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A MODEL OF PRODUCTION WITH SUPPLY MANAGEMENT FOR THE CANADIAN AGRICULTURAL SECTOR

Supply management designates those regulatory activities that involve controlling the level of production and fixing the price paid to producers. While this is a characteristic feature of Canadian agriculture (supply management is a prerogative of marketing boards dealing with milk, poultry meat, eggs, and tobacco, which account for about one fourth of total farm cash receipts), it is becoming more and more relevant in a wider setting. Supply control policies have recently been implemented for the dairy industry in the European Community, and are increasingly advocated in the United States as a means of dealing with a seemingly chronic overproduction problem. Despite the considerable research attention devoted to Canadian supply management policies (Barichello, Forbes et al., Schmitz), some interesting questions are still open for empirical analysis. One such question concerns the resource allocation effects of supply management. A characteristic of most of the above studies is their partial equilibrium framework, which neglects the possibility that the effects of restricting supply reach beyond the boundaries of regulated industries, with distorting impacts on the supply of other commodities and on the demand for inputs of production. The assessment of the inter-industry effects of supply management will be the focus of this paper, with the analysis being cast within a dual model of production for the Canadian agricultural sector.

The existence of supply management also has interesting implications for the specification of such a production model. Duality theory has spurred a number of empirical studies, and the flexibility of the dual approach is illustrated by the function that is chosen to represent aggregate technology and production behaviour. Among others, Ray adopts a cost function approach, Antle and Lopez (1984) use an unrestricted profit function, Shumway employs an input-restricted profit function, and Lee and Chambers utilize an expenditure-constrained profit function. While the symmetric nature of duality theory can justify all of the above, the choice of a dual form for empirical applications can move from the consideration that the selected form should be defined in terms of the variables exogenous to the agricultural sector in order to achieve desirable properties for the econometric results. This type of choice also offers a natural framework to evaluate the relevant elasticities and comparative statics effects, and their stochastic properties, since interest usually centers on the reaction of endogenous variables given changes in the exogenous ones. These considerations are reflected in the derivation of a particular form of restricted profit function, as will be clarified below.

Given the above, the objectives of this paper are: (i) to develop a broad theoretical framework suitable to analyze the resource allocation effects of supply restricting policies; and (ii) to analyze empirically the effects of supply restrictions on the production structure of the agricultural sector, with emphasis on the impact of supply constraints on the supply of unrestricted commodities and on the demand for inputs of production. These objectives are pursued by deriving a model of production under supply constraints, which is estimated with aggregate data for the Ontario agricultural sector.

A model of production with supply constraints

In this section, the supply management constraints are introduced explicitly in a model of profit maximization. More specifically, the model developed is of a static equilibrium nature, and is consistent with: (i) joint production; (ii) the constraints of a technology that changes over time; (iii) the hypothesis of profit maximization with price expectations; (iv) the existence of rigidities in the utilization of some factors; and (v) the existence of constraints on the supply of some commodities.

Consider a multiproduct production process in which vector y of I outputs is produced during a given period of time using a vector x of J variable inputs and a vector z of K fixed inputs. Given cost minimizing behaviour, duality theory allows the production technology to be described by a joint cost function, which is defined as:

$$(1) \qquad C(y,w,z) \equiv \min_{(x)} \{ w'x : (y,x,z) \text{ is in } V \}$$

where V is the production possibility set, and the superscripted prime indicates vector transposition. Under fairly general conditions, C(y,w,z) is shown to be non-negative, non-decreasing in w and y, and positively linearly homogeneous, concave, and continuous in w (McFadden). In addition to these properties, for analytical purposes it is useful to assume that C(y,w,z) is twice continuously differentiable in its arguments. In a competitive environment, where the output price vector p is given, the behavioural assumptions can be extended to profit maximization. It can be noted at the outset that if the output prices are not known at the moment production decisions are made, a case very pertinent to agricultural production, the vector p should represent a vector of expected prices. The required behavioural assumption is expected profit maximization, which is consistent with a more general framework of utility maximization if producers are risk neutral. Also, if a subset of the vector z, say z^{τ}, is taken as defining the state of technology at a particular point in time, this representation of production is general enough to accomodate technical change.

If there is a constraint on the maximum allowable output for some component of the vector y, as in the case of supply management, total variable profit is maximized when the profit from the unconstrained outputs is maximized. Thus, if the output vector y is partitioned into a subvector y^o of I^o products for which the constraint is binding and a subvector y^1 of I^1 unconstrained products, and if the output prices vector is similarly partitioned into p^o and p^1, the maximum profit attainable is given by:

$$\Pi \equiv p^{o\prime}y^o + G(p^1,w,y^o,z)$$

where the restricted profit function $G(p^1,w,y^o,z)$ is defined as

$$(2) \qquad G(p^1,w,y^o,z) \equiv \max_{(y^1)} \{ p^{1\prime}y^1 - C(y^1,y^o,w,z) \}$$

Given the properties of the cost function, the restricted profit function $G(p^1,w,y^o,z)$ is non-decreasing in p^1 and in z, non-increasing in w and in y^o, convex in (p^1,w), positively linearly homogeneous in (p^1,w), and continuous and twice differentiable. $G(p^1,w,y^o,z)$ can be viewed as a form of McFadden's restricted profit function, and of Diewert's variable profit function, with the explicit extension of the constraints to the output side, which implies that the restricted profit function $G(p^1,w,y^o,z)$ does not satisfy the property of non-negativity.

