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Will a government find it financially easier to neutralize a looming protest if more groups are involved?

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Abstract

We study a policy response to an increase in post-merger social stress. If a merger of groups of people is viewed as a revision of their social space, then the merger alters people's comparators and increases social stress: the social stress of a merged population is greater than the sum of the levels of social stress of the constituent populations when apart. We use social stress as a proxy measure for looming social protest. As a response to the post-merger increase in social stress, we consider a policy aimed at reversing the negative effect of the merger by bringing the social stress of the merged population back to the sum of the pre-merger levels of social stress of the constituent populations when apart. We present, in the form of an algorithm, a cost-effective policy response which is publicly financed and does not reduce the incomes of the members of the merged population. We then compare the financial cost of implementing such a policy when the merger involves more or fewer groups. We show that the cost may fall as the number of merging groups rises.

Keywords: Merger of populations; Revision of social space; Aggregate relative deprivation; Social stress; A cost-effective policy response

JEL classification: D04; D63; F55; H53; P51

1. Introduction

It has been shown that when integration is viewed as a revision of social space and, thereby, of people's set of comparators, integration increases the population-wide social stress measured by aggregate relative deprivation: the social stress of an integrated population is higher than the sum of the levels of social stress of the constituent populations when apart (Stark, 2013). Governments must be aware that an increase in social stress could translate into social unrest, and there have been plenty of episodes, historical and current, to remind them of the short distance between social stress and social protest, and between social protest and social upheaval. We therefore view social stress as a proxy measure for looming social protest.

In the context of the current paper, integration takes place as a consequence of political, administrative, communication technology, military, and other processes. For example, the internet, mobile phones, social media, and other modern means of communication help to integrate groups of people (not merely facilitate coordination between them), intensifying interpersonal comparisons. Provinces consolidate into regions, and small municipalities merge into a larger municipality (as is currently happening increasingly in Italy and in Japan). Adjacent villages that experience population growth coalesce into one town. East Germany and West Germany become united Germany. And European countries integrate financially by adopting a common currency; although a "super-government" does not take over from national governments, a body is formed - the European Central Bank - which takes on some of the functions of a national government.[1] Wars and conquests can join together regions, nations, and peoples.

Rising social stress can cascade into social unrest. A government seeking to forestall a possible social protest can respond to the increase in social stress, but the response will not be cost-free. If a government is to maintain social stress at the pre-merger level in order to counteract any looming protest, it will have to allocate funds to placate the integrated, more

[1] It is noteworthy that the introduction of a common currency is an instrument of fundamental change in economic and social relations in general, and in interpersonal comparisons of earnings, pay, and incomes in particular. Although, prior to the introduction of the euro as a common currency, individuals in specific European countries were able to compare their incomes with the incomes of individuals in other European countries, the comparison was not immediate, it required effort to convert incomes denominated in different currencies, and it was presumably not done very often. When a single currency is introduced, the comparison environment changes, easing, indeed inviting, comparisons with others. For example, with currency unification, workers who perform the same task and who are employed by a manufacturer with plants located in different European Monetary Union countries can compare their earnings with each other directly, effortlessly, and routinely.

distressed population. The main, and surprising, result reported in this paper is that the minimum funds required for keeping the post-merger level of social stress at its pre-merger level may decrease with the number of the integrating groups involved. When more groups merge, the increase in social stress and the minimum funds needed to contain this increase can move either in the same direction, or in opposite directions. One implication of this result is that when, in response to looming social protest, a government seeks to discourage the coming together of more groups, it may well need to rethink its stance: discouraging wider protest may exacerbate the associated financial burden rather than alleviate it.

It is not the purpose of this paper to present a menu of governmental responses to a rising social stress and / or to assess the relative feasibility of such responses. Suffice to remark that perhaps the very processes that bring about the expansion of social space may make devolution a rather ineffective response. The purpose of this paper is defined more narrowly: to show how a government that seeks to respond to post-merger intensified social stress by disbursing funds can optimize this disbursement, and how the amount expended relates to the number of groups joining.

The toolkit at the disposal of governments that can be applied to preserve social peace and maintain social order can obviously range from granting political rights and expanding general welfare programs to repression and the use of police and other coercive powers. As already noted, it is unlikely that any policy response will be costless. Nor is it clear whether concessions, if granted, will not signal weakness and encourage stronger protest. The analysis undertaken in this paper calculates the lowest price tag of one specific policy response. Hence, when assessing how to react, a government will be able to compare the outlay involved in other measures with the precise outlay specified here.

In the next section we present a measure of social stress, and the superadditivity theorem which shows that the social stress of a merged population is greater than the sum of the levels of social stress of the constituent populations when apart. For the sake of completeness, we present in the Appendix the rationale and logic for the measure of social stress that we use in this paper. In Section 3 we construct an algorithm for a cost-effective government response. In Section 4 we show that the expenditure needed to keep the post-merger social stress at its pre-merger level can either increase or decrease with the number of the merging groups. In Section 5 we conclude.

