FONDAZIONE ENI ENRICO MATTEI

NOTA DI LAVORO

58.2016

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Guiomar Martín-Herrán, Department of Applied Economics and IMUVa, University of Valladolid
Santiago J. Rubio, Department of Economic Analysis and ERI-CES, University of Valencia

Economic Theory
Series Editor: Carlo Carraro

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Summary

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Keywords: Monopoly, Commitment, Emission Tax, Abatement, Stock Pollutant

JEL Classification: H23, L12, L51, Q52, Q55

The authors would like to thank participants at the 17th International Symposium on Dynamic Games and Applications (Urbino) for stimulating discussion. Guiomar Martín-Herrán gratefully acknowledges financial support from the Spanish Ministry of Economics and Competitiveness under project ECO2014-52343-P and COST Action IS1104 "The EU in the new economic complex geography: models, tools and policy evaluation". Santiago J. Rubio also gratefully acknowledges financial support from the Spanish Ministry of Economics and Competitiveness under project ECO2013-45045-R, and Valencian Generality under project PROMETEO II/2014/054.

Address for correspondence:
Santiago J. Rubio
Department of Economic Analysis
University of Valencia
Edificio Departamental Oriental
Avda. de los Naranjos s/n
46022 Valencia
Spain
E-mail: santiago.rubio@uv.es

The Strategic Use of Abatement by a Polluting Monopoly*

Guiomar Martín-Herrán† and Santiago J. Rubio‡

August 23, 2016

*The authors would like to thank participants at the 17^{th} International Symposium on Dynamic Games and Applications (Urbino) for stimulating discussion. Guiomar Martín-Herrán gratefully acknowledges financial support from the Spanish Ministry of Economics and Competitiveness under project ECO2014-52343-P and COST Action IS1104 "The EU in the new economic complex geography: models, tools and policy evaluation." Santiago J. Rubio also gratefully acknowledges financial support from the Spanish Ministry of Economics and Competitiveness under project ECO2013-45045-R, and Valencian Generality under project PROMETEO II/2014/054.

†Department of Applied Economics and IMUVa, University of Valladolid, Spain.

‡Department of Economic Analysis and ERI-CES, University of Valencia, Spain.

Abstract

This paper evaluates the effects of the lack of regulatory commitment on emission tax applied by the regulator, abatement effort made by the monopoly and social welfare comparing two alternative policy games. The first game assumes that the regulator commits to an ex-ante level of the emission tax. In the second one, in a first stage the regulator and the monopolist simultaneously choose the emission tax and abatement respectively, and in a second stage the monopolist selects the output level. We find that the lack of commitment leads to lower taxation and abatement that yield larger emissions and, consequently, a larger steady-state pollution stock. Moreover, the increase of environmental damages because of the increase in the pollution stock more than compensates the increase in consumer surplus and the decrease in abatement costs resulting in a reduction of social welfare. Thus, our analysis indicates that the lack of commitment has a negative impact of welfare although this detrimental effect decreases with abatement costs.

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1 Introduction

The analysis of the incentives provided by environmental policy for both adoption and development of pollution-reduction technology has been extensively addressed in the literature (see for instance the survey published by Requate (2005)). Among the different issues studied, this paper focuses on the effects that the lack of environmental regulator to commit has on emission tax applied by the regulator, abatement effort made by the firm and social welfare when faced with the strategic behavior of a firm with market power. As noted by Biglaiser et al. (1995) and Gersbach and Glazer (1999), when the regulator is not able to commit to the stringency of the policy instrument, firms may strategically use innovation to ratchet down regulation and increase profits. One expects this behavior to have a negative effects on welfare relative to the case of regulatory commitment.

Interestingly, Petrakis and Xepapadeas (2001) find for a polluting monopoly in a *second-best policy* setting, that the strategic behavior of the firm has a beneficial effect on social welfare noting that it may induce more abatement than under regulatory commitment. In their static model, the tax rate is a *strategic substitute* of the abatement from the regulator's point of view whereas the abatement is a *strategic complement* of the tax rate for the monopolist. Then, if the monopolist moves first, it may increase strategically the abatement to rachet down the emission tax. However, this reduction expands production and hence consumer surplus and this increment in consumer surplus more than compensates the increase in abatement costs and the (possible) rise in environmental damages yielding an increase in social welfare. The lack of commitment is welfare improving. The result derived by these authors has implications for the design of environmental policy. Policy makers often believe that the inability to commit to limit future regulation discretion plays against the implementation of optimal environmental regulation. According to Petrakis and Xepapadeas' (2001) paper, commitment is not necessarily better than discretion for controlling the emissions of a monopoly when a tax is used to control emissions.

The aim of this paper is to asses the scope of this result when damages depend on a stock pollutant.[1] In the dynamic version of Petrakis and Xepapadeas' (2001) model, for a given value of the pollution stock the abatement is also a strategic complement of the emission tax but the optimal tax rate is *independent* of the abatement from the regulator's point of view. In other words, the regulator has a dominant strategy for any abatement level. In this case, the first mover advantage of the monopolist vanishes and the feedback Stackelberg equilibrium of the game coincides with the Markov-perfect Nash equilibrium. Thus, to address the consequences of the lack of commitment we compare the feedback Stackelberg equilibrium when the regulator is the leader of the game with the Markov-perfect Nash equilibrium.

The comparison establishes that the steady state for the pollution stock under commitment is lower than under non-commitment. Therefore, the lack of commitment increases the accumulation of emissions in the environment yielding larger damages at the steady state. Moreover, we find that the tax rule of the commitment solution gives larger taxes for all values of pollution stock lower than the steady-state value of the pollution stock for the non-commitment solution. In order words, the lack of commitment moves down the tax rule used by the regulator. In fact, the lack of commitment can lead to a change in the sign of the optimal policy converting a tax in a subsidy.[2] The welfare consequences of the lack of commitment cannot be determined analytically except when the initial pollution stock is zero. In this case, the comparison of the regulator's value functions

[1] Moner-Colonques and Rubio (2016) show that the result obtained by Petrakis and Xepapadeas (2001) does not hold if the regulator uses emission standards instead of an emission tax. Puller (2006) also finds that the lack of regulatory commitment lowers welfare when a performance standard is used to control emissions. However, he shows that in oligopoly settings the lack of commitment may increase welfare if firms invest in abatement technology to reduce the marginal cost of compliance.

[2] As in Benchekroun and Long (1998) we also find that for the commitment solution the tax increases with the stock of pollution but that it is negative when the pollution stock is low. Nevertheless, we show that if environmental damages are not very low the steady-state tax is positive. The subsidy operates to correct the market power of the firm when this distortion is more important than the distortion caused by the negative externality (pollution), i.e. when the pollution stock is low. However, for the non-commitment solution it cannot be discarded that a subsidy applies at the steady state. Notice that a subsidy is compatible with a positive abatement because this depends not only on the tax but also on the shadow price of the pollution stock.

shows that social welfare is lower when the regulator is unable to commit whereas net profits are larger. To advance in the comparative analysis, we have studied a numerical example that shows that the main difference between the static model and the dynamic model is that in the static model, the firm increases the abatement to obtain a reduction in the tax whereas in the dynamic model, as the abatement cannot be used to influence the tax, the firm decreases the abatement. The result is that, like in the static model, the reduction of the tax (or the rise of the subsidy) increases output and consumer surplus. Moreover, now we have a reduction in abatement costs. However, the increase in output (gross emissions) and the decrease in abatement cause an increase in emissions that lead to larger pollution stocks and environmental damages. The theoretical analysis shows that the increase in environmental damages is larger than the sum of the increase in consumer surplus and the reduction in abatement costs resulting in a fall of the social welfare when the initial pollution stock is zero, and the numerical exercise suggests that this result holds when the initial pollution stock is positive. Nevertheless, it should be noticed that the numerical exercise also shows that this negative effect on welfare diminishes with abatement costs and that for large abatement costs both equilibria practically yield the same welfare level. Thus, our research indicates that the lack of commitment has no cost in welfare terms if the abatement costs are large or reduces welfare if this is not the case. Then the idea that discretionality is better than commitment does not appear in the case of the regulation of a polluting monopoly with a stock pollutant. The lack of commitment is not welfare improving in any case.

The literature addressing the regulation of firms with market power in the context of stock dynamics includes Bergstrom et al. (1981), Karp and Livernois (1992), Karp (1992) and Benchekroun and Long (1998, 2002). The focus of these papers is on efficiency-inducing tax schemes. Bergstrom et al. (1981) show that there exists a continuum of tax schemes that induce a monopolist to exploit efficiently a non-renewable resource. However, these tax schemes are not subgame perfect. Karp and Livernois (1992) design a subgame perfect tax scheme that is efficiency inducing and is unique. Karp (1992) extends this result to the case of a common property oligopoly and Benchekroun and Long (1998) to the case of polluting oligopolists. Finally, Benchekroun and Long (2002) show that there exists a continuum of tax rules that guide a polluting monopolist to achieve the efficient emission path and ensure subgame perfectness. In this paper we extend the linear-quadratic case studied by Benchekroun and Long (2002) to address the effect of the lack of commitment of an environmental regulator on the tax rule, abatement and social welfare.

Only a pair of papers have studied this issue in the context of stock dynamics. Biglaiser et al. (1995) examines regulation of competitive firms with a flow pollutant that can invest in abatement capital. When the regulator cannot commit, firms and the regulator play a simultaneous game where firms choose investments and inputs and the regulator permit levels or taxes. They show that the first-best tax is equal to marginal damage and consequently does not depend on firm investments, then firms do not have a strategic incentive to change their investment decisions to influence future regulations and the emission tax is time consistent in this framework. More recently, Wirl (2014) has investigated a policy game between a monopoly that provides a clean technology for a polluting competitive industry and a regulator that uses an emission tax or emission permits to control a flow pollutant. The author finds that although the monopoly can be enforced to price taking behavior, the inability of the regulator to commit leads to too slow and to too little expansion of the clean technology regardless of the instrument applied to control emissions. Thus, to best of our knowledge, our paper addresses for the first time the analysis of the effects of the lack of regulator to commit on emission tax for a *stock* pollutant. The majority of papers studying the regulation of a stock pollutant assumes that firms take aggregate emissions as exogenous and behave non-strategically. Since firms take both the current and future policies as exogenous, firms solve a sequence of static optimization problems. Then, the regulator optimization problem becomes a optimal control problem where the optimal tax rate maximizes social welfare: private benefits of emissions minus environmental damages.[3] This approach has been adopted by Hoel and

[3]In Benford (1998) and Baudry (2000) the objective of the regulator is to minimize the sum of abatement costs and environmental damages. Benford (1998) describes a scheme for the control of a stock pollutant that is both incentive compatible and induces efficiency. The scheme is an extension to a dynamic setting of the scheme proposed by Kwerel (1977) for a flow pollutant. Baudry (2000)

Karp (2001, 2002), Karp and Zhang (2005, 2006) and, more recently, by Masoudi and Zaccour (2014) to compare the effects of taxes and quotas on welfare under uncertainty. Another paper where firms behave non-strategically is Xabadia et al. (2006). These authors consider a competitive industry made of heterogeneous production units, which produces a good using a fixed asset and polluting variable inputs that can be allocated to two different technologies. The production units differ by quality of the asset. They also assume that the pollution stock may be reduced by various abatement activities. In this setting, they find that the first-best policy consists of a quality differentiated input tax plus a quality differentiated technology subsidy or tax per unit of asset.[4] Finally, Yanase (2009) examines non-cooperatively policy games between national governments in a model of international pollution control in which polluting duopolists compete in a third country market. He finds that the emission tax game produces a more distortionary outcome than that in the quota game i.e. it generates more pollution and lower welfare.

Another branch of the literature has focused on the dynamics of abatement capital stock. The seminal paper is Beavis and Dobbs (1986). They solve a policy game where the regulator decides the standard and the time at which it comes into force to minimize the sum of environmental damages and firm's adjustment costs in a model where the level of a flow pollutant depends on the abatement capital stock. The authors assume that the regulator is the leader of the policy game. In the posterior literature this feature is lost and it is assumed that the environmental policy is exogenously determined. See, for instance, the papers by Xepapadeas (1992), Hartl and Kort (1996), Kort (1996), Stimming (1999), Farzin and Kort (2000) and Feenstra et al. (2001). In all these papers, the objective of the firms is to maximize the present value of net profits subject to the constraints imposed by environmental policy and the dynamics of the capital stocks, and the focus of the research is on the effects of a stricter environmental policy and on the comparison of taxes vs emission standards. Recently, Arguedas et al. (2016) has retrieved

investigates how threshold-based environmental policies may result from technological change applying the "real option" theory.

[4] In a second paper, Xabadia et al. (2008), they derive second-best policies and compare them to the efficient policy and also among themselves.

Beavis and Dobbs' (1986) approach to solve a policy game where given a quadratic fine rule, the regulator decides the standard to maximize social welfare including the social cost of sanctioning, and the firm selects the use of a polluting factor and the investment in a productive capital stock to maximize net profits: profits minus the fine for non-compliance. The authors calculate the stagewise feedback Stackelberg equilibrium where the regulator has a stagewise first-mover advantage, i.e. an instantaneous advantage at each time. An equilibrium concept we also use in this paper.

Finally, we would like to comment the papers by Saltari and Travaglini (2011) and Karp and Zhang (2012) where both the abatement capital stock and the pollution stock are taken into account in the analysis of the firm's investment decisions. Saltari and Travaglini (2011), following the approach adopted by the previous literature, assume that the environmental policy is exogenously determined. In their model, the pollution stock affects negatively the production function, and the firm has to decide about the use of a polluting factor and the abatement investment taking has given the dynamics of the pollution stock that evolves according to a geometric Brownian motion. Karp and Zhang (2012) extend Hoel and Karp (2002) allowing the representative firm to invest in abatement capital. They analyze a two-stage game where in the first stage the firm selects in each period the emissions that maximize its current profits and in the second stage the firm and the regulator play a simultaneous non-cooperative game. In this second stage, the firm decides the level of investment and the regulator the level of the policy instrument. Both papers focus on comparing taxes vs emission standards.

The remainder of the paper is organized as follows. Section 2 presents the model. Section 3 analyzes the committed regulator policy game and Section 4 the non-committed regulator policy game. In Section 5 we compare policy games to evaluate the effects of the lack of commitment. Section 6 offers some concluding remarks and points out lines for future research.

