the technologies (Cherchye *et al.*, 2014).

If we consider a system that produce m different products which are related by μ production equations, the *degree of assortment* of the system equals $\alpha = m - \mu$. In the case $\mu = 1$, we fall into a standard representation of the production technology with a single production relation with the maximum degree of assortment. In this situation, the maximal flexibility implies that inputs can be directed towards the production of any outputs without generating additional costs. In the presence of *pure factor bands* (relationships between inputs, independently of outputs) α can be negative. A special case is obtained when the degree of assortment equals zero. This situation has been coined *factorially determined multi-ware production* by Frisch (1965). In factorially determined technologies, given the levels of inputs, all the outputs are determined. Irrespective of m, μ, α, relations involving only product quantities can be present in the system. The number of these relations (κ) represents the *degree of coupling* of a multi-ware production. κ is simply the number of outputs relations that can be deduced from μ independently from the production factors.

2.2.The MRL model

Grounded in ideas put forward by Frisch (1965) and Førsund (1998), the by-production approach is then driven by the view that a production system should be described by several relations (transformation functions), and that this suits bad output-generating technologies particularly well. Murty (2010b) describes the five attributes inherent to these polluting technologies. First, the use of pollution-causing goods (inputs or outputs) necessarily triggers the nature of the emission generation mechanism. Second, variables aimed at producing intended outputs must be clearly separated from the emission-causing goods. The latter verify the non-rivalness and jointness properties, i.e., for the inputs, their use in the production of good outputs does not exclude them from the generation of pollution. Besides, the use of some intended outputs as inputs in the pollution mechanism does not reduce the amounts of these good outputs available to the producers.[3] Third, pollution-generating technologies violate the free disposability in incidental outputs. In general, the BP approach posits cost disposability, taken from Murty (2010a), for undesirable outputs. Free disposability is maintained for non-polluting inputs and some good outputs,[4] while pollution-generating goods violate the free disposability assumption. According to Murty (2015), intuitively, emission-causing goods can no longer be freely disposable since it implies larger amounts of emissions to be generated as a response. Murty (2015) proposes the conditional free and cost disposability assumptions for pollution-generating goods (and abatement outputs) which appears in both sub-technologies.[5] To be more precise, the approach states that with fixed quantities of some inputs and/or some

3 We can also add the non-exclusiveness property which states that none of the production processes (good and bad) can be excluded from the use of the joints inputs (polluting inputs).

4 This free disposability assumption is clearly described in Murty (2012 p8) as the fact that 'nature's emission generating mechanism is unaffected by changes in the usage of non-emission causing inputs and outputs. Changes in these goods affect only intended production'.

5 The conditional free disposability refers to the changes in the minimal amount of pollution given that higher (lower) levels of polluting inputs (abatement outputs) are feasible under the intended output production. The conditional cost disposability assumption implies the opposite, i.e. with lower (higher) levels of polluting inputs (abatement outputs) feasible under the emission generating technology, the maximum amount of intended outputs has to change consequently.

good outputs, a minimal amount of pollution can be simultaneously generated as a by-product of the technology. In the presence of inefficiencies, a higher level than this minimal level of undesirable outputs may be reached.[6] However, this assumption must coexist with the free disposability of the good outputs, which expresses that a set of maximal good output vectors can be produced if levels of inputs are held fixed (here also the presence of inefficiencies can lead to the production of lower levels of these intended outputs).[7] The positive monotonicity hypothesis (free or strong disposability) states that an increase in input consumption will not reduce the production of these good outputs, but will inevitably raise the level of minimum attainable bad outputs. Fourth, the correlation between intended and unintended outputs is systematic given the previous three attributes. Murty (2010b p8) tones down these trade-offs by demonstrating the existence of 'a positive correlation between the emission and any non-emission generating intended output when some inputs are the cause of the emission[8] and a negative correlation between the emission and any non-emission generating intended output when some intended outputs are the cause of the emission'.[9] Fifth, resources can be diverted from the production of intended outputs to the mitigation of incidental outputs. As pointed out in Murty (2010b) this comes at the cost of lower production of good outputs. More on the axiomatization of the by-production approach is discussed in Murty (2012, 2015).

Two production technology sets are constructed (see **Figure 1**): an intended-output production technology, which is a standard neoclassical production function, and a residual-generation technology, which reflects the nature of the polluting emission. The intended-output technology satisfies standard free disposability assumptions and is independent of the level of pollution.

6 As underlined in Murty (2010b) it also makes sense to imagine that there exists a maximal level of by-products which can be generated given the fixed levels of all inputs, good and abatement outputs. Nevertheless, as stressed in Murty (2012), given the fact that pollution generate external societal negative effects, one is more interested in the lower bounds of detrimental outputs.

7 The case where incidental outputs can generate external effects (positive or negative) that affect the levels of intended outputs is not considered here. Murty (2010b) refers to this situation as the weak by-production.

8 Murty (2015 p245) points out that 'in standard production theory, input free-disposability and output free-disposability imply a positive relationship between an input and an output along the production frontier. Hence, a positive trade-off between emission and intended output will be true in the input approach to emission generation.'

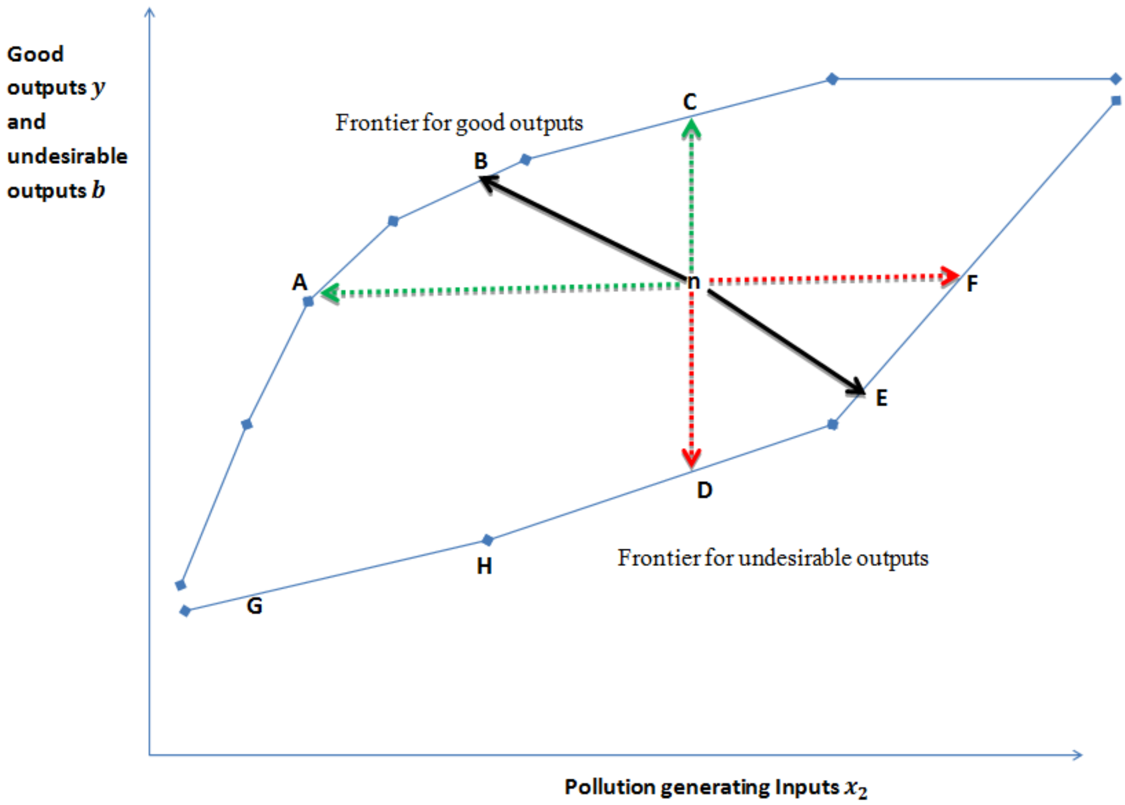
9 Similarly we can also prove the existence of a negative correlation between emissions and non-pollution-causing inputs when some inputs are the cause of pollution, and the existence of a positive correlation when some outputs are the cause of pollution.

