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BOUNDS ON THE VARIANCES OF SPECIFICATION ERRORS IN MODELS WITH EXPECTATIONS

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ABSTRACT

This paper develops a general method for placing a lower bound on the variance of the misspecification or noise in expectations based models. Correct specification of a model corresponds to consistency of a lower bound of zero with the observable data. Optimal bounds estimates are constructed through a signal extraction framework. The lower bound on admissable model noise captures the complete set of testable restrictions on an expectations based model. Many specification tests for asset prices are easily interpreted as estimates of this lower bound. As a result, the power of different tests may be ranked according to the information restrictions employed in constructing noise estimates. Our results strongly suggest that specification tests which use the history of lagged dependent variables are better able to uncover model noise than tests which exploit other information sets.

Introduction

Robert Shiller [1981a,b] was the first to note that expectational models have simple testable implications derived from general properties of expectations. A large literature has developed from his original idea (see the survey by Gilles and LeRoy [1988] for many references). Shiller's idea was that the expectation or prediction of a random variable must necessarily have a lower variance that its realization. His basic test compared the variance of the two and rejected the simple expectational model if the variance of the prediction exceeded the variance of the realization. In the stock market, the comparison is between the realized present discounted value of dividends and the forecast, which is observable in the form of the current market value of the stock. Shiller concluded that the stock market has a good deal of noise or model error from his finding that stock prices are much more variable than are realized discounted dividends.

Shiller looked at only two moments of the data—the variances of the realization, P^*, and the variance of the forecast or actual price, P. Subsequent researchers, notably West [1987], have proposed tests of the hypothesis of no noise that rely on covariances as well as variances. Our purpose in this paper is to carry out a systematic investigation of the sharpening of results that is available through the use of covariances. We depart from the almost exclusive focus of the earlier research on testing the null hypothesis of the absence of noise or model error. Our objective is to obtain lower bounds on the amount of noise. The null hypothesis of no noise is equivalent to the hypothesis that this lower bound may be set equal to zero.

Before Shiller, research on noise in expectation models tended to look at one-period-ahead relations. In the stock market, for example, numerous investigators looked for evidence of excess returns over brief holding periods. The finding of a relationship between an observed variable and excess returns supported a conclusion of market inefficiency or other type of noise relative to the model. One of the contributions of this paper is to compare the one-period-ahead or flow approach to the Shiller or stock approach. The stock approach appears far superior as a method for detecting and measuring slow-moving noise. In particular, noise components that grow at a rate near the rate of interest escape detection in the flow approach. This point has been emphasized by authors whose concern was the detection of speculative bubbles. Our work here is not directed toward bubbles in particular, but rather to slower-moving noise components in general. In applications of our ideas to investment and consumption, there is no reason to look for speculative bubbles.

Our results can be explained intuitively in a regression framework. The difference between the realization and the actual price is the sum of an expectation error and noise. The actual price contains the noise plus the true expectation of the realization. The regression of the difference on the price would have zero explanatory power if there were no noise, under rational expectations. On the other hand, if there were no variation in the true expectation but some noise, then the fitted value of the regression would measure the noise exactly. In the general case, with some noise and some variability of the unobserved true expectation, the variance of the fitted value of the regression is a lower bound on the variance of the noise.

The analysis is organized as a series of signal extraction problems. Different vectors of time series are decomposed according to an unobserved components framework. A lower bound is constructed on the variance of the unobserved component defined as model noise. The null hypothesis of correct specification is equivalent to the case where the variance of the noise component may be set equal to zero, given restrictions imposed by the data. Section 2 of the paper solves the signal extraction problem when contemporaneous moments on predictions and realizations are all the available information. Section 3 extends the analysis to knowledge of the complete spectral density matrix of various sets of observable variables. The constant geometric discount model is explored in some detail. Section 4 relates our results to Hausman specification tests. Summary and conclusions follow.

1. Information contained in the covariance of the actual and perfect-foresight variables

Define the following variables

$$P_t^e = E(P_t^* | \phi_t) \tag{1.1}$$

$$P_t^* = P_t^e + \nu_t \tag{1.2}$$

$$S_t \quad = \quad P_t \; - \; P_t^e \tag{1.3}$$

Here P_t^* is the realized value of a random variable constructed by the econometrician (the "perfect foresight" present discounted value of realized dividends in the application to the stock market, for example), P_t^e is the expectation formed in the market by participants with the unobserved vector of information, ϕ_t, ν_t is the impact on the realization of information not available at the time the price is set in the market, and S_t is the noise or model error, the difference between the observed price and the unobserved expectation.

In this unobserved components model, we seek to make inferences concerning P_t^e, ν_t, and S_t given data on P_t and P_t^*. Our particular interest is in finding the smallest admissable value of σ_s^2, the variance of S_t. The null hypothesis of no specification error is $P_t = P_t^e$ or $\sigma_s^2 = 0$.

It will help the ensuing discussion to consider the following expression of the observed variables in terms of underlying unobserved variables:

$$P_t^* \quad = \quad X_t \; + \; \beta Y_t \; + \; \nu_t \tag{1.4}$$

and

$$P_t \quad = \quad X_t \; + \; Y_t \tag{1.5}$$

where

$$X_t = P_t^e - \left(\frac{\sigma_{sp^e}}{\sigma_s^2}\right) S_t \tag{1.6}$$

and

$$Y_t = \left(1 + \frac{\sigma^2_{sp^e}}{\sigma_s^2}\right) S_t \tag{1.7}$$

Because P_t and P_t^e are both observed by market participants, ν_t is already orthogonal to the other variables in the decomposition. The observable moments of the data have the following relation to the moments of the unobserved components structure:

$$\sigma^2_{p^*} = \sigma_x^2 + \beta^2\sigma_y^2 + \sigma_\nu^2 \tag{1.8}$$

$$\sigma_p^2 = \sigma_x^2 + \sigma_y^2 \tag{1.9}$$

$$\sigma_{pp^*} = \sigma_x^2 + \beta\sigma_y^2 \tag{1.10}$$

Solving these equations yields:

$$\sigma_x^2 = \frac{\beta}{\beta-1}\sigma_p^2 - \frac{1}{\beta-1}\sigma_{pp*} \tag{1.11}$$

$$\sigma_y^2 \;=\; -\;\frac{1}{\beta-1}\,\sigma_p^2 \;+\; \frac{1}{\beta-1}\sigma_{pp*} \tag{1.12}$$

$$\sigma_\nu^2 \;=\; \sigma_{p*}^2 \;+\; \beta\sigma_p^2 \;-\; (1+\beta)\,\sigma_{pp*} \tag{1.13}$$

