Evolving Seasonal Pattern of Tenerife Tomato Exports

Gloria Martín Rodríguez
gmartinr@ull.es
José Juan Cáceres Hernández
jcaceres@ull.es
University of La Laguna
Department of Economía de las Instituciones, Estadística Económica y Econometría
Faculty of Ecnomics, Campus de Guajara
38071, La Laguna, Tenerife, Spain

Paper prepared for presentation at the XI[th] *International Congress of EAAE*
(European Association of Agricultural Economists),
'The Future of Rural Europe in the Global Agri-Food System', Copenhagen, Denmark, date as in: August 24-27, 2005

EVOLVING SEASONAL PATTERN OF TENERIFE TOMATO EXPORTS

Abstract

The aim of this paper is to analyse the long term movements and, particularly, the seasonal pattern of *Tenerife* (Canary Islands) tomato exports throughout the last two decades. In order to observe more clearly the exporter's decisions, weekly data has been used. The instabilities in the long term behaviour of the series and the specific nature of the seasonal pattern should be taken into account in order to capture the performance of exports accurately. Thus, this analysis is carried out inside the frame delimited by the structural approach to time series and the usefulness of evolving splines as a tool capable of modelling seasonal variations in which either the period or the magnitude of the fluctuations do not remain the same over time is shown.

Key words: Tenerife tomato exports, weekly data, structural models, evolving splines.
JEL classification: C22, Q17.

1. Introduction

The European tomato market is characterised by a constant process of dynamic adjustment towards equilibrium. Furthermore, Canary tomato exports cause a high seasonal impact on market prices in the winter period. In these circumstances, an adequate distribution of weekly shipments throughout the year could contribute to maximise producers' profits. In this paper, the evolution of the weekly exports of *Tenerife* tomatoes is analysed in the last twenty four harvests.

Before building an econometric model capturing the variations of this series, it is useful to point out some features of the tomato export activity in *Tenerife*. Firstly, the seasonal pattern of exports is characterised by concentration in winter and disappearance in summer. This pattern is a rational response guided by the search for profitability; there are no exports in summer because Northern European countries and Canary supplies converge in this season and so Canary tomato prices would be low. Secondly, the development of greenhouse technology in Northern Europe, the increase in mainland Spain supply and the third country supplies sharing the same export period as Canary product entry to the European market have led to a growing overlap of the different supplies in spring and autumn. Finally, Spain's full integration into the *CAP* has brought about the abolition of the mechanisms used by European producers as custom barriers against production from the Canary Islands[1] and these changes have encouraged Canary producers to increase their exports, despite the fact that Moroccans have also benefited from a significant reduction in these barriers[2]. During the last harvests, quality problems, low prices and exporters' expectations brought about a new decrease of export levels. From these remarks, it follows that the volumes of *Tenerife* tomatoes exported in the different weeks of the year have not kept stable throughout the last two decades. In order to handle these instabilities structural models appear to be an appropriate tool. The statistical techniques used in this paper can be framed inside this approach. A similar series was analysed in Martín and Cáceres (2004) by using structural models and two fixed splines. In this new paper, a method of dealing with a changing seasonal pattern by means of spline functions is proposed[3].

The plan of this paper is as follows. In the next section, the data used is identified and some interesting features of its nature and preliminary processing are discussed. In the case under study, some properties of the series do not appear to remain the same over time; then, structural models are an appropriate class of models to cope with this kind of situation. The basic statistical framework for handling a seasonal pattern in which either the period or the magnitude of the seasonal variations do not remain the same over time is outlined in the third section. In section four, this procedure is applied to the weekly series of *Tenerife* tomato exports. Section five presents the conclusions.

2. Data

This section is concerned with the series of weekly *Tenerife* tomato exports (measured in 6 kg boxes) from 1980/1981 to 2003/2004 harvests[4]. According to data identification purpose, each harvest is considered to start in week 27 of a year and conclude in week 26 of the following year[5]. The series is shown in Figure 1.

---

[1] For a detailed explanation of these protection mechanisms, see Cáceres (2000, pp. 292-305).

[2] The access conditions to European markets for Moroccan products are analysed in Cáceres (2000, pp. 278-281, 308-312). Recently, a new *EU*-Morocco agreement has been reached.

[3] Cáceres (2001) and Martín et al. (2002) use a more rigid specification of changing deterministic components to capture this kind of instabilities.

[4] Export statistics have been obtained from weekly data published by the provincial exporter association of *Santa Cruz de Tenerife* (*ACETO*) in its export season reports. In those weeks where this source did not register any data, a zero value has been assigned.

[5] When there are 53 observations that correspond to the same year, the starting point of the harvest has been moved forward by one week in these cases. In this way a series is obtained with 52 on each year of the period under study.

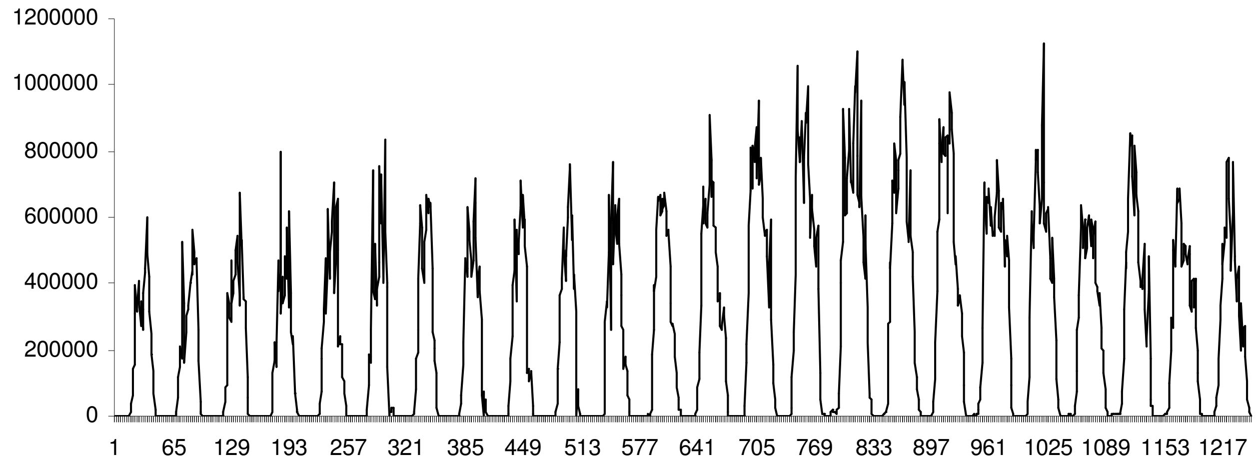


Figure 1. *Tenerife* tomato exports from 1980/1981 to 2003/2004 harvests.