The restricted profit function satisfies the derivative property (Hotelling's lemma):

$$y^1(p^1,w,y^o,z) = \nabla_{p^1}G(p^1,w,y^o,z)$$

$$x(p^1,w,y^o,z) = - \nabla_{w}G(p^1,w,y^o,z)$$

where ∇ indicates a vector of partial derivatives, and $y^1(p^1,w,y^o,z)$ and $x(p^1,w,y^o,z)$ are the vectors of restricted output supply and variable input demand that maximize profits. From a functionally specified restricted profit function $G(p^1,w,y^o,z)$, Hotelling's lemma allows the derivation of an estimable system of output supplies and input demands consistent with the constraint of the underlying technology and with the hypothesis of profit maximization under supply constraints. This makes it explicit that the supply of products not subject to supply management and the demand of variable inputs in general depend on the level of restricted commodities, and this dependency can be quantified and tested in empirical applications. The restricted profit function can also be illuminating on the economic value of the restrictions. From an extension of the derivative property we have:

$$p^o(p^1,w,y^o,z) = - \nabla_{y^o}G(p^1,w,y^o,z)$$

$$r(p^1,w,y^o,z) = \nabla_{z}G(p^1,w,y^o,z)$$

where $p^o(p^1,w,y^o,z)$ and $r(p^1,w,y^o,z)$ are the shadow or virtual price vectors for the restricted outputs y^o and for the fixed inputs z. These virtual prices can be interpreted as those prices which would have resulted in y^o and z being chosen as profit maximizing levels of outputs and inputs. It can also be verified that:

$$\nabla_{y^o} G(p^1,w,y^o,z) \equiv -\nabla_{y^o} C(y^1(p^1,w,y^o,z),y^o,w,z)$$

that is, the virtual prices of the supply managed outputs are given by their marginal costs evaluated at the optimal level of y^1.

Some general results

Supply management constraints have unambiguous effects on the size of direct elasticities of unrestricted output supplies and variable input demands as a consequence of Samuelson's le Chatelier principle. Note that, in the absence of supply management, an unrestricted profit function π could be defined as:

$$\text{(3)} \qquad \pi(p^1,p^o,w,z) \equiv \max_{(y^o)} \{ G(p^1,w,y^o,z) + p^{o\prime}y^o \}$$

Using the approach in terms of Hessian identities developed by Lau (1976), one finds:

$$\nabla^2_{p^1p^1}\pi(.) = \nabla^2_{p^1p^1}G(.) - \nabla^2_{p^1y^o}G(.) \, [\nabla^2_{y^oy^o}G(.)]^{-1} \, \nabla^2_{y^op^1}G(.)$$

The size of the diagonal elements of the Hessian matrices of unrestricted and restricted profit functions can now be related. For the generic i^{th} unrestricted output we have:

$$\partial^2\pi/\partial p^1_i\partial p^1_i = \partial^2 G/\partial p^1_i\partial p^1_i - \nabla^2_{p^1_i y^o}G(.) \, [\nabla^2_{y^oy^o}G(.)]^{-1} \, \nabla^2_{y^op^1_i}G(.)$$

The second term of the right-hand-side of the above is the negative of a quadratic form around a negative semi-definite matrix (G is concave in y° from the sufficient conditions for problem (3)), and is therefore non-negative. Thus, if we let ε^{π}_{ii} denote the direct supply elasticity of the i^{th} output without supply constraints, and ε^{G}_{ii} denote the same elasticity when supply constraints are binding, it follows that:

$$\varepsilon^{\pi}_{ii} \geq \varepsilon^{G}_{ii}$$

This shows that the effect of having binding constraints on the supply of some outputs is that of decreasing the direct supply elasticity of the unrestricted outputs, and this conclusion is independent of whether restricted and unrestricted outputs are complements or substitutes. Similarly, one can prove that the effect of supply constraints is to make the direct input demand elasticities less negative (more inelastic).

The comparative statics of the restricted profit function can be expressed in terms of the more familiar joint cost function. The relevant comparative statics questions in this case concern the effect of supply managed output on the supply of unrestricted commodities and on the demand for variable input of production. For the effects of restricted output on the supply of unrestricted commodities, one gets:

$$\nabla_{y^{\circ}} y^{1}(p^{1}, y^{\circ}, w, z) = - [\nabla^{2}_{y^{1}y^{1}} C(.)]^{-1} \nabla^{2}_{y^{1}y^{\circ}} C(.)$$

while the effects of supply constraints on variable input demand are given by:

$$\nabla_{y^o} x(p^1,y^o,w,z) = \nabla^2_{wy^o} C(.) + \nabla^2_{wy^1} C(.)\ \nabla_{y^o} y^1(.)$$

It is apparent that the sign of these effects depends crucially on the type of jointness that characterizes production. Two instances of joint production are particularly relevant to agricultural production, and they provide interesting benchmarks in the case of input normality as defined in Sakai and discussed in Hertel.[1] The first one is described by Heady as an interdependent production process. This case of interdependent production has been neatly formalized by Baumol et al. using the concept of a public input, that is an input which, when acquired to produce one good, is available costlessly in other production processes. In this fashion they show that, if the public input in question is normal, the joint cost function exhibits cost complementarities, i.e. $\partial^2 C/\partial y_i \partial y_m \leq 0$ $(i \neq m)$. These cost complementarities, together with the convexity of C(.) in y^1, imply that the Hessian $\nabla^2_{y^1y^1} C(.)$ is a M-matrix. The inverse of a M-matrix is a matrix with all elements of non-negative sign (Graybill), which implies that all the elements of $\nabla_{y^o} y^1(.)$ will be non-negative. Thus, in this case (called the normal case by Sakai), restricting the output of supply management commodities will unambiguosly reduce the production of the unrestricted outputs. Similarly, the elements of $\nabla_{y^o} x(.)$ will also be non-negative, and supply constraints will decrease input use.

