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A PROBABILISTIC DECISION MODEL FOR AGRICULTURAL PEST MANAGEMENT:
AN APPLICATION TO THE SOYBEAN CYST NEMATODE

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Abstract

A Markov decision model is used to determine optimal management strategies for controlling an agricultural pest. The management options evaluated include the use of resistant varieties, nonhost crops, and chemical control. Decision rules were sensitive to expected product prices and yield of nonhost crop.

Keywords: Agricultural Production Planning, Markov Decision Process, Agricultural Pest Management

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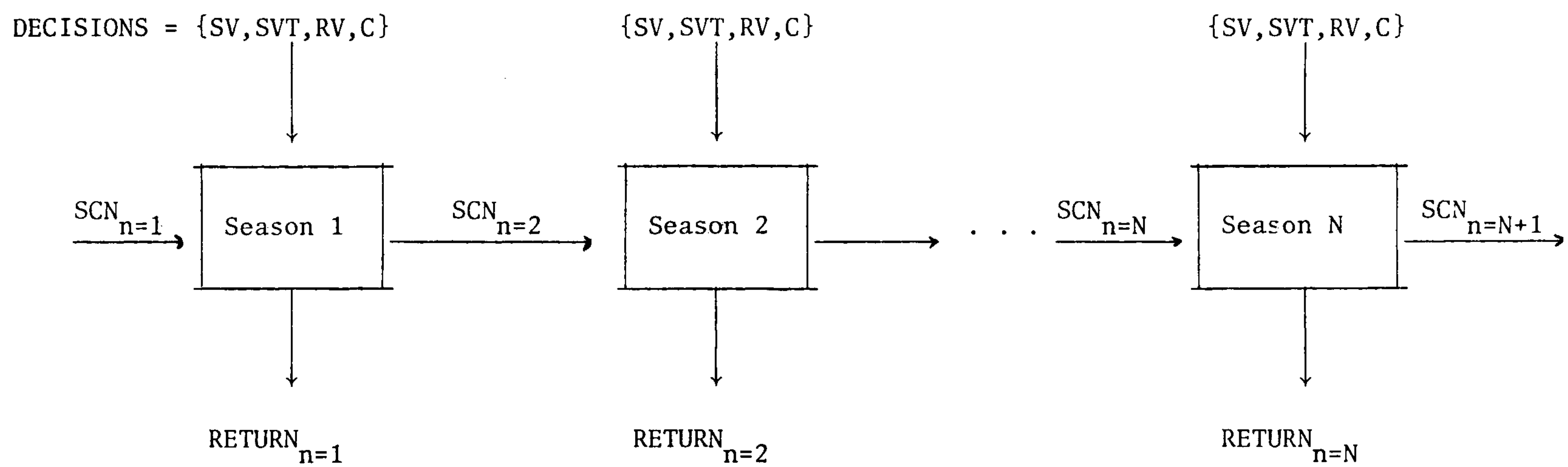

Figure 1. Block Diagram Representation of a Multi-Season Control Process

$$X_h = g(X_p, d_p, e_h) \qquad (1)$$

where X_h, the level of infestation at harvest, is a function of X_p and d_p the infestation level and decision occurring at planting, respectively. In Illinois the fall temperatures are too cool to allow further hatch of the SCN, thus the life cycle is terminated for the season with the infestation at harvest providing a good estimate of preplant density in the upcoming season. e_h is a stochastic disturbance term which represents the factors that we are unable to incorporate into our model given our present understanding of the system. More will be said about the statistical properties of the disturbance term in later sections.

The data required to estimate Eq. (1) were obtained from SCN population dynamics experiments performed by members of the Department of Plant Pathology at the University of Illinois. By combining the results from two experimental plots, fourteen observations relating preplant infestation to infestation level at harvest were available for each of the three soybean management alternatives. Statistical results using ordinary least squares (OLS) are reported below.

Intuitively, we expect that a zero level of SCN at planting would be associated with a zero level at harvest. Thus, the regressions relating SCN at harvest to preplant density were fit through the origin. For the no treatment alternative, the data indicated that SCN counts at harvest reached a maximum, then begin to decline at higher levels of preplant infestation. A quadratic equation was used to characterize this behavior

$$X_h = \underset{(0.91)}{3.0343}X_p - \underset{(-1.54)}{0.0682}\,X_p^2 \qquad (2)$$

where X_h and X_p are cyst counts per 250 cc^3 of soil at harvest and at planting, respectively. Coefficients in parentheses are computed t-values.

Under the chemical control alternative a simple linear equation appeared to fit the data quite well.

$$X_h = \underset{(3.79)}{1.2465}\, X_p \; . \tag{3}$$

Lastly, a semi-logarithmic form was found to perform well for the resistant variety option.

$$X_h = \underset{(2.63)}{3.2717}\, \ln\,(X_p) \; . \tag{4}$$

Unfortunately, the data set did not contain observations relating SCN density to crop rotation. However, the University of Illinois Cooperative Extension Service reported that when corn is used as a nonhost crop for SCN control, the population is reduced by 50 to 90 percent (Edwards et al. 1982). The midpoint of this range, 0.70, provides an estimate of the rotation effect.

