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THE MEASUREMENT OF PRODUCTIVITY:
A LECTURE IN MEMORY OF PROFESSOR TA-CHUNG LIU

by

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Abstract

Issues in industrial policy, international trade, research and development, science, and taxation depend importantly on the measurement of productivity, productivity improvement and productivity comparisons. This paper explicates some of the difficulties encountered in productivity measurement. They include inefficiency, variations in the rate of utilization, changes in the quality of input, non-constant returns to scale, non-neutrality of technological progress, vintage effects, and price-induced obsolescence.

THE MEASUREMENT OF PRODUCTIVITY
A LECTURE
IN MEMORY OF PROFESSOR TA-CHUNG LIU

by

Lawrence J. Lau*

Mr. Chairman, Ladies and Gentlemen: I am very honored today to be invited to present the Professor Ta-Chung Liu Memorial Lecture. Professor Liu was a great teacher and a great scholar. I personally learned a great deal from him. The subject I am going to discuss today is the "Measurement of Productivity". This is a subject close and dear to Professor Liu's heart. His book with George H. Hildebrand, Manufacturing Production Functions in the United States, 1957: An Interindustry and Interstate Comparison of Productivity[1], is still a standard reference on the subject.

1. Introduction

The measurement of productivity means different things to different people. In this lecture, we consider one very specific aspect, namely, the measurement of productivity that is intended to reflect the change, if any, in the productive potential of an economy. We represent the productive potential of an economy (or an industry) in terms of the set of all feasible input-output combinations, that is, all input-output combinations which can be realized under the technology of the economy at

[1]Cornell University Press, Ithaca, N.Y., 1965.

the time. We refer to such a set of input-output combinations as a set of production possibilities. By comparing the sets of production possibilities of an economy at two or more different points in time, we infer whether there has been a change in the productive potential, that is, whether there is any input-output combination that is feasible at the later date but not feasible at the earlier date or vice versa. In other words, the changes, if any, of the set of production possibilities are identified with changes in the productive potential. The expansion of the set of production possibilities over time is sometimes referred to as "technological progress".

Why are we interested in the measurement of productivity defined in this particular way? Well, everyone is interested in having higher output. However, higher output can always be obtained with more resources, and so is not very interesting in and of itself. What is interesting, in a world of scarcity, is whether we can obtain the same output with less resources, or a higher output with the same resources. This is where improvement in productivity or "technological progress" becomes important. The principal reason for our interest in the measurement of productivity is to identify and quantify technological progress.

We begin our discussion by introducing the concept of a "set of production possibilities". The set of production possibilities of an economy is defined as the set of all technologically feasible combinations of inputs and outputs. It thus provides an indicator of the productive potential of the economy. For example, the combination of zero inputs and zero outputs is always technologically feasible (although not at all desirable). The actual combination of inputs and outputs at any given point in time is obviously feasible. Many combinations of inputs and outputs which are technologically feasible are, however, never actually chosen. There are, of course, also combinations of inputs and outputs which are obviously infeasible. For example, the combination of zero inputs and positive outputs is obviously infeasible. (This is sometimes called, after Paul Samuelson, the "no free lunch" assumption.) We should, however, distinguish clearly between the concept of technological feasibility which depends only on the technology from that of actual feasibility which depends on not only the technology but also the actual availability of inputs and other nontechnological constraints at the time.

We shall illustrate the problems of the measurement of productivity within the context of a very simple economy in which there is only a single output, denoted Y, and a single input, say labor, denoted L. The set of production possibilities for this simple economy consists of all combinations of output and labor which are technologically feasible, that is, which can actually be realized if chosen. It is thus a two-dimensional set. Each element of the set of production possibilities consists of a pair of real numbers, (L, Y), meaning that the quantity of output Y can be produced with the quantity of input L.

Corresponding to the same level of input L, many different levels of outputs can be produced, depending on the level of efficiency. A feasible combination of labor and output (L^*, Y^*) is said to be <u>efficient</u> if Y^* is the maximum quantity of output that can be produced with L and L^* is the minimum quantity of labor that is required to produce Y^*. In other words, given L^*, Y^* is the maximum value of Y such that the input-output combination (L^*, Y) is technologically feasible; given Y^*, L^* is the minimum value of L such that the input-output combination (L, Y^*) is technologically feasible. The distinction between a feasible input-output combination and an efficient input-output combination is worth noting. There are many feasible combinations that are not efficient. For example, the combination (L, 0), that is, a positive quantity of labor L and a zero quantity of output, is not in general efficient but clearly feasible.

The locus of all efficient combinations of labor and output is called the <u>production-possibility frontier</u> (See the illustration in Figure 1). An economy is <u>efficient</u> only if it operates at a point on the production-possibility frontier. If an economy operates at an input-output combination inside the production-possibility frontier, then it is possible to increase output without increasing labor, or to decrease labor without decreasing output, leading to a more efficient use of resources and thus demonstrating that the initial input-output combination cannot have been efficient. The production-possibility frontier may be represented by a function giving the maximum output Y that can be obtained from a given labor L, Y = F(L). This function is generally referred to as the production function. It is customary to assume that the set of production possibilities is convex and hence the production-possibility frontier, or equivalently the production function, is concave, as depicted in Figure 1. However, the assumption of concavity is not essential for the discussion here.

The set of production possibilities in Figure 1 may change over time. This is sometimes referred to as technological change or progress. Generally, the set of production possibilities at a later date is expected to be "larger" than the set of production possibilities at an earlier date, reflecting an expansion of the productive potential of the economy. This is illustrated in Figure 2. The set of production possibilities at time 0 is completely contained in the set of production possibilities at time 1. In other words, any input-output combination that is feasible at time 0 is also feasible at time 1. The converse is not true: because of technological progress, some input-output combinations that are not feasible at time 0 become feasible at time 1. Note that an expansion of the set of production possibilities is always associated with a shift in the production-possibility frontier.

As mentioned earlier, our purpose in the measurement of productivity is to <u>identify</u> and <u>quantify</u> the changes in the set of production possibilities--or equivalently, in the technological productive potential of the economy--over time. In the next section we shall indicate and illlustrate the difficulties associated with such an effort.

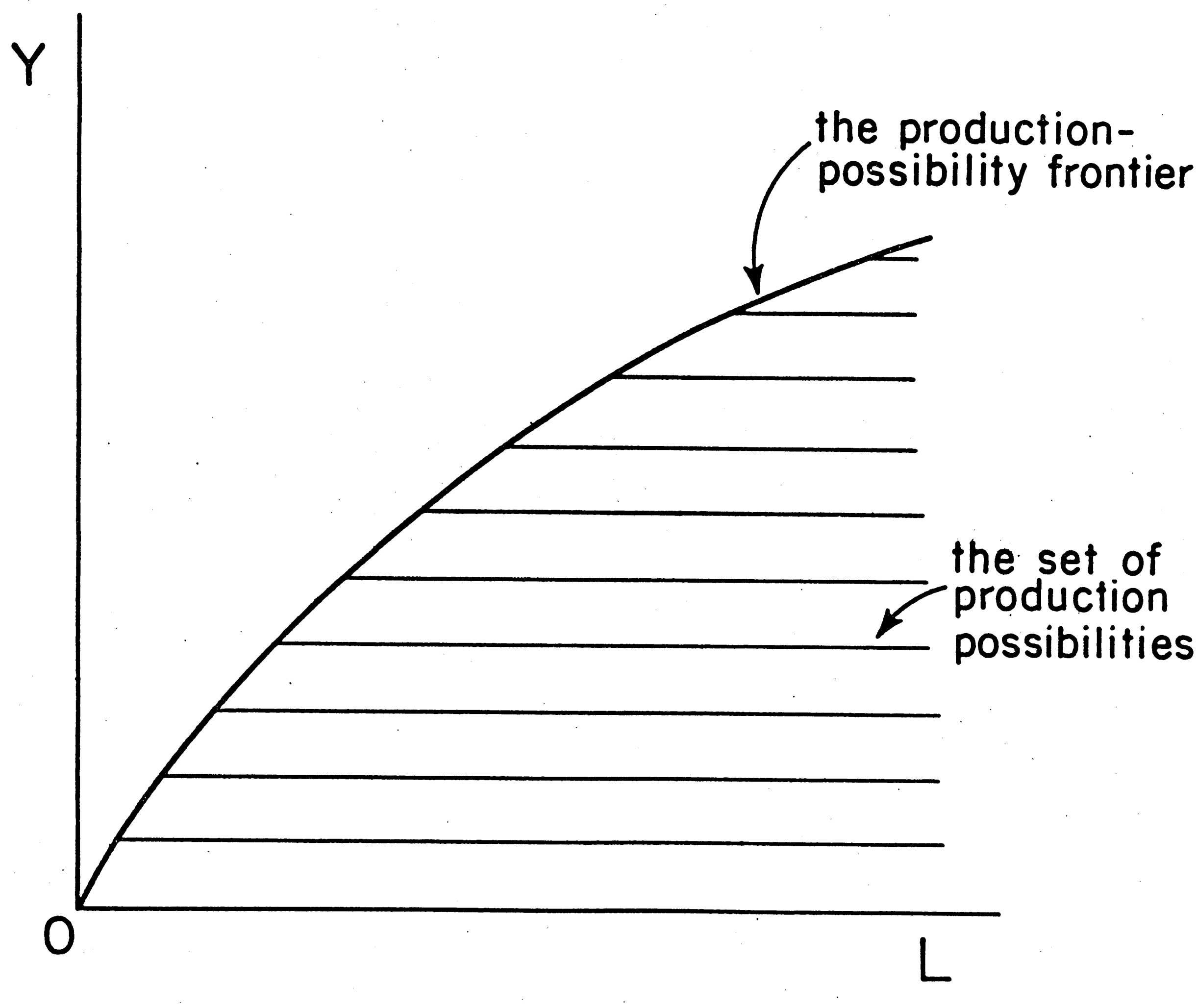
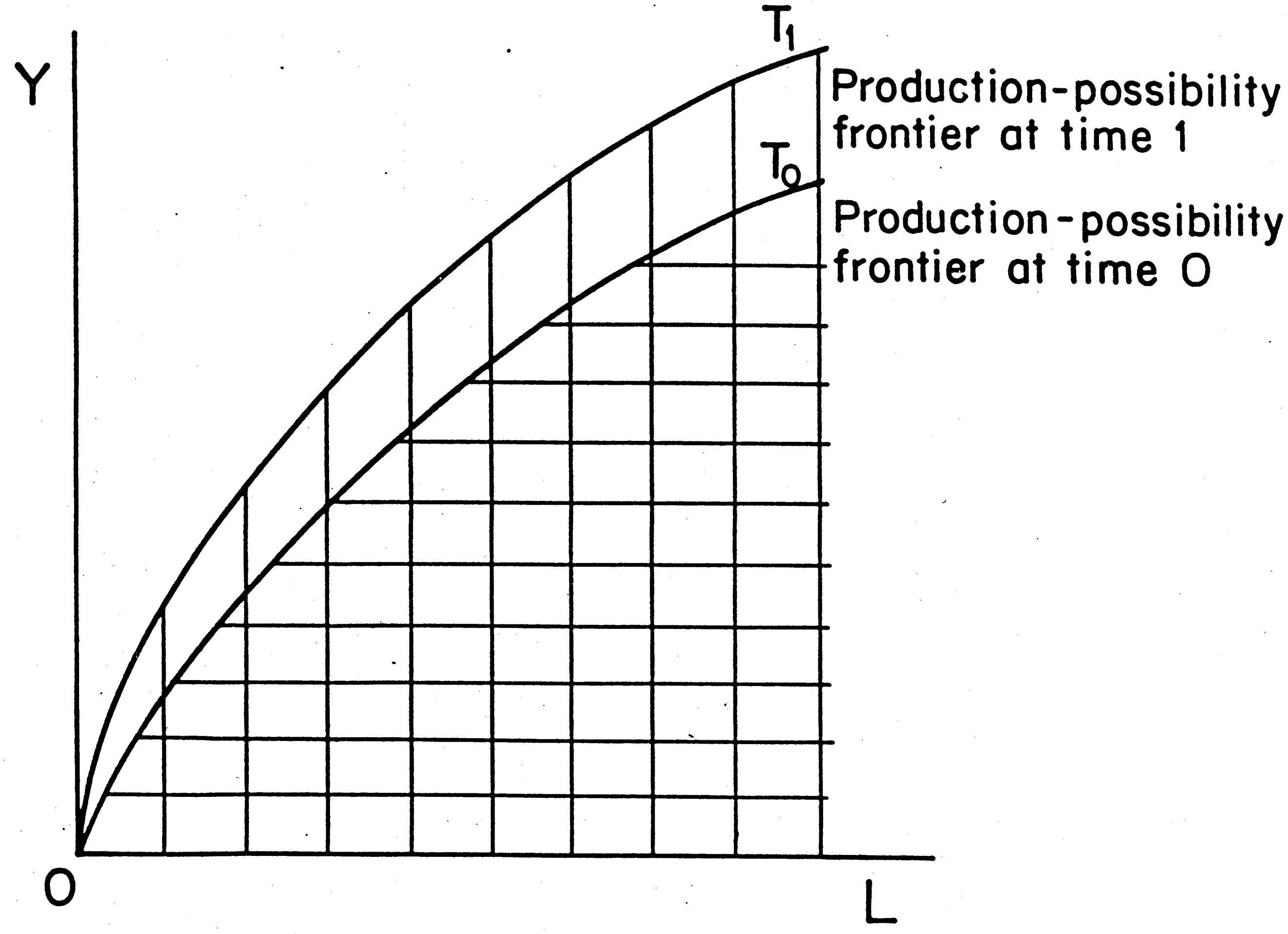

