GUELPH

WP 87/7

Working Papers Series

Working Paper WP87/7

July 1987

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A MICRO-ECONOMETRIC ANALYSIS

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THE COST STRUCTURE OF ONTARIO DAIRY FARMS: A MICRO-ECONOMETRIC ANALYSIS

Over the past twenty years, the Canadian dairy industry has developed an elaborate and distinctive regulatory system. The heart of this structure is a supply management scheme, implemented through production quotas at the provincial and farm level, by which the supply of milk is restricted to what is demanded at set prices (Barichello). The economic consequences of this supply management scheme have been the object of considerable debate. The general conclusion is that, as prices are set above production costs, the system entails a sizable income transfer, from consumers and taxpayers to producers. The best evidence for this proposition is the fact that farmers actively bid for the right to produce at privileged prices, so that milk quotas are traded at extraordinarily high prices (Forbes et al.). This state of affairs poses some interesting problems for the applied economist. An unresolved issue is the size of the departure of administered prices from production costs. Given that this spread is capitalized into quota values, its size is usually inferred by discounting these observed quota values. The arbitrariness of choosing an appropriate discount rate is however a serious limitation of this approach. Also, this type of regulation means that historical records on milk production and prices are uninformative on the supply conditions in the Canadian dairy industry, a problem of some relevance, for instance, in assessing the free trade deal being negotiated between Canada and the U.S. (Warley and Barichello).

This paper undertakes to shed some light on these issues by econometrically estimating the cost structure in dairy production using a large body of farm-level data. The methodology relies on the principles of duality and flexible functional forms (Diewert). This framework of analysis has given renewed momentum, in recent years, to empirical studies in production economics. Interest in propositions that are meaningful at the aggregate level, and ready availability of data, have resulted in most of these studies being devoted to modeling the agricultural sector as a production unit (Antle, Lopez, Shumway, Lee and Chambers), although some microeconomic applications can be found (Sidhu and Baanante, Akridge and Hertel). Yet, considering that duality theory mostly applies to principles strictly pertinent to individual production units, there appears to be scope for further empirical applications at the farm level. The results of this study seem to justify the interest that has been given to this methodology, and provide new insights into the cost structure of the largest of the Canadian agricultural industries subject to supply management. As this regulatory tool has recently been adopted for the dairy sector of the European Community, and is increasingly advocated for the U.S., quantitative information that leads to a better assessment of the Canadian experience can prove useful in a wider setting.

In what follows, a micro-econometric model of production based on a cost function approach is specified. The data set utilized is then presented, followed by a brief description of the estimation technique used. The estimation results are reported, with emphasis on testing some general hypothesis, on evaluating price and output elasticities, and on the determination of the degree of returns to scale. The pricing system of the dairy industry is compared to the estimated cost structure, and the paper ends with a summary of empirical results.

Model specification

Duality theory allows great flexibility in the choice of independent variables in terms of which to model a production system. Given that supply management does not prevent individual farms from adjusting the scale of production by buying and selling production quotas, a model of profit maximization would appear the most desirable one, except possibly for the allowance of dynamic adjustments. Unfortunately, the implementation of such a model is problematic. Because producers must hold production quotas which have a considerable market value, the relevant (shadow) price of production for milk is not observable. Although both the price of milk and the capital values of quotas are known, the exploitation of this information in a profit maximization setting would require estimation of an intertemporal model of production and investment, for which the available data is ill suited. To estimate this type of model one needs repeated observations on the same economic unit in different time periods, a requirement not satisfied by the predominantly cross-section nature of the sample. Also, because of the strict pricing policy administered by the marketing board, the price variability of milk production is extremely limited in the sample, which adds another difficulty to the profit function approach. To overcome this, the optimization behaviour of the farms is estimated conditional on the observed output vector, which leads to a cost function approach. To be consistent with the long-run interpretation of cross-section models (Kuh), all inputs are considered as variable, and the resulting cost function is best regarded as a long-run cost function for the representative farm of the Ontario dairy industry.

Owing to the heterogeneity of farm level production structures, assume that the production conditions are farm-specific. Thus, for the f^{th} farm, given a vector y of I outputs, a vector x of J inputs, and a vector w of J input prices, the joint cost function is defined as:

$$(1) \qquad C^f(y,w) \equiv \min_{(x)} \{ w'x : (y,x) \text{ is in } V^f \}$$

where the superscripted prime denotes vector transposition, and V^f is the farm-specific production possibility set, the set of all feasible input-output combinations. Under fairly general conditions, $C^f(y,w)$ is a continuous, non-negative, non-decreasing function, positively linearly homogeneous and concave in w (McFadden). In addition to these properties, for analytical convenience it will be assumed that $C^f(y,w)$ is twice continuously differentiable. While general, this approach is not very useful empirically, since it would require the estimation of a different cost function for each farm. A more workable approach is provided by Panzar and Willig (1978) by assuming that the technology is the same for all farms, but individual farms are characterized by an "efficiency" parameter continuously distributed across farms. While this analytic device is suitable for theoretical analysis (Chambers), it is not very practical empirically as long as this efficiency parameter is not directly observable. Building on this approach, however, one can proceed by assuming that the individual farms have access to the same production technology, but farm-specific structural factors put constraints on the attainable points of the production possibility set. If we let z be the K-dimensional vector of these attributes, the farm-specific production possibility set can then be written as $V^f \equiv V(z)$. It follows that the farm-specific cost function dual to this production possibility set is defined as:

(2) $$C^f(y,w) = C(y,w,z) \equiv \min_{(x)} \{ w'x : (y,x) \text{ is in } V(z) \}$$

Thus, C(y,w,z) allows the description of the optimizing behaviour of all farms by a single function, and thus offers an empirically tractable framework within which to analyze the problems addressed in this study.

