Working Papers Series

Working Paper WP87/6 July 1987

A COMPARISON OF TWO FUNCTIONAL FORMS IN DYNAMIC ANALYSIS

by

Wayne H. Howard
and
C. Richard Shumway

UNIVERSITY of GUELPH

Department of Agricultural Economics and Business

University of Guelph
Guelph, Ontario
Canada
N1G 2W1

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Wayne H. Howard and C. Richard Shumway*

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Department of Agricultural Economics and Business
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* The authors are, respectively, an Assistant Professor, Department of Agricultural Economics and Business, University of Guelph, (a former Research Associate, Texas A&M University) and a Professor, Department of Agricultural Economics, Texas A&M University.

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Abstract

The robustness of dynamic dual model results to choice of functional form is examined for the U.S. dairy industry. Modified generalized Leontief (GL) and normalized quadratic (NQ) functional forms are compared by examining their consistency with properties of the competitive firm, estimated rates of adjustment for cows and labor, tests of technological change, and elasticities. Homogeneity and symmetry are maintained in both models. Convexity is not rejected by the GL but is rejected by the NQ. Absence of technological change is rejected by both models, but a quality index on labor fully embodies technological change occurring within labor in the NQ but not in the GL. Policy-relevant elasticities differ greatly between the functional forms. Dynamic dual models are found to be non-robust in important ways to choice of functional form.

A COMPARISON OF TWO FUNCTIONAL FORMS IN DYNAMIC ANALYSIS

Dynamic adjustment in the U.S. dairy industry has received considerable recent attention (Chavas and Klemme; LaFrance and deGorter; Howard and Shumway). One method for estimating rates of adjustments of the quasi-fixed inputs is the dynamic dual model (Epstein). Given a flexible functional form, the dynamic dual allows testing and/or maintaining theoretical properties while examining the structure of the industry. However, the robustness of the dynamic dual model to choice of functional form has not been investigated.

This study examines the robustness of dynamic dual model results to the functional form employed for estimation. Epstein suggests four functional forms that meet the required conditions for an intertemporal cost or profit function. Three have been used for estimation in different economic studies (Epstein and Denny; Vasavada and Chambers; Taylor and Munson). However, none of these studies report results for more than one functional form with the same data.

Research comparing functional forms of static dual models has found significant differences in (a) tests of theoretical restrictions, (b) estimated price elasticities (Swamy and Binswanger), and (c) elasticities of substitution (Chalfant; Baffes and Vasavada). This study compares two of the functional forms suggested by Epstein in a dynamic dual analysis of the U.S. dairy industry for the period 1951-1982. Robustness is examined by comparing consistency with theoretical properties, structural tests, and elasticities to see if the choice of functional form substantially affects important results.

The Dynamic Dual Model

Assume a competitive industry consisting of firms maximizing their net discounted value of production over an infinite planning horizon. The firms face exogenous input and output prices and have static price expectations. Also assume an industry production function, $F(X,Z,\dot{Z})$, that is twice continuously differentiable, concave, with F_X, $F_Z > 0$ and $F_{\dot{Z}} < 0$, where $\dot{Z}$ is the net change in investment in quasi-fixed inputs and subscripts are derivatives.

Given the above assumptions, a value function, $J(P,W,C,Z)$ exists that is twice continuously differentiable, linearly homogeneous and convex in (P,W,C) and concave in Z (Epstein). Moreover, if $J_{ZC} \neq f(P,W,C)$, net investment in quasi-fixed inputs can be expressed in the form of a flexible accelerator model.

Functional forms that maintain linear homogeneity in prices, concavity in quasi-fixed inputs, and flexible accelerator investment in quasi-fixed inputs are employed to estimate the aggregate behavioral equations for the U.S. dairy industry. A modified generalized Leontief (GL) and a normalized quadratic (NQ) as used by Vasavada and Chambers (1982 and 1987, respectively) meet the above requirements.