The Influence of SCN on Soybean Yield

Since the management alternatives we are considering are available only at the beginning of the season, it seems reasonable that an estimate of crop yield as a function preplant infestation level would be extremely useful in determining the optimal decision. Regression analysis of the yield data obtained from the SCN-population experiments did not indicate a statistically significant relationship between preplant infestation and commercial soybean yield due to the low densities observed, however, data from other experimental plots in which densities were higher do indicate significant yield effects when susceptible varieties are grown. Using a semi-logarithmic transformation, the following results were obtained for the treated and untreated susceptible variety alternatives, respectively,

$$Y = \underset{(23.53)}{57.571} - \underset{(-5.89)}{3.698} \ln (X_p) \qquad (5)$$

$$Y = \underset{(19.02)}{45.031} - \underset{(-6.31)}{4.293} \ln (X_p) \qquad (6)$$

where Y is soybean yield and X_p is the preplant infestation level of SCN. Since the available data did not indicate a yield effect for the resistant variety alternative, a mean yield of 33.91 bushels per acre computed from the experimental data was used as an estimate of expected soybean yield when planting resistant varieties.

The Economic Optimization Model

Having developed the biological component of the system we are now in a position to discuss the economic model and its optimization. Let us initially consider the deterministic net returns realized at any stage in the planning horizon. These net returns are functions of product prices, crop yields, and production costs. Keeping in mind that soybean yield is a function of both the numbers of SCN present and the decision at planting we can write the following net return function for the three soybean alternatives.

$$= \delta^{*} \, (PSB^{*}Y(X_p, d_p) - c(d_p)) \qquad (7)$$

For corn, the net return function is simply

$$\pi = \delta^* \ (PSB^*YC - c\ (C)) \tag{8}$$

where δ is the discount factor. PSB and PC are the product prices of soybeans and corn, respectively. Similarly, Y and YC are the yields of soybeans and corn, respectively. The function $c(\cdot)$ denotes the respective non-land variable cost for each alternative. $c(\cdot)$ will reflect any differences due to crop selection, pesticide application, and costs associated with susceptible or resistant soybean seed.

Our interest is in the inter-temporal solution which is characterized by the population dynamics of the SCN. Given the discrete nature of the decision structure, it is convenient to view the management problem as a sequential Markov decision process solvable via dynamic programming. Within the Markov decision framework, a particular level of infestation is defined to be a state-of-the-system. Thus, Eq. (1) can be thought of as the state transformation function relating changes in the level of SCN to a particular decision and stochastic disturbance.

For purposes of solution, the transformation function is mathematically described in terms of discrete conditional probabilities. The fundamental dynamic programming recurrence relation for this optimization is written as follows

$$V_i(n) = \max_{d^k} \pi_i^k + \delta \sum_{i=1}^{M} p_{ij}^k \ V_j(n-1) \ . \tag{9}$$

Defining terms, we have $V_i(n)$ as the total expected return resulting from an n-stage process for i = 1,...,M discrete levels of the state variable. We define n as the number of stages remaining in the process. For large n we expect the optimal policy to converge, thereby providing the optimal decision rule for any state of the process regardless of stage. k is the subscript denoting the decision variables d, where k = 1,2,3,4 for this application and δ has been previously defined as the discount factor. The p_{ij}^{k} are the conditional Markovian transition probabilities. For a given decision k and given state i, p_{ij}^{k} is the probability that the state of the process will transit to state j in a specified time interval. Further discussion of the transition probabilities and their estimation is reserved for a later section.

The Role of Product Prices in a Pest Management Model--A Digression

At any stage in the decision-making process expectations concerning product prices are an important feature of the management system. In an economic setting, information is gathered and decisions are based on the relative profitability or the utility associated with the relative profitability for each of the available management alternatives. Since the traditional economic threshold commonly encountered in the pest management literature is price dependent it seems reasonable to incorporate a price expectation or forecast mechanism directly into our model, especially since the decision rules we wish to derive are inter-temporal and involve the choice between multiple products. Little is known about the process by which farmers formulate expectations even though it is widely recognized that price expectations are an important component of economic behavior models (Fisher and Tanner 1978). The question then becomes, what is an adequate formulation of expectations in an applied model? Two criteria are advanced here. First, to be viable, the expectations model or mechanism must possess some predictive ability, and second, in order for the overall pest management model to be operational, the mechanism should be as simple as possible.

One possibility for a price expectation mechanism is use of the futures market. Several agricultural economists have examined the explanatory role of future market prices and found that the futures prices for storable commodities perform quite well in relation to either sophisticated expectations mechanisms, such as the adaptive expectation and partial adjustment models developed by Nerlove (1958) or large scale econometric models (Gardner 1976, Just and Rausser 1981). The literature suggests that future prices provide an unbiased point estimate of the subsequent spot price.

Some of the early empirical work in this area was done by Tomek and Gray (1970). In their article, it was argued that for commodities such as corn and soybeans in which continuous inventories are maintained "the spring-time price of the harvest-time contract is, . . . , a reasonable forecast of the subsequent harvest-time price." The authors then tested the following linear statistical models

$$HTPC = a + b * STHC + u \tag{10}$$

$$HTPSB = a + b * STHSB + u \tag{11}$$

where HTPC and HTPSB are the closing prices on expiration date of corn and soybeans for the December and November futures contracts, respectively. STHC and STHSB are the closing corn and soybean prices for the December and November contracts on the preceeding April 30, respectively. The variable u is a stochastic disturbance term.