For the purpose of empirical estimation, a flexible functional form must be chosen for the cost function. The form utilized in this paper is the Hybrid-Translog proposed by Caves et al. This is a generalization of the well known translog functional form which utilizes the transformation proposed by Box and Cox for the output variables, thus overcoming some undesirable properties of the translog for the case of multiproduct cost functions (Baumol et al.). Thus, the cost function for the multiproduct dairy farm is written as:

(3) $$\begin{aligned} \ln C = {} & \alpha_o + \Sigma_i\, \alpha_i\, [(y_i^\lambda-1)/\lambda] + \Sigma_j\, \beta_j\, \ln w_j + \Sigma_k\, \gamma_k\, z_k \\ & + \tfrac{1}{2}\, \Sigma_i \Sigma_m\, \alpha_{im}\, [(y_i^\lambda-1)/\lambda][(y_m^\lambda-1)/\lambda] + \tfrac{1}{2}\, \Sigma_j \Sigma_n\, \beta_{jn}\, \ln w_j \ln w_n \\ & + \Sigma_i \Sigma_j\, \mu_{ij}\, [(y_i^\lambda-1)/\lambda]\, \ln w_j + \Sigma_i \Sigma_k\, \rho_{ik}\, [(y_i^\lambda-1)/\lambda]\, z_k \\ & + \Sigma_j \Sigma_k\, \psi_{jk}\, \ln w_j z_k \end{aligned}$$

Since it will be convenient to express the vector of attributes as a vector of binary variables, the logarithmic transformation is not used for the variable z_k, and the interaction terms between these attributes are ignored. Applying Shephard's lemma to this cost function yields a set of input share equations:

(4) $$S_j = \beta_j + \Sigma_n\, \beta_{jn}\, \ln w_n + \Sigma_i\, \mu_{ij}\, [(y_i^\lambda-1)/\lambda] + \Sigma_k\, \psi_{jk}\, z_k$$

where $S_j \equiv w_j x_j^*/C = \partial \ln C/\partial \ln w_j$, and x_j^* is an element of the vector of input demands that solves problem (2). This set of input share equations can be used together with the cost equation (3) to estimate the parameters of the cost function. The theoretical properties of linear homogeneity in input prices and symmetry of the cost function can be imposed on the hybrid-translog by parametric restrictions. The restrictions required are exactly the same as those necessary for the multiproduct translog cost function (see, for instance, Brown et al.).

Data

The data used in this study consists of farm-level data collected by the Ontario Dairy Farm Accounting Project (ODFAP).[1] For each farm, an extensive body of information on disaggregate sources of revenue and costs, and on the physical and financial structure of the farm, is collected. A typical production plan includes fluid and industrial milk, livestock production associated with the dairy herd, and grain and hay production. For these production processes, ODFAP provides a detailed breakdown of revenue and expenditures. For most items, both dollar and quantity information are available, so that it is possible to compute implicit prices. In addition to the above production processes, some farms also have beef feedlot and swine operations, or other crop productions. The information available on these specialized enterprises is far less detailed, and not sufficient to permit an adequate specification for these products. Thus, a sub-sample of farms was selected for which the revenue from specialized enterprises was not more than 10 percent of the farms total revenue. Also, a few farms were dropped from the sample because of inconsistencies in the recorded data.

The resulting sample spans the period 1978 to 1983, the most recent six years at the time the analysis was undertaken, and comprises a total of 612 observations. The farms of this sample are fairly specialized in milk production which provides, on average, almost 80 percent of total farm revenue.[2]

For the purpose of estimating the cost function, the production of the sample farms is aggregated into three outputs: milk, livestock products, and crops and other products. Although milk is classified into fluid milk and industrial milk for the purpose of pricing, at the production level milk is really an undifferentiated commodity. There is therefore no aggregation error in considering milk as a single commodity, which is measured in hectolitres of 3.6 percent fat content milk. The livestock output aggregate represents the net sales of livestock products. Since livestock purchases are really an addition to the livestock capital, and their services will not be exhausted within one production period, they are considered in this aggregate, which is then defined as total sales plus inventory change minus livestock purchases. The revenue from these net livestock sales is deflated by a Fisher price index of actual livestock sales (computed using averages for the whole sample as weights) to obtain an index of livestock production. The last output aggregate is made up of crops and other products. For sales of hay and of each grain it is possible to compute an implicit price from the sample data. The price of soybeans was used as a proxy to deflate specialized crop revenue, and a price index of all agricultural products is used to deflate other farm income. Positive inventory changes of feed and crops are allocated to this commodity group, while negative inventory changes are allocated to feed input purchased. With the above data, a Fisher quantity index is computed based on sample mean values.

On the input side, four aggregate inputs are specified: labour, feed, intermediate inputs, and capital services. A price index for labour is derived using the actual wage paid to hired labour computed from the sample data, and by imputing the hourly wage for hired farm labour from Statistics Canada to self-employed labour. The feed input group aggregates purchases of dairy rations, by-product feeds, brewer-grains, protein supplement, salt and minerals, milk replacer, calf ration, and other feed grain and forages purchased, together with negative changes of feed and crop inventories. For each of these items it is possible to obtain an implicit price index from sample data, and a Fisher price index based on sample mean values was computed. Intermediate inputs comprise fertilizers, herbicides and pesticides, seed, gasoline and diesel fuel, hydro and telephone, veterinary and drugs expenses, artificial insemination, insurance, and miscellaneous expenditures. For gasoline, diesel oil, and fertilizers, the sample data allow the computation of implicit prices. For the remaining inputs, price indices from Statistics Canada were used to deflate expenditures, and for the whole group a Fisher price index based on mean values was computed. The capital input group aggregates the services of land and buildings, machinery, and livestock herd capital.[3] For each of the above types of capital, a price index of capital services was computed by deflating the cost of capital services by a quantity index. Using these service prices for each component, a Fisher price index for the capital group was computed based on the sample mean values.

Prior to estimation, all output quantity and input price indices were normalized to one at the median of the sample. Table 1 summarises these indices, together with the cost shares to be used in the estimation, and gives a measure of their sample variability. The most important input share is capital, which accounts on average for 36 percent of total costs. The fact that all output quantity indices have a mean greater than one displays the fact that output tends to be concentrated in large farms, so that the distribution is skewed. For instance, for milk it takes about 70 percent of the smallest farms to produce 50 percent of total output.