The dual value function in the GL form is:

$$(1)\quad J(P,W,C,Z,T) = [P\ W]'AZ + C'M^{-1}Z + [P^{.5}\ W^{.5}]'EC^{.5} + C^{.5}{}'FC^{.5} + [P^{.5}\ W^{.5}]'G[P^{.5}\ W^{.5}] + TH[P\ W\ C],$$

where P is the average blend price of milk in the U.S., W is the price of concentrates, Z is two dimensional and includes the number of dairy cows in the U.S. that have calved and labor used in the dairy sector, C is two dimensional and includes the annual average rental price of a dairy cow in the U.S. and the agricultural labor wage rate. T is a trend variable included to capture the effects of disembodied technological change. Parameters A, M^{-1}, E, F, and G are each (2 x 2), and H is (1 x 4).

The dual value function in the NQ form is:

$$(2) \quad J(w,c,Z,T) = [1\ w\ c\ Z]'a + c'm^{-1}Z + .5\ g\ w^2 + w\ d\ Z$$
$$+ w\ e\ c + .5c'f\ c + .5Z'q\ Z + [1\ w\ c]'h\ T$$

where w = W/P, and c = C/P. Parameter a is (1 x 6), m, f, and q are (2 x 2), d and e are (1 x 2), g is a scaler, and h is (1 x 4).

The behavioral equations are obtained by applying the envelope theorem to the value function. For the GL, output supply, variable input demand, and quasi-fixed input demand are, respectively:

$$(3) \quad F(P,W,C,Z) = -rJ_p - J_{zp}\dot{Z},$$
$$(4) \quad X(P,W,C,Z) = -rJ_w - J_{zw}\dot{Z},$$
$$(5) \quad \dot{Z}(P,W,C,Z) = J_{zc}^{-1}(rJ_c + Z).$$

For the NQ, variable and quasi-fixed input demands are (4) and (5) with normalized prices. Output supply is obtained by adding normalized expenditures to the normalized value function, which yields:

$$(6) \quad f(w,c,Z) = rJ + wX + c'Z - J_z\dot{Z}.$$

Equations (3), (4), and (5) are the estimation equations for the GL, and (6), (4), and (5) for the NQ. The equations are appended with error terms to account for errors in optimization and estimated using nonlinear three stage least squares estimation (SYSNLIN, the nonlinear estimation on program is SAS). $\dot{Z}$ is approximated discretely as $Z_t - Z_{t-1}$. Lagged milk price is used as a proxy for expected milk price. Instruments for the jointly dependent variables are estimated using current and lagged input price, lagged milk price, and lagged quantities. Several very different starting values produced estimates for each functional form identical to the fourth decimal, which suggests that global optima were likely achieved.

Data

The model was estimated using annual U.S. data for the 1951-1982 period. Quantity of fluid milk marketed was used as output quantity and the average blend price of milk as output price. Concentrate quantity and price were used for the feed quantity and price.

The rental price of cows was computed as a discounted stream of payments on a replacement heifer kept for three lactations that would make a producer indifferent between paying three annual payments or a cash purchase price. The salvage value was assumed equal to the maintenance costs of the cow[1]. Quantity of cows and their rental price were adjusted for genetic quality changes as outlined in Howard and Shumway.

Quantity of hired and family labor was measured as the average number of workers per year in the U.S. dairy industry. A labor wage index was computed by dividing total expenditures on labor by the number of hired workers. The labor quantity and wage series were adjusted for quality using the Ball estimates and extended by Howard and Shumway.

Major data sources were <u>Agricultural Prices</u> (USDA, 1965, 1984), <u>Agricultural Statistics</u> (USDA, 1956-1983a), <u>Economic Indicators of the Farm Sector: Income and Balance Sheet Statistics</u> (USDA, 1965, 1983a). <u>Economic Indicators of the Farm Sector: Production and Efficiency</u>

Statistics (USDA, 1965, 1983b). Farm Income Statistics (USDA, 1979), Milk: Production, Disposition, and Income (USDA, 1951-1983b), and Milk Production (USDA, 1951-1983a). For details of data construction, see Howard and Shumway.

Results and Discussion

The parameter estimates for (1) and (2) with symmetry restrictions maintained are reported in Table 1. Thirteen of 22 parameter estimates were significant at the five percent level in the GL model; only four of 25 were significant at the five percent level in the NQ model. The NQ explained more variation in the output supply, but less in the input demand for labor and cows. The R^2's from the GL and NQ for milk supply and input demand for feed, cows and labor were, respectively, .14, .97, .98, .99, and .67, .97, .84, .67. The rate of adjustment of labor was not significantly different in the two models, but the rate of adjustment of cows was very different. The GL estimated that cows adjust 14 percent of the extent of disequilibrium per year ($M_{ii} = -.14$). This is a stable adjustment, i.e., between -1 and 0[2]. The NQ estimated a nonstable adjustment for cows, a positive 9 percent, which indicates adjustment moved away from an equilibrium level.