Y
the production-possibility frontier
the set of production possibilities
O
L

Figure 1



||| : set of production possibilities at time 1.

$\equiv$: set of production possibilities at time 0.

Figure 2

2. Measurement Problems in a Single-Output Single-Input Economy

In the measurement of productivity for a single-output, single-input economy the data typically consist of a time series of quantities of output and labor and possibly the prices of output and labor, denoted P and W, respectively. Thus, we are typically given:

$$\begin{array}{cccc} Y_0 & L_0 & P_0 & W_0 \\ Y_1 & L_1 & P_1 & W_1 \\ Y_2 & L_2 & P_2 & W_2 \\ \vdots & \vdots & \vdots & \vdots \\ Y_T & L_T & P_T & W_T \end{array}$$

where the subscripts indicate the time period.

From the given data, we are to determine whether the set of production possibilities has expanded over time and if so how much it has expanded. An expansion in the set of production possibilities (or equivalently a shift in the production-possibility frontier), is identified with technological progress.

Let us consider how we may do this by taking up one of the most favorable cases for this measurement. Let the quantity of labor, L, be fixed. Then all that we have to do to see if productivity has increased is to compare the values of the quantity of output, Y, over time. The difference $(Y_1 - Y_0)$ or the ratio (Y_1/Y_0) and more generally $(Y_t - Y_{t-1})$ or (Y_t/Y_{t-1}) can be used as a measurement of the shift in the production-possibility frontier over time. (See the illustration in Figure 3). It

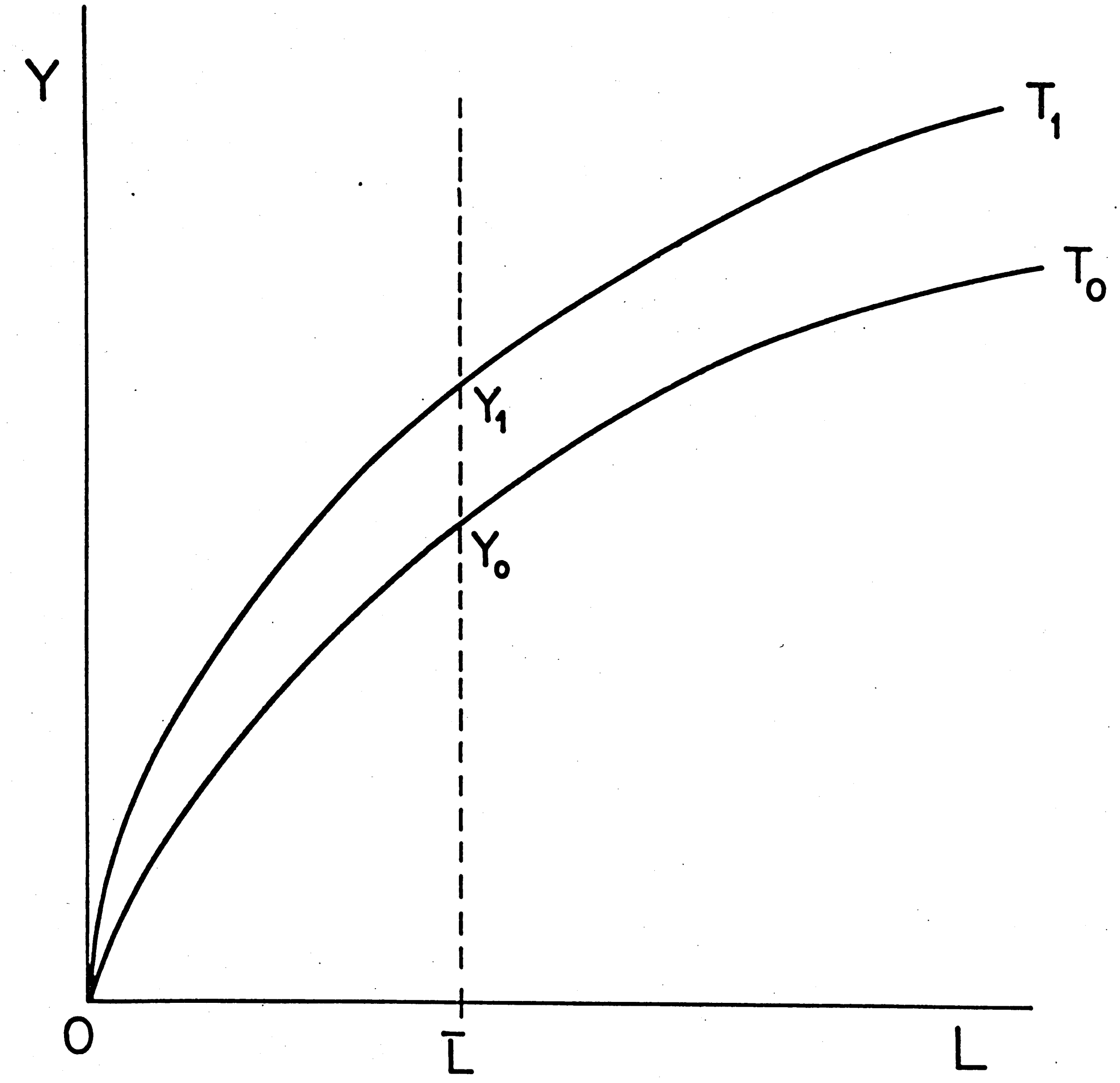


Figure 3

is possible that we may find $Y_t \leqq Y_{t'}$ even though $t \geqq t'$, which would imply technological retrogression--less output produced with the same input. One way to handle this problem would be to use $\underset{t' \leqq t}{\mathrm{Max}} \{Y_{t'}\}$ as an index of the level of technology in period t, which would prevent temporary lapses in efficiency from being identified as technological retrogression.[1]

However if the quantity of labor, L, were not fixed, the situation would be much more complicated. We would need to distinguish between movements within a set of production possibilities (including movements along the production-possibility frontier) and expansions of the set of production possibilities (represented by shifts of the production-possibility frontier). No increase in productivity is associated with the former. An increase in productivity is only associated with the latter.

The possible ambiguities are illustrated in Figure 4. (L_0, Y_0) and (L_1, Y_1) may be visualized as lying in different production-possibility frontiers T_0 and T_1 or on the same production-possibility frontier T_0^*. However, in the first case there is technological progress whereas in the second case there is none.

