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Measures of market power and returns to scale are obtained for the Canadian dairy processing industry with the estimation of a cost system. Assumptions of homothetic technologies and equal conjectures across firms allow the firm's profit maximizing problem to be aggregated to the market level. A two stage production process results in a cost function which is additively separable between raw milk costs and processing costs for other inputs. The processing cost component is multiplicatively separable between an input price index and an aggregate input which is a transformation of output. The input price index has a generalized Leontief functional form. The transformation function, which is non-joint in outputs, is additive between outputs and each output has an exponent which is the reciprocal of its scale elasticity.

ANALYSIS OF THE COST STRUCTURE FOR PROCESSED PRODUCTS: THE CASE OF THE CANADIAN DAIRY PROCESSING SECTOR

Previous studies have examined the Canadian dairy industry from the perspective of either the farm level (e.g. Moschini [1987] and Oxley et.al. [1989]) or have examined the welfare implications of supply management for society and its agents including consumers and primary producers (e.g. Barichello [1981], Schmitz [1983] and Veeman [1982]). For the most part previous research has excluded a detailed analysis of the processing sector. This paper addresses this deficiency by examining the cost structure of the Canadian dairy processing industry. This information should have value not only for conducting welfare analysis of the current regime, but also for examining potential reforms for the industry either through trade liberalization or domestic reform.

The dairy processing sector represents a non-trivial share of the economy. In 1992 dairy processing accounted for approximately 2.5 percent of the total value of Canadian manufacturing activity or $7.64 billion in shipments. Total 1992 employment was 24,614 which is approximately 4 percent of employment in Canada's manufacturing sector.

Over the last three decades the industrial milk processing sector has experienced a significant rationalization through corporate re-organizations, and the introduction of multi-product plants. The number of plants has declined from 1710, in 1961, to 308 by 1992. The rationalization in the number of plants has been accompanied by a considerable growth in investment in plant and equipment. For instance, between 1982 and 1992 investment grew by more then 200 percent.

This rationalisation has occurred concurrently with an increase in industry concentration. In 1960 the largest four firms in Canadian milk processing accounted for 14 percent of the market. By 1990 four firm concentration ratio was over 50 percent of shipments.

The significant rationalization in the dairy processing industry suggests potential for scale economies. The increased level of concentration suggests an imperfectly competitive market structure.

The objective of this study is provide information concerning the technology and market structure of the Canadian dairy processing sector. The application of duality theory allows several equivalent ways of characterising technology (see Diewert [1974]). The approach used in this paper is to discover the underlying technology via the industry's cost structure. The estimation of a cost function[1] in conjunction with assumptions concerning pricing behaviour allow inferences to be drawn concerning market structure for the dairy processing sector.

Modelling and Theoretical Considerations

The cost function represents the minimum cost required to produce given a vector of outputs, with input prices predetermined. So the cost function is the solution to the firm's cost minimization problem:

$$C(w,y) = \min_x \{w'x: (x,y) \text{ is in the production possibilities set}\}$$

To be an accurate representation of the firm's underlying technology the cost function must exhibit the following properties: C(w,y) is a non-negative function, linear homogenous in input prices w, non-decreasing in output y and w; is continuous, and is quasi-concave in input prices w.

The unit of analysis in this paper is the industry, rather than the individual firm. This requires aggregation of inputs, outputs and firms. In order for consistent aggregation to hold across firms the production function must be quasi-homothetic.[2] A technology is globally homothetic in output if the cost minimizing expansion path is a ray from the origin and the slopes of the isoquants are invariate

[1] A decision was made to use a cost function rather than a profit function for this analysis. Cost minimization is a necessary condition for profit maximization for a risk neutral firm. With supply management output can be considered fixed and the cost function captures the optimizing behaviour of the industry.

[2] See Blackorby, Primont and Russell [1978] for a more detailed discussion.

to the level of output[3]. So for given input price ratios all firms, no matter what their level of output, will have the same factor intensity. As input price ratios change, all firms will change their factor proportions in the same manner. Without this assumption each firm's input-output combinations at each price ratio will have to be known in order to proceed with aggregation. With a homothetic technology it is not necessary to know where each individual firm is in input space or to associate specific levels of output with specific firms; the only requirement is that individual production sums to aggregate production and individual costs sum to the aggregate costs. A quasi-homothetic function has a linear expansion path, which does not emanate from the origin which implies that firms face fixed costs of production.