A second instance of joint production, first considered by Pfouts, has its origin in the existence of constraints on allocatable factors. In contrast with the public input case, jointness due to allocatable factors does not require jointness in the primal production functions. Even if the efficient transformation frontier of the production possibility set can be represented by individual production functions, the dual cost and profit functions are joint. While this case of jointness is consistent with the existence of multiproduct firms, it can also be invoked to justify the adoption of dual joint cost or profit functions for an agricultural sector with specialized production units, as long as these specialized firms utilize factors (such as land) that are available in the aggregate only in fixed quantity. Shumway et al. consider the case of allocatable fixed factors extensively. Although they prove jointness in terms of factor requirement functions, they do not analyze the most useful implications in terms of the joint cost function. In fact, this type of jointness involves cost substitutability, i.e. $\partial^2 C/\partial y_i \partial y_m \geq 0$ $(i \neq m)$, when the further assumption of normality of the allocatable inputs is made. To see this, it is convenient to derive the joint cost function C(y,w,z) in two steps. In the first stage, the use of variable factors is optimized for each production process conditional on the availability of fixed resources for that process.[2] Thus the conditional cost functions are defined as:

$$(4) \qquad C^i(y_i,w,z^i) \equiv \min_{(x^i)} \{w'x^i : y_i \leq f^i(x^i,z^i)\}$$

where $f^i(x^i,z^i)$ is the production function for the i^{th} product, and (x^i,z^i) are the vector of variable inputs and the allocatable input used exclusively in the production of y_i. The second step involves the optimization of the allocations of fixed factors among the various products. The joint cost function can be defined as:

$$(5) \qquad C(y,w,z) \equiv \min_{(z^i)} \{\Sigma_i C^i(y_i,w,z^i) : \Sigma_i z^i \leq z\}$$

If z^{i*} are the solutions to problem (5), then

$$C(y,w,z) \equiv \Sigma_i C^i(y_i,w,z^{i*})$$

Differentiating this yields, via the envelope theorem:

$$\partial^2 C/\partial y_i \partial y_m = (\partial^2 C^i/\partial y_i \partial z^{i*})(\partial z^{i*}/\partial y_m)$$

Differentiating the first order conditions for problem (5) with respect to y_m and solving yields:

$$\partial z^{i*}/\partial y_m = (\partial^2 C^m/\partial z^{m*}\partial y_m)/(\Sigma_i \partial^2 C^i/\partial z^{i*}\partial z^{i*})$$

Combining these results gives:

$$\partial^2 C/\partial y_i \partial y_m=(\partial^2 C^i/\partial y_i \partial z^{i*})(\partial^2 C^m/\partial y_m \partial z^{m*})/(\Sigma_i \partial^2 C^i/\partial z^{i*}\partial z^{i*})$$

The second order conditions for the problem in (5) require the denominator of the above to be positive. Moreover, if the fixed input is a normal input the two cross derivatives in the numerator are non-positive. Hence $\partial^2 C/\partial y_i \partial y_m \geq 0$, with this inequality holding strictly if z is a strongly normal input for the two outputs involved. In this case, however, the comparative statics of supply management cannot be determined unambiguosly. However, in the special case of only one unrestricted output, allocatable normal inputs will imply $\partial y^1/\partial y_m^o \leq 0$. The effect of supply constraints on variable input demand, on the other hand, is indeterminate even in this special case.[3] In the absence of any general guideline, whether or not supply restricting policies will increase or decrease the supply of unrestricted output and the demand of variable inputs is a question that will be pursued at the empirical level.

The empirical model

Econometric estimation of the parameters of the normalized restricted profit function requires that we observe the solution vector of the postulated optimization problem at different values of the exogenous variables. Given that we are dealing with one sector, this can only be achieved by assuming that the outcome of the system at different points in time represent such observations. A consistent set of time series data was obtained for the period 1961-1983, the limiting factor being the lack of input prices prior to 1961. The choice of applying the model to the province of Ontario, rather than to Canada as a whole, stems from a consideration of the enormous structural differences in production between the provinces of Canada. The assumptions of the model are therefore more likely to be satisfied at the provincial level. This need not be too restrictive, especially because Ontario is the largest and most diversified province in terms of agricultural production.

Given the limitation of this data base, it is apparent that for estimation to be feasible outputs and inputs have to be expressed in terms of some meaningful aggregates.[4] Wishing to discriminate between commodities that are supply managed and commodities that are not, the model postulates the existence of five output groups: crop products (all crops except tobacco); red meat products (cattle, calves, hogs, and sheep and lambs); dairy products; poultry and eggs; and tobacco. The first two of these aggregate commodities are treated as unrestricted outputs, while the last three are considered restricted commodities. The products included in the first two groups are not subject to supply management, and they are traded in a virtually open market, which makes the assumption of exogenous prices reasonable for a small country like Canada. The last three groups of outputs correspond closely to the commodities that are controlled under supply management. These quantities are fixed by marketing board policies, and thus can safely be assumed exogenous in the estimating system.

On the input side, the model postulates the existence of five input groups: capital; labour; energy; materials for animal production; and materials for crop production. Capital includes land and buildings, implements and machinery, and livestock and poultry stocks. The quantity of land is given at the aggregate level, and thus corresponds closely to what we have called allocatable fixed inputs. Buildings and machinery, on the other hand, are likely to be quasi-fixed in the short run, an argument that could also be made for livestock capital. Thus the capital input group is treated as a fixed input. The remaining four groups of inputs are assumed to be variable in the short run. Labour includes both hired labour and self-employed labour. Energy includes petroleum, oil and lubricants, and electricity and telephone. Materials for animal production include feed, feeder cattle and weanling pigs, and other livestock expenses. Materials for crop production include fertilizers and lime, pesticides, seeds, an other miscellaneous expenditures. Finally, an additional exogenous variable is specified as a linear trend to account for the effects of (possibly biased) technical change.