In addition to the variables summarized in Table 1, to estimate the cost function model the characteristics z_k that define the farm-specific production possibility set need to be specified. The relevant set of these variables is not easily defined, and its choice has to be based on the available information. The variables selected meet the two criteria of being in principle correlated to some definition of farm-specific efficiency, and of being informative in the sense of having a sufficient variability within the sample. Some of these variables are discrete by definition, and must therefore be introduced in the model in terms of binary variables, while other variables are continuous. However, given the fairly arbitrary selection process, the latter are also expressed in terms of binary variables by splitting the sample at the median point. Twelve characteristics are considered: regional location; education of the operator; milking technique; feeding technique; quality of land; quality of buildings; type of dairy cows; age of the operator; seasonality of milk shipment; horsepower of the largest tractor; capacity utilization; and, debt to equity ratio. Following Fox and

Table 1 - Summary description of variables

	symbol	mean	standard deviation
Cost	C	91373	44256
Cost shares			
Labour	S_1	0.209	0.068
Feed	S_2	0.176	0.073
Int. inputs	S_3	0.257	0.057
Capital	S_4	0.358	0.068
Quantity indices			
Milk	y_1	1.153	0.615
Livestock	y_2	1.234	1.020
Crops	y_3	1.613	1.934
Input price indices			
Labour	w_1	1.017	0.219
Feed	w_2	0.991	0.170
Int. inputs	w_3	1.015	0.156
Capital	w_4	1.001	0.213

Driver, milking technique and horsepower of the largest tractor have been extensively used to classify Ontario dairy farms, although there appears to be little to be gained by limiting attention to these two characteristics. Regional location specifies 6 regions, education and milking technique take 3 levels each, and all the other characteristics take 2 levels. Thus, these twelve characteristics are completely classified into thirty binary variables, but to avoid singularity one dummy for each set of dummy variables has to be dropped. This leaves 18 variables z_k, whose coefficients are estimated simultaneously with the coefficients of the structural variables summarized in Table 1.

Estimation procedure

The parameters of the cost function are estimated using the cost equation (3) together with (J-1) input share equations (4), where one of the share equations (the labour share) has been dropped to avoid the well known singularity problem of share systems (Barten). The stochastic version of the resulting non-linear system can be written as:

(5) $$Y_{ft} = F(X_{ft},\theta) + e_{ft}$$

where f indexes the individual farm observations, t indexes the sampling year, Y_{ft} is a J×1 vector of dependent variables for the f^{th} farm in year t (J-1 input share equations and the logarithm of the cost), X_{ft} is the corresponding vector of all exogeneous variables, θ is the vector of all coefficients to be estimated, and e_{ft} is a J×1 vector of random errors. Before estimation can be carried out, assumptions about the distribution of the residual vector e_{ft} that take into account the time-series/cross-section nature of the sample are needed. The problem of pooled (panel) data is well researched, and the many possible solutions can be broadly classified as to whether the individual and time specific components of e_{ft} are constant (covariance or dummy variable models) or random (error component models). Given that individual farms are not surveyed every year because of the rotating nature of the sample, the simplification of ignoring the individual-specific component of the error is adopted, and a covariance model for the time-specific component is specified. The vector of error terms e_{ft} is therefore written as:

(6) $$e_{ft} = d_t + u_{ft}$$

where d_t is assumed constant and estimated through a set of dummy variables, and the random vector u_{ft} is distributed according to:

(7.1) $$E(u_{ft}) = 0,$$

(7.2) $$E(u_{ft}u_{ft}') = \Omega,$$

(7.3) $$E(u_{ft}u_{rs}') = 0 \qquad (f \neq r \text{ or } t \neq s)$$

Thus, it is assumed that the error terms of cost and share equations will be correlated for each individual farm at the same point in time (Ω is the J×J contemporaneous variance-covariance matrix), will be uncorrelated for any two different farms at any point in time, and (perhaps more stringently) will be uncorrelated for any one farm at different points in time. Given the above, the system (5) can be estimated using a minimum distance estimator or, if the further assumption of multinormal distribution for the residual vector is made, a maximum likelihood estimator (Amemiya). The maximum likelihood estimator, or its numerically equivalent iterated minimum distance estimator, has the desirable property of being independent of which equation is dropped to eliminate the singularity of the covariance matrix (Barten). Thus, the maximum likelihood estimator is used here, and it is computed using an algorithm derived by Berndt et al. and implemented in TSP 4.0. Under the stated stochastic assumptions, the maximum likelihood estimator is known to be consistent, asymptotically normal, and asymptotically efficient.

Results

With the restrictions of symmetry and linear homogeneity in prices maintained, the estimated system of cost and share equations contains 175 free parameters. To save space, only the estimated parameters pertaining to the structural variables are reported individually in Table 2. These show that 22 of the 29 structural parameters are significantly different from zero. In particular, the parameter λ of the Box-Cox transformation of the output variables is positive and significantly different from zero. This implies that average and marginal costs have the usual U shape, declining at low output levels and increasing at high output levels. As for the goodness of fit of the estimated equations, a simple measure is given by the squared correlation coefficient between fitted and observed system dependent variables. The computed R^2's are: 0.91 for the cost equation, 0.50 for the labour share equation, 0.29 for the feed share equation, 0.33 for the intermediate inputs share equation, and 0.31 for the capital share equation. Thus, except for the cost equation, the R^2 of the estimated equations is not very high, but this is to be expected given the

Table 2 - Estimated structural parameters

Parameter	Estimate	t-statistic
λ	0.20487	3.600
α_o	11.17775	353.804
α_1	0.64967	8.706
α_2	0.06230	1.114
α_3	0.04409	1.421
β_1	0.22229	21.096
β_2	0.16454	12.033
β_3	0.25944	27.158
β_4	0.35374	27.966
α_{11}	0.12803	1.158
α_{12}	-0.01988	-0.461
α_{13}	-0.03926	-1.478
α_{22}	0.00877	0.331
α_{23}	0.01190	0.799
α_{33}	0.06134	5.170
β_{11}	0.12457	12.641
β_{12}	-0.02646	-3.255
β_{13}	-0.02988	-3.659
β_{14}	-0.06823	-6.882
β_{22}	0.02950	2.279
β_{23}	-0.00796	-0.900
β_{24}	0.00493	0.432
β_{33}	0.02331	1.473
β_{34}	0.01454	1.015
β_{44}	0.04877	2.525
μ_{11}	-0.04590	-6.285
μ_{12}	0.05685	6.040
μ_{13}	0.01335	2.043
μ_{14}	-0.02430	-2.824
μ_{21}	0.00158	0.394
μ_{22}	0.00040	0.078
μ_{23}	0.00167	0.470
μ_{24}	-0.00365	-0.779
μ_{31}	-0.00815	-3.413
μ_{32}	-0.02722	-8.838
μ_{33}	0.01171	5.473
μ_{34}	0.02366	8.371

predominantly cross-sectional nature of the sample.