2. Relative deprivation as a measure of social stress, and the superadditivity of aggregate relative deprivation

We quantify the social stress of a population by the sum of the levels of social stress experienced by the individuals who constitute that population. As in Stark (2013), we measure the social stress of an individual by his relative deprivation. In line with the definition of relative deprivation in Stark (2013), we resort to income-based comparisons, namely an individual feels relatively deprived when others in his comparison group earn more than he does. To concentrate on essentials, we assume that the comparison group of each individual consists of all members of his population. Thus, we measure the social stress of an individual by the extra income units that others in the population have, we sum up these excesses, and we divide the sum by the size of the population. This approach tracks the seminal work of Runciman (1966) and its articulation by Yitzhaki (1979), Hey and Lambert (1980), Ebert and Moyes (2000), and Bossert and D'Ambrosio (2006). Summing over the levels of relative deprivation (social stress) experienced by all the individuals belonging to a given population yields the social stress of the population. We refer to this sum as the aggregate relative deprivation (*ARD*) of the population.

Formally, for population *P* consisting of n individuals whose incomes are represented by the following ordered vector $x=\left(x_1,...,x_n\right)$, where $x_1 \le x_2 \le ... \le x_n$, we define the relative deprivation of individual *i*, RD_i, earning income x_i as

$$RD_i\left(x\right) \equiv \begin{cases} \dfrac{1}{n}\displaystyle\sum_{j=i+1}^{n}(x_j - x_i) & for\ \ i=1,...,n-1, \\ 0 & for\ \ i=n. \end{cases} \tag{1}$$

To ease the analysis that follows, an alternative representation of the relative deprivation measure is helpful.

Lemma 1. Let $F\left(x_i\right)$ be the fraction of the individuals in population *P* of size n with an ordered income vector $x=\left(x_1,...,x_n\right)$ whose incomes are smaller than or equal to x_i. The relative deprivation of individual $i \in P$ earning x_i, where $i<n$, is equal to the fraction of those whose incomes are higher than x_i times their mean excess income. Namely

$$RD_i\left(x\right)=\left[1-F(x_i)\right]\cdot E\left(x-x_i \mid x>x_i\right). \tag{2}$$

Proof. We multiply $\frac{1}{n}$ in (1) by the number of the individuals who earn more than x_i, and we divide $\sum_{j=i+1}^{n}\left(x_j - x_i\right)$ in (1) by the same number. We then obtain two ratios: the first is the fraction of the population who earn more than individual *i*, namely $\left[1-F(x_i)\right]$; the second is the mean excess income, namely $E\left(x - x_i \mid x > x_i\right)$. □

The aggregate relative deprivation of population *P*, ARD^P, is the sum of the levels of relative deprivation experienced by the individuals belonging to *P*, that is,

$$ARD^P = \sum_{i=1}^{n} RD_i\left(x\right) = \frac{1}{n}\sum_{i=1}^{n}\sum_{j=i+1}^{n}\left(x_j - x_i\right). \tag{3}$$

ARD^P is our measure of the level of social stress of population *P*.

We now consider two populations, *A* of size n_A, and *B* of size n_B, with ordered income vectors $x^k = \left(x_1^k, ..., x_{n_k}^k\right)$, where $k = A, B$. When these two populations merge, the total population size is $n = n_A + n_B$. The ordered income vector of the merged population is denoted by $x^A \circ x^B$, and is the *n*-dimensional income vector obtained by merging the two income vectors and ordering the resulting *n* components from the lowest to the highest.

In the following claim we state that in comparison with the sum of the levels of aggregate relative deprivation of two populations when apart, a merger of the two populations increases the aggregate relative deprivation or leaves it unchanged. Namely if we conceptualize the merger of two income vectors as an addition operator, then *ARD* is a superadditive function of the income vectors.[2]

Claim 1. We denote by $ARD^{A\cup B}$ the aggregate relative deprivation in a population that constitutes the merger of population *A* and population *B*. Then, $ARD^{A\cup B} \geq ARD^A + ARD^B$.

Proof. A proof for the case of the merger of two populations with two members each is in Stark (2010); a proof for the case of the merger of two populations of any size is in Stark (2013). □

Consider now $l \geq 2$ merging populations, where l is a natural number. The size of each constituent population P_k is n_k, where $k = 1, ..., l$, and the corresponding ordered vector

[2] A function *H* is superadditive if for all *x, y* it satisfies $H(x+y) - H(x) - H(y) \geq 0$.

of incomes is $x^k = \left(x_1^k, ..., x_{n_k}^k\right)$. The merged population is then of size $n = n_1 + \ldots + n_l$, and its ordered income vector is $x^{P_1} \circ \ldots \circ x^{P_l}$.

Corollary. The aggregate relative deprivation of the merged population exhibits the superadditivity property, namely $ARD^{P_1 \cup \ldots \cup P_l \cup P_{l+1}} \geq ARD^{P_1 \cup \ldots \cup P_l} + ARD^{P_{l+1}}$.

Proof. The proof is by induction with respect to the number of the merged populations. □

3. A cost-effective policy response to the post-merger increase in social stress

An increase in social stress brought about by a merger can translate into social unrest, which may subsequently lead to social protest. We now ask how a government that is concerned about the increase in social stress will be able to respond in a cost-effective manner in order to obviate possible social protest.