In Figure 1 three periods differing by the long-term movement can be distinguished. The new trade situation of the Canary Islands with regard to the *EU* since July 1991 (reference prices were substituted by supply prices) and the full integration into the *EU* since January 1st 1993 (abolition of reference/supply prices) brought about a significant export boost. The general growth in exports in this second period was interrupted in 1996, coinciding with the introduction of a trade agreement between the *EU* and Morocco. As regard the seasonal pattern, a harvest by harvest rising movement is observed that begins in October and finishes in January or February, followed by another downward movement that continues until May or June. However, two periods differing by the extent of the harvest can be distinguished. From the 1991/1992 harvest, the harvests, often finished in early May, continued until June. Although the export period in each harvest has kept stable since the 1991/1992 harvest, changes of seasonal behaviour are observed. Therefore, and as a preliminary hypothesis, it could be assumed that there is a changing seasonal component around a stochastic trend component. In this paper structural models are used as a tool capable of capturing these instabilities. Once the conclusion is reached that the seasonal pattern in each of the periods mentioned is not fixed, these patterns will be modelled by a evolving periodic cubic spline. This methodology is briefly explained in the next section.

3. Evolving periodic cubic splines

In a structural time series model[6] formulated as

$$y_t = \mu_t + \gamma_t + \varepsilon_t \, , \; t = 1,...,T \, , \tag{1}$$

where μ_t and γ_t are the trend or level component and the seasonal component, and ε_t is the irregular component, modelling the seasonal pattern by means of a set of regressors defining a spline function could be interesting[7]. This section deals with the appropriate specification of a periodic cubic spline able to capture a seasonal pattern in which either the period or the magnitude of seasonal variations do not remain the same. In order to do this, an adequate procedure where a function related to adjustment error is optimised, but crossing given points is not required, is proposed in the following paragraphs.

When the seasonal pattern is fixed, $\gamma_t = \gamma_w$ if the observation at time t corresponds to the season w, $w = 1,...,s$; then, this component can be modelled by a periodic cubic spline. That is,

[6] See Harvey (1989) and Durbin and Koopman (2001).
[7] See Poirier (1973, 1976), Marsh (1983, 1986), Marsh et al. (1990), Koopman (1992), Harvey et al. (1997), Martín and Cáceres (2004).

$$\gamma_w = g(w) + \xi_w, \tag{2}$$

where ξ_w is a residual term and $g(w)$ is a third degree piecewise polynomial function,

$$g_i(w) = g_{i,0} + g_{i,1}w + g_{i,2}w^2 + g_{i,3}w^3,\ w_{i-1} \le w \le w_i,\ i = 1,...,k-1, \tag{3.a}$$

$$g_k(w) = g_{k,0} + g_{k,1}w + g_{k,2}w^2 + g_{k,3}w^3,\ w_{k-1} \le w \le s, \tag{3.b}$$

where w_0 is the first season.

Koopman (1992) and Harvey et al. (1997) propose to use the following procedure in order to obtain the previous spline. Let $\nabla^2 g_i(w_i) = a_i$, $i = 1,...,k$, be the values of the second derivative of the spline evaluated at the break points w_i, $i = 1,...,k$, with $w_k = s+1$. Then, the continuity of the second derivative of the spline function is enforced by the following conditions

$$\nabla^2 g_i(w_{i-1}) = a_{i-1},\ i = 2,...,k, \tag{4.a}$$

and

$$\nabla^2 g_k(w_k) = a_0, \tag{4.b}$$

in such a way that $a_0 = a_k$. The second derivative of the spline is a linear function such as

$$\nabla^2 g_i(w) = [(w_i - w)/(w_i - w_{i-1})]a_{i-1} + [(w - w_{i-1})/(w_i - w_{i-1})]a_i,\ i = 1,...,k. \tag{5}$$

Now, spline can be obtained taking into account that

$$\nabla g_i(w) = -\frac{1}{2}\left[(w_i - w)^2/(w_i - w_{i-1})\right]a_{i-1} + \frac{1}{2}\left[(w - w_{i-1})^2/(w_i - w_{i-1})\right]a_i + k_{1,i},\ i = 1,...,k, \tag{6}$$

in such a way that

$$g_i(w) = -\frac{1}{6}\left[(w_i - w)^3/(w_i - w_{i-1})\right]a_{i-1} + \frac{1}{6}\left[(w - w_{i-1})^3/(w_i - w_{i-1})\right]a_i + wk_{1,i} + k_{2,i},\ i = 1,...,k. \tag{7}$$

If it is assumed that the spline crosses the knots (w_i, γ_i^+), $i = 0,...,k-1$, that is to say, $\mathrm{g}_i(w_{i-1}) = \gamma_{i-1}^+$ and $\mathrm{g}_i(w_i) = \gamma_i^+$, then

$$k_{1,i} = \frac{(\gamma_i^+ - \gamma_{i-1}^+)}{(w_i - w_{i-1})} - \frac{1}{6}(w_i - w_{i-1})(a_i - a_{i-1}) \tag{8.a}$$

and

$$k_{2,i} = \gamma_i^+ - \frac{1}{6}(w_i - w_{i-1})^2 a_i - \frac{(\gamma_i^+ - \gamma_{i-1}^+)}{(w_i - w_{i-1})}w_i + \frac{1}{6}(w_i - w_{i-1})(a_i - a_{i-1})w_i. \tag{8.b}$$