Shephard [1981 p.p. 43-45] has shown that if a production function is homothetic the associated cost function can be written as:

$$c(w,y)=c(w)\bullet h(y)+g(w).$$

This cost function can be thought of as unit cost, c(w), times an aggregate input or a transform of output, h(y), plus g(w) which is an overhead cost which makes the cost function quasi-homothetic.

When the production process involves multiple products the relationship between these products can either be described as joint or non-joint. With joint production, levels of output among products are linked through resource constraints, non-allocatable factors (e.g. wool and mutton) or some form of technical interdependence. With non-joint production, the output of one product does not depend on the outputs of other products. In terms of the cost function this independence of outputs can be stated formally as:

$$\partial C^2/\partial y_i \partial y_j = 0$$

The cost functions, for non-joint products are the sum of the individual cost functions.

This study approaches the measurement of market power (i.e. the ability to mark prices above

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[3] See Silberberg [1978] p. 89 for a discussion of radial blowups of isoquants.

marginal costs) from the industry level, rather than the more conventional approach of examining strategic interactions among individual firms. Appelbaum [1982] measures market power at the industry level of aggregation by imposing the conditions for consistent aggregation (a homothetic production technology) and assuming that the industry behaves the same as a representative firm.[4] In this context market power is an average concept across an entire industry.

Appelbaum describes the representative firm's profit maximization problem as:

(1) $\max_y[py^i - C^i(y^i,w)]$

where:			
	p	=	price dependent market demand equation $p=p(Y,z)$
	y^i	=	representative firm i's output
	Y	=	market demand
	z	=	exogenous demand shifter
	w	=	vector of exogenous prices
	C^i	=	$c(w)\bullet h(y^i) + g^i(w)$

The first order conditions to this optimization problem can be aggregated across a firms and be written in the form of a Lerner index:

(2) $L=[(p-mc)/p] = \theta/\eta$

where:			
	mc	=	marginal cost for industry
	θ	=	$(\partial Y/\partial y^i v)(y^i/Y)$ conjectural elasticity of industry output with respect to representative (average) firm i's output
	η	=	market demand elasticity

Conjectural variation, $(\partial Y/\partial y^i)(y^i/Y)$, is a measure of how the representative firm (industry average) perceives the rest of the industry adjusting to a change in its output. The measure of conjectural variation is transformed into proportional rates of change to provide an elasticity measure which allows a re-interpretation of the market demand elasticity in terms of the representative (average) firm's perceived elasticity of demand. The θ parameter has a lower bound of zero (perfect

[4] He is able do this since the cost function is quasi-linear homogenous so that marginal costs are constant and identical across all firms. Since all firms face the same output price and market demand elasticity the degree of market power must be identical across firms

competition) and an upper bound of 1 (monopoly).[5] Since demand elasticity determines the degree of market power which can be exercised, the perceived inverse demand elasticity, θ/η, will determine the industry average perceived market power. The Lerner index can be rewritten as:

(3) $p = mc/(1-\theta/\eta)$

This equation can be inverted, given a tractable functional form, in order to obtain the industry's pseudo-supply function.

Empirical Model and Results

The criterion to select the functional form for the cost function was to a large extent determined by data considerations. The resulting specification had to account for the fact that there was insufficient data, in terms of primary factor prices and quantities to develop separate cost functions for each product. The industry is treated as one multi-product plant where capital, labour, and fuel are shared in the production of the five dairy products: butter, cheese, concentrated milk, soft products (an amalgam of ice cream, yoghurt, and cottage cheese) and fluid milk. The estimation used aggregate data at the national level.

A multi-product technology is a reasonable assumption given that many plants currently operate multi-product facilities. The five products are assumed to be technically non-joint[6].

There is no accurate national data on the volumes of milk used to produce individual dairy products. To circumvent this problem quantities of product have been converted to milk equivalents. This conversion assumes a fixed proportions technology. A sensible way to proceed is to assume that

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[5] An alternative interpretation is that $\theta \in [0,1]$ is the outcome of some unknown game.