We study a publicly-financed, cost-effective policy aimed at counteracting the increase in social stress. We consider the following target for a government policy that seeks to reverse the deleterious effect of the merger: to bring down the aggregate level of relative deprivation of the merged population to a level equal to the sum of the pre-merger levels of aggregate relative deprivation of the constituent populations when apart. Naturally, the government is keen to minimize the cost of implementing its chosen policy, which it enacts subject to the condition that, in the process, no income of any member of the merged population is allowed to fall.[3] We refer to this problem as Π. We show that the government can design an optimal policy response to the post-merger increase in aggregate relative deprivation by choosing carefully a subset of the individuals for whom the marginal increase in income yields the highest marginal decrease in relative deprivation.

Consider a merged population N of size n with an ordered income vector $x = \left(x_1, ..., x_n\right)$. We denote by Ω a subset of individuals from N whose incomes are the lowest. We analyze what happens when marginally, and by the same amount, we increase the incomes of the individuals in Ω, where a marginal increase refers to such an increase that the incomes of these individuals will not become higher than the income of any individual outside the set Ω.

[3] We resort to the condition "no income is allowed to fall" because of an implicit assumption that an individual's utility depends positively on his income and negatively on his relative deprivation, although we do not know the exact rate of substitution between income and relative deprivation. For example, we do not know how much income we could take away from an individual whose relative deprivation falls in the wake of the merger.

First, suppose that the set Ω consists of just one individual out of the *n* members of the merged population, meaning that there is only one individual earning the lowest income; that is, $x_1 < x_i$ for $i = 2,...,n$. Suppose that the government appropriates a sum ε to increase the income of this lowest-earning individual (namely individual 1), where ε is small enough to satisfy our definition of a marginal increase in income; that is, $\varepsilon \le x_2 - x_1$. Using (2), this individual's relative deprivation decreases by $\frac{n-1}{n}\varepsilon$, because the mean excess income of the fraction of $\frac{n-1}{n}$ individuals earning more than him is reduced by the amount ε. At the same time, as this individual's income was, and continues to be, the lowest in the population, this expenditure does not increase the relative deprivation of any other individual belonging to *N*. Therefore, the change in the aggregate relative deprivation of the merged population is equal to the decrease in the relative deprivation of individual 1, namely

$$\Delta ARD^N = -\frac{n-1}{n}\varepsilon . \tag{4}$$

We next show that upon spending ε on a single individual, the term on the right hand side of (4) is the highest marginal decrease in aggregate relative deprivation achievable. We do this by contradiction. Suppose that we were to increase by ε not the income of the lowest-earning individual, x_1, but, rather, the income of an individual earning $x_i > x_1$, where $i \in N$ and $i > 1$, such that $x_i + \varepsilon \le x_{i+1}$, so as to abide by the condition of a marginal change. Then, the relative deprivation of individual *i* would decrease as a result of his income getting closer to the incomes of the individuals earning more than he does, but the relative deprivation of those individuals who earn less than individual *i* would increase. Namely when $\overline{n}_i$ ($\tilde{n}_i$) is the number of the individuals earning strictly more (less) than x_i, the change in the aggregate relative deprivation of the merged population would be

$$\Delta ARD^N = -\frac{\overline{n}_i}{n}\varepsilon + \frac{\tilde{n}_i}{n}\varepsilon = -\frac{\overline{n}_i - \tilde{n}_i}{n}\varepsilon , \tag{5}$$

because the mean excess income of the fraction of $\frac{\overline{n}_i}{n}$ individuals earning more than x_i would fall by the amount ε, yet, at the same time, the relative deprivation of each of the $\tilde{n}_i$

individuals earning less than x_i would increase by $\frac{\varepsilon}{n}$. Because $\tilde{n}_i \geq 1$ and $\overline{n}_i < n$, comparing (4) and (5) yields

$$\frac{\overline{n}_i - \tilde{n}_i}{n}\varepsilon < \frac{n-1}{n}\varepsilon\,. \tag{6}$$

Thus, channeling the transfer ε to an individual who is not the lowest income recipient in the merged population yields a lower decrease in aggregate relative deprivation than increasing by ε the income of the individual who earns the lowest income.

Second, we consider a merged population *N* in which there are several individuals who earn the same income which constitutes the lowest income in the population. Hence, the set Ω includes more than one individual. We denote by $|\Omega|$ the size of this set. Suppose again that the government appropriates the sum ε to increase the earnings of each member of the subset Ω by $\frac{\varepsilon}{|\Omega|}$. Because every member of Ω receives a transfer of the same amount, the aggregate relative deprivation of the individuals belonging to Ω does not change.[4] Thus, the change in the aggregate relative deprivation in *N* arises only from a decrease of the relative deprivation sensed by the lowest-earning individuals in Ω whose incomes become closer to the incomes of the individuals earning more than they do. The fraction of the individuals in *N* who earn more than members of the set Ω is equal to $\frac{n-|\Omega|}{n}$, and the mean excess income of each individual who receives the transfer is reduced by $\frac{\varepsilon}{|\Omega|}$. Therefore, each of the members of Ω experiences a decrease in his relative deprivation equal to $\frac{n-|\Omega|}{n}\frac{\varepsilon}{|\Omega|}$. With no individual in *N* experiencing an increase in his relative deprivation (which occurs because the transfer $\frac{\varepsilon}{|\Omega|}$ marginally increases the incomes of the lowest-earning individuals) this expenditure yields the following change in the aggregate relative deprivation:

[4] If the set Ω were expanded to include several individuals who differ in their income levels, then, in the wake of the transfer under consideration, the aggregate relative deprivation of the individuals belonging to Ω would also not change. Upon each of the individuals in Ω receiving the positive transfer $\frac{\varepsilon}{|\Omega|}$, their incomes increase by the same amount and, thus, the aggregate relative deprivation within the set Ω does not change.

$$\Delta ARD^{N} = -|\Omega|\frac{n-|\Omega|}{n}\frac{\varepsilon}{|\Omega|} = -\frac{n-|\Omega|}{n}\varepsilon . \tag{7}$$

As in the case of the set Ω consisting of a single individual, this is obviously the optimal use of ε for any subset of individuals in the merged population.

Drawing on the preceding protocol, we present the optimal solution to problem Π, that is, the cost-effective policy response to the post-merger increase in social stress, in the form of an algorithm as follows.

Algorithm:

1. Include in the set Ω all the individuals who earn the lowest income in the merged population.
2. Proceed to increase simultaneously the incomes of the members of the set Ω, until either
    a. the aggregate relative deprivation of the merged population is brought down to the sum of the pre-merger levels of the aggregate relative deprivation of the constituent populations when apart,

    or

    b. the incomes of the members of the set Ω reach the income of the lowest-earning individual(s) who is (are) not a member (members) of this set, in which case expand the set Ω by including him (them) in Ω. Start from step 2 once again. Notice that the incomes of the pre-expansion members of the set Ω should be increased from the level already reached, that is, from the level equal to the income(s) of the newly included individual(s).[5]

It is easy to ascertain the optimality of the Algorithm: at each step, we increase the incomes of those individuals who earn the least, so the decrease in the aggregate relative deprivation of the merged population is most effective, and no one experiences an increase of their relative deprivation in the process. We raise incomes from the bottom, and we

[5] If, in the wake of employing the Algorithm, the set Ω is expanded to include individuals who prior to the government's transfer differed in their incomes, then, upon transferring funds to the individuals in Ω, the aggregate relative deprivation of the merged population falls via two channels. First, the incomes of the individuals in Ω become closer to the incomes of the individuals earning more than they do (namely the individuals outside Ω) and, second, the aggregate relative deprivation within the set Ω is reduced. The latter consequence follows from the fact that the transferred funds equalize the incomes of the individuals belonging to Ω who, prior to the transfer, differed in their incomes.

simultaneously gauge the aggregate relative deprivation response. The two processes move in tandem, and in opposite directions. The raising of incomes from below is ratcheted up the hierarchy of the individuals, and it ceases when the aggregate relative deprivation reaches its pre-merger level.

Example 1: *Application of the Algorithm*

We consider the merger of populations A and B with income vectors $x^A = (4,6)$ and $x^B = (2,3)$. The pre-merger levels of aggregate relative deprivation of each population are $ARD^A = 1$ and $ARD^B = \frac{1}{2}$. Because in the merged population with the ordered income vector $x^A \circ x^B = (2,3,4,6)$ we have that $ARD^{A\cup B} = \frac{13}{4} > \frac{3}{2} = ARD^A + ARD^B$, the government seeks to lower the aggregate relative deprivation of the merged population back to $ARD^A + ARD^B = \frac{3}{2}$.

Applying the Algorithm, we first include in the set Ω the individual earning 2, and we increase his income. Upon the maximal possible transfer satisfying the condition of a marginal change, that is, upon his income reaching the income of the lowest-earning individual outside Ω (namely the individual who earns 3), we obtain the post-transfer income vector $\left(x^A \circ x^B\right)_T = (3,3,4,6)$ with

$$ARD_T^{A\cup B} = \frac{2+2\cdot\left[(6-3)+(4-3)\right]}{4} = \frac{5}{2}.$$

We see that giving the individual earning 2 an additional unit of income does not suffice to bring down the aggregate relative deprivation to its pre-merger level. We, therefore, add the next lowest-earning individual (namely the individual who earns 3) to the set Ω, and we proceed to simultaneously and equally increase the incomes of each of the two individuals in the set Ω. We do so from the level of incomes already reached, that is, we start from each individual in Ω who now has income 3. At the point where these two incomes are elevated to 4 each, we obtain the income vector $\left(x^A \circ x^B\right)_T = (4,4,4,6)$ with

$$ARD_T^{A\cup B} = \frac{3\cdot 2}{4} = \frac{3}{2} = ARD^A + ARD^B.$$

Thus, in order to bring the aggregate relative deprivation in the merged population down to the sum of the pre-merger levels of aggregate relative deprivation of the constituent populations, the government has to transfer 2 to the individual earning 2, and 1 to the individual earning 3, which sums up to 3 as the total cost of implementing the policy.