For the purpose of empirical estimation of the restricted profit function, a specific functional form must be postulated. In doing so, it is important to choose a functional form that imposes the least restrictions on the function being estimated, while being able to satisfy the regularity conditions of the function itself. The functional form selected in this study is the normalized version of the quadratic form originally proposed by Lau (1974), and already applied in profit function estimation by Shumway, Lopez (1985), and Huffman and Evanson. This functional form satisfies accepted definitions of flexibility (Barnett), and has the advantages of having an Hessian of constants, so that the curvature property of convexity can be tested globally. Also, the normalized quadratic is not affected by the fact that the restricted profit function can take on negative values, which prevents the use of forms, such as the translog, that require logarithmic transformation of profit.

Choosing the last input as numeraire, let $q \equiv (y^1, -x_1, \ldots, -x_{J-1})$ be the $M=(I^1+J-1)$ component variable netput vector, v the corresponding normalized prices obtained by deviding the M prices $(p^1, w_1, \ldots, w_{J-1})$ by w_J, and $c \equiv (y^o, z)$ the $N=(I^o+K)$ component vector of constraints (restricted outputs and fixed resources, including the variables defining the state of technology). The normalized quadratic profit function can be written as:

$$g(v,c) = \alpha_o + \Sigma_i \alpha_i v_i + \Sigma_j \beta_j c_j + \tfrac{1}{2} \Sigma_i \Sigma_m \alpha_{im} v_i v_m + \tfrac{1}{2} \Sigma_j \Sigma_n \beta_{jn} c_j c_n + \Sigma_i \Sigma_j \gamma_{ij} v_i c_j$$

with $i,m = 1,\ldots,M$ and $j,n = 1,\ldots,N$, and the left-hand-side satifies $g = G/w_J$. Thus, G is linearly homogeneous by construction, while symmetry can be imposed by requiring $\alpha_{im} = \alpha_{mi}$ and $\beta_{jn} = \beta_{nj}$. By exploiting the derivative property we get:

$$(6) \qquad q_i = \alpha_i + \Sigma_m \alpha_{im} v_m + \Sigma_j \gamma_{ij} c_j \qquad i = 1,\ldots,M$$

This system of M equations can be used jointly with the normalized quadratic profit function to estimate all the parameters of the restricted profit function. However, even with this specification, the estimation of the whole set of parameters of the profit function is problematic. This is the case because the β_j and β_{jn} parameters of the fixed resources and of the restricted outputs only appear in the profit function. When the number of constraining variables is relatively high, the estimation burden that falls on this one equation is unmanageable. To proceed, the estimation is restricted to the set of M equations in (6). While this allows the estimation of all the direct and cross price elasticities for all variable quantities, it is clear that without estimating the full model it is not possible to estimate the shadow price of the constraints.

The system of equations (6) requires some transformation if we wish to test, and possibly impose, the property of convexity of the restricted profit function. Letting A be the M×M matrix of the α_{im} coefficients, the restricted profit function will be convex if A is positive semi-definite. Following Lau (1978b), a test for global convexity is possible if we estimate A in its Cholesky factorization. Lau shows that almost every real symmetric square matrix A can be represented in its Cholesky factorization as A=LDL', where L is a unit lower triangular matrix and D is a diagonal matrix whose elements are the Cholesky values. The matrix A will be positive semidefinite if and only if all Cholesky values are non-negative. To be able to test this proposition statistically, the parameters α_{im} in the system of equations (6) are therefore substituted by the above non-linear factorization.

The data necessary to estimate the system was obtained from published and unpublished Statistics Canada data. A detailed description is available from the author on request. Fisher ideal indices were used for the variable inputs, while available output indices are of the Laspeyres type. Output prices are ex-post implicit price indices, and include federal and provincial payments. Given the scaling problem involved by the use of price indices, for (6) to remain meaningful all price indices are normalized to 1 in 1980, and all quantity variables are measured in billions of 1980 dollars.

Estimation results

When the Cholesky factorization is applied, the system of equations (6) is non-linear in the parameters, so that an estimation technique suitable for a system of nonlinear, seemingly unrelated regressions is needed. The stochastic version of the nonlinear system of output supply and input demand equations can be written as:[5]

$$q_t = f(X_t, \theta) + u_t \qquad t = 1, \ldots, T$$

where t indexes time series observations, q_t is a M×1 vector of output supply and input demand quantities at time t, X_t is the vector of all exogenous variables at time t, θ is the vector of all coefficients to be estimated, and u_t is a M×1 vector of random errors with zero expectations and non-diagonal covariance matrix. More precisely, u_t is assumed to satisfy:

$$E(u_t) = 0 , \quad E(u_t u_t^{'}) = \Omega , \quad \text{and} \quad E(u_t u_s^{'}) = 0 \qquad (t \neq s)$$

The estimation procedures available for this multivariate non-linear regression model are reviewed in Amemiya. If, in addition to satisfying the above stochastic assumptions, the vector of disturbances is multinormally distributed, maximum likelihood estimation can be performed. Under the stated stochastic assumptions, the obtained maximum likelihood estimators are known to be consistent, asymptotically normal, and asymptotically efficient. The method used to obtain the maximum likelihood estimator in this paper is a generalized Gauss-Newton algorithm as derived by Berndt et al., and implemented in TSP 4.0 .