Therefore

$$g_i(w) = -\frac{w_i - w}{6(w_i - w_{i-1})}\left[(w_i - w)^2 - (w_i - w_{i-1})^2\right]a_{i-1} + \frac{w - w_{i-1}}{6(w_i - w_{i-1})}\left[(w - w_{i-1})^2 - (w_i - w_{i-1})^2\right]a_i + \\ + \frac{w - w_{i-1}}{(w_i - w_{i-1})}\gamma_i^+ + \frac{w_i - w}{(w_i - w_{i-1})}\gamma_{i-1}^+ \tag{9}$$

for $i = 1,...,k$. Now, by demanding continuity conditions of the first derivative of the spline, that is to say,

$$\nabla g_i(w_i) = \nabla g_{i+1}(w_i)\,,\; i = 1,...,k-1\,, \tag{10.a}$$
$$\nabla g_1(w_0) = \nabla g_k(w_k)\,, \tag{10.b}$$

k equations are obtained and $a_0,...,a_k$ parameters can be expressed as functions of $\gamma_0^+,...,\gamma_k^+$ parameters. Bearing in mind that $a_0 = a_k$ and $\gamma_0^+ = \gamma_k^+$, it is obtained that

$$\frac{w_i - w_{i-1}}{6}a_{i-1} + \frac{2\left[(w_i - w_{i-1}) + (w_{i+1} - w_i)\right]}{6}a_i + \frac{w_{i+1} - w_i}{6}a_{i+1} = \\ \frac{1}{w_i - w_{i-1}}\gamma_{i-1}^+ - \left(\frac{1}{w_i - w_{i-1}} + \frac{1}{w_{i+1} - w_i}\right)\gamma_i^+ + \frac{1}{w_{i+1} - w_{i-1}}\gamma_{i+1}^+ \tag{11.a}$$

for $i = 1,...,k-1$, and

$$\frac{2\left[(w_k - w_{k-1}) + (w_1 - w_0)\right]}{6}a_0 + \frac{w_1 - w_0}{6}a_1 + \frac{w_k - w_{k-1}}{6}a_{k-1} = \\ -\left[\frac{1}{w_k - w_{k-1}} + \frac{1}{w_1 - w_0}\right]\gamma_0^+ + \frac{1}{w_1 - w_0}\gamma_1^+ + \frac{1}{w_k - w_{k-1}}\gamma_{k-1}^+ \tag{11.b}$$

In matrix form,

$$W_1 A = W_2 \Gamma^+\,, \tag{12}$$

where $A:(a_0,...,a_{k-1})$, $\Gamma^+:(\gamma_0^+,...,\gamma_{k-1}^+)$,

$$W_1 = \begin{bmatrix} \frac{z_1}{6} & \frac{z_1 + z_2}{3} & \frac{z_2}{6} & 0 & \cdots & 0 \\ 0 & \frac{z_2}{6} & \frac{z_2 + z_3}{3} & \frac{z_3}{6} & \cdots & 0 \\ 0 & 0 & \frac{z_3}{6} & \frac{z_3 + z_4}{3} & \cdots & 0 \\ 0 & 0 & 0 & \frac{z_4}{6} & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \frac{z_k + z_1}{3} & \frac{z_1}{6} & 0 & 0 & \cdots & \frac{z_k}{6} \end{bmatrix} \tag{13}$$

and

$$W_2 = \begin{bmatrix} \frac{1}{z_1} & -\left(\frac{1}{z_1}+\frac{1}{z_2}\right) & \frac{1}{z_2} & 0 & \cdots & 0 \\ 0 & \frac{1}{z_2} & -\left(\frac{1}{z_2}+\frac{1}{z_3}\right) & \frac{1}{z_3} & \cdots & 0 \\ 0 & 0 & \frac{1}{z_3} & -\left(\frac{1}{z_3}+\frac{1}{z_4}\right) & \cdots & 0 \\ 0 & 0 & 0 & \frac{1}{z_4} & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ -\left(\frac{1}{z_k}+\frac{1}{z_1}\right) & \frac{1}{z_1} & 0 & 0 & \cdots & \frac{1}{z_k} \end{bmatrix} \quad (14)$$

with $z_i = w_i - w_{i-1}$, $i = 1,...,k$. Therefore $A = (W_1)^{-1} W_2 \Gamma^+$ and $G = W_A A + W_B \Gamma^+$, where $G = (g_1(w), ..., g_k(w))'$, $A = (a_0, ..., a_{k-1})'$, $\Gamma^+ : \left(\gamma_0^+, ..., \gamma_{k-1}^+\right)$ and W_A and W_B are matrices defined as

$$W_A = \begin{bmatrix} \frac{\delta_1}{6z_1}\left[\delta_1^2 - z_1^2\right] & \frac{-\delta_0}{6z_1}\left[\delta_0^2 - z_1^2\right] & 0 & 0 & \cdots & 0 \\ 0 & \frac{\delta_2}{6z_2}\left[\delta_2^2 - z_2^2\right] & \frac{-\delta_1}{6z_2}\left[\delta_1^2 - z_2^2\right] & 0 & \cdots & 0 \\ 0 & 0 & \frac{\delta_3}{6z_3}\left[\delta_3^2 - z_3^2\right] & \frac{-\delta_2}{6z_3}\left[\delta_2^2 - z_3^2\right] & \cdots & 0 \\ 0 & 0 & 0 & \frac{\delta_4}{6z_4}\left[\delta_4^2 - z_4^2\right] & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \frac{-\delta_{k-1}}{6z_k}\left[\delta_{k-1}^2 - z_k^2\right] & 0 & 0 & 0 & \cdots & \frac{\delta_k}{6z_k}\left[\delta_k^2 - z_k^2\right] \end{bmatrix} \quad (15)$$

and

$$W_B = \begin{bmatrix} \frac{\delta_1}{z_1} & \frac{-\delta_0}{z_1} & 0 & 0 & \cdots & 0 \\ 0 & \frac{\delta_2}{z_2} & \frac{-\delta_1}{z_2} & 0 & \cdots & 0 \\ 0 & 0 & \frac{\delta_3}{z_3} & \frac{-\delta_2}{z_3} & \cdots & 0 \\ 0 & 0 & 0 & \frac{\delta_4}{z_4} & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \frac{-\delta_{k-1}}{z_k} & 0 & 0 & 0 & \cdots & \frac{\delta_k}{z_k} \end{bmatrix} \quad (16)$$

with $\delta_i = w_i - w$, $i = 0,...,k$. Finally, it is obtained that

$$G = W_A A + W_B \Gamma^+ = W_A \left[(W_1)^{-1} W_2\right]\Gamma^+ + W_B \Gamma^+ = W\Gamma^+ . \quad (17)$$