Under this sub-technology, a decision making unit (DMU) n is dominated by all the observations located in the area delimited by arrows $\overrightarrow{nA}$ and $\overrightarrow{nC}$. The residual-generation technology satisfies, to quote MRL, 'the polar opposite condition' that is to say cost disposability, and is independent of the good output and the non-material inputs (i.e. non-polluting inputs). For this sub-technology, DMU_n is dominated by the points located in the area delineated by arrows $\overrightarrow{nD}$ and $\overrightarrow{nF}$. These dominating observations are characterized by the fact that they use more inputs to produce less undesirable outputs. Thereby, the sub-frontier for any inefficient DMU can be reached by increasing the consumption of inputs and simultaneously decreasing the generation of undesirable outputs. This situation is similar to that described by Sueyoshi and Goto (2010) and Sueyoshi *et al.* (2010) and later termed as 'managerial disposability' or positive adaptation (Sueyoshi and Goto, 2012a, b, c). The positive adaptation refers to a situation where input consumption can be increased and pollution reduced by simultaneously raising the production of good outputs. Yet this occurs by means of managerial efforts that lead to structural business transformations and the adoption of new technologies such as high quality inputs and other innovative technologies that can mitigate the levels of pollution. As pointed out by these authors, this concept ties in with the idea developed by Porter and van der Linde (1995) that regulation might offer innovation opportunities to secure the production of more good outputs and decrease the generation of bad outputs.

In view of the above, the intersection of these two sub-technologies then violates the free disposability assumption for pollution-causing inputs because of their opposite direction with regard to the two sub-technologies.[10] To understand this situation, it is useful to bear in mind that, due to their nature, the levels of good outputs need to be increased while the quantities of undesirable outputs are minimized. To sum up, by-production technology modeling has essentially three options to reduce the levels of detrimental outputs for a fixed technology: first an increase in abatement by means of resource diversion (which is accompanied by a reduction in the production of good outputs); second, a reduction in pollution-causing inputs (which decreases the levels of intended outputs except in the case of a substitution with non-polluting inputs to maintain the same amount of good output production); and third, the use of cleaner inputs, i.e. inputs that generate fewer bad outputs and maintain at least the same level of good output production.

10 However, it supports the free disposability of non-polluting inputs and good outputs, and the cost disposability of undesirable outputs.

Figure 1: Good output and undesirable output sub-technologies representation



Source: authors' own compilation based on MRL

Formally, MRL divide the input vector x into two input sub-vectors, where x_1 ($x_1 \in \mathbb{R}_+^{K_1}$) is the sub-vector of non-pollution-causing inputs and x_2 ($x_2 \in \mathbb{R}_+^{K_2}$) is the sub-vector of pollution-causing inputs. This input breakdown into two groups based on whether or not they generate pollution makes perfect sense, especially in a world where pollution is ruled by materials balance principle. Actually when we model emission-generating technologies and employ such models for empirical works, it is important to distinguish between emission-causing and non-emission causing inputs. This is required so as to be consistent with the way data on emission is collected.

For generalization, let's also split y the vector of good outputs into the set of non-pollution-causing $y_1 \in \mathbb{R}_+^{Q_1}$, and the sub-vector or pollution-causing outputs $y_2 \in \mathbb{R}_+^{Q_2}$. Denoting b the vector of bad outputs ($b \in \mathbb{R}_+^{R}$), y^a the cleaning up activities which is considered as pollution abatement output ($y^a \in \mathbb{R}_+^{S}$), and N the number of DMUs. The general production technology Ψ can be theoretically represented following MRL as

$$\Psi = \Psi_1 \cap \Psi_2 = \left[(x_1, x_2, y_1, y_2, b, y^a) \in \mathbb{R}_+^{K+Q+R+S} \mid f(x_1, x_2, y_1, y_2, y^a) \leq 0 \ \wedge \ u(b, x_2, y_2, y^a) \geq 0 \right] \tag{1}$$

where

$$\Psi_1 = \left[(x_1, x_2, y_1, y_2, b, y^a) \in \mathbb{R}_+^{K+Q+R+S} \middle| f(x_1, x_2, y_1, y_2, y^a) \leq 0\right] \tag{2}$$

$$\Psi_2 = \left[(x_1, x_2, y_1, y_2, b, y^a) \in \mathbb{R}_+^{K+Q+R+S} \middle| u(b, x_2, y_2, y^a) \geq 0 \right] \tag{3}$$

and f and u are both continuously differentiable functions. The cost disposability assumption with respect to the undesirable outputs can be expressed as follows:

$$(x_1, x_2, y_1, y_2, b, y^a) \in \Psi_2 \ \wedge \ \bar{b} \geq b \ \wedge \ \bar{x}_2 \leq x_2 \ \wedge \ \bar{y}_2 \leq y_2 \ \wedge \bar{y}^a \geq y^a \Rightarrow \left(x_1, \bar{x}_2, y_1, \bar{y}_2, \bar{b}, \bar{y}^a\right) \in \Psi_2 \tag{4}$$

Cost disposability implies that it is possible to pollute more given the levels of x_2 and y^a, i.e. that the set of technology Ψ_2 is bounded below (**Figure 1**) (Murty, 2010a). The Ψ_1 technology, however, satisfies the standard disposability assumptions:

$$(x_1, x_2, y_1, y_2, b, y^a) \in \Psi_1 \ \wedge \ \tilde{x}_1 \geq x_1 \ \wedge \ \tilde{x}_2 \geq x_2 \ \wedge \ \tilde{y}_1 \leq y_1 \ \wedge \ \tilde{y}_2 \leq y_2 \ \wedge \ \tilde{y}^a \leq y^a \ \Rightarrow (\tilde{x}_1, \tilde{x}_2, \tilde{y}_1, \tilde{y}_2, \tilde{y}^a, b) \in \ \Psi_1 \tag{5}$$

The degree of assortment of the production system in **(1)** equals $\alpha = (Q + R + S) - 2$. The model can be a factorially determined multi-ware production in the case $Q_1 = 1 \ \wedge \ Q_2 = 0 \ \wedge \ R = 1 \ \wedge \ S = 0$.[11] In their theoretical demonstration MRL propose $Q_1 = 1 \ \wedge \ Q_2 = 0 \wedge \ R = 1 \ \wedge \ K_1 = 1 \ \wedge \ K_2 = 1 \ \wedge \ S = 1$. In the MRL demonstration, the technology is not factorially determined. Using implicit function theorem MRL prove how the positive correlation between good and bad outputs is accounted for and all the other trade-offs are consistent with the by-production.

Empirically in the DEA framework, the unified technology under variable returns to scale (VRS) is represented by model **(6)** with two intensity variables $\nu$and ξ, which represent the two different sub-technologies.[12]

11 Other combinations are possible. However, we simply want to stress here that the MRL approach is a factorially determined production technology under some very special cases.

12 Actually in Murty et al. (2012), the authors assume constant returns to scale (CRS) for both sub-technologies while in Murty and Russell (2010) the authors impose decreasing returns to scale (DRS). However these latter

$$\Psi = \left[(x_1, x_2, y_1, y_2, b, y^a) \in \mathbb{R}_+^{K+Q+R+S} \middle| \, y_1 \leq Y_1\nu \,; \;\; y_2 \leq Y_2\nu \,; \; y^a \leq Y^a\nu \,; \right.$$

$$x_1 \geq X_1\nu \,; \;\; x_2 \geq X_2\nu \,; \tag{6}$$

$$\left. x_2 \leq X_2\xi \,; \;\; y_2 \leq Y_2\xi \,; \; b \geq B_i\xi \;\; ; \; y^a \geq Y^a\xi \;\; ; \; \nu'\mathbb{1} = 1 \;\; ; \;\; \xi'\mathbb{1} = 1 \;\; ; \;\; \nu, \xi \geq 0\right]$$

where (X, Y, Y^a, B) denote the matrix of inputs, good outputs, abatement outputs, and undesirable outputs of the benchmark of N DMUs, i.e. the reference set.

2.3. Limits of the MRL approach

We discuss the limits of the MRL approach through two lines of thoughts: first, theoretical aspects and the formulation in **(1)**; and second, practical features regarding efficiency estimation in the DEA framework (model in **(6)**).

