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ON THE FORMULATION AND SOLUTION OF ECONOMIC EQUILIBRIUM MODELS

by

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This is the introductory paper presented at the Workshop on the Application and Solution of Economic Equilibrium Models, June 1984. Helpful comments have been received from Victor Ginsburgh, Scott Rogers, Herbert Scarf, Gunter Stephan and John Weyant. The author is solely responsible for the views expressed here. The resulting volume of papers from the Workshop is to appear as a Mathematical Programming Study (see the attached Economic Equilibrium: Model Formulation and Solution). This work was supported by the National Science Foundation and the Center for Economic Policy Research.

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Abstract

This paper is addressed to the formulation and solution of CGE (computable general equilibrium) models. Topics are explored in the following sequence: (1) fundamental concepts of the Scarf equilibrium model; (2) a numerical example; (3) time paths of adjustment; (4) extensions - the production possibility set; (5) further extensions; (6) dimensionality reduction; (7) fixed-point and other general-purpose solution methods; and (8) a survey of numerical models.

Keywords: economic equilibrium, CGE model formulation, fixed-point and other solution methods.

Economic Equilibrium: Model Formulation and Solution

Edited by A.S. Manne

CONTENTS

This volume contains revised versions of papers presented at a workshop held on June 25, 26, 1984. The meeting was in the Koret Conference Series, Center for Economic Policy Research, Stanford University.

Except for several appendices (available upon request from individual authors), these papers are to appear as a Mathematical Programming Study.

Preface

This Mathematical Programming Study is intended both for economists and for mathematical programmers who wish to become better acquainted with recent developments in the area of economic equilibrium. The authors have done their best to avoid jargon that is specific to one or the other discipline. Some of the papers are expository in nature; some contain contributions to model formulation; and others advance the state-of-art of computations.

The volume is centered around CGE (computable general equilibrium) models. The CGE framework provides a logically consistent way to look at problems involving more than one economic agent. For public finance applications, these agents may be located within a single country, and the model will then focus on equity-efficiency tradeofffs between individuals in different income groups. For international trade models, the agents may consist of individual countries or regions. Trade models may be concerned with capital flows, Third World debt issues and with tariff and non-tariff barriers to trade. They may also refer to international trade in agriculture or in energy.

Typically, CGE models are microeconomic in nature. They deal with a sufficiently long horizon so that it is plausible to assume that prices are flexible, and that money is neutral. None of the papers in this monograph deal with inflation, exchange rates or other monetary phenomena.

Computable equilibrium models require close communication between several distinct disciplines: policy analysis, algorithm design and the estimation of consumers' demands and of producers' supplies. Each of these groups is represented here, and each has something useful to contribute.

The first paper is by A.S. Manne. This is a broad survey intended for non-specialists. It shows how the formulation of a CGE model may be influenced - consciously or otherwise - by the tools that are available for their solution. It also discusses the pros and cons of introducing expectations and adjustment costs into CGE models.

V. Ginsburgh and L. Van der Heyden show how mathematical programming techniques, together with Negishi weights, may be used to compute equilibria. The authors present a numerical model dealing with the short-run impact of income and exchange policies upon employment in Belgium.

A.L. Bovenberg illustrates the importance of adjustment costs through an aggregate model in which a consumption tax is introduced in place of a tax on capital income.

A.S. Manne and P.V. Preckel describe three different applications of a dynamic CGE model - one dealing with OPEC and future world oil prices, another dealing with OECD energy demand elasticities, and the third with constraints on North-South capital flows.

T. Condon, S. Robinson and S. Urata show how a CGE model can be applied to analyze Turkey's medium-term structural adjustment problems.

C.J. Heady and P.K. Mitra describe a procedure for computing optimal taxes and public production in an economy where the government has limitations on its ability to levy taxes and to provide subsidies.

M. Broadie provides an introduction to fixed-point techniques. These provide a rigorous, general-purpose method for computing economic equilibria. Convergence is guaranteed.

L. Mathiesen describes how economic equilibria may be computed by SLCP (a sequence of linear complementarity problems). SLCP provides a fast way to obtain numerical solutions. Although the method is not guaranteed to converge, failure occurs only rarely. The approach handles cases characterized by weak inequalities, complementary slackness and "non-integrability".

P.V. Preckel compares the methods of Broadie, Mathiesen and two others. He concludes that Mathiesen's SLCP approach is the most efficient technique among those considered.

J.C. Stone provides a comparison between SLCP and two approaches based on a sequence of optimization problems. Each of these methods is promising, but the experimental results are inconclusive at this point.

R.L. Phillips shows how Gauss-Seidel solution methods (combined with under-relaxation) may be highly efficient for large-scale nonlinear systems in which there are exactly as many unknowns as equations.

O.J. Blanchard describes how rational expectation models have been applied to stock market speculative phenomena such as bubbles and crashes. They are also applicable to anticipated changes in tax rates and in social security benefits. This paper provides a survey of numerical methods that are applicable to medium and large-scale problems.

B.C. Eaves analyzes a special case of CGE - a pure exchange model in which each agent has Cobb-Douglas utility functions. For this case, he shows how an economic equilibrium may be computed by solving a single set of simultaneous linear equations.

T.J. Kehoe reviews what is known about non-uniqueness - the case in which there is more than one solution associated with a CGE model.

Alan S. Manne

1. Introduction

Price-guided equilibrium continues to be a lively topic. It is a concept that goes back to the beginnings of economic thought. Smith's invisible hand, the Walras system of interdependent equations, Edgeworth's contract curve and the Arrow-Debreu existence proof - these are among the landmark achievements in the history of economic analysis.

Market equilibrium is a fundamental but an elusive concept. There are always unforeseen shocks which inhibit smooth adjustment. Markets do not clear instantaneously. Over the short run, prices may be rigid, and factors of production may be tied to specific industries and regions. Time is required for the operation of these adjustment processes. There is no easy way to integrate short-run theories of income, employment and money with long-run theories of equilibrium price formation. There is still a wide gulf between macro and microeconomics - in large part because of the difference in their time perspectives.

Despite these difficulties, computable equilibrium models are coming into widespread use for policy analysis. They provide a unified framework for analyzing the tradeoffs between economic efficiency and equity. Typically, the applications are microeconomic in nature - public finance, international trade and capital flows, agriculture and energy. Virtually all are computer-oriented. They follow in the tradition that began during the 1940s with Leontief's input-output work. These consistency models were capable of dealing with large-scale systems, but proved to have a number of shortcomings. Two difficulties - the lack of price-induced substitution and the absence of economic efficiency criteria - were soon overcome by the development of linear activity analysis. For an account of the early work in this area, see Dantzig (1951) and Koopmans (1951).

By the 1970s, it became routine to handle linear programming systems with thousands of variables and constraints. See Taylor (1975) for a general survey - and also a review of the idiosyncrasies of these models: flip-flop behavior, ad hoc constraints, etc. Fortunately, there soon occurred a rapid evolution in the art of <u>non</u>linear programming, and the new algorithms took care of some of the objections that had been raised by critics of the early linear models. MINOS, a nonlinear optimization system written by Murtagh and Saunders (1983), has proved its reliability at processing large-scale systems.