[6] The assumption of non-jointness is not as arbitrary as it might first appear for although each product shares a public input, raw milk, regulations (such as Ontario's system of Plant Supply Quotas) currently prevent much movement of milk between products so dairy products cannot be considered joint as a result of this shared input. Butter and skim milk powder are technically joint products; for this reason skim milk powder is excluded from this analysis.

the dairy processing technology is strongly (additively) separable between raw milk costs and processing costs because the combination of primary factors, labour, fuel, and capital, which determine the processing technique is separable from amount of milk used in the production process.[7] Production can be thought of as occurring in two stages. In the first stage the firm decides how much milk, x_m, and real value added[8] to use in the production process. The second stage determines how much labour (x_l) and capital (x_k) to use to produce a given level of real value added through the relationship $f(x_l,x_k)$. If the first stage follows a recipe which is fixed proportions and the second stage allows factor substitution, then the production function can be written as:

(4) $y = \min(x_m/\alpha, f(x_l,x_k))$

where: $\alpha \equiv$ technical conversion factor

The corresponding cost function can be written as:

(5) $C = w_m \bullet \alpha \bullet y + c(w_l,w_k) \bullet h(y)$

where: $c \bullet h \equiv$ cost function for the second stage

The two stage process ensures that the cost associated with raw milk, $w_m \bullet \alpha \bullet y$, is strongly separable from the costs associated with processing, c(w).

Since the underlying processing technology is assumed to be homothetic, processing costs are the product of an input price index and an aggregate input which is a transformation function of outputs. The input price index is assumed to have a generalized Leontief functional form.

The Generalized Leontief function tends to be a good representation of technology when the elasticity of substitution between factors is relatively small. Diewert [1971] describes the properties

[7] See for instance Bruno [1978] for a discussion when intermediate goods can be separated from primary factors in the production process. Bruno considers the instances where real value-added can be used in place of output in production function analysis.

[8] A physical measure of the labour and capital added through the production process. See Bruno [1978] for a discussion of production functions with real value added.

of a Generalized Leontief cost function which is multiplicatively separable in output and in the square root of prices:

(6) $c(w,y) = h(y) \bullet \Sigma_i\Sigma_j b_{ij}(w_i w_j)^{1/2}$ $\qquad w_i \geq 0; \quad y \geq 0$

Where b_{ij} are non-negative elements of a symmetric matrix of input price coefficients, and h(y) is a continuous monotonically increasing transformation function of outputs. For a single output, the h(y) term is related to the elasticity of scale, $\epsilon(y)$, by the definition[9]:

(7) $\epsilon(y) = h(y)/(y \bullet dh/dy)$

Integration of this differential equation allows the researcher to work backwards to the underlying transformation function. If the production technology is homogenous, the elasticity of scale will be invariant to the level of output and the transformation function will equal:

(8) $h(y) = H \bullet y^{1/\epsilon}$

where H $\equiv$ constant of integration

Given these functional forms for the price index c(w) and the output index h(y) the cost function now be written as:

(9) $C = w_m \bullet \alpha \bullet y + H \bullet y^{1/\epsilon} \bullet \Sigma_i\Sigma_j b_{ij}(w_i w_j)^{1/2}$

The cost function for a non-joint multi-product production process is simply the sum of the separate single product cost functions. Hall [1974] established that a non-joint homothetic cost function, for k products, has the form:

(10) $C(w,y) = [h_1(y_1) + ... + h_k(y_k)] \bullet c(w)$

Therefore the cost function for multiple output non-joint technology which is strongly separable between milk and primary inputs can be written as:

(11) $C = \Sigma_\tau w_{m\tau} \bullet \alpha_\tau \bullet y_\tau + \Sigma_\tau H_\tau \bullet y^{1/\epsilon\tau} \bullet \Sigma_i\Sigma_j b_{ij}(w_i w_j)^{1/2}$ $\quad \forall$ 1,...,k products

The derivative property, known as Shephard's lemma, allows conditional input demand

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[9] See Forsund [1975] for the derivation.

functions to be obtained by differentiation of the cost function with respect to input prices. The marginal cost function is obtained as the derivative of the cost function with respect to output. Pseudo-supply functions can be obtained by combining the Lerner index, equation 3, with the marginal cost function for each product. The system which is estimated includes the cost function, conditional input demand functions, and pseudo-supply functions. Cross equation restrictions help to identify the conjectural elasticity parameters.