2.3.1. Theoretical aspects

- **Limit 1:** Lack of coupling between the good and the bad outputs in the MRL approach and the problem of implicit functions theorem

The coupling between both types of outputs in the system represented in **(1)** is $\kappa = 0$ since there is no equation linking the outputs independently of the inputs (factors) levels. Thus both types of outputs can be considered as separable. Frisch (1965) discusses this situation of separable outputs represented with two separate substitution regions in the isoquants space (see Fig. 14b.2 in Frisch (1965 p272)). More clearly, Frisch (1965 p362) states that 'if the ratio between product quantities can be changed by changing the factor quantities, the products are separable'. Yet the case of undesirable outputs has met the adhesion of the scientific community on the fact that those detrimental outputs can be seen as a joint (coupled) production to intended outputs. We use the concrete example proposed in MRL (p123, footnote 17) without the presence of abatement output (y^a) to show this lack of coupling. We have $y = x_1^{\alpha_1} x_2^{\alpha_2}$ and $b = \beta x_2$,[13] and the system represented by these two equations is factorially determined. Following Frisch (1965

authors explicitly state that 'extensions to constant or variable returns can be done in the usual way' Murty and Russell (2010 p22).

[13] $(\alpha_1, \alpha_2, \beta > 0)$.

p276) the degree of coupling can be derived by the computation of the rank of the matrix of marginal productivities of both outputs:

$$\kappa = m - \text{Rank}(MP) = 2 - Rank\begin{bmatrix} \frac{dy}{dx_1} & \frac{dy}{dx_2} \\ \frac{db}{dx_1} & \frac{db}{dx_2} \end{bmatrix}$$
$$\kappa = 2 - rank\begin{bmatrix} \alpha_1 x_1^{\alpha_1-1} x_2^{\alpha_2} & \alpha_2 x_1^{\alpha_1} x_2^{\alpha_2-1} \\ 0 & \beta \end{bmatrix} \tag{7}$$
$$\kappa = 2 - 2 = 0$$

Another proof of this lack of coupling can be obtained by computing the ratio of good on bad outputs:

$$\frac{y}{b} = \frac{x_1^{\alpha_1} x_2^{\alpha_2}}{\beta x_2} = \frac{x_1^{\alpha_1} x_2^{\alpha_2-1}}{\beta} \tag{8}$$

From **(8)**, the ratio of the good on the bad outputs cannot be modified without altering the levels of both inputs. Thereby in the system example good and bad outputs are both separable outputs and not joint (coupled) outputs as it could be expected. Let's consider now the extreme case where the production system runs with only one input (x_2). The ratio of good on bad outputs is equivalent to $y/b = x_2^{\alpha_2-1}/\beta$. This ratio is factor independent if and only if $\alpha_2 = 1$. Although this situation yields a case where coupling is present there are no reasons in practice to believe that all the inputs x_2 can be fully transformed into good output: first in that case there is physically no room for pollution to be generated from those inputs, and second this one ratio between y and x_2 simply means that there is no transformation.

Using implicit function theorem, MRL demonstrate the existence of a positive trade-off between good and bad outputs. We argue here that this trade-off is simply a correlation between two dissociated variables and not a trade-off as it can be found in the economic literature (opportunity cost). For the system example defined above, we can derive the following relation:

$$\frac{dy}{db} = \frac{dy}{dx_2}\frac{dx_2}{db} = \alpha_2 x_1^{\alpha_1} x_2^{\alpha_2-1} \times \frac{1}{\beta} = \alpha_2 \frac{x_1^{\alpha_1} x_2^{\alpha_2-1}}{\beta} \tag{9}$$

The equation in **(9)** simply indicates the positive relation existing between y and b. From **(9)** it is impossible to derive by how much y will change in response to a change in b since **(9)** is not independent from the levels of inputs. As argued in Førsund (2009) an explicit introduction of product coupling (as a materialization of the jointness between ordinary outputs and residuals generation) will result in the social planning problem to a solution where an extra resource cost is added (which is absent from the MRL approach).

About the use of the implicit function theorem to estimate the trade-off between y and b it raises one concern: the demonstration carried out by MRL suggests the equality between two implicit functions, yet this is not clearly stated. Using the good and bad output previous production relations, we can derive the following implicit functions for the polluting input x_2

$$\begin{gathered}\hat{x}_2 = \left(\frac{y}{x_1^{\alpha_1}}\right)^{\frac{1}{\alpha_2}} = h(y, x_1) \\ \hat{x}_2 = \frac{b}{\beta} = g(b)\end{gathered} \tag{10}$$

There is no technical justification for equality between the two functions in **(10)** since y and b are generated under two independent processes (they are also separable given the lack of coupling), yet in MRL it is implicitly assumed so.[14]

Actually the existence of coupling or a technical interrelation between y and b implies the existence of a relationship of the form $F(y, b) = 0$.[15] By differentiating this functional representation, we obtain:

$$\frac{dy}{db} = -\frac{\delta F/\delta b}{\delta F/\delta y} \tag{11}$$

Given that incidental outputs are joint outputs (by-products) of intended ones, they are unavoidable. Frisch (1965 p346) defines joint production as the situation where 'a given set of production factors will – if used – necessarily produce more than one kind of production simultaneously'. To account for this property of bad outputs, following Førsund (2009) the constraints in **(12)** must be added to **(11)**

$$\frac{\delta F}{\delta b} \geq 0\,, \frac{\delta F}{\delta y} \leq 0 \;\text{ or }\; \frac{\delta F}{\delta b} \leq 0\,, \frac{\delta F}{\delta y} \geq 0 \tag{12}$$

Finally the lack of coupling between good and bad outputs under the MRL approach means that what may be identified as a tradeoff between these two types of output is simply a correlation (a partial correlation, computed in **(9)** through the polluting input x_2).

- **<u>Limit 2:</u>** The problem of positive assortment (not factorially determined multi-ware production)

14 More evidence on this situation is provided later in the paper.

15 This coupling relation can be simple or complicated.

Before the discussion of this limit, we would like to state the argument put forward by Førsund (2009) about factorially determined multi-ware production. He argues that the special case of zero (or no) assortment (factorially determined multi-ware production) is of particular relevance (‘best suited’) when modeling pollution-generating technologies.

The case of abatement outputs

In the system representation in **(1)**, abatement outputs y^a are represented as part of the intended outputs production sub-technology. Let’s recall that in assorted production, ‘ a given set of production factors can be used optionally for either one kind of product or for another’ (Frisch, 1965 p346). In this case abatement outputs are substitutes to intended production, even if there is no reason to observe it in reality, especially with end-of-pipe technologies which can be considered as a separate activity from the one we have described till now. Besides, the MRL approach does not provide explicit information on how those abatement outputs are produced.

The case of multiple pollutants

In the MRL representation in **(1)**, all undesirable outputs are treated under the same sub-technology. Hence, only substitution possibilities between these undesirable outputs are considered. However, it is possible to find some examples where bad outputs generated by different technical processes are complements rather than substitutes (Moslener and Requate, 2007). For instance consider the case of livestock farming where methane emissions are associated to animal enteric fermentation and nitrous oxide are associated to manure management. In this case methane and nitrous oxide are complements since a decrease in the number of animals will simultaneously decrease the levels of both incidental outputs. Nevertheless this case is somehow excluded from the MRL approach.

2.3.2. Practical considerations (DEA framework)

- **Limit 3:** Inconsistent benchmark in the DEA representation of the MRL approach

The lack of coupling between good and bad outputs in the MRL approach results in independent benchmarks between both sub-technologies involved (practical case of the situation summarized in **(10)**). To prove this inconsistency, we consider the following numerical illustration presented in **Table 1** where we have considered three polluting inputs, two good outputs and one bad output. We do not make any reference to materials balance principle here in computing the bad outputs levels as the data in **Table 1** can be associated to a system where materials balance principle do not apply (for instance banking system with non performing loans).

