WORKING PAPER
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FIRM RESPONSE TO PRICE UNCERTAINTY:
TRIPARTITE STABILIZATION
AND THE WESTERN CANADIAN CATTLE INDUSTRY

(Working Paper 2/90)

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ABSTRACT

This paper examines the optimal response of individual firms in the Western Canadian cattle industry to a participatory stabilization scheme. The type of stabilization scheme analyzed here is a stylized version of the National Tripartite Price Stabilization Program (NTPSP). The theoretical approach employs a static partial equilibrium model of firm behaviour under uncertainty, where competitive risk-neutral firms employ a two-stage production technology. The principal empirical findings are that a marginal increase in the size of a firm's breeding herd will increase the short-run own-price elasticity of supply. Accordingly, the optimal response of a risk-neutral firm to a mean-preserving decrease (increase) in the variability of output prices would be a reduction (increase) in the size of the breeding herd. These adjustments would be associated with enrolment in (or withdrawal from) a participatory stabilization scheme which is actuarially fair. These findings support the proposition that risk-neutral firms may keep larger herds and produce more cattle during periods of greater price variability and challenge the suggestion that all stabilization programs which affect cattle production necessarily increase it.

This paper is based on part of a doctoral dissertation written for the Department of Economics at Queen's University, Kingston.

Key Words: uncertainty, agricultural prices, economic stabilization
FOREWORD

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I. INTRODUCTION

The instability of output and input prices in the Canadian agriculture sector has prompted several distinct types of government intervention in recent years. This paper focuses on the optimal response of individual firms in the Western Canadian cattle industry to a participatory stabilization scheme such as the National Tripartite Price Stabilization Program (NTPSP).

From a theoretical and an empirical perspective, an unresolved question has been whether participatory stabilization schemes might increase output, in particular through shifting outward the supply curve of the firm. The general objective of this paper is to examine this aspect of firm response to price uncertainty, and to public initiatives which reduce the perceived variability of prices.

The line of inquiry pursued in this paper is sketched in the two parts of Figure 1. It has been suggested that some features of the NTPSP make it similar to a deficiency payment scheme with voluntary participation by individual producers. The operation of the NTPSP can be portrayed as in Figure 1.a, where $S_t$ and $D_t$ are market supply and demand curves at a point in time, t. (A number of alternate assumptions is possible about the shapes of these curves and about the source and nature of output price uncertainty). The market-clearing quantity and price would be $(Q_0, P_0)$ in the absence of an operative stabilization (or deficiency payment) program. However, if $P_s$ is known to be the support price under an industry program, then firms would be assured of receiving $P_s$ per unit output and would supply $Q_1$. This implies that market price would fall to $P_1$, and that producers would claim $(P_s - P_1)$ per unit output as a stabilization payment from the program administrators. (Total payments would be given by the rectangle of dimensions $(P_s - P_1) \cdot Q_1$.) Under such a program, the quantity which the market would choose to supply at any time, t, would be $Q_1$ unless price were above $P_s$, in which case $S_t$ would portray the desired price-quantity pairs.

Some analysts have suggested that this type of analysis misses some potentially important effects of these programs. For example, Martin and Goddard (1987, p.6) suggest that the supply curve, $S_t$, (and the vertical portion at $Q_1$) might shift to the right when such a stabilization scheme is
Price

\[ P \]

\[ D_t \]

\[ S_t \]

\[ P_S \]

\[ P_0 \]

\[ P_1 \]

0

\( Q_0 \)

\( Q_1 \)

Industry Output

Figure 1.a - Price Stabilization through a Deficiency Payment Program

Price

\[ S(\bar{K}_1, P) \]

\[ S(\bar{K}_2, P) \]

\[ P_S \]

\[ q_1 \]

\[ q_2 \]

Firm Output

Figure 1.b - Short-Run (Restricted) Supply Curves for the Firm with Different Levels of Investment in the Fixed Factor, \( K \)
established because:
- risk-averse producers may produce more in response to reduced market risk (for example, risk-averse firms with marginal costs below $P_s$ may expand),
- new entrants may join the industry while firms which would have exited without the program, remain in production.

Other analysts, such as Choi and Johnson (1986) (citing Hartman (1986)), suggest that firms may have chosen higher output levels under uncertainty (even if risk-neutral or risk-averse) through some affinity for variability of output prices. If these firms were now exposed to less output price variability, their planned output levels would fall shifting the supply curve in the opposite direction.\(^1\) The possibility of firms increasing output as a response to price uncertainty initially appears counter-intuitive, and it begs such questions as what response firms might have to stabilization programs and whether the programs have effects which are intended.

Figure 1.b illustrates the source of such a positive output response (to price uncertainty) at the firm level, for firms which have a two-stage decision-making process. It employs the construct of a short-run or restricted supply function for firms which follow a two-stage decision-making process. In stage one, firms choose the level of a "fixed" or predetermined factor, $K$, based on its price and on expectations about the (uncertain) output price. In stage two, the output price, $P$, is revealed, and firms vary the output level through choice of the quantity of variable factors to use. The firm's desired (or optimal) quantity to supply, once its first-stage commitment is made, is shown by its short-run supply curve, $S(K,P)$. Except in the special case of perfect complementarity or substitutability of fixed and variable factors, these curves will not be vertical or horizontal (respectively) but will have a positive short-run own-price elasticity of supply. That is, if output price is higher or lower than expected, there will be some ability to adjust output to it in the short run, even though the firm has already committed itself to using amount $K$ of the fixed factor.

\(^{1}\)This begs an explanation for the voluntary participation of such firms in the program. A sufficient explanation is an expectation by them of eventually receiving a net transfer of income through participation, such as when the program offers a better return than a fair bet.
It is hypothesized that for some firms, their short-run supply curves will be as shown in Figure 1.b. These curves have the important feature that higher levels of use of the fixed (or predetermined) factor, $\bar{R}$, cause the short-run supply curves to become more price-elastic (flatter). Equivalently, investments in the predetermined asset are investments in increased ex post production flexibility. When firms such as these encounter increased variability of output prices, they face an incentive to increase their use of the predetermined input, in order to gain greater production flexibility in stage two (when output price is revealed). As a result, there will be an increase in the planned level of production (except in the special case where there is no monotonic relationship between use of the predetermined input and the level of output).