Notwithstanding the improvements in nonlinear programming, there was a more fundamental difficulty. Virtually all the early work was based upon a planning viewpoint - as though there were a single decision-maker maximizing an economy-wide objective function subject to technical and behavioral constraints. This may be appropriate for calculations of economic <u>efficiency</u>, but is clearly inappropriate for analyzing <u>equity</u> issues among individual socioeconomic groups within a market economy. "Who gets what?" is a key issue - whether we are concerned with United States tax policy or with income distribution and dualism in the developing nations.

Given the importance of distributive issues, there has been growing recognition of the contributions of Scarf - both for his workable formulation of equilibrium problems and his rigorous procedures for obtaining numerical solutions. His first book on this topic - Scarf (1973) - has already been made obsolete by his own subsequent work and by that of his associates. See Scarf and Shoven (1984). This introductory paper is based largely upon their work - plus that of Ginsburgh and Waelbroeck (1981).

CGE (computable general equilibrium) models need no longer be limited to 10 or 20 equations. There are good prospects for overcoming the "curse of dimensionality". There is continual interplay between policy issues and the tools of economic analysis. Just as linear programming revolutionized our ability to analyze economic efficiency issues during the 1950s and 1960s, a similar process is likely to occur with equilibrium models and with equity/distribution issues during the next decade.

The formulation of CGE models may be influenced - consciously or otherwise - by the tools available for their solution. It is therefore highly productive to maintain close ties between those who formulate quantitative models for policy analysis, those who estimate their parameters and those who provide efficient methods for numerical solution. Unfortunately, this introductory survey cannot do justice to the topics of policy analysis and of parameter estimation.

Formulation and solution will be explored in the following sequence: (1) fundamental concepts of the Scarf equilibrium model; (2) a numerical example; (3) time paths of adjustment; (4) extensions - the production possibility set; (5) further extensions; (6) dimensionality reduction; (7) general-purpose solution methods; and (8) a survey of numerical models.

2. Fundamental concepts

Scarf's equilibrium model employs linear activity analysis to describe the supply side of the economy. Both technological process information and also econometric estimates may be employed for this purpose. Data and unknowns are defined as follows:

n: a finite number of commodities, defined in terms of their physical attributes, locations and dates; this rules out models with a continuum of locations or an infinite planning horizon;

w: an n-dimensional nonnegative vector of resource endowments;

A: a linear activity analysis matrix (outputs positive and inputs negative), with n rows and k columns, including disposal activities;

π, y: the unknowns of the equilibrium model - respectively, an n-dimensional price vector and a k-dimensional vector of activity levels; these unknowns are to be nonnegative.

Economy-wide demands are summarized by a system of demand functions. The following equations may be estimated econometrically or by other methods:

$x(\pi)$: market demands for each of the n commodities, expressed as continuous nonlinear functions of the price vector π.

Typically, it is stipulated that the market demand functions satisfy the Walras law at all values of the price vector π:

$$\pi \, x(\pi) = \pi \, w.$$

Loosely speaking, the Walras law states that "individuals always remain on their budget constraints". This conclusion follows from non-satiation, utility-maximizing behavior on the part of individual consumers and no economic rents on constant-returns activities.

To see this, let the vector w_h denote the resource endowment initially available to consumer h. Similarly, let x_h denote the consumer's demands (to be determined in response to the market price vector π), and let the utility function $u_h(x)$ define the individual's preference ordering among alternative demand vectors. Given a price vector π, the demand reactions $x_h(\pi)$ are determined by the individual's tastes, resource endowment and budget constraint as follows:

$$\begin{aligned} &\text{maximize } u_h(x_h) \\ &\text{subject to } \pi\, x_h \leq \pi\, w_h \\ &\qquad\qquad\quad x_h \geq 0. \end{aligned}$$

Non-satiation - that is, $u_h(x_h)$ is a monotone increasing function of x_h - implies that the consumer's optimal strategy is to use up one's entire income (in a single-period model) or to exhaust one's wealth (over a finite number of time periods). This is the meaning of the Walras law.

The individual optimization model has a further important consequence. If market demands are derived by adding over individual households, the functions $x(\pi)$ must be homogeneous of order zero. This means that money is "neutral", and that it is inappropriate to employ this type of model for analyzing monetary phenomena such as inflation and international exchange rates. Since the Scarf model generates <u>relative</u>

prices only, the π vector may be normalized in terms of a numéraire commodity whose price is unity. Alternatively, the price vector may be normalized so that the sum of its components equals unity.

Definition of economic equilibrium: A non-zero price vector π^* and a nonnegative vector of activity levels y^* are said to represent equilibrium prices and activity levels if:

1. $x(\pi^*) = Ay^* + w$ (balancing bundle of supply-demand choices)
2. $\pi^*A \leq 0$ (non-positive excess profits)
3. $\pi^*Ay^* = 0$ (complementary slackness; a direct consequence of (1) and the Walras law)

Together, (2) and (3) imply that there are zero excess profits on activities at positive levels; zero activity levels on unprofitable activities; and zero prices on commodities for which the disposal activity is at a positive level. It can also be shown that this model leads to an economically efficient (Pareto-optimal) solution. That is, there is no reallocation of resources that can lead to an increase in the utility of one consumer without a reduction in someone else's utility.

An economic equilibrium may be defined in a straightforward way, but its existence is not easy to establish. For this purpose, Scarf relies upon the Kakutani fixed-point theorem. This theorem ensures the existence of a solution to the nonlinear system of equilibrium equations and weak inequalities (1)-(3), but it does not show how to compute π^* and y^*. Before considering computational methods, we will examine a numerical example, and then consider a number of issues that arise in the formulation of such models.

3. A numerical example of economic equilibrium

In order to fix ideas, it is helpful to consider a numerical example. This is a "pure trade" case in which there is no production matrix A, and no vector of activity levels y. Consider the case of two consumers - Ann and Jack - who are isolated in the back country of the Sierra Nevada. If there are only two commodities - bread and wine - it is possible to solve this problem through a diagram originated by Edgeworth. Let π_1 and π_2, respectively, denote the price of commodity 1 (bread) and commodity 2 (wine). These two prices will be normalized so that their sum is unity.

The initial endowment data are as follows:

w = economy-wide endowment vector: 100 units of each commodity;
w_A = endowment vector, Ann's household;
w_J = endowment vector, Jack's household;
w_{ih}= endowment, commodity i (i = 1,2), household h (h = A,J)

For simplicity, suppose that $w_{1A} = w_{2A}$. Then the set of all possible initial distributions of purchasing power is indicated by the 45° line extending northeast from the origin in Figure 1. At point B, for example, Ann owns 25 units of bread and also 25 units of wine. Jack owns the balance. At point C, Ann owns 50 units of each resource, and at point D she owns 75 units.