2 The Model

Our model is a dynamic extension of Petrakis and Xepapadeas' (2001) model.[5] It considers a monopolist that faces a linear (inverse) demand function given by $P(t) = a - q(t)$, where P is price and $q(t)$ is total output at time t. The firm operates with a technology that presents constant returns to scale so that the marginal cost of production, c, is constant with $a > c > 0$. The production process generates pollution emissions. After an appropriate choice of measurement units we can say that each unit of output generates one unit of pollution. The emissions can be reduced without declining output if the monopoly employs an abatement technology. The abatement technology is assumed to be the end-of-the-pipe type. Abatement costs are given by $c(w(t)) = \gamma w(t)^2/2,\ \gamma > 0$, which indicates that the abatement technology has decreasing returns to scale, with the parameter γ measuring the extent of such decreasing returns and $w(t)$ standing for emission abatement. Thus, we can write firm's (net) emissions as $s(q(t), w(t)) = q(t) - w(t)$. The stock of pollution at time t is denoted by $x(t)$, which follows the dynamic equation:

$$\dot{x}(t) = s(q(t), w(t)) - \delta x(t) = q(t) - w(t) - \delta x(t), \quad x(0) = x_0 \geq 0, \tag{1}$$

where $\delta > 0$ stands for the decay rate of pollution. The disutility from environmental deterioration is given by the damage function $D(x(t)) = dx(t)^2/2,\ d > 0$.[6]

In what follows we shall analyze two alternative policy games, each featuring a multi-stage game of complete and perfect information at each period or *instant* of time between a welfare maximizing regulator and a profit maximizing firm.[7] To be more precise, in the first policy game, which will be labeled as the *committed (or ex-ante) regulator game,* the regulator sets the level of an emission tax, then the monopolist, taking that level as given, chooses the level of abatement and finally the output. In the second policy game, the *non-committed (or ex-post) regulator game,* the monopolist and the firm simultaneously

[5] It could be seen as well as an extension of the linear-quadratic case analyzed by Benchekroun and Long (1998) to include abatement activities.

[6] Yanase (2009) uses a similar model to study the strategic effects of environmental policy in a model of international pollution control in which two polluting firms located in two different countries compete in a third country market.

[7] Notice that in our model time is a continuous variable.

choose the emission tax and abatement respectively and in a second stage the monopolist selects its output level.

Finally, we would like to point out that the focus in this paper is on *second-best policies.* As is well known, since there are two control variables to adjust because of the distortions that characterize a polluting monopoly, the regulator would need two instruments to implement the first-best or efficient solution: a subsidy per unit of production could be used to correct for market power and a tax on emissions to correct for the pollution externality. In this case, it is easy to show that the tax set by the regulator equals the marginal value of pollution stock. As already mentioned we assume that the regulator can use only one policy instrument, a tax.

2.1 The reference differential game

In addition to the committed and non-committed regulator games and to facilitate the comparison of the outcomes of these two games, we present a reference two-stage game in which in the first stage the firm and the regulator simultaneously decide abatement and the emission tax rate, respectively, and in the second stage the monopolist selects its output.

Output selection occurs in the last stage which is common to both policy games, so we begin with the analysis of this stage. The monopolist chooses its output to maximize the discounted present value of net profits:

$$\max_{\{q(t)\}} \int_0^\infty e^{-rt} \left\{ (a - q(t))q(t) - cq(t) - \frac{\gamma}{2} w(t)^2 - \tau(t)(q(t) - w(t)) \right\} dt, \tag{2}$$

subject to differential equation (1) where r is the time discount rate. We assume that the firm acts strategically at this stage because it is aware that the dynamics of the stock will be taken into account by the regulator to set up the tax.

The solution to this dynamic optimization problem must satisfy the following Hamilton-Jacobi-Bellman (HJB) equation

$$\begin{aligned} rV(x(t)) &= \max_{\{q(t)\}} \Big\{ (a - q(t))q(t) - cq(t) - \frac{\gamma}{2} w(t)^2 - \tau(t)(q(t) - w(t)) \\ &\quad + V'(x(t))(q(t) - w(t) - \delta x(t)) \Big\}, \end{aligned}$$

where $V(x(t))$ stands for the optimal current value function associated with the dynamic optimization problem, i.e. it denotes the maxima of the objective (2) subject to (1) for the current value of the state variable.

From the first-order conditions for the maximization of the right-hand side of the HJB equations, we get

$$q(\tau(t), x(t)) = \frac{1}{2}\left(a - c - \tau(t) + V'(x(t))\right). \tag{3}$$

This condition establishes that for a given pollution stock, the output of the firm decreases with the emission tax. Using this expression the present value of net profits can be written as follows

$$\int_0^\infty e^{-rt} \Big\{ (a - q(\tau(t), x(t)))q(\tau(t), x(t)) - cq(\tau(t), x(t)) - \frac{\gamma}{2}w(t)^2 \tag{4}$$
$$-\tau(t)(q(\tau(t), x(t)) - w(t)) \Big\}\, dt,$$

where $q(\tau(t), x(t))$ is given by (3).

Next, consider the *reference differential game.* In the first stage, the monopolist chooses its abatement to maximize net profits defined by (4). On the other hand, the regulator selects the welfare maximizing emission tax rate, which is defined as the present value of the sum of consumer surplus and monopoly profits minus environmental damages, that is

$$\max_{\{\tau(t)\}} \int_0^\infty e^{-rt} \left\{ (a - c)q(\tau(t), x(t)) - \frac{1}{2}q(\tau(t), x(t))^2 - \frac{\gamma}{2}w(t)^2 - \frac{d}{2}x(t)^2 \right\} dt, \tag{5}$$

subject to the differential equation (1), where $q(\tau(t), x(t))$ is given again by (3).

The optimal paths for the emission tax and the abatement are defined by the solution of the differential game between the monopolist and the regulator defined by (1), (4) and (5). Next we propose to calculate the solution to this differential game through the computation of a Markov-perfect Nash equilibrium. We use Markov strategies because they provide a subgame-perfect equilibrium that is dynamically consistent.

Markov strategies must satisfy the following system of HJB equations

$$\begin{aligned} rV(x) &= \max_{\{w\}} \left\{ (a - q(\tau, x))q(\tau, x) - cq(\tau, x) - \frac{\gamma}{2}w^2 - \tau(q(\tau, x) - w) \right. \\ &\quad \left. + V'(x)(q(\tau, x) - w - \delta x) \right\}, \qquad (6) \\ rW(x) &= \max_{\{\tau\}} \left\{ (a - c)q(\tau, x) - \frac{1}{2}q(\tau, x)^2 - \frac{\gamma}{2}w^2 - \frac{d}{2}x^2 \right. \\ &\quad \left. + W'(x)(q(\tau, x) - w - \delta x) \right\} \qquad (7) \end{aligned}$$

where $W(x)$ stands for the optimal current value function associated with the dynamic optimization problem for the regulator.[8]

We get the *instantaneous* reaction functions of the regulator and the monopolist from the first-order conditions for the maximization of the right-hand sides of the HJB equations:

$$\begin{aligned} w(\tau, x) &= \frac{1}{\gamma}\left(\tau - V'(x)\right), \qquad (8) \\ q(\tau, x) &= a - c + W'(x), \qquad (9) \end{aligned}$$

where (9) yields

$$\tau(x) = V'(x) - 2W'(x) - (a - c), \qquad (10)$$

using (3). These expressions establish that the optimal tax rate is *independent* of the monopolist abatement, and that for a given value of the pollution stock the abatement is a *strategic complement* of the emission tax i.e. an increase in the tax leads the firm to increase abatement to reduce taxes.

Once the reference differential game has been defined, it follows that the committed regulator game corresponds to the game where the regulator is the Stackelberg leader and the monopolist the Stackelberg follower, while the non-committed regulator game corresponds to the game where their roles are reversed. In the next two sections, the feedback Stackelberg equilibrium of those two policy games are characterized.

[8] Time argument will be eliminated when no confusion arises.

3 The Committed Regulator Game

To study this policy game, we assume that the regulator can move first in each period. To find the regulator's optimal policy, we apply backward induction, substituting the monopolist's reaction function given by (8) in the regulator's HJB equation, and computing the optimal strategy by maximizing the right-hand side of this equation. The resulting outcome is a stagewise feedback Stackelberg solution, which is a Markov-perfect equilibrium.[9]

Operating in this way, we get the solution for the emission tax as a function of the first derivatives of the value functions

$$\tau(x) = V'(x) - \frac{\gamma(a-c) + 2(\gamma+2)W'(x)}{\gamma+4}. \quad (11)$$

Next, substituting $\tau - V'$ in (3) and (9) we can write both output and abatement also as functions of the first derivatives of the value functions

$$q(x) = \frac{\gamma+2}{\gamma+4}(a-c+W'(x)), \quad (12)$$

$$w(x) = -\frac{\gamma(a-c) + 2(\gamma+2)W'(x)}{\gamma(\gamma+4)}. \quad (13)$$

Notice that as both output and abatement depend on $\tau - V'$, finally the optimal strategies of these two control variables are independent of the first derivative of the monopolist's value function. The emissions can be obtained as the difference between output (gross emissions) and abatement

$$s(x) = q(x) - w(x) = \frac{\gamma(\gamma+3)(a-c) + (\gamma+2)^2 W'(x)}{\gamma(\gamma+4)}. \quad (14)$$

Now, substituting the optimal strategies (12) and (13) in the HJB equation (7), the following nonlinear differential equation is obtained

$$rW(x) = \frac{\gamma+3}{2(\gamma+4)}(a-c)^2 + \frac{\gamma+3}{\gamma+4}(a-c)W'(x) + \frac{(\gamma+2)^2}{2\gamma(\gamma+4)}W'(x)^2 - \frac{d}{2}x^2 - \delta x W'(x). \quad (15)$$

[9]This concept of equilibrium has been used recently by Arguedas et al. (2016) to study the dynamic interaction between a polluting firm and a regulator who sets standards overtime. An detailed explanation of the Markovian Stackelberg equilibria can be found in Haurie et al. (2012) and several examples in Long (2010).

In order to solve this equation, we guess a quadratic representation for the value function W:

$$W^c(x) = \frac{A_1^c}{2}x^2 + B_1^c x + C_1^c, \tag{16}$$

which implies that $dW^c(x)/dx = A_1^c x + B_1^c$ and where A_1^c, B_1^c and C_1^c are unknowns to be determined.[10]

The substitution of $W^c(x)$ and $dW^c(x)/dx$ into (15) yields a system of Riccati equations that must hold for every x. Then if this system of equations for the coefficients of the value function has a solution, the optimal strategies for output and abatement would be

$$q^c(x) = \frac{\gamma+2}{\gamma+4}\left(a - c + B_1^c + A_1^c x\right), \tag{17}$$

$$w^c(x) = -\frac{1}{\gamma(\gamma+4)}\left(\gamma(a-c) + 2(\gamma+2)B_1^c + 2(\gamma+2)A_1^c x\right), \tag{18}$$

which are obtained from (12) and (13). Finally, we obtain the dynamics of the state variable in terms of the coefficients of the value function substituting (17) and (18) in (1)

$$\dot{x} = \frac{\gamma+3}{\gamma+4}(a-c) + \frac{(\gamma+2)^2}{\gamma(\gamma+4)}B_1^c + \left(\frac{(\gamma+2)^2}{\gamma(\gamma+4)}A_1^c - \delta\right)x. \tag{19}$$

Thus, if we look for a stable solution, the following condition should be satisfied

$$\frac{d\dot{x}}{dx} < 0 \rightarrow \frac{d\dot{x}}{dx} = \frac{(\gamma+2)^2}{\gamma(\gamma+4)}A_1^c - \delta < 0.$$

Applying this stability condition, we find that the system of Riccati equations has only one stable solution given by the following values for the coefficients of the regulator's value function

$$A_1^c = \frac{\gamma(\gamma+4)(r+2\delta) - \sqrt{\gamma(\gamma+4)[4d(\gamma+2)^2 + \gamma(\gamma+4)(r+2\delta)^2]}}{2(\gamma+2)^2} < 0, \tag{20}$$

$$B_1^c = \frac{\gamma(\gamma+3)(a-c)A_1^c}{\gamma(\gamma+4)(r+\delta) - (\gamma+2)^2 A_1^c} < 0, \tag{21}$$

$$C_1^c = \frac{(\gamma+3)(a-c)^2[\gamma(\gamma+4)(r+\delta)(\gamma(r+\delta) - 2A_1^c) + (\gamma+2)^2(A_1^c)^2]}{2r(\gamma(\gamma+4)(r+\delta) - (\gamma+2)^2 A_1^c)^2} > 0.$$

[10]The superscript c stands for the case of a committed regulator.

Using these coefficients and taking into account the Riccati equation for A_1^c, the steady-state pollution stock can be calculated resulting in

$$x_{SS}^c = \frac{(a-c)\gamma(\gamma+3)(r+\delta)}{(\gamma+2)^2 d + \gamma\delta(\gamma+4)(r+\delta)}, \tag{22}$$

that clearly establishes an inverse relationship between the pollution stock at the steady state and d the slope of the marginal damages curve. Thus, we can conclude the larger the marginal damages the lower the accumulation of emissions at the steady state.

Moreover, we can obtain that

Proposition 1 *The optimal production is defined by the following rule*

$$q^c(x) = \frac{(a-c)(\gamma+2)(A_1^c - \gamma(r+\delta))}{(\gamma+2)^2 A_1^c - \gamma(\gamma+4)(r+\delta)} + \frac{(\gamma+2)A_1^c}{\gamma+4}x,$$

where A_1^c is given by (20). The production is positive for $x = 0$ and decreases with the pollution stock until becomes zero for

$$\hat{x}^c = \frac{(a-c)(\gamma(r+\delta) - A_1^c)}{\gamma(\delta A_1^c + d)} > x_{SS}^c.$$

Proof. The optimal linear strategy is calculated substituting B_1^c by (21) in (17). The intersection point with the vertical axis is positive and the slope is negative provided that A_1^c is negative. Thus, the production is positive for $x = 0$ and decreases with the pollution stock. On the other hand, making $q^c(x) = 0$, $\hat{x}^c$ is obtained. The difference of this value with the steady-state pollution stock is

$$x_{SS}^c - \hat{x} = (a-c)\frac{(d+\gamma(r+\delta)\delta)(A_1^c(\gamma+2)^2 - \gamma(r+\delta)(\gamma+4))}{\gamma((\gamma+2)^2 d + \gamma(\gamma+4)(r+\delta)\delta)(\delta A_1^c + d)},$$

that is negative for $A_1^c < 0$ since $\delta A_1^c + d$ is positive. According to (20) $\delta A_1^c + d$ is positive if and only if

$$\delta\frac{\gamma(\gamma+4)(r+2\delta) - \sqrt{\gamma(\gamma+4)[4d(\gamma+2)^2 + \gamma(\gamma+4)(r+2\delta)^2]}}{2(\gamma+2)^2} + d > 0,$$

that can be rewritten as

$$\gamma\delta(\gamma+4)(r+2\delta) + 2(\gamma+2)^2 d > \delta\sqrt{\gamma(\gamma+4)[4d(\gamma+2)^2 + \gamma(\gamma+4)(r+2\delta)^2]},$$

taking square in both sides of the inequality and simplifying yields

$$2\left(\gamma+2\right)^{2}d(r\gamma\delta\left(\gamma+4\right)+2(\gamma+2)^{2}d)>0,$$

that established that δA_1^c+d is positive. ■

On the other hand, the calculation of the optimal abatement yields

Proposition 2 *The optimal abatement is given by the following rule*

$$w^c(x)=\frac{(a-c)(\gamma(r+\delta)+(\gamma+2)A_1^c)}{(\gamma+2)^2A_1^c-\gamma(\gamma+4)(r+\delta)}-\frac{2(\gamma+2)A_1^c}{\gamma(\gamma+4)}x, \tag{23}$$

where A_1^c is given by (20). If environmental damages are large enough, in particular if d is larger than

$$d'=\frac{\gamma(r+\delta)[(r+\delta)(\gamma+2)+(\gamma+4)(r+2\delta)]}{(\gamma+4)(\gamma+2)}, \tag{24}$$

then the abatement is positive for $x=0$ and increases with the pollution stock.

