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**The Role of Buffalo production in Sustainable Development of  
Rural Regions: A case study from Egyptian Agriculture**

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### ***ABSTRACT***

Rice is the main summer crop in Egypt. It is a cash exportable crop that provides a main source of income to the Egyptian farmers and the national economy. However, the farmers used to burn the rice straw at the farm borders and violate the law that forbids such action, which causes socio-economic negative externalities due to the generated smoke from burning the rice straw produced from 1.8 million feddans of rice i.e. about 0.75 million ha, which causes social costs due to the probability of premature-mortality and morbidity of rural and urban individuals. When chopped rice straw silage mixed with dissolved urea and molasses at 2% and 3% of straw weight, respectively, was provided as feed to buffalo-feeder calves for meat production at 40% of the S.E. of the daily ration with concentrate feed mix, raised the marketing weight derived from the estimated feed-response model that maximizes the gross margin above the feed costs, from 384 to 518 Kg live eight. Using rice straw as feed, for buffalo calves, seems more feasible to the rural communities, than use it as soil fertilizer or in manufacturing wood and paper. While Egypt imported red meat of 963 million dollars in 2013, due to lack of sufficient feed supply, enriched rice straw silage as feed would provide additional 80,000 tons carcass weight from fed buffalo calves, rather than slaughtering them as rearing veal calves. The estimated income generated from one buffalo fed calves reached 50% of the average annual per capita income in Egypt, with more employment opportunities for the rural communities and saving the probable social losses from burning rice straw. 92% of the Egyptian farmers holding 88% of livestock are with small farms they should be the target of a training program on how to make enriched rice straw silage.

### **INTRODUCTION**

Till early eighties of the last century, wheat straw was the major roughage feed for Egyptian livestock and the farmers had negative impacts towards using rice straw for livestock feeding. Till that time Egypt was almost under planned economy system. Whereas the domestic and imported wheat price and supply distribution were controlled by government, wheat straw was at free market price. Therefore, up to seventies of the last century, its price surpassed wheat grains price in many years, (Soliman and Nawar, 1986). This was mainly due to limited area cultivated with wheat and the low yield of wheat grains, leading to low yield of wheat straw. The wheat area in Egypt was 1,395,382 Feddans in 1961, (1 Feddans = 4200M<sup>3</sup>), due to controlled cropping pattern, and the yield per feddan was about 1.029 tons, (FAOSTAT, 2015). Since 1986/1987 Egypt has started the economic reform program which implied liberalization of the prices and marketing of crops and agricultural inputs. The program was associated with introducing new high yield varieties of grains and expansion in agricultural mechanization systems. Therefore, the yield per feddan of wheat increased rapidly to reach about 2.778 tons per feddan in 2013, associated with expansion in the wheat area to reach 3,404,899 Feddans due to free decision to cultivate and providing high farm guaranteed price. Therefrom, the wheat straw production and supply has extremely increased. The major summer crop, i.e. rice has exposed to the same policies and then showed the same performances over the same period. Its area and yield increased from 941,667 feddans and 1.213 tons per feddan, respectively, in 1961 to 2,916,667 feddans and 2.314 tons per feddan, respectively, in 2013. Accordingly, the farmers have faced, recently, with abundant supply of rice straw. Thereof, the majority of them preferred to burn the rice straw at the farm border, which caused the phenomenon of "The black cloud" all over the Nile delta governorates, at the rice harvesting season (September-October) of every year. As burning of rice straw has harmful impacts on the environment and human health, farmers have thus been encouraged refraining from burning rice straw and adopting more environment and human-friendly rice straw management practices?

### **The negative effects of open-field rice straw burning**

Burning causes atmospheric pollution and results in nutrient loss, even though, it is a cost-effective method of straw disposal and also helps reduce pest and disease populations, (Dobermann. and Fairhurst, 2002) The environmental consequences of rice straw burning in terms of greenhouse gas (GHG) emissions are, mainly, carbon dioxide, methane and nitrous oxide gases, that require adoption of selected rice straw management alternatives, (Launio, Cheryl C., et. al., 2013). The compositions of biomass among fuel types are variable, especially with respect to inorganic constituents. Alkali and alkaline earth metals, in combination with silica, sulfur and chlorine, are responsible for many undesirable reactions in combustion of straw, (Jenkins, 1999)

Burning of rice straw causes almost complete N loss, P losses of about 25%, K losses of 20%, and S losses of 5 to 60%. The amount of nutrients lost depends on the method used to burn the straw. When straw is heaped into piles at threshing sites and burned after harvest, the ash is usually not spread on the field, results in large losses of minerals K, Si, calcium (Ca), magnesium (Mg) leached from the ash piles.