4. The government cost of forestalling increased social stress in relation to the number of integrating groups

We now inquire how the number of integrating groups impinges on what the government spends in order to keep the level of social stress at its pre-merger level. We show that this financial burden can either increase or decrease with the number of merging groups. Claim 2 states that when more groups merge, the increase in social stress and the minimum funds needed to contain this increase can move either in the same direction or in opposite directions. The "breakdown" in the intuitive logic is caused by the fact that the superadditivity property that characterizes the increase in social stress upon the merger of $n+1$ populations as opposed to the merger of n populations does not replicate onto the domain of the government's financing. In other words, subadditivity of the government financial cost can coincide with superadditivity of social stress.[6]

Claim 2. Not allowing any individual's income to be reduced, the minimum funds required for keeping post-merger social stress at its pre-merger level do not necessarily increase with the number of integrating groups (populations).

Proof. By means of two examples, we show that two opposite directions of a change in the government expenditure are possible. In each example, we consider three populations, *A*, *B*, and *C*, with income distributions of the same type, in that, income-wise, population *A* and population *B* do not overlap, whereas population *C* overlaps with populations *A* and *B*. The two examples differ only with respect to dispersion of the income distributions. In both examples we compare the minimum financial outlay needed to keep at bay the social stress when two populations, *A* and *B*, merge with that needed to keep at bay the social stress when three populations merge, *A*, *B*, and *C*. We show that for the income distributions in the first example, the minimum necessary funds are larger when three populations merge than when two populations merge, whereas for the income distributions in the second example, the

[6] A function *H* is subadditive if for all *x*, *y* it satisfies $H(x)+H(y)-H(x+y)\geq 0$.

minimum necessary funds are smaller when three populations merge than when two populations merge.

Example 2.1: *An increase in the government cost of forestalling increased social stress upon the merger of more groups*

Let there be populations *A*, *B*, and *C* of two individuals each, with income vectors $x^A = (a+3\varepsilon, a+5\varepsilon)$, $x^B = (a+\varepsilon, a+2\varepsilon)$, and $x^C = (a, a+4\varepsilon)$, where $a > 0$ and $\varepsilon > 0$. We note that for $a = 1$ and $\varepsilon = 1$, the income distributions of *A* and *B* reduce to the distributions referred to in *Example 1*. When the three populations are apart, $ARD^A = \varepsilon$, $ARD^B = \frac{\varepsilon}{2}$, and $ARD^C = 2\varepsilon$.

We consider first the merger of populations *A* and *B*. For the post-merger ordered income vector $x^A \circ x^B = (a+\varepsilon, a+2\varepsilon, a+3\varepsilon, a+5\varepsilon)$,

$$ARD^{A\cup B} = \frac{13}{4}\varepsilon > \frac{6}{4}\varepsilon = ARD^A + ARD^B .$$

In order to bring, at the minimum cost, the *ARD* of the merged population down to the pre-merger level of the sum of the levels of *ARD* of the constituent populations without lowering the income of any individual belonging to the merged population, the government's transfer has to yield the income vector of the merged population $\left(x^A \circ x^B\right)_T = \left(a+3\varepsilon, a+3\varepsilon, a+3\varepsilon, a+5\varepsilon\right)$, which requires funds of 3ε. Then, indeed, we obtain that $ARD_T^{A\cup B} = \frac{6}{4}\varepsilon = ARD^A + ARD^B$.

We now consider the merger of the three populations, *A*, *B*, and *C*. The ordered income vector of the merged population is $x^A \circ x^B \circ x^C = \left(a, a+\varepsilon, a+2\varepsilon, a+3\varepsilon, a+4\varepsilon, a+5\varepsilon\right)$, and

$$ARD^{A\cup B\cup C} = \frac{35}{6}\varepsilon > \frac{21}{6}\varepsilon = ARD^A + ARD^B + ARD^C.$$

Employing the Algorithm, we derive the optimal post-transfer income vector of the merged population $\left(x^A \circ x^B \circ x^C\right)_T = \left(a+\frac{19}{9}\varepsilon, a+\frac{19}{9}\varepsilon, a+\frac{19}{9}\varepsilon, a+3\varepsilon, a+4\varepsilon, a+5\varepsilon\right)$, as then,

$$ARD_T^{A\cup B\cup C} = \frac{21}{6}\varepsilon = ARD^A + ARD^B + ARD^C .$$

In order to keep the social stress of the integrated population at its pre-merger level without lowering the income of any individual, the government needs to spend minimum

$$\frac{19}{9}\varepsilon + \left(\frac{19}{9}\varepsilon - \varepsilon\right) + \left(\frac{19}{9}\varepsilon - 2\varepsilon\right) = \frac{10}{3}\varepsilon = 3\frac{1}{3}\varepsilon .$$

Because $3\frac{1}{3}\varepsilon$ is higher than 3ε, it follows that the strain on the government's finances is more severe when three populations, *A*, *B*, and *C*, merge than when two populations, *A* and *B*, merge.