To make the hypothesis of constrained profit maximization more suitable to the case of agricultural production characterized by uncertain prices, it is argued that the optimization problem is solved conditional on expected output prices. Given that these expected prices are typically unobservable, the problem of their estimation is solved prior to the estimation of the restricted profit function. In a similar context, Shumway measures expected prices with a geometric lagged function of the previous seven years' prices, while Weaver uses futures prices. Gordon adopts a univariate ARIMA model to extrapolate expected prices, since such models display many of Muth's rational expectations behaviour while retaining the simplicity of not requiring the specification of a structural model of price expectation formation. For these reasons, it is assumed that the relevant expected prices for the unrestricted outputs can be approximated by the predictions of an ARIMA(p,d,q) model. Using the Ljung-Box Q statistic as the main diagnostic checking procedure (together with the fulfilment of the stationarity and invertibility conditions), the models that were accepted were an ARIMA(1,1,2) for the price of crops, and an ARIMA(0,1,2) for the price of red meat products.

Using the expected prices fitted with the ARIMA models, and the data described in previous section, the system of equations (6) is estimated by the method of maximum likelihood. Price homogeneity is maintained by deflating all prices by the price of labour. Accordingly, the system has 5 equations, the dependent variables of which are: crop production, red meat production, negative of energy consumption, negative of crop materials consumption, and negative of animal materials consumption. Symmetry is also maintained by requiring $\alpha_{im} = \alpha_{mi}$. The estimation results for this system of equations are reported in Table 1. Given that the Choleski reparameterization was employed, the estimated parameters of the price variables in Table 1 are calculated from a non-linear combination of the estimated Cholesky factorization parameters. The standard errors of the α_{im} parameters are computed by linearizing these non-linear functions by a Taylor series expansion of the first order, and then applying the standard results for variance and covariance of linear functions of random variables (Goldberger).

The estimated model fits the data very well, as indicated by the R^2 coefficients, ranging from 0.90 and 0.98 . Since we are using time series data, a legitimate worry concerns the presence of autocorrelation in the residuals, which would violate the stochastic assumptions of the model. A general test for randomness is the Q statistic already used for the ARIMA model, and this test was computed based on the first 20

Table 1 - Maximum likelihood estimates of the normalized quadratic restricted profit function coefficients

Dependent Variable	Constant	Crops	Red Meat	Energy	Crop Materials	Animal Materials	Poultry & Eggs	Dairy Products	Tobacco	Capital	Time	R2	Q(20)
		Normalized Price of					Quantity of						
Crops	1.3809 (0.7765)	0.0988 (0.1156)	-0.0693 (0.0620)	0.0052 (0.0106)	0.0379 (0.0392)	0.2006 (0.0557)	-0.6324 (0.7905)	-1.0351 (0.6888)	-1.9598 (0.4374)	0.3689 (0.3741)	0.0379 (0.0072)	0.93	6.27
Red meat	1.1358 (0.4337)		0.7429 (0.1037)	-0.1808 (0.0260)	-0.0338 (0.0612)	0.2020 (0.0581)	-0.9875 (0.4274)	-0.3038 (0.4031)	-0.2766 (0.2326)	-0.2722 (0.2153)	0.0412 (0.0042)	0.95	13.86
Energy	-0.1600 (0.0592)			0.2347 (0.0262)	0.0778 (0.0414)	-0.1087 (0.0181)	-0.2119 (0.0744)	-0.2685 (0.0566)	-0.0309 (0.0306)	0.1537 (0.0351)	-0.0062 (0.0006)	0.98	19.26
Crop Materials	0.5526 (0.2446)		symmetric		0.3755 (0.1053)	-0.2004 (0.0564)	-0.0748 (0.2698)	-1.2384 (0.2259)	-0.1348 (0.1317)	-0.3331 (0.1300)	-0.0140 (0.0023)	0.96	24.54
Animal Materials	0.2390 (0.4013)					0.1034 (0.0685)	-0.6452 (0.4042)	-1.0727 (0.3629)	0.1820 (0.2197)	-0.0009 (0.1996)	-0.0139 (0.0039)	0.90	14.31

Notes: Dependent variables are positive quantities for outputs and negative quantities for inputs.
Asymptotic standard errors are reported in parentheses under the corresponding parameter estimates.
Maximized Log-likelihood = 272.401

autocorrelations of the estimated residuals. For 20 degrees of freedom, the χ^2 critical value at the 5 percent level is 31.410, and therefore the Q statistics reported in Table 1 do not lead to the rejection of the hypothesis that the estimated residuals are independently distributed.

Although nineteen of the forty five estimated parameters reported in Table 1 are not significantly different from zero at the 5 percent level, these results appear satisfactory considering the large number of parameters of the model. The sign of the estimated parameters are in general consistent with the theoretical model. The own price response of the output supply equations is positive, while the own price response of the input demand equations is negative. The relationships of greatest interest, however, are those involving the supply managed commodities. With respect to the effects of these constraints on the supply of unrestricted commodities, we note that the relationships are uniformly negative. This result (together with the estimated negative effect between crops and red meat supply) is sufficient to generate the cost substitutabilities typical of allocatable normal factors, thus providing support for the relevance of this kind of jointness. The effect of constraining the supply managed commodities below equilibrium level would result in an increased supply of both groups of unrestricted output. Individually, the strongest effects on the supply of crops involve the quantity of dairy products and tobacco, while the supply of red meat products is more directly affected by the restrictions on poultry and eggs production. As to the effects of supply management on input use, the estimated relationships in general imply input use increasing with the level of restricted outputs, with the exception of the effects of tobacco on the animal input group. Among the most significant estimated parameters are those of the time variable, indicating the presence of a strong autonomous component in the trend of output supply and input demand. In all cases, this trend is associated with an increase in output supplies and input demands, suggesting that the effect of technical change is to increase the scale of production. A more significant discussion of these results can be performed by converting the estimated parameters into elasticities. Before turning to that, however, the possibility of restricting the parameter space by imposing some economically meaningful restrictions will be investigated.