Multi-variate models of cost functions, utilizing time series data, frequently also account for technical change by including a time variable which is included in a manner similar to input prices.[10] Additive dummy intercepts have been added to all equations to account for the introduction of supply management which is assumed to start in 1975[11]

$$(12)\quad C = \Sigma_\tau \alpha_\tau w_{m\tau} y_\tau + \Sigma_\tau H_\tau \cdot y_\tau^{a\tau} \cdot (\Sigma_i \Sigma_j b_{ij} \cdot (w_i \cdot w_j)^{1/2} + \Sigma_i b_{it} \cdot w_i \cdot t)$$

$$(13)\quad x_i = \Sigma_\tau H_\tau \cdot y_\tau^{a\tau} \cdot \Sigma_j b_{ij} \cdot (w_j/w_i)^{1/2} + b_{it} \cdot t$$

$$(14)\quad p_\tau = [\alpha_\tau \cdot w_{m\tau} + a\tau H_\tau \cdot y_\tau^{(a\tau-1)} \cdot (\Sigma_i \Sigma_j b_{ij} \cdot (w_i \cdot w_j)^{1/2})]/[1-\theta/\eta_\tau]$$

where:	C	$\equiv$	total cost
	α	$\equiv$	conversion factor
	w_m	$\equiv$	price of milk
	y	$\equiv$	production level
	w	$\equiv$	primary input prices
	x	$\equiv$	primary input usage
	t	$\equiv$	time
	τ	$\equiv$	index for butter, cheese, concentrated milk, soft product, and fluid
	i	$\equiv$	labour, capital, and fuel
	θ	$\equiv$	conjectural elasticity
	η	$\equiv$	market demand elasticity for product τ
	a	$\equiv$	cost elasticity or the inverse of the scale elasticity $1/\epsilon$

The system (1) to (3) consists of nine equations and was estimated using the non-linear multi-

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[10] See Morrison [1988] for an example of such a specification.

[11] This is not entirely correct as the market sharing quota for industrial milk was initiated in 1970. However, not all provinces joined the plan initially. Formula pricing and global import quotas were introduced in 1975.

variate regression procedure in Shazam version 6.2. With this procedure the coefficients converge to maximum likelihood estimates which differ from the conventional seemingly-unrelated-regression[12] approach in that the estimates iterate over both the residual covariance matrix and the parameter matrix. The system is not treated as simultaneous equation model. The problem of simultaneity bias is avoided since the final demand side of the model, η, is treated as pre-determined and the parameters are obtained exogenously.[13] The market demand elasticities are:

	Hassan & Johnson*	Moschini & Moro	Al-Zand & Andriamanjay	Cluff & Stonehouse
Butter	-1.06	-0.92	-1.08	-0.77
Cheese	-0.86	-0.41	-1.01	-0.57
Concentrated milk	-0.83	-1.03		
Ice Cream	-0.85	-1.03		
Fluid	-0.44	-0.36	-0.21	-0.14

* Selected for this study

The Hassan and Johnson set elasticities fall within the mid-range of final demand elasticities. For this reason and the wide acceptance of this study of Canadian food consumption patterns, this set of elasticities where selected for use in this study.

Homogeneity of degree one in input prices, for the cost function, has been imposed by the generalized Leontief functional form which uses relative input prices. Symmetry is maintained by

[12] The assumption of contemporaneous covariance means the error terms between equations are correlated at any point in time.

[13] Predetermined market demand elasticities also circumvent the problem of identification of conjectural elasticities in a system of equations. See Lau [1982] for a discussion of the identification problem.

imposing cross equation restrictions on the parameters such that $b_{ij}=b_{ji}$.[14]

The estimation was conducted with data from 1961 to 1986. Appendix 1 describes the sources of data for this analysis.

Table 1 presents the results of the estimation of the system of equations 12 to 14. The H_i's parameters have been restricted to equal the physical shares[15], in terms of milk equivalents, for each dairy product.

The surface of the cost function appears to be relatively flat with many local optima[16]. Initial attempts to estimate the complete system 12 to 14 resulted in very unstable parameters. In an attempt to circumvent the problem an iterative routine was devised which estimates the scale parameters, a_τ's, separately from the θ_i's, the conjectural elasticities. This is a step wise process. In the first step the a_τ's are restricted to one which is consistent with constant returns to scale. The estimated θ_i's from this step are then used as constants in the next step. In the second step all the other parameters are allowed to vary. The estimated a_τ's are treated as constants in the third step which estimates the remaining parameters and the θ_i's. This process is then repeated until the parameter values become relatively stable with less than five percent variation between steps. The same set of starting values is used for every iteration. The model stabilized after eight steps. In the final iteration the θ_i's are held constant and the rest of the free parameters are estimated. So in effect this is a method to obtain credible values for the conjectural elasticity parameters which are

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[14] Arguments for imposing theoretical restrictions include a reluctance to depart from theory, and practical matter of minimizing the number of parameters which are estimated.

