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# FORECASTING IRRIGATION TECHNOLOGY WITH TRANSITION PROBABILITIES IN THE PACIFIC NORTHWEST

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## INTRODUCTION

Competing water demands in the 17 Western States has effectively reclassified water as a scarce resource. Irrigated agriculture accounts for approximately 90 percent of Western water consumption. However, population and industrial growth in lower-basin States in the last two decades has significantly increased nonagricultural water demands for energy, municipal, commercial and industrial uses. More recently, water quality and equity issues involving the use of instream flows for fish and wildlife habitat, recreation, and Indian and federal reserved rights has also significantly expanded water demands in the West.

Market transfers, while capable of efficiently reallocating water resources to satisfy many competing demands, will not likely be the only solution to the Western water problem. Upper-basin States, with water-dependent agriculture, are unlikely to support water transfers to lower-basin States. Therefore, the large non-agricultural demands means that supplies to meet these demands will likely come from conservation in agricultural water use.

Water conservation in agriculture depends upon the adoption of water-conserving irrigation technologies and water management alternatives. Initial irrigation technologies in the West involved the use of gravity techniques. The adoption of sprinkler technologies expanded irrigated cropland to non-riparian areas as well as allowed for water conservation through better irrigation management. The wide-spread adoption of center-pivot sprinkler irrigation systems during the 1960's and early 1970's continued to significantly increase the efficiency of irrigated agriculture.

In 1984, sprinkler irrigation accounted for 36.1 percent of irrigation in the West with gravity irrigation accounting for 61.6 percent (5). Center-pivot systems accounted for 54.6 percent of sprinkler irrigation. Recent conservation investments in irrigated agriculture in the southern Plains States indicates further water saving potential of adopting new water saving technologies and water management. Irrigation efficiencies have increased by 20 to 30 percent. Newer irrigation application systems involve such techniques as low-pressure sprinklers, low-energy precision application (LEPA) systems, and surge or cablegation techniques. Management techniques will involve more extensive adoption of irrigation scheduling, deficit irrigation, limited irrigation-dryland (LID) farming, laser field leveling, and such furrow options as furrow diking, spacing, compacting and alternative furrow irrigations.

The transition to more extensive use of water saving irrigation technologies in the West most likely means greater quantities of water available to meet non-agricultural demands (assuming no expansion in irrigated agriculture). Estimation of a transition matrix will provide useful information on potential agricultural water conservation.

Madansky proposed the use of a Ordinary Least Squares (OLS) estimation procedure for estimating the transition probabilities with aggregate time

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series data. Lee, Judge and Takayama improved the OLS procedure by introducing the probability constrained least squares estimation procedure. In an article by Lee, Judge, and Zellner, it was shown how one can estimate transition probabilities, equivalent to Aitken's generalized least squares estimates, with only aggregate time-series data. However, in the problem at hand, neither the time-ordered data of microeconomic units nor a sufficient number of aggregate time series data are available.

In this paper, we forecast irrigation technology with limited data, by modifying the model developed by Lee, Judge and Zellner. The model is applied using State level cross-section, time series data on irrigated crop acreage by technology (1974-1986) for the Pacific Northwest, published annually by the Irrigation Journal.

## THE MODEL

The stochastic process of a finite Markov chain in discrete time can be written as:

$$\begin{aligned} Pr(S_i(t), S_j(t+1)) &= Pr(S_i(t)) \cdot Pr(S_j(t+1)/S_i(t), \ldots, S_i(0)) \qquad (1) \\ &= Pr(S_i(t)) \cdot Pr(S_j(t+1)/S_i(t)) \\ &\quad \text{for all } i \text{ and } j, \end{aligned}$$

where $Pr(S_i(t))$ represents the probability that state $S_i$ occurs in year t, $Pr(S_i(t), S_j(t+1))$ is the joint probability of $S_i(t)$ and $S_j(t+1)$, and $Pr(S_j(t+1)/S_i(t))$ represents the conditional probability for state $S_j$. Equation (1) explains that the probability of going to each state depends only on the present state and is independent of how we arrived at that state (1).