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[1]Using $\underset{t' \leqq t}{\mathrm{Max}} \{Y_{t'}\}$ as an index of the level of technology would not, however, eliminate the difficulties caused by inefficiency as discussed below.

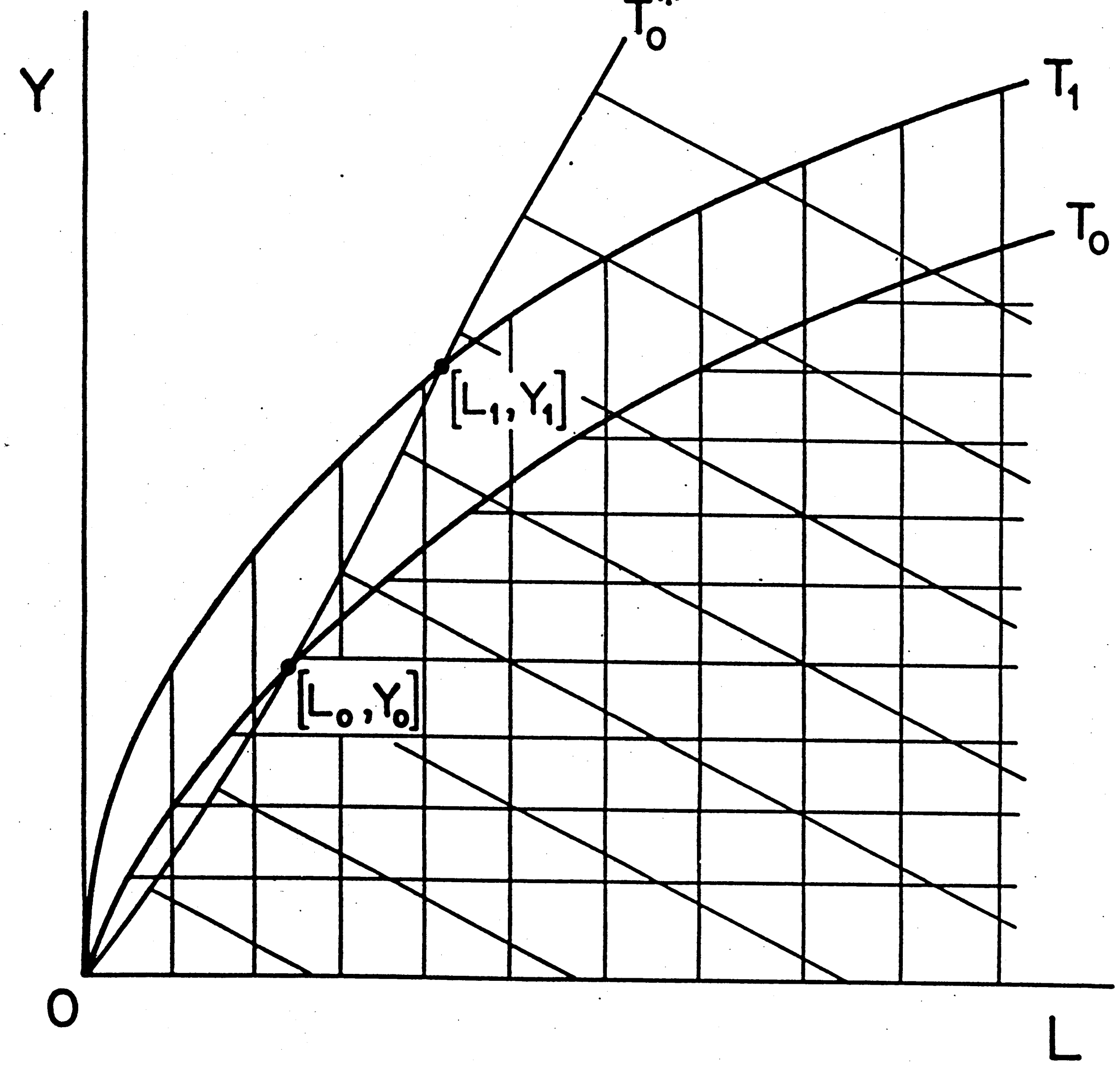

Figure 4

Is it possible for us to have an unambiguous answer in this case? We can have an unambiguous answer only if we know the shape up to a scale factor of the production-possibility frontiers at different points in time. For example, the production-possibility frontier may be known to have the form: $\ln Y_t = \alpha_t \ln L_t$. Changes in α_t over time can then be identified as technological change. Without this knowledge, it is not possible to distinguish unambiguously between a shift of the production-possibility frontier from a movement along a stationary production-possibility frontier.

This ambiguity is not the only problem we face. A number of other problems have received extensive discussions in the economics literature. We shall identify only a few of the more important ones here.

(1) Inefficiency;

(2) Variations in the rate of utilization;

(3) Changes in the quality of input[1];

(4) Non-constant returns to scale;

(5) Non-neutrality of technological progress;

(6) Vintage effects;

(7) Price-induced obsolescence.

We shall discuss each of these problems in turn.

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[1]As Professor John Fei pointed out, changes in the quality of the output can also be a serious problem.

(1) Inefficiency

In order to determine unambiguously whether the set of production possibilities has expanded over time (or equivalently whether the production-possibility frontier has shifted) it is necessary to maintain the assumption that the observed actual input-output combinations lie on their respective production-possiblity frontiers. Otherwise we may mistakenly interpret a movement from the interior of a set of production possibilities to the production-possibility frontier as a shift of the production-possibility frontier. For example, in Figure 5, (L_0, Y_0) is an inefficient but feasible input-output combination and (L_1, Y_1) is an efficient combination. However, since the production-possibility frontier has not shifted, there has been no technological progress (but an improvement in efficiency).

In other words it is necessary to assume that the economy operates on the production-possibility frontier, at least most of the time, in order that one can identify shifts of the production-possibility frontier. Otherwise such shifts will be confounded by changes in the degree of efficiency.

(2) Variations in the rate of utilization.

Another potential measurement problem is caused by variations in the rate of utilization of the input. Suppose that in period 0 the rate of utilization of the input is 100% and in period 1 it is $100\mu\%$, $1 > \mu > 0$. Then unless the variation in the rate of utilization is taken into account, a shift of the production-possibility frontier may fail to be identified.

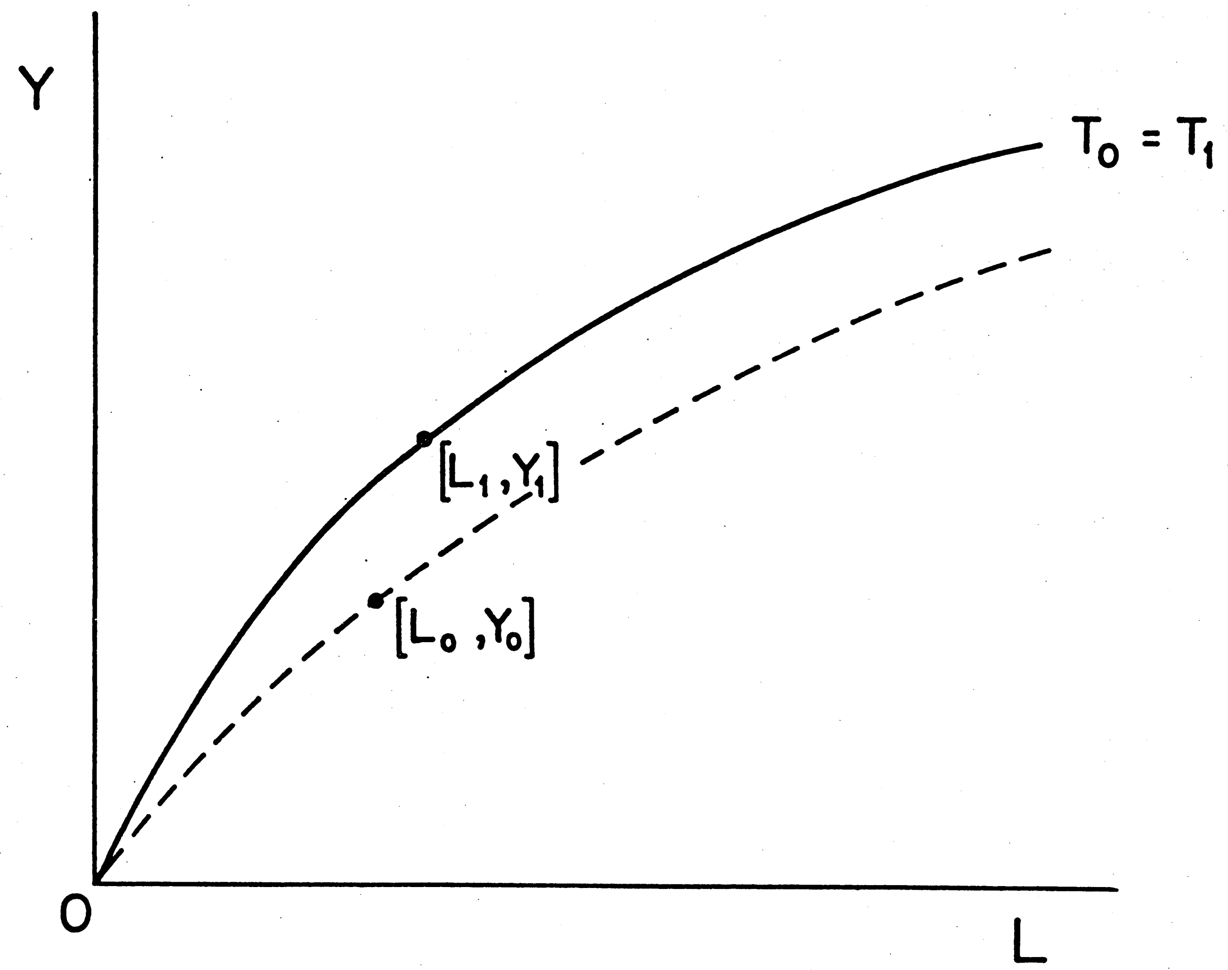

Figure 5

For example, in Figure 6, if the input in (L_1, Y_1) is not adjusted by the rate of utilization, it appears that (L_1, Y_1) may be an inefficient input-output combination and in any case does not indicate a shift of the production-possibility frontier. However, if it is properly adjusted for the rate of utilization so that the input-output combination becomes $(\mu L_1, Y_1)$, a movement of the production-possibility frontier from T_0 to T_1 is indicated.