From the earlier definitions,

$$S_t \;=\; (1 \;-\; \beta)\; Y_t \tag{1.14}$$

so, from (1.12),

$$\sigma_s^2 \;=\; (1-\beta)(\sigma_p^2 - \sigma_{pp*}) \tag{1.15}$$

From these expressions, it is clear that $\sigma_s^2 = 0$ if and only if $\sigma_p^2 = \sigma_{pp*}$, which is equivalent to a t-test on the coefficient α in the regression

$$P_t - P_t^* \;=\; \alpha P_t + \epsilon_t \tag{1.16}$$

where the null hypothesis is that $\alpha = 0$.

The minimal value of σ_s^2

To minimize σ_s^2 given the observed moments is equivalent to minimizing (1.15) with respect to β subject to non-negativity of all variances. The Cauchy-Schwartz relations, which might appear to impose additional constraints, are already embedded in the problem through the orthogonalization. Some manipulation of (1.11) through (1.13) reveals that non-negativity of variances requires

$$\frac{\sigma_{p*}^2 - \sigma_{pp*}}{\sigma_{pp*} - \sigma_p^2} \leq \beta \leq \frac{\sigma_{pp*}}{\sigma_p^2} \quad \text{if } \sigma_{pp*} \leq \sigma_p^2 \tag{1.17}$$

or

$$\frac{\sigma_{pp*}}{\sigma_p^2} \leq \beta \leq \frac{\sigma_{p*}^2 - \sigma_{pp*}}{\sigma_{pp*} - \sigma_p^2} \quad \text{if } \sigma_{pp*} \geq \sigma_p^2 . \tag{1.18}$$

If the data do not preclude the existence of noise, that is, if σ_p^2 differs from σ_{pp*}, then minimizing σ_s^2 subject to (1.17) and (1.18) requires that $\beta = \sigma_{pp*}/\sigma_p^2$. If $\sigma_{pp*} \leq \sigma_p^2$, (1.15) shows that the minimum of σ_s^2 occurs at the largest permitted value of β, which, from (1.17), is σ_{pp*}/σ_p^2. If $\sigma_{pp*} \geq \sigma_p^2$, (1.15) shows that the minimum of σ_s^2 occurs at the smallest permitted value of β, which, from (1.18), is σ_{pp*}/σ_p^2. Consequently, we have established

Theorem 1.1. The bound on the variance of the noise is

$$\sigma_s^2 \geq \frac{(\sigma_{pp*} - \sigma_p^2)^2}{\sigma_p^2} \tag{1.19}$$

This bound is attainable and is therefore the tightest possible bound.□

Relation to regression tests

The bounding variance in Theorem 1.1 is the variance of the ordinary least squares predictor, αP_t, in the regression

$$P_t - P_t^* = \alpha P_t + \epsilon_t \tag{1.20}$$

From (1.2) and (1.3), this is

$$S_t + \nu_t = \alpha(S_t + P_t^e) + \epsilon_t \tag{1.21}$$

The true, unobserved expectation, P_t^e, has the role of an error in a variable in this regression. If P_t^e were zero, then the variance of αP_t would be exactly the variance of S_t (α would be one because ν_t is uncorrelated with S_t by hypothesis). The presence of the unobserved P_t^e biases the explanatory power of the regression downward to an unmeasurable extent. Because the direction of the bias is known, however, the regression provides a lower bound on the variance of S_t. The idea that noise can be detected by regressing the difference between the perfect-foresight variable and the actual variable on variables known at the time the actual variable is formed is implicit in the work of West [1987] and is the basis for the more general results in the next section of this paper.

The contribution of Theorem 1.1 is to show that the simple regression procedure gives the tightest possible bound on the variance of the noise if only the contemporaneous moments are available.

Inference without the covariance

If σ_{pp*} were unobservable, then any choice for this covariance is admissable in the above problem that satisfies the Cauchy-Schwartz inequality, $(\sigma_{pp*})^2 \leq \sigma_p^2 \; \sigma_{p*}^2$. We can therefore employ the same analysis and treat the covariance as a choice variable.

The Cauchy-Schwartz constraint means that if $\sigma_p^2 \leq \sigma_{p*}^2$, then $\sigma_{pp*} = \sigma_p^2$ is admissable, and the data are consistent with the hypothesis that the model specification error S_t is zero. Alternatively, if $\sigma_p^2 \geq \sigma_{p*}^2$ then σ_s^2 must be positive. The bound in Theorem 1.1 is minimized subject to the Cauchy-Schwartz inequality when $\sigma_{pp*} = \sigma_p \sigma_{p*}$. This establishes

Theorem 1.2. Absent information on the covariance, σ_{pp*}, if $\sigma_p^2 \leq \sigma_{p*}^2$, there is no informative bound on the noise variance, σ_s^2. If $\sigma_p^2 \geq \sigma_{p*}^2$, then

$$\sigma_s^2 \geq (\sigma_p - \sigma_{p*})^2 \, . \tag{1.22}$$

□

This bound is by necessity less informative than the bound in Theorem 1.1. This derivation formally justifies the excess volatility test derived by Shiller [1981], in the sense that his test is optimal when the variances are the only observable moments. This derivation also illustrates how the Shiller test has zero power against alternative hypotheses which represent large deviations from the null. His test is incapable of detecting noise that makes the variance of P less than the variance of P^*. Only the covariance can tell whether this situation arises because of expectation errors or because of noise.