[15] These parameters, H_i's, can be thought of as weights in an output aggregator function.

[16] This could be a result of the large number of parameters, the non-linear nature of the model, or the large number of restrictions which are imposed upon the model.

Table 1 : Maximum Likelihood Estimates of Cost System

Dependent Variable	Explanatory Variablies: Cost elasticity a_τ	Conjectural elasticity θ	labour price b_{il}	capital price b_{ik}	fuel price b_{if}	time b_{it}	Dummy variable D_i	R-Squared	Durbin-Watson
butter	0.42325	0.2291					35.674	0.9939	1.8691
	(.0159)	(.0374)					(8.9102)		
cheese	0.51615	0.2229					7.6852	0.987	2.4252
	(.0301)	(.0362)					(12.815)		
softproduct	0.61367	0.28341					2.9735	0.964	2.258
	(.0289)	(.0524)					(6.9901)		
evap milk	0.45827	0.58978					-3.8149	0.9956	1.4266
	(.0609)	(.007.68)					(4.5474)		
fuild milk	0.59246	0.05611					9.1603	0.9946	1.5239
	(.0159)	(.0171)					(1.7784)		
labour			49.903	1.8837	148.6	1.2635	75.35	0.4461	1.2795
			(16.862)	(4.4183)	(33.427)	(0.4877)	(344.76)		
capital				53.791	-24.888	1.0779	-140.81	0.9224	1.4362
				(8.8939)	(11.014)	(0.3044)	(118.35)		
fuel					112.04	7.76e-02	-4521.3	0.9316	1.6324
					(127.33)	(3.6269)	(815.79)		
Cost								0.9991	1.183

Notes In order to facilitate illustration of parameters the b_{ij}'s are only shown with the primary factor demand equations, but these parameters are actually associated with every equation. As well all of the parameters were estimated with the cost equation,but are not shown as such.
Standard errors are reported in parenthesis under parameter estimates; maximized log-likelihood function = -1234.449
Corrected for second order auto-correlation $RHO_1 = 0.5812$ (.0871) $RHO_2 = 0.2366$ (0.845)

then treated as exogenous information in the final regression.[17]

The overall explanatory power is good. Each of the equations, except labour, has a R-squared of at least 0.92.

Auto-correlation proved to be a problem. An attempt was made to correct this problem with a second order vector auto-regressive specification. Each auto-covariance matrix of rho's is restricted to be a diagonal matrix with the same value of rho across all equations[18] The RHO parameters are statistically significant at both the first and second orders.

The signs of the coefficients are as expected. Monotonicity in output is satisfied as the a_τ's are positive. The b_{ij}'s, $i \neq j$, are generally required to be non-negative since this is a sufficient condition for the cost function to be concave in prices. All of these coefficients are positive except b_{kf}. This indicates a complementary relationship between capital and fuel which holds in both sets of results. Most coefficients are statistically significant at the 5 percent level. Exceptions are b_{lk} and b_{ff}. A number of dummy variables and time trend variables are not significant.

Since not all the b_{ij}'s are positive it is necessary to check for strict quasi-concavity of the cost function in input prices. Concavity requires that the matrix $\nabla^2 C_{ww}(w)$ is negative semi-definite. This regularity condition can be checked by determining if the matrix of Allen partial elasticities of substitution is negative semi-definite. Table 2 presents the Allen partial elasticities for the three primary factors of production. These elasticities are computed as:

$$\sigma_{ij} = (C \cdot C_{ij})/(C_i \cdot C_j)$$

[17] This method is similar to the approach used by Lopez [1984] when he was unable to obtain convergence of the conjectural elasticity for the Canadian food processing industry. Lopez's method, however only involves two steps.

[18] See Berndt [1991] p.p. 475-79 for a general description of auto-regressive processes in multi-variate equation systems. Hendry [1971] provides a more detailed description of maximum likelihood non-singular systems of equations with errors generated by an auto-regressive process.