Example 2.2: *A decrease in the government cost of forestalling increased social stress upon the merger of more groups*

Let there be populations *A*, *B*, and *C* of two individuals each, with income vectors $x^A = (a+4\varepsilon, a+8\varepsilon)$, $x^B = (a+2\varepsilon, a+3\varepsilon)$, and $x^C = (a, a+6\varepsilon)$, where $a > 0$ and $\varepsilon > 0$. Then, when the three populations are apart, $ARD^A = 2\varepsilon$, $ARD^B = \frac{\varepsilon}{2}$, and $ARD^C = 3\varepsilon$.

We consider first the merger of populations *A* and *B*. For the post-merger ordered income vector $x^A \circ x^B = (a+2\varepsilon, a+3\varepsilon, a+4\varepsilon, a+8\varepsilon)$,

$$ARD^{A\cup B} = \frac{19}{4}\varepsilon > \frac{10}{4}\varepsilon = ARD^A + ARD^B.$$

In order to keep the social stress of the integrated population at its pre-merger level without the income of any individual being reduced, we again follow the Algorithm's method of increasing incomes "from the bottom." The cost-effective government response requires transferring funds to the three lowest-earning individuals by raising their incomes to the common level of $a + \frac{14}{3}\varepsilon$. Thus, the optimal, post-transfer income vector of the merged population is $\left(x^A \circ x^B\right)_T = \left(a + \frac{14}{3}\varepsilon, a + \frac{14}{3}\varepsilon, a + \frac{14}{3}\varepsilon, a + 8\varepsilon\right)$ and, indeed, we obtain that

$$ARD_T^{A\cup B} = \frac{10}{4} = ARD^A + ARD^B.$$

This policy response requires spending $\left(\frac{14}{3}\varepsilon-2\varepsilon\right)+\left(\frac{14}{3}\varepsilon-3\varepsilon\right)+\left(\frac{14}{3}\varepsilon-4\varepsilon\right)=5\varepsilon$, which constitutes the lowest possible cost of the policy of keeping at bay the *ARD* after the merger of populations *A* and *B*.

We now consider the merger of the three populations, *A*, *B*, and *C*, with the post-merger ordered income vector $x^A \circ x^B \circ x^C = (a, a+2\varepsilon, a+3\varepsilon, a+4\varepsilon, a+6\varepsilon, a+8\varepsilon)$. We have that

$$ARD^{A\cup B\cup C}=\frac{53}{6}\varepsilon>\frac{33}{6}\varepsilon=ARD^A+ARD^B+ARD^C.$$

In order to bring the social stress of the merged population down to its pre-merger level, we invoke the Algorithm and increase incomes "from the bottom." We stop doing so upon assigning the income of $a+\frac{29}{9}\varepsilon$ to each of the three lowest-earning individuals. This yields the post-transfer income vector

$$\left(x^A \circ x^B \circ x^C\right)_T=(a+\frac{29}{9}\varepsilon, a+\frac{29}{9}\varepsilon, a+\frac{29}{9}\varepsilon, a+4\varepsilon, a+6\varepsilon, a+8\varepsilon)$$

and then, indeed,

$$ARD_T^{A\cup B\cup C}=\frac{33}{6}\varepsilon=ARD^A+ARD^B+ARD^C.$$

In sum, this policy increases the government's spending aimed at keeping at bay the *ARD* of the three merged populations, *A*, *B*, and *C*, by $\frac{29}{9}\varepsilon+\left(\frac{29}{9}\varepsilon-2\varepsilon\right)+\left(\frac{29}{9}\varepsilon-3\varepsilon\right)=\frac{14}{3}\varepsilon=4\frac{2}{3}\varepsilon$, which is *less* than 5ε, the spending required to keep at bay the *ARD* of the two merged populations, *A* and *B*. □

5. Conclusions

The integration of populations is often a process that in and by itself governments have little or no means to control. Social stress, caused by integration and measured by aggregate relative deprivation, is subject to superadditivity: the social stress in the integrated population increases as compared to the sum of the levels of the social stress in the constituent populations when apart. The corresponding minimum financial outlay needed to reduce the post-merger social stress to its pre-merger level does not necessarily share a similar property. In fact, this outlay can be subject to subadditivity.

The consequence of increased aggregate relative deprivation can be dire. Gurr (1970, p. 12) writes: "Discontent arising from the perception of relative deprivation is the basic, instigating condition for participants in collective violence." He is also of the opinion (p. ix) that "to understand protest and rebellion in general, … we should analyze … popular discontent (relative deprivation) … and the government's capacity to repress or channel [people's] anger." Interestingly, Gurr argues (p. 24) that "the potential for collective violence varies strongly with the intensity and scope of relative deprivation among members of a collectivity." However, Gurr does not study or develop the quantitative perspective that is the focus of our paper.

There is an intriguing similarity between the mechanism of our Algorithm, which aims at increasing social welfare via the reduction of aggregate relative deprivation, and the Rawlsian program which aims at increasing social welfare directly. The Rawlsian approach to social welfare, built on the foundation of the "veil of ignorance,"[7] measures the welfare of a society by the wellbeing of the worst-off individual (the maximin criterion). Rawls argues that if individuals were to select the concept of justice by which a society is to be regulated without knowing their position in that society - the "veil of ignorance" - they would choose principles that involve the least undesirable condition for the worst-off member over utilitarian principles. This hypothetical contract is the basis of the Rawlsian society, and of the Rawlsian maximin social welfare function. To see vividly the analogy between our Algorithm and the Rawlsian welfare-increasing policy, we revisit *Example 1* where the post-merger ordered income vector is $x^A \circ x^B = (2,3,4,6)$, and the pre-merger sum of the levels of the aggregate relative deprivation of the two populations is 3/2.