One of the most troublesome issues in empirical applications of flexible functional forms concerns the fulfillment of the curvature properties of monotonicity and convexity. Monotonicity requires that the estimated output supply and input demand quantity be positive, and this property is satisfied at each observation point by the estimated equations. Convexity in prices will be globally satisfied by the normalized quadratic profit function if the matrix $A \equiv [\alpha_{im}]$ is positive semi-definite. Given the reparameterization of this matrix by the Cholesky factorization, convexity will be satisfied if the estimated Cholesky values δ_{ii} are all non-negative. The parameters of the Cholesky diagonal matrix D, directly estimated by the model, are reported in Table 2. Despite the fact that the own price responses of output supplies and input demands all have the correct sign, one of the estimated Cholesky values turns out to be negative, thus violating the property of convexity. To assess this violation statistically, following Lau (1978b) the null hypothesis of convexity can be expressed as:

Table 2 - Estimated Cholesky values

Parameter	Estimate	t-statistic
δ_{11}	0.0988	0.8545
δ_{22}	0.6942	5.7111
δ_{33}	0.1892	8.2569
δ_{44}	0.3320	2.1311
δ_{55}	-0.6840	-0.6914

H_0: $\delta_{ii} \geq 0$ $\qquad$ $i = 1,\ldots,5$

which is tested against the alternative:

H_1: $\delta_{ii} < 0$ $\qquad$ for at least one i

Thus, H_0 will be rejected if at least one δ_{ii} is significantly negative (see also Morey). To test for the significance of the individual δ_{ii}, given that H_0 involves simultaneously 5 inequalities, the Bonferroni t-statistics can be used (Savin). If the overall level of significance of the test is 0.05, the fact that our test involves 5 simultaneous restrictions implies that the one-tailed critical value of the Bonferroni t-statistic is given by the Student t distribution at the 0.01 significance level. Using the Student t distribution with ∞ degrees of freedom, given the asymptotic nature of the estimates, the critical value for the individuals δ_{ii} t-statistics is 2.326 . It follows that the estimated δ_{55} is not significantly negative, and that the hypothesis of convexity cannot be rejected at the 5 percent probability level.

Given that the theoretical results of the le Chatelier effects, and those of the comparative statics of supply constraints, crucially depend on the existence of jointness in production, it is important to verify that the estimated model satisfies this structural property. Since non-jointness implies that the marginal cost of producing an output is independent of the quantity of other outputs, for the restricted profit function it requires that the unrestricted output supplies must be independent of other unrestricted product prices and of restricted product quantities, and this involves that 7 of the estimated parameters be set to zero. Another test of interest, which is nested in this general test of non-jointness, concerns non-jointness between unrestricted outputs and supply managed commodities, and this requires that 6 of the estimated parameters be set to zero. These parametric tests were performed using the likelihood testing procedure (Engle), conditional on the maintained hypothesis of symmetry and price homogeneity, and are reported in Table 3. In all cases, the likelihood ratio statistics lead to a rejection of the null hypothesis at the 5 percent level. Consequently, the aggregate technology of the Ontario agricultural sector is characterized by jointness in production, and constraining the supply of supply managed commodities will have a significant impact on the supply of other outputs.

A further hypothesis of considerable economic interest is that of constant returns to scale. Panzar and Willig extend the concept of scale economies to the case of multiple outputs. As in the single output case, scale economies are easily defined when dealing with homogeneous functions. Constant returns to scale can be imposed globally by requiring the restricted profit function to be homogeneous of degree one in the restricted outputs and fixed inputs (y°, z). This can be done along the same lines used to maintain linear homogeneity in prices. Let c° be the vector of (N-1) restricted outputs and fixed inputs deflated by the N^{th} fixed input, i.e. $c_n^o \equiv c_n/c_N$, $n = 1,\ldots,N-1$. Then the normalized restricted linearly homogeneous in c profit function is defined by :

$$g(v,c) \equiv c_N \, g^\circ(v,c^\circ)$$

Thus, by specifying a flexible functional form for $g^\circ(v,c^\circ)$, we

Table 3 - Likelihood ratios for non-jointness tests

Test	LR Statistics	$\chi^2_{0.05}$
Non-jointness	25.713	14.067
Non-jointness between unrestricted and restricted outputs	24.552	12.592

guarantee that G(.) will be linearly homogeneous in prices and linearly homogeneous in constrained outputs and fixed inputs. When the quadratic form is chosen, however, the approximation for $g^{\circ}(v,c^{\circ})$ will not be parametrically nested within the approximation for g(v,c), which calls for a non-nested hypothesis testing. This framework of analysis, reviewed by MacKinnon, allows the testing of the model under the null hypothesis against the model under an alternative hypothesis by nesting them in an artificial meta-model. Let $f_{it}(\theta)$ be the model for the i^{th} dependent variable at time t of the unrestricted system of equations, and let $h_{it}(\theta^{\circ})$ represent the model for the same variable at time t when constant returns to scale are imposed. By stacking the MT observations into vectors, these two models can be rewritten as:

$$H_0: \quad q = h(\theta^{\circ}) + u^{\circ}$$

$$H_1: \quad q = f(\theta) + u$$

where q, h, f, u°, and u are vectors of dimension (MT×1). These two models can be combined into the artificial regression:

$$q = (1-\beta)\ h(\theta^{\circ}) + \beta\ f(\theta) + e$$

where β is a nesting parameter, and e is a vector of residuals. Thus, testing the null hypothesis of constant returns to scale reduces to the test of β = 0. Using Davidson and MacKinnon's P_1 test (β, θ°, and θ are not simultaneously identified), the t-value for the estimated β is 3.526, and thus the non-nested test clearly rejects the hypothesis of constant returns to scale.[6]