Let $w_{p,q}$ be the element of W in row p and column q. Each $w_{p,q}$ is a function of the season w, in such a way that

$$g(w) = [w_{1,1}D_{1,w} + w_{2,1}D_{2,w} + ... + w_{k,1}D_{k,w}]\gamma_0^+ + \\ [w_{1,2}D_{1,w} + w_{2,2}D_{2,w} + ... + w_{k,2}D_{k,w}]\gamma_1^+ + ... + [w_{1,k}D_{1,w} + w_{2,k}D_{2,w} + ... + w_{k,k}D_{k,w}]\gamma_{k-1}^+ \tag{18}$$

where $D_{i,w} = \begin{cases} 1, w_{i-1} \le w < w_i \\ 0, in\ other\ case \end{cases}$, $i = 1,..., k-1$, and $D_{k,w} = \begin{cases} 1, w_{k-1} \le w \le s \\ 0, in\ other\ case \end{cases}$. So, the spline function can be expressed as a linear function

$$g(w) = \gamma_0^+ X_{0,w} + ... + \gamma_{k-1}^+ X_{k-1,w}. \tag{19}$$

Koopman (1992) and Harvey et al. (1997), based on Poirier (1973, 1976), propose to obtain the values of such a function by using the observed values of the seasonal pattern, $\Gamma^+ : (\gamma_0^+, ..., \gamma_{k-1}^+)$. This approach can be generalised to allow the seasonal pattern to evolve over time by letting spline be stochastic[8]. However, in this paper it is proposed to specify the seasonal pattern as $\gamma_w = g(w) + \xi_w$, where the spline $g(w)$ is expressed as

$$g(w) = \gamma_0^* X_{0,w} + ... + \gamma_{k-1}^* X_{k-1,w}, \tag{20}$$

where $X_{0,w}, ..., X_{k-1,w}$ are the regressors previously defined and $\gamma_0^*, ..., \gamma_{k-1}^*$ are free parameters which must be estimated. In this way, the seasonal pattern can be incorporated into a structural model as a function of such regressors.

The previous specification is flexible enough to capture a non-fixed seasonal pattern. The period under study could be divided in sub-periods of s time units (seasons). Suppose that there are m sub-periods. For the sub-period c, $c = 1,..., m$, appropriate regressors $X_{0,w}^c, ..., X_{k_c-1,w}^c$ can be defined as functions of the break points w_i^c, $i = 0,..., k_c - 1$. Although the break points would be assumed to be the same for different sub-periods, in such a way that regresssors $X_{0,w}^c, ..., X_{k_c-1,w}^c$ are also the same, changes in the magnitude of seasonal variations can be captured by defining different parameters $\gamma_0^{c*}, ..., \gamma_{k_c-1}^{c*}$ for each sub-period. Furthermore, when the period s in which the seasonal variation is completed does not remain the same over time, the length of the sub-period c can be defined as s_c, $c = 1,..., m$. That is to say, the seasonal pattern can be formulated as $\gamma_t = g(t) + \xi_t$, where the spline $g(t)$ is expressed as

$$g(t) = \sum_{c=1}^{m} [\gamma_0^{c*} X_{0,t}^c + ... + \gamma_{k_c-1}^{c*} X_{k_c-1,t}^c] D_{c,t}^c, \tag{21}$$

where $D_{c,t}^t = \begin{cases} 1, t \in sub-period\ c \\ 0, in\ other\ case \end{cases}$, $c = 1,..., m$, and $X_{i,t}^c = X_{i,w}^c$, $i = 0,..., k_c - 1$, if the observation at time t corresponds to the season w, $w = 1,..., s_c$. When the length s_c and the break points w_i^c are the same for

[8] See Koopman (1992) and Harvey et al. (1997).

all sub-periods, then $X^{c}_{i,w} = X_{i,w}$, $i = 0,...,k-1$, but the seasonal variations are able to evolve over time. When $\gamma^{c*}_{i,w} = \gamma^{*}_{i,w}$, $i = 0,...,k-1$, the seasonal pattern is fixed. Obviously, these assumptions lead to a more parsimonious formulation.

The critical point is the selection of the number and position of knots. The problem is very complicated when these coefficients are treated as parameters to be estimated. Experience shows that iterative estimation procedures lead to local minima and, generally, better results are obtained through heuristic methods involving successive adjustments in which it is assumed that locations are known[9]. In this sense, the decision has been adopted to select the combination of locations that minimises the residual sum of squares when the regression model

$$\gamma^{1}_{t} = \sum_{c=1}^{m} \left[\gamma^{c*}_{0} X^{c}_{0,t} + ... + \gamma^{c*}_{k_c-1} X^{c}_{k_c-1,t} \right] D^{c}_{c,t} + \xi_t , \tag{22}$$

is estimated, γ^{1}_{t} being a previous seasonal component approximation. For the chosen locations, the regressors $X^{c}_{0,t} D^{c}_{c,t}, ..., X^{c}_{k_c-1,t} D^{c}_{c,t}$, $c = 1, ..., m$, can be incorporated into the structural model as exogenous variables. However, given that $\sum_{c=1}^{m} \sum_{i=0}^{k_c-1} X^{c}_{i,t} D^{c}_{c,t} = 1$, $\forall t$, one of these regressors should be dropped.

4. Structural model for the export series

In this section the previously described methodology is, firstly, adapted to the specific nature of *Tenerife* export levels and, then, it is applied to this particular series. Because exports are almost or exactly zero for some weeks in each year of the sample and the non-export period is longer until the 90/91 harvest, a model with a fixed seasonal period throughout the sample fails. In the first period, there are regular exports from week 43 of the year to week 17 of the following year. In the other one, the export activity could be considered to start in week 42 and conclude in week 24. So, if only observations corresponding to these weeks are considered, a new series is obtained and it will be referred to as $\{y_t\}_{t=1,...,752}$, hereafter. It is appropriate to specify a model for the new series capable of capturing a seasonal pattern in which the period is 27 until the 90/91 harvest and other one in which the period is 35 from the 91/92 harvest. Note that in this way, the seasonal pattern throughout the harvest is not being described but the distribution of exports over the export period. In this paper, the proposal for coping with these two seasonal patterns consists of using a evolving spline function, because available statistical packages are not be able to estimate conventional stochastic formulations in order to cope with a changing seasonal period. Such an analysis is shown in detail in the following subsections.

