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A Practical Way to Obtain Near-Optimal Solutions (NOS) in Linear Programming

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The nature of agricultural production is such that the functional relationships that describe technology and resources requirements cannot always be modeled adequately in a deterministic way. These functional relationships, along with the objective functions that are associated with them can be linear or nonlinear and continuously differentiable (smooth) or nondifferentiable (nonsmooth). In addition, the decision variables may be continuous, restricted to integer values, or in certain situations, both. Also, production may take place at a fixed point in time (static) or during an interval of time (dynamic). Even with these complexities many agricultural model builders have opted to represent farm problems using linear-deterministic-smooth-continuous-static models (LP). Mathematical linear programming (LP) also assumes additivity, divisibility, finiteness, and single-valued expectations. The use of LP in agriculture offers an indispensable degree of operational simplicity.

Using this optimization technique, the researcner approaches a complex decision problem by concentrating on a single objective designed to quantify performance. This one objective is minimized (or maximized) subject to the constraint set. If one can isolate and characterize a problem by one objective, be it net returns or net loss in a farm situation, transferring resources or farm commodities between various locations, or social welfare in the context of government planning, LP may provide a useful procedure and basis for analysis.

It is, however, a rare situation in which the model builder can fully represent all the complexities of interactions, constraints, and appropriate objectives when faced with a complex decision setting such as agriculture. Thus, as with all quantitative techniques of analysis, a particular LP formulation should be regarded only as an approximation (Luenberger). This has caused many practical farm planners to reject the idea of a single unique optimal solution to a linear programming model of a particular farm situation. Instead, these planners prefer to compute a number of solutions for the farmer's consideration. According to Powell and Hardaker, the most restrictive assumption of LP is its deterministic nature. The authors also assert that LP has the limitation of permitting only one objective function - normally maximum expected profit. They point to the work of Officer, Halter, and Dillon who have indicated the importance of higher order moments of the profit criterion and the importance of risk attitudes in farmers' decisions. Based on research by others, Powell and Hardaker indicated that the farmer's utility function may be multidimensional, perhaps nonlinear, or even impossible to represent adequately using an LP formulation. In addition, according to the authors, farmers may have difficulty in articulating their objectives precisely enough to be incorporated into any formal model, but may be able to determine which of a set of plans suits their needs the best. Based on a study by Renborg, the authors indicate that the solution space is often relatively "flat" in the optimum region. This will often signify that solutions exist which would enable the farmer to satisfy better some secondary objective at the expense of relatively little reduction in expected earnings.

With some modification, a range of suboptimal solutions that are of interest to farmers can be generated using procedures that are described in the literature. For example, Powell and Hardaker exchanged the objective function of their LP problem with a constraint to determine new, perhaps more acceptable, solutions. In their study, sacrificing 15 percent of income allowed for either labor use to be reduced to one-third of that required for the optimum, or for 40 percent of the farm to be in lucerne. Willis and Perlack illustrated in two different studies the use of two approaches to formalize multiple objective functions; generating techniques which use the weighting and constraint methods and goal programming. Paris studied multiple optimal solutions (MOS) in Linear Programming and asserted that the polyhedral nature of the solution set in LP models may cause MOS if some plausible conditions are realized. These conditions are related to the phenomenon of degeneracy which, in the case of the primal solution, occurs when a set of activities employ inputs in exactly that proportion which exhausts completely two or more available resources. Burton et al. proposed a procedure in which the vertices of a convex polytope can be found by using a pivoting method of vertex enumeration. This method was used to generate all extreme-point nearly-optimal solutions of an example problem involving selection of a marketing strategy for beef cows. The results showed that compared to the optimal solution, nearly-optimal solutions have more diversity or use less cash or hired labor.

This study discusses and demonstrates the use of two additional procedures; 1) the Hop, Skip, and Jump method and 2) the Random Generation Method (RGM). The alternative solutions are compared with respect to stochastic dominance.

The Hop, Skip, and Jump method (Brill, 1979) (HSJ) and the Random Generation Method (RGM) are only two of the many techniques that are capable of generating such suboptimal solutions (for an elaborate review of other available techniques, see Brill, 1982).

Methodology

The HSJ Method

This method, as discussed by Brill et al., can be applied to a range of problems and is designed to explore the full range of discrepancies among solutions with respect to the values of decision variables. The steps in the HSJ approach as they relate to the example chosen for this study are:

Step 1. Obtain an initial solution by any method.