(3) Changes in the quality of input

A third potential measurement problem has to do with changes in the quality of the input. If the quality of an input changes over time, then inputs at different times are basically noncomparable. In order to compare them we need quality adjustment factors that are <u>independent</u> of the changes in outputs. If such independent adjustment factors are unavailable one cannot distinguish between a change in <u>measured</u> productivity due to a change in the quality of input and a change due to a shift of the production-possibility frontier.

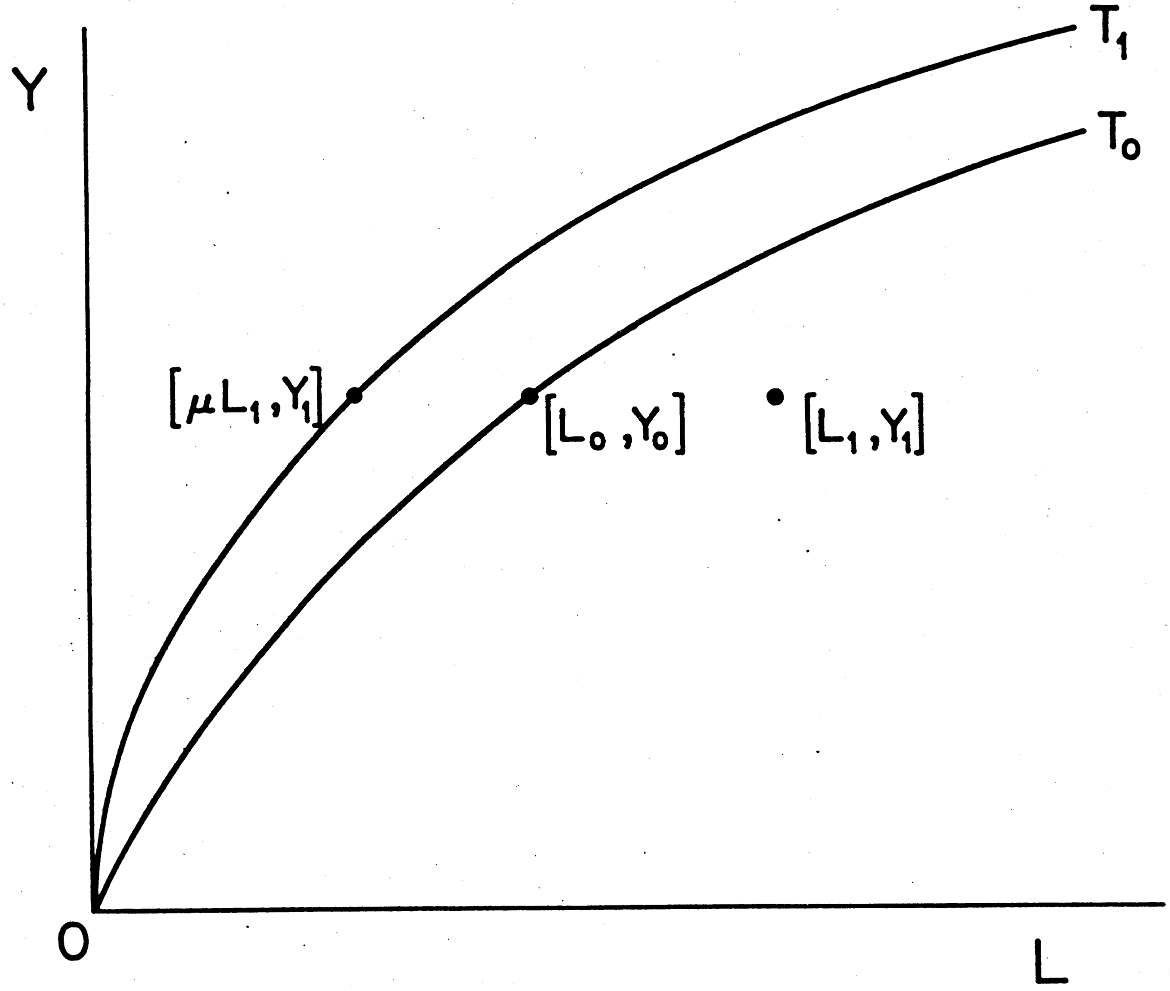

Figure 6

For example, in Figure 7, (L_0, Y_0) and (L_1, Y_1) are the observed input-output combinations. T_0 is given and assumed to be known. If the quality of the input is assumed unchanged, then (L_1, Y_1) must lie on the production-possibility frontier T_1, indicating technological progress. However, if there were a quality improvement of unknown magnitude, then it is always possible to choose a quality improvement factor λ, such that the quality-adjusted input-output combination $((1 + \lambda) L_1, Y_1)$ lies on the production-possibility frontier T_0, thus indicating no technological progress. $(1 + \lambda)L_1$ is to be interpreted as the equivalent number of units of labor of the quality of period 0 in period 1. Only when λ is independently determined is it possible to say whether the production-possibility frontier has shifted.

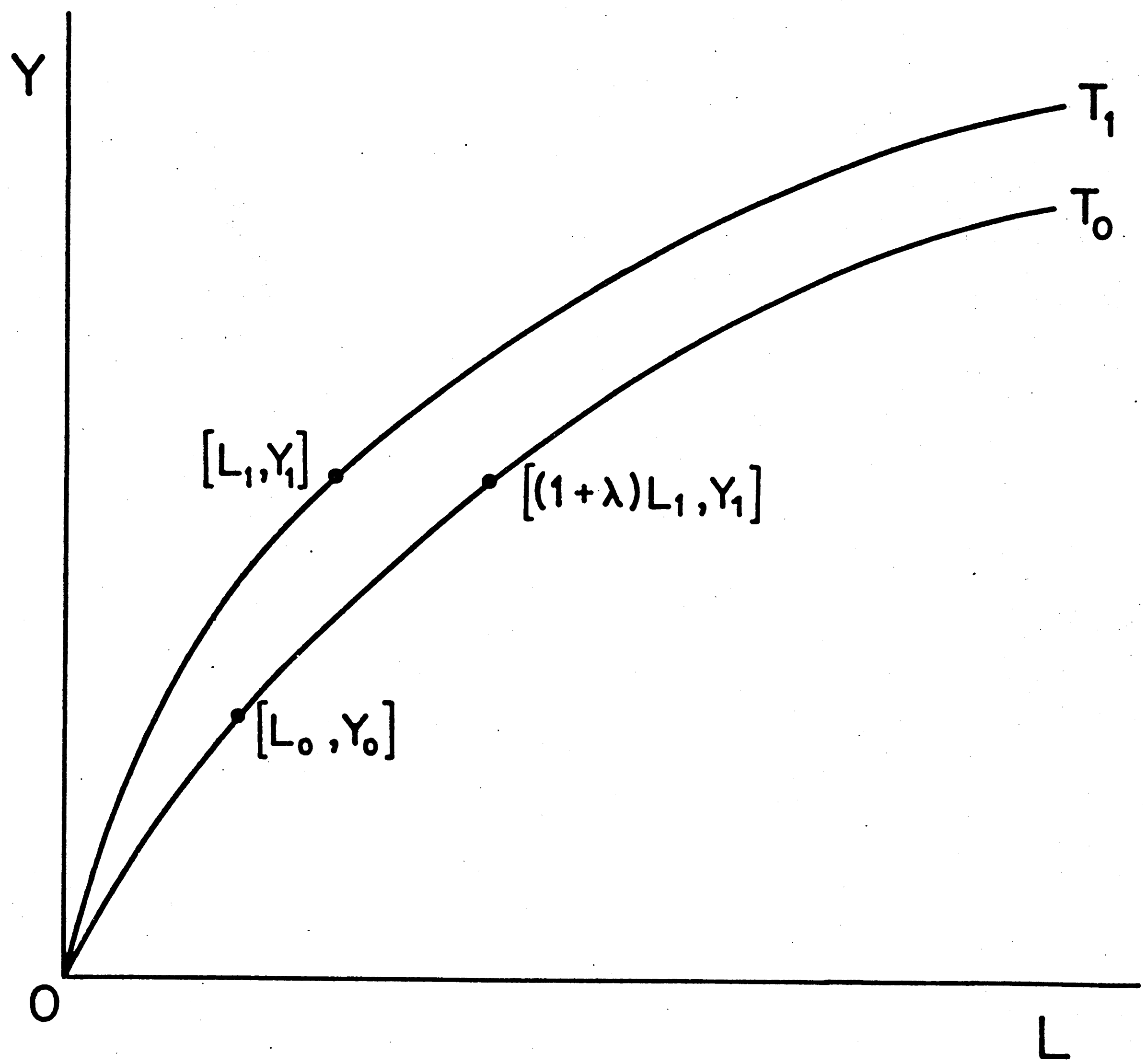

Figure 7

(4) Non-constant returns to scale

A fourth potential measurement problem has to do with the deviation of the technology from constant returns to scale. In the single-output, single-input case, it is customary to identify an increase in the measured output per unit input as an increase in productivity and a decrease in the measured output per unit input as if not a decrease in productivity at least a decrease in efficiency. However, this common-sense interpretation is rooted in the implicit assumption of constant returns to scale--output per unit input can be maintained constant at all scales of operation. If there are non-constant returns to scale, such a common-sense interpretation breaks down.

For example, if there were decreasing returns to scale as portrayed in Figure 8, then even though the production-possibility frontier T_1 represents an outward shift of the production-possibility frontier T_0, $Y_1/L_1 < Y_0/L_0$ as indicated by the ray from the origin through (L_0, Y_0). Along the ray output per unit input is constant and equal to Y_0/L_0. Thus, technological progress may be mistakenly identified as technological retrogression or inefficiency.