### **Recycling Rice Straw as Fertilizer**

There are several approaches to utilize rice straw nutrient components. The most common approach is to be removed from the field, burned in situ, piled or spread in the field, incorporated in the soil, or used as mulch for the following crop.

About 40% of the nitrogen (N), 30% to 35% of the phosphorus (P), 80% to 85% of the potassium (K), and 40% to 50% of the sulfur (S) taken up by rice remains in vegetative plant parts at crop maturity. Each of these measures has a different effect on overall nutrient balance and long-term soil fertility.

When straw is the only organic material available in significant quantities to most rice farmers and where S-free mineral fertilizers are used, straw may be an important source of S; thus, straw burning should not be practiced. However, spreading and incorporation of straw are labor-intensive tasks and farmers consider burning to be more expedient, (Dobermann, and Fairhurst, 2002)

In contrast, burning effectively transforms straw into a mineral K nutrient source, and only a relatively small amount of K is lost in the process. Therefore, effect of straw removal on long-term soil fertility is much greater for K than for P, (Launio et al, 2013)

Straw is also an important source of micronutrients such as zinc (Zn) and the most important influence on the cumulative silicon (Si) balance in rice. Straw (Nelson, et. al., 1980)

### **Other Alternative Recycled Products of Rice Straw**

In addition to being a fertilizer, rice straw can be used as fuel for cooking, ruminant fodder, and stable bedding or as a raw material in industrial processes (e.g., papermaking), (Nelson, et al, 1980)

### **The study importance and Problem**

The study focused on the socio-economic evaluation of using rice straw as fodder for fattening buffalo male calves for two feasible reasons. First, The Egyptian budget has faced an increasing burden on its budget due to the speed increase net imports of red meat from around 303 million dollars in 2000 to more than 963 million dollars in 2011 ((FAOSTAT, 2015), while the value of the Egyptian pound is decreasing fast in front of the dollar, at least over the last four years. The exchange rate was less 5.6 EGP/ \$1 in 2009 and reached 7.47 EGP/\$1 in January 2015, (Central Bank of Egypt, 2015). Secondly, buffalo population in Egypt yields around 700,000 heads of male calves. At least 500,000 of them are slaughtered at rearing period (around 2-3 months old) to save both buffalo milk for sale and green fodder (clover) in winter for dairy buffalos feeding. Such number of buffalo veal yields around 20,000 tons of carcass weight (40 Kg per head). If such large number of buffalo calves were kept for feeding, they yield around 100,000 tons of carcass weight (about 200 Kg carcass weight/head at around 24-30 months old). The net added meat production would be 80,000 tons carcass weight. Such amount might share, significantly, in filling the market gap between domestic production and demand for red meat in Egypt.

### **Objective of the Study**

Thereof, the objective of this study was to estimate an econometric model for the feed response of using a ration composes of treated rice straw silage with the common concentrate feed mix for feed-lot system of male buffalo feeder calves to reach the optimum marketing weight which maximize the gross margin above the least cos ration.

### **DATA BASE**

The study used the data of a field sample survey from progressive large livestock farms in the north east Nile delta region of Egypt. The sample composed of 60 buffalo male calves. A subsample of 30 heads were fed (1) chopped rice straw with concentrate feed mix as a control and the other 30 heads were fed (2) Chopped rice straw silage treated with dissolved Urea (2% of the straw weight) and Molasses (3% of the straw weight) with concentrate feed mix. The data covered the agricultural year (2013-2014). The first group was named Feeding System (1) and the second one was called Feeding System (2) allover, this study. The ration combination in both systems as Starch Equivalent quantity (Kg S.E.) was 60% concentrate feed mix and 40% rice straw. The cumulative live weight till marketing with the associated feeds intake data was recorded bi-weekly. The feeder calf weight (initial weight) was around 185 Kilogram live weight. The annual average inputs and output prices were collected for the year (2013/2014), as shown in (Table, 1). The concentrate feed mix composes of corn, and oilseed meal, bran and molasses. Its S.E. equivalent weight is .52%, and 12% crude protein.