At prices π_i, household h chooses x_{ih}, its level of consumption of commodity i. Ann's demand choices are measured to the north and east of her origin at 0_A. Jack's are measured to the south and west of his origin at 0_J. Suppose that Ann and Jack have Cobb-Douglas utility functions. These may be written:

$$u_A = x_{1A}^{\alpha} \, x_{2A}^{1-\alpha}$$

$$u_J = x_{1J}^{\beta} \, x_{2J}^{1-\beta}$$

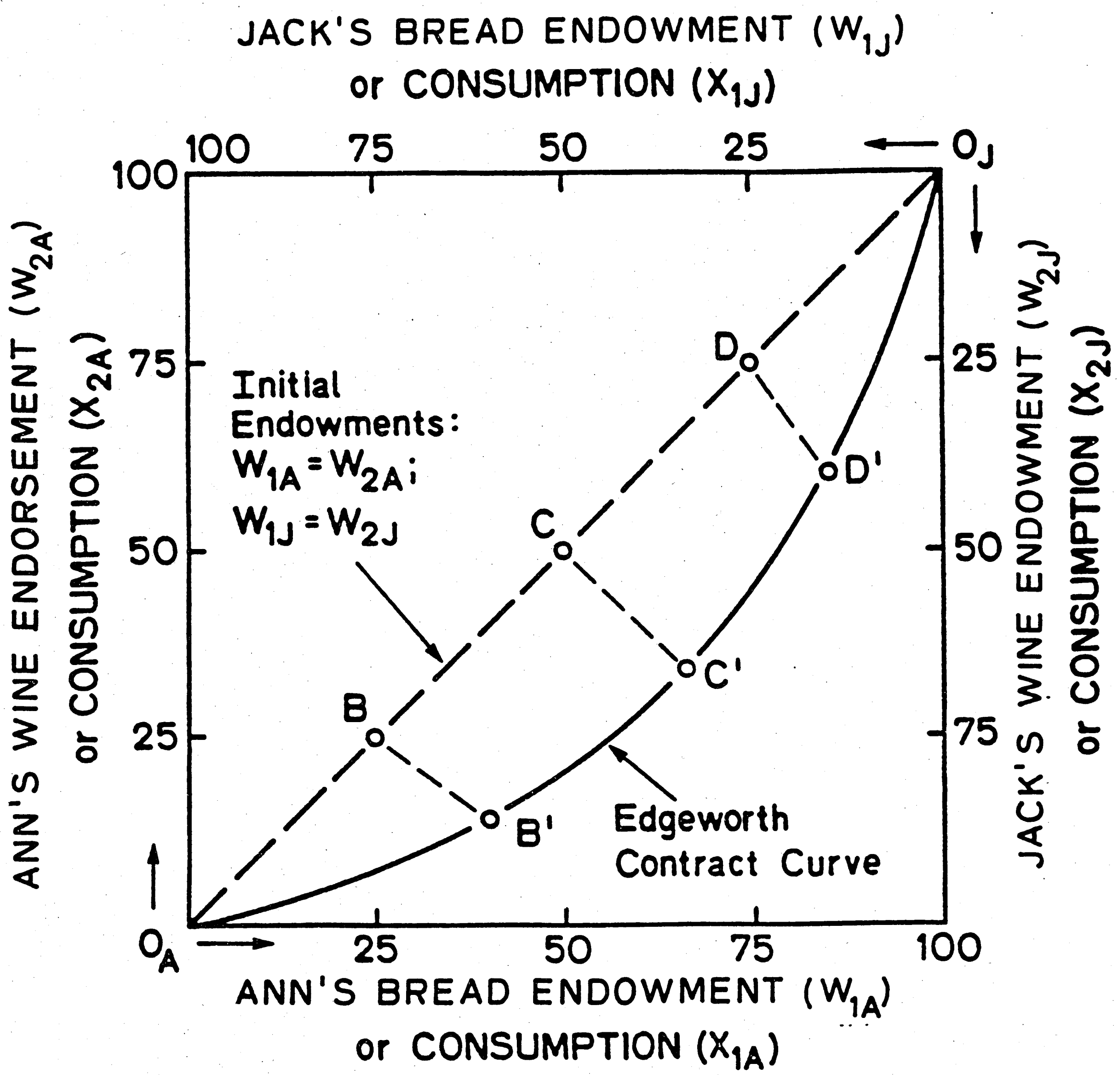

Figure 1. An Edgeworth diagram

With these utility functions and endowment distributions, Ann's demand functions depend as follows upon the two prices:

$$x_{1A} = \alpha(\pi_1 w_{1A} + \pi_2 w_{2A}) / \pi_1 = \alpha(w_{1A})/\pi_1$$

$$x_{2A} = (1-\alpha)(\pi_1 w_{1A} + \pi_2 w_{2A}) / \pi_2 = (1-\alpha)(w_{1A})/\pi_2$$

Suppose that $\alpha = 2/3$, $\beta = 1/3$, and the initial endowments are at point D (with $w_{1A} = w_{2A} = 75$). With these numerical data, the equilibrium price vector is $\pi_1 = 7/12$, and $\pi_2 = 5/12$. These two prices determine the slope of the dashed budget line from the endowment point D to the equilibrium demand point D', where the utility contour lines (indifference curves of Ann and Jack) are tangent to each other. By symmetry, if the initial endowments were at point B, the bread and wine prices would be 5/12 and 7/12 respectively. The demands of the two households would then come into equilibrium at the point B'. At the equilibrium C', $\pi_1 = \pi_2 = .5$. Each point along the Edgeworth contract curve B'C'D' leads to a different set of relative prices between the two commodities.

This example illustrates the general proposition that the price structure depends not only upon tastes and technology, but also upon the initial distribution of resources. Prices and demands are independent of this distribution only when Ann and Jack have identical values for their taste parameters α and β. The economic equilibrium problem may then be replaced by an optimization model in which there is a single representative consumer. In this case, the demand functions $x(\pi)$ are "integrable" into an economy-wide utility function, and the equilibrium conditions may be replaced by a simpler optimization model.

The Ann-Jack parable confirms Bernard Shaw's first maxim for revolutionists: "Do not do unto others as you would that they should do unto you. Their tastes may not be the same."

4. Time paths of adjustment

It requires <u>time</u> to adjust to market forces, but there is no general agreement on ways of introducing dynamics into models of economic equilibrium. One approach is that described by Ginsburgh and Waelbroeck (1981, p. 166):

> Our feeling is that it is not appropriate to use general equilibrium models to describe short-run economic behavior which is dominated by expectational phenomena and other types of disequilibria. The ten-year span of GEM (a general equilibrium model of the world economy) is, however, long enough for equilibrium forces to assert themselves. The model describes economic behavior in a one-period framework. This requires that some way is found to cut out a meaningful static equilibrium from the dynamic fabric of possible future developments.

The term "counterfactual equilibrium analysis" is sometimes employed to describe a comparison between the status quo and the hypothetical situation that would arise as a consequence of a substantial policy change. For example, Whalley (1984, Table 1) examines a change in tariff rates "under complete adjustment to the change in the trade policy regime in one or more regions. Neither time for adjustment to occur, nor costs of adjustment (are) analyzed." Similarly, in their review of empirical general equilibrium models of taxation, Fullerton, Henderson and Shoven (1984, p. 392) note out that only a few authors have treated time in an explicit way.

The counterfactual method is a useful first step. A static analysis is all that is needed if, for example, there are only minor long-run gains available from the removal of tariff barriers to international trade. In this case, it would hardly be worthwhile to examine the details of the time path of adjustment to a proposed change in policy.