Of the many tested restrictions on the restricted profit function, the only one that was not rejected is the hypothesis of convexity in prices. Before computing relevant elasticities, therefore, it would seem desirable to impose the convexity property. In principle this can be done by the method of squaring (Lau, 1978b). The problem with this technique, as already noted by Talpaz et al., is that achieving convergence in the maximum likelihood estimation may be difficult. In fact, convergence was not achieved using a reasonable amount of computer time. Altough this need not work in general, in our case convexity can be maintained by simply imposing the constraint $\delta_{55} = 0$. Thus, the model was re-estimated with this constraint, and Table 4 reports the elasticities derived from this restricted model and evaluated at the exogeneous variables' mean point. The elasticities of variable quantities with respect to the deflator (the price of labour) are retrieved from the homogeneity condition, while the elasticities of labour demand with respect to the price variables are retrieved from the Cournot aggregation condition. Since elasticities are nonlinear functions of the estimated parameters, the standard errors reported in Table 4 are obtained by the linearization method already mentioned.

It can be seen that most price elasticities are significantly different from zero. Both unrestricted outputs have inelastic supply, although red meat supply seems more responsive to own price movements. Energy, crop materials, and animal materials have inelastic demand. The demand of animal materials, the main component of which is feed, is the most inelastic one. Labour demand, on the other hand, displays an elastic own price response. This finding, together with the fact that all variable inputs turn out to be gross substitutes with the labour input and that labour price has risen more than other input prices, can

Table 4 - Elasticity estimates at the mean point
convexity in prices maintained

Elasticity of	with respect to the price of					
	Crops	Red meat	Energy	Crops materials	Animal materials	Labour
Crops	0.257 (0.138)	-0.089 (0.075)	0.009 (0.013)	-0.090 (0.041)	0.144 (0.047)	-0.313 (0.144)
Red meat	-0.059 (0.050)	0.531 (0.076)	-0.132 (0.019)	-0.027 (0.046)	0.137 (0.044)	-0.451 (0.080)
Energy	-0.021 (0.030)	0.445 (0.066)	-0.625 (0.070)	-0.203 (0.110)	0.292 (0.047)	0.111 (0.051)
Crops materials	0.011 (0.053)	0.051 (0.089)	-0.116 (0.063)	-0.605 (0.158)	0.221 (0.076)	0.438 (0.098)
Animal materials	-0.161 (0.052)	-0.229 (0.073)	0.144 (0.023)	0.191 (0.066)	-0.250 (0.066)	0.304 (0.128)
Labour	0.204 (0.094)	0.442 (0.078)	0.032 (0.015)	0.222 (0.050)	0.178 (0.075)	-1.079 (0.163)

Notes: Asymptotic standard errors are reported in parentheses.

partly explain the dramatic decline of labour input use during the observation period. The cross effects between input demands show that animal materials are, like labour, gross substitutes for all other inputs, while energy and crop materials display the only complementarity relationships. The cross effects between input prices and output supplies indicate that crop supply is affected by animal materials and labour prices, while red meat is affected by energy, animal materials, and labour prices. While energy and labour prices have a negative effect on supply, animal materials price enters positively in the supply of both outputs.

Similar to the price elasticities discussed above, one could compute the elasticities of unrestricted outputs and variable inputs with respect to the restricted outputs and fixed resources. These elasticities, however, may not add much to the information conveyed by the gradients reported in Table 1. What is perhaps more interesting, as emphasized by Weaver, is to determine whether the constraining variables have a neutral or bias effect on the unrestricted choice variables. Several notions of neutrality can be defined. The most useful in the present context is what Lau (1978a) calls indirect Hicks neutrality, which implies that the ratios of the derived demand (supply) of any two inputs (outputs) is independent of the constraining variable. Thus, indirect Hicks neutrality implies:

$$\partial(q_i/q_m)/\partial c_n = (q_i/q_m c_n)(\eta_{in} - \eta_{mn}) = 0$$

where (i,m) represent any pair of variable inputs or unrestricted outputs, and η_{sn} is the elasticity of the s^{th} variable quantity (s=i,m) with respect to the n^{th} constraining variable. Thus, a measure of bias between unrestricted outputs (inputs) q_i and q_m due to the exogenous variable c_n can be defined as $B_{im}^n \equiv (\eta_{in} - \eta_{mn})$, and this measure is independent of the unit of measurement of c_n. If $B_{im}^n = 0$, then the constraining level of c_n does not bias the optimal mix between quantities q_i and q_m, while $B_{im}^n > 0$ implies a bias against quantity q_m, and $B_{im}^n < 0$ implies a bias against quantity q_i.