2. Bounds on the noise variance based on observed predictors—the general case

The results of Section 1 illustrate the powerful information restrictions implicit in both regression and excess volatility tests of model noise. We generalize our results in this section to the restrictions on model noise which are generated by data on some of the information available to the market. In this section, we maintain the assumption that P_t^* has a general form and consequently ν_t has arbitrary time-series properties. In the next two sections, we consider the case where P_t^* is a geometric weighted average of a future variable. In that case, ν_t follows a prescribed AR(1) process.

Throughout our discussion, we shall assume that there exists a vector time series x_t observable to the econometrician which is a strict subset of the information set Φ_t employed by market participants to construct forecasts. Let $L_x(t)$ be the linear space of time series running up to time t, each of which is a linear combination of x_t, x_{t-1}, x_{t-2},... with square-summable weights. We let $M_x(t)$ denote the projection operator for $L_x(t)$. Recall from the previous section that the difference between noise and the expectation error is observed after the fact:

$$P_t - P_t^* = S_t - \nu_t \tag{2.1}$$

Because ν_t is orthogonal to any element in $L_x(t)$, arguments of the previous section verify that a lower bound on the variance of noise is provided by the variance of the projection of $P_t - P_t^*$ onto $L_x(t)$. We define $S_{t|t}$ as $M_x(t)S_t$. Then we have

$$P_t - P_t^* = S_{t|t} + [1 - M_x(t)]S_t - \nu_t \tag{2.2}$$

Because the residual term $[1-M_x(t)]S_t$ is orthogonal to the projection, $S_{t|t}$, the variance of the projection is less than the variance of S. Hence $\sigma^2_{s_{t|t}}$ is a lower bound for the noise variance, σ^2_s. Further, because the model imposes no restrictions on the relation between ν_t and any element of $L_x(t+j) - L_x(t)$, it is possible to find covariances of this type such that the bound is attained. Thus, we have

Theorem 2.1 Absent restrictions on the covariance structure of ν_t, the variance of the model error must satisfy

$$\sigma^2_s \geq \sigma^2_{s_{t|t}} \; . \tag{2.3}$$

This bound is attainable.□

The Appendix contains a proof that the bound is attainable.

The idea of exploiting the unpredictability of a random variable which is a forecast error under some null hypothesis as a specification test appears in a large number of studies in many different areas of research. For example, numerous authors have explored the hypothesis that forward exchange rates are forecasts of future spot exchange rates. Fama [1984], in particular, constructs a regression in an attempt to estimate a time varying risk premium (corresponding to model noise in our framework) which is equivalent to the projection of $P_t - P_t^*$ onto $P_t - P_{t-1}^*$. Tests of foreign exchange market efficiency such as Hansen and Hodrick [1980], Bilson [1981], and Hodrick and Srivastava [1984], to name a few, can be interpreted in our framework as well. Our results demonstrate how the orthogonality of forecast errors represents the total set of testable restrictions in these models. The power properties of all such proposed tests may therefore in principle be ranked on the basis of which information restrictions are imposed.

This Theorem underscores the results of Frankel and Stock [1987] comparing regression and excess volatility tests. Frankel and Stock demonstrated that regression tests of model misspecification were optimal relative to any excess volatility test which could be constructed with a given information set. Our results extend this optimality by deriving the circumstances under which the regression formulation exhausts all testable implications of the model.

3. Inference with geometric discounting

When P_t^* has the particular form of the discounted value of a future stream, with a constant discount rate, further sharpening of noise variance bounds is possible. Models with this expectational structure include the stock price-dividend relation in Shiller's original work, as well as models of hyperinflations (Cagan [1956], Sargent and Wallace [1973], Sargent [1977]), exchange rates (Meese [1986]) and output and investment (Hall [1988]). The methods developed in this section could apply to any of these models. On the other hand, the methods do not apply to the consumption problem as in Flavin [1981], because consumption is a function of the stock of assets as well as of discounted future earnings.

The constant discount model hypothesizes that P_t^e reflects the discounted value of a future flow, D_{t+i}, with a constant discount factor β:

$$P_t^e = \sum_{i=0}^{\infty} \beta^i E(D_{t+i}|\Phi_t) . \tag{3.1}$$

The perfect-foresight variable, P_t^*, has the same form without the expectations. As before, we assume that the observed price, P_t, is the sum, $P_t^e + S_t$.

A long tradition of empirical work on the constant discount rate model has studied the excess return,

$$r_t = D_t - P_t + \beta P_{t+1}$$

$$= \eta_t + S_t - \beta S_{t+1} \ . \tag{3.2}$$

Here

$$\eta_t \ = \sum_{i=0}^{\infty} \beta^i \ [E(D_{t+i}|\Phi_{t+1}) - E(D_{t+i}|\Phi_t)] \ , \tag{3.3}$$

the innovation in the valuation. (D_t is assumed to be observable at $t+1$.) The ***intertemporal arbitrage condition*** states that the expectation of the excess holding return conditional on information available in the market is zero. Let

$$N_t \ = \ M_x(t) r_t \ = \ M_x(t)(S_t \ - \ \beta S_{t+1}) \ . \tag{3.4}$$

N_t is a measure of the extent of the failure of the arbitrage condition. Under certain strong conditions, finding a non-zero N_t supports a conclusion of failure of market efficiency.

Within the framework of this paper, N_t is a measure of the backward quasi-difference, $S_t - \beta S_{t+1}$ of the noise, S_t. As such, it does not directly serve our purpose of measuring the level of noise. In one important case, the noise vanishes from the arbitrage condition: This happens when S_t grows or is expected to grow in proportion to β^{-t}, a speculative bubble. Even when the noise does not take the exact form of a bubble, any slow-moving component will be essentially erased by the backward quasi-difference operation. The problem is particularly acute if the time interval is short, say weekly or monthly. Specification tests will have low power against slow-moving noise if the tests use quasi-differenced data.