Where C_i and C_{ij} are the first and second partial derivatives of the cost function with respect to input prices. Since this definition includes both exogenous variables and parameters the value of σ_{ij} will vary with each observation. The values in table 2 are measured at the means.

Table 2 Elasticities of Substitution

	Labour	Capital	Fuel
Labour	-.55816 (.1640)	.026746 (.0640)	1.77217 (.2955)
Capital		.028809 (.0304)	-.27414 (.1020)
Fuel			-4.4348 (1.024)

* standard errors are in parentheses and are calculated as second order approximations see Kmenta [1986] p.p.486-87

The own Allen partial elasticity of substitution for capital is positive, but very small and not statistically different from zero. As a result the matrix of Allen partial elasticities of substitution is not negative semi-definite, at the point of expansion, so the cost function is not quasi-concave in prices. However this does not constitute a violation of concavity in a statistical sense because of the statistical insignificance of σ_{kk}.[19] The own Allen elasticities of substitution, for labour and fuel, both have the correct negative sign and they are statistically significant. Labour and capital are net substitutes as indicated by the positive sign on the Allen partial elasticity of substitution. However this substitution elasticity is not statistically different from zero. The strongest substitution possibilities exist between labour and fuel. The quantity of labour used in dairy processing increased dramatically after the energy crisis. This could be coincidental as supply management was introduced at approximately that

[19] The usual method of imposing concavity is to force the offending parameters to be equal to zero, however this approach has not be used in this study because (1) the parameters of primary interest belong to the transformation function and not the input price index, and (2) the own elasticity of substitution for capital is already not statistically different from zero.

time. Capital and fuel are net substitutes. This corresponds to the negative sign on b_{lk}.

The θ or conjectural elasticity parameters are all statistically different from zero. Table 3 summaries these elasticities, the implied Lerner indexes and the only published previous estimate of measure of market power for Canadian food processing.

Table 3 Measure of Market Power

	Conjectural Elasticity	Lerner Index (θ/η)
Butter	0.23	22%
Cheese	0.22	26%
Concentrated Milk	0.28	35%
Soft Products	0.56	66%
Fluid Milk	0.06	14%
Food Processing*	0.192	50%

* See Lopez [1988]

The magnitude of the conjectural elasticities suggests a significant amount of market power has been exercised in the dairy processing sector.

Previous studies (Lopez [1988]) have found market power in Canadian food processing. However, this approach imposed constant returns to scale in order to derive the estimates of market power. Empirical analysis of market power which does not account for potential scale economies (dis-economies) will bias the estimates of market power upwards (downwards).[20]

[20] The upward bias in the conjectural elasticity parameter follows because the numerator of the Lerner index, equation 2, is larger with constant returns to scale then with increasing returns. Rearrange the Lerner index and replace marginal cost with a homothetic functional form.

$$p=h(y)\bullet c(w)/(1+\theta/\eta)$$

with	IRTS	$h'(y)\bullet c(w)<c(w)$	(increasing returns to scale)
	CRTS	$h'(y)\bullet c(w)=c(w)$	(constant returns to scale)
	DRTS	$h'(y)\bullet c(w)>c(w)$	(decreasing returns to scale)

The cost elasticities with respect to output, a_τ's, are the reciprocal of the elasticities of scale for the value added portion of the two stage production function. All products exhibit statistically significant increasing returns to value added for proportional increases in primary inputs.

Processing occurs over two stages of production: conversion of primary factors of production to value added and conversion of milk to product utilizing the processing capacity created with the value added from the first stage. In order to measure returns to scale across both stages of production, value-added returns to scale and the fixed proportions constant returns to scale conversion of milk to product are combined as a weighted average.

$$(13) \quad 1/\varepsilon_\tau = \{\alpha_\tau \bullet w_{m\tau}/(\alpha_\tau \bullet w_{m\tau} + H_\tau \bullet y_\tau{}^{A\tau\text{-}1} \bullet c(w)\} \bullet 1 + \{H_\tau \bullet y_\tau{}^{A\tau\text{-}1} \bullet c(w)/(\alpha_\tau \bullet w_{m\tau} + H_\tau \bullet y^{A\tau\text{-}1} \bullet c(w)\} \bullet A_\tau$$

where:

τ	$\equiv$	index of product type
H	$\equiv$	physical share
c(w)	$\equiv$	primary input price index

Table 4 shows the combined cost elasticities (inverse of scale elasticities) over the value-added and milk conversion stages of production.