It is not always possible to avoid the problem of "how to get from here to there". For this reason, when Auerbach and Kotlikoff (1983) analyzed the effects of changes in tax structure upon national savings, they found it useful to employ a 150-year intertemporal model of the transition from one tax regime to another. To analyze the announcement effects of future changes in tax and expenditure policy (e.g., changes in social security legislation), they defined individual age cohorts separately - each with its life-cycle pattern of consumption and savings.

A long time scale is also needed to evaluate energy policy issues. Energy-consuming devices are durable, and it takes time for energy demands to adapt to higher prices. Intertemporal issues arise with respect to energy supplies. According to the Hotelling-Nordhaus depletion model, each economic agent correctly anticipates a sequence of future prices. Today's energy markets are influenced by the length of time that it will take to make a transition from low-cost exhaustible resources to high-cost "backstop" sources of supply.

In formulating dynamic models, two distinct approaches have been employed. One may be labeled "myopic" and the other "clairvoyant". In more technical terms, the former is "recursive", and the latter is "intertemporal" dynamics. Each of these approaches has its own difficulties.

With a clairvoyant model, each commodity is dated. All households and producers make consistent projections of future prices. With an equilibrium sequence of prices, the supplies and demands balance for each commodity at each point of time. Typically, it is supposed that the planning horizon is sufficiently long so that relative prices and quantities remain constant over an infinite horizon subsequent to the terminal period.

From a descriptive/institutional viewpoint, the clairvoyant model leads to all sorts of difficulties - e.g., the legal enforceability of a contract for the delivery of a thousand barrels of Middle East oil 10 or 50 years from now. A myopic model handles these difficulties by dealing with only one period at a time - neglecting the impact of subsequent changes in prices, tastes, technologies or resource endowments. If one assumes that today's prices will persist into the indefinite future, this is sometimes termed "static expectations".

To date, there has been no systematic comparison of the clairvoyant versus the myopic approach. Much depends upon the value of information. Are today's supply and demand decisions significantly influenced by <u>expectations</u> as to changes in prices, tax rates, etc.?

A myopic approach can easily lead to inconsistent expectations and to cobweb cycles - with prices and quantities overshooting and then undershooting their long-run equilibrium levels. A clairvoyant model may avoid these difficulties, but generally requires additional data. Almost certainly there will be an increase in computing costs. These costs increase exponentially with the number of time periods. Thus, the practicality of an intertemporal model depends upon the state-of-art of numerical solution techniques. We will return to this issue later, but first will explore some extensions of Scarf's basic model.

5. Extensions - the production possibility set

Scarf's model is based upon the idea that production may be described in terms of constant or diminishing returns to scale. (Price-guided decentralization becomes difficult or impossible in the case of increasing returns.) The activity analysis model of production may be extended to handle nonlinear production functions - provided that the production sets remain convex. Suppose, for example, that labor and capital may be substituted for each other in the production of "value added" through a Cobb-Douglas production function:

$$Q = K^{\alpha} L^{1-\alpha}$$

where Q = value added (output variable)
K = capital (input variable)
L = labor (input variable)
α = elasticity of value added with respect to capital; $0 < \alpha < 1$

This nonlinear production function implies a continuum of activities - not the finite number postulated by Scarf. Several methods are available for including continuous substitutability within an equilibrium model. One method came into widespread use during the period when linear programming was the only practical method available for the computation of large-scale systems. A nonlinear production function may always be approximated by prespecifying an arbitrary number of combinations of capital and labor inputs per unit of output. In geometric terms, this means that the production possibility set is represented by a polyhedral approximation to the original cone.

By prespecifying the substitution activities in this way, there are several difficulties: (a) there is flip-flop behavior if too few activities are selected - that is, small price variations can lead to large changes in quantities; and (b) because of the exponential increase in the number of possible combinations, it becomes awkward to handle simultaneous substitution between three or more inputs in this way.

For these reasons, there has been an increasing tendency to avoid the prespecification of activities, but instead to use "column generating" techniques. (Hudson and Jorgenson (1974) were among the earliest to apply this idea to economic equilibrium models.) At each iteration of a Newton method, fixed-point or other algorithm, there is available a tentative set of prices for the inputs of capital, labor, etc. Given these values, it is straightforward to calculate an optimal mix of inputs per unit of output. These input-output coefficients are then included as a new vector in Scarf's production matrix A. Column-generating methods are easy to describe, but can be tedious to implement. Typically, however, they are preferable to the earlier methods for approximating nonlinear production functions within linear activity analysis models.

When there are <u>diminishing</u> returns to scale - and a <u>unique</u> response of producers to each price vector π, Scarf's basic model may be reformulated in still another way. For concreteness, again consider the Cobb-Douglas production function - but with fixed inputs of capital. This means that there are diminishing marginal returns to the variable input of labor. At any given price of labor relative to output, there is a unique optimal response - one which maximizes the returns to the fixed factor of capital. We may then include the net demands for labor (positive) and output (negative) within $x(\pi)$, the demand equations of the economy. Supplies differ from demands only in their algebraic sign!

With diminishing returns to scale, there are economic rents. Care must be taken to attribute the ownership of the fixed input (capital) to the resource endowment of one or more households. This is essential if the Walras law is to be satisfied by the demand equations $x(\pi)$. The issue of ownership does not arise in the case of activities with constant returns to scale. Scarf's condition (2) ensures that excess profits - the rents on fixed factors of production - can never be positive when there are constant returns.

In designing an equilibrium model, it is convenient to choose functional forms that simplify the estimation of numerical parameters. It is typical to employ "benchmarking" procedures to evaluate parameters such as α, the elasticity of output with respect to capital. That is, one observes a specific combination of prices of capital, labor and output during a statistical base year. One also observes the quantities K, L and Q. Provided that prices and quantities were in equilibrium at that time, the Cobb-Douglas specification immediately implies that capital's historical value share is identical to the elasticity value α.

Benchmarking is a convenient shortcut for parameter estimation. This accounts for the popularity of Cobb-Douglas and of nested CES (constant elasticity of substitution) functional forms. More general production functions (e.g., translog) provide greater flexibility, but they also require more data than is provided by a single year's statistical observations. On the pros and cons of benchmarking versus more elaborate econometric estimation procedures, see the interchange between Lau (1984) and Mansur and Whalley (1984).

6. Further extensions of the basic model

Scarf's model is sufficiently flexible so that it may be extended in a number of directions. This brief introduction cannot do justice to all these possibilities. Just two examples will be reviewed - one dealing with wage rigidities and unemployment; the other with taxes, subsidies and public finances. Both of these cases may lead to "system constraints" in addition to the usual supply-demand balances.

Unemployment is compatible with long-run market equilibrium - provided that there is a lower bound on real wages and also rationing on the supply side of the labor market. The basic model is modified by dropping a price variable (the wage rate) and introducing a quantity variable in its place (the rate of unemployment). These two variables are complementary. Unemployment is not positive unless the wage constraint is binding.