Summing both sides of equation (1) over all possible outcomes of the state $S_i$, the stochastic process then may be written in the following form.

$$\begin{aligned} Pr(S_j(t+1)) &= \sum_i^r Pr(S_i(t)) \cdot Pr(S_j(t+1)/S_i(t)) \qquad (2) \\ &= \sum_i^r Pr(S_i(t)) \cdot P_{ij}, \end{aligned}$$

where $P_{ij}$ represents the transition probability and has the following properties.

$$P_{ij} \geq 0 \qquad \text{for all } i \text{ and } j \qquad (3)$$

$$\sum_j P_{ij} = 1 \qquad \text{for all } i \qquad (4)$$

By replacing $Pr(S_j(t+1))$ and $Pr(S_i(t))$ in (2) with $y_j(t)$ and $x_i(t-1)$, respectively, equation (2) can be rewritten in conventional notation for regression analysis as:

$$y_j(t) = \sum_i^r x_i(t-1)\cdot P_{ij} + e_j(t), \qquad \text{for } j=1,2,\ldots,r \text{ and } t=1,2,\ldots,T \tag{5}$$

where $y_j(t)$ is the observed proportion in state j in time t, $x_i(t-1)$ is the observed proportion in state i in time (t-1), and e is a random disturbance.

The regression equation (5), using pooled observations on M cross-section units over T periods of time may be represented in equation form as:

$$y_{jk}(t) = \sum_i^r x_{ik}(t-1)\cdot P_{ij} + e_{jk}(t), \qquad \text{for } j = 1,2,\ldots,r;\ k = 1,2,\ldots,M; \text{ and } t = 1,2,\ldots,T; \tag{6}$$

or compactly in matrix notation as follows:

$$Y_j = X_i P_j + e_j \qquad (j = 1,2,\ldots,r) \tag{7}$$

where $Y_j$ is an (MT x 1) vector, $P_j$ is an (r x 1) vector, $e_j$ is an (MT x 1) vector, and $X_i$ is an (MT x r) matrix such that

$$X_i = \begin{bmatrix} y_{11}(0) & y_{21}(0) & \cdots & y_{r1}(0) \\ \vdots & \vdots & & \vdots \\ y_{1M}(0) & y_{2M}(0) & \cdots & y_{rM}(0) \\ \vdots & \vdots & & \vdots \\ y_{1M}(T-1) & y_{2M}(T-1) & \cdots & y_{rM}(T-1) \end{bmatrix}.$$

When dealing with pooled cross-sectional and time-series data, it is reasonable to assume that the disturbances are heteroskedastic and also have contemporaneous covariances. Since the variance-covariance matrix in equation (7) is singular, Aitken's generalized least squares method can not be used.[1]

Following Lee, Judge and Zellner, the reduced model is expressed as follows.

$$\begin{bmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_{r-1} \end{bmatrix} = \begin{bmatrix} X_1 & & & \\ & X_2 & & \\ & & \ddots & \\ & & & X_{r-1} \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \\ \vdots \\ P_{r-1} \end{bmatrix} + \begin{bmatrix} e_1 \\ e_2 \\ \vdots \\ e_{r-1} \end{bmatrix} \tag{8}$$

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[1] Summing both sides of equation (7) over all possible outcomes, $\sum_j Y_j = \sum_j X_i P_j = d_{MT} - X_i d_r = d_{MT} - d_{MT} = 0$, where d is a column vector with all elements 1. This result implies that the variance-covariance matrix associated with equation (7) is a singular matrix.

or compactly as

$$Y_* = X_* P_* + e_* \tag{9}$$

with

$$E(e_*) = 0 \tag{10}$$

and

$$E(e_* e_*') = W_* \tag{11}$$

where $Y_*$ is an $((r-1)MT \times 1)$ vector, $X_*$ is an $((r-1)MT \times r(r-1))$ block diagonal matrix, $P_*$ is an $(r(r-1) \times 1)$ vector, $e_*$ is an $((r-1)MT \times 1)$ vector, and $W_*$ is an $((r-1)MT \times (r-1)MT)$ nonsingular matrix.

The variable $y_{jk}(t)$ in equation (6) follows a multinomial distribution with a mean $q_{jk}(t)$, a variance equal to $q_{jk}(t)(1-q_{jk}(t))/N_k(t)$, and a covariance equal to $q_{ik}(t)q_{jk}(t)/N_k(t)$, where $N_k(t)$ is the number of observations from the kth cross-sectional unit at time period t. In order to derive the variance-covariance matrix $W_*$ in equation (11), we define $V_k(t)$ to be an $((r-1) \times (r-1))$ cross-sectional matrix at time period t such that;

$$V_k(t) = \frac{1}{N_k(t)} \cdot Z(t)$$

where

$$Z(t) = \begin{bmatrix} q_{1k}(t)[1-q_{1k}(t)] & -q_{1k}(t) \cdot q_{2k}(t) & \cdots & -q_{1k}(t) \cdot q_{r-1,k}(t) \\ -q_{2k}(t) \cdot q_{1k}(t) & q_{2k}(t)[1-q_{2k}(t)] & \cdots & -q_{2k}(t) \cdot q_{r-1,k}(t) \\ \vdots & \vdots & \ddots & \vdots \\ -q_{r-1,k}(t) \cdot q_{1k}(t) & -q_{r-1,k}(t) \cdot q_{2k}(t) & \cdots & q_{r-1,k}(t)[1-q_{r-1,k}(t)] \end{bmatrix}.$$