Proof. The optimal linear strategy for abatement is obtained substituting B_1^c by (21) in (18). The slope is positive provided that A_1^c is negative what means that the abatement increases with the pollution stock. However, the intersection point with the vertical axis depends on the sign of $\gamma(r+\delta)+(\gamma+2)A_1^c$. Substituting A_1^c by (20) this expression is negative if and only if

$$\gamma(r+\delta)+(\gamma+2)\frac{\gamma(\gamma+4)(r+2\delta)-\sqrt{\gamma(\gamma+4)[4d(\gamma+2)^2+\gamma(\gamma+4)(r+2\delta)^2]}}{2(\gamma+2)^2}<0,$$

that can be reordering yielding

$$2\gamma(r+\delta)(2+\gamma)+\gamma(4+\gamma)(r+2\delta)<\sqrt{\gamma(4+\gamma)[4d(2+\gamma)^2+\gamma(4+\gamma)(r+2\delta)^2]}.$$

Taking square in both side of the inequality gives

$$\gamma(r+\delta)[(r+\delta)(2+\gamma)+(4+\gamma)(r+2\delta)]-(4+\gamma)(2+\gamma)d<0,$$

that is negative when $d>d'$. Thus, if $d>d'$, then $\gamma(r+\delta)+(\gamma+2)A_1^c$ is negative and the abatement is positive for $x=0$. ■

Finally, using (14) and the coefficient B_1^c we calculated the optimal strategy for emissions.

Proposition 3 *The optimal emissions are defined by the following rule*

$$s^c(x) = \frac{(a-c)\gamma(\gamma+3)(r+\delta)}{\gamma(\gamma+4)(r+\delta) - (\gamma+2)^2 A_1^c} + \frac{(\gamma+2)^2 A_1^c}{\gamma(\gamma+4)} x, \tag{25}$$

where A_1^c is given by (20). The emissions are positive for $x = 0$ and decrease with the pollution stock until become zero for

$$\check{x}^c = \frac{\gamma(a-c)(\gamma+3)(r+\delta)}{(\gamma+2)^2(\delta A_1^c + d)} > x_{SS}^c, \tag{26}$$

being $\check{x}^c$ lower than $\hat{x}^c$ the value of the stock that makes zero the output.

Proof. As A_1^c is negative, the intersection point with the vertical axis is positive and the slope is negative. Thus, the emissions are positive for $x = 0$ and decrease with the pollution stock. Moreover, making $s^c(x) = 0$, $\check{x}^c$ is obtained. The difference of this value with the steady-state pollution stock is

$$x_{SS}^c - \check{x}^c = \frac{(a-c)\gamma(\gamma+3)(r+\delta)[(\gamma+2)^2\delta A_1^c - \gamma(\gamma+4)(r+\delta)\delta]}{[(\gamma+2)^2 d + \gamma(\gamma+4)(r+\delta)\delta](\gamma+2)^2(\delta A_1^c + d)},$$

that is negative for A_1^c negative since $\delta A_1^c + d$ is positive as has been shown in the proof of Prop. 1. Finally, $\hat{x}^c$ and $\check{x}^c$ are compared

$$\hat{x}^c - \check{x}^c = \frac{(a-c)(\gamma(r+\delta)(\gamma+4) - (\gamma+2)^2 A_1^c)}{\gamma(\gamma+2)^2(\delta A_1^c + d)}.$$

The difference is positive for A_1^c negative since $\delta A_1^c + d$ is positive establishing that $\hat{x}^c > \check{x}^c$.
■

The features of the model allow to calculate the optimal strategies for production, abatement and net emissions without solving the monopolist's HJB equation. However, the next step, the calculation of the regulator optimal policy, cannot be given without solving this equation. With this aim, we substitute the tax given by (11), the output defined by (12) and the abatement specified by (13) in the monopolist's HJB equation given by (6) obtaining the following differential equation

$$\begin{aligned} rV(x) &= \frac{2\gamma^2 + 9\gamma + 8}{2(\gamma+4)^2}(a-c)^2 + \frac{2(\gamma+3)(\gamma+2)}{(\gamma+4)^2}(a-c)W'(x) \\ &+ \frac{(\gamma+2)^3}{\gamma(\gamma+4)^2} W'(x)^2 - V'(x)\delta x. \end{aligned} \tag{27}$$

In order to solve this equation, we also guess a quadratic representation

$$V^c(x) = \frac{A_2^c}{2}x^2 + B_2^c x + C_2^c, \tag{28}$$

that yields $dV^c(x)/dx = A_2^c x + B_2^c$. The substitution of $V^c(x)$ and $dV^c(x)/dx$ into (27) yields a system of Riccati equations whose solution is

$$A_2^c = \frac{2(\gamma+2)^3}{\gamma(r+2\delta)(\gamma+4)^2}(A_1^c)^2 > 0,$$

$$B_2^c = \frac{2(a-c)\gamma(\gamma+3)(\gamma+2)A_1^c}{(\gamma+4)(\gamma(\gamma+4)(r+\delta)-(\gamma+2)^2 A_1^c)} < 0,$$

$$C_2^c = \frac{(a-c)^2[(2\gamma^2+9\gamma+8)\gamma^2(r+\delta)^2 - 2\gamma(\gamma+4)(\gamma+2)(r+\delta)A_1^c + (\gamma+2)^3(A_1^c)^2]}{2r(\gamma(\gamma+4)(r+\delta)-(\gamma+2)^2 A_1^c)^2} > 0.$$

Then eliminating $V'(x)$ and $W'(x)$ in (11) using the coefficients of the value functions, the optimal policy is obtained.

Proposition 4 *The optimal policy is given by the following rule*

$$\tau^c(x) = -\frac{\gamma(a-c)}{\gamma+4} + \frac{2(\gamma+2)d}{(r+2\delta)(\gamma+4)}x. \tag{29}$$

If environmental damages are large enough, in particular if d is larger than

$$d'' = \frac{\gamma\delta(\gamma+4)(r+2\delta)(r+\delta)}{(r(\gamma+4)+2\delta)(\gamma+2)},$$

there exists a threshold value for the stock of pollution, x'', given by the following expression

$$x'' = \frac{\gamma(a-c)(r+2\delta)}{2(\gamma+2)d} < x_{SS}^c$$

such that the optimal policy consists of applying a decreasing subsidy for $x < x''$ and an increasing tax for $x > x''$.

Proof. The value of the pollution stock x'' is calculated making $\tau^c(x) = 0$ and the difference of this value with the steady-state pollution stock is

$$x_{SS}^c - x'' = \frac{(a-c)\gamma[(r(\gamma+4)+2\delta)(\gamma+2)d - \gamma\delta(\gamma+4)(r+2\delta)(r+\delta)]}{2(\gamma+2)[d(\gamma+2)^2 d + \gamma(\gamma+4)(r+\delta)\delta]d}$$

that is positive for $d > d''$. ■

The intuition of this result is straightforward. When the pollution stock is zero, the marginal damages are also zero and the inefficiency of the monopoly is caused only by its power market. It is well known that in this case the monopoly reduces its output to take advantage of a larger price selecting a level of production lower than the efficient level. Then, the optimal policy consists of setting up a subsidy to stimulate production.[11] Thus, when the pollution stock is zero or when is low the environmental problem is not relevant and the regulator applies exclusively an industrial policy. However, once the emissions accumulate causing environmental damages, the inefficiency of the polluting monopoly is also caused by a negative externality. In other words, there are two market failures operating at the same time. The point is that a negative externality induces the firm to produce more than the efficient level and in this case, as is also well known, the optimal policy, when the firm is competitive, consists of applying an emission tax to reduce the output and the emissions of the firm. Thus, the sign of the optimal policy applied by the regulator when the two market failures are acting at the same time can be negative (a subsidy) or positive (a tax) depending on the stock pollution level and also on the importance of the marginal damages. Prop. 4 defines a threshold value for d that implies that a tax is applied at the steady state. In other words, it implies that environmental damages are serious enough to justify that the environmental policy (taxation) dominates the industrial policy (subsidization) at the steady state.

The comparison of d' and d'' yields an ambiguous sign. For this reason, we assume that $d > \max\{d', d''\}$. The consequences of this assumption are that the non-negative constraint is satisfied by the control variables of the model in the interval $[0, \check{x}^c]$ because $d > d'$, and that the optimal policy at the steady state consists of setting a tax on emissions given that $d > d''$.

Finally, we characterize the dynamics of the pollution stock. Substituting B_1^c in (19),

[11] Observe that a subsidy is compatible with a positive abatement effort because it depends not only on the tax but also on the shadow price of the pollution stock. According to (8), w depends on the difference $\tau - V'$ with V' negative so that this difference can be positive even when τ is negative.

we obtain the following differential equation for the pollution stock

$$\dot{x} = \left(\frac{(\gamma+2)^2}{\gamma(\gamma+4)} A_1^c - \delta \right) x + \frac{(a-c)\gamma(\gamma+3)(r+\delta)}{\gamma(\gamma+4)(r+\delta) - (\gamma+2)^2 A_1^c},$$

whose solution is

$$x^c(t) = (x_0 - x_{SS}^c) e^{\alpha^c t} + x_{SS}^c, \quad \text{with } \alpha^c = \frac{(\gamma+2)^2}{\gamma(4+\gamma)} A_1^c - \delta < 0, \tag{30}$$

for x_0 in the interval $[0, \check{x}^c]$. For $x_0 = 0$, the dynamics of the pollution stock is

$$x^c(t) = x_{SS}^c (1 - e^{\alpha^c t}), \tag{31}$$

that establishes that

Remark 1 *The equilibrium tax and abatement increase asymptotically to their steady-state values whereas the production and emissions decrease when the initial value for the pollution stock is zero.*

4 The Non-committed Regulator Game

If there is a lack of commitment, the regulator will choose the tax rate after the firm has selected the level of abatement. This means that the monopolist moves first in each period and that it could use this strategic advantage to influence the environmental policy in its own interest. However, according to (10) the optimal policy does not depend on abatement and the firm cannot exercise this influence. In other words, the regulator has a dominant strategy for any level of abatement that only depends on the pollution stock. In this case, the feedback Stackelberg equilibrium coincides with the Markov-perfect Nash equilibrium of the reference differential game.[12] In this section we compute the Nash equilibrium to evaluate the consequences of the lack of commitment.

Substituting (10) in (9) and (8) we obtain the output and the abatement as a function of the first derivatives of the value functions

$$q(x) = a - c + W'(x), \tag{32}$$

$$w(x) = -\frac{1}{\gamma}(a - c + 2W'(x)). \tag{33}$$

[12] In Rubio (2006) conditions for obtaining this coincidence in differential games are defined.

Notice that again the optimal strategies of both variables are independent of the first derivatives of the monopolist's value function. The emission can be calculated as the difference between output (gross emissions) and abatement yielding

$$s(x) = q(x) - w(x) = \frac{1}{\gamma}\left((\gamma+1)(a-c) + (\gamma+2)W'(x)\right). \tag{34}$$

Now, substituting the optimal strategies (32) and (33) in the HJB equation (7), the following nonlinear differential equation for the regulator's value function is obtained

$$rW(x) = \frac{\gamma-1}{2\gamma}(a-c)^2 + \frac{\gamma-1}{\gamma}(a-c)W'(x) + \frac{1}{2}W'(x)^2 - \frac{d}{2}x^2 - \delta x W'(x). \tag{35}$$

We also guess in this section a quadratic representation for the value function W[13]:

$$W^{nc}(x) = \frac{A_1^{nc}}{2}x^2 + B_1^{nc}x + C_1^{nc}, \tag{36}$$

whose first derivative is $dW^{nc}(x)/dx = A_1^{nc}x + B_1^{nc}$.

The substitution of the first derivative and the proposed value function in (35) gives a system of Riccati equations. If this system of equations for the coefficients of the value function has a solution, the optimal strategies for the output and the abatement are

$$q^{nc}(x) = a - c + B_1^{nc} + A_1^{nc}x, \tag{37}$$

$$w^{nc}(x) = -\frac{1}{\gamma}\left(a - c + 2B_1^{nc} + 2A_1^{nc}\right), \tag{38}$$

that are derived from (32) and (33) by substitution of the first derivative of the value function. Using these two optimal strategies to eliminate the output and the abatement in (1), the dynamics of the pollution stock is

$$\dot{x}^{nc} = \frac{1+\gamma}{\gamma}(a-c) + \frac{\gamma+2}{\gamma}B_1^{nc} + \left(\frac{\gamma+2}{\gamma}A_1^{nc} - \delta\right)x, \tag{39}$$

so that the stability condition implies the following constraint on A_1^{nc}

$$\frac{d\dot{x}}{dx} < 0 \rightarrow \frac{d\dot{x}}{dx} = \frac{\gamma+2}{\gamma}A_1^{nc} - \delta < 0.$$

Only one of the roots of the first equation of the system of Riccati equations satisfies this constraint

$$A_1^{nc} = \frac{1}{2}\left(r + 2\delta - \sqrt{(r+2\delta)^2 + 4d}\right) < 0. \tag{40}$$

[13]The superscript nc stands for the case of a non-committed regulator.

The other two coefficients of the value functions can be written as a function of A_1^{nc}

$$B_1^{nc} = \frac{(\gamma-1)(a-c)A_1^{nc}}{\gamma(r+\delta-A_1^{nc})}, \tag{41}$$

$$C_1^{nc} = \frac{(\gamma-1)(a-c)^2(\gamma(r+\delta)^2-2(r+\delta)A_1^{nc}+(A_1^{nc})^2)}{2r\gamma^2(r+\delta-A_1^{nc})^2}.$$

Notice that the sign of both coefficients depends on γ. If $\gamma > 1$, B_1^{nc} is negative and C_1^{nc} is positive.