Consider, for example, an economy with a fixed endowment of labor (100 units) - and with producers' demand for labor inversely proportional to w, the wage rate. Suppose that full employment requires wage rates of 40 units (expressed in terms of the numéraire good). With flexible wages and full employment, the labor supply and demand equation may be written:

$$\text{demand for labor} = \text{supply of labor}$$
$$4000/w = 100$$

Now let there be institutional constraints or time lags of adjustment so that the wage rate w must be at least 50 units - still measured in terms of the numéraire good. If this wage constraint is binding, there is a positive value of the complementary quantity variable u, the fraction of the labor force that is unemployed. The binding wage constraint and the unemployment variable may then be combined as follows in the modified labor supply-demand equation:

$$\text{demand for labor} = \text{supply of labor} - \text{unemployment}$$
$$4000/50 = 100\ (1-u)$$

In this example, equilibrium is reached when the unemployment rate $u = 20\%$. For further details on unemployment and wage rigidities, see Kehoe and Serra-Puche (1983). Existence of an equilibrium is proved by noting that supplies and demands are <u>continuous</u> functions of the unemployment rate and the price variables. A similar existence proof applies for public finance applications such as the one described below.

Consider an ad valorem excise tax which drives a <u>wedge</u> between the prices paid by consumers and those received by producers. Let there be just two commodities. The numéraire good is not taxed, and it is employed solely for government consumption. The taxed good is sold by producers at the price π, and it is purchased by consumers at the price $\pi(1+t)$.

Let the supply and demand curves for the taxed commodity both be linear. Choose units of measurement so that both the equilibrium quantity and the price (expressed in terms of the numéraire) are 1.00 when the tax rate is zero. At this point, let the price elasticities of supply and demand for the taxed good be, respectively, +1 and -1. As can be seen from Figure 2, the supply-demand balance equation for the taxed commodity may be expressed as the following function of the producer price variable and the tax rate parameter:

$$\text{supplies} = \text{demands}$$
$$\pi = 2 - (1+t)\pi.$$

Unlike a large-scale public finance model, this one is sufficiently simple so that one may solve analytically for the market-clearing price as a function of the tax rate:

$$\pi = 2/(2+t).$$

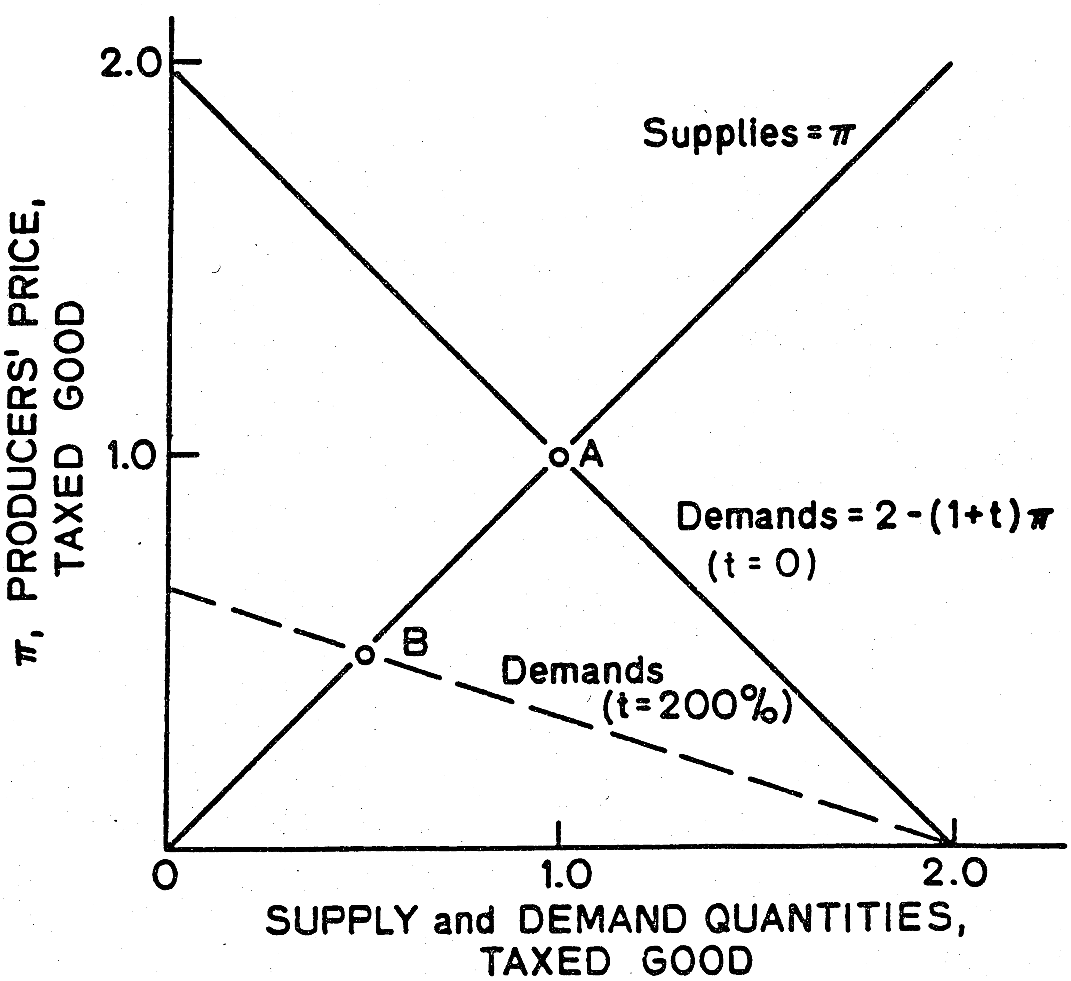

Figure 2. Effects of an excise tax

Two alternative tax rates are illustrated on Figure 2: zero (shown with a solid line) and 200% (shown with a dashed line). This model may be used to illustrate several ideas related both to model formulation and to computation:

(1) Public finance models may be employed to calculate optimal tax rates. For example, an excise tax rate of 200% will maximize the government's total income and its expenditures on the numéraire good. (Compare points A and B, Figure 2.) Instead of maximizing the government's revenues, a more reasonable goal might be to set tax rates so as to maximize a social welfare function. For an instructive example along these lines, see Heady and Mitra (1984). More typically, tax rates are viewed as the outcome of a political process - with several alternative scenarios but no explicit social welfare function. This is the procedure followed in all eight of the empirical public finance models reviewed by Fullerton, Henderson and Shoven (1984).

(2) Figure 2 illustrates why one must be cautious about Gauss-Seidel solution algorithms for solving a simultaneous system of demand and supply equations. With this method, one might begin with an approximate equilibrium price, insert this value in the supply equation to obtain an approximate equilibrium quantity, insert this quantity in the demand equation to obtain a new approximation to the equilibrium price, and so on. From Figure 2, the reader may verify that this procedure will cycle endlessly when the tax rate is zero, but that it will converge when the tax rate is 200%.

Even with a 200% tax rate, Gauss-Seidel may work poorly. It will diverge explosively away from the equilibrium if one initiates the algorithm by inserting an approximate price into the demand equation, and then continues by inserting the resulting quantity into the supply equation. The performance of Gauss-Seidel algorithms will depend upon the analyst's skill at decomposing a system of variables and equations into a form that is (nearly) block-triangular.

Analogies with Gauss-Seidel may also account for the success of the PIES algorithm for energy modeling. See Hogan (1975) and Ahn and Hogan (1982). PIES is initiated by inserting an approximate price vector into an econometric submodel to determine energy demands. The demand equations are then approximated by separable (and therefore integrable) functions. These are transmitted into a process model of energy supplies - which may be solved by a linear or nonlinear optimization algorithm. This results in a new approximation to the vector of market-clearing prices. The price vector is inserted back into the demand submodel, and the process is repeated. Hogan reports that convergence is achieved in just a few iterations.