4.1 Previous approximations to the seasonal pattern

The adequate specification of the spline, according to the previous section, requires obtaining a previous approximation of the seasonal component in each of the two periods. The stochastic formulation of the seasonal component requires a fixed seasonal period. Then, for each period, a basic structural model

$$y_t = \mu_t + \gamma_t + \varepsilon_t , \tag{23}$$

where $\varepsilon_t \sim NID(0,\ \sigma^2_{\varepsilon})$, is estimated. The level component is assumed to be generated by the random walk $\mu_t = \mu_{t-1} + \beta_{t-1} + \eta_t$, where $\beta_t = \beta_{t-1} + \zeta_t$, $\eta_t \sim NID(0, \sigma^2_{\eta})$ and $\zeta_t \sim NID(0, \sigma^2_{\zeta})$. The seasonal component is modelled by a set of trigonometric terms at the seasonal frequencies, $\lambda_j = 2\pi j/s$,

[9] See Nielsen (1998:46-47).

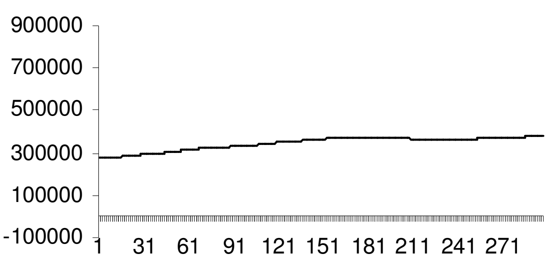
$j = 1, ..., [s/2]$, in such a way that the seasonal effect at time t is $\gamma_t = \sum_{j=1}^{[s/2]} \gamma_{j,t}$, where each $\gamma_{j,t}$ term is constructed by the recursion formulae $\begin{pmatrix} \gamma_{j,t} \\ \gamma^*_{j,t} \end{pmatrix} = \begin{pmatrix} \cos\lambda_j & \text{sen}\,\lambda_j \\ -\text{sen}\,\lambda_j & \cos\lambda_j \end{pmatrix} \begin{pmatrix} \gamma_{j,t-1} \\ \gamma^*_{j,t-1} \end{pmatrix} + \begin{pmatrix} \omega_{j,t} \\ \omega^*_{j,t} \end{pmatrix}$. $\gamma^*_{j,t}$ appears as a matter of construction and $\omega_{j,t}$ and $\omega^*_{j,t}$ are zero mean white noise processes which are uncorrelated with each other with a common variance σ^2_ω. For the model to be identifiable, the disturbances in all three components, level, seasonal and irregular, are assumed to be mutually uncorrelated.

For each period, the results of estimating the basic structural model by maximum likelihood indicate that the slope is constant. In the second period, the significance test of the slope in the final state vector suggests that the trend could be reduced to a random walk without drift. Bearing in mind these results, a model is proposed for each period where the seasonal component retains its stochastic formulation and the slope term is eliminated in the model for the second period. The estimates for the two models are summarised in table 1 and figures 2 and 3[10]. Hyperparameter estimates suggest the stochastic nature of the level component, although the variability is noticeably higher in the second period. The estimated variance of the seasonal disturbance terms is null only for the second period. Accordingly, the conclusion is reached that the seasonal pattern has a deterministic nature only in the second one. On the other hand, the estimates of seasonal components confirm that there are different export patterns[11].

Table 1. Disturbance variances: structural models (23) with fixed slope (period I) and without slope (period II)

	σ^2_η	σ^2_ω	σ^2_ε
Period I	1.0548×10^7	4.5992×10^6	6.9098×10^9
Period II	5.8844×10^9	0.0000	2.2112×10^9

Period I: 80/81 to 90/91



Period II: 91/92 to 03/04

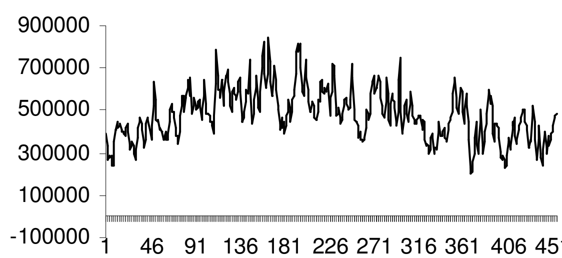


Figure 2. Level component: structural models (23) with fixed slope (period I) and without slope (period II)

[10] The smoothed option of the *STAMP 6.0* statistical package was used (Koopman et al., 2000).

[11] Despite the statistical non-significance of the parameters corresponding to some seasons, the joint test of the seasonal effects indicates the significance of this component. The values of this statistic, which evaluates the joint statistical significance of the seasonal effects at the end of the sample, were 355.551 in the first period, and 242.531 in the other one. This statistic is asymptotically chi-square with 26 and 34 degrees of freedom, respectively.

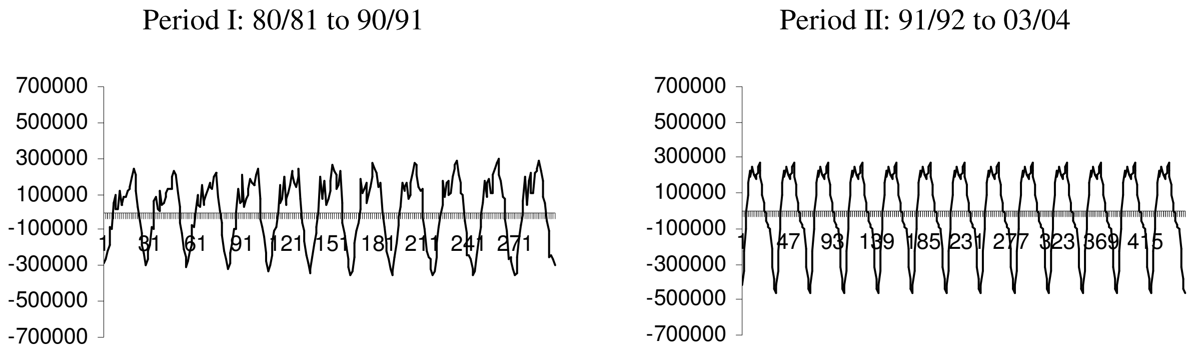


Figure 3. Seasonal component: structural models (23) with fixed slope (period I) and without slope (period II)