The regularity conditions for the cost function require that the estimated cost function be monotonically increasing in input prices and output levels, and concave in input prices. Monotonicity requires that the estimated cost share equations and the estimated elasticity of cost with respect to output be non-negative. Concavity requires that the Hessian matrix $[\partial^2 C/\partial w \partial w']$ be negative semidefinite. In the case of the hybrid translog cost function, this matrix depends not only on estimated parameters, but also on the value of the exogeneous variables. Thus, both monotonicity and concavity can be evaluated at each observation point. However, given the large number of observations, and the greater incidence of measurement errors in cross-section data, in the present case it seems reasonable to check these conditions for some normal value of the exogenous variables. Thus, monotonicity and concavity were checked at the expansion point (the median) for the structural variables, and at the mean of the characteristic variables. Both monotonicity in input prices and output levels, and concavity in input prices were found to hold at this point.

Given the large number of parameters involving the characteristics z_k, they are not explicitly reported in this paper. Some tests, however, can summarize the significance of these variables for the estimated cost stucture. For these tests, as for others in this paper, the Wald test is used, which has the advantage of allowing several tests to be carried out in the same estimation run, an attractive feature for large models. If a set of (possibly nonlinear) M restrictions over the N-dimensional parameter vector θ is written as $g(\theta) = 0$, where g is M-dimensional, the

Wald test W is:

$$W = [g(\theta)]'[G\ V(\theta)\ G']^{-1}[g(\theta)] \tag{8}$$

where $V(\theta)$ is the variance-covariance matrix of the parameter vector θ, and G is the M×N Jacobian matrix $[\partial g(\theta)/\partial\theta']$, and all the functions are evaluated at the estimated value of θ in the unrestricted model (Judge et al.). Similar to the more commonly used likelihood ratio test, the Wald test is asymptotically distributed as χ^2 with M degrees of freedom, although conflicting results between these two statistics are possible (Breusch).

Table 3 reports the Wald tests for the joint significance of the coefficients of each characteristic. Of the twelve characteristics considered, seven have a significant impact. They are: regional location, debt-equity ratio, milking technique, building quality, cow type, education, and horse power of the largest tractor. The characteristics that do not have any significant impact are: feeding technique, land quality, age, seasonality, and capacity utilization. Table 3 also reports the test on all the coefficients involving variable characteristics, and this test indicates an overall significant impact of these variables. Thus, the results support the hypothesis that farm-specific characteristics have a significant impact on the estimated cost structure, and that it is insufficient to limit the attention to milking technique and horse power of the largest tractor.

Some interesting structural tests can be performed by simple parametric restrictions in the hybrid-translog cost function. These include tests for separability and non-jointness. Input-output

Table 3 - Wald statistics for tests on characteristics

Set of Variables	Number of Restrictions	Wald Statistics	$\chi^2_{0.05}$
Regional location	35	106.098	49.517
Debt-equity ratio	7	40.454	14.067
Milking technique	14	44.176	23.685
Feeding technique	7	10.466	14.067
Land quality	7	8.998	14.067
Building quality	7	13.851	14.067
Cow type	7	18.916	14.067
Education	14	26.716	23.685
Age	7	6.759	14.067
Seasonality	7	11.596	14.067
Horse power	7	56.615	14.067
Capacity	7	12.461	14.067
All characteristics	126	432.357	152.91

separability of the cost function allows the cost function to be written as $C(H(y),w,z)$, which implies that the ratio of marginal costs of any two outputs is independent of input prices and of variable characteristics (Hall). For the hybrid-translog this requires:

$$(9.1) \qquad \mu_{ij}\, q_m - \mu_{mj}\, q_i = 0 \qquad i,m=1,\dots,I;\ j=1,\dots,J$$

$$(9.2) \qquad \rho_{ik}\, q_m - \rho_{mk}\, q_i = 0 \qquad i,m=1,\dots,I;\ k=1,\dots,K$$

where q_i satisfies $Q_i \equiv q_i y_i^{\lambda}$, and $Q_i = \partial \ln C/\partial \ln y_i$ is the elasticity of cost with respect to output y_i, which from the cost equation (3) can be written as:

$$(10) \qquad Q_i = [\alpha_i + \Sigma_m\, \alpha_{im}[(y_m^{\lambda}-1)/\lambda] + \Sigma_j\, \mu_{ij}\, \ln w_j + \Sigma_k\, \rho_{ik}\, z_k]\, y_i^{\lambda}$$

Thus, a sufficient condition for global separability is:

$$(11.1) \qquad \mu_{ij} = 0 \qquad i=1,\dots,I;\ j=1,\dots,J$$

$$(11.2) \qquad \rho_{ik} = 0 \qquad i=1,\dots,I;\ k=1,\dots,K$$

A weaker condition for separability evaluates q_i and q_m in (9) at some point of the explanatory variables. This yields a set of non-linear restrictions on the estimated parameters which gives a local test for separability. Non-jointness in input quantities allows the cost function to be written as a summation of individual output cost functions, i.e. $\Sigma_i C^i(y_i,w,z)$, which implies that the marginal cost of each output is independent of any other output (Kholi). This requires:

$$(12) \qquad \alpha_{im} + q_i\, q_m = 0 \qquad i,m=1,\dots,I$$

By evaluating q_i and q_m at some point of the explanatory variables one gets a set of non-linear restrictions which provide a local test for non-jointness.