The experimental evidence remains inconclusive. As a general-purpose tool, one must be cautious about Gauss-Seidel methods. According to Fullerton, Henderson and Shoven (1984, p. 402):

> The costs of the Gauss-Seidel method depend on efficient ordering of equations into simultaneous and recursive blocks. We have not found any discussion of this in the applied general equilibrium literature, however.

See, however, Phillips (1984) for an encouraging report on Gauss-Seidel combined with successive under-relaxation methods. Excellent empirical results have also been obtained by Mercenier and Waelbroeck (1984).

7. Dimensionality reduction

Two-dimensional diagrams provide useful insights, but are inherently limited. Particularly when a model is designed to analyze the differences between individual political interest groups, there is a tendency to include many details - even though many of them will eventually turn out to be immaterial to the main points at issue. This is why a quantitative policy analysis will often require large-scale models with many sectors, many primary factors of production, locations, time periods, etc.

Large models have their limitations. One is never quite sure whether a counter-intuitive result is a genuine discovery - or whether it is a consequence of errors in data or in computer programming. If such a model generates puzzling results, it is always good practice to attempt to replicate the paradox in a smaller more easily understandable system. When in doubt, it is prudent to observe the maxim: "If the answer is counterintuitive, it is wrong." Hogan (1978).

The principal limitations on model size are the ability to manage a large-scale data base and to interpret the results in a meaningful way. Computing costs cannot, however, be neglected. If a model is so large that a few runs will exhaust one's computer budget, the system is bound to be inflexible. There will be no possibility for last-minute inclusion of variables and constraints that turn out to be important, but were not anticipated in the initial model design.

Whenever it becomes difficult to solve a large-scale system through a single algorithm (Newton method, fixed-point, etc.), it is worthwhile to make an attempt at dimensionality reduction. This works particularly well in the case of block-triangular linear systems when one can eliminate some of the variables and constraints by solving for them as linear functions of other variables.

Consider a case in which there are 20 commodities produced by three primary factors of production - capital, skilled and unskilled labor. At first glance, it looks as though this requires us to solve a 23-dimensional system. Suppose, however, that interindustry material flows may be described by a square Leontief matrix of fixed input-output coefficients (independent of relative prices). The input-output matrix may then be inserted and used to translate the final demands for the 20 commodities into the direct and indirect requirements for the three primary factors of production. Instead of 23 individual supply-demand balances, the system has been reduced to just three equations. For a typical application of these ideas, see Serra-Puche (1984).

A special form of dimensionality reduction is employed in intertemporal models. In order to apply control theory and the discrete-time "maximum" principle, it is typical to assume that capital stocks are homogeneous (not specific to sectors or to regions), and that there are only a few types of capital stock variables. It is also convenient to suppose that all gradients are well-defined, and that there are interior solutions. These assumptions lead to a block-triangular form of dynamic interdependence. The system may be solved as a two-point boundary value problem - that is, with specified levels of the initial and terminal capital stocks. Through recursive calculations based on first-order optimality conditions, the net demands for the terminal period's capital capital stocks are expressed as functions of the initial period's price vector. The initial prices are then adjusted so that the terminal period's net demands are non-positive.

Dimensionality reduction need not be applied directly to the entire planning horizon. Instead, with many periods, it may be advisable to divide and conquer - that is, to divide the planning horizon into sub-periods, solve each separately, and then combine the results. Lipton, Poterba, Sachs and Summers (1982) report that this variation on the "shooting" algorithm leads to rapid convergence for growth models involving up to 100 time periods.

Decomposition methods provide another example of dimensionality reduction. In the Dantzig-Wolfe (1961) decomposition algorithm for linear programs, an overall optimization problem is divided into a "master" and "subproblems". Prices are transmitted from the master to the subproblems, and quantity responses are transmitted in the reverse direction. Both the master and the subproblems are each of much lower dimension than the original model. Similarly, Mansur and Whalley (1982) have shown that this is a promising approach to the analysis of international trade. In this case the common block consists of tradeable commodities, and the individual subblocks consist of non-tradeables within each individual country. The decomposition principle has been applied on a large scale within the IIASA (International Institute for Applied Systems Analysis) model of trade in food and agricultural products. See Keyzer (1980).

Decomposition and dimensionality reduction have been applied in still another way by Ginsburgh, Van der Heyden and Erlich (1984). They assign "Negishi weights" to each of a small number of individual agents. (In their static model of the Belgian economy, there are only five such agents: households, government, the foreign sector, etc.) They then solve a nonlinear optimization system to determine prices and quantities for all of the n goods. If the resulting prices and quantities are inconsistent with the initial estimate of the Negishi weights, a tâtonnement calculation is used to revise these five weights, and the process is repeated. Rapid convergence is reported - despite theoretical counterexamples.

There may be preferable alternatives to tâtonnement, but the general idea of Ginsburgh et al. appears to be a promising direction for future research. In effect, they solve a sequence of nonlinear optimization models. The beauty of their approach is that the dimensionality of the search is reduced to just five Negishi weights. There may be n goods in the equilibrium system, but all of these supply-demand balances are handled through conventional nonlinear optimization methods. The curse of dimensionality is no longer as threatening as it once appeared.

8. General-purpose solution methods

Dimensionality reduction is useful, but tends to be problem-specific. If one of these techniques works well on an intertemporal public finance problem, there is no reason to believe that it will also work well on a static model of international trade - and vice versa. Work on dimensionality reduction may divert one's efforts away from the more substantive issues involved in model formulation. This accounts for the continuing importance of general-purpose solution methods.

Among general-purpose methods, it is useful to distinguish between two broad classes - those that depend upon gradients and those that do not. In both cases, function evaluations are required. Newton methods, for example, employ gradient information, whereas piecewise linear fixed-point algorithms rely only upon function values.

Newton methods tend to require less computer time, but the following advantages of fixed-point methods should be considered: (1) Convergence is guaranteed within a finite (but perhaps large) number of steps. (2) Because Newton methods depend upon gradient information, they may require more effort at programming the gradients during the initial stages of work upon a specific problem. And (3) Newton techniques cannot easily be applied to models involving linear activity analysis and/or weak inequalities.

To illustrate why it is difficult to employ activity analysis together with a Newton approach, consider an economy with just two commodities - the numéraire good and the one whose supply and demand quantities are indicated along the horizontal axis of Figure 3. The vertical axis indicates π, the price of this commodity (expressed in terms of the numéraire). The solid line demand curve is identical to that employed in the public finance model described earlier in Figure 2.