Given a time series for N_t, a corresponding estimated noise series, $\hat{S}_t$, is available from the recursion,

$$\hat{S}_t \;=\; \beta \hat{S}_{t+1} + N_t \;. \tag{3.5}$$

Although $\hat{S}_t$ is potentially useful as an indication of the amount of noise in the level of P_t, its variance is not necessarily less than the variance of the actual noise, S_t. Because of its dependence on future N's, it does not satisfy the basic orthogonality condition needed to prove that its variance places a lower bound on the noise variance. It is a topic for further research to see if useful bounds on the noise variance can be derived from the arbitrage condition. For now, the only result we have obtained is that the projection of $\hat{S}_t$ on variables known at time t, resulting in a measure $\hat{S}_{t|t} = M_x(t)\hat{S}_t$ has a variance less than the noise variance.

Our approach to developing sharp bounds on the noise variance focuses on the level of noise, rather than its backward quasi-difference. However, as will be seen, the excess return plays an important role in our method.

Recall that our basic approach to finding lower bounds on the noise variance is to find variables in the information set of market participants that have predictive power for $P_t - P_t^*$. The explanatory power of these regressors raises and thus sharpens the bound on the variance of the noise. The additional regressors we choose are the excess return lagged 1, 2, ... periods. We assume that these lagged excess returns have not already been included in the X-variables. The explanatory power arises from the presence of the backward quasi-difference of noise, $S_{t-i} - \beta S_{t-i+1}$, in the excess return, which helps predict the noise, S_t, in $P_t - P_t^*$.

Let $L_r(t)$ denote the linear space generated by r_t. The elements of $L_r(t-1)$ are orthogonal to the forecast error ν_t. Denoting the projection of S_t onto $L_x(t) \oplus L_r(t-1)$ as $S_{t|\infty}$ and its variance as $\sigma^2_{s_{t|\infty}}$, we have:

Theorem 3.1 In the constant discount rate model, if the discount rate is known, the variance of the model noise satisfies the lower bound

$$\sigma_s^2 \geq \sigma^2_{s_{t|\infty}} \tag{3.6}$$

This bound is the tightest bound available, in that it is attainable.□

The Appendix contains a formal proof of the attainability of the bound, which is derived in a Kalman filtering framework.

Information contained in regression residuals

It might appear that a second way to add explanatory variables to the regression to measure noise might be based on the observation that so long as D_{t-1} (and by implication $L_D(t-1)$) is known at time t, absent noise, the disturbance in the regression is AR(1) with parameter β. To the extent that the actual serial correlation of the residual is different, added predictive power would be available. However, adding the lagged excess returns as recommended by Theorem 3.1 precludes that source of predictive power.

In order to see that the properties of the innovations will not augment the lower bound, whenever lagged innovations are included as regressors, we derive the additional explanatory power of the lagged innovations for an arbitrary regressor set. It is easy to show that

$$\nu_t - \beta\nu_{t+1} = \eta_t , \tag{3.7}$$

where η_t is the value innovation mentioned earlier. Thus ν_t can be viewed as a time series process in reverse time with Wold innovation η_t. From equation 3.7 it is plainly an AR(1) process. It can also be viewed as an AR(1) process in normal time, in which case its Wold innovation, generated by projecting onto the past, is a different white noise random variable, w_t.

The requirement that the forecast error, ν_t, be AR(1) allows us to increase the noise bound through a two step procedure. Consider the autoregressive transformation of the residuals from the noise-detecting regression. The residuals are:

$$[1 - M_x(t)]S_t - \nu_t . \tag{3.8}$$

and the autoregressive transformation of the residuals is:

$$z_t = (1 - \beta L^{-1})\{[1 - M_x(t)]S_t\} - w_t . \tag{3.9}$$

Because the transformation turns the forecasting error into its white-noise innovation, w_t, any serial correlation in z_t must come from the other term, which is the part of the noise that escaped detection in the regression, $[1-M_x(t)]S_t$. In the univariate Wold decomposition of z_t,

$$z_t = \pi(L)z_{t-1} + \epsilon_t \,, \tag{3.10}$$

the extra predictive power is measured by $\pi(L)z_{t-1}$. To restate this predictive power in terms of the level of the noise variable, we multiply by the inverse of the quasi-differencing operator:

$$u_t \;=\; (1 - \beta L^{-1})^{-1}\pi(L)z_{t-1} \,. \tag{3.11}$$

and can refilter the innovations in order to recapture the model noise

$$[1 - M_x(t)]S_t - \nu_t \;= u_t + \;(1 - \beta L^{-1})^{-1}\epsilon_t \tag{3.12}$$

The $\pi(L)z_{t-1}$ variable is orthogonal to the forecast error and also to $L_x(t)$. Forward filtering does not preserve this orthogonality. Consequently, we cannot argue that the u_t component is smaller in variance than the left hand side of (3.13). In order to bound the variance, we must construct the Wiener-Kolmogorov projection of u_t, which we denote as $u_{t|t-1}$

$$u_{t|t-1} \;= \sum_{i=0}^{\infty} \beta^i \left(\frac{\pi(\mathrm{L})}{\mathrm{L}^i}\right)_+ \pi(L)z_{t-1}. \tag{3.13}$$

By construction, the forecast error $\zeta_t = u_t - u_{t|t-1}$ must possess the autoregressive decomposition

$$\zeta_t = \beta\zeta_{t-1} + \mu_t. \tag{3.14}$$

since it is the forecast error of a constant discount model. Rewriting

$$[1 - M_x(t)]S_t - \nu_t = u_{t|t-1} + \zeta_t + (1 - \beta \mathrm{L}^{-1})^{-1}\epsilon_t \tag{3.15}$$

we have now decomposed the left hand side into a component $u_{t|t-1}$ which does not fulfill the requirements of the forecast error and an orthogonal component that does fulfill the AR(1) requirement.