Conceptually, if the intercept term, "a", is not significantly different from zero and the slope term, "b", is not significantly different from 1, the expectation or forecast is self-fulfilled. In their analysis, Tomek and Gray estimated Eqs. (10) and (11) and obtained meaningful results. Based on their results and the findings of other studies, it seems that the Tomek and Gray specification would be appropriate for our purposes here. The same equations were re-estimated using data for the period 1969-1982. A discussion of the analysis is presented below.

Statistical results for the corn and soybean price equations are shown in Table 1. Both intercept terms were not significantly different from zero using a standard hypothesis test. Significance of the slope term was determined using a linear restrictions test under the null hypothesis that the slope was not significantly different from one (Johnston 1972). The results support the Tomek and Gray hypothesis, further still, the price equations fulfill the criteria established earlier for an expectations formulation. Statistically, the equations perform well. In addition, the functional form is quite simple. The reader will note, however, that the above relation is non-Markovian, this, of course, has implications for purposes of solution.

To determine the optimal set of decision rules using the future price specification the following optimization problem will be solved. First, the long run average spring-time futures price can be used as a deterministic value in a multi-stage model with SCN infestation as the only probabilistic element. Upon convergence, the total expected return from the multi-stage optimization is used as a terminal value for the one-stage problem. The one-stage problem is then solved for various combinations of corn and soybean prices. Thus, the decision rule is conditional upon product prices even though the expectations mechanism is non-Markovian. An example of this type

Table 1 Statistical Results for the Corn and Soybean Price Equations

	Intercept	Slope	$\hat{\sigma}^2$	R^2	F
Corn					
$df^c = 12$	0.7776	0.6659	0.3344	0.47	10.69
	(1.59)[a]	(-1.64)[b]			
Soybeans					
$df^c = 12$	1.3194	0.7947	1.612	0.60	18.05
	(1.27)[a]	(-1.10)[b]			

[a] Computed t value under the null hypothesis that the intercept is not significantly different than zero. The levels of significance were approximately 14 percent and 22 percent for corn and soybeans, respectively.

[b] Computed t value under the null hypothesis that the intercept is not significantly different than one. Levels of significance were approximately 11 percent and 30 percent for corn and soybeans, respectively.

[c] Degrees of freedom on 14 observations for the years 1969 through 1982.

of problem formulation is found in Burt and Allison (1963). In terms of computational burden, the above formulation is quite advantageous. If the price relationships were Markovian, two additional state variables would be required for the multi-stage problem. Also, since expectations enter the objective function linearly, and the decision rule is one-stage, we need not be concerned with the bivariate probability distribution of product prices.

An Illustrative Example

To illustrate the operation of the model, a numerical example is presented in this section. In our model, the stage of the process is defined to be a single growing season for either corn or soybeans. At the beginning of the season the preplant infestation level of the SCN is known. With this information, expectations for the current crop yield and the infestation level in the next crop season can be formulated. Let us designate four discrete state variable levels of preplant infestation levels as low, moderate, severe, and very severe indexed i = 1,...,4, respectively. Expected yields and net returns for each management alternative and each state level are given below.

Chemical Control Alternative (Susceptible Variety)

Infestation Level (i)	Expected Yield Y(i,SVT)	Expected Return (π_i^{SVT})
Low (1)	40.0	135.8
Moderate (2)	36.0	111.8
Severe (3)	33.0	93.8
Very Severe (4)	30.0	75.8

Resistant Variety Alternative

Infestation Level (i)	Expected Yield Y(i,RV)	Expected Return (π_i^{RV})
Low (1)	34.0	129.8
Moderate (2)	31.5	114.8
Severe (3)	29.0	99.8
Very Severe (4)	28.0	93.8

In this example it is assumed that susceptible varieties are higher yielding, yet at higher infestations the expense of nematicides favors the use of resistant varieties. Since corn is a nonhost, its yield and return are independent of infestation level. A net return of $100.2 for corn is used in this example. The transition matrices under each alternative are as follows:

Chemical Control Alternative

State i/j	1	2	3	4
1	0.9674	0.0326	0.0	0.0
2	0.8095	0.1905	0.0	0.0
3	0.6079	0.3921	0.0	0.0
4	0.5146	0.4854	0.0	0.0

Resistant Variety Alternative

State i/j	1	2	3	4
1	0.991	0.009	0.0	0.0
2	0.4956	0.5044	0.0	0.0
3	0.0	0.8324	0.1676	0.0
4	0.0	0.3902	0.6098	0.0

Nonhost Crop Alternative

State i/j	1	2	3	4
1	1.0	0.0	0.0	0.0
2	0.521	0.479	0.0	0.0
3	0.479	0.4272	0.0938	0.0
4	0.0938	0.4272	0.479	0.0