It was opted for obtaining other two approaches of the seasonal pattern in each period. First, a model is estimated with a three-segment linear spline capturing the trend component (the break points divide the sample under study in three periods: 80/81-90/91, 91/92-95/96 and 96/97-03/04). Then the residual term of this regression model is a rough approximation of seasonal variations, $\{\gamma_t^{1a}\}_{t=1,\ldots,752}$. Second, moving averages with period 27 until the last observation of 90/91 harvest and a moving average with period 35 since the first observation of 91/92 harvest are calculated. Then the difference between $\{y_t\}_{t=1,\ldots,752}$ series and moving average series is another approximation of seasonal variations, $\{\gamma_t^{1b}\}_{t=14,\ldots,735}$. The approximations shown in Figure 4 suggest that the structural model approximation does not capture all the seasonal behaviour. Perhaps, as a response of the iterative estimating procedure, the variance of the trend component is high enough to capture some seasonal variations. This fact could explain the different behaviour in the trend component during the second period. In this sense, it is opted for using the last two approximations in order to specify an evolving spline, which also captures the seasonal variations, as a useful alternative to the foregoing formulation.

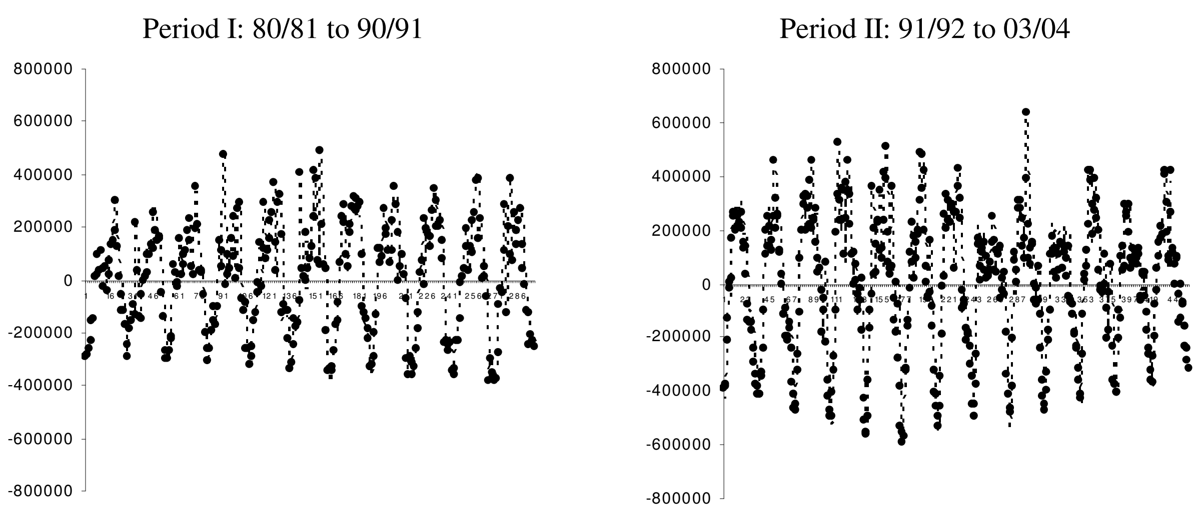


Figure 4. Seasonal components: $\{\gamma_t^{1a}\}_{t=1,\ldots,752}$ (• • •) and $\{\gamma_t^{1b}\}_{t=14,\ldots,735}$ (——)

4.2. Evolving spline

An alternative way of modelling the seasonal pattern described in previous subsection is by a evolving periodic cubic spline formulated as it was indicated in methodological section. That is to say, the seasonal pattern could be formulated as $\gamma_t = g(t) + \xi_t$, where the spline $g(t)$ is expressed as

$$g(t) = \sum_{c=1}^{24} \left[\gamma_0^{c*} X_{0,t}^{c} + \ldots + \gamma_{k_c-1}^{c*} X_{k_c-1,t}^{c} \right] D_{c,t}^{c} \,, \tag{24}$$

where $D_{c,t}^{t} = \begin{cases} 1, t \in \textit{harvest } c \\ 0, \textit{in other case} \end{cases}$, $c = 1,\ldots,24$, and $X_{i,t}^{c} = X_{i,w}^{c}$, $i = 0,\ldots,k_c - 1$, if the observation at time t corresponds to the season w, $w = 1,\ldots,s_c$, where $s_c = \begin{cases} 27, c = 1,\ldots,11 \\ 35, c = 12,\ldots,24 \end{cases}$.

The spline is specified as a function of week *w* of the export period. That is to say, $w = 1$ being the corresponding week of the year in which the export period is considered to start and $w = s$ being the last week of the following year in which the export period is considered to conclude. So, until 90/91 harvest, the length of the seasonal period is 27 in such a way that $w = 1$ corresponds to week 43 of a year and $w = 27$ corresponds to week 17 of the following year. From 91/92 harvest, the length of the seasonal period is 35 in such a way that $w = 1$ corresponds to week 42 of a year and $w = 35$ corresponds to week 24 of the following year.

In order to obtain a more parsimonious formulation, the break points are assumed to be the same for all harvests from 80/81 to 90/91. The same assumption is taken for all harvests from 91/92 to 03/04. Then $X_{i,w}^{c} = X_{i,w}^{I}$, $c = 1,\ldots,11$, and $X_{i,w}^{c} = X_{i,w}^{II}$, $c = 12,\ldots,24$, but the seasonal variations $\gamma_{i,w}^{c}$, $i = 0,\ldots,k_c - 1$, could evolve over time. The resulting model is

$$\gamma_t = \sum_{c=1}^{11} \left[\gamma_0^{c*} X_{0,t}^{I} + \ldots + \gamma_{k-1}^{c*} X_{k_1-1,t}^{I} \right] D_{c,t}^{c} + \sum_{c=12}^{24} \left[\gamma_0^{c*} X_{0,t}^{II} + \ldots + \gamma_{k_2-1}^{c*} X_{k_2-1,t}^{II} \right] D_{c,t}^{c} + \xi_t \,. \tag{25}$$

