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CONVENTIONAL CAPITAL BUDGETING VERSUS STOCHASTIC DYNAMIC ANALYSIS OF OPTIMAL FARMLAND PURCHASE AND SELL DECISIONS

by

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Conventional Capital Budgeting Versus Stochastic Dynamic Analysis of Optimal Farmland Purchase and Sell Decisions

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Abstract

The purpose of this paper is to compare optimal farmland purchase and sell decisions from two dynamic programming (DP) models and a capital budgeting model. Significant reductions in net worth are incurred by following the capital budgeting decision rule rather than a DP decision rule.

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The current period of financial stress in agriculture has led economists to reexamine farmland price movements (Alston; Burt; Scott), farmland return movements (Burt; Phipps), and the financial structure of agricultural firms (Jolley, et. al.; Hughes, Richardson, and Rister; Melichar; and U.S. Department of Agriculture). Besides analyzing these dimensions of financial stress, it also behooves us to examine the accuracy of conceptual frameworks used in analyzing farmland purchase and sell decisions.

Frameworks used to analyze farmland purchase (sell) decisions and related firm growth issues include multiperiod linear programming models (Barry; Boehlje and White; Boussard and Petit; Ellinger, Barry, and Lins), stochastic linear programming models (Johnson, Tefertiller, and Moore), expected utility models (Harris and Nehring), multiperiod MOTAD models (Kaiser and Boehlje), multiperiod quadratic programming models (Johnson and Boehlje), simulation models (Held and Helmers; Hinman and Hutton), dynamic programming (DP) models (Minden; Larson, Stauber, and Burt), and static deterministic capital budgeting (SDCB). Possibly because of its ease of use the SDCB framework is the most popular analysis framework. SDCB is detailed in agricultural finance textbooks (Barry, Hopkin and Baker; Lee, Boehlje, Nelson and Murray), used in professional research articles (Lerner; Lee; Lee and Rask), and appears to be used by many lenders and landowners.

Although the SDCB framework is quite popular, it is not without conceptual faults. In applying SDCB, all random variables, such as farmland returns, are replaced by their expected values, resulting in suboptimal

decisions if certainty equivalence requirements (Simon; Theil) are not met. Undoubtedly these requirements are violated because of progressive income taxes (Taylor). A second and perhaps more severe problem is that the SDCB framework as conventionally used (see Barry, Baker, and Hopkin) does not consider future purchase and sell decisions. That is, the framework does not consider, for example, the advisability of waiting a year before purchase or sale of land. Consequently, decisions may be suboptimal.

This paper's purpose is to examine the accuracy of the SDCB framework relative to a DP framework. In addition, the costs of only selling farmland under financial stress relative to considering selling farmland at any time are addressed. In order to accomplish this, a DP model incorporating Markovian farmland returns and a farmland capitalization formula is developed and then numerically solved. This basic framework and numerical parameters for a Central Illinois high-quality farm are given in the next section. Then, comparisons of the DP model and a SDCB model are made in the second section.

Dynamic Programming Model of a Crop Farm

Assumptions used in constructing the DP model are that: (a) all land that is farmed is owned; (b) any amount of farmland can be purchased or sold at the current price of farmland; and (c) farmland returns and prices can be represented by Markovian relationships estimated on the basis of historical time-series data. The presumed objective is to maximize the expected value of after-tax net worth over a T-year planning horizon subject to equations giving expected farmland direct returns and prices, and relationships describing the movements of farm size and debt-to-asset position over time.

Formulation of this problem as a stochastic DP model leads to the following recursive equation:

$$V_t(OA_t, DR_t, P_t, P_{t-1}, DFA_t) = \max_{XOA_t} E[V_{t+1}(OA_{t+1}, DR_{t+1}, P_{t+1}, P_t, DFA_{t+1})] \quad (1)$$

where $V_t(\cdot)$ is the optimal value function (i.e., maximum expected after-tax net worth from t to T), OA_t = number of acres owned, DR_t = stochastic direct returns, P_t and P_{t-1} = current and previous price of farmland, DFA_t = debt-to-farm-asset ratio, XOA_t = number of acres purchase (sold), and E is an expectation operator. Returns during a year are compounded forward to the terminal time point, thus a current returns function is not included in the recursive equation.