For another example, if there were increasing returns to scale as portrayed in Figure 9, then even though there is no shift in the production-possibility frontier T_0, one may mistakenly conclude that there is technological progress because $Y_1/L_1 > Y_0/L_0$.

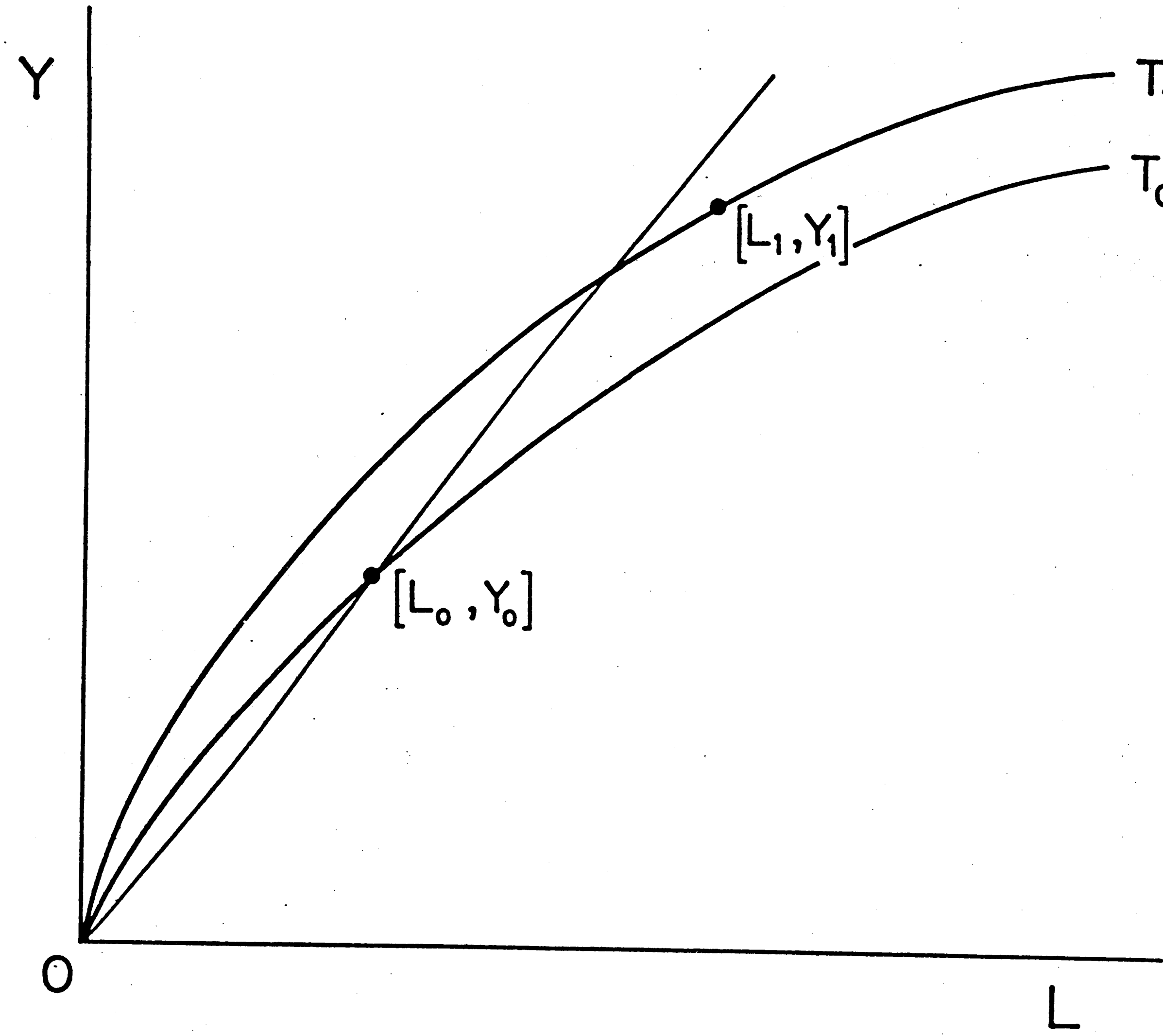

Figure 8

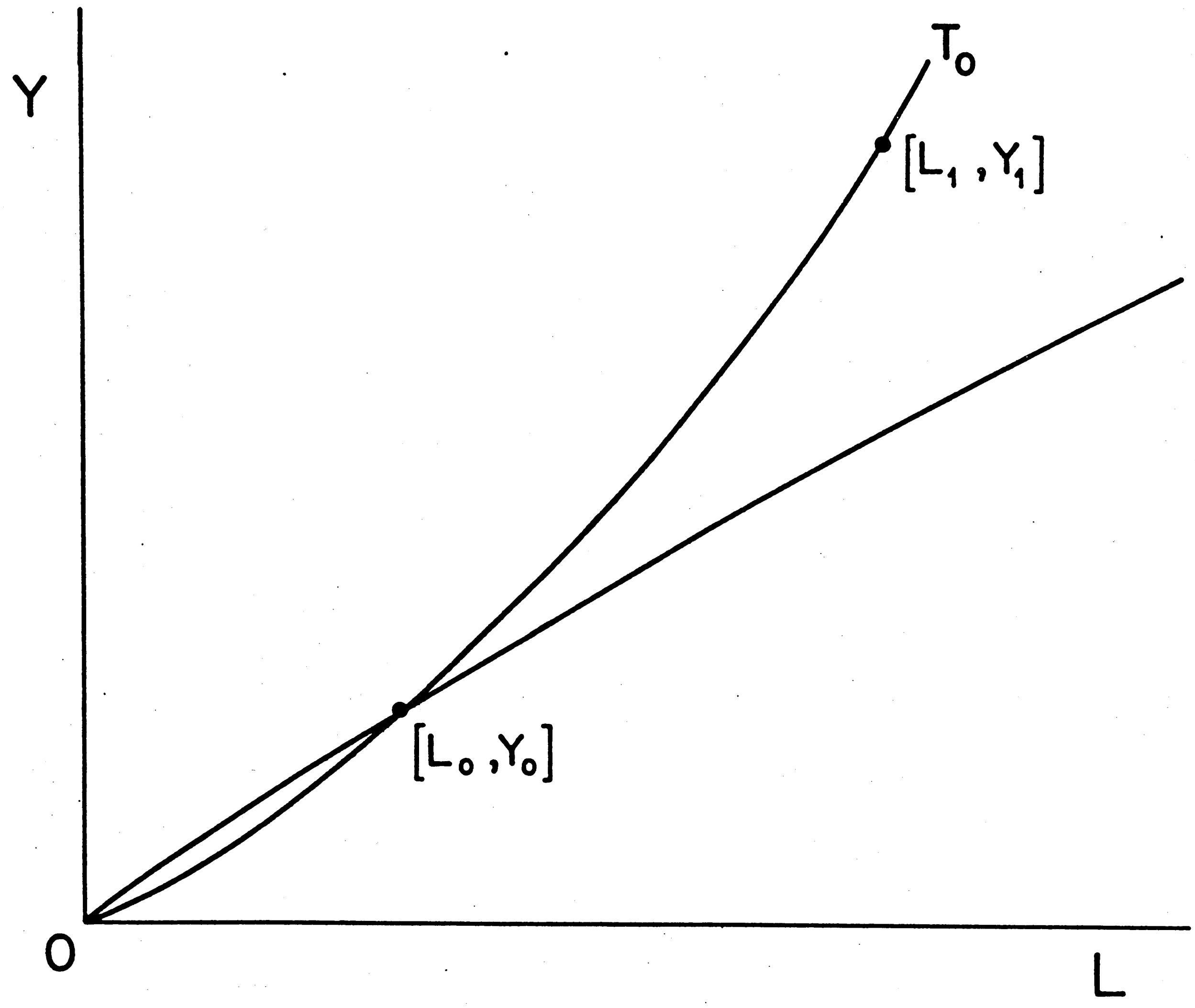

Figure 9

Constant returns to scale in the single-output, single-input case imply that the production-possibility frontier (or equivalently the production function) is a ray through the origin. (See Figure 10.) In general, in a growing economy, if the returns of scale are unknown a priori, the scale effect tends to be confounded with the technological progress effect and neither can be separately identified.

(5) Non-neutrality of technological progress

A fifth potential measurement problem has to do with the nature of the technological progress. Technological change is said to be neutral in the single-output, single-input case if the production-possibility frontier (or equivalently the production function) can be represented in the form:

$$Y_t = F(L_t, t) = A(t) f(L_t), \ \forall\ t.$$

In other words, the production functions of different periods are identical up to a multiplicative factor. Otherwise technological change is said to be non-neutral.