For this solution of the Riccati equations, the steady-state pollution stock is

$$x_{SS}^{nc} = \frac{(a-c)(\gamma(\gamma+1)(r+\delta)-2A_1^{nc})}{\gamma(r+\delta-A_1^{nc})(\gamma\delta-(\gamma+2)A_1^{nc})} > 0. \tag{42}$$

Moreover, we can get that

Proposition 5 *The optimal production is defined by the following rule*

$$q^{nc}(x) = \frac{(a-c)(\gamma(r+\delta)-A_1^{nc})}{\gamma(r+\delta-A_1^{nc})} + A_1^{nc}x,$$

where A_1^{nc} is given by (40). The production is positive for $x=0$ and decreases with the pollution stock until becomes zero for

$$\hat{x}^{nc} = \frac{(a-c)(\gamma(r+\delta)-A_1^{nc})}{\gamma(\delta A_1^{nc}+d)} > x_{SS}^{nc}.$$

Proof. The optimal linear strategy is obtained eliminating B_1^{nc} in (37) using (41). The intersection point with the vertical axis is positive and the slope is negative since A_1^{nc} is negative. Thus, the production is positive for $x=0$ and decreases with the pollution stock. The value of $\hat{x}^{nc}$ is calculated making $q^{nc}(x)=0$. The difference of this value with the steady-state pollution stock is

$$x_{SS}^{nc} - \hat{x}^{nc} = -\frac{(a-c)\gamma((r+\delta)\gamma\delta+d)(r+\delta-A_1^{nc})}{\gamma(\delta A_1^{nc}+d)(r+\delta-A_1^{nc})(\gamma\delta-(\gamma+2)A_1^{nc})}.$$

The sign of this difference is negative for A_1^{nc} negative since $\delta A_1^{nc}+d$ is positive. According to (40) $\delta A_1^{nc}+d > 0$ if and only if

$$\delta\frac{2\delta+r-\sqrt{(2\delta+r)^2+4d}}{2}+d > 0,$$

that can be rewritten as

$$2\delta^2 + \delta r + 2d > \delta\sqrt{(2\delta + r)^2 + 4d}.$$

Taking square in both sides of the inequality and simplifying terms, the following expression is obtained

$$4d(d + \delta(r + \delta)) > 0,$$

that allows us to conclude that $\delta A_1^{nc} + d$ is positive. ∎

On the other hand, the derivation of the optimal strategy for abatement gives

Proposition 6 *The optimal abatement is given by the following rule*

$$w^{nc}(x) = -\frac{(a - c)(\gamma(r + \delta) + (\gamma - 2)A_1^{nc})}{\gamma^2(r + \delta - A_1^{nc})} - \frac{2A_1^{nc}}{\gamma}x,$$

where A_1^{nc} is given by (40). If γ is larger than 2 and environmental damages are large enough, in particular if d is larger than

$$d''' = \frac{\gamma^2(r + \delta)^2 + \gamma(r + \delta)(2\delta + r)(\gamma - 2)}{(\gamma - 2)^2},$$

then the abatement is positive for $x = 0$ and increases with the pollution stock.

Proof. The optimal linear strategy for abatement is obtained substituting in (38) B_1^{nc} by (41). The slope is positive for A_1^{nc} negative. Thus, the abatement increases with the pollution stock. The intersection point with the vertical axis is negative for $\gamma \leq 2$ regardless of the value of d. For $\gamma > 2$, it is positive provided that $\gamma(r + \delta) + (\gamma - 2)A_1^{nc}$ is negative. This implies, substituting A_1^{nc} by (40), that

$$\gamma(r + \delta) + (\gamma - 2)\frac{2\delta + r - \sqrt{(2\delta + r)^2 + 4d}}{2} < 0,$$

that can be reordering giving

$$2\gamma(r + \delta) + (2\delta + r)(\gamma - 2) < (\gamma - 2)\sqrt{(2\delta + r)^2 + 4d}.$$

Taking square in both sides of the inequality yields

$$\gamma^2(r + \delta)^2 + \gamma(r + \delta)(2\delta + r)(\gamma - 2) < (\gamma - 2)^2 d,$$

that is satisfied when $d > d'''$. Thus, if we assume that $d > d'''$, $\gamma(r+\delta) + (\gamma - 2)A_1^{nc}$ is negative and the abatement is positive for $x = 0$. ■

Finally, we calculate the optimal strategy for emissions eliminating W' in (34).

Proposition 7 *The optimal emissions are defined by the following rule*

$$s^{nc}(x) = \frac{(a-c)(\gamma(\gamma+1)(r+\delta) - 2A_1^{nc})}{\gamma^2(r+\delta - A_1^{nc})} + \frac{(\gamma+2)A_1^{nc}}{\gamma}x,$$

where A_1^{nc} is given by (40). The emissions are positive for $x = 0$ and decrease with the pollution stock until become zero for

$$\check{x}^{nc} = \frac{(a-c)(\gamma(\gamma+1)(r+\delta) - 2A_1^{nc})}{\gamma(\gamma+2)(d+\delta A_1^{nc})} > x_{SS}^{nc},$$

being $\check{x}^{nc}$ lower than $\hat{x}^{nc}$, the stock value that makes zero the output.

Proof. The intersection with the vertical axis is positive and the slope is negative provided that A_1^{nc} is negative. Thus, the emissions are positive for $x = 0$ and decrease with the pollution stock. Moreover, $\check{x}^{nc}$ is obtained making $s^{nc}(x) = 0$. The difference of this value of the pollution stock with the steady-state value is

$$x_{SS}^{nc} - \check{x}^{nc} = \frac{(a-c)\delta\gamma^2(\gamma+1)(r+\delta)^2}{\gamma(\gamma+2)(d+\delta A_1^{nc})(r+\delta - A_1^{nc})((\gamma+2)A_1^{nc} - \gamma\delta)},$$

that is negative for A_1^{nc} negative provided that $\delta A_1^{nc} + d$ is positive as has been showed in the proof of Prop. 5. Finally, $\hat{x}^{nc}$ and $\check{x}^{nc}$ are compared yielding

$$\hat{x}^{nc} - \check{x}^{nc} = \frac{(a-c)((r+\delta)\gamma - \gamma A_1^{nc})}{\gamma(\gamma+2)(d+\delta A_1^{nc})}.$$

This difference is positive for A_1^{nc} negative since $\delta A_1^{nc} + d$ is positive, concluding that $\hat{x}^{nc} > \check{x}^{nc}$. ■

As occurs for the solution of the committed regulator the optimal strategies for production, abatement and net emissions can be calculated without solving the monopolist's HJB equation. However, to derive the optimal policy it is necessary to solve this equation. With this aim, we substitute the tax given by (10), the output defined by (32) and the abatement specified by (33) in the monopolist's HJB equation obtaining the following differential equation for the monopolist's value function

$$rV(x) = \frac{1+2\gamma}{2\gamma}(a-c)^2 + \frac{2(\gamma+1)}{\gamma}(a-c)W'(x) + \frac{\gamma+2}{\gamma}W'(x)^2 - V'(x)\delta x. \quad (43)$$

We also guess a quadratic representation for this case

$$V^{nc}(x) = \frac{A_2^{nc}}{2}x^2 + B_2^{nc}x + C_2^{nc},$$

whose first derivative is $dV^{nc}(x)/dx = A_2^{nc}x+B_2^{nc}$. The substitution of $V^{nc}(x),\ dV^{nc}(x)/dx$ and also $dW^{nc}(x)/dx$ into (43) results in a system of Riccati equations whose solution is

$$\begin{aligned}
A_2^{nc} &= \frac{2(\gamma+2)}{\gamma(r+2\delta)}(A_1^{nc})^2 > 0, \\
B_2^{nc} &= \frac{2(a-c)((\gamma+1)(r+\delta)\gamma A_1^{nc} - 2(A_1^{nc})^2)}{\gamma^2(r+\delta)(r+\delta-A_1^{nc})} < 0, \\
C_2^{nc} &= \frac{(a-c)^2[\gamma^2(1+2\gamma)(r+\delta)^2 - 2\gamma(\gamma+2)(r+\delta)A_1^{nc} + (\gamma^2-2\gamma+4)(A_1^{nc})^2]}{2r\gamma^3(r+\delta-A_1^{nc})^2} > 0. \quad (44)
\end{aligned}$$

Next, we derive the optimal policy eliminating $V'(x)$ and $W'(x)$ using the coefficients of the value functions.

Proposition 8 *The optimal policy is given by the following rule*

$$\begin{aligned}
\tau^{nc}(x) &= \frac{(a-c)[\gamma^2(r+\delta)^2 - \gamma(\gamma+4)(r+\delta)A_1^{nc} + 4(A_1^{nc})^2]}{\gamma^2(r+\delta)(A_1^{nc}-r-\delta)} \\
&+ \frac{2(\gamma+2)(A_1^{nc})^2 - 2\gamma(r+2\delta)A_1^{nc}}{\gamma(r+2\delta)}x. \quad (45)
\end{aligned}$$

where A_1^{nc} is given by (40). $\tau^{nc}(x)$ increases with the pollution stock but is negative for $x=0$, i.e. the optimal policy consists of setting up a subsidy for $x=0$.

Although in this case it is not possible to derive an explicit value for the parameter d above which the optimal policy at the steady state is a tax, the fact that τ^{nc} increases with the pollution stock suggests that this is a feasible outcome of the non-committed regulator game. The numerical analysis developed in the next section confirms the conjecture. The intuition behind this result is the same that in the case of a committed regulator. Notice that there are no qualitative differences between the optimal strategies obtained for both policy games: output and emissions decrease with the pollution stock, whereas abatement and the tax augment. In this section is assumed that $\gamma > 2$ and that $d > d'''$. These two assumptions guarantee that abatement is positive for $x=0$ so that we can conclude that the control variables of the model satisfy the non-negative constraint in the interval $[0, \check{x}^{nc}]$.

Finally, we derive the dynamics of the pollution stock. Substituting B_1^{nc} in (39), the following differential equation for the pollution stock is obtained

$$\dot{x}^{nc} = \left(\frac{\gamma + 2}{\gamma} A_1^{nc} - \delta \right) x + \frac{(a-c)(\gamma(\gamma+1)(r+\delta) - 2A_1^{nc})}{\gamma^2 (r + \delta - A_1^{nc})},$$

whose solution is

$$x^{nc}(t) = (x_0 - x_{SS}^{nc}) e^{\alpha^{nc} t} + x_{SS}^{nc}, \quad \alpha^{nc} = \frac{\gamma+2}{\gamma} A_1^{nc} - \delta < 0, \tag{46}$$

for x_0 in the interval $[0, \check{x}^{nc}]$. If $x_0 = 0$ then this dynamics reduces to

$$x_\tau^{nc}(t) = x_{\tau SS}^{nc}(1 - e^{\alpha^{nc} t}), \tag{47}$$

and we can conclude that

Remark 2 *The equilibrium tax and abatement increase asymptotically to their steady-state values, whereas the production and emissions decrease when the initial value for the pollution stock is zero.*

5 Comparing Policy Games

In this section we address which are the effects of the lack of commitment on the different variables of the model and payoffs. In particular, we are interested in finding out whether the lack of commitment might be welfare improving. Usually, the argument to expect a larger welfare when the regulator revises its policy once the firm has selected its level of abatement is that discretionality (flexibility) yields a larger welfare than commitment. Nevertheless, it should be taken into account that in our model the strategic advantage of the firm when it moves first disappears because the optimal policy of the regulator does not depend on abatement. Thus, what we really investigate in this section are the differences between the feedback Stackelberg equilibrium when the leader is the regulator and the feedback Nash equilibrium. In this last case, both the regulator and the firm select simultaneously in each period the tax rate and the level of abatement respectively.

The focus of the comparison is on *interior solutions* of both policy games. To guarantee an interior solution we assume that $\gamma > 2$ and that $d > \max\{d'', d'''\}$. It is easy to show that $d''' > d'$. Then if d is larger than d''' is also larger than d' and, according to Prop. 2, the optimal abatement is positive for the commitment solution. Furthermore, $d > d''$ guarantees that the optimal policy at the steady state is to tax emissions for the committed regulator game as has been established in Prop. 4. Finally, $d > d'''$ along with $\gamma > 2$ yield a positive abatement for the non-commitment solution according to Prop. 6.

We begin the comparison calculating the difference between the steady-state values of the pollution stock. The result is

Lemma 1 *The steady state for the pollution stock under commitment is always lower than under non-commitment, i.e.* $x^c_{SS} < x^{nc}_{SS}$.

Proof. See Appendix. ■

Next, we study the effects of the lack of commitment on the optimal policy. First, we compare the steady-state values of the emission tax. The comparison gives the following result

Proposition 9 *If environmental damages are large enough, in particular if d is larger than*

$$d^{iv} = \frac{\gamma(\gamma+4)(r+\delta)(r+2\delta)(2\gamma^2(r+\delta)+2\gamma(3r-\delta)-16\delta)}{4(\gamma+2)[\gamma^2(r+3\delta)+\gamma(5r+13\delta)+8(r+2\delta)]}, \tag{48}$$

then the steady-state tax of the commitment solution is larger than the steady-state tax of the non-commitment solution, i.e. $\tau^{nc}(x^{nc}_{SS}) < \tau^c(x^c_{SS})$.

To show this result we calculate first the steady-state value of the emission tax for the commitment solution substituting the steady-state value of the pollution stock in the tax rule (29). Then using the tax rule of the non-commitment solution given by (45), we calculate the value of the pollution stock for which this tax rule yields the steady-state value of the emission tax for the commitment solution, and show that this value of the pollution stock is larger than the steady-state pollution stock corresponding to the non-commitment solution. Then taking into account that the tax rules increase with respect to the pollution stock, it can be concluded that the steady-state value of the emission tax for the non-commitment solution must be lower than the value that the tax takes at the steady state for the committed policy game. This is the sketch of the proof. Nevertheless, for the interested reader the details can be checked at the Appendix.

This proposition defines a sufficient condition so that this result could hold for values of d below the lower bound (48). The numerical exercise that closes this section shows that this condition is not very restrictive. Thus, we can expect that the steady-state tax rate for the committed regulator game is larger than the steady-state tax rate for the non-committed regulator game for a wide constellation of parameter values.

Now, we study the effects of the lack of commitment on the optimal tax rule evaluating how the intersection point with the vertical axis, i.e. the subsidy for $x = 0$, and the slope of the optimal tax rule change. The results are

Lemma 2 *The slope of the optimal tax rule is lower under commitment than under non-commitment, while the optimal tax for a null pollution stock is greater under commitment than under non-commitment i.e.* $\tau^{nc}(0) < \tau^{c}(0) < 0,\ 0 < m^{c} < m^{nc}$, *where* m^{c} *and* m^{nc} *denote the slopes of the optimal tax strategy for the commitment and non-commitment solutions, respectively.*

Proof. See Appendix. ■

If sufficient condition (48) is satisfied it is immediate to conclude from the previous results that

Corollary 1 *The optimal policy of the commitment solution gives larger taxes than the optimal policy of non-commitment solution for* x *in the interval* $[0, x_{SS}^{nc}]$, *i.e.* $\tau^{c}(x) > \tau^{nc}(x),\ \forall x \in [0, x_{SS}^{nc}]$.