### **METHODS & ANALYTICAL PROCEDURES**

A feed response model was estimated for the two feeding systems. However, the two feed items were fed at a constant combination, i.e. 40% rice straw and 60% concentrate feed mix as S.E. equivalent value. Therefore, the estimated linear correlation between the intake quantities of both feed items was about 0.924. It was an evidence mutlicolnearity problem, (Intriligator, 1978), which might cause biased estimates of the production response model and violation of the statistical inferences if both feed variables were introduced to such model as explanatory variables. Accordingly, the study aggregated the feed items as one explanatory variable that expressed feeding level as S. E. in Kg.

The literature showed that the livestock feed response is often not a linear relationship. It follows the principal of dimensioning return of inputs, (Soliman, 2006). Therefrom, the best fitted feed response function was the quadratic function, as shown by Equation (1)

$$\dot{y}_i = \alpha + b_1x - b_2x^2 \dots\dots\dots (1)$$

Several functions were derived from the response function (1) to estimate the technical and economic efficiency of each feeding system. Equation 2 is the estimated physical marginal product "MPP" The average physical production is presented by equation 3. The production elasticity function (equation 4) is derived from (equations 2 and 3). The estimated optimum marketing weight that recognizes the maximum gross margin above the least cost feed intake (equation 5) is derived from (equation 2) to express the condition of the equilibrium economic point when the value of marginal product (VMP) equals the marginal cost (the price of 1-kg S.E) of the ration. The market prices are presented in (Table 1), and the economic efficiency coefficient "EE" (equation 6) is derived from (equation 5).

In addition the model of the fattening time function as presented by (equation 7) was used to estimate the required period to reach the optimum marketing weight. The first derivative of (equation 7) is the marginal time required to consume additional unit of feed (equation 8). From the two parametric equations (2 & 8) a Cartesian equation (9) is derived which estimates the marginal daily gain of live weight. The gross margin (equation 10) presents a measure of profitability per head of one fed buffalo calf. It is not the normal profit, but the margin left above the feed cost

$$MPP_x = \frac{dy}{dx} = b_1 - 2b_2x \dots\dots\dots (2)$$

$$APP_x = Y/X \dots\dots\dots (3)$$

$$\xi_x = MPP_x / APP_x \dots\dots\dots (4)$$

$$\text{Maximum gross margin when: } VMP_x = P_y \text{ (MPP)} = P_x \dots\dots\dots (5)$$

$$EE = VMP_x / P_x \dots\dots\dots (6)$$

$$T'_1 = c_0 + C_1X + C_2X^2 \dots\dots\dots (7)$$

$$MPP_{x,t} = \frac{dT}{dx} \dots\dots\dots (8)$$



$$MPP_t = MPP_x / MPP_{x,t} = dy/dx / = dT/dx = = \frac{dy}{dx} * dx / dy = dy / dt \dots\dots (9)$$

$$GM = P_y (MW) - \sum P_{xi} X_i \dots\dots\dots (10)$$

Where:

$\dot{y}$  = Estimated cumulative weight gain in Kgs Live weight of calf i

$\alpha$  = Estimated intercept

x =Feed intake in Kg of S. E. of (40%rice straw and 60% Conc. mix)

$b_i$  = estimated feed response coefficient (regression coefficient)

$MPP_x$ = marginal physical product estimates the additional weight gain due to additional 1-kg of ration

$APP_x$  = estimates the average live weight gain per 1-kg of ration

$\xi_x$  = Production elasticity = estimates the relative change in weight gain due to 1% change in feed intake,

$VMP_x$  = estimates the marginal revenue per additional 1-kg of feeds combination

$P_y$  = Price of 1-kg live weight in EGP

$P_x$  = Price of 1-kg S.E. of ration in EGP

EE = Economic Efficiency coefficient = marginal revenue generated by spending additional 1-EGP on feeds. If it is > 1 means it is feasible to expand in feed use for more weight gain and if it is less than one then less feed should be used and less marketing weight is more feasible.

T = Time period of feeding a calf till the marketing weight in days

$C_i$  = estimated regression coefficients of the time function of the feeds consumption

$MPP_{x,t}$  = Marginal time for each additional feed unit intake

$MPP_t$  = Marginal Daily Gain

GM = Gross Margin in EGP = Total revenue – Feed costs

## ***RESULTS and DISCUSSION***

The estimated response functions and the derived functions are presented under each feeding system, with associated statistical inferences estimates.