As will be shown, it is the firm's production technology (as characterized by the interaction of fixed and variable factors in a firm's production function) which determines whether the firm's short-run supply curves will be as illustrated in Figure 1.b. Other possibilities are that the curves are as drawn, but that $\bar{R}_1$ is greater than $\bar{R}_2$, in which case a reduction in the level of use of the predetermined factor has the effect of increasing production flexibility. It is also possible that for some firms the short-run supply curve would be invariant to any (positive) level of use of the fixed factor.

The approach followed in his paper to address these issues will be (i) to develop a theoretical model of firm behaviour in the presence of a stabilization scheme, (ii) to propose an econometric framework within which firm behaviour can be examined, and (iii) to prepare econometric estimates using recent firm-level data from which one can draw inferences about optimal firm response. These steps are described in Sections II through V.

The theoretical approach employs a static partial equilibrium model of firm behaviour under uncertainty, where competitive risk-neutral firms employ a two-stage production technology. A participatory stabilization scheme is introduced, where the effect of such a scheme on returns is mean-preserving by design. The comparative static effects of the scheme under these assumptions are those which obtain in more general models of firm behaviour under uncertainty. The direction of optimal firm output response is shown to be an
empirical question which will depend on specific characteristics of the two-stage production technology.

The empirical analysis is notable for its use of a panel micro-data set, constructed for this study, which describes in considerable detail the receipts, expenditures, and factor use of thousands of individual firms over time. These data were assembled from the Statistics Canada National Farm Survey for the years 1983 through 1987, and cover the three Prairie provinces plus the Peace River region of British Columbia. A flexible functional form (a modified translog) is employed to represent the restricted profit function of the firm from which net output equations are derived and estimated.

Section VI interprets several aspects of firm behaviour which may be inferred from the econometric estimates. These include short-run elasticities of supply and factor demand and the specific influence of the level of fixed factors on these short-run elasticities. The values obtained are related to optimal firm response to price variability, and to participation in a specific type of stabilization scheme.

The principal empirical findings are that many of the short-run supply and factor demand responses of firms are elastic with respect to within-season price variability. The evidence does not support a vertical or negatively-sloped short-run supply curve for cattle. Further, a marginal increase in the size of a firm's breeding herd will increase the short-run own-price elasticity of supply. Accordingly, the optimal response of a risk-neutral firm to a mean-preserving decrease (increase) in the variability of output prices would be a reduction (increase) in the size of the breeding herd. These adjustments would be associated with enrolment in (or withdrawal from) a participatory stabilization scheme which is actuarially fair.

Although these empirical findings are not formally tested as statistical hypotheses, they provide empirical support for the proposition that risk-neutral firms may keep larger herds and produce more cattle during periods of greater price variability. This challenges the suggestion that all stabilization programs which affect cattle production necessarily increase it.
II. A MODEL OF FIRM BEHAVIOUR IN THE PRESENCE OF A STABILIZATION SCHEME

The principal effect of stabilization schemes to be examined here is the way in which they will alter the profit-maximizing choices of individual firms. In order to isolate the critical features of the stabilizing influence of these programs and to show what determines response at the firm level, a number of simplifying assumptions will be made. In particular, it will be assumed that firms are risk-neutral and that the stabilization schemes under consideration are designed so as to influence the dispersion of market returns but not their mean.

The operational details of the NTPS programs are provided in a series of publications which includes Agriculture Canada, Agricultural Stabilization Board (1987a, 1987b, 1987c, 1988, 1989) and Tan (1988). Other aspects of Canadian price stabilization policies are covered by Hassan and Huff (1985), Cluff and Huff (1986) and Van Kooten, Spriggs and Schmitz (1989).²

Two principal differences between the NTPSP and the stabilization scheme characterized in the following analysis are that the NTPSP is not self-financing (that is, it does not provide actuarially fair insurance against market risk) and that the NTPSP has separate program components which cover cow-calf and slaughter cattle production.

A. A MODEL OF OPTIMIZING BEHAVIOUR WITH RISK NEUTRALITY

Assume the representative agent is a risk-neutral firm which views output price as a random variable. The firm employs two factors which may be denoted capital, K, and labour, L, and which have the following properties. Capital decisions must be made before a random output price is revealed, and, once made, represent sunk costs to the firm. Labour decisions are not binding until after the output price has been revealed, but before production is in fact completed. Production is a single-period process, with capital decisions made before the start of a period and labour inputs chosen at the start of the

To illustrate the application of the model, consider a cow-calf producer. The breeding herd at year end represents an irreversible capital commitment. Labour corresponds to the feed, labour, and other variable inputs which will be used during the period up until weaning the next fall. This captures the idea that if feeder calf or slaughter cattle prices should unexpectedly change, one could vary only such things as the length of time the animals were on farm, the ration, or the weight at which one intended to sell the animals, treating as fixed the number of animals. One could not immediately acquire additional cows or calves to feed in mid-winter or spring if a more favourable selling price were revealed. Critical to what follows is the possibility that it may be relatively costly to increase marketable output ex post by increasing use of a variable input (such as feed) alone. With hindsight, it may have been less costly to produce the increased output by keeping (ex ante) a larger number of animals over winter. The question is whether the cost savings (additional profit) which would result if output prices should unexpectedly rise, sufficiently offset the potential costs of being caught with "too many" animals if prices should unexpectedly fall.

1. Introduction of the Stabilization Scheme

The stabilization program is characterized as follows. The firm which has chosen to enroll in a participatory stabilization scheme pays a premium, $R$, per unit output for the full duration of some subscription period. In return, the firm receives a deficiency payment per unit output for any output sold when the market price, $p$, is below the support price, $p_s$.