Proof. According to Lemma 2, the optimal tax rules must intersect once. Then if we denote by x^{ip} the value of the pollution stock defined by the intersection point, $\tau^{c}(x)$ must be larger than $\tau^{nc}(x)$ for all x in the interval $[0, x^{ip})$ also by Lemma 2. Suppose that $x_{\tau SS}^{nc}$ is larger than or equal to x^{ip}. In this case according to Prop. 10, x_{SS}^{nc} should be lower than x^c_{SS} but this contradicts Lemma 1. Thus, x^{nc}_{SS} is lower than x^{ip} and $\tau^c(x)$ is larger than $\tau^{nc}(x)$ for all x in the interval $[0, x^c_{SS}]$. ■

After obtaining these results, to advance in the comparative analysis we need to assume that the initial stock is zero. This assumption allows us to compare the optimal paths of the pollution stock and the payoffs of both players.

Proposition 10 *The pollution stock under commitment is always lower than under non-commitment, i.e.* $x^c(t) < x^{nc}(t)$ *for* $t > 0$.

Proof. Firstly, we show that α^c is lower than α^{nc}. Suppose that $\alpha^c \geq \alpha^{nc}$ then substituting A^c_1 defined by (20) in α^c given by (30) and A^{nc}_1 defined by (40) in α^{nc} given by (46), this inequality implies that

$$\begin{aligned} & \frac{2\delta + r - \sqrt{(2\delta + r)^2 + 4d}}{2} \\ \geq \; & \frac{\gamma + 2}{\gamma +} \frac{\gamma(\gamma+4)(r+2\delta) - \sqrt{\gamma(\gamma+4)[4d(\gamma+2)^2 + \gamma(\gamma+4)(r+2\delta)^2]}}{2(\gamma+2)^2}, \end{aligned}$$

that simplifying terms yields

$$\begin{aligned} & (\gamma+2)(\gamma+4)(\sqrt{(2\delta+r)^2+4d} - 2\left(\gamma+4\right)\left(r+2\delta\right) \\ \leq \; & \sqrt{\gamma(\gamma+4)[4d(\gamma+2)^2 + \gamma(\gamma+4)(r+2\delta)^2]}, \end{aligned}$$

where the left-hand side is positive for $d \geq 0$. Notice that the expression is positive for $d = 0$ and increasing with d. Taking square in both sides of the inequality gives

$$\left(\gamma+4\right)\left(r+2\delta\right)^2 + 4d\left(\gamma+2\right) \leq \left(r+2\delta\right)\left(4+\gamma\right)\sqrt{(2\delta+r)^2+4d},$$

that taking square again yields the following contradiction

$$4d\gamma\left(r+2\delta\right)^2\left(\gamma+4\right) + 16d^2\left(\gamma+2\right)^2 \leq 0.$$

Thus, we can conclude that α^c is lower than α^{nc}. To show that $x^c(t) < x^{nc}_{\tau}(t)$ for all $t > 0$, we suppose that there exists an intersection point between the temporal paths of the pollution stock $x^c(t) = x^{nc}(t)$. Then according to (31) and (47) it must be satisfied that

$$x^c_{SS}(1 - e^{\alpha^c t}) = x^{nc}_{SS}(1 - e^{\alpha^{nc} t}),$$

where $\alpha^{nc} < \alpha^{c} < 0$ and $x_{SS}^{c} < x_{SS}^{nc}$. This equality implies the following inequality

$$\frac{1-e^{\alpha^{c}t}}{1-e^{\alpha^{nc}t}} = \frac{x_{SS}^{nc}}{x_{SS}^{c}} > 1,$$

that requires that

$$e^{\alpha^{c}t} < e^{\alpha^{nc}t},$$

which is a contradiction for $\alpha^{nc} < \alpha^{c} < 0$. Thus there is no intersection point between the temporal paths of the stock of pollution and as x_{SS}^{c} is lower than x_{SS}^{nc} it can be concluded that $x^{c}(t)$ is lower than $x^{nc}(t)$ for all $t > 0$. ■

The comparison of players' payoffs yields the following result

Proposition 11 *The social welfare is lower when the regulator is unable to commit to a specific emission tax level whereas net profits are larger.*

Proof. See Appendix. ■

Thus, the lack of commitment has a cost in welfare terms and the idea that discretionality is better than commitment does not show in the case of the regulation of a polluting monopolist with a pollutant stock. The lack of commitment benefits the firm but does not increase social welfare.

Unfortunately, the comparison of the steady-state values for the rest of variables is ambiguous because it involves the coefficients A of the value functions corresponding to the two solutions. For this reason, we develop a numerical exercise to explore what the model predicts about the steady-state values of production, abatement and emissions and also about the comparison of social welfare and net profits for values of the initial pollution stock different from zero.

In this numerical example we keep constant r and δ and consider variations on γ and d, parameters that determine the slope of the marginal abatement costs and the marginal environmental damages respectively. We assume two reasonable values for r and δ : $r = 0.05$ and $\delta = 0.10$. First we compute the lower bounds for parameter d defined in the paper to get an idea whether the conditions imposed on this parameter are very restrictive.

$\gamma\backslash d$	d'	d''	d'''	d^{iv}
2.50	2.95×10^{-2}	2.58×10^{-2}	0.750	2.20×10^{-2}
5.00	3.93×10^{-2}	3.71×10^{-2}	0.125	2.08×10^{-2}
10.00	4.73×10^{-2}	4.86×10^{-2}	0.082	0.061
20.00	5.28×10^{-2}	5.84×10^{-2}	0.069	0.142

Table 1. Lower bounds of d for r=0.05 and δ=0.10.

From Table 1 it can be concluded that for the values of this example it would be enough with assuming that $d > 1$ to guarantee that abatement is positive for $x = 0$ for both policy games (Prop. 2 and 6), that the steady-state value of τ is positive when there is commitment (Prop. 4) and that the steady-state value of the tax is larger when the regulator is able to commit with the environmental policy (Prop. 10).

In Table 2 the steady-state values for production are represented for $a = 1000$ and $c = 20$. In each box, the first figure stands for the steady-state value corresponding to the commitment solution, whereas the second figure represents the steady-state value for the non-commitment solution.

$\gamma\backslash d$	2.50	5.00	10.00	20.00
2.50	219.98, 226.17	218.88, 223.24	218.33, 221.40	218.05, 220.22
5.00	143.41, 146.01	141.71, 143.53	140.86, 142.13	140.43, 141.33
10.00	86.06, 86.98	83.87, 84.50	82.77, 83.23	82.22, 82.53
20.00	49.60, 49.88	47.08, 47.27	45.81, 45.95	45.18, 45.27

Table 2. Steady-state values of production.

According to figures that appear in this table, the steady-state value of production when there is non-commitment is always larger than the corresponding value of the commitment solution. Moreover, for both regimens the output decreases both with respect to d and γ. However, an increase in d has a lower effect on the steady-state value of production that the effect caused by an increase in γ. In other words, a variation in abatement costs causes a stronger change in the level of production than the change caused by the same variation in environmental damages. Finally, it could be pointed out that the higher γ and d, the lower the differences in production at the steady state.

$\gamma\backslash d$	2.50	5.00	10.00	20.00
2.50	216.01, 211.07	216.89, 213.41	217.34, 214.88	217.56, 215.82
5.00	138.63, 137.59	139.31, 138.59	139.66, 139.15	139.83, 139.47
10.00	80.79, 80.60	81.23, 81.10	81.45, 81.36	81.56, 81.49
20.00	44.04, 44.01	44.29, 44.27	44.42, 44.40	44.48, 44.47

Table 3. Steady-state values of abatement.

The figures in Table 3 establish that the steady-state value of abatement when the regulator is able to commit is larger than the steady-state value corresponding to the non-commitment solution. They also establish that the abatement increases with d and decreases with γ. In other words, the larger the damages the larger the abatement, whereas the contrary occurs with the abatement costs. Again, the effect of a change in damages on the steady-state values is significantly weaker that the effect of a change in abatement costs, and the differences in the abatement at the steady state are minimal for big values of γ and d.

As the output is lower and the abatement is larger at the steady state for the commitment solution, the emissions are higher when there is not commitment. Moreover, as the output decreases with environmental damages and the abatement increases both things for both solutions, the emissions are decreasing with respect to d for both solutions. However, the larger the abatement costs the larger the emissions for the committed policy game, although the contrary occurs for the non-committed policy game. This difference is explained by the fact that the reduction in production for the non-commitment solution is larger than the reduction in production for the commitment solution when the abatement costs increase whereas the contrary occurs for the abatement. The result is that the net effect on emissions of an increase in abatement costs is different for each policy game.

$\gamma\backslash d$	2.50	5.00	10.00	20.00
2.50	2647049, 2334931	2627978, 2320129	2618304, 2312605	2613422, 2308803
5.00	1623579, 1589922	1596112, 1563607	1582164, 1550232	1575118, 1543472
10.00	934921, 931795	901383, 898491	884341, 881563	875730, 873007
20.00	531603, 531326	494454, 494218	475574, 475357	466032, 465825

Table 4a. Social welfare for x_0=0.

In Table 4a we represent social welfare. The figures illustrate the result in Prop. 11: the social welfare is lower for the non-commitment solution. The figures also show that welfare decreases with the environmental damages and also with abatement costs regardless of whether the regulator is able to commit or not. In Table 4b we show the welfare losses in relative terms associated to the lack of commitment. In concordance with the results obtained for the previous variables, the differences between the two policy games are minimal when both damages and abatement costs are high. Moreover, it can be checked that welfare losses are almost perfectly inelastic to the environmental damages. For instance, when $\gamma = 2.5$ an increase in d from 2.50 to 20.00 only reduces the welfare losses in 0.13 percentage points. However, the contrary occurs when the abatement costs increase. When $d = 2.5$ an increase in γ from 2.50 to 20.00, the welfare losses go down by 11.74 percentage points.

$\gamma\backslash d$	2.50	5.00	10.00	20.00
2.50	11.79	11.71	11.67	11.66
5.00	2.07	2.04	2.02	2.01
10.00	0.33	0.32	0.31	0.31
20.00	0.05	0.05	0.04	0.04

Table 4b. Welfare losses (%).

The next exercise is a welfare comparison for different levels of the pollution stock. To do the comparison we have selected the values of γ and d that yield the biggest difference between the two solutions: $\gamma = d = 2.5$. For these two values the regulator's value functions are

$$W^c(x) = -\frac{1.3196}{2}x^2 - 609.8084x + 2647049, \quad \text{for } x \in [0, \breve{x}^c = 42.1509],$$
$$W^{nc}(x) = -\frac{1.4611}{2}x^2 - 533.2538x + 2334931, \ \text{for } x \in [0, \ \breve{x}^{nc} = 156.7148].$$

The difference $y(x) = W^c(x) - W^{nc}(x)$ is plotted in Figure 1 for the interval $[0, 42.1509]$ that guarantees that the different variables of the model satisfy the non-negativity constraint in both cases.

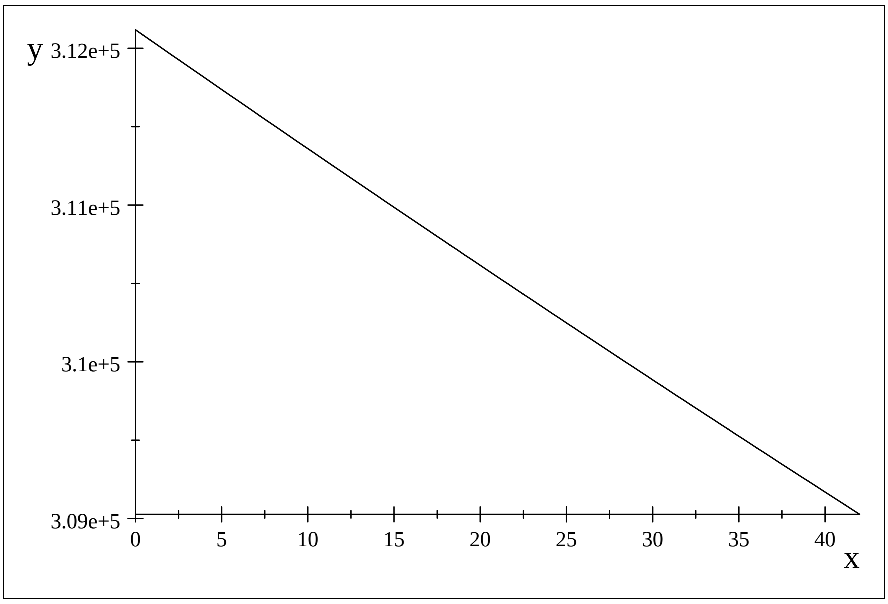

Figure 1. $W^c_T(x) - W^{nc}_T(x)$ for $x \in$ [0,42]

Thus, it does not seem that the assumption $x_0 = 0$ behind the result of Prop. 11 is a necessary condition to obtain a positive difference between the discounted present value of social welfare of the commitment solution and the discounted present social welfare of the non-commitment solution. In this example, this difference is positive provided that $x_0 \in [0, 42]$. This constraint on the interval is imposed because we focus on the comparison of the interior solutions of the policy games studied in the paper.

$\gamma \backslash d$	2.50	5.00	10.00	20.00
2.50	2187607, 4021576	2161200, 3770117	2147806, 3601324	2141046, 3486704
5.00	1458456, 1837910	1415731, 1698172	1394033, 1613485	1383073, 1560914
10.00	916722, 1032963	859229, 934453	830014, 880541	815252, 850546
20.00	574440, 619272	506334, 533391	471719, 487731	454226, 464264

Table 5a. Net profits for $x_0 = 0$.

To complete the comparison of payoffs we present in Table 5a the discounted present value of net profits for $x_0 = 0$. The figures support the result in Prop. 11: net profits are larger when there is non-commitment. As occurred with social welfare, net profits decrease both with environmental damages and abatement costs regardless whether the solution corresponds to the committed policy game or to the non-committed policy game. Table 5b shows the increase in net profits in relative terms caused by the lack of commitment. The percentages also decrease both with damages and abatement costs as it happened with social welfare. However, the increments of the net profits in percentage points are significantly larger than the reductions of social welfare also in percentage points for all the cases. For instance, for $\gamma = d = 2.5$ the reduction in welfare is 11.79% whereas the increase in net profits is 83.83%. This difference is explained by the variations in the optimal policy caused by the lack of commitment. As shown in Fig. 2 just for this example, when there is commitment the optimal policy consists of taxing emissions for $t > 1$ whereas without commitment the firm receives a subsidy all the time what accounts for the substantial increase in net profits the firm obtains when the regulator does not commit.[14] Nevertheless, the larger the abatement costs and the larger the damages, the lower is this effect because the difference between the two solutions substantially decreases.

$\gamma \backslash d$	2.50	5.00	10.00	20.00
2.50	83.83	74.44	67.67	62.85
5.00	26.02	19.95	15.74	12.86
10.00	12.68	8.75	6.09	4.33
20.00	7.80	5.34	3.39	2.21

Table 5b. Increase in net profits (%).