Using a discount rate of 0.03 which is equivalent to a δ of 0.97 and assuming the terminal values, the $V_i(0)$, to be zero, we can write the recurrence relation occurring at each state level for the first stage.

$$V_1(1) = \max_{\{SVT,RV,C\}} [(\pi_1^{SVT} + \delta^* \sum^M p_{ij}^{SVT} V_j(0)), (\pi_1^{RV} + \delta^* \sum^M p_{ij}^{RV} V_j(0)),$$

$$(\pi_1^C + \delta \sum^M p_{ij}^C V_j(0))]$$

$$= \max[(135.8 + 0.0), (129.8 + 0.0), (100.2 + 0.0)]$$

$$= 135.8; \text{ for which the optimal decision is } d_1^* (1) = SVT.$$

$$V_2(1) = \max[(\pi_2^{SVT} + \delta^* \sum^M p_{2j}^{SVT} V_j(0)), (\pi_2^{RV} + \delta^* \sum^M p_{2j}^{RV} V_j(0)),$$

$$(\pi_2^C + \delta \sum^M p_{2j}^C V_j(0))]$$

$$= \max[(111.8 + 0.0), (114.8 + 0.0), (100.2 + 0.0)]$$

$$= 114.8; \text{ for which the optimal decision is } d_2^* (1) = RV.$$

$$V_3(1) = \max[(\pi_3^{SVT} + \delta^* \sum^{M} p_{3j}^{SVT} V_j(0)), (\pi_3^{RV} + \delta^* \sum^{M} p_{3j}^{RV} V_j(0)),$$

$$(\pi_3^{C} + \delta^* \sum^{M} p_{3j}^{C} V_j(0))]$$

$$= \max[(93.8 + 0.0), (99.8 + 0.0), (100.2 + 0.0)]$$

= 100.2; for which the optimal decision is $d_3^*(1) = C$.

$$V_4(1) = \max[(\pi_4^{SVT} + \delta^* \sum^{M} p_{4j}^{SVT} V_j(0)), (\pi_4^{RV} + \delta^* \sum^{M} p_{4j}^{RV} V_j(0)),$$

$$(\pi_4^{C} + \delta \sum^{M} p_{4j}^{C} V_j (0))]$$

$$= \max [(75.8 + 0.0), (93.8 + 0.0), (100.2 + 0.0)]$$

= 100.2; for which the optimal decision is $d_4^*(1) = C$.

Summarizing the results for Stage 1:

$V_1(1) = 135.8$, $V_2(1) = 114.8$, $V_3(1) = 100.2$, $V_4(1) = 100.2$

Second Stage Recurrence Relation for State 1

$$V_1(2) = \max[(\pi_1^{SVT} + \delta^* \sum^{M} p_{1j}^{SVT} V_j(1)), (\pi_1^{RV} + \delta^* \sum^{M} p_{1j}^{RV} V_j(1)),$$

$$(\pi_1^{C} + \delta^* \sum^{M} p_{1j}^{C} V_j (1))]$$

$$= \max[(\pi_1^{SVT} + \delta*(p_{11}^{SVT} V_1(1) + p_{12}^{SVT} V_2(1) +$$

$$p_{13}^{SVT} V_3(1) + p_{14}^{SVT} V_4(1))), (\pi_1^{RV} + \delta*(p_{11}^{RV} V_1(1) +$$

$$p_{12}^{RV} V_2(1) + p_{13}^{RV} V_3(1) + p_{14}^{RV} V_4(1)),$$

$$(\pi_1^{C} + \delta* (p_{11}^{C} V_1(1) + p_{12}^{C} V_2(1) + p_{13}^{C} V_3(1) + p_{14}^{C} V_4(1)))]$$

$$= \max[(135.8 + (0.97)*(0.9674* V_1(1) + 0.0326* V_2(1) +$$

$$0.0* V_3(1) + 0.0* V_4(1))), (129.8 + (0.97)*(0.991* V_1(1) +$$

$$0.09* V_2(1) + 0.0* V_3(1) + 0.0* V_4(1)),$$

$$(100.2 + (0.97)*(1.0* V_1(1) + 0.0* V_2(1) + 0.0* V_3(1) +$$

$$0.0* V_4(1)))]$$

$$= \max[267.66, 261.48, 232.06]$$

$= 267.66$; for which the optimal decision is $d_1^*(2) = SVT$.

Stage 2; State 2

$$V_2(2) = \max[(\pi_2^{SVT} + \delta* \sum^{M} p_{2j}^{SVT} V_j(1)), (\pi_2^{RV} + \delta* \sum^{M} p_{2j}^{RV} V_j(1)),$$

$$(\pi_2^{SVT} + \delta^* \sum^{M} p_{2j}^{C} V_j (1))]$$

$$= \max[(\pi_2^{SVT} + \delta^* (p_{21}^{SVT} V_1(1) + p_{22}^{SVT} V_2(1) + p_{23}^{SVT} V_3(1) +$$

$$p_{24}^{SVT} V_4(1))), (\pi_2^{RV} + \delta^* (p_{21}^{RV} V_1(1) + p_{22}^{RV} V_2(1) +$$

$$p_{23}^{RV} V_3 (1) + p_{24}^{RV} V_4(1))), (\pi_2^{C} + \delta^* (p_{21}^{C} V_1(1) +$$