The variance-covariance matrix $W_*$ can then be represented by

$$W_* = (V_k(t) \underset{t}{\overset{T}{\otimes}} I) \underset{k}{\overset{M}{\otimes}} I \tag{12}$$

and the inverse matrix $W_*^{-1}$ is then given by

$$W_*^{-1} = (V_k^{-1}(t) \underset{t}{\overset{T}{\otimes}} I) \underset{k}{\overset{M}{\otimes}} I \tag{13}$$

where

$$
V_k^{-1}(t) = \begin{bmatrix}
\frac{N_k(t)}{q_{1k}(t)} + \frac{N_k(t)}{q_{rk}(t)} & \frac{N_k(t)}{q_{rk}(t)} & \cdots & \frac{N_k(t)}{q_{rk}(t)} \\
\frac{N_k(t)}{q_{rk}(t)} & \frac{N_k(t)}{q_{2k}(t)} + \frac{N_k(t)}{q_{rk}(t)} & \cdots & \frac{N_k(t)}{q_{rk}(t)} \\
\vdots & \vdots & \ddots & \vdots \\
\frac{N_k(t)}{q_{rk}(t)} & \frac{N_k(t)}{q_{rk}(t)} & \cdots & \frac{N_k(t)}{q_{r-1,k}(t)} + \frac{N_k(t)}{q_{rk}(t)}
\end{bmatrix}.
$$

Given equations (9) and (13), the nonnegativity constrained Aitken's generalized least squares estimators are obtained by solving the following quadratic programming problem.

$$\text{Minimize } \; Z = (Y_* - X_*P_*)'W_*^{-1}(Y_* - X_*P_*) \tag{14}$$

subject to the following constraints

$$RP_* \le d_r \tag{15}$$

$$P_* \ge 0 \tag{16}$$

where $d_r$ is an $(r \times 1)$ vector of ones and R is an $(r \times r(r-1))$ matrix such that $R = (I_1 \; I_2 \; \ldots\ldots \; I_{r-1})$ with each $I_i$ an $(r \times r)$ identity matrix. The deleted parameter $P_r$, from equation (7), can be estimated from the following identity.

$$P_r = 1 - RP_* \tag{17}$$

## Application to Irrigation Technologies in the Pacific Northwest

In 1984, approximately 6.6 million cropland acres were irrigated in the Pacific Northwest (1.5, 1.8 and 3.3 million acres in Washington, Oregon, and Idaho, respectively) (5). The majority of these acres were irrigated with sprinkler systems (56 percent), with nearly 30.0 percent of sprinkler irrigated acres irrigated using center-pivot sprinklers and the remaining sprinkler irrigated acres using conventional systems such as mechanical or hand move systems and solid/permanent set systems. Approximately 42.1 percent of cropland was irrigated using a gravity flow system (gated pipe, siphon tube or flood systems). The remaining irrigated cropland (1.9 percent) used either drip, trikle or subirrigation systems.

Nearly 12.5 million acre-feet of water was applied to produce a variety of crops in the Pacific Northwest. Table 1 indicates aggregate gross water use efficiency of alternative irrigation systems. Idaho, while using the larger quantity of water (3.31 million acre-feet), is also the more efficient. Average per acre application for all irrigation systems was 1.7 acre-feet in Idaho, compared to 2.0 and 2.2 acre-feet for Oregon

Table 1--Water application rates in the Pacific Northwest

| | Irrigation Technology | | |
|---|---|---|---|
| State | All systems | Gravity systems | All Sprinkler systems |
| | | acre-feet/acre | |
| Washington | 2.2 | 2.4 | 2.0 |
| Oregon | 2.0 | 2.2 | 1.5 |
| Idaho | 1.7 | 1.8 | 1.5 |

Source: U.S. Department of Commerce, Bureau of the Census. 1984 Farm and Ranch Irrigation Survey, Special Report Series AG84-SR-1. June 1986.

and Washington, respectively. In all three States, per acre application rates were lower for sprinkler irrigation systems than for gravity flow systems.

In order to apply the quadratic programming model in equations (14) through (17), irrigated acreage data from the Irrigation Journal for the Pacific Northwest was grouped into three technology states (classifications). Data were grouped separately for Washington, Oregon and Idaho for the years 1974-1986. Technology states consisted of gravity systems (GR), conventional sprinkler systems (SPK1) (including gun, boom, traveler systems; hand, mechanical, wheel move systems; and towline and sideroll systems), and center-pivot, drip and subirrigation systems (SPK2).