Under such a scheme the net payment or benefit, $S$, faced by the firm, per unit output, is given by:

$$ S = \begin{cases} (p_s - p) - R & \forall \; p \in (0, p_s] \\ 0 - R & \forall \; p \in (p_s, \infty) \end{cases} $$

It will prove convenient to assume that $(p_s - R) > 0$. That is, the exogenous support price which triggers any payout under the scheme is strictly greater than the per period premium which is paid. Equivalently, this implies that in periods where the insured event occurs the insurance indemnity or payout is
sufficient that, when combined with the market price, it at least covers the cost of the insurance premium in that period. It follows that \((p + S) > 0\). A firm's total return from market plus indemnity (stabilization payment) will be strictly positive, even after premiums are paid. Figure 2 shows the relationship between total market return and market price for a participatory stabilization program with a positive premium, \(R\), under the assumption \((p_s - R) > 0\).

Assume there is a random output price, such that \(p \sim F(p)\), where \(F(p)\) is the continuous cumulative distribution function of \(p\), \((p > 0)\) with \(E[p] = \bar{p}\). This implies, by (1), that net payments, \(S\), are also random. The expected net stabilization payment, \(S\), becomes:

\[
E[S] = \int_{0}^{p_s} (p_s - p - R) dF(p) + \int_{p_s}^{\infty} (0 - R) dF(p) \quad \text{or} \quad E[S] = -R + \int_{0}^{p_s} (p_s - p) dF(p).
\]

Expected total market return, \(E[p + S]\), can be expressed as:

\[
E[p + S] = (p + S) - R + \int_{0}^{p_s} (p_s - p) dF(p)
\]

which will be strictly positive. Assuming that the values \(p_s\) and \(R\) are known with certainty before any input decisions are made—especially the decisions to participate in the scheme or to commit specific capital—then the value of total market return will also be random but will follow a different distribution than \(p\).

The effect of incorporating a net stabilization benefit, \(S\), into the firm's decision making is similar to redefining the price it expects to receive from \(E[p]\) to \(E[p + S]\) per unit output, given the assumptions made about \(S\). There may be an effect on the firm's behaviour associated with both the expected value and the dispersion of the total market return, \((p + S)\). Most of the analysis which follows will focus on the latter effect.
Figure 2. Total Market Return With Stabilization as a Function of Market Price

The diagram shows the relationship between total market return and price for a participatory stabilization program with a positive premium, R, under the assumption \((p_s - R) > 0\). The 45 degree line facilitates comparison of total market return under stabilization with market price for any production period.
2. Determination of an Optimum for the Firm

Since the relevant production environment employs two-stage decision making, and because the net benefit of the stabilization scheme can be characterized as shown here, it will prove convenient to apply an existing analytical framework to questions of firm behaviour. One appropriate approach has already been established in the literature for the firm with production flexibility under price uncertainty. In particular, the analysis and results of Hartman (1976), which were independently developed and generalized by Epstein (1977, 1978), may be directly applied to some aspects of the stabilization problem.³

In the section which follows, Hartman's analysis is applied to the determination of optimal firm response under specific assumptions about expected net payout of such stabilization schemes. The results shown here closely follow the presentation of Hartman (1976), which illustrates general comparative static effects of price uncertainty in the absence of any type of stabilization program. These results are presented here principally for completeness and to frame the empirical approach. To foreshadow the question of appropriate functional form, illustrations of the comparative static results are tabulated for some common functional forms.

A two-stage optimization process is employed, solving the second stage first. The solution or optimal level of the variable input, labour, L, is defined in terms of any positive values of \((p + S)\) and K which might occur. These values will be known with certainty when the choice of L is actually made. Solving the first stage next provides the optimal level of K by taking into account price expectations and the optimal adjustment in L to follow.

Assume that input prices and the production technology are known in advance with certainty (i.e., there is a non-random production function). The

³This research follows a line of inquiry started by Turnovsky (1973) and contributed to by Wright (1984), Hiebert (1984) and Devadoss and Choi (1987).
The firm's problem is to:

\[(4)\quad \text{Maximize:} \quad E[n(f(K,L))] = E[(p + S)f(K,L) - wL - cK] \quad \{K,L\}\]

where output, \(q = f(K,L)\) and where \(f\) is strictly concave.\(^4\)

The second stage of the maximization problem is given by:

\[(5)\quad \text{Maximize:} \quad \hat{n} = (p + S)f(K,L) - wL. \quad \{L\}\]

Solution of (5) gives \(L^* = L^*(K,p,w,S)\).\(^5\) The maximized short-run profit function \(\hat{n}\) when \(L^*\) is employed will be denoted by \(g(K,p,w,S)\), where:

\[(6)\quad g(K,p,w,S) = (p + S)f(K,L^*) - wL^*.\]

In the first stage of the decision process, the risk-neutral firm will choose \(K\) to maximize the expected value of current profits, where the value of current profits is contingent on also choosing the optimal level of variable inputs, \(L^*\). That is:

\[(7)\quad \text{Maximize:} \quad E[\pi] = E[(p + S)f(K,L^*) - wL^*] - cK \quad \{K\}\]

\[(8)\quad \frac{\partial E[\pi]}{\partial K} = E[(p + S)f_1(K,L^*)] - c = 0, \quad \text{or}\quad \text{(8')}\]

\[E \left[ \frac{\partial g(K,p,w,S)}{\partial K} \right] = c.\]

The assumption of strict concavity of \(f\) ensures that \(K^*\) will yield a maximum for \((p + S) > 0\). Firms choose capital such that the certain cost per unit,

\(^4\)Strict concavity of the production function \(f(K,L)\) implies \(f_{11}(K,L) < 0,\) \(f_{22}(K,L) < 0,\) and \(f_{11}f_{22} - f_{12}^2 > 0.\)

\(^5\)For \(L^*\) to provide a maximum for \(\hat{n}\), it is sufficient that \(\partial^2\hat{n}/\partial L^2 < 0,\) where \(\partial^2\hat{n}/\partial L^2 = (p + S)f_{22}(K,L).\) By assumption \((p + S) > 0\) and by the assumption of strict concavity \(f_{22}(K,L) < 0,\) which together imply \(\partial^2\hat{n}/\partial L^2 < 0\) and short-run profit maximization.