Table 4 reports the Wald statistics for the above structural tests. The local tests of separability and non-jointness are computed by evaluating q_i and q_i at the expansion point of the structural variables (the whole sample median), and at the mean of the variable characteristics. Separability is rejected both at the global and local level. Given the implication of separability for the problem of consistent aggregation, this result suggests that no output aggregate H(y) is admitted by the data, and therefore that the multi-output specification adopted in this paper cannot be simplified further by aggregating outputs. Non-jointness, on the other hand, is not rejected, and this finding is somewhat surprising given the type of production process analyzed. A possible explanation is that in this case all inputs are treated as variables, which again points to the existence of constraints on some inputs as the more likely source of jointness in agricultural production (Shumway et al.). Thus, the cost function for milk could be studied separately from that of the other outputs given the appropriate information on costs allocation.

Input elasticities

The production structure of Ontario dairy farms can be illustrated further by computing relevant elasticities. Although Allen partial elasticities of substitution are often reported to illustrate estimated substitution possibilities, when there are more than two inputs they are not necessarily the best measure of substitutability (Lau). A better alternative is offered by price and output elasticities of compensated

Table 4 - Wald statistics for structural tests

Test	Number of Restrictions	Wald Statistics	$\chi^2_{0.05}$
Separability (global)	63	261.291	82.245
Separability (local)	63	202.263	82.245
Non-jointness (local)	3	1.502	7.815

demand, the size of which is also amenable to a more direct interpretation. The own-price elasticity ε_{jj} for the hybrid translog cost model is computed as:

$$(13) \qquad \varepsilon_{jj} = -1 + S_j + \beta_{jj}/S_j$$

The elasticity of demand of the j^{th} input with respect to the n^{th} input price is computed as:

$$(14) \qquad \varepsilon_{jn} = S_n + \beta_{jn}/S_j$$

Finally, the elasticity of the j^{th} input with respect to the i^{th} output is computed as:

$$(15) \qquad \eta_{ji} = Q_i + \mu_{ij}(y_i^{\lambda}/S_j)$$

Table 5 reports the estimated elasticities of compensated input demand, evaluated at the expansion point of the structural variables, and at the mean of the characteristics variables. Given that the estimated elasticities are non-linear functions of the estimated parameters, the standard errors reported in Table 5 are computed as follows. Let the typical elasticity be represented by $\varepsilon = h(\theta)$, where h is a non-linear function. From a linearization of this function around the true values of θ, the variance $V(\varepsilon)$ of this elasticiy can be approximated by:

$$(16) \qquad V(\varepsilon) = [\partial h/\partial\theta]'[V(\theta)][\partial h/\partial\theta]$$

where $V(\theta)$ is the variance-covariance matrix of the estimated parameters, and the vector $[\partial h/\partial\theta]$ is evaluated at the estimated value of θ. An example of this methodology applied to the elasticity of substitution for the translog functional form is given in Toevs.

Table 5 - Estimated price and output elasticities of input demand

Input Demand	Input Price				Output Quantity		
	Labour	Feed	Intermediate inputs	Capital	Milk	Livestock Products	Other Products
Labour	-0.219 (0.044)	0.050 (0.037)	0.128 (0.037)	0.041 (0.045)	0.397 (0.037)	0.065 (0.020)	0.021 (0.013)
Feed	0.066 (0.050)	-0.656 (0.077)	0.215 (0.054)	0.376 (0.070)	0.941 (0.064)	0.060 (0.035)	-0.104 (0.021)
Intermediate Inputs	0.109 (0.031)	0.138 (0.033)	-0.649 (0.060)	0.402 (0.055)	0.654 (0.033)	0.064 (0.018)	-0.046 (0.015)
Capital	0.026 (0.029)	0.182 (0.032)	0.304 (0.041)	-0.513 (0.055)	0.533 (0.036)	0.047 (0.020)	0.126 (0.013)

Note: Asymptotic standard errors are reported in parentheses.

All the estimated own-price elasticities have the expected negative sign, which is implied by the fact that the estimated cost function satisfies the concavity condition at the evaluation point. All inputs have inelastic demands, the most inelastic being labour demand. The substitution possibilities between inputs are illustrated by the cross-price elasticities of input demand. All inputs are net substitutes, as indicated by the positive sign of the estimated cross elasticities. The magnitude of these elasticities indicate low substitution possibilities of labour for all other inputs, while the strongest substitution possibilities exist between capital and feed, and capital and intermediate inputs. Since capital includes land, the large substitution possibilities between capital and feed shows that farmers can readily substitute farm-grown forages and grains for purchased feed. Finally, Table 5 reports the estimated output elasticities of input demand, evaluated at the expansion point for the structural variables and at the mean point for the characteristic variables. All the input elasticities with respect to milk output are positive and significantly different from zero, indicating that all inputs are normal with respect to milk output. Feed demand is the most responsive to changing the scale of milk production, while labour demand is the least responsive. The magnitude of input demand elasticities with respect to the remaining outputs is much smaller, which reflects the minor importance of these products for the fairly specialized dairy farms of this sample. Two of the input elasticities with respect to the other product's output aggregate are negative, which suggests that feed and intermediate inputs are inferior inputs for this output aggregate. While this cannot be ruled out a priori (for instance, as crop production is expanded with all inputs being variable, some of the crop production can substitute for purchased feed), this occurrence may also signal some measurement problem for this output group, which aggregates a fairly heterogeneous set of products.