We omit the comparison of the monopolist's value functions because it confirms, as occurred with social welfare, that the discounted present value of the net profits are larger when there is non-commitment regardless of the value of the initial pollution stock.

Finally, also for $\gamma = d = 2.5$ and $x_0 = 0$ we compare the optimal paths of the different variables of the model. In Fig. 2 we plot the optimal paths of the tax. In

[14] Notice that if the firm receives a subsidy, net profits include subsidies.

green the temporal trajectory of the commitment solution and in black that of the non-commitment solution.

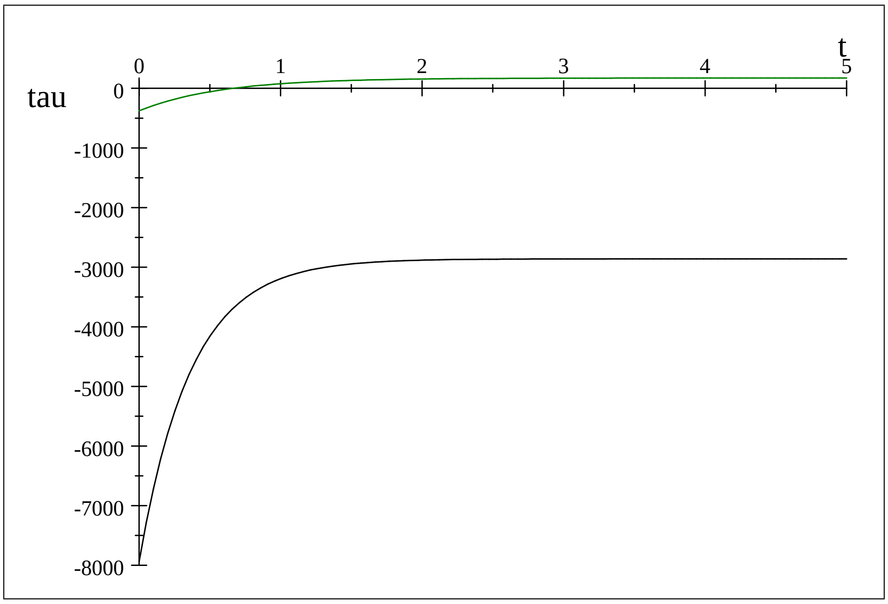

Figure 2. The optimal paths of the emission tax.

The figure shows that the lack of commitment drastically changes the sign of the optimal policy. When there is commitment, the optimal policy for $x_0 = 0$ consists of subsidizing emissions to correct the effect of the monopolist's power market on production but in less that one period the subsidy becomes a tax and continues being a tax until reaching its steady state value of 173.25. However, when we solve the non-committed regulator game the optimal path yields a subsidy for all t. The path converges to a steady-state value for τ equal to -2860.42. As pointed out above this difference in the sign of the optimal policy explains the significant increase in net profits when there is non-commitment. This numerical example shows a case where a subsidy applies at the steady state. In this case the severity of environmental damages does not justify a tax be applied but observe that the accumulation of emissions induces a reduction of the subsidy. Figure 3 shows that the optimal path of production for the non-commitment solution is above the optimal path for the commitment solution. The subsidy incentives production whereas the tax has the opposite effect. The result is a level of production for the commitment solution that is lower than the level of production for the non-commitment solution for all t.

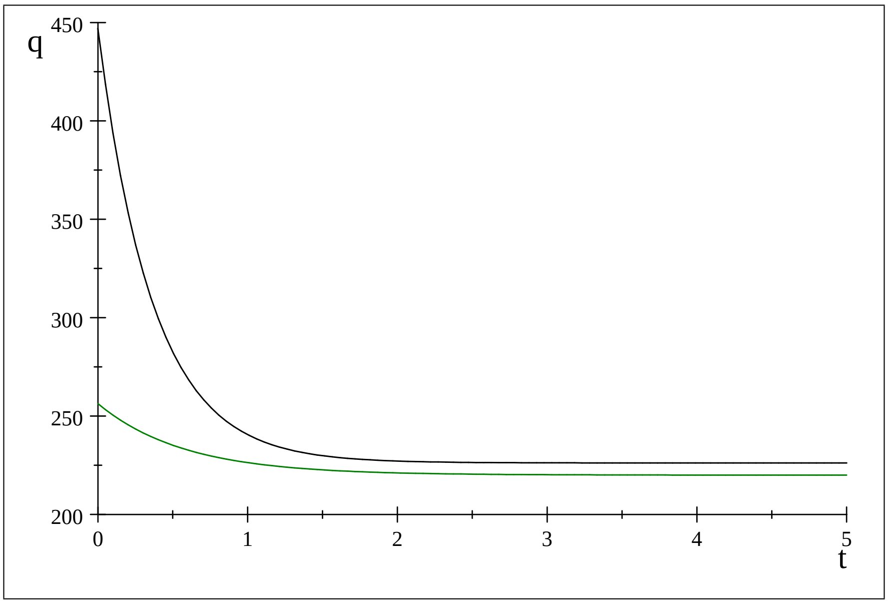

Figure 3. The optimal paths of production.

Fig. 4 shows that the abatement effort is lower for all t when there is non-commitment. The subsidy is an incentive to reduce abatement, and therefore, to emit more. Notice that if the production is larger and the abatement is lower when the regulator does not commit, the emissions are bigger for the non-commitment solution.

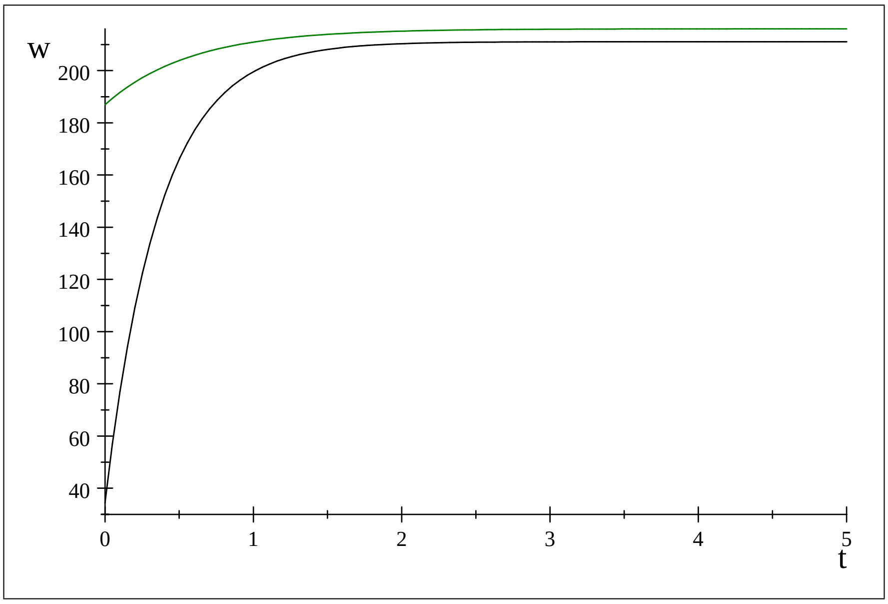

Figure 4. The optimal paths of abatement.

Finally, in Fig. 5 we represent the optimal paths for the pollution stocks. As emissions are larger for the non-commitment solution, the pollution stock is larger for the non-commitment solution as Prop. 10 established.

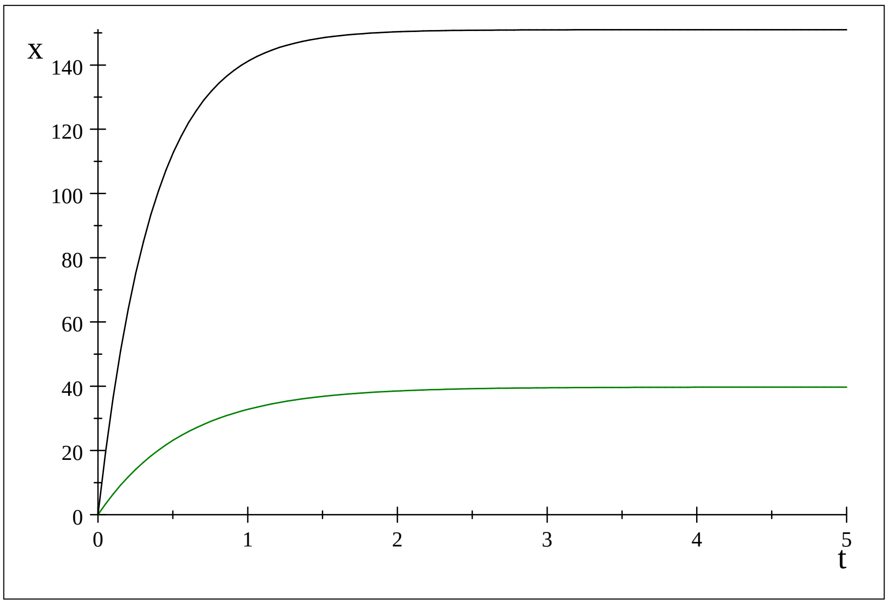

Figure 5. The optimal paths of pollution stock.

Summarizing, the lack of commitment implies a change in the sign of the optimal policy turning a tax in a subsidy, increases output and reduces abatement causing an increase in emissions that leads to a larger pollution stock. The expansion of output increases consumer surplus and the decrease of abatement reduces abatement costs. Both changes are welfare improving but, as can be seen in Fig. 5, there is an important augmentation of the pollution stock that results in an increase of damages big enough as to yield finally a reduction in social welfare.

6 Conclusions

This paper studies how the lack of regulator to commit affects the emission tax for a stock pollutant and its welfare implications. To evaluate these effects we compare the feedback Stackelberg equilibrium when the regulator is the leader of the policy game, that we have called in the paper the committed regulator game, with the feedback Stackelberg equilibrium when the monopolist is the leader of the policy game, that we have called the non-committed regulator game.[15] The comparison of these two equilibria establishes that

[15] In our model, this second Stackelberg equilibrium coincides with the Markov-perfect Nash equilibrium.

the steady-state pollution stock under commitment is lower than under non-commitment. Thus, the lack of commitment has a clear consequence on the accumulation of emissions leading to larger damages at the steady state. Moreover, we show that the lack of commitment moves down the tax rule applied by the regulator and that if environmental damages are not very low, the steady-state tax of the commitment solution is larger than the steady-state tax of the non-commitment solution that, on the other hand, could be negative, i.e. the optimal policy would be to apply a subsidy. The welfare implications of the lack of commitment are unclear except when the initial pollution stock is zero. For this case, the social welfare is lower when the regulator is unable to commit whereas net profits are larger. To progress in the comparison we have calculated a numerical example that shows that the lack of commitment has a negative impact on social welfare also when the initial pollution stock is different from zero. Thus, our analysis shows that the lack of commitment has a *detrimental* effect on welfare. However, the numerical exercise also shows that this negative effect decreases with the abatement costs and that for large abatement costs the difference in welfare between the two equilibria is practically zero. Finally, we find that the steady-state value of abatement for the non-commitment solution is lower than the corresponding steady-state value for the commitment solution. Thus, the lack of commitment leads to lower taxation and abatement. To conclude, we would like to point out that our findings do not give support to the idea that discretion (no commitment) may be welfare improving in the regulation of a polluting monopoly.

A limitation of our analysis is that we have assumed the simplest form of the emission function, i.e. one that is additively separable in production (gross emissions) and abatement. To overcome this limitation, an interesting extension to develop would be to consider that abatement expenditures can reduce the emissions-to-output ratio as in Hartl and Kort (1996) or that this coefficient can be reduced by investing in abatement capital as in Beavis and Dobbs (1996) or Farzin and Kort (2000). This second approach would allow to study the dynamic interaction between the accumulation of emissions and the accumulation of abatement capital. A further step in this line of research would be to analyze the effects of the lack of commitment when the abatement technology is subject to stochastic innovation. It would be also interesting to know how the results would change if the market structure is an oligopoly although according to the first results obtained for oligopolistic firms by Petrakis and Xepapadeas (2001), it seems that competition plays for regulatory commitment.

Appendix

Proof of Lemma 1

The steady state for the pollution stock under commitment is given by (22) and the steady state for the pollution stock under non-commitment by (42). Taking into account that the first Riccati's equation for the non-committed regulator game establishes that $(A_1^{nc})^2 = (2\delta + r)A_1^{nc} + d$ and using (40) for eliminating A_1^{nc}, (42) can be written as follows

$$x_{SS}^{nc} = \frac{(a-c)\left(\gamma(1+\gamma)(r+\delta) + \sqrt{4d+(r+2\delta)^2} - (r+2\delta)\right)}{\gamma\left(d(2+\gamma) + \delta\left(r+2\delta+\gamma(r+\delta) - \sqrt{4d+(r+2\delta)^2}\right)\right)}.$$

Easy computations lead to the following equivalence:

$$x_{SS}^{c} < x_{SS}^{nc} \Leftrightarrow Num/Den < 0,$$

where

$$\begin{aligned} Num &= (2+\gamma)(d+\gamma\delta(r+\delta))\left(2(r+2\delta) - \gamma r - (2+\gamma)\sqrt{4d+(r+2\delta)^2}\right), \\ Den &= \left(d(2+\gamma)^2 + \gamma(4+\gamma)\delta(r+\delta)\right) \times \\ &\quad \left(d(2+\gamma) + \delta\left(r+2\delta+\gamma(r+\delta) - \sqrt{4d+(r+2\delta)^2}\right)\right). \end{aligned}$$

Den is positive, because the second factor can be proved to be positive:

$$\begin{aligned} &d(2+\gamma) + \delta\left(r+2\delta+\gamma(r+\delta) - \sqrt{4d+(r+2\delta)^2}\right) > 0 \Leftrightarrow \\ &d(2+\gamma) + \delta\left(r+2\delta+\gamma(r+\delta)\right) - \delta\sqrt{4d+(r+2\delta)^2} > 0 \Leftrightarrow \\ &\left[d(2+\gamma) + \delta\left(r+2\delta+\gamma(r+\delta)\right)\right]^2 - \delta^2(4d+(r+2\delta)^2) > 0 \Leftrightarrow \\ &(d+\delta(r+\delta))(d(2+\gamma)^2 + \gamma\delta(r(2+\gamma)+(4+\gamma)\delta)) > 0. \end{aligned}$$

The sign of Num is the same as the sign of the following expression:

$$2(r+2\delta) - \gamma r - (2+\gamma)\sqrt{4d+(r+2\delta)^2}. \tag{49}$$

The expression above is always negative. If $2(r+2\delta) - \gamma r < 0$, then the expression in

(49) is negative. If $2(r+2\delta)-\gamma r>0$,then

$$\begin{aligned}
&2(r+2\delta)-\gamma r-(2+\gamma)\sqrt{4d+(r+2\delta)^2}<0 \ \Leftrightarrow\\
&2(r+2\delta)-\gamma r<(2+\gamma)\sqrt{4d+(r+2\delta)^2} \ \Leftrightarrow\\
&(2(r+2\delta)-\gamma r)^2<(2+\gamma)^2(4d+(r+2\delta)^2) \ \Leftrightarrow\\
&4d(2+\gamma)^2+4\gamma(r+\delta)(2r+(4+\gamma)\delta)>0.
\end{aligned}$$

Therefore, $Num/Den<0$, and $x^c_{\tau SS}<x^{nc}_{\tau SS}$.