Maximization of (1) is subject to the following state transition equations:

$$DR_{t+1} = f_1(DR_t) + u_t \quad (2\text{-a})$$

$$P_{t+1} = f_2(DR_t, P_t, P_{t-1}) \quad (2\text{-b})$$

$$OA_{t+1} = OA_t + XOA_t \quad (2\text{-c})$$

$$DFA_{t+1} = \frac{[(P_t*OA_t + OFA(OA_t))*DFA_t] + FLOW_t}{P_{t+1}*OA_{t+1} + OFA(OA_{t+1})} \quad (2\text{-d})$$

Expectations of (1) are taken over the Markovian probability density function of direct returns given in equation (2-a). Direct returns per acre equal expected revenue per acre minus variable costs. The stochastic Markovian relationship is estimated for a 1954 through 1984 time period using data from Illinois Agricultural Statistics, Gallager and Green, and the Illinois Farm Bureau Farm Management record-keeping service. Ordinary least squares parameter estimates and standard errors (in parentheses) are:

$$\ln(DR_t) = \underset{(.4196)}{1.0330} + \underset{(.0795)}{.79962}*\ln(DR_{t-1}) \qquad (3)$$

where ln is an natural logarithmic operator. This equation has 27 degrees of freedom, a standard error of estimate of .1323 and an adjusted R-square of .8134. A dummy for 1972 and 1973 is included because residuals associated with these years are outliers.

Equation (2-b) gives the dynamic movements of expected farmland price. Burt developed the theoretical foundation for this equation by using lagged returns as measures of expected returns. The expected returns are discounted in a capitalization formula to arrive at the farmland price. Parameter estimates and standard errors (in parentheses) for this equation using direct returns as a measure of residual returns and a farmland price series constructed by Reiss and Scott (modified by Burt) are:

$$\begin{aligned}\ln(P_t) = {} & \underset{(.0168)}{.2302} + \underset{(.0167)}{.0503}*\ln(DR_t) + \underset{(.0231)}{.0934}*\ln(DR_{t-1}) \\ & + \underset{(.0269)}{1.6921}*E(\ln(P_{t-1})) - \underset{(.0225)}{.8208}*E(\ln(P_{t-2})) + \underset{(.2900)}{.8000}*MA \end{aligned} \qquad (4)$$

where MA is a moving average error component (see Burt, et. al. for a discussion of the estimating procedure). The coefficient on the MA term is fixed at .8000 due to the upward bias of maximum likelihood estimates of this parameter (Sargan and Bhargava). This equation has 18 degrees of freedom, a .0215 standard error of estimate, and a .9957 adjusted R-square.

To include the price equation in the DP model, the coefficients associated with the two direct returns are added together and treated as one variable. This is done to avoid specifying a lagged direct return state variable (farmland price predictions are not seriously affected). Also, the price equation is treated as a deterministic relationship. Significant biases should not be introduced into numerical results because the standard

error of (4) is low relative to the size of the land variable's state increments.

Equation (2-c) gives the changes in the number of acres owned based on the farmland purchase (sell) decision at the beginning of the year. Positive XOA_t values indicate purchases and negative values indicate sales.

Movements of the debt-to-farm-asset ratio over time are given by equation (2-d). This ratio differs slightly from the typical debt-to-asset ratio. Positive values of DFA_t equal debt over total assets--the typical debt-to-assets measure, while negative values represent positive holdings of financial instruments. The dominator in (2-d) gives the holdings of farm assets at the end of the year, where the function OFA(·) relates other farm assets to the number of acres farmed. (Estimates of this relationship suggest a linear function where other farm assets equal $320 per acre of owned land.) The bracketed term in the numerator gives the value of assets at the beginning of the year, while the $FLOW_t$ represents the changes in assets during the year.