Table 4 Cost Elasticities

	value added	std error	over-all	std error *
butter	0.4233	0.0159	0.9885	0.0022
cheese	0.5162	0.0300	0.9817	0.0045
Concentrate	0.4583	0.0608	0.9735	0.0058
Soft Product	0.6137	0.0289	0.9279	0.0174
fluid	0.5925	0.0159	0.8735	0.0141

* Standard errors are measures as a second order Taylor series approximation

The cost elasticities across both stages of production are less than unity for all five products. Constant returns to scale can be rejected since each of these cost elasticities is statistically different from one. The production process for each product exhibits increasing returns to scale with scale elasticities which range from 1.0116 for butter to 1.1448 for fluid milk.

There is a paucity of previous research into the Canadian dairy processing sector making it difficult to affirm the order of magnitude for the scale elasticities[21] However, there are econometric estimates of the scale elasticity for the aggregate Canadian dairy processing sector. For comparison purposes the scale elasticities reported in table 3 were converted in the following manner. Baumol, Panzer, and Willig [1982 p.74] define an overall multi-product scale elasticity for non-joint products:

$$OSE = \sum_i (y_i \bullet MC_i / (\Sigma_i y_i \bullet MC_i)) \bullet PSE$$

where: OSE $\equiv$ overall scale elasticity

PSE $\equiv$ product specific scale elasticity

The mean value of the OSE is 1.03 with a standard error of .0045. Robidoux and Lester [1988] show that the average cost curve of the Canadian dairy processing sector is L-shaped and that at half the minimum efficient scale the scale elasticity is 1.02. Only 2.37 percent of the industry is at minimum efficient scale. These results would seem to confirm the findings of this study, however the lack of cross sectional information precludes a judgment whether most firms are at half the minimum efficient scale. Salem [1987] finds conflicting results and describes an industry with decreasing returns to scale and a scale elasticity of .925. [22]

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[21] Certain engineering studies have examined the economies of scale for constructing new dairy processing facilities. For example, the engineering firm APV estimates that scale economies exit for cheese, ice cream and yoghurt plants (1.05 for cheese, and an average of 1.18 for soft product plants).

[22] Since Salem uses labour as a proxy for scale his specification is not consistent with the approach used in this stud.

Conclusions

This article has developed an industry model of the Canadian dairy processing sector which is suitable to analyze the effects of market power and returns to scale on this industry. Potential applications with this framework could include: measuring tariff equivalents in the presence of market power, extending the welfare analysis of supply management to include rents accruing to the processing sector, and measuring the impacts of changes in domestic policy or trade liberalization on the market structure and product composition of the Canadian dairy processing sector.

The cost function results are satisfactory from an econometric point of view. The equations fit well, and the theoretical restriction of monotonicity could not be rejected. The concavity of the cost function remains in question but this property can not be rejected in the statistical sense.

The article found that Canadian dairy processors operate with increasing returns to scale. No reasons for these economies were given, but it is possible to speculate that milk supply restrictions have caused processors to operate at capacities that are less than minimum efficient scale. Increasing returns to scale also has implications for market power, for if producers price at marginal cost they will lose money (i.e. at each level of output average cost exceeds marginal cost with increasing returns). For this reason some degree of market power can be expected when increasing returns occur. In addition the border measures, associated with supply management may have allowed domestic processors to exercise more market power. The presence of market power has implications for the measurement of tariff equivalents, for if these duties are measured with a price gap method the tariff estimates may overstate the cost differences between countries because the price gap will include the effects of market power.

This study has illustrated the importance of measuring scale economies when attempting to measure market power. Most previous attempts to measure market power have employed functional forms which impose constant returns to scale. Lopez finds evidence of market power by Canadian food processors, but he imposes a constant returns to scale technology. If scale economies (diseconomies) exist, then estimates of market power will be biased upwards (downwards) when constant returns are imposed in the estimation.

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A.1 Data Sources

The major data sources are found in table A.1. Since some data sources have changed over the sample period, 1961-1986, several sources may be given for one particular item.