Consider a Rawlsian social planner who seeks to increase social welfare by adhering to the maximin principle and who has at his disposal three units of income. This planner will allocate the first unit of income to the individual who earns 2; the income vector will then become (3,3,4,6). Thereafter, the Rawlsian social planner will reach out to the now worst off, namely to the two individuals who earn 3 each, and increase the incomes of each of them to 4, thereby obtaining income vector (4,4,4,6). Clearly, as the allocation proceeds, the identity of the worst off individuals changes (first it is the individual whose income was initially 2, then

[7] "[N]o one knows his place in society, his class position or social status; nor does he know his fortune in the distribution of natural assets and abilities, his intelligence and strength, and the like." (Rawls, 1999, p. 118.)

these are the two individuals whose incomes were initially 2 and 3). However, the principle guiding the allocation of the income available for disbursement does not change: the sequence of attending to the individuals is from the bottom up. And this procedure is analogous to the one specified by our Algorithm.

Naturally, there are differences between the Rawlsian procedure and the Algorithm protocol in that the rationales for interference differ, and the reasons for proceeding from the bottom up differ. Still, in the configuration of *Example 1*, a Rawlsian social planner with a "policy budget" of three units of income will allocate those units in the same way as will a government applying our Algorithm.

An interesting question is to what extent the likelihood of implementing our Algorithm depends on the degree of autocratic power that a government has. Given the view in political economy that autocratic governmental power hinders civic participation, it could be anticipated that individuals in autocratically governed societies will not be inclined to resort to collective action. Consequently, social protest will not be likely, and a governmental response will not be required. However, the inevitability of such a scenario has been doubted by Acemoglu et al. (2014) who, drawing on the example of Sierra Leone, conclude that an autocratically governed society can hold large scale community meetings, exhibit intensive participation in social groups, and frequently undertake collective action. This conclusion implies that even in such societies, "integrate and protest" can well occur and, therefore, the need for governmental response cannot be assumed away.

Appendix. The rationale and construction of the measure of social stress

Several recent insightful studies in social psychology (for example, Callan et al., 2011; Smith et al., 2012) document how sensing relative deprivation impacts negatively on personal wellbeing, but these studies do not provide a calibrating procedure; a sign is not a magnitude. For the purpose of constructing a measure, a natural starting point is the work of Runciman (1966), who argued that an individual has an unpleasant sense of being relatively deprived when he lacks a desired good and perceives that others with whom he naturally compares himself possess that good. Runciman (1966, p. 19) writes as follows: "The more people a man sees promoted when he is not promoted himself, the more people he may compare himself with in a situation where the comparison will make him feel deprived," thus implying that the deprivation from not having, say, income *y* is an increasing function of the fraction of people in the individual's reference group who have *y*. To aid intuition and for the sake of concreteness, we resort to income-based comparisons, namely an individual feels relatively deprived when others in his comparison group earn more than he does. An implicit assumption here is that the earnings of others are publicly known. Alternatively, we can think of consumption, which could be more publicly visible than income, although these two variables can reasonably be assumed to be strongly positively correlated.

As an illustration of the relationship between the fraction of people possessing income *y* and the deprivation of an individual lacking *y*, consider a population (reference group) of six individuals with incomes {1,2,6,6,6,8}. Imagine a furniture store that in three distinct compartments sells chairs, armchairs, and sofas. An income of 2 allows you to buy a chair. To be able to buy any armchair, you need an income that is a little bit higher than 2. To buy any sofa, you need an income that is a little bit higher than 6. Thus, when you go to the store and your income is 2, what are you "deprived of?" The answer is "of armchairs," and "of sofas." Mathematically, this deprivation can be represented by $P(Y>2)(6-2)+P(Y>6)(8-6)$, where $P(Y>y_i)$ stands for the fraction of those in the population whose income is higher than y_i, for $y_i=2,6$. The reason for this representation is that when you have an income of 2, you cannot afford anything in the compartment that sells armchairs, and you cannot afford anything in the compartment that sells sofas. Because not all those who are to your right in the ascendingly ordered income distribution can afford to buy a sofa, yet they can all afford to buy armchairs, a breakdown into the two (weighted) terms $P(Y>2)(6-2)$ and $P(Y>6)(8-6)$ is needed. This way, we get to the very essence of the measure of *RD* used in this paper: we take into account the fraction of the comparison group (population) who possess some good which you do not, and we weigh this fraction by the “excess value” of that good. Because income enables an individual to afford the consumption of certain goods, we refer to comparisons based on income.