\(^6\)Equation (8) makes use of the necessary condition that \((p + S)f_2(K,L^*) = w,\) to eliminate terms with \(\partial L^*/\partial K.\)
Figure 3. Expected Value of Marginal Product as a Function of Capital Use
3. Response to a Mean-Preserving Reduction in the Dispersion of Expected Returns

Consider how the optimal choice of K and L will vary in the presence of a stabilization scheme (which provides the net payment, \( S \)) compared to the situation without such a scheme (setting \( S = 0 \)). One way to proceed is to examine the special case where the value of the premium, \( R \), is set such that the program is implemented as a mean-preserving reduction in the dispersion or spread of the total market return. This will require choosing \( R \) such that \( E[S] = 0 \), or:

\[
E[p] = E[p + S]
\]

\[
\bar{p} = (\bar{p} - R) + \int_0^{p_S} (p_S - p)dF(p)
\]

\[
R = \int_0^{p_S} (p_S - p)dF(p)
\]

The value of \( K^* \) when \( R \) is set as in (9) such that \( E[S] = 0 \), can be contrasted with the case where \( S = 0 \) in the absence of any program. In particular, changes in \( E[\partial g/\partial K] \) can be evaluated (for a given level of capital stock) when the distribution or spread of total market returns is narrowed in a mean-preserving way.

As illustrated in Figure 4., if \( \partial g/\partial K \) is strictly convex in total market returns, \((p + S)\), then narrowing the dispersion of \((p + S)\) in a mean-preserving way will lower the value of \( E[\partial g/\partial K] \) for any level of \( K \). Conversely, if \( \partial g/\partial K \) is strictly concave in total market returns, the value of \( E[\partial g/\partial K] \) will rise. It will be invariant to the change if \( \partial g/\partial K \) is linear in \((p + S)\). This is an application of the Rothschild and Stiglitz (1970) result.

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7 This is equivalent to offering producers an insurance policy against market risk which is actuarially fair and where any administrative costs associated with the program are covered by government contributions.
Figure 4. $\frac{\partial g}{\partial K}$ as a Convex Function of Total Market Return
that the expected value of a convex (concave) function increases (decreases) as its argument undergoes a mean-preserving spread.

If $E[\partial g/\partial K]$ should fall for a given level of $K$ as a consequence of narrowing the spread of (i.e., stabilizing) total market returns, then the value of $K^*$ which satisfies (8') must also decrease. This would appear as a leftward shift of the curve shown in Figure 3.

More generally, when there is reduced uncertainty about the value of $(p + S)$ caused by a stabilization scheme which satisfies (9), then its effect on the desired capital stock, $K^*$, will depend on the sign of $\frac{\partial^3 g}{\partial K \partial (p + S)^2}$. If this value is strictly positive, then $E[\partial g/\partial K]$ is convex in total market return, and the optimal capital stock will be lower. To characterize the effect of the stabilization program on $K^*$, therefore, one needs to characterize the conditions under which

$$\frac{\partial^3 g}{\partial K \partial (p + S)^2} > 0.$$

Under the stated assumptions, it can be shown that $\frac{\partial^3 g}{\partial K \partial (p + S)^2}$ will have the opposite sign to $\frac{\partial (f_{12}/f_{22})}{\partial L^*}$ which itself could take either sign given the set of assumptions which has been made about the production function. That is:

$$(10) \quad \frac{\partial^3 g}{\partial K \partial (p + S)^2} > 0 \quad \text{as} \quad 0 > \frac{\partial (f_{12}/f_{22})}{\partial L^*}.$$

4. Illustration of Comparative Static Effects

It is clear from the foregoing and from (10) that optimal adjustment of capital use to the introduction of a stabilization scheme will depend on the degree of factor interdependence exhibited by the production function.

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8 In evaluating the properties of $\partial g/\partial K$, as in Figure 4, the sign of its first derivative with respect to total market return, $\partial^2 g/\partial K \partial (p+S)$, will sign the slope of $\partial g/\partial K$, whereas the sign of the second derivative with respect to total market return will indicate its curvature.
Table 1 shows how this result can be interpreted when the production function takes each of four common functional forms.\(^9\) The direction of change of the optimal values of \(K^*\) and \(L^*\) is shown when the introduction of a stabilization scheme causes a mean-preserving decrease in the dispersion of the total market return, \((p + S)\).

For a generalized Cobb-Douglas production function, there will be a decrease in \(K^*\) and \(L^*\) provided the production function is strictly concave. For a transcendental production function (of which the Cobb-Douglas is a special case), either sign is possible. A tighter set of parameter restrictions than those needed to ensure strict concavity would have to hold to determine the direction of adjustment in \(K^*\).

With a quadratic production function, the expressions given in (10) both equal zero. Therefore, \(E[\partial g/\partial K]\) will be invariant to changes in the price spread, so, for a risk-neutral firm, capital and labour decisions would not be affected by the introduction of this type of stabilization scheme.

The generalized Constant Elasticity of Substitution (CES) production function is consistent with either response in \(K^*\), depending on the values taken by the exponential parameters, \(\beta\) and \(\mu\). However, the optimal value of \(L^*\) will generally fall (and will never increase) provided the assumption of strict concavity is maintained. In the special case of \((\beta + \mu) = 0\) or \(f_{12} = 0\), input use will be invariant to the price spread.\(^{10}\)

Hartman (1976) shows for a CES production function which is homogeneous of degree \(\mu\), that if the elasticity of factor substitution is above some critical level, \(\sigma^\alpha, \left(\sigma^\alpha = \frac{1}{1-\mu}, \mu \neq 1\right)\), then firms will respond to reduced uncertainty by increasing \(K\). Equivalently, these are firms for which capital use is reduced and labour input increased when output price uncertainty increases. Intuitively, for production processes which exhibit a sufficiently

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\(^9\) The properties of these and other commonly-employed functions are reviewed by Beattie and Taylor (1985) and Debertin (1986).