$$p_{22}^{C} V_2(1) + p_{23}^{C} V_3(1) + p_{24}^{C} V_4(1)))]$$

$$= \max[(111.8 + (0.97)*(0.8095* V_1(1) + 0.1905* V_2(1) +$$

$$0.0* V_3(1) + 0.0* V_4(1))), (114.8 + (0.97)*(0.4956* V_1(1) +$$

$$0.5044* V_2(1) + 0.0* V_3(1) + 0.0* V_4(1))),$$

$$(100.2 + (0.97)*(0.5210* V_1(1) + 0.479* V_2(1) +$$

$$0.0* V_3(1) + 0.0* V_4(1)))]$$

$$= \max[239.78, 236.25, 222.17]$$

$= 239.78$; for which the optimal decision is $d_2^*(2) = SVT$.

Stage 2; States 3 and 4

$V_3(2) = 220.11;\ d_3^*(2) = C$

$V_4(2) = 206.77;\ d_4^*(2) = C$.

Stage 3; All States

$V_1(3) = 394.19;\ d_1^*(3) = SVT$.

$V_2(3) = 366.02;\ d_2^*(3) = SVT$.

$V_3(3) = 343.89;\ d_3^*(3) = C$.

$V_4(3) = 326.35;\ d_4^*(4) = C$.

Thus, for one stage the optimal policy is chemical control when infestations are low, and at moderate levels select the resistant variety. As infestations becomes severe, farmers should rotate to corn. Solving for the next two stages we find that the resistant variety is relatively less profitable over time at moderate infestation levels than the option to use chemical control. For stages 2 and 3 the optimal policy was to use chemical control at low and moderate infestations while higher infestations required rotation to the nonhost crop. Inspection of the second row of the transition matrices for the resistant variety and chemical control options reveals why this is indeed the case. If the process is in state 2 the probability of transiting to state 1 under chemical control is quite high, 0.8095, relative to the 0.4956 probability associated with the use of the resistant variety.

The example just shown was purely hypothetical and served only to illustrate to the reader the structure of the model and the optimization procedure. In a later section, an empirical application of the model involving more state variable designations and an additional decision variable will be presented. Numerical solution of this larger problem is obtained by computer.

III. ESTIMATION OF PROBABILISTIC ELEMENTS

The transition probabilities referred to earlier are a key feature of our decision model. In this section a method for systematically estimating these elements will be presented. The method is general in its approach, however, only a special case will be shown here since our primary focus in this paper is the overall modeling effort.

Mathematically, we are interested in the cummulative distribution function (cdf) of the preplant infestation level in the current season which is dependent upon the infestation and decision taken in the previous season. Since the decision taken is deterministic we can ignore its presence in the discussion which follows. For a first order Markov system, the level of SCN infestation, X_h, is a random variable which takes on discrete values as the production process transits from state to state. Following the notation of Lee et al. we can write.

$$Pr\ (X_h = X_j | X_p = X_i) = p_{ij}(t) = p_{ij} \text{ for all } t. \qquad (12)$$

The above probability statement tells us that the value of the random variable, X_h, is conditional only on its value in the preceeding time interval; further, this conditional probability is constant across all time intervals. For the probability statement to be valid, we require

$$0 < p_{ij} < 1.0$$

and

$$\sum_{j=1}^{M} p_{ij} = 1.0 \ .$$

Anderson and Goodman (1957) have shown that the maximum likelihood (ML) estimator of p_{ij} is

$$\hat{p}_{ij} = x_{ij} / \sum_{i=1}^{M} x_{ij} \tag{13}$$

where the right hand side of (13) is the sample frequency of going from i to j for all states i = 1,..., M. In certain applications to decision analysis the Anderson and Goodman estimator is undesirable since the data required to estimate equation (13) may be few relative to the number of state variable designations one might wish to specify. The estimated transition probabilities would be "rough" as indicated by a substantial number of empty cells and trapping states in the transition matrices.

Taylor (1981) has recently proposed an alternative procedure for estimating the probability parameters of Markov-type decision processes which yields a continuous cdf and is readily applicable for computer-based models. The method requires an hyperbolic trigonometric (HT) transformation of the data and allows the researcher to statistically estimate the cdf directly using either OLS or ML. The HT transformation is sufficiently flexible so that the more common theoretical distributions can also be estimated Additionally, the procedure constrains values to fall within the range zero-one.