Finally, notice that by construction, $u_{t|t-1}$ is identically equal to the projection of $[1-M_x(t)]S_t-\nu_t$ onto $L_z(t-1)$. This means that the two-step procedure will generate identical projections to the procedure in Theorem 2.1, since the two estimators are in fact projecting onto the same space. We summarize our results in

Theorem 3.2 In the constant discount rate model, if the discount rate is known, the variance of the model noise must fulfill

$$\sigma_s^2 \geq \sigma^2_{s_{t|t}} + \sigma^2_{u_{t|t-1}} = \sigma^2_{s_{t|\infty}}. \tag{3.16}$$

This bound is attainable.□

The residuals in the regression could also be filtered backwards in order to generate AR(1) innovations. We analyze the case where the projection residuals are a regular, stationary process. Defining

$$y_t = (1- \beta L)(1 - M_x(t))S_t + v_t. \tag{3.17}$$

one may think of the construction of the model noise estimate as a projection of $(1-M_x(t))S_t+\nu_t$ onto that part of $L_y(t)$ which is orthogonal to the forecast error. (A quick reexamination of the derivation of Theorem 3.2 reveals that $u_{t|t-1}$ is a projection of $(1-M_x(t))S_t+\nu_t$ onto $L_z(t-1)$, all of whose elements are orthogonal to the forecast error. Notice that the dating of the two linear spaces is different. This occurs because y_t is known as of t whereas z_t is not.)

The bound in Theorem 3.2 is optimal, as verified by Thoerem 3.4 below. Therefore the backwards filter cannot increase the variance estimate of the noise residual. On the other hand, since

$$\frac{1 - \beta L^{-1}}{1 - \beta L}\, y_t = z_t. \tag{3.18}$$

it follows that $L_z(t-1) \in L_y(t)$. Therefore the backwards filter must generate a variance estimate at least as large as the forward filter, since the forward projection is an admissable solution. In fact, since $L_z(t-1)$ is the only subspace of $L_y(t)$ whose elements are necessarily orthogonal to the forecast error, the predictions of the model noise residual are identical.

Similar results may be obtained for different assumptions on the observability of dividends. If the available information set on dividends at time t is equal to $L_D(t-i-1)$, then the forward quasi-difference of the regression residuals is now MA(i) under the null. This means that $L_r(t-i-1)$ should be included in the regressor set. One can replicate the appropriate two step procedure we have outlined and show that Theorem 3.2 generalizes to this case as well. If no assumptions are made on the market participants' knowledge of the dividends history, one can also show that the residuals contain no implications for the model noise.

The decomposition of the bound in Theorem 3.2 shows how information on the residuals may be extremely valuable when the information set of market participants is unknown or the predictive power of the available X-variables is weak, as is the case for example when dividends are used to predict stock prices. This result emphasizes that the departure of P_t^* from P_t in the class of expectations models we have been discussing must be AR(1) if there is no model noise. The structure of the disturbances in the $P_t^*-P_t$ regression should not be treated as a nuisance parameter. Rather, this structure should be incorporated as part of the specification testing.

A similar expression to $u_{t|t-1}$ was derived in a different context by Campbell and Shiller [1987] who studied the behavior of excess holding returns in the dividend stock price model. These authors treated the present discounted value of expected excess holding returns as a measure for violations of the model. Our analysis provides a precise metric which justifies the use of the measure in assessing models deviations and also demonstrates how the measure may serve as a comprehensive model specification test.

West [1987] proposed a test of the specification of the constant geometric stock price model which essentially consists of comparing the variance of η_t defined in (3.7) to the variance of

$$\hat{\eta}_t \quad = \quad (1 - \beta L^{-1})E((P_t^*|L_D(t)) - P_t^*) \tag{3.19}$$

(The information set employed in (3.8) can in principle be any subset of agents information which includes current and lagged dividends.) The variance of $\hat{\eta}_t$ must at least as great as the variance of η_t in the absence of noise, since the two random variables are white noise transformations of forecast errors based upon nested information sets. Our analysis shows that this test only captures a subset of the many implications for innovations of the constant geometric discount model. When a sufficiently rich information set is analyzed, there is no need to analyze the residuals (and implicitly the innovations) in isolation. When the information set of agents is unknown, the residuals should be rendered AR(1) by extracting an additional noise estimate. West's innovation analysis is the equivalent of an excess volatility test for the model innovation, if predictions are compared to inferior predictions, rather than realizations. The set of alternative hypotheses against which the test is powerful is therefore quite restrictive. The test does, however, contain information beyond the AR(1) restriction as it implicitly contains elements of a test for unpredictability of the forecast errors.

Optimality of stock versus flow approach

One should not conclude from this discussion that an analysis of the model noise through the excess holding returns from stocks is equivalent to an analysis of price levels. The flow approach to model noise will in general be less informative than the stock approach.

To see this, notice that the random variable $P_t - P_t^*$ is used to construct the holding returns r_t. This does not ensure that the present and future holding returns may be used to construct $P_t - P_t^*$. This nonequivalence occurs, for example, whenever there is a rational bubble. If

$$B_t = \beta^{-1} B_{t-1} + \iota_t. \tag{3.20}$$

then B_t is not an element of $L_\iota(\infty) - L_\iota(t-1)$, since the bubble is unbounded in variance whereas the linear spaces after filtering only contain bounded elements. Whenever the projections of current and future r_t are employed to artificially construct the model noise, it therefore can never improve the bound, and if nonequivalence occurs, can cause a diminution of the bound. It is clear, further, that the use of the realized excess returns is only one of an infinity of ways to filter $P_t - P_t^*$. So long as the original series is recoverable from the transformation, it is arbitrary (in a projection sense) whether one constructs the model noise estimate or through the reversal of the filter. Formally, *Theorem* 3.3. For any (possibly two sided) filter $\mu(L)$ such that

$$\mu(L)(P_t - P_t^*) = \xi_t \tag{3.21}$$

if $P_t - P_t^* \in L_\xi(\infty)$ i.e. $\kappa(L)\xi_t = P_t - P_t^*$ for some $\kappa(L)$, then the variance of the flow prediction of model noise

$$\sum_{j=-\infty}^{\infty} \kappa_j M_x(t)\xi_j. \tag{3.22}$$

will equal the stock prediction of the lower bound on the admissable level of noise variance. If $P_t - P_t^* \notin L_\xi(\infty)$, the flow prediction of model noise will not identically equal the stock prediction of the lower bound on the admissable level of noise variance. The stock estimate of the model error will always be at least as great as the flow error in this case.□