\(^{10}\) It can be shown that for the strictly concave CES production function, \(f_{12}\) will have the same sign as \((\beta + \mu)\), whereas \(\partial(f_{12}/f_{22})/\partial L^*\) will have the opposite sign.
Table 1: Comparative Static Effects of Reduced Uncertainty

<table>
<thead>
<tr>
<th>FUNCTIONAL FORM</th>
<th>CONDITIONS</th>
<th>RESPONSE IN OPTIMAL LEVEL OF</th>
<th>CAPITAL</th>
<th>LABOUR</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) Cobb-Douglas</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>[ Q = K^{1-a}L^{b} ]</td>
<td>Strict Concave</td>
<td>Falls</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Strict concavity requires: ( 0 &lt; b_1 &lt; 1, (i = 1,2) ) and ( 0 &lt; (b_1 + b_2) &lt; 1 )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(2) Transcendental</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>[ Q = K^{a_1}e^{b_1}L^{a_2}e^{b_2} ]</td>
<td>Either result possible depending on parameter restrictions</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(3) Quadratic</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>[ Q = a_1K + a_2L + \frac{1}{2}b_1K^2 + \frac{1}{2}b_2L^2 + b_3KL ]</td>
<td>Input use is invariant to the price spread</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(4) Constant Elasticity of Substitution</td>
<td>( \beta + \mu &gt; 0 )</td>
<td>Falls</td>
<td></td>
<td></td>
</tr>
<tr>
<td>[ Q = \left[ aK^{-\beta} + bL^{-\beta} \right]^{-\mu/\beta} ]</td>
<td>( \beta + \mu = 0 )</td>
<td>Constant</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Strict concavity requires: ( a &gt; 0, b &gt; 0, \beta &gt; -1, \beta \neq 0, 0 &lt; \mu &lt; 1 )</td>
<td>( \beta + \mu &lt; 0 )</td>
<td>Rises</td>
<td>Falls</td>
<td></td>
</tr>
</tbody>
</table>

Note:

The last column shows the response in the optimal level of input use to a mean-preserving decrease in the dispersion of total market return (output price plus net stabilization payment). For a risk-neutral firm, a two-stage production process takes one of four common functional forms.
high degree of factor substitutability, firms will commit less capital \textit{ex ante} and substitute more labour \textit{ex post} in the case of output price uncertainty. Introduction of a stabilization scheme removes the incentive to make such adjustments when there is a high degree of factor substitutability.

If it is optimal for a firm to adjust its use of capital when it perceives more or less variability in total market returns, this adjustment can have two components. One component (a scale effect) involves changing the planned level of output (which might result in changes in the level of use of either or both inputs). The other component of this adjustment may involve substituting variable inputs for capital inputs to take advantage of the inherent flexibility in production in this two-stage production environment.

The essence of the arguments presented here is that, given a sufficiently high degree of factor substitutability in production, firms will commit less capital \textit{ex ante} and employ more labour \textit{ex post} when faced with (some or increasing) output price uncertainty (Hartman, 1976, p.678). In the extreme case of perfect factor substitutability, the firm's problem disappears because there is no call for any commitment of capital \textit{ex ante}; labour can be deployed equally efficiently once all price uncertainty is resolved. At the other extreme of no possible factor substitution \textit{ex post} (perfect complementarity), the results are those of the single-stage production process: output price uncertainty has no effect on the production decisions of risk-neutral firms.

When there is more than one fixed factor, then it is also necessary to examine whether they are gross substitutes or complements. Where increased use of one fixed factor in response to price uncertainty causes short-run behaviour to become more flexible, then there may also be an increase in the level of use of fixed factors which are gross complements with it (Epstein, 1978, p.257).

**B. SUMMARY**

The results presented here rely heavily on the assumption that production is a single-period process with production flexibility due to two distinguishable phases of input use or commitment. Some input decisions represent irreversible commitments made during a period of price uncertainty, whereas other input decisions can be varied even after an uncertain output
price is revealed.

The possibility of optimal input use changing in either direction contradicts results generated by models which describe single-stage production processes. It also challenges the notion that all stabilization programs which affect production will necessarily increase it.

III. THE EMPIRICAL MODEL

The focus of this section is on identifying an implementable model with which to measure and test aspects of the short-run behaviour of firms which produce cattle in Western Canada. 11

A. CHOICE OF A MEASURE OF RESTRICTED PROFIT

In choosing a measure of restricted profit, both the breeding herd and available land area will be treated as fixed factors of production whose level of use is constant within a production period. There is little scope in cattle production for endogenous short-run adjustments in the stock of this capital, although inventory adjustments may play a role.

Some general conclusions may be drawn about the definition of restricted profit. First, restricted profit should be defined within a single production year, so that factors such as the breeding herd and available land area are constant. In the case of farm land area, the assumption of a constant within-season level of use is intended to provide a general representation of actual practice. In the case of the breeding herd, it is acknowledged that, in practice, firms may complete the production season with more or fewer animals than they held at the start, so that the assumption is a simplification of the actual behaviour.

11 A number of studies have been reported which examine the production structure of various parts of the Canadian agricultural sector, employing cost and profit function approaches. Examples include Lopez (1980a, 1980b, 1984), Gordon (1984), Adamowicz (1986), Moschini (1988a, 1988b) and Cloutier and Wesa (1988). The research reported by Gordon examines the production structure of the cattle industry in Western Canada, but it does not employ a functional form which is sufficiently flexible to test the hypotheses advanced here about response to output price uncertainty. The Gordon study also differs from this one in that it does not employ directly firm-level (micro) data.
Second, the definition of restricted profit should reflect the inventory decision made by the firm in each period. Specifically, the decision by firms about the portions of the total output plus inventory on hand to sell and to carry over for sale in subsequent periods will be influenced by firms' expectations of future prices. Adjustments in the inventories of all classes of animals should be reflected in the measure of restricted profit. In other words, given an opening stock of capital, which may in practice incorporate available land area and all cattle inventories (including the breeding herd), firms maximize the value of all produced output less expenditures on variable inputs.