For example,

$$(1)\quad \begin{array}{lrl} \text{Maximize} & z & = c'x \\ \text{s,t} & Ax & \le b \\ & x & \ge 0 \end{array}$$

Step 2. Convert the objective function into a constraint and obtain an alternative solution by solving

$$\text{(2)}\quad \text{Maximize } P = \sum_{k=K} x_k$$
$$\text{s,t}\quad Ax \leq b$$
$$c' \geq (1 - \alpha)\, z^*$$
$$x \geq 0$$

where z = the scalar objective function value
c' = the price, return, and cost vector
K = set of indices of the decision variables that are zero in the initial solution
A = constraint coefficient matrix
b = requirement or resource vector
z^* = optimal objective function from Step 1
α = tolerance level i.e. α = 20%, 10%, or 5%.

Equation (2) is designed to produce an alternative solution that is different from the first one by maximizing the sum of the decision variables that are zero in the original plan. The target specified by $(1 - \alpha)z^*$ will ensure that the alternative solution will be "good" with respect to modeled objectives. Brill et al. note that the target would generally be relaxed in comparison to the respective values of the objective function in solution of Step 1.

Step 3. Additional solutions are obtained by maximizing the sum of the zero variables that appeared in previous solutions.

Brill et al. describe the stopping criteria of the HSJ as:

(1) when no new decision variables enter the basis because all decision variables are included in the current HSJ objective function.

(2) when no new decision variable enters the basis even though there are variables not included in the HSJ objective function.

(3) The model builder terminates the procedure when a large number of alternative solutions have been generated or when the difference between each new alternative and the one that precedes it becomes negligible.

This method has been applied on land use planning problems by Brill et al. and on water resources planning problems by Chang et al. (1982a).

The Random Generation Method

The random generation method has many variants. For example, one form of the method was used by Chang et al. (1982b) on a land planning problem in which an optimal feasible solution was reached using multiobjective LP (as is depicted in formulation 1 above). Targets were also set for the planning objectives included in the model to reduce the space further to include only solutions that were good with respect to the modeled objectives. An extreme point solution is located by maximizing an objective function that is generated at random. In this way, a solution that is feasible and good can be generated in an efficient manner and a different solution may be found randomly by selecting different objective functions to be optimized.

The specific formulation for this method is similar to Equation (2) above with the exception that the objective function is formed by selecting a specified number of decision variables at random. These variables are then placed in the objective function and assigned a coefficient of one while all other variables are assigned a coefficient of zero. The other exception is the addition of more target rows to restrict the remaining objective functions.

Another variant of the procedure was used by Harrington and Gidley on a water resources planning problem. In their study, an optimal feasible solution was first reached, then alternative optima and near-optima were generated by converting the objective function into a constraint (as in Equation (2) above with the exception that their problem was a minimization problem). The authors have used, however, random objective functions that were generated by choosing a coefficient from a uniform distribution on [-1, 1] for each structural and slack variable.

In this paper, a similar procedure was used with the exception that the random objective function was generated by choosing a coefficient from a uniform distribution with mean equal to zero and variance equal to one for only the decision variables. By randomizing this objective function for each different run at certain levels of $c'x \geq (1 - \alpha)z^*$, different suboptimum alternative plans were generated.

Farm Setting, Data and Economic Assumptions

The data used for this study portrays a representative farm situation of the central region of the Santa Fe province in Argentina. It is a region characterized by temperate climate with annual rainfall averaging 900 mm (approx. 36 inches) and mixed soils for crop and livestock production. Dominant production systems include dairy and beef fattening, grain sorghum, corn, and soybeans. Of less importance are cow-calf operations, sunflower, wheat, linseed and double cropping of wheat and soybeans. Most farms are of medium size, ranging from 100 to 400 hectares and are predominantly family operated, with the exception of dairy, where share contracts are common.

The representative farm model has 180 ha of usable land which can be used for livestock or crop production. Labor is provided by the owner, who supplies 2400 hours per year available for all cropping and husbandry operations with the exception of milking which is carried out by labor hired under share contract. The farmer owns the machinery set except for harvesters which are contracted. Although working capital requirements are specified for each activity, a specific amount available (or a credit constraint) was not set.