Fama and French [1988] and Lo and MacKinlay [1987] have argued in a related context that the stock approach also possesses superior power properties in finite samples in uncovering model noise to the flow approach. Specifically, they demonstrate that for a time series exhibiting long run mean reversion, the first differences of the series might appear to be uncorrelated at short lags. These authors conclude that the behavior of long changes in the series is a superior way of uncovering noise, as opposed to analyzing first differences in isolation. The first differences correspond to our measure of excess holding returns. Our Theorem exclusively deals with asymptotic noise identification, and therefore says nothing about finite sample properties. For actual empirical work, the finite sample based critique may very well be more important in justifying the use of the stock approach.

Optimal model noise bounds

Finally, we are able to characterize the full set of restrictions which the constant discount model places on a given linear space of observable variables. This set of restrctions in turn defines the largest admissable bound on the model noise variance. Our results in the Appendix show that if the information set x_t contains the price series P_t and the dividend series D_t, then the projection $S_{t|t}$ will equal $S_{t|\infty}$. This follows immediately from the two equivalent formulations of r_t in (3.2) above. Lagged r_t variables are contained in the history of P_t and D_t. Therefore, the impact of adding residuals to the regression will be the same as adding lagged dividends and prices. Asymptotically, the sequences of projections on $L_x(t)$ will render the residuals AR(1) with a coefficient of β. This generates the result that absent any assumptions on the structure of model noise which imply the existence of a component of future noise which is unaffected by contemporaneous dividend forecast errors, then orthogonality of $P_t - P_t^*$ to information available at t will represent the complete set of restrictions for the model. Restating this result in the language of Kalman filter/signal extraction framework of the Appendix, the optimal predictor and optimal smoother will coincide. As the Appendix verifies, one can construct a structural model which is consistent with the data, such that the optimal prediction of model noise is perfect. As a result, the one-sided projection framework will exhaust all the implications of the model. The optimality of the regression formulation is summarized in

Theorem 3.4 For any time series vector x_t which includes current dividends and prices,

A. Orthogonality of $P_t - P_t^*$ to $L_x(t)$ constitutes all of the testable restrictions which the constant dividend model places on the spectral density matrix of the data.

B. The variance of model noise must fulfill

$$\sigma_s^2 \geq \sigma_{s_{t|t}}^2 = \sigma_{s_{t|\infty}}^2. \tag{3.23}$$

This bound is attainable. No greater bound may be constructed which is consistent with all potential structural models which could have generated the observables.□

The case of an unidentified discount rate

The previous discussion has assumed that the discount rate, β, is known. In effect, we are assuming that it is econometrically identified—all of our previous results would apply if β were not known from prior considerations but could be estimated by the use of an instrumental variable known to be orthogonal to the noise in the equation at hand. An example of such a variable in a macroeconomic setting is military spending, which is unlikely to be a response to noise.

A more challenging task is putting bounds on noise when the discount rate is not identified. When noise is present, the standard approach to estimation of β will fail. That approach is to use the set of variables known in the market at time t as instruments in the excess return equation,

$$P_t - D_t = \beta P_{t+1} + \eta_t + S_t - \beta S_{t+1} . \tag{3.24}$$

The approach will work only if $S_t - \beta S_{t+1}$ is zero or uncorrelated with the instruments.

Although the value of β cannot be known if the identifying condition fails, it is still possible to put a bound on the noise variance. Unfortunately, the approach we favored in the previous section fails if the discount rate is not known, because $P_t^* - P_t$ is no longer observable. Instead, we will apply the idea developed in the previous section of projecting the excess return on the X-variables, cumulating in reverse time, and then projecting again on the X-variables. This approach will be even weaker than before because the excess return will be measured with the biased estimate of β obtained by using all of the X-variables as instruments in equation 3.24. When the residuals from the instrumental estimates are projected on the X-variables, the projection is non-zero to the extent that the "overidentifying restrictions" fail. Note that the bound is vacuous if there is only a single X-variable.

Let μ_t be the residuals from estimating equation 3.24 by two-stage least squares with all of the X-variables as instruments. From earlier arguments, it is clear that the projection N_t of μ_t on $L_X(t)$ will provide a lower bound on the variance of $(1-\beta L^{-1})S_t$ because

$$M_X(t)\mu_t \quad = \quad M_X(t)(1 - \beta L^{-1})S_t \tag{3.25}$$

As before, let

$$\hat{S}_t = (1 - \beta L^{-1})^{-1} N_t \quad . \tag{3.26}$$

and let

$$\hat{S}_{t|t} \quad = \quad M_X(t)\hat{S}_t \; . \tag{3.27}$$

Then we have

Theorem 3.1. In the constant discount rate model with unknown β, the variance of the model noise satisfies the bound,

$$\sigma_s^2 \geq \sigma_{\hat{s}_{t|t}}^2. \tag{3.28}$$

The bound of theorem 3.1 is not sharp. The problem is that the projections have been constructed off of filtered observations of the model noise. As a result some information is lost. For example, as discussed in Theorem 2.5, if the model noise equals a rational bubble B_t then the bound generated by the procedure would be consistent with zero noise. In the unidentified discount case the lagged price term is the sole instrument capable of generating a consistent estimate of β. The more robust methods for measuring noise developed in the previous sections require the identifiability of β or, more generally, the identifiability of the coefficients needed to calculate P_t^*.