If technological change is non-neutral, shifts of the production-possibility frontier may fail to be identified. For example, in Figure 11, although the production-possibility frontier has shifted between period 0 and period 1 the shift cannot be identified from the observed input-output combinations (L_0, Y_0) and (L_1, Y_1). In addition, if technological change is non-neutral, technological progress, even if its existence is identifiable, cannot be unambiguously measured. For example,

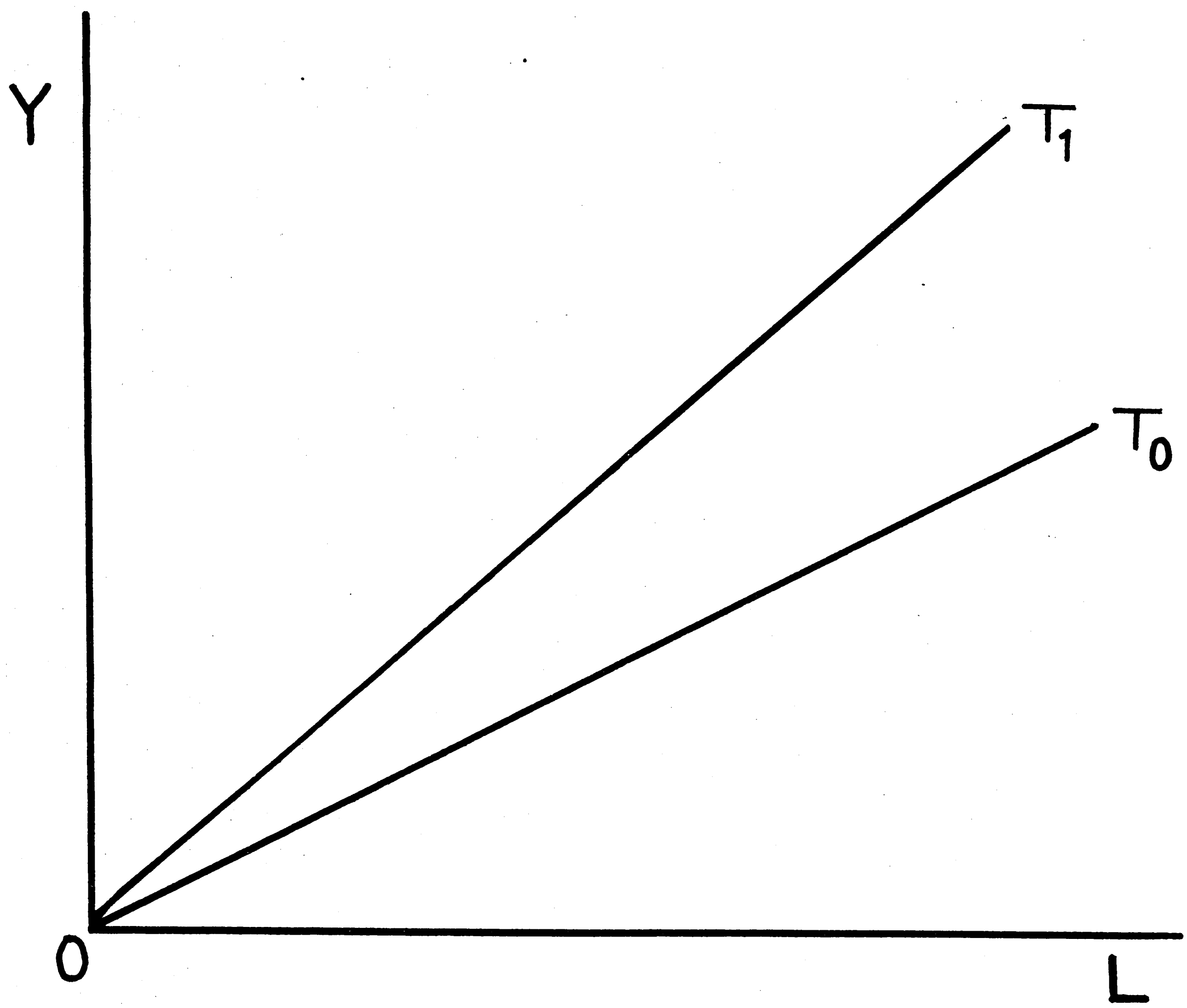

Figure 10

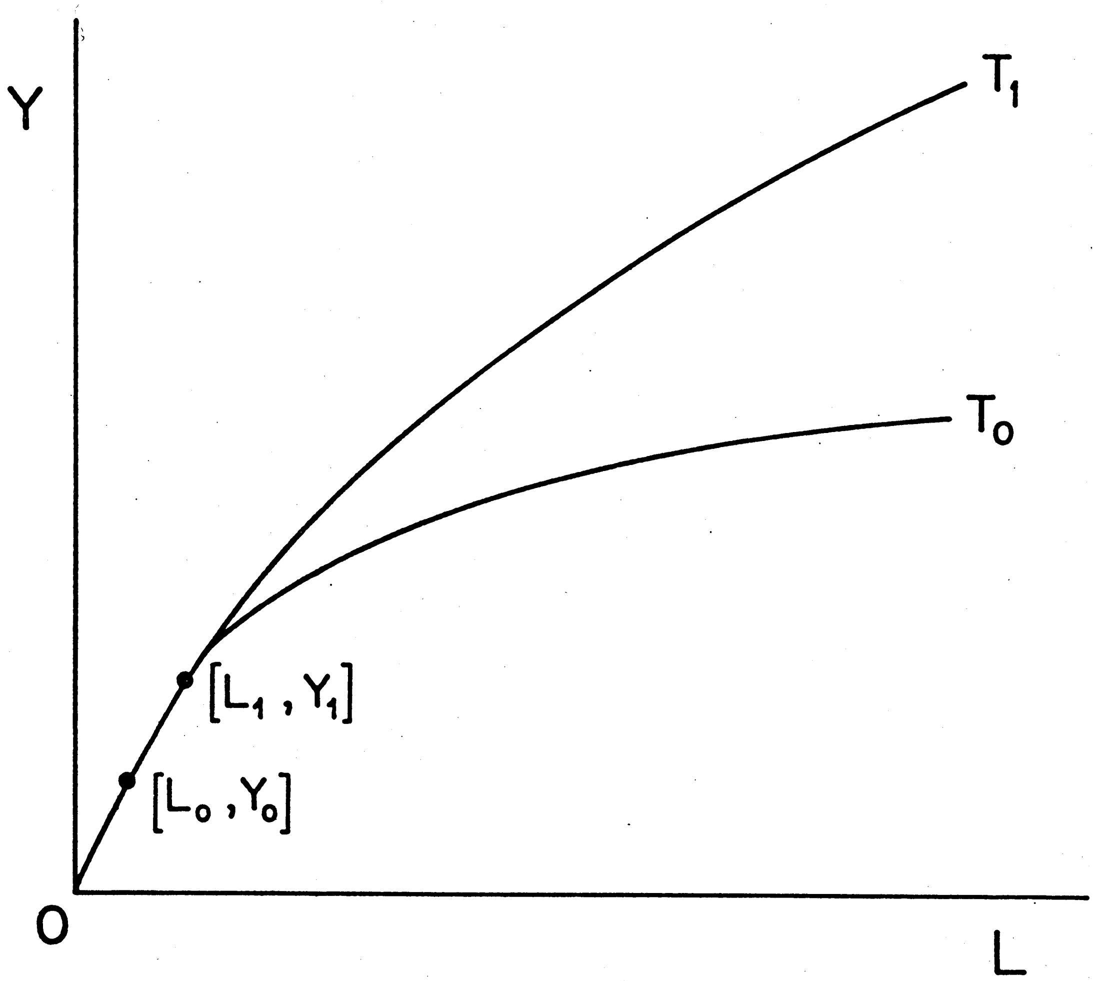

Figure 11

in Figure 12, the degree of technological progress between period 0 and period 1 can be measured by either

$$\frac{F\ (L_0,\ t_1)}{Y_0} - 1 \quad \text{or} \quad \frac{Y_1}{F\ (L_1,\ t_0)} - 1$$

depending on whether L_0 or L_1 is used as the reference level of input. However, as is obvious from Figure 12, the two alternative measurements are not identical.

In order that the two alternative measurements yield identical answers independently of the reference level of input one must have:

$$\frac{F\ (L_0,t_1)}{F\ (L_0,t_0)} = \frac{F\ (L_1,t_1)}{F\ (L_1,t_0)} \quad \forall\ L_0,L_1.$$

which implies, since the left-hand-side is independent of L_1 and the right-hand-side is independent of L_0, that

$$\frac{F\ (L_0,t_1)}{F\ (L_0,t_0)} = \frac{F\ (L_1,t_1)}{F\ (L_1,t_0)} = \alpha(t_1,t_0),$$

a function independent of both L_0 and L_1.