Although the GL has higher R^2's for most equations and more significant parameters, results of both models are examined further for two reasons. First, given the nonlinear and simultaneous nature of the model, it is difficult to judge the model's theoretical consistency solely by its parameter estimates. Second, the GL maintains concavity of the value function in quasi-fixed inputs as a byproduct of maintaining linear homogeneity in prices; the NQ allows explicit examination of the concavity conditions. Hence, the theoretical and structural properties of both models were examined.

Tests of Competitive Behavior, Differentiability, and Structure

The models were estimated with linear homogeneity in prices and symmetry maintained in both models and concavity in quasi-fixed inputs maintained in the GL. Examinations of the theoretically expected properties of monotonicity and convexity in prices were conducted. Concavity in quasi-fixed inputs was examined for the NQ. Production structure was investigated by examining adjustment and technological change.

The necessary monotonicity conditions on the value function, i.e., J() increasing in output price and decreasing in input prices, held at all observations for both models.

The tests for competitive behavior and structure of J() for the NQ are reported in Table 2. They are compared with comparable tests for the GL reported in Howard and Shumway. The test statistic used was the Gallant and Jorgenson T^0, which is approximately Chi-square, with degrees of freedom equal to the number of restrictions. Global convexity in prices in the NQ is satisfied when the matrix of normalized price parameters is positive definite. Although a statistical test of convexity in the NQ was not conducted because of the inequality constraints required, a positive definite matrix was achieved by adjusting each of the estimated price parameters less than one standard error which suggests that convexity was not seriously violated. Neither was convexity in prices in the GL rejected (five percent level) by statistical test ($X^2 = 4.87$ with critical value 12.59).

Global concavity of J() in quasi-fixed inputs was maintained by functional form in the GL. A sufficient condition for global concavity in quasi-fixed inputs in the NQ is that Q_{11}, $Q_{22} < 0$ and $Q_{11}Q_{22} - Q_{12}^2 > 0$. Although violated by the estimated parameters, the violation was not statistically significant. A change of less than 0.1 standard deviation in q_{22} was sufficient to obtain concavity of J() in quasi-fixed inputs.

Independent adjustment, instantaneous adjustment, and several technological change hypotheses were tested as nested hypotheses while maintaining homogeneity and symmetry of the value functions. These tests for the NQ are also reported in Table 2 and are compared to those of Howard and Shumway for the GL.

Independence of adjustment occurs when $m_{12} = m_{21} = 0$ ($M_{12} = M_{21} = 0$ for the GL), and means that each quasi-fixed input adjusts towards its desired level independently of the other. The null hypothesis of independence was strongly rejected in the NQ but was not rejected in the GL (in the latter, $X^2 = 4.16$).

If $m_{ii} = -1$ and $m_{ji} = 0$ (M_{ii}, M_{ji} for the GL), the i^{th} quasi-fixed input adjusts instantaneously to its desired level and would correctly be modeled as a variable input. Instantaneous adjustment was tested separately for cows and for labor. It was rejected for both in the NQ. Because independent adjustment was not rejected in the GL, it was maintained while testing for instantaneous adjustment, which was strongly rejected for labor in that model ($X^2 = 140.32$ with critical value 3.84). Convergence was not attained with the GL while maintaining instantaneous adjustment for cows, so no test statistic for that case is available.

When testing alternative technological change hypotheses, homogeneity and symmetry were maintained in both models. Independent adjustment was also maintained in the GL. The null hypothesis that there had been no change in technology over the data period 1951-1982, i.e., $h_i = 0$ (H_i for the GL), i = 1,..4, was strongly rejected in both models ($X^2 = 245.86$ in the GL). The hypothesis of no disembodied technological change in cows, $h_3 = 0$, i.e., the quality index fully embodied the technological changes that occurred, was rejected at the five percent level in both models. It would not have been rejected at the one percent level in the NQ. The null hypothesis of no disembodied technological change in labor, $h_4 = 0$, was rejected at the .05 level only in the GL ($X^2 = 15.71$).