For each period, the decision has been adopted to select the number of knots from the two approximations to the seasonal pattern previously obtained; that is to say, by estimating the regression models

$$\gamma_w^1 = \gamma_0^* X_{0,w}^{I} + \ldots + \gamma_{k-1}^* X_{k-1,w}^{I} + \xi_w \,, \; w = 1,\ldots,27 \,, \tag{26.a}$$

where γ_w^1 corresponds to the averages values per week calculated from $\left\{\gamma_t^{1a}\right\}_{t=1,\ldots,752}$ and $\left\{\gamma_t^{1b}\right\}_{t=14,\ldots,735}$ series for the first period, and

$$\gamma_w^1 = \gamma_0^* X_{0,w}^{II} + \ldots + \gamma_{k-1}^* X_{k-1,w}^{II} + \xi_w \,, \; w = 1,\ldots,35 \,, \tag{26.b}$$

where γ_w^1 corresponds to the averages values per week calculated from $\left\{\gamma_t^{1a}\right\}_{t=1,\ldots,752}$ and $\left\{\gamma_t^{1b}\right\}_{t=14,\ldots,735}$ series for the second period. Figure 4 shows the seasonal effects estimated in the previous sub-section, and the results of estimating previous models suggest a six-segment spline for both the first and the second period as an adequate specification. The combinations of locations that minimises the residual sum of squares when the regression models previously defined are estimated are the same using either of two approximations. For the first period, $c = 1,\ldots,11$, the break points are $w_1^I = 7$, $w_2^I = 11$, $w_3^I = 13$, $w_4^I = 14$,

$w_5^I = 21$. For the second period, $c = 12,...,24$, the break points are $w_1^{II} = 5$, $w_2^{II} = 16$, $w_3^{II} = 17$, $w_4^{II} = 24$, $w_5^{II} = 30$.

So, the final model of the seasonal pattern is

$$\gamma_t = \sum_{c=1}^{11} \left[\gamma_0^{c*} X_{0,t}^{I} + ... + \gamma_5^{c*} X_{5,t}^{I} \right] D_{c,t}^{c} + \sum_{c=12}^{24} \left[\gamma_0^{c*} X_{0,t}^{II} + ... + \gamma_5^{c*} X_{5,t}^{II} \right] D_{c,t}^{c} \,. \qquad (27)$$

The regressors $X_{0,t}^{I} D_{c,t}^{c},..., X_{5,t}^{I} D_{c,t}^{c}$, $c = 1, ...,11$, and $X_{0,t}^{II} D_{c,t}^{c},..., X_{5,t}^{II} D_{c,t}^{c}$, $c = 12, ...,24$, can be incorporated into the structural model as exogenous variables; but, in order to avoid multicolinearity problems, the regressor $X_{5,t}^{II} D_{24,t}^{c}$ is dropped and the following model

$$\begin{aligned} y_t &= \mu_t + \\ &\sum_{c=1}^{11} \left[\gamma_0^{c*} X_{0,t}^{I} + ... + \gamma_5^{c*} X_{5,t}^{I} \right] D_{c,t}^{c} + \sum_{c=12}^{23} \left[\gamma_0^{c*} X_{0,t}^{II} + ... + \gamma_5^{c*} X_{5,t}^{II} \right] D_{c,t}^{c} + \left[\gamma_0^{24*} X_{0,t}^{II} + ... + \gamma_4^{24*} X_{4,t}^{II} \right] D_{24,t}^{c} \\ &+ \varepsilon_t \end{aligned} \qquad (28)$$

is estimated where $\mu_t = \mu_{t-1} + \eta_t$. The results of this model's estimation are shown in table 2 and figures 5 and 6.

Table 2. Disturbance variances (structural model, Equation (28))

	σ_η^2 (q ratio)	σ_ε^2 (q ratio)
Periods I-II	6.0421x10^9 (1.0000)	3.2100x10^9 (0.5313)

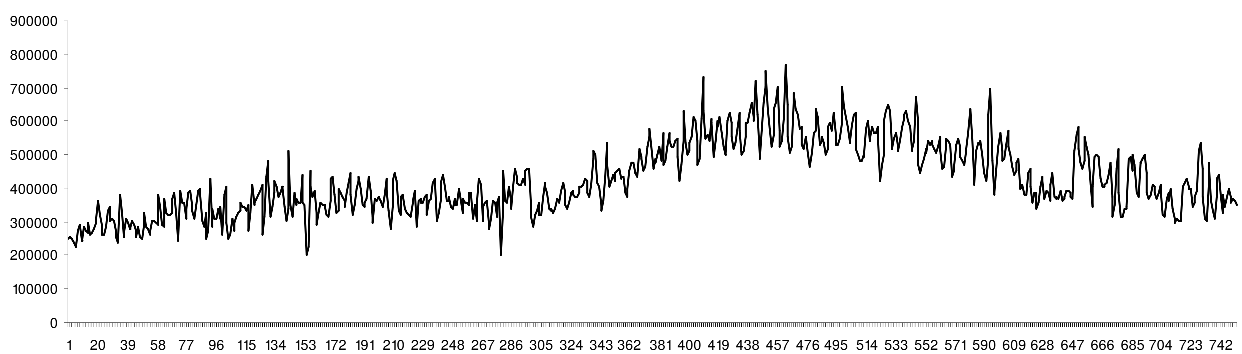


Figure 5. Level component (structural model, Equation (28))

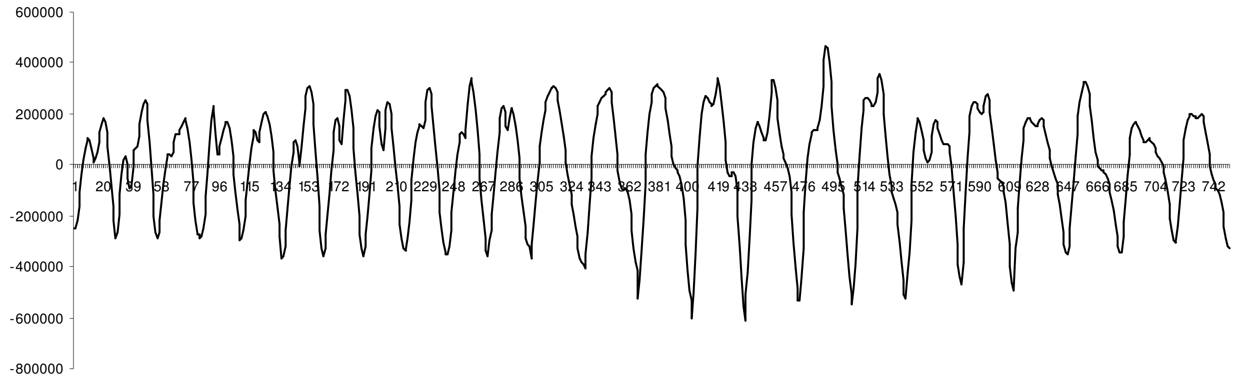


Figure 6. Seasonal component (structural model, Equation (28))