Returns to scale and average costs

An important feature of the technology being modeled concerns the possible existence of scale economies. For the hybrid-translog functional form, a global test for constant returns to scale is possible only jointly with the functional form reducing to a translog, a hypothesis which is rejected by the fact that the estimated λ is significantly different from zero. However, a local measure of the degree of returns to scale for a multiproduct technology, that can be computed directly from the cost function, has been derived by Panzar and Willig (1977). The degree of returns to scale is defined as:

$$R = C(y,w,z)/[\Sigma_i \, y_i \, (\partial C/\partial y_i)] \tag{17}$$

Thus, R measures the ability of the revenue obtained from marginal cost pricing (the denominator) to compensate all input of production (the total cost in the numerator). If $R = 1$ we have locally constant returns to scale, while $R > 1$ indicates increasing returns to scale, and $R < 1$ shows decreasing returns to scale. It is clear that R is a local measure of returns to scale, which depends on the point at which the explanatory variables are evaluated. An obvious candidate for this point is the expansion point for the structural variables, and the mean for the characteristic variables. However, it is also interesting to explore how the degree of returns to scale changes as output levels change. Thus, while we evaluate the price variables at the expansion point, and the characteristic variables at the mean, the output levels are evaluated at ten representative points which are defined by scaling the unit vector (the output vector at the expansion point) by a scalar. This is in keeping with the definition of ray average costs (RAC) given by Baumol et al., which is:

$$(18) \qquad RAC \equiv C(sy^1,w,z)/s$$

where y^1 is the unit bundle, and s is an arbitrary scalar. RAC will be decreasing in s under increasing returns to scale and increasing in s under decreasing returns to scale, a multiproduct extension of the well known relationship of the single output case.

Table 6 reports the estimated degree of returns to scale at ten points, where s is chosen to correspond to the milk output index at the 25, 50, 75, 85, 90, and 95 percent percentile distribution, and at the minimum and maximum milk output levels observed in the sample.[4] It is apparent that the estimated cost function displays increasing returns to scale. The degree of returns to scale is not different from one - the constant returns to scale case - only above the 85 percent evaluation point, which suggests that only the largest farms have exploited the increasing returns existing in milk production. This result is very revealing of the structure of the Ontario dairy industry. It depicts a case in which most of the farms are in long-run disequilibrium. As undersized farms strive to achieve an optimal long-run scale, with the aggregate output virtually constant or decreasing, a number of farms will have to leave the industry. For example, if we take 5000 Hl to be

Table 6 - Estimated degree of returns to scale

Percentile Evaluation Point	Milk Quantity (Hl)	Estimated R	Standard Error
minimum	348	2.558	0.499
25 percent	1505	1.555	0.074
50 percent	2072	1.392	0.041
75 percent	2897	1.238	0.049
85 percent	3675	1.138	0.065
90 percent	4330	1.074	0.076
95 percent	5113	1.012	0.086
maximum	7986	0.862	0.106

the approximate size at which scale economies are fully exploited, the total Ontario milk production for the dairy year 1985-86 would be produced by 4670 producers, roughly 50 percent of the present number of producers. The number of licensed milk producers in Ontario, in the ten years from 1976-77 to 1985-86, has changed from 13821 to 9768, a decline of about 30 percent. The above analysis suggests that this decline is likely to continue for several years.[5]

Estimated costs and milk prices

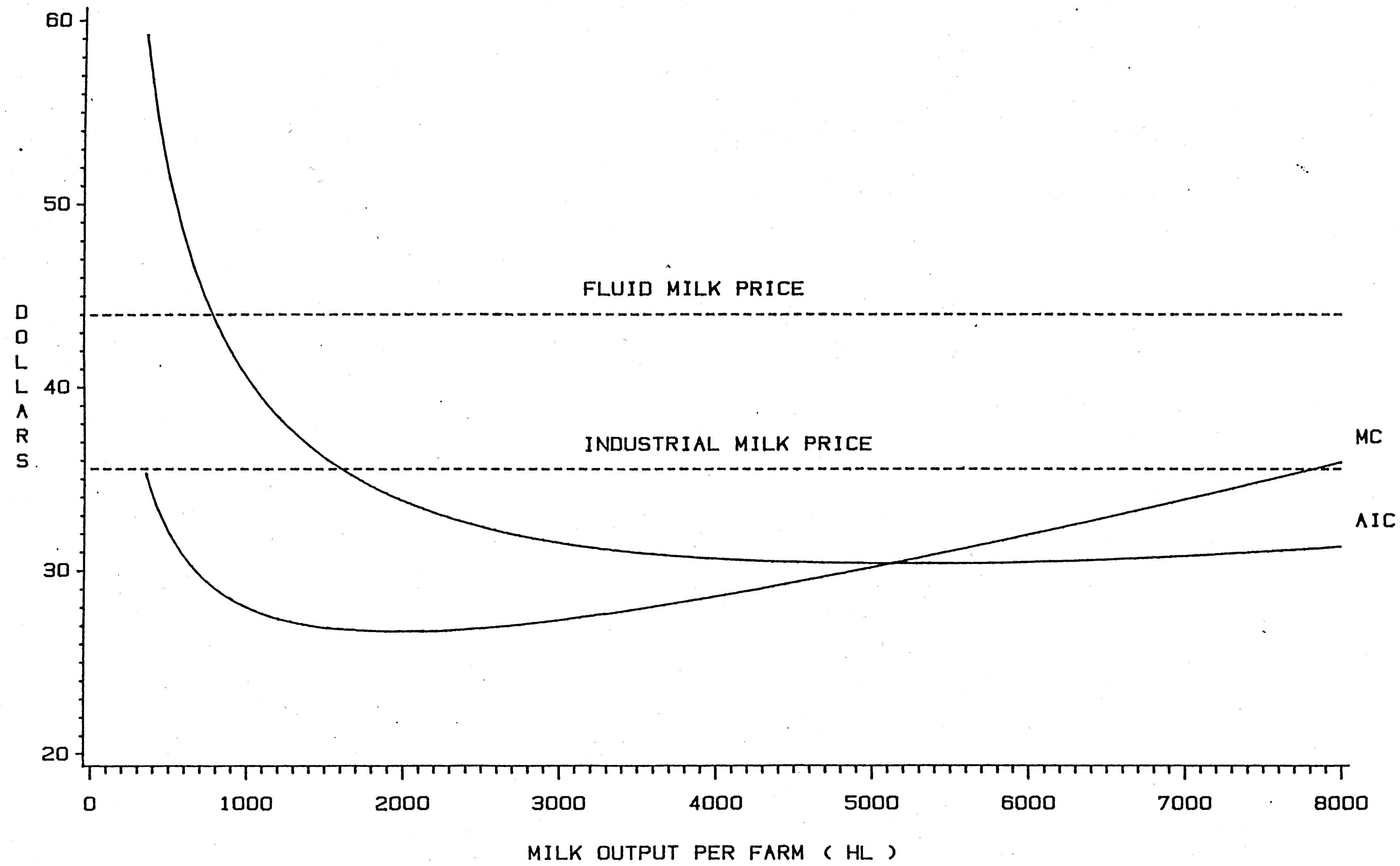
The ray average cost curve that underlies the measure of the degrees of returns to scale in Table 6 could be plotted. However, since RAC refers to a composite commodity, it cannot be compared directly to milk prices. For this purpose, a more useful concept is the notion of average incremental cost (Baumol et al.). In a multi-output framework, the incremental cost of producing milk (y_1) is defined as the difference between the cost of producing the whole output vector, and the cost of producing an output subvector where the milk output is set to zero. The average incremental cost expresses this incremental cost in terms of milk units. Thus, letting $y^\circ \equiv (0, y_2, y_3)$, the average incremental cost of milk (AIC_1) is defined as:

$$(19) \qquad AIC_1 \equiv [C(y,w,z) - C(y^\circ,w,z)]/y_1$$

Clearly, AIC_1 is conditional on the level of the other outputs. However, given that the hypothesis of non-jointness was not rejected by the Wald test, the level of other outputs is not likely to be crucial. Choosing the median level for the two remaining outputs, the AIC of producing milk was computed from the estimated parameters (at 1983 median input prices), and it is reported in Figure 1 together with the estimated marginal cost (MC). The shape of AIC is very much like that of RAC, achieving a minimum only at an output level of 5095 Hl per farm. At this point, average and marginal costs are estimated at 30.33 \$/Hl. In 1983, the price of industrial milk, net of the applicable subsidies and deductions, averaged 35.50 \$/Hl, while the price of fluid milk averaged 43.97 \$/Hl. A farm that produced the maximum allowed fraction of fluid milk, in 1983 could have received an average price for its total milk of about 40 \$/Hl , which would have put the break-even scale at 1042 Hl, a production size achieved by about 90 percent of the sample farms.

It is apparent that farms that have achieved an optimal long run scale of production enjoy a sizable rent, but even for smaller scales the advantage of the pricing system is clear. Indeed, in an industry with varying degrees of scale efficiency in production, the crucial issue for a cost-of-production pricing scheme, such as that used in the Ontario dairy industry, concerns whose costs are to be covered. The results in this section suggest that the present system covers the cost of virtually all but the most inefficient producers. An interesting additional issue is the implication of the estimated cost structure for quota values. As farmers bid for quotas to increase production towards their long-run equilibrium, the shadow rental price of quota tends to be the difference between price and marginal (and average) cost at the optimal long-run scale. For sub-optimal production scales, on the other hand, not all inputs can be rewarded at their marginal contribution to production. This can explain why milk quotas tend to be considered

FIGURE 1 — ONTARIO MILK PRICES AND ESTIMATED COSTS IN 1983



overpriced by most farmers, a case that Tweeten had postulated for the parallel case of land prices given an heterogeneous farm size structure.

Conclusions

This paper has developed a multiproduct cost function approach to analyzing the cost structure of Ontario dairy farms, and the empirical application utilized a hybrid-translog functional form estimated with farm-level data. The regularity conditions of monotonicity in output quantities and input prices, and of concavity of the cost function in input prices, were found to be satisfied by the estimated model at the expansion point. Parametric structural tests rejected the hypothesis of input-output separability, while the hypothesis of non-jointness could not be rejected. The farm-specific characteristics specified in the cost function were also tested, and seven of the twelve characteristics were found to have a significant impact. Compensated input demands were found to be inelastic with respect to own-price changes. All inputs were net substitutes, and substitution possibilities were found to be particularly strong between feed and intermediate inputs, and between feed and capital. A multiproduct measure of returns to scale was computed, and the results indicate the existence of large economies of scale for a wide range of output levels, suggesting that the long-run configuration of the Ontario dairy industry is characterized by a sharp reduction in the number of farms. The results of the estimated cost function qualify and substantiate the claim of large economic rents from over-pricing of milk.

NOTES

[1] ODFAP is a joint undertaking of the Ontario Milk Marketing Board, Agriculture Canada, the Canadian Dairy Commission, the University of Guelph, and the Ontario Ministry of Agriculture and Food.

[2] One should also note the rotating nature of the sample, due to the fact that the composition of the sample is systematically changed by dropping some of the farms each year, and replacing them with newly recruited ones. For example, of the 105 farms surveyed in 1983, 35 were newly recruited in the program, 29 were recruited in 1982, and 41 were recruited in 1981. Individual farms are typically followed for a period of three years, although several farms enter the sample for a shorter period.

[3] Given that most of these inputs are typically owned by the farmers, an implicit cost is imputed based on the notion of user cost of capital, which is defined as:

$$r_j = R_j(i + \delta_j + \tau_j - \dot{R}_j),$$

where j indexes the capital input, r is the user cost (rental price), R is the capital (replacement) price, i is the interest rate (the opportunity cost of holding capital), δ is the physical depreciation rate, τ is the tax rate, and $\dot{R}$ is the unitary expected capital gain. To obtain a user cost of capital that has an intermediate to long run interpretation, the interest rate and expected price changes used are calculated as ten-year averages, for the last ten years up to 1983, of the prime lending rate and of the actual inflation rates for the various types of capital considered. The depreciation rate used is 0.15 for machinery, 0.05 for buildings, and zero for land and livestock. Applying these rates to the value of farmer-owned capital stock obtains a partial measure of the cost of capital services for each capital item. To obtain a full measure, for land and buildings these values were added to the cost of repairs to buildings, rent, and real estate taxes paid, and for machinery they were added to the cost of machinery operations.