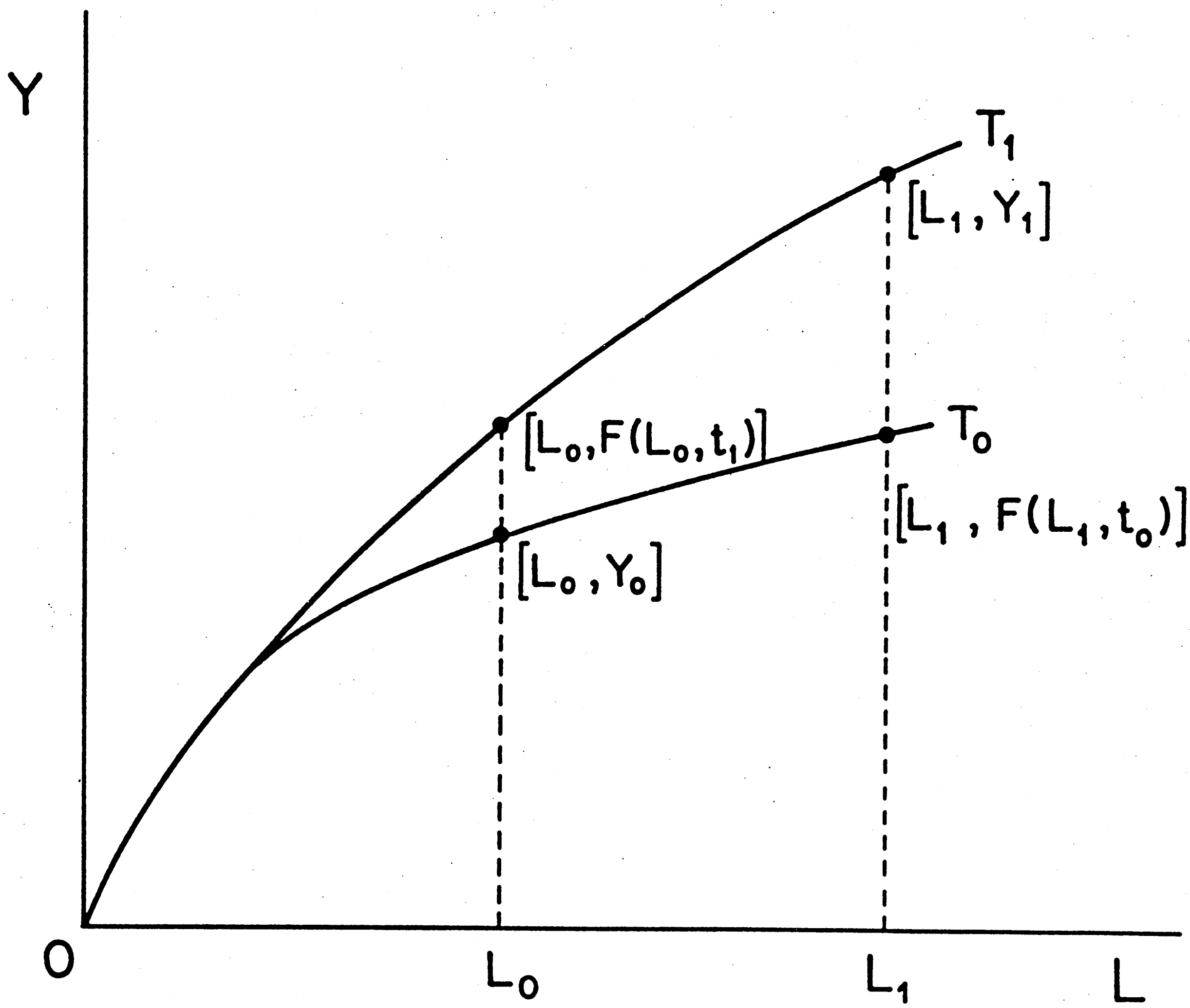

Figure 12

Thus,

$$F(L_0,t_1) = \alpha(t_1,t_0)\ F(L_0,t_0).$$

Let $$A(t) \equiv \alpha(t,t_0)$$

$$f(L) \equiv F(L,t_0)$$

then

$$Y_t = F(L_t,t) = A(t)f(L_t),\ \forall t.$$

In other words, technological progress must be neutral. To the extent that actual technological progress is non-neutral, its measurement will, in general, be ambiguous, depending on the reference level of the input chosen for the measurement.

(6) Vintage effects.

A sixth potential measurement problem has to do with the non-malleability of durable inputs of different vintages. In the multiple-input context, this non-malleability is sometimes also referred to as ex post nonsubstitutibility. Non-malleability in the single-output, single-input context implies that the efficient input-output proportions are fixed over time for each vintage of input (but may be different across different vintages). For example, consider an economy of two periods. In period 0 the quantities of output and input are Y_0 and L_0 respectively; and the input-output coefficient is given by $l_0 \equiv L_0/Y_0$. In period 1 the quantity of the new input is $\Delta L \equiv L_1-L_0$. The input-output coefficient for the new input is l_1. It is assumed that there has been technological progress so that $l_1 < l_0$. The maximum quantity of out-put that can be produced by the new input is therefore equal to $\Delta L/l_1$. The maximum quantity of total output that can be produced by the economy with a total quantity of input L_1 is therefore:

$$Y_1 = \frac{L_0}{l_0} + \frac{\Delta L}{l_1}$$

$$= Y_0 + \frac{\Delta L}{l_1}$$

However, for a given total quantity of input L_1 in period 1, the maximum quantity of total output that can be produced depends on the vintage composition of L_1. If there is no new input in period 1, $Y_1 = L_1/l_0$. If there is no input in period 0 so that all input is new input in period 1, $Y_1 = L_1/l_1$. The actual maximum quantity of total output that can be produced in period 1 depends on the composition of L_1 and lies between these two values.

What implication does non-malleability have on the measurement of productivity? We illustrate the difficulties caused by non-malleability with an example that is most favorable to the measurement of productivity, namely: the production-possibility frontier is always a ray through the origin (thus, its shape is always known *a priori*; constant returns to scale always hold and technological progress, if any, is always neutral).

We begin by identifying the set of *actual* production possibilities available in period 1. We use the word "actual" to indicate that all the input-output combinations contained in the set can in fact be realized given the actual inputs available in period 1. We reserve the word "potential" to describe the set of production possibilities containing all input-output combinations feasible under the technology available in period 1.

In Figure 13, the production-possibility frontiers of periods 0 and 1, T_0 and T_1 respectively, are drawn. The sets of potential production possibilities of the two periods are then represented by the triangular areas under the respective production-possibility frontiers. The set of actual production possibilities, however, is given by the area bounded by the origin, $(\Delta L, \Delta L/l_1)$, $(L_1, L_0/l_0+\Delta L/l_1)$ and $(L_1, 0)$.

If we assume efficiency and full utilization, we should observe $Y_1 = L_0/l_0+\Delta L/l_1$. If we do not take into account the non-malleability of the inputs, we would identify the ray from the origin through $(L_1, L_0/l_0+\Delta L/l_1)$, T_1^*, as the new production-possibility frontier in period 1. However, the actual production-possibility frontier is given by the line segments joining the origin, $(\Delta L, \Delta L/l_1)$ and $(L_1, L_0/l_0+\Delta L/l_1)$. The potential production-possibility frontier is given by T_1, which is higher than T_1^*. Thus, if non-malleability is ignored, both the actual and potential production-possibility frontiers in period 1 would be underestimated.

Nevertheless, if we take into account the non-malleability of the inputs, it is possible to identify the new production-possibility frontier T_1, under the assumption of efficiency and known rate of utilization of total input. The slope of T_1 is given by $1/l_1$. In order to measure this

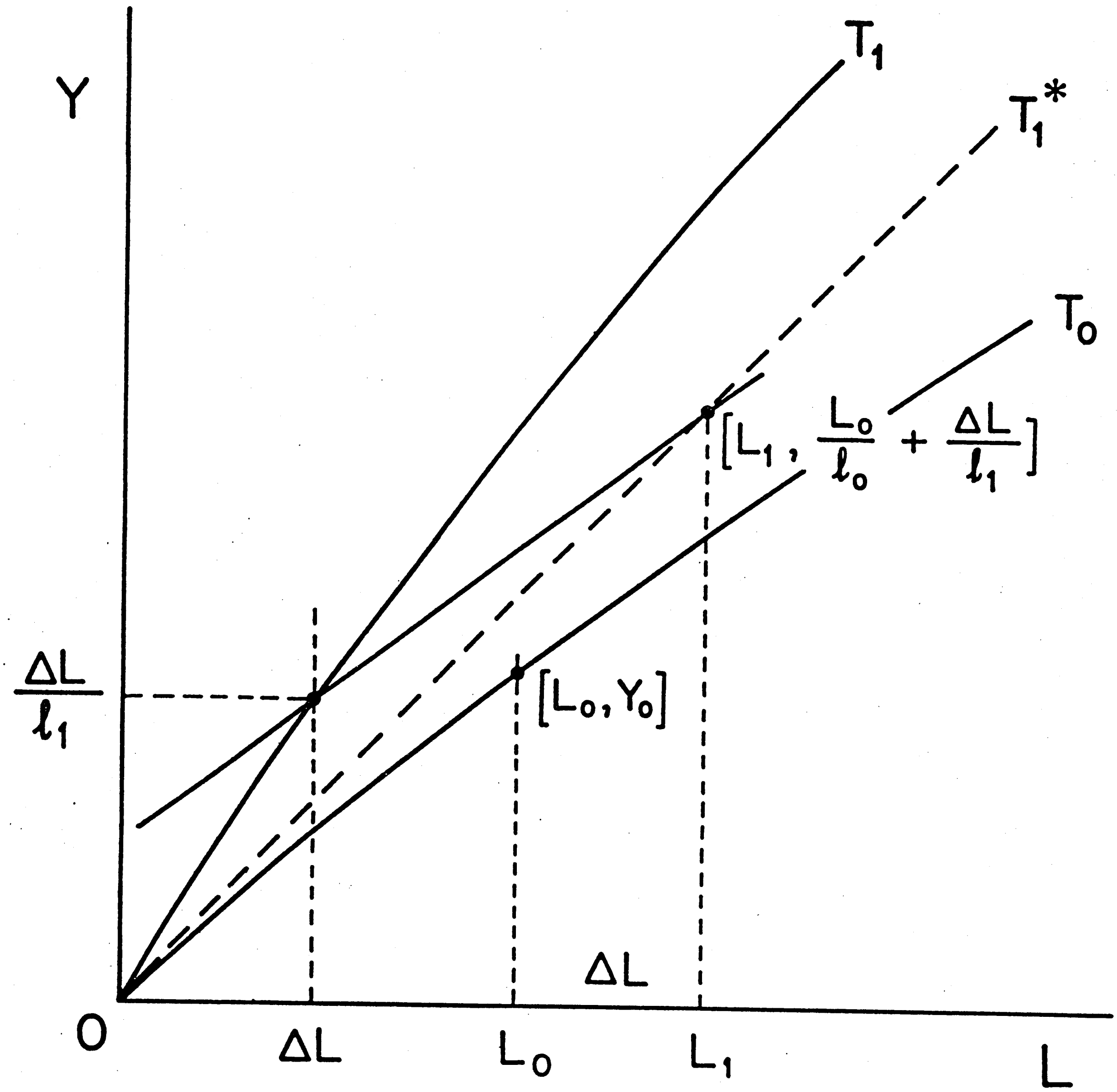
Y
T_1
T_1^*
T_0
$\left[L_1, \frac{L_0}{\ell_0} + \frac{\Delta L}{\ell_1}\right]$
$\frac{\Delta L}{\ell_1}$
$[L_0, Y_0]$
ΔL
O
ΔL
L_0
L_1
L

Figure 13

slope, we set

$$\frac{1}{l_1} = \frac{Y_1 - l_0(\lambda L_1 - \Delta L)}{\Delta L} = \frac{Y_1 - (Y_0/L_0)(\lambda L_0 + (\lambda - 1)\Delta L)}{\Delta L}, \text{ if } \lambda L_1 \geqq \Delta L$$

$$= \frac{Y_1}{\lambda L_1}, \text{ if } \lambda L_1 \leqq \Delta L,$$

where Y_1 is the observed quantity of output in period 1, $L_1 \equiv L_0 + \Delta L$ is the quantity of total input in period 1, and λ is the rate of utilization of total input in period 1. This measurement is based on the assumption of efficiency in production which implies that the new input will be fully utilized before any of the old input is utilized.