For the problem of determining the probabilistic movement of SCN infestations consider the following transformation.

$$F(X_h | X_p) = 0.5 + 0.5 \tanh [G(X_h, X_p)] \tag{14}$$

where F(•) is a continuous cdf, tanh is an hyberbolic tangent transformation, and G(•) is an implicit function. Two approaches for estimating the parameters of F(•) have been suggested Taylor (1983)[1]. The first is to estimate (14) directy[2]. Direct OLS estimation for the susceptible variety alternative resulted in the following cdf

$$F(X_h|X_p) = 0.5 + 0.5 \tanh [\underset{(-0.29)}{-1.1709} + \underset{(0.31)}{0.0065} X_h \tag{15}$$

$$+ \underset{(1.39)}{1.078\text{ E-7}} (X_h \cdot X_p)^2 - \underset{(-1.29)}{4.928\text{ E-6}} X_p X_h^2 + \underset{(1.06)}{1.53\text{ E-7}} X_h^3].$$

The second approach which is shown here is somewhat simpler and applies the state transformation equations estimated in Section II. To simplify matters it was assumed that the stochastic disturbances were normally distributed. Using the estimated equation which relates SCN at planting to SCN at harvest for the chemical control alternative we can write the following conditional probability density function (pdf).

$$f(X_h|X_p) = \frac{1}{\sigma_e\sqrt{2\pi}} \exp - 1/2 \left[\frac{X_h - \hat{\alpha}X_p}{\sigma_e}\right]^2 \tag{16}$$

where $\hat{\alpha}$ is the estimated coefficient of the regression equation and an estimate of σ_e is given by $\hat{\sigma}_e$. The standardized error is:

$$Z = \frac{X_h - \hat{\alpha}X_p}{\hat{\sigma}_e} . \tag{17}$$

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1 Personal Communication, September 30, 1983.

2 A full description of the direct estimation procedure is somewhat tedious and inappropriate here. The interested reader should see Taylor (1981).

Since a particular state designation for a transitional probability (p_{ij}) is merely a range of values, upper and lower limits of the standardized error can be determined in order to specify the upper and lower limits for the range of the future state, state j, given the current state, state i. This can be done by computing two values for the standardized error.

$$ZU = (X_h(U) - \hat{\alpha}\, X_p)/\hat{\sigma}_e \qquad (18)$$

$$ZL = (X_h(L) - \hat{\alpha}\, X_p)/\hat{\sigma}_e \qquad (19)$$

where U and L respectively denote the upper and lower limits of the future state as determined by the researcher and X_p is set equal to some measure of central tendency for the range of the current state, usually the midpoint.

An hyperbolic tangent transformation which approximates the normal cdf (Taylor, 1981) is $F(Z) = 0.5 + 0.5 \tanh (0.7971*Z + 0.0371*Z^3 - 0.003923*Z^5$. Hence, the cumulative distribution values for ZU and ZL are

$$F(ZU) = 0.5 + 0.5 \tanh(0.7971*ZU+0.0371*ZU^3- 0.003923*ZU^5) \qquad (20)$$

$$F(ZL) = 0.5+0.5 \tanh(0.7971*ZL+0.0371*ZL^3-0.003923*Z^5) \qquad (21)$$

where ZU and ZL are the respective upper and lower values of the approximated cdf. For any p_{ij}, ZU represents the upper bound on the cdf for state j given state i. Likewise, ZL represents the lower bound on the cdf for state j given state i. Since p_{ij} is simply the area under the normal curve,

$$p_{ij} = F(ZU) - F(ZL). \qquad (22)$$

Sensitivity of the optimal decision with respect to the estimated transition probabilities can be determined by altering (i) the upper and lower limits of the state variable designations; (ii) the estimate of the standard deviation used to normalize the disturbance terms; or (iii) by respecifying the form of the cdf. The above procedure is easily implemented for computer applications in which a nontrivial number of state variable designations is desired. Transition matrices for each alternative at ten discrete levels of the state variable are presented in the Appendix. The reader should be warned that the data used in estimating the parameters of the transition matrices are quite limited and do not encompass the entire range of possible SCN infestation levels. Also, the assumption of normality for the SVT, RV, and C alternatives may or may not be appropriate. For these reasons, the distributions depicted should not be considered as an adequate description of the actual process, but rather an initial attempt to model the system.

IV. THE ANALYSIS

Two sets of results are presented in this section to illustrate the applicability of the model in analyzing the SCN control process. Because the confidence intervals encompassing certain estimates of parameter values used in the model are quite wide, the extension of these results to actual farm practice would be premature. As more and better data become available, the estimation and optimization procedures can be re-performed to generate results with narrower confidence intervals.

Nature of the Decision Rule

The optimal solution is found by solving Eq. (9) for a relatively large number of stages, sufficiently large such that convergence in policy space is obtained[3]. That is, upon convergence, the selection of an alternative does not change across stages for any given state of the system. For a discounted process as we have described here the value of $V_i(n)$ approaches a finite number as $n \to \infty$. Moreover, for a stochastic multistage process, the decision rule is a single stage rule and is dependent upon both the state of the process and the outcome associated with the random variable.

Results

Table 2 contains the numerical values of the coefficients used in the model. Generally speaking, these coefficients assume values which are representative of farming operations located in Southern Illinois. However, a brief statement concerning product prices is in order.

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[3] A computer code developed at Montana State University was used in calculating the numerical solutions reported in this section.