Short and Long-run Elasticities

Short and long-run elasticities obtained from the GL and NQ for 1982 are reported in Table 3. Homogeneity and symmetry were maintained in both models. Concavity in quasi-fixed inputs was maintained in the GL.

The models estimated elasticities with different signs in 11 of the reported 32 pairs of elasticities. Magnitudes of many of the elasticities with the same sign also differed substantially. The larger elasticity (in absolute value) was more than double the smaller elasticity in 23 pairs.

The Le Chatelier principle held for the GL but not for the NQ. Some elasticities from both models changed signs from the short run to the long run. Because convexity in prices was not satisfied by either initial model, not all long-run own-price elasticities have the signs expected for competitive behavior. Unlike static models, dynamic models do not yield testable sign hypotheses on short-run own-price elasticities for competitive behavior (Treadway, p. 344-345).

Summary and Conclusions

The robustness of dynamic dual model results to choice among two functional forms has been examined for the U.S. dairy industry. Robustness of results for modified generalized Leontief and normalized quadratic functional forms was evaluated by examining structural parameters, elasticities and consistency with competitive behavior. Homogeneity and symmetry were maintained in both models.

Statistical characteristics of the estimated models differed substantially. More than half of the estimated parameters in the GL model were significant at the five percent level; only 16 percent in the NQ model were. R^2 values differed substantially between models for milk supply and labor demand. Calculated 1982 elasticities also differed substantially with respect to both magnitude and sign. A full third of the elasticities differed in sign between models. Two-thirds of the elasticities differed in absolute value by more than 100 percent, thus documenting the extreme sensitivity of this important practical empirical result to functional form.

Theoretical properties were not clearly rejected with either model. Monotonicity conditions were satisfied at all observations for both functional forms. Convexity in prices was not rejected in the GL and was not seriously violated in the NQ. Concavity in quasi-fixed inputs was maintained in the GL and not rejected in the NQ.

Of five statistical tests of structure completed with both models, however, consistent results were obtained on only three at the five percent level (two at the one percent level). In the GL, independent adjustment was not rejected. In the NQ, fully embodied technical change for labor was not rejected; at the one percent level, fully embodied technical change for cows was not rejected either. The remaining structural hypotheses were rejected in both models. Every structural hypothesis was rejected in at least one model.

Although only two functional forms were examined, results from this dynamic dual analysis of the U.S. dairy industry documented a serious lack of robustness across functional forms in several important ways. This lack of robustness is consistent with that previously documented in static dual models but specific areas of nonrobustness differ. Extreme sensitivity of policy-relevant elasticities to functional form was documented. Robustness across functional forms was found in our study with respect to theoretical restrictions (which was contrary to Swamy and Binswanger) but not with respect to technological change hypotheses (contrary to Baffes and Vasavada). The need for model specification searches previously noted for static dual models applies equally to dynamic dual models.

Table 1. Nonlinear Three Stage Least Squares Parameter Estimates of the Generalized Leontief and Normalized Quadratic Value Functions, Homogeneity and Symmetry Maintained.

Leontief		Quadratic	
Parameter	Estimate	Parameter	Estimate
A_{11}	14.47 (3.610)	a_1	-14.87 (25.29)
A_{12}	1.534 (0.4531)	a_2	-0.4349 (0.7293)
A_{21}	0.3959 (1.034)	a_3	-8.351 (7.132)
A_{22}	-0.1121 (0.1149)	a_4	47.30 (53.48)
M_{11}	-0.1401 (0.05452)	a_5	-61.92 (66.81)
M_{12}	-0.01008 (0.01568)	a_6	-27.53 (37.28)
M_{21}	0.003587 (0.3900)	m_{11}	0.09116 (0.06023)
M_{22}	-0.3688 (0.1271)	m_{12}	-0.4022 (0.01802)
E_{11}	-8.065 (2.644)	m_{21}	0.6568 (0.4614)
E_{12}	-4.921 (4.076)	m_{22}	-0.4330 (0.1373)
E_{21}	-1.122 (0.6969)	g	0.0202 (0.01356)
E_{22}	0.1588 (1.551)	d_1	0.02975 (0.1153)
F_{11}	-9.122 (1.879)	d_2	0.6689 (0.8575)
F_{12}	4.178 (1.807)	e_1	-0.001558 (0.01735)
F_{22}	-37.21 (4.400)	e_2	-0.008718 (0.003955)
G_{11}	19.95 (4.773)	f_{11}	-0.007836 (0.01197)
G_{12}	-0.1029 (0.3639)	f_{12}	0.003339 (0.003839)
G_{22}	0.7911 (0.7562)	f_{22}	-0.000159 (0.001452)
H_1	0.1772 (0.04610)	q_{11}	-35.07 (22.79)
H_2	-0.1651 (0.01272)	q_{12}	186.8 (119.7)
H_3	0.1139 (0.01746)	q_{22}	-945.0 (800.6)
H_4	0.3841 (0.05885)	h_1	0.1619 (0.008091)
		h_2	0.004836 (0.004836)
		h_3	-0.003215 (0.001694)
		h_4	-0.000023 (0.00022)