For the purpose of implementing a model of short-run behaviour, it will be sufficient to employ a general definition of restricted profit (\( \bar{\pi} \)) which incorporates the features described above. Consider:

\[
(11) \quad \bar{\pi}(p,w;K) = \max_{\{y,z\}} \left\{ p'y - w'z; \, (y,z;K) \in T \right\}
\]

where \( y \) is a vector of outputs which can be produced using the amounts of variable inputs, \( z \), and fixed factors, \( K \), given that \( T \) is the production possibilities set.

The interpretation of the vector of outputs in this case will need to encompass all cattle production, distinguishing between that which is sold and that which is produced but carried over for sale in a subsequent period. Cattle sold in the current period will be valued at current market prices, whereas cattle held in inventory for future sale will be valued according to the (discounted) expectation of future prices. To reflect accurately the nature of joint production by these firms in Western Canada, other outputs such as crops can also be represented.

Hotelling's Lemma can be applied to a restricted profit function described in this way, which will yield the short-run output supply curves and short-run derived input demand curves of the firm.

B. FORMATION OF EXPECTATIONS ABOUT FUTURE CAPITAL VALUES

Estimation of the expected dollar value in future periods of the end-of-period herd size or inventory is critical to measuring accurately the level of restricted profits when these may incorporate accruals or reductions to a
given herd. When cattle is the capital asset in question, some forward or futures market prices may be available upon which to base such expectations. However, as Gordon notes (1984, p.92), futures market prices are not available for all classes of cattle, and they are based on United States markets and currency, which introduces an exchange rate uncertainty to their application in Canada.

Gordon’s solution, and that of Epstein (1977) and others before them, is to hypothesize that each firm’s expectations of future prices are precisely those defined by the predictions of a univariate time-series model. Following the methods of Box and Jenkins (1970), one can fit an autoregressive integrated moving average (ARIMA) model to a time series of prices, and use such a model for prediction purposes.

C. TREATMENT OF CROSS-SECTIONAL AND TIME-SERIES EFFECTS

The data available for analysis provide a multivariate statistical history for a large number of individual farms, although the time period covered for specific farms varies from one to four years. There exist a number of alternative ways of employing these data, such as to estimate behavioural relationships for individuals or to summarize these relationships for a number of individuals.

The approach which will be followed in this study employs a model which is known alternatively as the analysis-of-covariance model, the least-squares-with-dummy-variables (LSDV) model, or the fixed-effects model.) The model is characterized by the feature that the intercept term in each equation to be estimated is allowed to vary spatially and temporally, whereas the vector of slope coefficients is held constant. The model aggregates in this way the behaviour of individuals from a number of regional cross sections over four years. The size of these cross sections varies from year to year. Such a model is thought appropriate to this analysis because the questions of interest concern the behaviour of short-run (restricted) profit and its determinants within a particular production period, and do not concern the intertemporal or dynamic adjustment behaviour of individuals or groups.

This approach measures or values the behaviour of a specific firm in any two years the same way it treats the behaviour of two unrelated firms in the
same year and location. No significance attaches, on average, to continuity of management or to adaptive behaviour of any sort. To the firm, each year is assumed to be the same, except that it starts the year with a new endowment of resources. Of these, only the cattle breeding herd and available land area are deemed significant to its specific short-run production decisions, given that it always faces the same market prices as its neighbours.

The short span of years serves to support the necessary assumption of common responses, especially if the years 1983 through 1986 were relatively similar in terms of external influences on the behaviour under study. For example, casual observation suggests that a period of peak nominal interest rates and labour unemployment preceded this interval, and a period of extreme drought conditions and crop failures followed. Clearly, a number of important influences, such as levels of world grain prices, inflation and interest rates did change during the study period, but these effects are incorporated, for the most part, in the price data included in the analysis. 12

D. FUNCTIONAL FORM FOR THE RESTRICTED PROFIT FUNCTION

It has been shown that for a risk-neutral firm with a two-stage production process, the optimal adjustment to increased price uncertainty will depend on the degree of substitutability or complementarity of various input pairs. Further, when using a dual approach these characteristics may only be identifiable in the third derivatives of a firm's restricted profit function. Accordingly, the functional form which is proposed should express higher-order terms associated with these derivatives, and should not arbitrarily impose restrictions on the substitutability or complementarity of input pairs. Ideally, such a functional form would also readily lend itself to econometric estimation. Although a functional form is said to be flexible if it can approximate an arbitrary twice continuously differentiable function to the second order at a point, it will here be necessary to consider a functional form which is flexible to the third order in some terms. Epstein (1980,

12 High interest rates will be internal to the analysis when reflected in input prices, output prices, and discount rates. They might be external if they are accompanied by credit rationing or other constraints imposed by imperfect capital markets.
Recall that the arguments of a restricted profit function are the prices (p) of the variable outputs and inputs, and the quantities (K) of the fixed factors. The true restricted profit function is assumed to be exactly represented by the following translog functional form, flexible to the third order, where \( \ln \) denotes the natural logarithm:

\[
\ln g(p;K) = \alpha_0 + \sum_{i=1}^{8} \alpha_i \ln p_i + \frac{1}{2} \sum_{i=1}^{8} \gamma_{ij} \ln p_i \ln p_j + \sum_{i=1}^{2} \beta_i \ln K_i + \sum_{i=1}^{2} \phi_{ij} \ln K_i \ln K_j + \frac{1}{2} \sum_{i=1}^{8} \sum_{j=1}^{2} \sum_{k=1}^{2} \delta_{ijk} \ln p_i \ln p_j \ln K_k
\]

As will be elaborated below, it is assumed that there are four variable outputs, four variable inputs, and two fixed factors. It will not generally be possible to estimate an equation like (12) directly, due to likely severe multicollinearity, so that one will wish to develop other related forms for direct estimation.