Proof of Proposition 10

First, we calculate the steady-state tax rate substituting the steady-state value of the pollution stock in (29):

$$\tau^c_{SS}=\tau^c(x^c_{SS})=-\frac{(a-c)\gamma(\gamma\delta F_2F_3F_4-dF_1(rF_2+2\delta))}{F_2F_4(F_1^2d+\gamma\delta F_2F_3)},$$

where

$$F_1=\gamma+2,\ F_2=\gamma+4,\ F_3=r+\delta \text{ and } F_4=r+2\delta.$$

Next using this tax, we derive the value of the stock of pollution, $\tilde{x}$, for which the optimal policy of the non-commitment solution yields the steady-state tax rate of the committed regulator game. Thus this value of the pollution stock must satisfy $\tau^{nc}(\tilde{x})=\tau^c_{SS}$.

$$\begin{aligned}
\tilde{x} &= (a-c)\left(\frac{F_2F_4(\gamma^2F_3^2-\gamma F_3F_2A_1^{nc}+4(A_1^{nc})^2)(F_1^2d+\gamma\delta F_2F_3)}{\gamma F_3F_2(F_3-A_1^{nc})(F_1^2d+\gamma\delta F_2F_3)(2F_1(A_1^{nc})^2-2\gamma F_4A_1^{nc})}\right.\\
&- \left.\frac{\gamma^3F_3(F_3-A_1^{nc})(\gamma\delta F_2F_3F_4-dF_1(rF_2+2\delta))}{\gamma F_3F_2(F_3-A_1^{nc})(F_1^2d+\gamma\delta F_2F_3)(2F_1(A_1^{nc})^2-2\gamma F_4A_1^{nc})}\right).
\end{aligned}$$

Using the steady-state value of the pollution stock for the non-commitment solution given by (42), $\tilde{x}>x^{nc}_{SS}$ if and only if

$$\begin{aligned}
&\frac{F_2F_4(\gamma^2F_3^2-\gamma F_3F_2A_1^{nc}+4(A_1^{nc})^2)(F_1^2d+\gamma\delta F_2F_3)(F_1A_1^{nc}-\gamma\delta)}{F_3F_2(F_1^2d+\gamma\delta F_2F_3)(2F_1(A_1^{nc})^2-2\gamma F_4A_1^{nc})(F_1A_1^{nc}-\gamma\delta)}\\
-&\frac{\gamma^3F_3(F_3-A_1^{nc})(F_1A_1^{nc}-\gamma\delta)(\gamma\delta F_2F_3F_4-dF_1(rF_2+2\delta))}{F_3F_2(F_1^2d+\gamma\delta F_2F_3)(2F_1(A_1^{nc})^2-2\gamma F_4A_1^{nc})(F_1A_1^{nc}-\gamma\delta)}\\
+&\frac{F_3F_2(F_1^2d+\gamma\delta F_2F_3)(2F_1(A_1^{nc})^2-2\gamma F_4A_1^{nc})(\gamma(1+\gamma)F_3-2A_1^{nc})}{F_3F_2(F_1^2d+\gamma\delta F_2F_3)(2F_1(A_1^{nc})^2-2\gamma F_4A_1^{nc})(F_1A_1^{nc}-\gamma\delta)}>0, \qquad (50)
\end{aligned}$$

where the denominator is negative since A_1^{nc} is negative.

The development of the numerator yields

$$
\begin{aligned}
&- \quad 4\gamma^4\delta^2(\gamma+4)(r+\delta)^3(r+2\delta) - 2d\gamma^3\delta(\gamma+2)(r+\delta)^2 G_1(\gamma,\delta,r) \\
&+ \quad A_1^{nc}\delta\gamma^2(r+\delta)(4(\gamma+2)G_2(\gamma,\delta,r)d - \gamma(\gamma+4)(r+\delta)(r+2\delta)G_3(\gamma,\delta,r)) \\
&- \quad 2(A_1^{nc})^2\gamma\delta(2(\gamma+2)^2 G_4(\gamma,\delta,r)d - \gamma(\gamma+4)(r+\delta)G_5(\gamma,\delta,r)) \\
&+ \quad 4(A_1^{nc})^3\delta(\gamma+2)(\gamma+4)(\gamma\delta(\gamma+4)(r+\delta) + d(\gamma+2)^2), \qquad (51)
\end{aligned}
$$

where

$$
\begin{aligned}
G_1(\gamma,\delta,r) &= \gamma^2(r+\delta) + \gamma(5r+7\delta) + 4(r+2\delta) > 0, \\
G_2(\gamma,\delta,r) &= \gamma^2(r+3\delta) + \gamma(5r+13\delta) + 8(r+2\delta) > 0, \\
G_3(\gamma,\delta,r) &= 2\gamma^2(r+\delta) + 2\gamma(3r-\delta) - 16\delta, \\
G_4(\gamma,\delta,r) &= 2\gamma^2(r+\delta) + \gamma(8r+9\delta) + 4(2r+3\delta) > 0, \\
G_5(\gamma,\delta,r) &= \gamma^3(r+\delta)^2 + \gamma^2(3r-\delta)(r+\delta) - 2\gamma\delta(8r+9\delta) - 8\delta(2r+3\delta).
\end{aligned}
$$

$G_3(\gamma,\delta,r)$ might be positive and in this case

$$A_1^{nc}\delta\gamma^2(r+\delta)(4(\gamma+2)G_2(\gamma,\delta,r)d - \gamma(\gamma+4)(r+\delta)(r+2\delta)G_3(\gamma,\delta,r))$$

might be positive too. However, if d is larger than d^{iv} given in (48) this possibility is eliminated. The same occurs for $G_5(\gamma,\delta,r)$. In this case d should be larger than

$$d^v = \frac{\gamma(\gamma+4)(r+\delta)(\gamma^3(r+\delta)^2 + \gamma^2(3r-\delta)(r+\delta) - 2\gamma\delta(8r+9\delta) - 8\delta(2r+3\delta))}{2(\gamma+2)^2(2\gamma^2(r+\delta) + \gamma(8r+9\delta) + 4(2r+3\delta))}$$

to avoid that

$$-2(A_1^{nc})^2\gamma\delta(2(\gamma+2)^2 G_4(\gamma,\delta,r)d - \gamma(\gamma+4)(r+\delta)G_5(\gamma,\delta,r))$$

be positive. As d^v is lower than d^{iv}, we can conclude that if d is large enough, in particular if d is larger than d^{iv} all the terms in (51) are negative and (50) is satisfied so that we can conclude that $\tilde{x} > x_{SS}^{nc}$. Thus, as the optimal policy is increasing with respect to the pollution stock we have that $\tau_{SS}^c = \tau^{nc}(\tilde{x}) > \tau^{nc}(x_{SS}^{nc})$, i.e. the steady-state tax rate of the non-commitment solution is lower than the steady-state tax rate of the committed regulator game.

Proof of Lemma 2

Substituting in the intersection point of the optimal policy defined by (45) A_1^{nc} given by (40) we obtain that $\tau^{nc}(0)$ is equal to

$$-(a-c)\frac{2\left(r+2\delta-\gamma(r+\delta)-\sqrt{4d+(r+2\delta)^2}\right)^2-\gamma^2(r+\delta)\left(r+2\delta-\sqrt{4d+(r+2\delta)^2}\right)}{\gamma^2(r+\delta)(r+\sqrt{4d+(r+2\delta)^2})}$$

and then using (29) in Prop. 4, the difference $\tau^c(0)-\tau^{nc}(0)$ can be written as follows

$$-\frac{(a-c)\gamma}{\gamma+4}$$
$$+(a-c)\frac{2\left(r+2\delta-\gamma(r+\delta)-\sqrt{4d+(r+2\delta)^2}\right)^2-\gamma^2(r+\delta)\left(r+2\delta-\sqrt{4d+(r+2\delta)^2}\right)}{\gamma^2(r+\delta)(r+\sqrt{4d+(r+2\delta)^2})},$$

that taking common factor yields

$$=-\frac{(a-c)}{(r+\sqrt{4d+(r+2\delta)^2})}\left(\frac{\gamma^3(r+\delta)(r+\sqrt{4d+(r+2\delta)^2})}{\gamma^2(r+\delta)(\gamma+4)}\right.$$
$$\left.-\frac{2(\gamma+4)\left(r+2\delta-\gamma(r+\delta)-\sqrt{4d+(r+2\delta)^2}\right)^2}{\gamma^2(r+\delta)(\gamma+4)}+\frac{\gamma^2(r+\delta)(\gamma+4)(r+2\delta-\sqrt{4d+(r+2\delta)^2})}{\gamma^2(r+\delta)(\gamma+4)}\right),$$

that developing the numerator results in the following expression

$$=-\frac{(a-c)\gamma}{\gamma(r+\sqrt{4d+(r+2\delta)^2})}\frac{4H_2+4H_1\sqrt{4d+(r+2\delta)^2}-8d\,(\gamma+4)}{\gamma^2(r+\delta)(\gamma+4)}, \tag{52}$$

where

$$\begin{aligned}4H_1 &= 4(4(r+2\delta)-\gamma(3r+2\delta)-2\gamma^2(r+\delta))<0 \text{ for } \gamma>2,\\ 4H_2 &= 4(2\gamma^2\delta\,(r+\delta)+\gamma\,(3r+2\delta)\,(r+2\delta)-4\,(r+2\delta)^2)>0 \text{ for } \gamma>2.\end{aligned}$$

As

$$4H_2+4H_1\sqrt{4d+(r+2\delta)^2}-8d\,(\gamma+4)=-8\gamma^2\,(r+\delta)^2 \text{ for } d=0,$$

and the expression decreases with d, we can conclude that

$$4H_2+4H_1\sqrt{4d+(r+2\delta)^2}-8d\,(\gamma+4)<0 \text{ for } d>0,$$

and hence that (52) is positive establishing that $\tau^c(0)-\tau^{nc}(0)$ is positive.

Next, we compare the slope of the optimal policies. According to the first Riccati's equation: $(A_1^{nc})^2 = (2\delta + r)A_1^{nc} + d$ which allows to write the slope of the optimal policy for the non-committed policy games given by (45) as follows

$$m^{nc} = \frac{2(2(r+2\delta)A_1^{nc} + d(2+\gamma))}{\gamma(r+2\delta)},$$

that substituting A_1^{nc} by (40) yields

$$m^{nc} = \frac{2[d(2+\gamma) + (2\delta+r)^2 - (r+2\delta)\sqrt{(r+2\delta)^2+4d}]}{\gamma(r+2\delta)},$$

so that the difference between the slopes is

$$m^{nc} - m^c = \frac{2}{r+2\delta}\left(\frac{4(2+\gamma)d + (\gamma+4)(2\delta+r)^2 - (\gamma+4)(r+2\delta)\sqrt{(r+2\delta)^2+4d}}{\gamma(\gamma+4)}\right). \quad (53)$$

This difference is positive provided that

$$4(2+\gamma)d + (\gamma+4)(2\delta+r)^2 - (\gamma+4)(r+2\delta)\sqrt{(r+2\delta)^2+4d} > 0.$$

Reordering terms and taking square, we get

$$16(2+\gamma)^2 d^2 + 4\gamma d(r+2\delta)^2(\gamma+4) > 0.$$

Therefore (53) is positive and consequently m^{nc} is larger than m^c.

Proof of Proposition 11

Notice that according to the expressions of the value functions (16) and (36), $W^c(0)$ is larger than $W^{nc}(0)$ provided that C_1^c is larger than C_1^{nc}. Using the expressions for these parameters given respectively by (21) and (42), C_1^c is larger than C_1^{nc} if and only if

$$\begin{aligned} &\Sigma_1(\gamma,\delta,d,r) + r\sqrt{4d+(r+2\delta)^2}\,\Sigma_2(\gamma,\delta,d,r) \\ +\ & r\sqrt{\gamma(4+\gamma)(4d(r+\gamma)^2+\gamma(4+\gamma)(r+2\delta)^2)}\,\Sigma_3(\gamma,\delta,d,r) > 0, \end{aligned} \quad (54)$$

where

$$\Sigma_1(\gamma,\delta,d,r) = 2[4d^2(2+\gamma^2) + 2d\left(4\gamma(2+\gamma)^2\delta^2 + 2r(2\gamma^3+7\gamma^2+4\gamma-2)(r+2\delta)\right)$$
$$+ \gamma(r^2+2r\delta+2\delta^2)(2\gamma(2+\gamma)^2\delta^2+r(r+2\delta)(2\gamma^3+8\gamma^2+7\gamma-4))] > 0 \text{ for } \gamma > 2,$$
$$\Sigma_2(\gamma,\delta,d,r) = 4d(2+\gamma)^2$$
$$+ \gamma[(\gamma^5+6\gamma^4+9\gamma^3+4\gamma+16)\delta^2 + r(r+2\delta)(\gamma^5+6\gamma^4+9\gamma^3+2\gamma+8)] > 0,$$
$$\Sigma_3(\gamma,\delta,d,r) = 4d - \gamma(\gamma^3+2\gamma^2-3\gamma-4)\delta^2 - r(r+2\delta)(\gamma^4+2\gamma^3-3\gamma^2-4\gamma+2)$$
$$+ 2r\sqrt{4d+(r+2\delta)^2}.$$

The sign of $\Sigma_3(\gamma,\delta,d,r)$ depends on the value of d. For a d large enough, $\Sigma_3(\gamma,\delta,d,r)$ is positive and therefore (54) is also positive that establishes that $C_1^c > C_1^{nc}$. Suppose now that this is not the case then (54) implies the following inequality

$$\left(\Sigma_1(\gamma,\delta,d,r) + r\sqrt{4d+(r+2\delta)^2}\Sigma_2(\gamma,\delta,d,r)\right)^2$$
$$- \quad r^2\gamma(4+\gamma)(4d(r+\gamma)^2+\gamma(4+\gamma)(r+2\delta)^2)\Sigma_3(\gamma,\delta,d,r)^2 > 0,$$

that yields

$$\chi_1(\gamma,\delta,d,r) + \chi_2(\gamma,\delta,d,r)\sqrt{4d+(r+2\delta)^2} > 0, \quad (55)$$

with

$$\chi_1(\gamma,\delta,d,r) = \Sigma_1(\gamma,\delta,d,r)^2 + r^2\left(4d+(r+2\delta)^2\right)\Sigma_2(\gamma,\delta,d,r)^2$$
$$-r^2(\gamma(4+\gamma)(4d(r+\gamma)^2+\gamma(4+\gamma)(r+2\delta)^2)) \times$$
$$\{[4d-\gamma(\gamma^3+2\gamma^2-3\gamma-4)\delta^2-r(r+2\delta)(\gamma^4+2\gamma^3-3\gamma^2-4\gamma+2)]^2+4r^2(4d+(r+2\delta)^2)\},$$
$$\chi_2(\gamma,\delta,d,r) = 2r\Sigma_1(\gamma,\delta,d,r)\Sigma_2(\gamma,\delta,d,r)$$
$$-4r^3(\gamma(4+\gamma)(4d(r+\gamma)^2+\gamma(4+\gamma)(r+2\delta)^2)) \times$$
$$\{4d-\gamma(\gamma^3+2\gamma^2-3\gamma-4)\delta^2-r(r+2\delta)(\gamma^4+2\gamma^3-3\gamma^2-4\gamma+2)\}.$$