Table 1: Data for the first numerical illustration (Data1)

DMUs	x_1	x_2	x_3	y_1	y_2	b
A	46	25	12	10	28	13
B	1	2	2	2	2	2
C	1	1	1	1	1	1
D	**35**	**13**	**34**	**20**	**33**	**23**
E	2	3	5	4	4	10
F	4	12	4	5	6	1
G	8	8	7	7	16	3
H	15	31	66	62	25	18
I	12	23	10	12	8	19
J	10	6	9	38	39	17
K	12	24	11	17	11	18
L	20	94	55	52	61	16
M	13	57	10	19	68	20
N	14	7	13	62	41	19
O	16	55	103	100	85	22

Source: authors

To assess the outputs inefficiency we consider the following Enhanced Russell-Based Directional Distance Measure (ERBDDM) (Chen *et al.*, 2014):

$$\begin{aligned}
&\vec{D}\left(x,y,b;\vec{g}_y,\vec{g}_b\right) = \max_{\theta,\phi,\nu,\xi} I^n_{ERBDDM} = \frac{1}{2}\left[\frac{1}{Q}\sum_{q=1}^{Q}\theta_q^n + \frac{1}{R}\sum_{q=1}^{R}\phi_r^n\right] \\
&s.t.\quad \sum_{i=1}^{N}\nu_i y_{qi} \geq y_{qn} + \theta_q^n \vec{g}_y^q \quad q = 1,\dots,Q \\
&\sum_{i=1}^{N}\nu_i x_{ki} \leq x_{kn} \quad k = 1,\dots,K \\
&\sum_{i=1}^{N}\xi_i b_{ri} \leq b_{rn} - \phi_r^n \vec{g}_b^r \quad r = 1,\dots,R \\
&\sum_{i=1}^{N}\xi_i x_{ki2} \geq x_{kn2} \quad k_2 = 1,\dots,K_2 \\
&\sum_{i=1}^{N}\nu_i = 1 \;\; ; \;\; \sum_{i=1}^{N}\xi_i = 1 \;\; ; \;\; \nu_i,\xi_i \geq 0; i = 1,\dots,N \; ; \; \theta,\phi \geq 0
\end{aligned} \tag{13}$$

In **(13)**, $\vec{g}_y, \vec{g}_b$ represents the directional vectors of good outputs and undesirable outputs respectively, and θ_q^n, ϕ_r^n the inefficiency scores associated with the q-th good output and the r-th undesirable output. Following the recommendation by Chung *et al.* (1997) for directional vectors $\vec{g}_y, \vec{g}_b$, we use the observed vectors for the different outputs: $\vec{g}_y = \vec{y}$ and $\vec{g}_b = \vec{b}$. The ERBDDM in model **(13)** is somewhat similar to the Färe–Grosskopf–Lovell (FGL) index applied by MRL, except that the results here are expressed in terms of inefficiency. We solve model **(13)** for DMU_D for instance. $\mathrm{I}_{\mathrm{ERBDDM}}^{\mathrm{D}} = 0.877$; $\theta_1^{\mathrm{D}} = 2.338$; $\theta_2^{\mathrm{D}} = 0.409$; $\phi_1^{\mathrm{D}} = 0.381$. The features of the benchmarks (reference sets) associated to the inefficient $\mathrm{DMU_D}$ are summarized in **Table 2**.

Table 2: Benchmarks features of DMU_D using the MRL approach

Variables	$\boldsymbol{\Psi_1}$	$\boldsymbol{\Psi_2}$
$\boldsymbol{x_1}$	$\mathbf{14.250}$	$\mathbf{35.000}$
$\boldsymbol{x_2}$	$\mathbf{13.000}$	$\mathbf{31.383}$
$\boldsymbol{x_3}$	$\mathbf{24.250}$	$\mathbf{34.000}$
$\boldsymbol{y_1}$	66.750	–
$\boldsymbol{y_2}$	46.500	–
$\boldsymbol{b}$	–	14.235

Source: authors

Inefficient DMU_D is projected onto benchmarks that are dissociated given the optimal levels of inputs (opposite direction). This inappropriate handling of the general inputs[16] in the MRL approach is linked to the disconnection between the good and the bad outputs sub-technologies.[17] Besides, it is hard to believe that DMU_D can become efficient by being projected towards two benchmarks using different levels of inputs: such situation sounds impossible. The independence between the two sub-technologies involved in the MRL approach can also be

16 We define as general inputs, inputs that are jointly shared by the two sub-technologies like the pollution-causing inputs x_2. In the case of factorially determined production system and several sub-technologies we refer to specific inputs those only used in one sub-technology.

17 Actually, in the program (13), the two sub-technologies involved are linked through the objective function. However this is not sufficient to connect the benchmarks of the sub-technologies which are still independent from each other especially in their use of polluting inputs.

seen in their FGL efficiency measure formula (p129) (their equation **(5.7)**), where the efficiency score obtained is a weighted addition of two independent (separate) output efficiency scores: an operational efficiency score based on Ψ_1 and an environmental efficiency score obtained from Ψ_2. In formula **(13)** as in formula **(5.5)** in MRL (p128), the good and bad outputs sub-technologies are unified in the objective function but not in the constraints, which allows efficient use of polluting generating inputs to differ in the two sub-technologies. One may conjecture that the introduction of input inefficiencies in the objective function of **(13)** may solve the issue. However as previously discussed, the conditional free and cost disposability assumptions might imply two opposite directions for inputs efficiency assessment. For instance under the intended outputs sub-technology, an efficient point can be attained by reducing the levels of inputs, while under the emission generating sub-technology, to reach an efficient point the levels of polluting inputs can be increased (**Figure 1**). Yet, from a society point of view (or normative sense), inputs saving may be viewed as the reasonable situation that preserves the nature from pollution emissions (besides, given that the changes in polluting-inputs need to be the same under both sub-technologies, only one direction can be retained).

For that case we consider the following model:

$$
\begin{aligned}
\vec{D}\left(x, y, b; \vec{g}_y, \vec{g}_b\right) = \max_{\theta,\phi,\nu,\xi} I^n_{ERBDDM} = \frac{1}{3}\left[\frac{1}{Q}\sum_{q=1}^{Q}\theta^n_q + \frac{1}{R}\sum_{q=1}^{R}\phi^n_r + \frac{1}{K}\sum_{k=1}^{K}\varphi^n_k\right] \\
s.t.\ \sum_{i=1}^{N}\nu_i y_{qi} \geq \theta^n_q \vec{g}^{\,q}_y + y_{qn} \quad q = 1, \dots, Q \\
\sum_{i=1}^{N}\nu_i x_{ki} \leq x_{kn} - \varphi^n_k\, \vec{g}^{\,k}_x \quad k = 1, \dots, K \\
\sum_{i=1}^{N}\xi_i b_{ri} \leq b_{rn} - \phi^n_r \vec{g}^{\,r}_b \quad r = 1, \dots, R \\
\sum_{i=1}^{N}\xi_i x_{ki2} \geq x_{kn2} - \varphi^n_{k_2}\, \vec{g}^{\,k}_{x_2} \quad k_2 = 1, \dots, K_2 \\
\sum_{i=1}^{N}\nu_i = 1 \ ;\ \sum_{i=1}^{N}\xi_i = 1 \ ;\ \nu_i, \xi_i \geq 0; i = 1, \dots, N\ ;\ \theta, \phi \geq 0
\end{aligned}
\tag{14}
$$

As previously, we set $\vec{g}_x = \vec{x}$. Still for DMU_D we obtain $\mathrm{I}^{\mathrm{D}}_{\mathrm{ERBDDM}} = 0.836$; $\theta^{\mathrm{D}}_1 = 2.100$; $\theta^{\mathrm{D}}_2 = 0.242$; $\phi^{\mathrm{D}}_1 = 0.776$

and $\varphi^D_1 = 0.600$; $\varphi^D_2 = 0.462$; $\varphi^D_3 = 0.618$.

The benchmark features of DMU_D are summarized in **Table 3**.

Table 3: Benchmarks features of DMU_D using the MRL approach with input inefficiencies

Variables	Ψ_1	Ψ_2
x_1	14.000	14.000
x_2	7.000	17.977
x_3	13.000	13.000
g_1	62.000	–
g_2	41.000	–
b	–	5.146

Source: authors

Under the conjecture in **(14)** the benchmark between good and bad output can still be different even when inputs inefficiency is accounted for. However for DMU_D two of the inputs (x_1, x_3) have the same levels under both benchmarks. Nevertheless the difference in input x_2 quantity may result in wrong assessment of the good and the bad outputs inefficiency. Actually an important implied property of the polluting inputs in the MRL approach is that there are nonallocable production factors where the distribution of inputs between good and bad outputs is not explicit (or unobservable). Hence the consumption of polluting inputs is known for the whole system and not for the specific processes. By analogy to the work in Cherchye *et al.* (2014) and cost minimization objective, we can say that the MRL approach in **(13)** and **(14)** can be viewed as a non-cooperative inefficiency assessment where each division (here each sub-technology) chooses the level of the inputs that yield its optimal objective leading thereby to a 'Nash-type' equilibrium where the joint inputs (polluting inputs) are inefficiently allocated.