The above measure of bias was computed for each pair of outputs and inputs, and relative to the three supply managed outputs, the fixed factor, and the index of technological change, and the results are reported in Table 5.[7] The unrestricted outputs are significantly affected only by tobacco, and increases in the quantity of this regulated output will bias the mix against the other crops aggregate. As for the variable input mix, the quantity of poultry and eggs does not have a significant impact, while the quantity of dairy products is more significant. In particular, raising the quantity of this regulated output will bias the input mix against labour, relative to all the other three inputs. To evaluate this finding, it should be kept in mind that in this model short run adjustments in milk production are conditional on a given dairy herd, and can be achieved by recombining variable quantities only. Capital does not affect significantly the output mix, while on the input side it is significantly bias against energy, and in favour of labour, relative to all remaining variable inputs. This shows that the estimated technology displays gross substitutability between energy and capital, while capital is a gross complement for labour. Technological change does not affect the relative importance of unrestricted outputs, and on the input side it has a significant impact only for input pairs involving labour. In particular, technological

Table 5 - Biasing effects of constraining variables at the mean point convexity in prices maintained

Pair of variable quantities		Fixed Quantities				
		Poultry & eggs	Dairy products	Tobacco	Capital	Time
Crops	Red Meat	0.031 (0.424)	-0.873 (0.840)	-0.541 (0.169)	0.544 (0.420)	0.168 (0.128)
Energy	Crop Materials	0.039 (0.168)	-0.568 (0.336)	-0.065 (0.058)	-0.510 (0.170)	-0.074 (0.047)
Energy	Animal Materials	-0.200 (0.184)	-0.016 (0.405)	0.050 (0.070)	-0.110 (0.194)	-0.033 (0.057)
Energy	Labour	-0.090 (0.197)	1.364 (0.450)	-0.038 (0.077)	-0.815 (0.212)	0.482 (0.063)
Crop Materials	Animal Materials	-0.239 (0.143)	0.552 (0.283)	0.115 (0.049)	0.400 (0.145)	0.041 (0.040)
Crop Materials	Labour	-0.130 (0.162)	1.932 (0.343)	0.027 (0.060)	-0.305 (0.168)	0.556 (0.049)
Animal Materials	Labour	0.110 (0.218)	1.380 (0.488)	-0.088 (0.086)	-0.705 (0.232)	0.515 (0.070)

Note: Asymptotic standard errors are reported in parentheses.

change is significantly biased against labour, and this appears another very important explanation for the diminution of labour use observed in the estimation period.

In addition to analyzing the significance of each individual entry in Table 5, it is possible to consider joint restrictions on these bias measures to test broader hypothesis. An appropriate test for this purpose is the Wald test (Engle). Table 6 summarizes the Wald tests for indirect Hicks neutrality of supply managed commodities (considered as a whole), capital, and technological change, with respect to two broad groups of variable choices: unrestricted outputs, and variable inputs. The conclusion is that the quantity of restricted outputs has an overall significant biasing effect on both groups of choices, while capital and technical change significantly bias only the variable input group.

Summary

This paper has developed a model of production under supply constraints that is suitable to assess some of the resource allocation effects of supply management on the Canadian agricultural sector. Given jointness, the supply of unrestricted outputs and the demand for variable inputs are affected by the level of regulated output. At the theoretical level, a le Chatelier effect of supply constraints was derived. The comparative statics of supply management depend crucially on the type of jointness. The model was thus estimated with time series data of the Ontario agricultural sector. The estimation results of the restricted profit function are satisfactory from an econometric point of view. The estimated equations fit the data well, the stochastic

Table 6 - Wald tests for indirect Hicks neutrality

output/input pairs	constraining variable		
	restricted outputs	capital	time
unrestricted outputs	10.96 (7.82)	1.67 (3.84)	1.72 (3.84)
variable inputs	47.98 (28.87)	18.35 (12.59)	146.4 (12.59)

Note: In parentheses are the critical values for $\chi^2_{0.05}$.

assumptions of the model are not violated, and the theoretical restrictions of monotonicity and convexity cannot be rejected. An application of non-nested tests was implemented to obtain a test of constant returns to scale for the normalized quadratic, and constant returns to scale were rejected. The time trend is very significant in explaining all estimated relationships, and technological change is significantly biased against labour use. The elasticity estimates show that, with the exception of labour, output supply and input demand are inelastic, and input substitution possibilities are limited. The results show that jointness in production at the aggregate level cannot be rejected. In particular, all outputs seem to be substitutable in production, which provides explicit support for the case of allocatable fixed inputs. This means that the reduction in output of regulated commodities, usually imputed to supply management, is partly offset by increased production of unrestricted output. There are however limits to this substitution, as reduction in regulated outputs in general reduces total input use.

NOTES

[1] An input is said to be normal if the output elasticity of the compensated input demand is non-negative.

[2] For notational simplicity the proof that follows is restricted to the case of one fixed input.

[3] In a multiple output framework, whenever Sakai's "normal" technology conditions are not satisfied, $\partial x_j/\partial p_i^1 = -\partial y_i^1/\partial w_j \leq 0$ is also permitted. The effects of supply constraints on shadow prices could similarly be studied. However, given the inability of the empirical model to estimate shadow prices, there is little scope for exploring this direction.

[4] Aggregation over commodities can be justified if the production function is characterized by what Leontief called functional separability, or if all price changes within an aggregate are proportional (Hicks' composite commodity theorem).

[5] The transition from the deterministic model of economic theory and the stochastic model of empirical analysis can be justified with arguments given in Weaver, or Chalfant and Gallant.

[6] Given this finding, it would be interesting to measure the actual degree of returns to scale. Unfortunately, this cannot be done satisfactorily by the estimated model, since a local measure of multiproduct scale economies would involve the unknown shadow prices of restricted outputs and fixed inputs. The alternative of measuring returns to scale relative to variable inputs only, as done by Weaver,

is not very useful, since decreasing returns to scale in variable inputs is a necessary condition for the very existence of a profit function.

[7] Again, the standard errors of B_{im}^{n} (a nonlinear combination of random variables) are computed by the aforementioned linearization method. To be able to assess the bias of input pairs involving labour (the numeraire), all the bias measures involving input pairs are calculated with the model re-estimated using an output price (the expected price of red meat) as the deflator.

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