Applying the optimization model in equations (14) through (17), the transition matrix for the Pacific Northwest is estimated as:

$$\begin{array}{c} \\ \mathrm{GR} \\ \mathrm{SPK1} \\ \mathrm{SPK2} \end{array} \begin{array}{c} \begin{array}{ccc} \mathrm{GR} & \mathrm{SPK1} & \mathrm{SPK2} \end{array} \\ \begin{bmatrix} .9211 & .0680 & .0109 \\ .0482 & .8972 & .0546 \\ 0 & .1300 & .8700 \end{bmatrix} \end{array} \qquad (18)$$

The estimated transition matrix (18) provides us some useful information about the dynamic nature of irrigation technology adoption in the Pacific Northwest. First, the large diagonal coefficients indicate that irrigators tend to remain with their initial investment. In other words, technology in-place is likely to remain in-use. Furthermore, the use of gravity flow systems tends to remain more stable than sprinkler systems. Approximately 92 percent of irrigators using gravity systems will likely continue to use these systems, while 6.8 percent of these irrigators will shift to using conventional sprinkler systems and only 1.1 percent will shift to using center-pivot sprinkler systems. Second, Pacific Northwest irrigators tend to shift from gravity flow to conventional sprinkler systems and from conventional sprinkler systems to center-pivot systems, rather than from gravity flow to center-pivot systems.

The transition matrix (18) also indicates that not all irrigators in the Pacific Northwest replace their aging irrigation system with center-pivot sprinkler systems. Some irrigators using a conventional sprinkler system will shift to a gravity flow system, and some irrigators using a center-pivot sprinkler system will shift to a conventional sprinkler system. However, irrigators do not replace center-pivot sprinkler systems with gravity flow systems. The shifts from conventional sprinkler to gravity flow and from center-pivot to conventional sprinkler systems are most likely due to soil and topographic characteristics and ease of irrigation management.

The transition matrix (18) traces the adoption of irrigation technology between three technology classes over time. In order to project the time path of technology proportions, let w(o) be the initial vector of proportions and w(t) the vector of proportions at time t. From the definition of a Markov process, the conditional expectation of w(t) is given by

$$w(t|t-1) = w(t-1) \cdot P \tag{19}$$

where P is the transition matrix. Using the observed proportions in 1986, the projected time path of technology proportions for the time period 1987-2010 are provided in Table 2 for the Pacific Northwest region.

## Conclusions

This paper estimates transition probabilities and forecasts irrigation technology proportions for the Pacific Northwest. The transition matrix is estimated using cross-section, time series data with a modified Lee, Judge and Zellner probability-constrained quadratic programming model. The transition matrix indicates that irrigation technology adoption in the Pacific Northwest is relatively slow. However, irrigators will continue to adopt less water-using technologies. Given increasing non-agricultural demands for water, the stability of irrigation technology adoption suggests the existance of institutional barriers and/or the lack of economic incentives for irrigators to invest more heavily in less water-using technologies.

Table 2--Projected probabilities of irrigation technologies for the Pacific Northwest

Washington:

| Year | GR | SPK1 | SPK2 |
|---|---|---|---|
| 1986 | 0.2388 | 0.5239 | 0.2373 |
| 1990 | 0.2599 | 0.5030 | 0.2371 |
| 2000 | 0.2827 | 0.4860 | 0.2313 |
| 2010 | 0.2898 | 0.4821 | 0.2281 |

Oregon:

| Year | GR | SPK1 | SPK2 |
|---|---|---|---|
| 1986 | 0.4642 | 0.4162 | 0.1196 |
| 1990 | 0.4066 | 0.4333 | 0.1601 |
| 2000 | 0.3328 | 0.4623 | 0.2049 |
| 2010 | 0.3070 | 0.4739 | 0.2191 |

Idaho:

| Year | GR | SPK1 | SPK2 |
|---|---|---|---|
| 1986 | 0.4998 | 0.3662 | 0.1340 |
| 1990 | 0.4259 | 0.4112 | 0.1629 |
| 2000 | 0.3380 | 0.4586 | 0.2034 |
| 2010 | 0.3086 | 0.4730 | 0.2184 |

Pacific Northwest:

| Year | GR | SPK1 | SPK2 |
|---|---|---|---|
| 1986 | 0.4271 | 0.4171 | 0.1558 |
| 1990 | 0.3805 | 0.4391 | 0.1804 |
| 2000 | 0.3232 | 0.4662 | 0.2106 |
| 2010 | 0.3036 | 0.4755 | 0.2209 |

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