4. Inference with overlapping forecast errors

In numerous expectations based models, the data on forecasts and realizations are both available and observable, but the horizons for forecasts overlap. For example, every month there may be a k-month ahead forward exchange rate observation. As emphasized by Hansen and Hodrick [1980], this implies that the forecast errors ν_t will be MA(k). The techniques of the previous theorems are easily applied in this case. If the autocovariance function of the regression errors does not die out after k periods, then $u_{t|t-1}$ will equal the projection in the regression

$$[1-M_x(t)]S_t-\nu_t=\pi(L)[[1-M_x(t-k-1)]S_{t-k-1}-\nu_{t-k-1}] + \zeta_t \quad (4.1)$$

which permits the construction of the two step smoothing procedures outlined above. If P_{t-k-1} and P^*_{t-k-1} are elements of $L_x(t)$, then the regression estimate of the model noise is optimal.

5. Relation to Hausman class specification tests

The bounds we have developed bear a close relation to previous tests of expectational models in the literature. In particular, we focus on a Hausman class specification test developed by West [1987], which was originally applied to stock prices and has been subsequently used to analyze expectations based models of exchange rates (Meese [1986]) and hyperinflations (Casella [1988]; Casella's paper derived the test independently from West.) West's idea was to test the expectations based dividends stock price model by comparing the reduced form coefficients in the regression

$$P_t = \pi(L)D_t + \mu_t \tag{5.1}$$

with the coefficients predicted by the model

$$P_t^e = \sum_{i=0}^{\infty} \beta^i E(D_{t+i}|\Phi_t) \tag{5.2}$$

$$D_t = \gamma(L)\delta_t \tag{5.3}$$

$$E(D_{t+i}|\Phi_t) = \left(\frac{\gamma(L)}{L^i}\right)_+ \gamma(L)^{-1}\; D_t \tag{5.4}$$

where P_t^e denotes the fundamental stock price (which equals P_t absent noise) and δ_t denotes the Wold innovation in D_t. West tests the model specification by comparing the projections of P_t and P_t^e onto $L_D(t)$.

The projections of $P_t^* - P_t$ and $P_t^e - P_t$ onto $L_D(t)$ are by construction identical, because the two time series can differ only by the expectation error $P_t^* - P_t^e$, which is necessarily orthogonal to the current and lagged dividend series. By the arguments in the previous section, the West analysis is equivalent to a procedure which constructs a lower model noise bound based upon the information set $L_D(t)$. West's test therefore exhausts only a subset of the testable implications of the stock price model. Either the expansion of the information set or the decomposition of the regression errors to isolate AR(1) components will enhance the ability of the test to discriminate between the null and alternative hypotheses.

Our formulation also shows that the West test is not necessarily more powerful than a simple regression test in uncovering model noise. The reason for this is that the information set specified does not include P_t. This omission is quite serious if the specification error contains a unit root, as a projection of stock prices onto the dividend information set will generate sample error variances which diverge to infinity. In fact, when dividends represent the complete information set, specification test statistics will be inconsistent, as a modification of the arguments in Durlauf [1988] will easily show. This latter paper also shows that the use of dividends will also generate inconsistent test statistics if the alternative hypothesis contains an exploding rational bubble. The basis of these arguments can be seen when one observes that when the model error possesses infinite variance due to a unit root or an explosive bubble, the projection of $P_t^* - P_t$ onto $L_D(t)$ is no longer well defined. As a result, as the latter paper verifies, the associated hypothesis test statistics will not diverge.

These theoretical observations are in fact supported by the data. In a companion paper (Durlauf and Hall [1988]), we demonstrate that the entire history of dividends provides relatively little information on the nature of stock price noise, even when compared to the first lag of prices. In fact, excess volatility tests, which possess no power against local alternatives, will generate greater estimates of noise variance than dividend based tests. Our empirical research has found the history of prices extremely effective in capturing nearly all potential model noise.

6. Summary and conclusions

This paper has explored a number of issues in assessing the degree of misspecification or model noise in expectations based models. We have sought to provide lower bounds on the magnitude of misspecification in expectational models, as opposed to merely detecting its presence. Our approach has relied upon treating the model noise and market expectations as the objects of interest in a sequence of unobserved components or signal extraction problems. We were thus able to derive lower bounds on the variance of model noise consistent with the data. Consistency of the lower bound with zero is equivalent to the acceptance of the null hypothesis of correct specification.

By varying the information set available to the econometrican, we have been able to characterize different conditionally optimal lower bounds on model noise. These characterizations have shown how various specification tests have simple regression interpretations. In addition, we have shown how for constant geometric discount models, which apply to many issues in asset prices, the autocovariance structure of regression residuals possesses implications for the degree of specification. Our optimal bound results further permitted a characterization of all testable implications of the model. Interestingly, the ability of the econometrician to explain the past using the future does not assist in uncovering the degree of model noise. Formally, the optimal prediction and optimal smoothing estimates coincide for these models.

Finally, we have compared a number of different asset price tests in the literature in terms of power. We have found that the much maligned excess volatility test has good power against many alternative hypotheses, when compared to dividend based specification tests. The reason is simple. If a stock price contains slowly moving noise, the history of prices is an effective way of capturing the noise. Against some alternative hypotheses, dividends may contribute nothing to the implicit signal extraction problem.

Areas for future research, which we hope to pursue, fall into two categories. First, the determination of appropriate instrumental variables for permitting the future to explain the past can break the equivalence between the optimal predictor and optimal smoother we have discussed above. Our bounds could also be extended to cases where the nature of the potential misspecification is at least partially parameterized. Second, the extremely large model noise components we have found for stock prices naturally warrant separate examination. An important undertaking would attempt to relate the model noise estimates for aggregate stock prices to the model noise estimates in other asset markets as well as commodity and labor markets (as found by Hall [1988]) in order to begin to develop a more complete characterization of the limitations of current expectations based theories.