Table 2 Numerical Values of the Coefficients Used in the Analysis

Variable	Numerical Value
Price of Soybeams (dollars per bushel)	5.24
Nonland Variable Production Costs for Soybeans (dollars per acre)	75.00
Nematicide Costs (dollars per acre)	30.00
Differential Cost Increase for Using SCN-Resistant Soybean Seed (dollars per acre)	5.00
Price of Corn (dollars per bushel)	2.28
Expected Yield of Corn (bushels pr acre)	105.00
Nonland Variable Production Costs for Corn (dollars per acre)	138.00
Discount Rate (net of inflation)	0.03

Since the estimated coefficients from Eqs. (10) and (11) were statistically insignificant, estimates of the expected product price for an infinite stage planning horizon were computed by taking the mean of the independent variable for the futures price equations (springtime futures price of harvesttime contract). Thus, the product prices depicted in the model represent the long run expected price the farmer would anticipate when using the futures market as a tool in decision making.

The values in Table 2 along with Eqs. (5) and (6) and the transition matrices (see Appendix) completely specify the model and allow us to determine an optimal strategy. An interesting aspect of Eqs. (5) and (6) is that for all levels of SCN infestation, the chemical control alternative always yields a higher level of return than nonchemical control when using fairly representative nematicide cost estimates. Although we might expect this to be the case at high levels of SCN infestation, this result is somewhat surprising at the lower levels. Hence, the nonchemical control alternative does not enter the optimal solution.

The resulting optimal decision rules for various nonhost crop yields are presented in Table 3. For the 95 bushel per acre level of corn the resistant variety alternative (RV) is more profitable at the higher levels of infestation; while at the 105 bushel per acre level the nonhost alternative, corn, should be used unless infestations are negligible. The decision rule is a bit more complex when the expected corn yield is 100 bushels per acre. At this yield level the net return associated with corn in $90.00, the net return of RV is $97.66. Inspection of the transition matrices (see Appendix) reveals that at intermediate levels of infestation the ability to suppress reproduction of SCN is greater for corn and offsets the net return differential. This is not the case at the higher state levels, a result which

Table 3 Optimal Decision Rules for Control of Soybeam Cyst Nematodes Under Alternative Assumptions Concerning the Expected Yield of the Nonhost Crop[1]

SCN Infestation Cysts per 250 cc[3]	Expected Corn Yield (bushels per acre) 95	100	105
0-5	SVT	SVT	SVT
6-10	SVT	SVT	SVT
11-15	SVT	SVT	C
16-20	SVT	C	C
21-25	RV	C	C
26-30	RV	RV	C
31-35	RV	RV	C
36-40	RV	RV	C
41-45	RV	RV	C
46-50	RV	RV	C

[1] Notation: SV--The decision to plant an SCN-suceptible soybean variety on untreated soil; SVT--The decision to plant a susceptible variety on nematicide treated soil; RV--the decision to plant an SCN-resistant soybean variety; C--the decision to plant corn, a nonhost crop.

conflicts with current thinking in SCN control and indicates an error in the present specification of the model. Two probable sources of error are (i) lack of sufficient experimental data to estimate the relationship between SCN and the nonhost crop and (ii) inappropriateness of the normality assumption.

Results for the one-stage optimization under alternative produce price specifications with expected corn yield at 105 bushels per acre are presented in Table 4. Price levels were arbitrarily selected from the futures price time series. As expected, only SVT and RV are optimal at a price ratio favorable to soybeans. Conversely, the model selects the rotation alternative when corn prices are high relative to soybeans.

Table 4 One Stage Optimal Decision Rules for Control of Soybean Cyst Nematode Under Alternative Product Price Specifications†

SCN Infestation		Range on Product Prices (dollars per bushel)		
	Corn	3.75	2.50	2.80
Cysts per 250 cc^3	Soybeans	8.20	7.10	6.75
(1) 0 - 5		SVT	SVT	SVT
(2) 6 - 10		C	SVT	SVT
(3) 11 - 15		C	SVT	SVT
(4) 16 - 20		C	SVT	C
(5) 21 - 25		C	SVT	C
(6) 26 - 30		C	SVT	C
(7) 31 - 31		C	RV C	
(8) 36 - 40		C	RV C	
(9) 41 - 45		C	RV C	
(10) 46 - 50		C	RV C	

† See Footnote Table 2. Corn yield assumed to be 105 bushels per acre.

V. SUMMARY

The model presented in this paper has potential application to a wide class of integrated pest management systems in field crop agriculture. Most notable is the model's ability to address the issues of rotation to a nonhost crop and the use of resistant varieties. With only slight modifications the optimal pesticide rate could also be determined within the model. This use of multiple controls in a dynamic and probabilistic setting is not encountered in most pest management studies (McCarl, 1981). Additionally, a relatively new procedure for estimating probability parameters has been implemented. Thus, the analysis shown here represents a unique contribution to the existing literature in terms of its application and modeling effort.

To be sure, the results presented here are tentative and reflect a narrow range of experimental conditions. In the future, more emphasis will be placed on statistical analysis of the available data and proper specification of the probabilistic elements in the model. Also, a thorough sensitivity analysis of the model must be performed, which may result in recommendations for further experimental work in areas revealed by the sensitivity analysis to be important. Finally, future efforts will also consider multiple pest infestations and the attitudes of farmers toward risk.