The difference occurs on the supply side of this economy. To illustrate the Newton method, there is a nonlinear supply function. Up to point B, supplies are a continuously differentiable function of the market price: $\pi^{.5}$. With the solid-line demand curve, the market equilibrium lies at point A. Even if the algorithm is initiated at point A' (far below the equilibrium at A), there is a well-defined tangent line. This leads to an

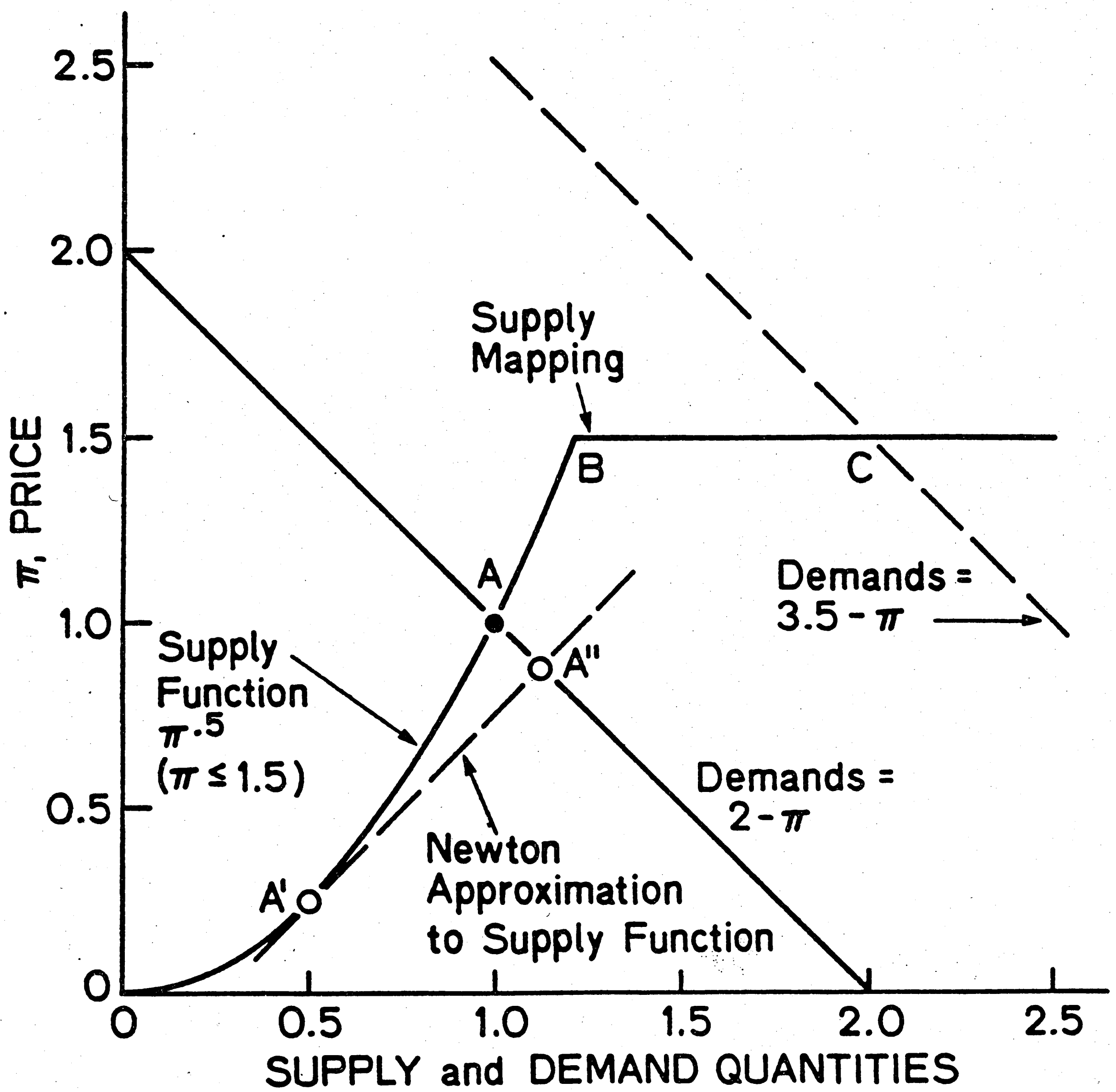

Figure 3. Supply functions and mappings

approximate equilibrium at A". From the price associated with A", we draw a new tangent approximation to the supply curve. The Newton process is repeated until there is a satisfactory degree of approximation to the equilibrium at A.

Now suppose that there is a constant-returns activity that converts 1.5 units of the numéraire into 1.0 unit of the commodity described in Figure 3. This is sometimes known as a "backstop" activity, for it prevents the price from rising above 1.5. In this case, we are dealing with a supply mapping (point-to-set) - not a supply function (point-to-point). This supply mapping is undefined for all values of π above 1.5. An indeterminate amount is supplied when $\pi = 1.5$.

Consider the dashed line demand curve (parallel to the original one). In this case, the equilibrium lies at C. The backstop activity would provide BC units of supply, and the balance would come from the lower-cost source.

When the equilibrium value of $\pi = 1.5$, a naive Newton method is doomed to fail. If the search is begun with a value of $\pi > 1.5$, the supply mapping is undefined, and a tangent line cannot be drawn. If the search is begun with a value of $\pi < 1.5$, a tangent line can be drawn to the initial point along the supply function, but the iterative process will eventually lead to a value of $\pi > 1.5$ - and again an undefined supply mapping.

In this two-dimensional example, a Newton method could easily be modified to deal with the discontinuity in the gradient of the supply mapping at point B, but ad hoc modifications are awkward with higher dimensional problems. It soon becomes prohibitively expensive to use brute-force enumeration to evaluate all possible combinations of those activities which are at positive intensities - and those weak inequality constraints which are binding at an equilibrium point. In these cases, a fixed-point approach may be advantageous - despite the curse of dimensionality.

Activity analysis provides a natural way to formulate models of technological choice where not all of the inputs and outputs can be substituted continuously for each other. Backstop technologies are one example, and joint-product relationships are another. Joint production is characteristic of dynamic investment planning. The decision to bring one unit of capacity onstream in period t implies that the same unit of capacity will be available in subsequent periods throughout the equipment's service life. In models of this type, a fixed-point algorithm permits a more flexible formulation than a Newton approach.

An ideal technique would be one that combines the strengths of Newton (rapid solution of systems of nonlinear equations) with the strengths of fixed-point methods (activity analysis and weak inequality constraints). Mathiesen (1984) has demonstrated the effectiveness of this hybrid approach. He solves for an economic equilibrium by an SLCP (sequence of linear complementarity problems) algorithm. During the initial stages, Mathiesen's method resembles fixed-point and linear programming. It identifies those activities that are at positive intensities and those weak inequalities that are binding constraints. Once these activities and constraints are identified, the SLCP algorithm resembles a Newton method. Mathiesen's approach has proven to be highly effective in practice - despite the absence of a proof that it will always converge.

Preckel (1984) reports on a controlled comparison between alternative solution techniques. In the specific case examined - a two-region intertemporal model of international trade and capital flows - he concludes that SLCP is more efficient than the Newton method, and that this in turn is more efficient than the Broadie (1984) fixed-point algorithm. Preckel warns, however, that "these experiments do not indicate which solution methods are best for all general equilibrium models".

The SLCP algorithm operates with a square matrix containing n+k rows and columns. There is one dimension for each of the n supply-demand balances and one for each of the k activities. It is possible that SLCP may be superseded by methods involving lower-dimensional matrices - e.g., through a sequence of n-dimensional linear programs. For several possibilities along these lines, see Hogan (1975), Manne, Chao and Wilson (1980) and Stone (1984). These methods are likely to be effective whenever k (the number of activities) greatly exceeds n (the number of supply-demand balances). More controlled experiments are needed in this area.