Appendix

Kalman Filtering Interpretation

The lower bound on the variance may be interpreted in a Kalman filtering framework. This interpretation also permits us to state an optimality result concerning the bound.

Information is structured as

$L_x(t)$ = Linear space generating information available to the econometrician at t.

$L_\phi(t)$ = Linear space generating information available to market participants at t.

$Y_{t|s}$ = projection of Y_t onto $L_x(s)$.

$\zeta_t = L_x(t) - L_x(t-1)$

We consider the mapping from the states to functions of the observables as

$$P_t - P_t^* = S_t - \nu_t \ . \tag{A.1}$$

$$P_t^* = P_t^e + \nu_t \ . \tag{A.2}$$

If the measurement error in the equation were white noise independent of the states, the optimal inference of S_t from the observables is the same as the optimal smoothing estimate of S_t in a Kalman filtering framework. To construct the optimal smoothing estimate, one can partition the available information into the mutually orthogonal subspaces $L_x(t)$ and $L_x(\infty)-L_x(t)$. The projection of S_t onto both the past, present and the future will generate an optimal estimate which consists of two orthogonal projections. This estimate is

$$S_{t|\infty} \quad = \quad S_{t|t} + \sum_{i=1}^{\infty} E(S_t \zeta_{t+i})\, Var(\zeta)^{-1} \zeta_{t+i}. \tag{A.3}$$

The optimal smoothing estimate thus projects S_t onto two orthogonal subspaces, one of which corresponds to the standard backwards regression projection. However, the terms in this expression containing the covariance of future observable innovations and current model noise in the optimal smoothing estimate are not identified, in that one does not know the values of $E(S_t \zeta_{t+i})$. One only knows the values of covariances of the imperfectly observed model noise with the observable different observable variables

$$E((P_t - P_t^*)\zeta_{t+i}) = E((S_t + \nu_t)\zeta_{t+i}). \tag{A.4}$$

In other words, using future observables to infer the current value of $P_t - P_t^*$ is not equivalent to predicting S_t because the use of future variables may capture the forecast error term ν_t as well as the unobservable state S_t. All admissable information for forming the optimal smoothing estimate must lie in $L_\phi(t)$. This means that consistent inferences on S_t can be made only by restricting the second projection in equation A.4 to that those vectors δ_t such that

$$L_\delta(t) = \{L_x(\infty) \; - \; L_x(t)\} \; \cap \; L_\phi(t) \; . \tag{A.5}$$

If there is no information on which elements of $L_x(\infty) - L_x(t)$ are known at time t, then the best estimate of the projection of S_t onto $L_\phi(t)$ will equal the projection onto $L_x(t)$, and Theorem 2.1 is verified. Alternatively, suppose that P_t^* fulfills the constant discount model and that forecast errors are orthogonal to $L_D(t-1)$, so that the filtered forecast error $\beta\nu_{t+1-i} - \nu_{t-i}$ is white noise. No such transformation of the forecast errors is observable under the alternative, however, because the price variable is contaminated by the model noise. Since the observables do not permit the model noise and the forecast to both be observed in isolation, these terms are not elements of $L_x(\infty)$. However, the set of lagged excess returns r_{t-i} defined in the text will be both observable to market participants and to the econometrician. Decomposing r_{t-i} as

$$r_{t-i} = M_x(t) r_{t-i} + \; [M_x(\infty) \; - \; M_x(t)] r_{t-i} \; , \; \; i > 0 \tag{A.6}$$

permits us to map the r_t variables into the smoothing decomposition, exclusively employing data which are observable to the econometrician. The only component of the available variables dated after t which may be used as estimates to extract the model noise are those which are orthogonal to the forecast error. The r_t variables are the only variables which fulfill this requirement. This is the sense in which the model noise estimate $S_{t|\infty}$ is an optimal smoothing solution. These arguments verify that Theorem 2.2 generates a bound that is consistent with the data.

A converse result also exists, suggesting that this is the largest possible bound which can be inferred from the data. The restrictions imposed by the covariance structure of the observable variables restricts the set of admissable structural models which could have generated the data. There exists a structural relationship between the model error estimate $S_{t|\infty}$ and the D_t process such that this estimate of the model error variance is equal to the true model error. Therefore, it is possible to construct an example of a structural model which is consistent with the data and possesses a model error variance equal to $\sigma^2_{s_{t|\infty}}$. The existence of such a model means that the bound is maximal. Intuitively, this model follows from the observation that it is possible for the variance of the model error residual which is not captured by the smoothing estimate to be zero.

Formally, the structural relationship between states and observables where the filter is perfect is a modification of the so-called innovations representation of the Kalman filter which was developed by Kailath [1968] and Son and Anderson [1973] and is discussed extensively in Anderson and Moore [1979]. A colored noise state space model which is consistent with the lower bound on noise variance coinciding with the actual noise variance is:

$$P_t - P_t^* = S_{t|\infty} - \nu_{t|\infty} \tag{A.7}$$

$$P_t^* = P^e_{t|\infty} + \nu_{t|\infty} \tag{A.8}$$

$$S_{t|\infty} = \alpha S_{t-1|\infty} + e_t \tag{A.9}$$

$$P^e_{t+1|\infty} = \beta^{-1} P^e_{t|\infty} - D_t + \beta^{-1}(1 - \beta L^{-1})^{-1} \nu_{t|\infty} \tag{A.10}$$

where α and e_t are implicitly defined. The reader may verify that this state space model fulfills all of the orthogonality restrictions in the text. Finally note that if current price and dividend variables are included in the information set at t, this will render r_t an element of $L_x(t)$ and render the optimal predictor and optimal smoother identical, which is Theorem 3.4 in the text.

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