Formally, let $y=(y_1,...,y_m)$ be the vector of incomes in a population of size *n* with relative incidences $p(y)=\left(p(y_1),...,p(y_m)\right)$, where $m\le n$ is the number of distinct income levels in *y*. The *RD* of an individual earning y_i is defined as the weighted sum of the excesses of incomes higher than y_i such that each excess is weighted by its relative incidence, namely

$$RD_i\left(y\right)\equiv\sum_{y_j>y_i}p(y_j)(y_j-y_i)\,. \tag{A1}$$

In the example given above with income distribution {1,2,6,6,6,8}, we have that the vector of incomes is $y=\left(1,2,6,8\right)$, and that the corresponding relative incidences are $p(y)=\left(\frac{1}{6},\frac{1}{6},\frac{3}{6},\frac{1}{6}\right)$. Therefore, the *RD* of the individual earning 2 is $\sum_{y_j>y_i}p(y_j)(y_j-y_i)=p(6)(6-2)+p(8)(8-2)=\frac{3}{6}\cdot 4+\frac{1}{6}\cdot 6=3$. By similar calculations, we have that the *RD* of the individual earning 1 is higher and is equal to $3\frac{5}{6}$, and that the *RD* of each of the individuals earning 6 is lower and is equal to $\frac{1}{3}$.

We expand the vector y to include incomes with their possible respective repetitions, that is, we include each y_i as many times as its incidence dictates, and we assume that the incomes are ascendingly ordered, that is, $y=(y_1,...,y_n)$ such that $y_1\le y_2\le ...\le y_n$. In this case, the relative incidence of each y_i, $p(y_i)$, is $1/n$, and, (A1) becomes exactly as given in (1):

$$RD_i\left(y\right)\equiv\begin{cases}\frac{1}{n}\sum_{j=i+1}^{n}(y_j-y_i) & for\;\; i=1,...,n-1,\\ 0 & for\;\; i=n.\end{cases}$$

Looking at incomes in a large population, we can model the distribution of incomes as a random variable *Y* over the domain $[0,\infty)$ with a cumulative distribution function *F*. We can then express the *RD* of an individual earning y_i as

$$RD_i(y) = [1 - F(y_i)]E(Y - y_i \mid Y > y_i). \tag{A2}$$

The formula in (A2) is quite revealing because it casts *RD* in a richer light than the ordinal measure of rank, which have been studied intensively in sociology and beyond. The formula informs us that when the income of individual A is, say, 10, and that of individual B is, say, 16, the *RD* of individual A is higher than when the income of individual B is 15, even though, in both cases, the rank of individual A in the income hierarchy is second. The formula also informs us that more *RD* is sensed by an individual whose income is 10 when the income of another is 14 (*RD* is 2) than when the income of each of four others is 11 (*RD* is $\frac{4}{5}$), even though the excess income in both cases is 4. This property aligns nicely with intuition: it is more painful (more stress is experienced) when the income of half of the population in question is 40 percent higher, than when the income of $\frac{4}{5}$ of the population is 10 percent higher. In addition, the formula in (A2) reveals that even though *RD* is sensed by looking to the right of the income distribution, it is impacted by events taking place on the left of the income distribution. For example, an exit from the population of a low-income individual increases the *RD* of higher-income individuals (other than the richest) because the weight that the latter attach to the difference between the incomes of individuals "richer" than themselves and their own income rises. The often cited example from a three tenors concert organized for Wembley Stadium in which Pavarotti reputedly did not care how much he was paid so long as it was one pound more than Domingo was paid does not invalidate the logic behind our measure because, in light of the measure, Pavarotti's payment request can be interpreted as being aimed at ensuring that no *RD* will be experienced when he looks to the right in the pay distribution.

Similar reasoning can explain the demand for positional goods (Hirsch, 1976). The standard explanation is that this demand arises from the unique value of positional goods in elevating the social standing of their owners ("These goods [are] sought after because they compare favorably with others in their class." Frank, 1985, p. 7). The distaste for relative deprivation offers another explanation: by acquiring a positional good, an individual shields himself from being leapfrogged by others which, if that were to happen, would expose him to *RD*. Seen this way, a positional good is a form of insurance against experiencing *RD*.

There can, of course, be other, quite intuitive ways of gauging *RD*, and in some contexts and for some applications, a measure simpler than (1) can be adequate. Suppose that an individual's income is I, and the average income of the individual's reference group is R. We can then define RD as a function of I and R, namely

$$RD(I,R) = \begin{cases} R - I & \text{if} \quad I < R \\ 0 & \text{if} \quad I \geq R. \end{cases} \tag{A3}$$

This representation captures the intuitive requirements

$$\frac{\partial RD(I,R)}{\partial I} < 0, \ \frac{\partial RD(I,R)}{\partial R} > 0 \ \text{ for } R > I \ ,$$

namely that, holding other things the same, for a relatively deprived individual (that is, for an individual whose income is lower than the average income of the individual's reference group), RD decreases with his own income, and increases with the average income of his reference group. Examples of the use of (A3) are in Fan and Stark (2007), Stark and Fan (2011), and Stark and Jakubek (2013). However, the advantage of using (1) is that it is based on an axiomatic foundation which is, essentially, a translation of Runciman's (1966) work, let alone that it is nice in economics to draw on a foundation laid out in social psychology.

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