The variable, $FLOW_t$, contains four components: investment (disinvestment), before-tax income, taxes, and withdrawals. Investment (disinvestment) results from farmland purchases (sales) and equals the value of farmland purchased (sold), changes in other farm asset investment, and transaction costs consisting of a three percent brokerage fee on farmland sales or an one percent surcharge on the acquisition of new debt. Before-tax income equals direct returns time the number of acres farmed, minus fixed costs ($54 per acre), plus (minus) any interest income (costs). Real interest rates as of 1986 on bank savings, production credit association loans, and federal land bank loans are used in the model. Taxes are

calculated based on the 1984 federal and Illinois tax codes. Withdrawals for family living equal $20,000 per year.

Comparisons of Three Purchase (Sell) Decision Models

Optimal decision rules -- the collection of optimal purchase and sell decisions for each state variable value -- are calculated for three different models. These models, and their abbreviations, are: (1) dynamic programming -- purchase and sell (DP-PS) allows purchase and sell decisions at the beginning of each year; (2) dynamic programming -- purchase only (DP-P) allows purchase decision at the beginning of each year and only allows sell decisions if the debt-to-farm-asset ratio exceeds .70; (3) capital budgeting -- purchase and sell (CB-PS) allows purchase and sell decisions only in the beginning year of the planning horizon.

The CB-PS model represents the traditional capital budgeting approach. In a capital budgeting approach, mutually exclusive investment alternatives are ranked according to their net present values (or equivalently terminal net values) generated by their periodic cash flows. The alternative with the highest net present value is selected as optimal (Barry, Hopkin, and Baker). For the CB-PS model, the ending expected net worth for each possible purchase and sell options is calculated using the state transition equations in (2-a) through (2-d) assuming that farm size remains constant after the first year in the planning horizon.

In order to numerically solve the DP models, the state variables are discretized. Farm sizes range in 80 acre increments from 500 acres to 1460 acres. A farm size of zero acres also is included to represent either farm bankruptcy or an optimal decision to liquidate the farm. The current price

of farmland ranges from $1000 to $3850 in equal $150 increments. The lagged price of land is allowed to be -$300, -$150, $0, +$150, and +$300 from the current price of land. Direct return values are $125, $150, $175, $205, and $245. Forty increments are associated with the debt-to-farm-assets variable, ranging in equal increments from 1 to -.5. This discretation results in 280,000 states. There are 14 decision variable possibilities, matching the number of possible farm sizes.

Optimal Decisions. For each model, yearly maximizations are performed backwards through time until the optimal decision rule converges to a steady state decision rule. All three models' optimal decision rules converge by the tenth stage.

Figure 1 shows portions of the three converged, optimal decision rules for a 740 acre farm having a debt-to-farm-asset ratio of .25. The x-axis of each panel shows the current price of farmland while the y-axis indicates the optimal number of acres to purchase or sell. The y-axis coordinate closest to the x-axis, labeled "Sell All", represents a decision to liquidate the farming operation. Each panel of the figure shows decisions for a given current direct returns level and a given change in farmland price. For example, panel A shows the optimal decision rules for a current direct return level of $125 and a current price minus lagged price of -$300 (i.e., the price of land in the current period is $300 below the price of farmland in the previous year). The favorableness of direct returns and price movements in each panel are arranged such that panel A represents extremely pessimistic conditions, panel C represents average conditions, and panel E represents extremely favorable conditions.

The DP-P rule lies on or below the DP-PS rule when the DP-PS rule indicates farmland purchases are advisable (see panels C, D, and E). This occurs because the DP-P model eliminates the ability to sell farmland, except under severe financial stress. Sales of farmland can provide expected funds for a farm facing financial adversity. When farmland sales are not considered, these funds are not available. Therefore, the farm must maintain lower debt-to-farm-asset ratios, thus resulting in smaller farmland purchases.