Note that this slope is always greater than

$$\frac{1}{1_1^*} + \frac{Y_1}{\lambda L_1} + \frac{Y_1}{\lambda (L_0+\Delta L)}$$

for all $\lambda L_1 > \Delta L$ if there is technological progress.[1]

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[1]This can be seen as follows:

$$\frac{Y_1}{\Delta L} - \frac{Y_0}{L_0} \frac{(\lambda L_1 - \Delta L)}{\Delta L}$$

$$= \frac{Y_1}{\Delta L} - \frac{Y_0}{L_0} \left(\frac{\lambda L_1}{\Delta L} - 1\right)$$

But since $\lambda L_1 > \Delta L$, $\lambda L_1/\Delta L > 1$, and hence $(\lambda L_1/\Delta L - 1) > 0$. In addition, since there is technological progress, $Y_1/\lambda L_1 > Y_0/L_0$. Thus,

$$\frac{Y_1}{\Delta L} - \frac{Y_0}{L_0} \left(\frac{\lambda L_1}{\Delta L} - 1\right)$$

$$> \frac{Y_1}{\Delta L} - \frac{Y_1}{\lambda L_1} \left(\frac{\lambda L_1}{\Delta L} - 1\right)$$

$$= \frac{Y_1}{\lambda L_1}$$

We should be careful in interpreting the potential production-possibility frontier T_1. It represents what is potentially feasible if all inputs have the same characteristics as the new input, through complete replacement or retraining or retrofitting of the old inputs. It does not necessarily represent what is actually feasible in period 1 given the composition by vintage of the existing inputs.

(7) Price-induced obsolescence.

A seventh potential measurement problem has to do with price-induced obsolescence. We illustrate this problem with the same production-possibility frontiers used to illustrate the problem of non-malleability. The production functions for period 0 and period 1 inputs are given by:

$$Y_1^0 = \frac{1}{l_0} L_0$$

$$Y_1^1 = \frac{1}{l_1} \Delta L$$

By assumption, there is technological progress and hence $1/l_1 > 1/l_0$.

If the producer maximizes profit, then the period 0 input L_0 is used only if:

$$\frac{1}{l_0} \geq \frac{W}{P}$$

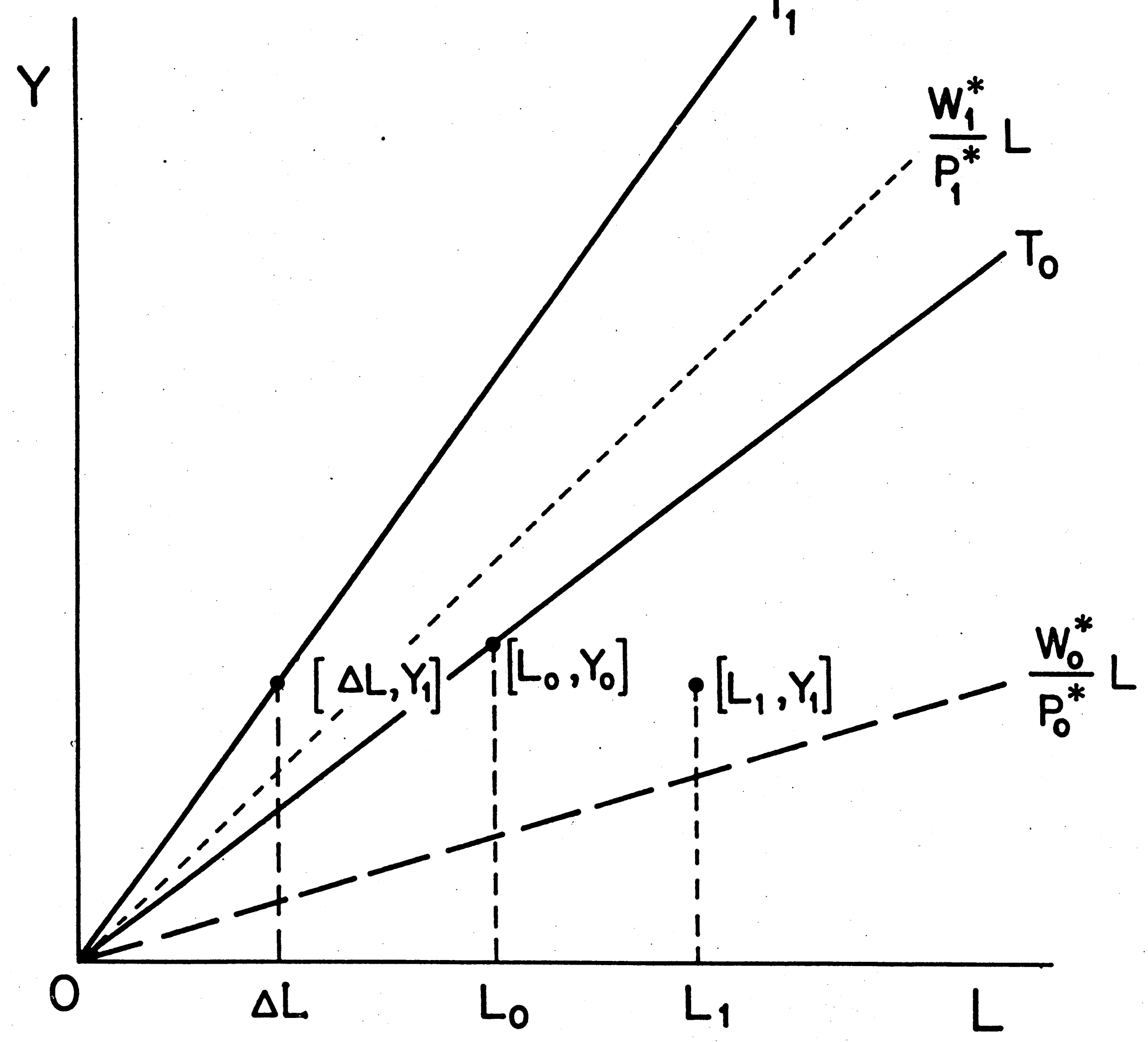
that is, output produced per unit of labor must be greater than or equal to the cost per unit of labor measured in terms of units of output.

If $W/P>1/l_1$, there is zero production, and either the point (0,0) or the point $(L_1,0)$ will be observed depending on whether information on the rate of utilization of total input is available or not. If $1/l_1>W/P>1/l_0$, the period 1 input will be employed but the period 0 input will remain idle, and either the point $(\Delta L,\Delta L/l_1)$ or the point $(L_1,\Delta L/l_1)$ will be observed depending on whether information on the rate of utilization of total input is available or not. If $1/l_0>W/P$, all inputs will be fully employed and the point $(L_1,L_0/l_0+\Delta L/l_1)$ will be observed. Depending on which point is observed, which in turn depends on the price ratio W/P in period 1, different measurements have to be used to identify technological progress correctly.

For example, in Figure 14, in period 0, $1/l_0>W_0/P_0$, and (L_0,Y_0) is observed. In period 1, $1/l_1 \geq W_1/P_1>1/l_0$, and $(L_1,Y_1(=\Delta L/l_1))$ is observed. If the rate of utilization of total input is not known, then one may conclude that there is inefficiency. If, however, the rate of utilization of total input is known in period 1, so that $(\Delta L,Y_1)$ is observed, then it is possible to identify and quantify technological progress. It can be measured simply as

$$\frac{Y_1}{\frac{Y_0}{L_0}\Delta L} - 1 = \frac{\frac{Y_1}{\Delta L}}{\frac{Y_0}{L_0}} - 1$$

all the variables of which are observable.



T_1
Y
$\frac{W_1^*}{P_1^*}L$
T_o
$[\Delta L, Y_1]$
$[L_o, Y_o]$
$[L_1, Y_1]$
$\frac{W_o^*}{P_o^*}L$
O
ΔL
L_o
L_1
L

Figure 14

3. Measurement Problems in a Multiple-Output Multiple-Input Economy.

The lack of space and time does not permit a detailed discussion of the general case. It suffices to say that all the measurement problems encountered in the single-output, single-input case carry over directly to the general case. Moreover, the problems of non-malleability and price-induced obsolescence are manifested in additional dimensions. For example, with two or more inputs, one of which is durable, it is no longer possible to identify the degree of technological progress with information on the rate of utilization of only total input. Instead, information on the rates of utilization of each vintage of input in each period is required. In addition, information on the prices of output and inputs in each period is also required. The additional information is needed because of the inability to rank order ex post technologies in its absence when there are two or more inputs.

Several new measurement problems not encountered in the single-output, single-input case also arise in the multiple-output, multiple-input case. First, there is the problem of the choice of a numeraire. In the single-output, single-input case, there is no problem. In the case of two or more inputs, many choices are available. For example, one may measure productivity in terms of output per unit of labor or output per unit of land (as is customarily done in agriculture). However, no one choice is completely appropriate. The preferred solution is to measure total factor productivity, thus avoiding the arbitrary selection of a single commodity as the numeraire.

Second, when there are multiple outputs, changes in the composition of the outputs can have a significant effect on the measurement of productivity. (This may be regarded as the numeraire problem on the output side).

Third, some outputs and inputs may be omitted either by mistake or due to the lack of appropriate measurements. They include externalities such as clean air, water, infrastructure and pollutants. They also include stochastic elements such as weather and quality of soil. If these non-measured or non-measurable outputs and inputs change over time, they can have a significant effect on measured productivity.

Finally, in the multiple-output, multiple-input case, differences between ex ante and ex post scale efficiencies can also have a significant effect on the measurement of productivity.

4. Conclusions

Even though we have only examined the single-output, single-input case in detail, the measurement problems are so legion and the assumptions necessary for a proper measurement so strong that one may well be quite pessimistic about our ability to identify and quantify technological progress in general.

However, it is possible to improve our measurement of productivity by using more information than is traditionally used. For example, one can supplement aggregate time-series data with cross-section data at the level of individual establishment on the input-output relations for newly installed capital in one or more periods. One can also supplement market data with engineering, process, or design data. Finally, one can perform a more sophisticated analysis of input-output relationships from market data by recognizing explicitly the difference between ex ante and ex post substitutibilities, that is, the "putty-clay" nature, of production technologies.

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