Standard Errors of the Estimates are in parentheses. MSE = 1.6382 with 106 degrees of freedom for the GL, 1.8385 with 103 degrees of freedom for the NQ.

Table 2. Tests of Hypotheses for the Normalized Quadratic Value Function.[a]

Hypothesis	Test Statistic	Critical Value
Convexity		
$\begin{bmatrix} g & e_1 & e_2 \\ e_1 & f_{11} & f_{12} \\ e_2 & f_{12} & f_{22} \end{bmatrix}$ positive definite	Ad hoc -- parameters within 1 standard deviation	
Independent Adjustment		
$m_{12}=m_{21}=0$	362.63	$X^2_{2,.05} = 5.991$
Instantaneous Adjustment of Labor		
$m_{22}=-1.0$, $m_{12}=0$	31.363	$X^2_{2,.05} = 5.991$
Instantaneous Adjustment of Cows		
$m_{11}=-1.0$, $m_{21}=-0$	292.785	$X^2_{2,.05} = 5.991$
No Technological Change		
$h_i=0$, $i=1,\ldots 4$	212.515	$X^2_{4,.05} = 9.448$
No Unobserved Technological Change in Cows		
$h_3=0$	4.419	$X^2_{1,.05} = 3.841$
No Unobserved Technological Change in Labor		
$h_4=0$	0.854	$X^2_{1,.05} = 3.841$

[a] Homogeneity and symmetry in prices maintained.

Table 3. Short and Long-Run Output Supply and Input Demand Elasticities for the U.S. Dairy Industry Derived from the Generalized Leontief and Normalized Quadratic Value Functions, 1982.[a]

Quantity	Elasticity with respect to price of			
	Labor	Milk	Feed	Cows
Short Run				
Milk				
GL:	−0.121	−0.007	0.098	0.030
NQ:	0.052	0.046	−0.033	−0.065
Feed				
GL:	0.012	−0.048	0.047	−0.011
NQ:	−0.036	−0.028	0.059	0.005
Cows				
GL:	0.127	0.006	−0.075	−0.058
NQ:	0.003	−0.005	−0.006	0.008
Labor				
GL:	0.206	−0.003	−0.305	0.102
NQ:	0.015	−0.024	−0.037	0.045
Long Run				
Milk				
GL:	0.114	0.001	−0.078	−0.037
NQ:	0.055	0.043	−0.043	−0.054
Feed				
GL:	0.007	−0.048	0.043	−0.002
NQ:	−0.037	−0.028	0.061	0.004
Cows				
GL:	1.066	0.057	−0.557	−0.556
NQ:	−0.021	0.054	−0.007	−0.026
Labor				
GL:	0.614	−0.007	−0.909	0.296
NQ:	0.050	−0.054	−0.188	0.192

a Homogeneity and symmetry maintained in both models; concavity in quasi-fixed inputs maintained in GL.

Footnotes

1. Dairy cows have a feed maintenance requirement that is much lower than the feed required for maximum milk production. It is assumed that the cost of maintaining the cow is recovered through the salvage value, and the remaining feed cost going for milk production.

2. The flexible accelerator is $\dot{Z} = M(Z - Z^*)$, where Z is the original endowment of quasi-fixed inputs and Z^* is the desired level. Elements of M between 0 and -1.0 indicate a stable adjustment towards the desired level of Z.

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