Hotelling's Lemma can be used to develop a set of equations which will be linear, although these will not be supply and derived demand equations per se. As shown below, these relations result when output revenues and input expenditures are expressed as shares of each firm's total restricted profit. These revenue and expenditure share equations are appropriate only if (12) obeys certain regularity conditions.¹³

By assumption, the translog functional form given in (12) is not just an approximation of the form of the true restricted profit function, but is

¹³For example, the property of symmetry of cross-price terms will be employed to collect like terms in deriving the expressions (13) from (12).
considered exact. The purpose of such an assumption is to allow interpretation of the error terms associated with the estimated equations to represent random errors in the optimizing decisions of firms. This assumption excludes from consideration the possibility that some component of the error will be directly due to approximation error in the choice of the functional form.

For estimation purposes, the equations (13) will each contain indicator or dummy variables which reflect the fixed effects of time and location, under the pooling assumptions employed. Accordingly, these shares are given by:

\[
S_i = \left. \frac{p_{i}x_i(p;K)}{g(p;K)} \right|_{\partial \ln g(p;K)} = \frac{\partial \ln g(p;K)}{\partial \ln p_i}
\]

\[
= \alpha_i + \sum_{j=1}^{8} \gamma_{ij} \ln p_j + \sum_{j=1}^{2} \delta_{ij} \ln K_j + \sum_{j=1}^{8} \sum_{k=1}^{2} \delta_{ijk} \ln p_j \ln K_k
\]

\[
+ \sum_{j=2}^{25} \theta_{ij} \text{Region}_j + \sum_{j=1}^{3} \rho_{ij} \text{Year}_j \quad \forall i = 1, \ldots, 8.
\]

The stochastic structure of the estimating equations consists of additive error terms on each share equation. It is assumed that these terms are normally distributed with mean zero and a positive semidefinite variance-covariance matrix. These errors are assumed to be temporally independent but contemporaneously correlated across equations. The form of the contemporaneous covariance matrix is due to the restriction that all input and output shares will sum to one by definition. Positive errors in estimating one share are necessarily offset by negative shares in one or more others. This assumption about the correlation of stochastic disturbances between share equations is critical to the choice of estimating method presented below. The assumption about the nature of the correlation of the stochastic disturbances between

---

14 The terms net supply share equation and net output share equation will be used to describe equations like (13). These terms encompass both short-run output supply and short-run derived input demand, expressed as a revenue or cost share of restricted profit, depending on whether the commodity \( x_i \) is an output or an input.
periods is a reflection of the assumptions of the LSDV model for dealing with cross-sectional and time-series data.

E. REGULARITY CONDITIONS FOR THE ESTIMATED EQUATIONS

A restricted profit function must possess certain properties in order for it to be an equivalent or dual description of the production technology under profit maximization. These properties, which may be exhibited by the postulated restricted profit function (and by the net supply share relationships derived from it), will be denoted regularity conditions. They include linear homogeneity of the profit function in prices, symmetry of cross-price effects, convexity in prices and concavity in the fixed factors.

Homogeneity and symmetry are readily expressed as testable linear restrictions on the equations to be estimated, as shown in Figure 5. Convexity in prices is an important property for a profit function which is consistent with both profit-maximizing behaviour and with a production technology which is well behaved. This behaviour rules out restricted supply share functions with discontinuities, kinks, or the wrong slopes.

It is a property of the translog functional form that, except in special limiting cases, convexity will not hold globally but may hold locally over some arbitrary range of data. Furthermore, attempts to impose such a restriction may remove the flexibility property of a functional form and may unintentionally impose restrictions on estimated coefficients, and through them, on elasticities of substitution, for example.

F. EMPIRICAL HYPOTHESES ABOUT FIRM RESPONSE TO REDUCED VARIABILITY OF TOTAL MARKET RETURN

The analysis of Section II makes it clear that output response to a mean-preserving change in the distribution of total market return will depend on whether or not the expected value of marginal product of capital function is convex in total market returns. Where there is more than one type of fixed capital, one will be concerned with the convexity of these functions for each capital input and with their gross substitutability or complementarity in production.
Specifically, if the sign of $\frac{\partial^3 g}{\partial K_k \partial p_1^2} \ (k=1,2)$ is positive, then a firm will reduce its use of $K_k$ when there is a mean-preserving reduction in the dispersion of total market returns. Expressions (14) and (15) present the own-price and cross-price third derivatives, which can be evaluated numerically. Because the translog form (12) employs logarithmic transformations of variables, some manipulation of terms is required to define these derivatives in terms of the underlying function. Separate expressions are obtained for own-price and cross-price third-order derivatives:

\[
\frac{\partial^3 g}{\partial K_k \partial p_1^2} = \left(2S_1 - 1\right)\left(\delta_{1k} + \sum_{j=1}^{8} \delta_{1jk} \ln p_j + \delta_{11k}\right) \frac{g}{p_1^2 K_k} + \frac{h_{1i}}{p_1^2} \left(\frac{\partial g}{\partial K_k}\right)_{i=1,...,8} \quad \forall \left(k=1,2\right)
\]

\[
\frac{\partial^3 g}{\partial K_k \partial p_1 \partial p_j} = \left(S_1 \left(\delta_{jk} + \sum_{m=1}^{8} \delta_{jmk} \ln p_m\right) + \delta_{1jk}\right) \frac{g}{K_k p_1 p_j} + \frac{h_{1j}}{p_1 p_j} \left(\frac{\partial g}{\partial K_k}\right)_{j=1,...,8} \quad \forall \left(k=1,2, j \neq i\right)
\]

where $\frac{\partial g}{\partial K_k} \ (k = 1,2)$ is the marginal value product of capital as given by:

\[
\frac{\partial g}{\partial K_k} = \frac{g}{K_k} \cdot \left(\beta_k + \sum_{i=1}^{8} \delta_{1ik} \ln p_i + \sum_{j=1}^{2} \phi_{jk} \ln K_j + \frac{1}{2} \sum_{i=1}^{8} \sum_{j=1}^{8} \delta_{ijk} \ln p_i \ln p_j\right) \quad \forall \ k = 1,2
\]