Several activities can be included in this model. The dairy activity (DAIRY) follows closely the parameters of the dairy production unit which has been under operation during the last 9 years within an Agricultural Experiment Station located in the region. The composition of the herd, stocking rate and productivity measures are averages of the period 1981-1986 (Comeron et al, 1988). The pasture production levels are from one year only (Comeron et al., 1986). In fact, the stocking rate is endogenously determined by the LP model, given animal requirements and the pasture supply. There are also six beef production activities. Two of these "buy" young steers from the dairy activity at approximately one year of age and 180 kg of liveweight. In one case they are kept for 12 months and then sold with 350 kg (BEEF1) and in the other the animals are retained for 24 months and sold at 580 kg of liveweight (BEEF2). Two other potential activities include the same productive sequence but the animals are bought outside the farm (BEEF3 and BEEF4). In the last two livestock activities, steers are bought outside the farm at 240 kg and 350 kg and kept until they reach 580 kg, at 18 (BEEF5) and 12 (BEEF6) months, respectively (Zehnder and Schilder, 1986). Pasture production is modeled as a separate activity and include oats (OATS) as annual winter pasture and sorghum (FSORG) and Millet (MILLET) as summer pastures. Perennial pastures are of two types, a mix based on alfalfa (ALFA1, ALFA2, ALFA3, AND ALFA4) and a mix based on Cychorium (CYCHO1, CYCHO2, and CYCHO3). Crop production activities include wheat (WHEAT) and linseed (LNSEED) as winter crops and grain sorghum (GSORG), corn (CORN), soybean (SOYB1) and sunflower (SNFLWR) as summer crops. The sequence of wheat and late soybean (WSOYB2) is also included.

Monthly prices for crop and livestock products were deflated using the nonagricultural wholesale price index provided by the Bureau of Census and Statistics. All prices are expressed in the local currency, the Austral (A) and are representative of July 1987 real values. For comparison purposes, at that time the relation with the U.S. dollar was about 2.15/US$. Budgeted data for livestock, gain and machinery are based on biannual budgets prepared by the Agricultural Economics group at the Agricultural Experiment Station.

Results and Discussion

The Optimal LP Solution

The optimal LP solution yielded a maximum return (to land, family labor, capital, and management) of A 44,431.56 and is summarized in Tables 1 and 2.

This solution uses 1640 hours of labor per year, and A 14,420 of working capital. As can be seen from Table 1, it includes sales from only two activities, dairy and corn. *In the rest of this section the organizations resulting from the two proposed methodologies will be contrasted to the optimal solution particularly with respect to diversification and its associated risk.*

Application of the HSJ Method

This method produced a set of alternative solutions. Not all of these are presented here, however, because in some cases the activities, particularly dairy, were at very low levels, incompatible with the implied fixed cost per output unit. In addition, some nearly-optimal solutions allow for positive levels of pasture activities where no livestock was present in the solution. This is clearly an undesirable feature of the method. One way to overcome this is by recognizing the implicit linkages which exist between livestock and pasture production when maximizing the new objective function and distribute the ones and zeros accordingly. Table 1 shows the seven solutions obtained with the application of the HSJ method.

Table 1 shows that this method generates a set of very different solutions near the optimum. It must be recognized that diversification potential (or flatness of the net returns function near the optimum) is largely problem specific. In other problems, one activity may be so dominant that these methods may not be able to find nearly-optimal solutions. In this case, for example, the solution with upper bound at 80 percent of the original objective function can dubiously be termed nearly optimal.

Application of the Random Generation Method (RGM)

There are several ways to randomly select nearly optimal solutions using RGM. One is by randomly selecting a set of activities to be forced into the solution. Another way is by assigning a random number for each variable in the new objective function (to the structural and slack variables or to the structural variables only). This last variant was used for this study. Random variables from a uniform distribution with mean zero and variance one were assigned to the structural variables in each run. The construction of the new row constraint containing the values of the original objective function is similar to the HSJ method. Table 2 shows a set of seven alternative solutions found by applying the random generating method.

There were more alternatives generated by this method but only a few of these are presented. Some solutions presented the same type of problems mentioned with the HSJ method, that is, activities which come at scale levels at odds with fixed cost implications and "waste" activities. In contrast with the HSJ method, random generation of nearly-optimal alternatives appears to have two disadvantages: (a) there is more replication, in the sense that within the near-optimal set there are several solutions which do not differ widely and (b) the random method has less power to force diversification away from the optimal solution. By comparing Tables 1 and 2, it can be appreciated that some activities like sunflower, wheat, linseed, and forage sorghum don't enter the solution set (the number of runs with the two methods was the same, 15 in total).

Risk Characteristics of HSJ and RGM Solutions

Based on the *distribution* of costs and returns for each activity the *distribution* of net returns for each solution (7 for each method) for the period 1978-79 to 1986-87 was calculated. A first degree stochastic dominance analysis was constructed and showed that three solutions from the HSJ set (HSJ951, HSJ952, AND HSJ953) and five solutions from the RGM set (RGM952, RGM953, RGM955, RGM902, AND RGM905) were undominated by the LP solution. Although these results are important more work is needed to explore the risk characteristics of nearly-optimal solutions. With respect to this setting in particular, additional data is required to represent the variability of pasture yields and thus of dairy and beef production.