Some spline parameters are not statistically significant but the joint significance test suggests that the seasonal variables $X^{I}_{0,t}D^{c}_{c,t},...,X^{I}_{5,t}D^{c}_{c,t}$, $c = 1, ..., 11$, $X^{II}_{0,t}D^{c}_{c,t},...,X^{II}_{5,t}D^{c}_{c,t}$, $c = 12, ..., 23$, and $X^{II}_{0,t}D^{c}_{24,t},...,X^{II}_{4,t}D^{c}_{24,t}$ must remain as regressors in the model[12]. The final estimates of the seasonal pattern have been obtained from the estimated regression coefficients for these regressors[13]. As noted in Figure 6, seasonal pattern can be similar in several harvests. So, some F tests can be applied in order to check this assumption and simplify the model. In fact, a F test was calculated for testing the hypothesis that the seasonal pattern is fixed in period I and different but also fixed in period II. However, the conclusion is obtained that neither of these seasonal patterns is fixed[14]; although the magnitude of the changes in the seasonal pattern is not very relevant from an economic point of view.

5. Conclusions

The study of the *Tenerife* tomato export weekly series using spline functions embedded into a structural time series model is an example for modelling a changing seasonal pattern. The data is almost or exactly zero for some weeks in each year of the sample and two periods differing by the extent of the export period can be distinguished. Therefore, the traditional approximation of the seasonal cycle fails. So, in order to model the seasonal pattern, a new series has been constructed in such a way that the seasonal period is different in the two sub-samples. This seasonal pattern, in which the period is not fixed throughout the sample and there are changing seasonal variations, has been modelled using a evolving specific spline function. It is also interesting to note the instability of what, in the classic approach to time series, is defined as trend component, whose behaviour has been adequately characterised by a random walk plus noise model.

Notwithstanding that the growth of export levels which started at the beginning of the last decade was interrupted several years ago, the main economic conclusion obtained from the previous analysis is that the seasonal pattern is more or less stable, but some changes are observed in the length of the non-export period and also in the magnitude of the seasonal variations. It would be true to say that, even though the economic agents involved in the production and export of *Tenerife* tomatoes have reacted to changes in market rules, this response has not brought about a significant modification of the weekly

---

[12] The value of the F statistic was $F_{143,608} = 3.6613$. However, this statistic is biased towards the non-rejection of the null hypothesis in a structural model due to the stochastic nature of some components. That is, the level component could be capturing some seasonal effects when the seasonal component is not explicitly specified.

[13] The estimates of the seasonal component obtained from the estimates of spline parameters are corrected in such a way that the seasonal variations sum up to zero over each harvest. Then, the estimates of the level component are also properly corrected so that the same variations are not captured simultaneously by trend and seasonal components.

[14] The value of the F statistic was $F_{132,608} = 1.4220$.

distribution of exports throughout the harvest. This conclusion is highly relevant in order to understand the marketing decision-making process of *Tenerife* producers.

References

Cáceres, J.J. (2000). La Exportación de Tomate en Canarias. Elementos para una estrategia competitiva. Ediciones Canarias.

Cáceres, J.J. (2001). Optimalidad del patrón estacional de las exportaciones canarias de tomate. Estudios de Economía Aplicada 18: 41-66.

Durbin, J. and Koopman, S.J. (2001). Time Series Analysis by State Space Methods. Oxford University Press.

Harvey, A.C. (1989). Forecasting, Structural Time Series Models and the Kalman Filter. Cambridge University Press.

Harvey, A.C., Koopman, S.J. and Riani, M. (1997). The modelling and seasonal adjustment of weekly observations. Journal of Business and Economic Statistics 15(3): 354-368.

Kalman, R.E. (1960). A new approach to linear filtering and prediction problems. Transactions ASME, Series D, Journal of Basic Engineering 82: 35-45.

Kalman, R.E. and Bucy, R.S. (1961). New results in linear filtering and prediction theory. Transactions ASME, Series D, Journal of Basic Engineering 83: 95-108.

Koopman, S.J. (1992). Diagnostic Checking and Intra-Daily Effects in Time Series Models. Thesis Publishers, Tinbergen Institute Research Series, 27. Amsterdam.

Koopman, S.J., Harvey, A.C., Doornik, J.A. and Shephard, N. (2000). STAMP: Structural Time Series Analyser, Modeller and Predictor. Timberlake Consultants.

Marsh, L. (1983). On estimating spline regression. Proceedings of SAS User's Group International 8: 723-728.

Marsh, L. (1986). Estimating the number and location of knots in spline regression. Journal of Applied Business Research 3: 60-70.

Marsh, L., Maudgal, M. and Raman, J. (1990). Alternative methods of estimating piecewise linear and higher order regression models. Proceedings of SAS User's Group International 15: 523-527.

Martín, G., Cano, V. and Cáceres, J.J. (2002). Exportación de tomate en Canarias: ¿un patrón estacional estable? Economía Agraria y Recursos Naturales 2(2): 53-72.

Martín, G. and Cáceres, J.J. (2004). Modelling weekly Canary tomato exports. Agricultural Economics (forthcoming).

Nielsen, H.B. (1998). Cubic Splines. IMM Department of Mathematical Modelling. Technical University of Denmark.

Poirier, D.J. (1973). Piecewise regression using cubic splines. Journal of the American Statistical Association 68: 515-524.

Poirier, D.J. (1976). The Econometrics of Structural Change with Special Emphasis on Spline Functions. North-Holland.