[4] For computational purposes note that, in view of (10), equation (17) can be reduced to $R = 1/(\Sigma_i Q_i)$. Again, given that R is a nonlinear function of the estimated parameters, the standard errors reported in Table 6 are computed by the linearization method alredy mentioned.

[5] From a methodological point of view, this finding also casts some doubts on the usefulness of a linear programming approach to modeling the representative farm of the dairy industry, since standard linear programming models assume linear homogeneous technologies, that is constant returns to scale when all inputs are variable.

REFERENCES

Akridge, J.T. and T.W. Hertel. "Multiproduct Cost Relationships for Retail Fertilizer Plants." Amer. J. Agr. Econ. 68(1986):928-938.

Antle, J.M. "The Structure of U.S. Agricultural Technology, 1910-78." Amer. J. Agr. Econ. 66(1984):414-421.

Amemiya, T. "Non-linear Regression Models", in Z. Griliches and M.D. Intriligator, eds., Handbook of Econometrics, Volume I. Amsterdam: North-Holland, 1983.

Barichello, R.R. The Economics of Canadian Dairy Industry Regulation. Techn. Rep. No. E/I2. Ottawa: Economic Council of Canada, 1981.

Barten, A.P. "Maximum Likelihood Estimation of a Complete System of Demand Equations." Eur. Econ. Rev. 1(1969):7-73.

Baumol, W.J., Panzar, J.C., and Willig, R.D. Contestable Markets and the Theory of Industry Structure. New-York: Harcourt Brace Jovanovich Inc., 1982.

Berndt, E.R., Hall, B.H., Hall, R.E., and Hausman, J.A. "Estimation and Inference in Nonlinear Structural Models." Ann. Econ. Soc. Meas. 3(1974):653-665.

Box, G.E.P., and Cox, D.R. "An Analysis of Transformations." J. Royal Stat. Soc., Series B, 26(1964):211-243.

Breusch, T.S. "Conflict among Criteria for Testing Hypothesis: Extensions and Comments." Econometrica 47(1979):203-207.

Brown, R.S., Caves, D.W., and Christensen, L.R. "Modelling the Structure of Cost and Production for Multiproduct Firms." South. Econ. J. 46(1979):256-273.

Caves, D.W., Christensen, L.R., and Tretheway, M.W. "Flexible Cost Functions for Multiproduct Firms." Rev. Econ. Stat. 62(1980):477-481.

Chambers, R.G. "Least-Cost Subsidization Alternatives." _Amer_. _J_. _Agr_. _Econ_. 67(1985):251-256.

Diewert , W.E. "Duality Approaches to Microeconomic Theory." in K.J. Arrow and M.D. Intriligator, eds., _Handbook of Mathematical Economics_, volume II. Amsterdam: North-Holland, 1982.

Forbes, J.D., Hughes, R.D., and Warley, T.K. _Economic Intervention and Regulation in Canadian Agriculture_. Ottawa: Economic Council of Canada, 1982.

Fox, G.C., and Driver, H.C. "Developing Farm Stratification Procedures for Representative Farm Linear Programming Analysis." _Can_. _J_. _Agr_. _Econ_., CAES Annual Meeting (1980):148-157.

Hall, R.E. "The Specification of Technology with Several Kinds of Output." _J_. _Pol_. _Econ_. 81(1973):878-892.

Judge, G.C., Griffiths, W.E., Carter Hill R., Lutkepohl H., and Lee T-C. _The Theory and Practice of Econometrics_, second edition. New-York: J. Wiley and Sons, 1985.

Kohli, U. "Non-Joint Technologies." _Rev_. _Econ_. _Stud_. 50(1983):209-219.

Kuh, E. "The Validity of Cross-Sectionally Estimated Behavior Equations in Time Series Applications." _Econometrica_ 27(1959):197-214.

Lau, L.J. "A Note on the Elasticity of Substitution Functions." _Rev_. _Econ_. _Stud_. 43(1976):353-358.

Lee, H. and R.G. Chambers. "Expenditure Constraints and Profit Maximization in U.S. Agriculture." _Amer_. _J_. _Agr_. _Econ_. 68(1986):857-865.

Lopez, R.E. "The Structure of Production and the Derived Demand for Input in Canadian Agriculture." _Amer_. _J_. _Agr_. _Econ_. 62(1980):38-45.

McFadden, D. "Cost, Revenue, and Profit Functions." in M. Fuss and D. McFadden, eds., Production Economics: A Dual Approach to Theory and Applications, volume I. Amsterdam: North-Holland, 1978.

Panzar, J.C., and Willig, R.D. "Economies of Scale in Multi-output Production." Quart. J. Econ. 91(1977):481-493.

Panzar, J.C., and Willig, R.D. "On the Comparative Statics of a Competitive Industry with Inframarginal Firms." Amer. Econ. Rev. 68(1978):474-478.

Shephard, R.W. Cost and Production Functions. Princeton: Princeton University Press, 1953.

Shumway, C.R. "Supply, Demand, and Technology in a Multiproduct Industry: Texas Field Crops." Amer. J. Agr. Econ. 65(1983):748-760.

Shumway, C.R., R.D. Pope, and E.K. Nash. "Allocatable Fixed Inputs and Jointness in Agricultural Production: Implication for Economic Modeling." Amer. J. Agr. Econ. 66(1984):72-78.

Sidhu, S.S., and Baanante, C.A. "Estimating Farm-Level Demand and Wheat Supply in the Indian Punjab Using a Translog Profit Function." Amer. J. Agr. Econ. 63(1981):237-246.

Toevs, A.L. "Approximate Variance Formulas for the Elasticities of Substitution Obtained from Translog Cost Functions." Econ. Letters 10(1982):107-113.

Tweeten, L.G. "Theories Explaining the Persistence of Low Resource Returns in a Growing Farm Economy." Amer. J. Agr. Econ. 51(1969):798-817.

Warley, T.K., and Barichello, R.R. "Agricultural Issues in a Comprehensive Canada-USA Trade Agreement: A Canadian Perspective." Policy Research Notes, Issue 22. USDA, ERS, November 1986.