9. Survey on computable equilibrium models

During 1984, an informal poll was taken among a number of groups working on CGE models. This represented a very incomplete tabulation of ongoing work in North America and Europe. It was restricted to English-language publications and to individuals who were known either to the author - or to his colleagues.

Table 1 summarizes the results of this survey. (The 19 individual responses are contained in an appendix that is available upon request.) On the basis of this poll, several generalizations have emerged:

Table 1. Survey on computable equilibrium models

Author(s)	Static/ dynamic	Regions	Major objectives	Solution method
Ballard, Goulder	recursive; intertemporal	USA, rest of world	role of expectations in tax policy analysis	Gauss-Siedel, tâtonnement
Benjamin, Devarajan	recursive	Cameroon	oil revenues, tariff policy, investment planning	Newton
Blanchard, Sachs	intertemporal	one	intertemporal equilibrium with rational expectations	multiple shooting, Gauss-Seidel
Bovenberg	intertemporal	two	introduce dynamic equilibria and adjustment costs	matrix inversion, eigen-values
Bovenberg, Keller	recursive	Netherlands	tax policy analysis	matrix inversion
Burniaux	intertemporal	10	international food and agricultural policy	Gauss-Seidel
Carrin, Gunning, Waelbroeck	recursive	12 World Bank regions	North-South interdependence: food, energy, aid, tariffs, migration	Gauss-Seidel
Centre for World Food Studies	recursive	Bangladesh, Thailand	food policy analysis; linked to IIASA model	LP; sequence of fixed-points points solved by Jacobi technique
Condon, Corbo, de Melo	recursive, closure rule	Chile	productivity gains from liberalization reforms; macroeconomic policies	Newton and steepest-descent
Dantzig, McAllister, Stone	intertemporal	USA	energy/economy interactions, technological change	MINOS, non-linear programming
Fullerton, Shoven, Whalley	recursive	USA, rest of world	tax policy evaluation	Merrill's fixed-point; tâtonnement
Ginsburgh, Van der Heyden, Erlich	both static and intertemporal	Belgium, rest of world	effects of downward wage rigidities on unemployment and the external deficit	sequence of LP, tâtonnement for Negishi weights
Heady, Mitra	static; intertemporal	rural, urban	optimal taxation policy	fixed-point; reduced-gradient
IIASA	recursive	36	international food and agricultural policy	complementarity + non-differential optimization
Kehoe, Serra-Puche	static	Mexico	effects of tax and expenditure policies on unemployment, income distribution and resource allocation; price controls on food and energy	Merrill fixed-point, pseudo-Newton
Lewis, Urata	recursive	Turkey	foreign exchange market	Powell
Manne, Preckel	intertemporal	OECD, OPEC, Other	international energy prices; constraints on North-South capital flows	fixed-point
Norman, Rutherford	static	3 OECD regions, OPEC/Mexico NIC, Other	international scenario analysis; macro input to disaggregated models; effects of factor mobility	sequence of linear complementarity problems
Whalley	static	EEC, US, Japan, Canada, Other developed, OPEC, NIC, LDC	global trade policy issues (GATT, North-South)	Newton

Table 2. Computing Limitations

Have computing limitations influenced the formulation of this model?

C. Ballard, L. Goulder: "Solving the model often requires 30-40 minutes of CPU time on a VAX 11/780. We are fortunate to have access to free computer services - otherwise we would be severely constrained."

N. Benjamin, S. Devarajan: "Yes."

O. Blanchard, J. Sachs: "Not drastically."

L. Bovenberg: "Yes, we intended to derive analytical expressions in the first phase of the project in order to clearly show the mechanisms at work in dynamic general equilibrium models with adjustment costs."

L. Bovenberg, W.J. Keller: "No, data problems are the restricting factor."

J.-M. Burniaux and G. Carrin, J.Gunning, J. Waelbroeck: "Tried to keep it small."

Centre for World Food Studies: "Yes, computational costs of linear program make it impossible to simultaneously solve exchange component with linear program (of supply component)."

T. Condon, V. Corbo and J. de Melo: "No. We were not developing the model software."

G.B. Dantzig, P.H. McAllister and J.C. Stone: "We have specified integrable demand functions for PILOT in order to be able (computationally) to incorporate a significant level of sectoral, technical, and intertemporal detail."

D. Fullerton, J.B. Shoven and J. Whalley: "Not much."

V. Ginsburgh, L. Van der Heyden and S. Erlich: "Not in the static and two-period model. The five-period version is harder to solve. It takes much more time, but seems to work."

C. Heady, P. Mitra: "Static version: no. Dynamic version: yes."

IIASA Food and Agriculture Program: "Yes. The major limitations have been: (1) predetermined supply during exchange; (2) limited number of commodities; (3) limited number of regions."

T. Kehoe, J. Serra-Puche: "To some extent."

J.D. Lewis, S. Urata: "Yes."

A.S. Manne, P.V. Preckel: "Yes. The fixed-point approach led to a good deal of effort at dimensionality reduction. This is why the planning horizon was limited to five time periods. For the analysis of debt repayment and of 'backstop' energy supply technologies, more time periods would have been desirable."

V. Norman, T. Rutherford: "Solution costs (of the static model) are low: less than one minute of CPU time on a DEC-2060. An intertemporal version of this model is under construction, and its solution costs will be considerably higher."

J. Whalley: "No."

(a) Money plays a neutral role in all of these models. They are addressed toward long-term issues - not short-run fluctuations in income, employment, price levels and exchange rates.

(b) The level of detail (number of time periods, regions and commodities) varies considerably from one of these models to the other - depending upon the objectives of the analysis. For example, an international trade policy study will require more regions than one directed toward the structure of taxation within a single country. Conversely, a tax analysis will require a more detailed description of socioeconomic or income groups, for each group may be affected differently by a proposed change in the country's tax structure.

(c) Several of these analyses employ comparative statics, but a majority allow for dynamic processes and for adjustment costs. The first generation of static CGE models is rapidly being superseded by a second generation that handles dynamics explicitly - either through recursive or through intertemporal ("rational expectations") methods.

(d) Among algorithms, there are no clear winners. A wide variety of methods are in use. Each has advantages for specific applications. Despite the potential pitfalls, the most frequently mentioned solution methods are those of Newton, Gauss-Seidel and t atonnement. Most CGE analyses can be formulated so as not to run into convergence difficulties. Only a few authors report the use of the more general-purpose techniques such as fixed-point or SLCP.

Since there is a wide range of opinions on whether computing limitations have influenced model formulation, the survey included a question on this topic. Typically, the designers of static models reported that they were not limited by computing restrictions, but that they ran into difficulties when they began to include dynamic elements. There were exceptions to this, but only a handful. Table 2 contains the exact wording of the question - and all of the comments received from individual respondents. A terse but informative summary of computing difficulties was that of Heady and Mitra: "Static version: no. Dynamic version: yes."

Dynamic equilibrium models can be difficult to solve, but they provide a systematic way to deal with adjustment costs, announcement effects, borrowing and debt repayment, resource depletion, and other expectations-related phenomena. For further progress, it will be essential to maintain a continuing dialogue between those who formulate dynamic models and those who provide the algorithms required for their solution.

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