- **Limit 4:** Misclassification of efficiency status and overestimation of the inefficiency in the DEA framework

So far we have not considered materials balance, which we do now. We assume that the production process that is intended to be modeled by MRL is ruled by materials balance principle, on the ground that all physical processes are governed by mass-energy conservation. We consider the same numerical example (Data1) as previously, except that the emission factors (pollution contents) associated to each of the inputs are respectively $a_1 = 0.5$; $a_2 = 0.35$; $a_3 = 0.75$ $(b = a_1 \times x_1 + a_2 \times x_2 + a_3 \times x_3)$.[18] The new data with the resulting new pollution level is summarized in **Table 4**. In this new example, we set the objective as being

18 We do not consider recuperation of pollution by the good outputs.

related only to the assessment of the environmental inefficiency (only the bad output inefficiency is considered in the objective function - $\max_{\phi,\nu,\xi} I^n_{ERBDDM} = \frac{1}{R}\sum_{q=1}^{R} \phi_r^n$ -). The application of the MRL approach with the data in **Table 4** yields an environmental efficiency score of one for all the DMUs, i.e. no interior point or no inefficient observation is feasible when materials balance is present and inputs levels are fixed. In other words, the output orientation in MRL approach is not sufficient for the evaluation of environmental efficiency. As underlined in Beattie *et al.* (1974 p161) 'for complementarity to arise from the by-product, phenomena, the usual assumptions regarding fixity of the resource base and simultaneous production periods must be altered'. It is worth noting that in the situation where DMUs do not share the same emission factors for the inputs, some environmental inefficiency can be identified by the MRL output orientation under fixed levels of inputs. However, this inefficiency will simply reflect measurement errors, or inefficiency due to aggregation of inputs (generally the case in DEA), or inefficiency in the transformation factors of inputs into good and bad outputs (due for instance to lack of maintenance of existing technologies or to the use of old technologies). As previously, one may consider situations where inputs minimization is introduced as an objective like in program **(14)**. At this point it is worth mentioning that the DEA program in **(14)** is a contribution to the MRL approach which reconciles with input measurement inefficiency and makes a choice regarding the directional ambiguity with respect to input measurement. In this case materials balance is verified (with constant emission factors for all DMUs), the MRL approach yields consistent benchmark. This important result is however limited as: (i) it is likely that in practice some DMUs do not share the same emission factors (for reasons we have earlier mentioned: input aggregation, old technologies…) and in this case the benchmarks are inconsistent like the one in **Table 3**; (ii) theoretical limit of MRL approach regarding the lack of coupling between the good and bad outputs, still holds; (iii) with inputs minimization we do not capture the inefficiency associated to different combinations of inputs (allocative efficiency). As discussed in Coelli *et al.* (2007), one way to assess the environmental efficiency is to endogenously determine the levels of polluting inputs that minimize the volume of pollution. Coelli *et al.* (2007) propose to solve the model in **(15)** to estimate the environmental efficiency. This efficiency is defined as the ratio of the minimum optimal level of pollution divided by the observed level. In **(15)**, the environmental efficiency is evaluated by using the mass-energy equation and estimating some kind of iso-environmental lines in the same way as iso-cost lines. However the use of model **(15)** requires knowledge on emissions factors.

$$\begin{aligned} CLV_n = \min_{x_k,\lambda} \sum_{k=1}^{K_2} a_k x_{kn} \\ s.t.\ \sum_{i=1}^{N} \lambda_n x_{kn} \le x_{kn} \quad k = 1, \dots, K \\ \sum_{i=1}^{N} \lambda_n y_{qn} \ge y_{qn} \quad q = 1, \dots, Q \\ \sum_{i=1}^{N} \lambda_i = 1 \\ x_k, \lambda \ge 0 \end{aligned} \tag{15}$$

To keep in line with the approach developed in Coelli *et al.* (2007), the MRL model can be solved by endogenizing the levels of polluting inputs.

The new objective function is equivalent to **(16)**.

$$\max_{\phi,\nu,\xi,x} I^n_{ERBDDM} = \frac{1}{R} \sum_{q=1}^{R} \phi^n_r \tag{16}$$

Table 4: Data for the second numerical illustration (Data2)

DMUs	x_1	x_2	x_3	y_1	y_2	b
A	46	25	12	10	28	41
B	1	2	2	2	2	3
C	1	1	1	1	1	2
D	35	13	34	20	33	48
E	2	3	5	4	4	6
F	4	12	4	5	6	9
G	8	8	7	7	16	12
H	15	31	66	62	25	68
I	12	23	10	12	8	22
J	10	6	9	38	39	14
K	12	24	11	17	11	23
L	20	94	55	52	61	84
M	13	57	10	19	68	34
N	14	7	13	62	41	19
O	16	55	103	100	85	105

Source: authors

The results of **(15)** and **(16)** are summarized in **Table 5**.

Table 5: Inefficiency results using numerical illustration data Data2

DMUs	Materials balance principle (Model (15))	MRL approach with endogenous level of inputs (Model (16))
A	0.747	0.961
B	0.286	0.407
C	0.000	0.000
D	0.749	0.966
E	0.554	0.724
F	0.651	0.826
G	0.466	0.867
H	0.717	0.976
I	0.778	0.926
J	0.000	0.884
K	0.726	0.929
L	0.495	0.981
M	0.000	0.953
N	0.000	0.917
O	0.000	0.985

Source: authors

In **Table 5**, the results show that under endogenously determined levels of polluting inputs, some DMUs which are actually efficient (DMUs M, N, O, for instance) are deemed inefficient under the MRL approach, resulting in misclassification of the efficiency status. Besides, for most of the DMUs the inefficiency levels obtained by the MRL approach is higher than the one obtained under model **(15)**, implying an overestimation of the inefficiency score.[19] Actually in this illustration, the MRL model simply considers the DMU with the minimum level of pollution (here DMU_C) and evaluates the environmental inefficiency for all the other DMUs relatively to this efficient one (DMU_C). There is no reason in practice to proceed this way especially if VRS are present. Again, the overestimation of the inefficiency scores displayed in **Table 5** in the case of the MRL approach is a consequence of the absence of coupling between good and bad outputs.

19 We borrow here the expression 'misclassification of the efficiency status' from Chen (2014).

In this section, we have showed and discussed the limits associated to the MRL approach, which relate mainly to the lack of coupling between good and bad outputs and to the incorrect way of handling generalized inputs within DEA implementation. In the next section we discuss some theoretical and practical solutions to the MRL issues.

3. Overcoming the issues associated to the MRL approach: our extensions of the by-production approach

3.1. Theoretical approach

From the previous developments, one of the obvious solutions to the problems identified for the MRL approach is the introduction of coupling between good and bad outputs. Førsund (2009) argues that the use of factorially determined multi-ware production augmented with materials balance conditions is the best alternative in modeling pollution-generating technologies. We start from this idea to show how theoretically materials balance principle can be used for the coupling of both desirable and undesirable outputs. We consider the following general relations:

$$\begin{gathered} y_q = g_q(x_1, x_2)\,, \left(x, y_q\right) \in \mathbb{R}_+^{K+1}, \frac{dg_q}{dx} \geq 0\,, q = 1, .., Q \\ b_r = u_r(x_2)\,, (x_2, b_r) \in \mathbb{R}_+^{K_2+1}, \frac{du_r}{dx_2} \geq 0\,, r = 1, .., R \\ b_r = \sum_{k=1}^{K_2} a_k^r x_{k2} - \sum_{q=1}^{Q} c_q^r y_q\,, r = 1, .., R \end{gathered} \tag{17}$$

In **(17)** the third type of relations represents the materials balance principle. The difference of **(17)** with the formulation of Førsund (2009) is that in **(17)** bad outputs are not summed. Each undesirable output is represented with its own mass-energy content equation. Besides, we do not consider the existence of purification inputs in the incidental outputs representation. However for simplicity we now assume for the next developments that $Q = K_1 = K_2 = R = 1$. Hence we have the simplified production system in **(18).**

$$\begin{gathered} y = g(x_1, x_2)\,, (x, y) \in \mathbb{R}_+^3, \frac{dg}{dx} \geq 0 \\ b = u(x_2)\,, (x_2, b) \in \mathbb{R}_+^2, \frac{du}{dx_2} \geq 0 \\ b = a_2 x_2 - cy \end{gathered} \tag{18}$$

It is easy to prove that the mass conservation equation ($b = a_2x_2 - cy$) is equivalent to the functional relationship $F(y,b) = 0$ when x_2 is replaced by the indirect function $x_2 = u^{-1}(b)$.[20] From **(18)**, the coupling between good and bad outputs is obtained through the materials balance relation where good and bad outputs are expressed in the same mass content. As indicated in Førsund (2009), the introduction of materials balance in **(17)** and **(18)** will results in putting some bounds on derivatives (also trade-offs) in the empirical modeling.