The CB-PS decision rule's range of optimal decisions tend to be larger than the DP-PS decision rule for a given return level and price movement. The CB-PS rule indicates larger purchases than the DP-PS rule at relatively low farmland prices. Then it crosses the DP-PS rule and indicates larger farmland sales at relatively higher farmland prices. This phenomenon is clearly illustrated in panels A, B, and C. In panels D and E the CB-PS rule does not have an opportunity to cross the DP-PS rule due to the upper constraint on farm size.

This phenomenon can be explained by viewing the decision rules shown in each panel as derived factor demand curves. Recall that short run derived demand curves have one or more fixed factors of production while a long run demand curve has no fixed production factors. Because of the fixed production factors, the slope of the short run demand curve is greater than the long run demand curve. In this analogy, the CB-PS rule is similar to a short run demand curve because all future purchase (sell) decisions are fixed at zero, and the DP-PS rule is similar to a long run demand curve because it does not have any fixed purchase (sell) decision levels.

Expected Net Worth Values After Ten Years. To compare the relative profitability of the three models, expected values of net worth after ten years are calculated for yearly applications of the converged decision rule. This is accomplished by constructing a transition matrix, associated with the converged decision rules, that gives the probability of moving from a current state to any of the other states in the next year. A given state is selected in year one and conditional probabilities of being in each state in year ten are calculated by ten postmultiplications of the transition matrix (see Howard for a discussion of this methodology). The probability of being in each state is then multiplied by the after-tax net wealth associated with each state to arrive at expected after-tax net wealth.

Note that this transition matrix application is an ex ante approach. Farmland direct returns and prices are being predicted into the future based on the past history of returns and price movements and the selected current return and price levels. In this application all three models use the same direct return and price expectations.

The values of each models' ending net worth are calculated for a beginning farm size and debt-to-farm-asset ratio of 660 acres and .25, respectively. These values are reported in Table 1 for four different sets of direct returns and current and lagged farmland price levels. The first three sets are for specific returns and price levels which represent pessimistic, average, and optimistic conditions. The last set is a composite of all possible combinations of farmland direct returns and prices. In this case, each possible direct return and farmland price combination is given an equal weight in the beginning year.

In all cases, the expected net worth associated with the DP-PS decision rule is significantly higher than the other two decision rules. Differences between the DP-PS and the DP-P and CB-PS rules for the composite case are $582,826 and $198,978, respectively. On a yearly basis, these differences suggest that the DP-PS outperforms the DP-P and CB-PS by $52,282 and $19,897, respectively.

Conclusions

Significant reductions in ending net worth are incurred when a farmer only considers selling farmland at high debt-to-farm-asset ratios. Optimal purchase decisions are less for farmers who do not sell farmland than for those farmers who do consider selling farmland.

Capital budgeting frameworks which do not consider future optimal decisions, the most common framework for farmland analysis, give substantially suboptimal purchase and sell decisions. This judgement is based on the lower expected values of ending net worth of the CB-PS model as compared to the DP-PS model. Thus, frameworks used to analyze purchase and sell decisions should consider future farmland purchase and sell decisions.