**System 1 Chopped Rice Straw with Conc. Feed Mix**

$$\hat{y}_1 = 12.155 + 0.283x - 0.000089x^2 \dots\dots\dots (10)$$

$$(6.16) \quad (-2.29) \quad R^2 = 0.93, \quad F = 185.05$$

$$MPP_t = (0.283 - 0.00018X) / (0.405 - 0.000024X) \dots\dots\dots (13)$$

$$MPP_x = 0.283 - 0.00018X \dots\dots\dots (11)$$

$$T_l = 4.972 + 0.405X - 0.000012X^2 \dots\dots\dots (12)$$

$$. \quad (80.23) \quad (-17.29) \quad R^2 = 0.988, \quad F = 2372.$$

**System 2 Rice Straw Silage with Urea and Molasses and Conc. Feed Mix**

$$\hat{y}_1 = 0.902 + 0.349x - 0.000079x^2 \dots\dots\dots (14)$$

$$(17.14) \quad (-2.98) \quad R^2 = 0.985, \quad F = 1634.1$$

$$MPP_x = 0.349 - 0.00016X \dots\dots\dots (15)$$

$$T_l = 4.075 + 0.386X - 0.00012X^2 \dots\dots\dots (16)$$

$$(99.95) \quad (-23.99) \quad R^2 = 0.988, \quad F = 2372.7$$

$$MPP_t = (0.349 - 0.00016X) / (0.386 - 0.00024X) \dots\dots\dots (17)$$

Table (2) presents the estimated techno-economic criteria of fed calves under the two tested feeding systems at the price levels of inputs and outputs shown in (Table 1). The results are presented in two sections, the first compares the optimum live weight, least cost ration and the gross margin of the two systems and the second section provides a comparative analysis of the estimated average techno-economic criteria of the two feeding systems

### **Maximum Gross Margin and Optimum Marketing Weight**

At the market price level of feeds and live weight of buffalo fed calves in 2013/2014, the estimated marketing weight that maximizes the gross margin above the feed costs was 383 Kg live weight under feeding system (1), while it reached about 517 Kg live weight under feeding system (2), (Table 2) and (Fig., 1). Both estimated weights were achieved at the least cost ration which maximizes the gross margin. Obviously, the feeding system (2) reaches a higher marketing weight at a larger quantity of (kg S.E) than the feeding system (1) of less marketing weight, i.e. 1381 Kg S.E. versus 872 Kg S.E, respectively. Thereof, the least cost ration of feeding system (2) was higher than that of the feeding system (1), i.e. 5012 EGP and 3092 EGP, respectively. The higher price Per Kg S.E. of the system (2) than System (1), i.e. 3.64 EGP versus 3.54 EGP, respectively was also behind the higher feeding cost under system (2). However, the higher total revenue at the larger marketing weight of the feeding system (2) surpassed much that of the feeding system (1), i.e. 14353 EGP and 10632 EGP, respectively. Therefrom, the acquired gross margin under feeding system (2) reached 9342 EGP while that generated under feeding system (1) was 7540 EGP. In other words the farmer would acquire a gross margin under system (2) about 124% of what he (she) could reach under system (1). As both subsamples had started with feeder calves of the same initial weight (185 Kg/head) at average age of calves around 14 months old, thereof, such higher marketing weight and gross margin of the feeding system (2) were due to better techno-economic performance resulted from the nutrient content of the second system, as was discussed in the following section.

### **Techno-Economic Efficiency of the Feed Response under Two Feeding Systems**

The estimated production elasticity showed that while 10% increase in feed intake under feeding system 1 raises the live weight gain by 7.2% it raises the live weight gain under feeding system 2 by 8.4%. On the average, additional 1-kg S.E. of the ration under the feeding system 1 adds only 0.15 kg live weight gain, but adds 0,231 Kg live weight gain under feeding system 2. The buffalo fed calf would reach an optimum marketing live weight of 383 Kg in 436 days under feeding system (1) and an optimum marketing weight of 517 kg live weight in 517 days under feeding system (2). Accordingly, the estimated average daily gain was 0.21 kg under system (1) and 0.27 kg, under system (2). However, the average marginal daily gain from the time function is a better measure for growth speed. It is the average increase in live weight per additional day of growth. While such gain reached 0.351 kg under feeding system (1) it was 0.881 kg under feeding system (2), i.e. under the feeding system (2) the buffalo calves grow faster.