Another solution can be a direct estimation of the technical connection between a good and a bad output. Using the properties in **(12)**, the functional relationship $F(y,b) = 0$ can be estimated. Additional properties like null-jointness or emission-causing input essentiality can also be added for the estimation of $\hat{F}(y,b)$.

3.2.Practical solutions in the DEA framework

Solution 1

We theoretically proved above how materials balance principle can be used to build coupling between products. From the previous discussions (**Limit 4**), we know that materials balance does not allow inefficiency since it is an accounting identity (it is absolute and holds at any point of the input-output space, including points located on the frontier (Førsund, 2009)). As earlier proposed, one way to assess for instance the environmental efficiency in this situation[21] is to endogenize the levels of polluting inputs. Let's consider the following model

$$\begin{aligned} &\max_{\phi,\nu,\xi,x} I^n_{ERBDDM} = \frac{1}{R}\sum_{q=1}^{R}\phi^n_r \\ s.t.\ &\sum_{i=1}^{N}\nu_i y_{qi} \geq y_{qn} \quad q = 1,\dots,Q \\ &\sum_{i=1}^{N}\nu_i x_{ki} \leq x_{kn} \quad k = 1,\dots,K \\ &\sum_{i=1}^{N}\xi_i b_{ri} \leq b_{rn} - \phi^n_r \vec{g}^r_b \quad r = 1,\dots,R \end{aligned} \tag{19}$$

20 By allusion to the situation described in (10) we assume equality between the implicit functions.

21 It is possible to imagine situations where the materials balance can allow for inefficiency. For example the lack of maintenance of an electric heater can create more energy losses. In this case, the inefficiency is related to the transformation factor of electricity for instance in heat.

$$\sum_{i=1}^{N} \xi_i x_{ki2} \geq x_{kn2} \quad k_2 = 1, \dots, K_2$$

$$\sum_{i=1}^{N} \nu_i = 1 \ ; \ \sum_{i=1}^{N} \xi_i = 1 \ ; \ \nu_i, \xi_i \geq 0; i = 1, \dots, N \ ; \ \phi \geq 0$$

Attempts to include mass conservation in the production technology can be seen in Rødseth (2015) where the author uses the concept of weak G-disposability to incorporate the materials balance using inputs and outputs slacks. However this approach does not consider the existence of an unintended output sub-technology. Since the materials balance holds at any point of the input-output space, at the optimality the following condition is verified for any bad output r:

$$b_r^* = \sum_{k=1}^{K_2} a_k^r x_{k2}^* - \sum_{q=1}^{Q} c_q^r y_q^* \tag{20}$$

where $*$ denotes optimal levels of the corresponding variables. Using model **(19)**, **(20)** can be rewritten as:

$$\sum_{i=1}^{N} \xi_i^* b_{ri} = \sum_{k=1}^{K_2} \sum_{i=1}^{N} a_k^r \nu_i^* x_{ki2} - \sum_{q=1}^{Q} \sum_{i=1}^{N} c_q^r \nu_i^* y_{qi} \tag{21}$$

The relation in **(21)** considers the optimal levels of polluting inputs under the intended outputs technology. Since **(19)** is made of two sub-technologies, the relation is also true for the optimal levels of these polluting inputs under the unintended outputs technology. We have

$$\textstyle\sum_{i=1}^{N} \xi_i^* b_{ri} = \sum_{k=1}^{K_2} \sum_{i=1}^{N} a_k^r \xi_i^* x_{ki2} - \sum_{q=1}^{Q} \sum_{i=1}^{N} c_q^r \nu_i^* y_{qi} \tag{22}$$

A practical inclusion of the materials balance in **(19)** is possible by adding the two equations **(20)** and **(21)**. Using the data in **Table 4**, we obtain the same results as the materials balance model in **(15)**.

Solution 2

Solution 2 is a simplification of the previous solution and is more general because it can even apply to situations where materials balance does not exist. The difference between the relation in **(21)** and the one in **(22)** is related to the optimal levels of emission-causing inputs. If we subtract **(22)** from **(21)**, we obtain the relation in **(23)**.

$$\sum_{k=1}^{K_2} \sum_{i=1}^{N} a_k^r \nu_i^* x_{ki2} - \sum_{k=1}^{K_2} \sum_{i=1}^{N} a_k^r \xi_i^* x_{ki2} = 0 \quad \Leftrightarrow \tag{23}$$

$$\sum_{k=1}^{K_2} a_k^r \left(\sum_{i=1}^{N} v_i^* x_{ki2} - \sum_{i=1}^{N} \xi_i^* x_{ki2} \right) = 0$$

Given that $a_k^r \neq 0 \; \forall \, r, k,$ from **(23)** we can derive a particular solution which simplifies the conditions in **(21)** and **(22)**.

$$\sum_{i=1}^{N} \boldsymbol{v_i^* x_{ki2}} - \sum_{i=1}^{N} \boldsymbol{\xi_i^* x_{ki2}} = \mathbf{0} \quad \boldsymbol{k} = \mathbf{1}, \dots, \boldsymbol{K_2} \tag{24}$$

We call the conditions in **(24)** as the dependence constraints (or interconnectedness constraints) which implicitly introduce couplings between bad and good outputs (since it is a simplification of the materials balance condition). These dependence constraints state that the efficient combination level of the polluting inputs should be equal in both sub-technologies. This systematically overcomes the issues identified in **Limit 3** and **4** in the previous section and also the implicit function theorem situation summarized in **(10)**. The dependence constraints can be viewed as the integration of residual production into the overall technology, implying that it is not just the production of good outputs that matters, but also the generation of detrimental outputs. By adding constraints **(24)** to model **(19)**, we link up the two sub-technologies involved. These dependence constraints implicitly account for the tradeoffs between operational and environmental performances (output orientation). Besides, the equality in **(24)** simply transcribes the idea stated by Førsund (2009) that 'obviously the direction of a coupling …is unrestricted in sign, as well as the corresponding direction within a factor band'. Let's stress again here that the dependence constraints are only related to polluting inputs. More simply these constraints do not apply to non-polluting inputs because they do not appear in both sub-technologies, whereas pollution generating inputs do.

In terms of dominance, as earlier pointed out, under Ψ_1 a DMU n is dominated by those DMUs that use fewer inputs to produce more good outputs [$x \leq x_n$ & $y \geq y_n$]. Under the second sub-technology Ψ_2, DMU_n is dominated by the set of observations that pollute less by using more inputs [$x \geq x_n$ & $b \leq b_n$]. In the by-production model proposed by MRL and reported in model **(6),** these two dominating sets of DMU_n are treated separately to build an overall efficiency score and, therefore, the interaction between the two objectives is ignored. At optimality, the model proposed by MRL will result in unexpected different levels of inputs between the two sub-technologies. This can be explained by the fact that the model relies on two independent benchmarks (**Limit 3** and **4**). In **Figure 1**, this independence can result, in the case of sub-technology Ψ_1, in a projection of DMU_n onto the upper sub-frontier in B corresponding to an input level x_B. In the case of sub-technology Ψ_2, however, DMU_n is projected towards the lower sub-frontier in point E associated with input quantities x_E. It is obvious (graphically) that the two optimal levels of inputs are not equivalent ($x_B \neq x_E$). By contrast, the introduction of constraints **(24)** can create a unique benchmark (a convex combination of different DMUs) and the tradeoff between the two objectives of operational and environmental performance improvements can be accounted for. It is important to note that these dependence constraints do not deform the initial sub-technologies Ψ_1 and Ψ_2, but modify the benchmarks in a way that coupling is present between good and bad outputs.

Advantages of **Solution 2** over **Solution 1** are its generalisation and the fact that no information on emissions factor is required for the model to be operated. Besides, equations in **(23)** can have multiple solutions other than the one in **(24)**. Even if the efficiency results are the same the independence between the benchmarks makes **Solution 1** less powerful than **Solution 2**. Another advantage of **Solution 2** over **Solution 1** lies in the case where DMUs do not share the same mass contents for polluting inputs (and good outputs abatement factors). In this situation, **Solution 1** can sometimes wrongly identify as benchmarks inefficient DMUs with respect to unintended outputs. The benchmark with respect to unintended outputs is obtained in the situation where the efficiency is simply evaluated when one only considers the bad output sub-technology and seeks for a minimization of bad outputs given the levels of polluting inputs. This program identifies inefficiency present in mass conservation equation since the transformation factors now differ among DMUs.