Table 4.1 Data Sources

VARIABLE CATEGORY	SOURCE
Input Values and Volumes - labour - fuel	- Statistics Canada 32-209 & 32-250B and for fuel quantity index see S.C. 57-208 & 62-001
Capital Stock	-perpetual inventory method
- user cost of capital	
- interest rate	-prime business rate (Bank of Canada Review)
- depreciation rate	-Statistics Canada 13-586 Text Table II
- machinery and equipment price index	-Statistics Canada 61-518, 61-206
Quantity of Output	-Statistics Canada 32-229 & 23-001
Wholesale Product Prices	-Statistics Canada 32-211 for value & qty of shipments
Raw Milk Prices by Class	-Agriculture Canada Dairy Market Review and S.C. 23-001
Conversion factors	-U.S.D.A., Dairy Farmers of Canada, U of G Dept of Food Science

The capital stock was derived using a perpetual inventory method which involves accumulation of investment over time given a straight line depreciation method (see Statistics Canada 13-568). Investment series were obtained from S.C. 61-205, 61-214, 61-518. The price deflator series was obtained from S.C. 62-007 and estimated economic lives for assets were obtained from S.C. 13-586.

The energy series is constructed from eight sources: natural gas, gasoline, kerosene, diesel fuel, light and heavy oil, liquid propane gas, and electricity. Given that the unit of measurement of this fuels are different a quantity index is constructed. Prices for each fuel are obtained as unit values from Statistics Canada 57-208 and 62-001. An overall price index is constructed by weighting each fuel price by that item's share of total fuel expenditure. The value of total expenditures are then deflated by the price index to give a quantity index.

Raw milk volumes are derived from production volumes for final dairy products via technical conversion factors.

Output levels of final products for cheese, soft products, and concentrated milk have all been constructed as aggregate commodities. The index is a Fisher expenditure index where the weights are expenditure shares. The commodity aggregates include: cheddar and variety cheese for cheeses; cottage cheese, yoghurt and ice cream for soft products; and evaporated and condensed milk for concentrated milk. All dairy products have been converted to milk equivalents to facilitate aggregation over different product types.

Table A.2 provides units of measurement and summary descriptive statics for each of the major variables in the cost system.

Table A.2 Summary of Variables

NAME	LABEL	UNIT	MEAN	ST. DEV	MINIMUM	MAXIMUM
LABOUR DEMAND	XL	000HOURS	13981.	484.54	12933.	14839.
WAGE RATE	WL	PER HOUR	11.198	7.6671	3.3392	25.989
CAPITAL DEMAND	XK	INDEX	8241.4	573.49	7332.2	9154.7
DEPRECIATION	DEP	YEARS	16.365	0.10614	16.170	16.560
PRICE OF CAPITAL	PIK	INDEX	1.5377	0.79012	0.74703	3.1460
RENTAL RATE	WK	INDEX	28.735	18.043	4.587	66.176
FUEL PRICE	WF	INDEX	0.59666	0.51480	0.19927	1.6132
FUEL DEMAND	XF	INDEX	82248.	12095.	61231.	98265.
QTY. OF BUTTER	QB	HLTR M.E.	29664.	5494.2	21823.	37820.
QTY. OF CHEESE	QC	HLTR M.E.	11354.	4341.0	5347.8	20304.
QTY. OF EVAP	QE	HLTR M.E.	3749.1	807.10	2625.6	5251.8
QTY. OF SOFT PRD.	QS	HLTR M.E.	2701.0	853.32	1734.1	4706.8
QTY. OF FLUID	QFLD	HLTR M.E.	11602.	1969.8	7971.9	14669.
INTEREST RATE	IR	% REAL	2.0993	2.4103	-3.2588	7.3185
PRICE OF BUTTER	PB	$/100 KG	248.54	132.50	133.92	512.7
PRICE OF CHEESE	PC	$/100 KG	230.53	149.78	78.226	522.86
PRICE OF EVAP.	PE	$/100 KG	70.143	46.507	27.541	165.07
PRICE OF SOFT PRD.	PS	$/100 KG	118.63	63.292	61.608	241.60
PRICE OF FLUID	PFLD	$/100 KG	42.874	28.679	16.772	100.40
MILK PRICE S.P.	WMS	$/HLTR	18.603	12.890	5.9157	42.267
MILK PRICE CHS.	WMC	$/HLTR	18.252	11.908	6.2136	40.364
MILK PRICE BTR.	WMB	$/HLTR	17.973	11.755	7.0943	39.75
MILK PRICE FLD	WMFLD	$/HL	24.664	14.249	203.04	10.384
	COST	$	0.448E+06	.311E+06	0.17E+06	0.11E+07