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Appendix Table 1 Transitional Probability Matrix for the SCN-Susceptible Soybean Variety Without Nematicide

SCN Infestation Cysts per 250 cc[3]		1	2	3	4	5	6	7	8	9	10
0 - 5	1	.093	.0055	.0057	.006	.0063	.0066	.0069	.0073	.0077	.8549
6 - 10	2	.093	.0053	.0053	.0053	.0053	.0053	.0054	.0054	.0054	.8643
11 - 15	3	.0929	.0051	.0049	.0048	.0046	.0045	.0043	.0041	.0039	.8708
16 - 20	4	.0929	.0050	.0047	.0045	.0042	.0048	.0037	.0034	.0031	.8746
21 - 25	5	.0929	.0049	.0047	.0044	.0041	.0038	.0035	.0031	.0028	.8759
26 - 30	6	.0929	.0051	.0049	.0045	.0042	.0039	.0036	.0033	.0030	.8749
31 - 35	7	.0929	.0051	.0049	.0048	.0046	.0044	.0042	.0040	.0038	.8714
36 - 40	8	.0930	.0052	.0052	.0052	.0052	.0052	.0052	.0052	.0052	.8653
41 - 45	9	.0930	.0055	.0057	.0059	.0061	.0064	.0067	.0070	.0074	.8563
46 - 50	10	.0931	.0058	.0063	.0068	.0073	.0081	.0087	.0095	.0104	.8440

Appendix Table 2. Transitional Probability Matrix for the SCN-Susceptible Soybean With Nematicide

SCN Infestation Cysts per 250 cc[3]		1	2	3	4	5	6	7	8	9	10
0 - 5	1	.5190	.0502	.0491	.0473	.0448	.0418	.0384	.0347	.0308	.1438
6 -10	2	.4561	.0505	.0504	.0495	.0478	.0455	.0426	.0393	.0356	.1828
11 -15	3	.3943	.0495	.0504	.0505	.0498	.0483	.0461	.0434	.0401	.2278
16 - 20	4	.3350	.0473	.0491	.0502	.0505	.0500	.0487	.0467	.0441	.2783
21 - 25	5	.2795	.0442	.0468	.0488	.0500	.0505	.0502	.0491	.0473	.3337
26 - 30	6	.2289	.0402	.0434	.0462	.0483	.0498	.0505	.0503	.0494	.3929
31 - 35	7	.1838	.0357	.0394	.0427	.0456	.0479	.04950	.0504	.0505	.4547
36 - 40	8	.1446	.0310	.0348	.0385	.0419	.0449	.0474	.0492	.0502	.5176
41 - 45	9	.1114	.0262	.0300	.0339	.0376	.0411	.0442	.0468	.0488	.5801
46 - 50	10	.0839	.0216	.252	.291	.0329	.0367	.0400	.0435	.0462	.6406

Appendix Table 3 Transitional Probability Matrix for an SCN-Resistant Soybean Variety

SCN Infestation Cysts per 250 cc^3		1	2	3	4	5	6	7	8	9	10
0 - 5	1	.6525	.2631	.0761	.0081	.0002	0.0	0.0	0.0	0.0	0.0
6 - 10	2	.3776	.3702	.2034	.0453	.0035	.0001	0.0	0.0	0.0	0.0
11 - 15	3	.2613	.3717	.2741	.0832	.0094	.0003	0.0	0.0	0.0	0.0
16 - 20	4	.1962	.3533	.3160	.117	.0168	.0007	0.0	0.0	0.0	0.0
21 - 25	5	.1546	.3309	.3419	.1465	.0248	.0013	0.0	0.0	0.0	0.0
26 - 30	6	.1258	.3087	.3579	.1723	.0032	.0021	0.0	0.0	0.0	0.0
31 - 35	7	.1048	.288	.3675	.1949	.0417	.0031	0.0	0.0	0.0	0.0
36 - 40	8	.0889	.2691	.3728	.2147	.0502	.0041	.0001	0.0	0.0	0.0
41 - 45	9	.0765	.252	.3752	.2322	.0587	.0053	.0001	0.0	0.0	0.0
46 - 50	10	.0667	.2365	.3754	.2477	.0669	.0066	.0002	0.0	0.0	0.0

Appendix Table 4 Transitional Probability Matrix for Corn

SCN Infestation Cysts per 250 cc[3]		1	2	3	4	5	6	7	8	9	10
0 - 5	1	.8023	.1656	.0300	.0021	.0001	0.0	0.0	0.0	0.0	0.0
6 - 10	2	.7087	.2307	.0552	.0052	.0002	0.0	0.0	0.0	0.0	0.0
11 - 15	3	.5986	.2957	.0934	.0117	.0006	0.0	0.0	0.0	0.0	0.0
16 - 20	4	.4801	.3488	.1455	.0240	.0015	0.0	0.0	0.0	0.0	0.0
21 - 25	5	.3633	.3788	.2085	.0454	.0039	.0001	0.0	0.0	0.0	0.0
26 - 30	6	.2580	.3788	.2748	.0791	.0090	.0004	0.0	0.0	0.0	0.0
31 - 35	7	.1711	.3488	.3332	.1267	.0190	.0011	0.0	0.0	0.0	0.0
36 - 40	8	.1056	.2957	.3719	.1867	.0371	.0029	.0001	0.0	0.0	0.0
41 - 45	9	.0605	.2307	.3822	.2530	.0664	.0069	.0003	0.0	0.0	0.0