<u>Solution 3</u>

As we argued in the theoretical approach, another solution can be the direct estimation of the relation $F(y, b) = 0$. Within the DEA framework, we got inspired by the literature on frontier eco-efficiency approach (Kuosmanen and Kortelainen, 2005) and we propose to augment the program in **(19)** with some constraints that give a description of $F(y, b) = 0$ and which share some properties with the sub-technologies Ψ_1 and Ψ_2 (convexity, VRS).

The following program can be solved

$$
\begin{aligned}
&\max_{\phi,\nu,\xi,x} I^{n}_{ERBDDM} = \frac{1}{R}\sum_{q=1}^{R}\phi_r^n \\
&s.t.\ \sum_{i=1}^{N}\nu_i y_{qi} \geq y_{qn} \quad q = 1,\ldots.,Q \\
&\sum_{i=1}^{N}\nu_i x_{ki} \leq x_{kn} \quad k = 1,\ldots,K \\
&\sum_{i=1}^{N}\xi_i b_{ri} \leq b_{rn} - \phi_r^n \vec{g}_b^r \quad r = 1,\ldots,R \\
&\sum_{i=1}^{N}\xi_i x_{ki2} \geq x_{kn2} \quad k_2 = 1,\ldots,K_2 \\
&\sum_{i=1}^{N}\mu_i y_{qi} \geq y_{qn} \quad q = 1,\ldots.,Q \\
&\sum_{i=1}^{N}\mu_i b_{ri} \leq b_{rn} - \phi_r^n \vec{g}_b^r \quad r = 1,\ldots,R \\
&\sum_{i=1}^{N}\nu_i = 1 \ ; \ \sum_{i=1}^{N}\xi_i = 1 \ ; \ \sum_{i=1}^{N}\mu_i = 1 \ ; \ \nu_i,\xi_i,\mu_i \geq 0; i = 1,\ldots,N \ ; \ \phi \geq 0
\end{aligned}
\tag{25}
$$

The constraints $\sum_{i=1}^{N}\mu_i y_{qi} \geq y_{qn}$, $\sum_{i=1}^{N}\mu_i b_{ri} \leq b_{rn} - \phi_r^n \vec{g}_b^r$ and $\sum_{i=1}^{N}\mu_i = 1$ are introduced in the MRL approach to represent an estimation of the coupling relation $\hat{F}(y,b) = 0$. As regard to conditions in **(12)**, we represent the coupling building on the conditions $\frac{\delta F}{\delta b} \leq 0$, $\frac{\delta F}{\delta y} \geq 0$ to keep consistency with the outputs relations in the other sub-technologies. As in the case of **Solution 1**, when DMUs do not share anymore the same emission contents the model in **(25)** tends to identify as benchmarks inefficient DMUs in the sense that they are inefficient when we consider only the unintended outputs generation sub-technology.[22] Again the particular solution in **(24)** offers better alternative under the situation of non-common pollution contents of inputs/outputs.

22 It is worth recalling that the introduction of a coupling relation in the MRL approach does not reshape the original sub-technologies (intended and unintended outputs production).

For simplicity, all the models considered in **Solutions 1** to **3** are not factorially determined. This is an easy extension especially in the case of multiple pollutants.

4. An empirical illustration

In this section we support all our above developments regarding the by-production modeling using an existing database of different countries around the world.

4.1. Dataset description

The database is made of 112 countries from the different continents independently of their income per capita levels (rich and poor countries are simultaneously considered) and observed in year 2011. We have retained three inputs for the analysis: labour which is the total labour force available in each country, gross capital formation, and fossil fuel energy consumption which is considered as the pollution-causing input. Gross Domestic Product (GDP) is the desirable output while carbon dioxide (CO_2) emissions are the by-products associated to burning fossil fuels and the manufacturing of cement. The summary statistics of the sample are displayed in **Table 6**.

Table 6: Summary statistics of the data used (112 countries in 2011)

Variables	Mean	Standard deviation	Relative standard deviation	Minimum	Maximum
Gross Domestic Product – GDP – (billions of US 2005 constant Dollars)	463	1,484	3.2	1.1	13,817
Carbon dioxide emissions – CO_2 – (kilotons)	270,890	1,012,941	3.7	520.7	9,019,518
Labour force (millions of people)	26	88	3.4	0.2	790
Gross capital formation (billions of US 2005 constant Dollars)	111	335	3.0	0.1	2,650
Fuel energy consumption (kilotons of oil equivalent)	85,107	296,783	3.5	166.2	2,428,787

Notes: The relative standard deviation is the ratio of the standard deviation to the mean. All the data are obtained from the world development indicators (WDI) of the World Bank. The list of countries can be found in Annex. Fuel *energy consumption is computed using the proportion of this energy consumed in the total energy use, the energy consumption per capita and the total population size.*

Source: World Bank (http://databank.worldbank.org/data/home.aspx)

4.2. Models and results

For the MRL approach we consider the model in **(13)** under first the fixed levels of pollution inputs and second by endogenizing the levels of the emission-causing inputs. We consider this latter assumption in light of the materials balance and the discussion in **Limit 4**. Regarding the solutions, we retain **Solution 2** because, first, we have no information about the emissions factor associated to the solely polluting inputs considered here, and, second, we cannot assert that the emission factors are the same among the different countries given that the fuel energy consumption is an aggregated input. The main results are summarized in **Table 7**.

Table 7: Descriptive statistics for ERBDDM inefficiency scores under several models

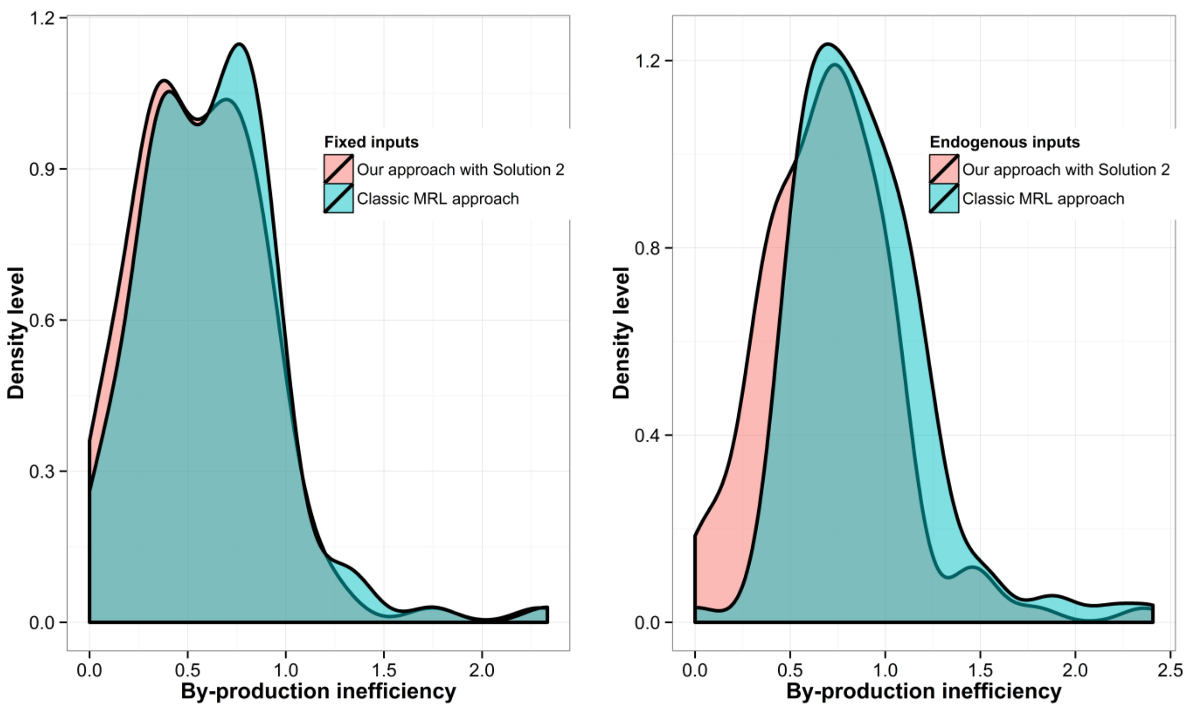
Models	Type of inefficiency	Minimum	Median	Mean	Standard deviation	Maximum
Classic MRL model with fixed levels of all inputs	**Overall**	**0**	**0.626**	**0.616**	**0.355**	**2.333**
	Operational	0	0.641	0.700	0.614	3.842
	Environmental	0	0.648	0.532	0.237	0.930
Classic MRL model with free levels of polluting inputs and fixed levels of non-polluting inputs	**Overall**	**0**	**0.837**	**0.881**	**0.360**	**2.407**
	Operational	0	0.702	0.815	0.719	3.842
	Environmental	0	0.986	0.947	0.110	1.000
Extension relying on Solution 2 **with dependence constraints and fixed levels of all inputs**	**Overall**	**0**	**0.565**	**0.568**	**0.355**	**2.292**
	Operational	0	0.501	0.604	0.591	3.760
	Environmental	0	0.648	0.532	0.237	0.930
Extension relying on Solution 2 **with dependence constraints, free levels of polluting inputs and fixed levels of non-polluting inputs**	**Overall**	**0**	**0.701**	**0.707**	**0.363**	**2.360**
	Operational	0	0.664	0.753	0.692	3.824
	Environmental	0	0.712	0.661	0.224	0.944