Once the expressions of $\Sigma_1(\gamma,\delta,d,r), \Sigma_2(\gamma,\delta,d,r)$ and $\Sigma_3(\gamma,\delta,d,r)$ have been replaced in $\chi_1(\gamma,\delta,d,r)$ and $\chi_2(\gamma,\delta,d,r)$ and after some tedious computations carried out with Mathematica 10.1, they can be rewritten as

$$\chi_1(\gamma,\delta,d,r) = 8d^4(2+\gamma)^2+16d^3p_1(r,\gamma,\delta)+4d^2p_2(r,\gamma,\delta)+2d\gamma p_3(r,\gamma,\delta)+\gamma^2(r+\delta)^2p_4(r,\gamma,\delta),$$
$$\chi_2(\gamma,\delta,d,r) = 8d^3(2+\gamma)^2+2d^2p_5(r,\gamma,\delta)+4d\gamma p_6(r,\gamma,\delta)+\gamma^2(r+\delta)^2p_7(r,\gamma,\delta),$$

where $p_i(r,\gamma,\delta), i=1,\ldots,7$ are the polynomials in terms of parameters r,γ and δ

$$
\begin{aligned}
p_1(r,\gamma,\delta) &= r^2\gamma(4+7\gamma+2\gamma^2)+2r\delta(-2+4\gamma+7\gamma^2+2\gamma^3)+2\gamma(2+\gamma)^2\delta^2,\\
p_2(r,\gamma,\delta) &= r^4(4-4\gamma-5\gamma^2+16\gamma^3+24\gamma^4+12\gamma^5+2\gamma^6)\\
&\quad+4r^3(4-4\gamma+3\gamma^2+24\gamma^3+17\gamma^4+6\gamma^5+\gamma^6)\delta\\
&\quad+2r^2(8-28\gamma+39\gamma^2+96\gamma^3+38\gamma^4+6\gamma^5+\gamma^6)\delta^2+4r\gamma(-12+27\gamma+40\gamma^2+11\gamma^3)\delta^3\\
&\quad+12\gamma^2(2+\gamma)^2\delta^4,\\
p_3(r,\gamma,\delta) &= r^6\gamma^4(2+\gamma)(3+\gamma)^2+2r^5(8-14\gamma+33\gamma^3+50\gamma^4+43\gamma^5+16\gamma^6+2\gamma^7)\delta\\
&\quad+r^4(80-140\gamma+32\gamma^2+329\gamma^3+242\gamma^4+135\gamma^5+48\gamma^6+6\gamma^7)\delta^2\\
&\quad+4r^3(32-72\gamma+44\gamma^2+159\gamma^3+76\gamma^4+24\gamma^5+8\gamma^6+\gamma^7)\delta^3\\
&\quad+r^2(64-272\gamma+320\gamma^2+613\gamma^3+208\gamma^4+26\gamma^5+8\gamma^6+\gamma^7)\delta^4\\
&\quad+16r\gamma(-6+15\gamma+19\gamma^2+5\gamma^3)\delta^5+16\gamma^2(2+\gamma)^2\delta^6,\\
p_4(r,\gamma,\delta) &= r^6\gamma^4(3+\gamma)^2+6r^5\gamma^4(3+\gamma)^2\delta+r^4(16-40\gamma+29\gamma^2+38\gamma^3+126\gamma^4+78\gamma^5+13\gamma^6)\delta^2\\
&\quad+4r^3(16-40\gamma+29\gamma^2+38\gamma^3+36\gamma^4+18\gamma^5+3\gamma^6)\delta^3\\
&\quad+4r^2(16-48\gamma+43\gamma^2+54\gamma^3+22\gamma^4+6\gamma^5+\gamma^6)\delta^4\\
&\quad+16r\gamma(-4+7\gamma+8\gamma^2+2\gamma^3)\delta^5+8\gamma^2(2+\gamma)^2\delta^6,\\
p_5(r,\gamma,\delta) &= r^2(-8+8\gamma+26\gamma^2+8\gamma^3+9\gamma^4+6\gamma^5+\gamma^6)\\
&\quad+2r(-8+24\gamma+30\gamma^2+8\gamma^3+9\gamma^4+6\gamma^5+\gamma^6)\delta\\
&\quad+\gamma(2+\gamma)^2(12-3\gamma+2\gamma^2+\gamma^3)\delta^2,\\
p_6(r,\gamma,\delta) &= r^4\gamma^4(3+\gamma)^2+r^3(-8+14\gamma+16\gamma^2-5\gamma^3+30\gamma^4+23\gamma^5+4\gamma^6)\delta\\
&\quad+2r^2(-12+25\gamma+22\gamma^2-4\gamma^3+21\gamma^4+17\gamma^5+3\gamma^6)\delta^2\\
&\quad+r(-16+60\gamma+40\gamma^2-\gamma^3+30\gamma^4+23\gamma^5+4\gamma^6)\delta^3+\gamma(2+\gamma)^2(6-3\gamma+2\gamma^2+\gamma^3)\delta^4,\\
p_7(r,\gamma,\delta) &= r^4\gamma^4(3+\gamma)^2+4r^3\gamma^4(3+\gamma)^2\delta+r^2(-16+40\gamma+3\gamma^2-6\gamma^3+12\gamma^4+42\gamma^5+7\gamma^6)\delta^2\\
&\quad+2r(-16+40\gamma+3\gamma^2-6\gamma^3+26\gamma^4+18\gamma^5+3\gamma^6)\delta^3+2\gamma(16+4\gamma+9\gamma^3+6\gamma^4+\gamma^5)\delta^4.
\end{aligned}
$$

It is straightforward to check that the polynomials $p_i(r,\gamma,\delta), i=1,\ldots,7$ always take positive values for any value of the parameters r,δ and $\gamma>2$. Therefore, $\chi_1(\gamma,\delta,d,r)$ is positive too ($\chi_2(\gamma,\delta,d,r)$ is also positive), and in consequence condition (55) is fulfilled, and inequality $C_1^c > C_1^{nc}$ is always satisfied.

Next we compare the present value of net profits for $x_0 = 0$. First, we show that A_1^{nc} is lower than A_1^c. Suppose that this is not the case. Then using (20) and (40) the following inequality must hold

$$\frac{\gamma(4+\gamma)(r+2\delta) - \sqrt{\gamma(4+\gamma)(4d(2+\gamma)^2 + \gamma(4+\gamma)(r+2\delta)^2)}}{2(2+\gamma)^2}$$
$$\leq \frac{1}{2}(r + 2\delta - \sqrt{(r+2\delta)^2 + 4d}),$$

that simplifying terms gives

$$(2+\gamma)^2\sqrt{(r+2\delta)^2 + 4d} - 4(r+2\delta)$$
$$\leq \sqrt{\gamma(4+\gamma)(4d(2+\gamma)^2 + \gamma(4+\gamma)(r+2\delta)^2)},$$

where the left-hand side is positive for $d \geq 0$. Notice that the expression is positive for $d = 0$ and increasing with d. Taking square in both sides of the inequality yields

$$(r+2\delta)^2 + 8d \leq (r+2\delta)\sqrt{(r+2\delta)^2 + 4d},$$

that taking square again leads to the following contradiction

$$3\,(r+2\delta)^2\,d + 16d^2 \leq 0.$$

Thus, we can conclude that A_1^{nc} is lower than A_1^c.

Next, we compare C_2^c and C_2^{nc}. Notice that for $x_0 = 0$, $V^c(0) = C_2^c$ and $V^{nc}(0) = C_2^{nc}$. Taking into account the expressions of C_2^c and C_2^{nc} given by (21) and (44) $C_2^c < C_2^{nc}$ if and only if

$$\begin{aligned}&-\left[(4-2\gamma+\gamma^2)\,(A_1^{nc})^2 - 2\gamma(2+\gamma)(r+\delta)A_1^{nc} + \gamma^2(1+2\gamma)(r+\delta)^2\right]\\ &\qquad\times\left[(2+\gamma)^2A_1^c - \gamma(4+\gamma)(r+\delta)\right]^2\\ &+\left[(A_1^c)^2\,(2+\gamma)^3 - 2\gamma(8+6\gamma+\gamma^2)(r+\delta)A_1^c + \gamma^2(8+9\gamma+2\gamma^2)(r+\delta)^2\right]\gamma^3(r+\delta-A_1^{nc})^2 < 0. \quad (56)\end{aligned}$$

With the help of Mathematica 10.1 this expression can be written as the following product

$$\begin{aligned}&\left[-\gamma(r+\delta)\,(A_1^c(1+\gamma)(2+\gamma) - 2\gamma(r+\delta)) + A_1^{nc}\left(2(2+\gamma)A_1^c + \gamma(-4+\gamma+\gamma^2)(r+\delta)\right)\right]\times\\ &\quad\left\{A_1^{nc}\left[2(2+\gamma)^2A_1^c - \gamma(8+\gamma(1+\gamma)(2+\gamma))(r+\delta)\right]\right.\\ &\quad\left.+\gamma(r+\delta)\left[-(1+\gamma)(2+\gamma)^2A_1^c + 2\gamma(2+4\gamma+\gamma^2)(r+\delta)\right]\right\}\end{aligned}$$

where the factor in curly brackets is positive provided that both A_1^c and A_1^{nc} are negative values. Then (56) is negative if and only if

$$-\gamma(r+\delta)\left(A_1^c(1+\gamma)(2+\gamma)-2\gamma(r+\delta)\right)+A_1^{nc}\left(2(2+\gamma)A_1^c+\gamma(-4+\gamma+\gamma^2)(r+\delta)\right)>0,$$

that can be rewritten as

$$-\gamma(r+\delta)\left(A_1^c(1+\gamma)(2+\gamma)-2\gamma(r+\delta)\right)+\gamma(-4+\gamma+\gamma^2)(r+\delta)A_1^{nc}+2(2+\gamma)A_1^{nc}A_1^c>0.$$

Taking into account that $A_1^c > A_1^{nc}$, a sufficient condition that ensures the fulfillment of the last inequality is given by

$$-\gamma(r+\delta)\left(A_1^c(1+\gamma)(2+\gamma)-2\gamma(r+\delta)\right)+\gamma(-4+\gamma+\gamma^2)(r+\delta)A_1^{nc}+2(2+\gamma)\left(A_1^{nc}\right)^2>0.$$

Substituting the expressions of A_1^c and A_1^{nc} and rearranging terms, the inequality above reads

$$\begin{aligned}&2(2d(2+\gamma)^2+r^2(4+2\gamma^2+\gamma^3)+r(16+4\gamma+3\gamma^2+\gamma^3)\delta+2(8+4\gamma+\gamma^2)\delta^2)\\&\quad+\gamma(1+\gamma)(r+\delta)\sqrt{\gamma(4+\gamma)[4d(r+\gamma)^2+\gamma(4+\gamma)(r+2\delta)^2]}\\&\quad-(2+\gamma)\left(r(4-2\gamma+\gamma^2+\gamma^3)+(8+\gamma^2+\gamma^3)\delta\right)\sqrt{4d+(r+2\delta)^2}>0.\end{aligned}$$

The first and second lines in the inequality above are positive, while the third one is negative. Therefore, the inequality above is equivalent to the following inequality

$$\begin{aligned}&\Big[2(2d(2+\gamma)^2+r^2(4+2\gamma^2+\gamma^3)+r(16+4\gamma+3\gamma^2+\gamma^3)\delta+2(8+4\gamma+\gamma^2)\delta^2)\\&\quad+\gamma(1+\gamma)(r+\delta)\sqrt{\gamma(4+\gamma)[4d(r+\gamma)^2+\gamma(4+\gamma)(r+2\delta)^2]}\Big]^2\\&\quad-\left[(2+\gamma)\left(r(4-2\gamma+\gamma^2+\gamma^3)+(8+\gamma^2+\gamma^3)\delta\right)\right]^2\left(4d+(r+2\delta)^2\right)>0.\end{aligned}$$

After some calculus, the expression above can be rewritten as:

$$\Omega_1(\gamma,\delta,d,r)+\Omega_2(\gamma,\delta,d,r)\sqrt{\gamma(4+\gamma)[4d(r+\gamma)^2+\gamma(4+\gamma)(r+2\delta)^2]}>0, \qquad (57)$$

where $\Omega_1(\gamma,\delta,d,r)$ and $\Omega_2(\gamma,\delta,d,r)$ are positive and given by

$$\Omega_1(\gamma,\delta,d,r) = \gamma(1{+}\gamma)(r{+}\delta)\times$$
$$\left[2d(2{+}\gamma)^2{+}r^2(4{+}\gamma^2(2{+}\gamma)) + r(16{+}\gamma(4{+}\gamma(3{+}\gamma)))\delta + 2(8{+}\gamma(4{+}\gamma))\delta^2\right],$$
$$\Omega_2(\gamma,\delta,d,r) = 4d^2(2{+}\gamma)^4$$
$$+4d\gamma(2{+}\gamma^2)(r{+}\delta)\left[r(4{+}\gamma(-1{+}\gamma{+}3\gamma^2{+}\gamma^3)) + (2{+}\gamma)(4{-}3\gamma{+}\gamma^3)\delta\right]$$
$$+\gamma^2\left[r^4(16{+}\gamma(1{+}\gamma)(-4{+}\gamma(2{+}\gamma)(4{+}\gamma))) + r^3(80{+}\gamma(1{+}\gamma)(-28{+}\gamma(32{+}\gamma(31{+}6\gamma))))\delta\right.$$
$$+\, r^2(128{+}\gamma(-104{+}\gamma(-40{+}\gamma(3{+}\gamma)(35{+}13\gamma))))\delta^2$$
$$\left.+4r(-4{+}\gamma(2{+}\gamma))(-4{+}9 + \gamma(10{+}3\gamma)))\delta^3{+}4\gamma(-24{+}\gamma(-16{+}\gamma(3{+}\gamma(5{+}\gamma))))\delta^4\right].$$

$\Omega_1(\gamma,\delta,d,r)$ is always positive, and it can be easily proved that Ω_2 is positive for any value of γ greater than 2. Therefore, the inequality in (57) is always satisfied for $\gamma > 2$, and consequently, $C_2^c < C_2^{nc}$ what implies that $V^c(0)$ is lower than $V^{nc}(0)$.

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