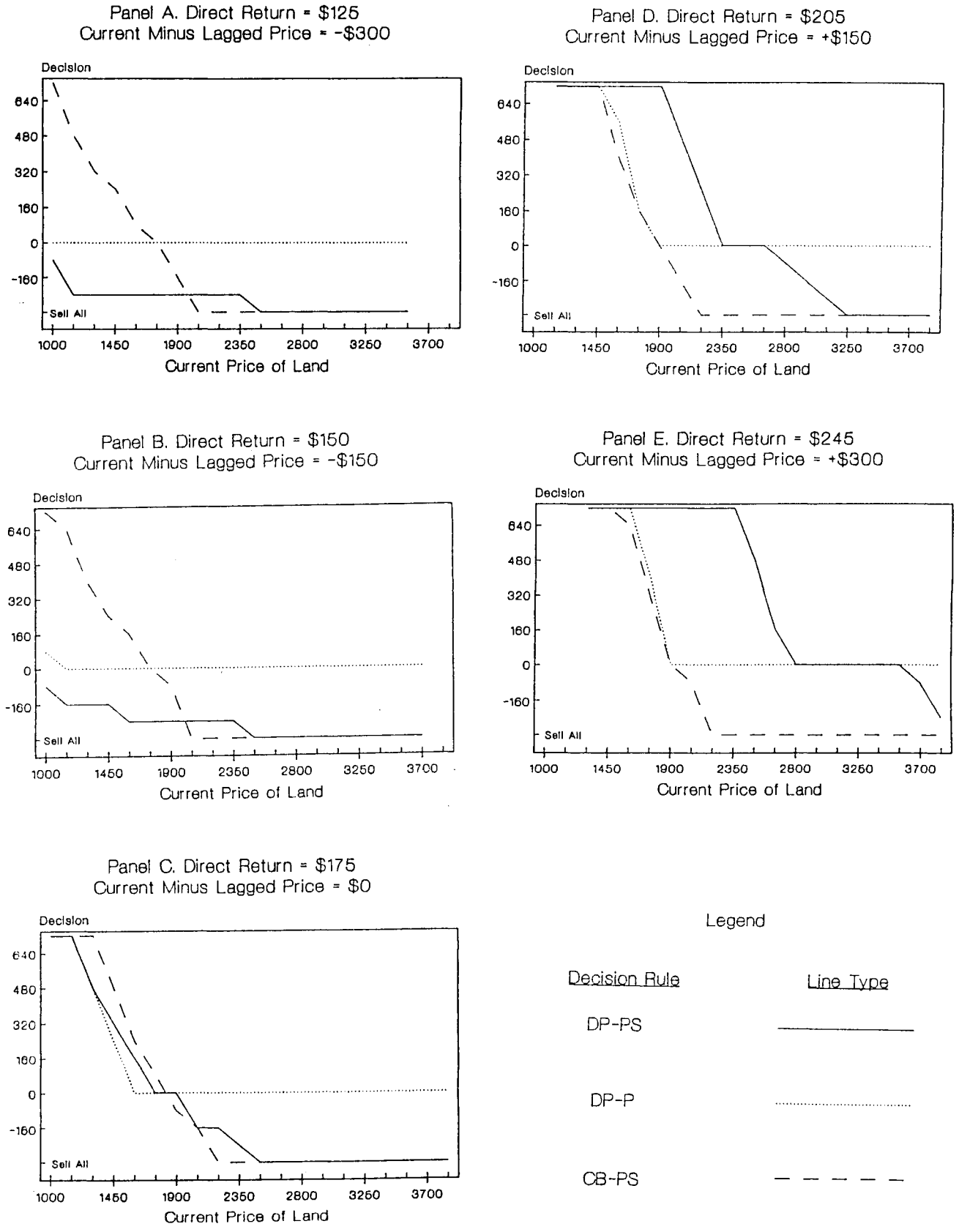

Figure 1. Comparison of Decision Rules From Three Farmland Puchase (Sell) Models.

Table 1. Expected Net Worth Values After Ten Years for Differing Farmland Direct Return and Price Combinations Using Three Farmland Purchase (Sell) Decision Models.

Return and Price Level	DP-PS	DP-P	CB-PS
	------ $ Expected Ending Net Worth ----		
Pessimistic Direct Return = $150 Current Price = $2,350 Lagged Price = $2,650	1,744,193	1,469,688	1,506,681
Average Direct Return = $175 Current Price = $1,750 Lagged Price = $1,750	1,883,598	1,680,327	1,584.587
Optimistic Direct Return = $205 Current Price = $1,450 Lagged Price = $1,300	2,630,270	2,072,207	2,050,771
Composite	2,151,766	1,568,940	1,952,788

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