Conclusion

This analysis has shown that these two methods of generating nearly optimal solutions are workable alternatives to more exhaustive vertex enumeration methods. For the same reason that LP continues to be the preferred OR technique due to its inherent simplicity and software availability, the HSJ and RGM methods presented here are likely to be preferred to more complicated alternatives. Problems that have to be taken into account when working in applied settings are those related to scale (where a mixed-integer programming approach could be of value), and the appearance of "waste" activities. In this case, the implicit complementarity of these activities should be recognized, perhaps by assigning the same coefficient to the whole livestock production subsystem (cattle and pastures).

The results of this paper are validated also by empirical observation on the region where the data come from. Not only dairy and corn are produced there, as the LP solution would suggest, but also beef and other crops, as the set of nearly optimal solutions predicts. These solutions also show that a high potential for diversification exists in the forage production subsystem, where generated solutions resemble the situation in the region where different types of perennial and annual pastures are used on livestock farms.

The two methodologies presented here offer a readily workable tool for farm management advice. They can be made even more useful by taking into consideration the performance of solutions with respect to some unmodeled objectives such as cash requirements, hired labor, rotation possibilities, etc. This is a good tool for interdisciplinary research with agronomists, for example, who could be able to find nearly optimal solutions (with respect to the modeled objective) which may perform better in terms of soil conservation or other more diffuse but no less important decision criteria.

Table 1. Nearly-Optimal Solutions Corresponding to the Application of the HSJ Method.

Activity	Units	LP SOL.	HSJ951	HSJ952	HSJ953	HSJ902	HSJ903	HSJ904	HSJ905
Dairy	cow	41.71	20.61	37.56	19.41	30.35	23.73	29.80	34.89
Beef1	head					16.69		4.98	
Beef3	head					4.81			
Beef4	head								
Beef5	head								
Wheat	ha			22.72			14.18		
Lnseed	ha								10.51
Corn	ha	71.58	8.18	31.08	15.14	94.06		81.49	
GSorg	ha		14.84		13.72		14.18		10.51
Soyb1	ha			22.72				35.83	
WSoyb2	ha		78.88		80.62		39.37		
Snflwr	ha		28.92	13.83	25.98		46.80		56.13
Oats	ha	25.32	12.51	22.81	11.78	21.50	14.41	18.80	21.18
FSorg	ha	5.30				6.69	9.45		13.90
Alfa1	ha	22.29	16.17	29.47	15.23	26.09		62.68	
Alfa2	ha	22.29	16.17	29.47	15.23	26.09		62.68	
Alfa3	ha	22.29	16.17	29.47	15.23				
Cycho1	ha	29.27	8.42	15.35	14.10	53.17	56.02		59.36
Cycho2	ha	29.27	8.42	15.35	14.10	53.17	56.02		59.36
Cycho3	ha	29.27	8.42	15.35					59.36
Returns	(A)	44,431	42,218	42,219	43,937	40,027	39,988	40,034	35,567

Table 2. Nearly-Optimal Solutions Corresponding to the Application of the RGM Method.

Activity	Units	LP SOL.	RGM952	RGM953	RGM955	RGM902	RGM905	RGM803	RGM805
Dairy	cow	41.71	21.89	23.99	36.25	14.18	29.32	22.70	15.47
Beef1	head							12.48	
Beef3	head								
Beef4	head								
Beef5	head							12.48	
Wheat	ha								
Lnseed	ha								
Corn	ha	71.58	56.51	60.19	52.12	62.12	42.23	90.15	22.29
GSorg	ha				37.17		64.46		119.04
Soyb1	ha			2.91				28.75	
WSoyb2	ha		73.66	65.68		82.72			
Snflwr	ha								
Oats	ha	25.32	28.96	14.57	22.01	49.04	17.80	18.33	9.38
FSorg	ha	5.30							
Alfa1	ha	22.29	22.29	40.04	30.21	16.08	24.43	61.10	12.89
Alfa2	ha	22.29	22.29	40.04	30.21	16.08	24.43	61.10	12.89
Alfa3	ha	22.29	22.29		30.21	16.08	24.43		12.89
Alfa4	ha				30.12		24.43		12.89
Cycho1	ha	29.27	5.26	5.59					
Cycho2	ha	29.27	5.26	5.59					
Cycho3	ha	29.27		5.59					
Returns	(A)	44,431	42,210	42,213	42,184	39,277	36,896	36,898	35,545

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