*Note: The ERBDDM is based on the directional distance function approach, and thus all scores in **Table 7** should be interpreted as inefficiency levels. The operational inefficiency is inefficiency associated to the good outputs' maximization while the environmental inefficiency is related to bad outputs' minimization. The overall inefficiency is the arithmetic average of both operational and environmental inefficiencies.*

Source: authors

The first difference between the two by-production approaches - the classic approach by MRL based on independent sub-technologies and ours with dependence constraints - is the higher level of inefficiency scores obtained with the classic MRL model. However, in the case of our empirical available data, the differences are quite small when all inputs are assumed to be fixed: for instance, on average, the overall inefficiency score with independent sub-frontiers is 0.616, while it is 0.568 when dependence constraints are introduced. This small difference is fairly understandable given the low flexibility present in this situation of fixed levels of inputs. For robustness check, we run the two-sample Kolmogorov-Smirnov and the Wilcoxon rank sum tests which both reveal no differences in the distributions of all inefficiency scores when all inputs are held fixed (see also **Figure 2** for inefficiency distribution comparison where density plots on the left panel, where inputs are held fixed, seem to overlap).

Figure 2: Inefficiency distribution comparison between two by-production approaches



Source: authors

When the constraints on emission-causing inputs are relaxed so that such inputs can be freely chosen, the difference between the two models is clearly accentuated (right panel on **Figure 2**).

The classic MRL model attributes higher inefficiency compared to our extension with dependence constraints. This situation can be explained by the inadequate handling of materials inputs in the MRL approach. The distribution tests reveal significant differences in the distribution of the overall and the environmental inefficiencies. Actually, it seems that the classic MRL model results are outliers deriving from the fact that the approach tends to cumulatively identify the best performers under each sub-technology independently on where those DMUs are located on each frontier. This can explain the higher inefficiency of the MRL approach. Still in the case of flexible assumption of free choice of polluting inputs, the classic MRL approach indicates that bad outputs management is the major source of inefficiency. By contrast, our *extended by-production approach* with **Solution 2** (dependence constraints), where the classic MRL model is augmented with dependence constraints, identifies good outputs production as the major source of inefficiency. Considering our model (and endogenous levels of polluting inputs), the top five best performers are Eritrea, France, Japan, the United Kingdom and the United States. The five worst performers are Mongolia, Nepal, Oman, Vietnam and Zimbabwe.

5. Conclusion

This paper proposes practical extensions of the by-production modeling formulated by MRL. The MRL approach, based on the estimation of distinct sub-technologies to characterize an overall pollution-generating technology, is one of the most promising models for capturing the production of undesirable outputs. The main advantage of this approach is that it is based on a full description of production processes. The theoretical aspects of this approach are now clearly defined (Murty, 2012). However, we argued in this paper that the practical use of DEA proposed by MRL fails to unify the two sub-technologies in opposite to what is developed in the theory, because of the lack of coupling between outputs. Following this observation, we developed three extensions of the MRL by-production approach by including: (i) materials balance through the mass-conservation equation; (ii) some dependence constraints; (iii) a direct estimation of the coupling relation between good and bad outputs. We also argued that the second extension, the one with the use of the dependence constraints, may surpass the other two under certain conditions. These additional constraints offer some interesting opportunities to theoretically and empirically discuss the nature of regulations designed to integrate detrimental output generation into managers' strategic decisions. Another interesting feature of the by-production approach is the possibility of explicitly incorporating abatement outputs in the production processes.

We argued that the introduction of coupling between good and bad outputs does not reshape the initial sub-technologies, which can be constructed with different DMUs. Actually solutions like **Solution 2** (case (ii) above) simply modify the relations between the benchmarks of an inefficient observation. Practically this leaves room for discussion on coupling that involves relations with the intensity variables (ν, ξ) in the DEA programming. For instance the relation can be set such that, for an inefficient observation, the DMUs that serve as benchmarks under the intended output sub-technology are the same under the emission-generating sub-technology.[23] It is however probable that the resulting sub-technologies would be different from the original independent ones. The question is therefore what the reality looks like? The choice of coupling as the one in our **Solution 2** or the one associated to the same benchmark under both sub-technologies can be assessed through expert knowledge of the systems that is to be represented. As we mentioned earlier a benefit of **Solution 2** is its general flexibility.

One limit is that we have not given much importance to inputs minimization expressly included in the objective function (except for the contribution discussed in Limits 3 and 4). First, given materials balance conditions, if inputs can be reduced by α % so, the bad outputs could be reduced by at least the same proportion. Second, by doing so, we do not capture the allocative inefficiency associated to the inefficient combinations of the different inputs. Thereby we have given much focus on endogenizing the levels of polluting inputs while seeking for the minimization of bad outputs.

Another point that worths more discussion is the returns to scale under the bad output sub-technology. Considering again that materials balance guides the physical processes, and the input essentiality property is maintained it seems intuitive to assume constant returns to scale under the bad outputs sub-technology. In other words with zero levels of polluting inputs there will be zero levels of pollution and the increase (decrease) in pollution will be proportional to the increase (decrease) in polluting inputs.

Given the sensitivity of nonparametric approaches to outliers, an extension to the estimation of robust versions is necessary (Cazals *et al.*, 2002; Aragon *et al.*, 2005; Daouia and Gijbels, 2011). It is also important to develop algorithms for the estimation of conditional inefficiency

23 Mixed integer linear programming can certainly be helpful in this achievement.

scores along with the derivation of statistical inference in light of the discussions by Simar and Wilson (2015) and Simar *et al.* (2013).

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Annex: List of countries

Angola; Argentina; Armenia; Australia; Austria; Azerbaijan; Bangladesh; Belarus; Belgium; Benin; Bolivia; Botswana; Brazil; Brunei Darussalam; Bulgaria; Cambodia; Cameroon; Canada; Chile; China; Colombia; Congo, Dem. Rep.; Congo, Rep.; Costa Rica; Croatia; Cuba; Cyprus; Czech Republic; Denmark; Ecuador; Egypt, Arab Rep.; El Salvador; Eritrea; Estonia; Finland; France; Gabon; Georgia; Germany; Greece; Guatemala; Haiti; Honduras; Hong Kong SAR, China; Hungary; Iceland; India; Indonesia; Ireland; Israel; Italy; Japan; Jordan; Kazakhstan; Kenya; Korea, Rep.; Kyrgyz Republic; Latvia; Lebanon; Lithuania; Luxembourg; Macedonia, FYR; Malaysia; Mauritius; Mexico; Moldova; Mongolia; Montenegro; Morocco; Mozambique; Namibia; Nepal; Netherlands; New Zealand; Nicaragua; Nigeria; Norway; Oman; Pakistan; Panama; Paraguay; Peru; Philippines; Poland; Portugal; Romania; Russian Federation; Senegal; Serbia; Singapore; Slovak Republic; Slovenia; South Africa; Spain; Sri Lanka; Sudan; Sweden; Switzerland; Tajikistan; Tanzania; Thailand; Togo; Turkey; Ukraine; United Arab Emirates; United Kingdom; United States; Uruguay; Uzbekistan; Venezuela, RB; Vietnam; Zimbabwe.

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