As the technical criteria of the feeding system (2) surpass much those criteria under feeding system (1), the estimated average economic efficiency coefficient under the feeding system (2) was significantly higher than such coefficient under the feeding system (1), i.e. 1.76 and 1.17, respectively. It means that while, each additional 1-EGP of ration costs under the system (2) generates marginal revenue of 1.76 EGP, and it generates only 1.17 EGP under system (1).

### **Socio Economic Impacts of the Feeding System (2) for Buffalo Fattening**

Each buffalo calf fed a ration combination of concentrate feed mix with rice straw silage enriched with dissolved urea and molasses generates direct and external benefits to the rural communities. (1) generates additional annual income to the farm household of about \$1251, equivalent to one-half of the average per capita income in Egypt, i. e. \$2500 (World Bank, 2012), (2) generates extra employment opportunities for the family labor on farm, which currently suffer from high unemployment, (Soliman, 2004), (3) Shares in decreasing the annual red meat imports, (4) utilization of potential livestock resources by stopping slaughtering of buffalo veal calves during rearing period, (5) stopping the burning of rice straw enables the communities to avoid the probability of pre-mature mortality and morbidity of not only rural but also urban human resources and (6) some studies measured the social costs of a probability of losing one's life by \$200,000, (Soliman, Soliman, 1995),

### **Causes of the Higher Efficiency of Rice Straw Silage than Chopped dray Straw**

It seems that, the high techno-economic efficiency of the ration composes of starch equivalent 60% concentrate feed mix and 40% chopped rice straw silage (with 2% dissolved urea and 3% molasses) is due to some physiological reasons: (1) the enriched rice straw with dissolved urea raises the protein content of the ration, (2) addition of molasses activates the bacterial activities, during silage preparation, which raises the digestibility of the rice straw, and thereof its starch equivalent value, and (3) to make silage from rice straw increases its palatability and gives it a preferable smell to the animals. Therefore increases the intake of the rice straw silage.

### ***Conclusion and Recommendations***

Therefore, the study recommends: (1) providing extension services to train the small farm managers on how to make such silage.as feed for buffalo feeder calves, as they represent 92% of the farm holdings in Egypt and hold 88% of livestock numbers, (2) providing options for reducing the cost of collection and transportation of rice straw, and intensifying of information campaigns and drives regarding environmental regulations and policies as well as increasing the demand for rice straw for other uses are also recommended

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*Table (1) Average Price of Inputs and Output for Buffalo Meat Fattening (2013/2014)*

<i>Item</i>	<i>Fed Male Calves (400Kg/Head)</i>	<i>Concentrate Feed Mix</i>	<i>Rice Straw</i>	<i>Rent of Chopping Machine</i>	<i>Urea</i>	<i>Molasses</i>
<i>Unit</i>	<i>1-Kg weight Live</i>	<i>1-Ton</i>	<i>1-Load = 250 Kg</i>	<i>1-hr for 1-ton straw</i>	<i>1-Sac = 50Kg</i>	<i>Tin (30Kg)</i>
<i>Egyptian Pound (EGP)</i>	28.75	2800	100	60	100	10

1-\$ = EGP 7.47

*Table (2) the Economic Analysis Profile of the Fed Buffalo Calves on Two different Feeding systems*

<i>Estimated Techno-Economic Criteria</i>	<i>Unit</i>	<i>Feeding System 1</i>	<i>Feeding System 2</i>
<i>Average Marginal Live weight Gain/ Kg S.E</i>	<i>Kg Live weight</i>	0.150	0.231
<i>Production Elasticity of Feed Intake</i>	<i>%Gain/1% feed</i>	0.72	0.84
<i>Least Cost Ration Quantity</i>	<i>Kg S.E.</i>	872	1381
<i>Optimum Marketing Weight</i>	<i>Kg Live Weight</i>	383	517
<i>Feeding Period to Reach Marketing Weight</i>	<i>Days</i>	436	382
<i>Total Revenue</i>	<i>EGP</i>	10632	14354
<i>Feed Costs</i>	<i>EGP</i>	3092	5012
<i>Gross Margin</i>	<i>EGP</i>	7540	9342
<i>Average Marginal Daily gain/Day</i>	<i>Kg Live weight</i>	0.351	0.881
<i>Average Daily Gain</i>	<i>Kg Live Weight</i>	0.21	0.27
<i>Average Economic Efficiency</i>	<i>EGP</i>	1.17	1.76

Source: Estimated from: equations (10–17), Table (1) and application of the Cartesian

equations (5, 6 and 9)