The terms $h_{1i}$ and $h_{1j}$ are elements of the (modified) Hessian matrix in prices of the restricted profit function, and are given by:

\[
\frac{\partial^2 g}{\partial p_1^2} \cdot \frac{p_1^2}{g} = h_{1i} = \left(\gamma_{1i} + S_1^2 - S_1 + \sum_{k=1}^{2} \delta_{11k} \ln K_k\right) \quad \forall \ i = 1,...,8
\]

\[
\frac{\partial^2 g}{\partial p_1 \partial p_j} \cdot \frac{p_1 p_j}{g} = h_{1j} = \left(\gamma_{1j} + S_j S_j + \sum_{k=1}^{2} \delta_{1jk} \ln K_k\right) \quad \forall \ \left(i = 1,...,8 \land j = 1,...,8 \land i \neq j\right)
\]

The expressions (14) and (15) are similar to the conditions which one would use to check for local compliance with curvature conditions in two
important respects. First, because these expressions rely on the sample observations, the numerical values which are obtained will vary over the range of the sample data. One might wish to calculate such numerical values at the sample mean, or at the point of approximation of the function (12) where \( p = K = 1 \). Second, given the complexity of the expressions to be evaluated, it will generally not be possible to estimate a standard error or confidence interval around each calculated value of a third-order derivative. These computed standard errors would be required to conduct a statistical test of alternative hypotheses about the significance or direction of optimal firm response to increased price uncertainty. Any finding that specific derivatives of the form \( \frac{\partial^3 g}{\partial k^p_1 \partial p_1} \) are positive or negative may be indicative or supportive of the local value, but their statistical significance cannot be tested readily.

The third-order derivatives given by (14) and (15) are not elasticity measures per se. However, the qualitative adjustments suggested by a positive or negative value will be of considerable importance to understanding firm behaviour under uncertainty.

IV. DATA CONSTRUCTION AND ECONOMETRIC TECHNIQUE

This research project represents the first reported attempt to perform econometric analysis of firm behaviour in the Canadian agricultural sector using micro data from the annual National Farm Survey undertaken by Statistics Canada.

A. THE NATIONAL FARM SURVEY

The primary source of data employed in this project is the "core survey" component of the National Farm Survey (hereafter denoted NFS) undertaken by Statistics Canada in the years 1983 through 1987 inclusive. The NFS consists of an individually-administered questionnaire recording many facets of each farm's annual operation including organization, ownership, land use, sales revenues, and detailed data on input expenditures by category.

The data collection methods vary regionally for the NFS, with different survey design parameters and questionnaires used across regions. The study area chosen for this project consists of the provinces of Manitoba,
Saskatchewan, and Alberta plus the Peace River Region of British Columbia, which are divided into twenty-five Sub Provincial Areas (SPA's). This specific area was surveyed over the years 1983-1987 using a common survey design and questionnaire which was relatively unchanged over the period. By contrast, before 1983 and in 1988 the types of survey instruments used to collect farm-level data in this region were considerably different.

B. CONSTRUCTION OF MEASURES OF OUTPUT SUPPLY, INPUT DEMAND, AND FIXED FACTORS

In order to calculate measures of output supply, input demand and fixed factors, several steps need to be taken. First, the variables of interest need to be defined. Then, those data which are capable of providing such measures need to be selected. Finally, there will be a number of specific assumptions and calculations used to construct these values.

1. **Choice of Output Aggregates**

   The output aggregates employed in this study provide an exhaustive classification of revenue by source. They include:
   
   (i) cattle sold,
   (ii) changes in the cattle inventory,
   (iii) grains and other field crops, and
   (iv) custom work and production of other livestock.

2. **Choice of Input Aggregates**

   So many previous econometric studies of aggregate profit functions and production technology have grouped inputs into capital, labour, energy, and materials that this scheme deserves mention. This classification is not employed here because it does not lend itself well to application to cattle production and because the present focus on variable inputs will exclude most capital. It is also the case that the NFS questionnaire does not clearly capture labour input use. For example, in one section, only wage labour is reported (although some operators pay themselves no wage). In another section, the labour component of repair costs is treated as a machinery expense and cannot be isolated. (It is also the case that many operators do their own repairs).

   In developing a set of aggregate inputs which is consistent with the
available data, and therefore with the cattle industry, the following exhaustive classification scheme was employed:

(i) machinery and motor vehicle expenses
(ii) crop production activities
   (seed, fertilizer, pesticides, crop insurance)
(iii) animal production activities
   (purchased feed, veterinary supplies, replacement animals)
(iv) supplies and services
   (includes hired labour, maintenance, utilities).

Based on the weighting diagram used by Statistics Canada to compute the Farm Input Price Index in Western Canada, these aggregates represent about 20, 20, 40 and 20 percent respectively of farm expenditures on variable inputs. (See Statistics Canada, Prices Division, 1987, p.17).

3. Restricted Profit and Fixed Factors

The definition of restricted profit constructed for each farm is simply revenues less variable input expenditures as defined above.

Those fixed factors which are thought most important to a firm's short-run output of cattle are the breeding herd and available land area. Conceptually, the measure of land area would be best expressed in terms of its crop production potential, reflecting agricultural productivity of the site and the soils, as well as reflecting its current state of improvement. It is a limitation of the available area measure that it places an equal value on productive crop land, pasture land, and uncultivable grazing lands of the Rocky Mountain Foothills, for example.

4. Sample Size

It is well known in econometric analysis that the selection or exclusion of sample observations by the researcher can seriously bias resulting measurements, especially where the selection criteria relate to the endogenous variables to be explained. It was necessary nonetheless to exclude some of the available NFS data. In this study, observations have been excluded from further analysis for one or more of the following three reasons:

29
1) the survey was not completed by a respondent, so that all reported data are estimates or extrapolations provided by Statistics Canada staff,

11) the respondent had no beef cows on July 1st of that year,

111) there was no reliable basis for estimating revenue from cattle sales in that calendar year.

By far, the largest number of exclusions were due to the absence of any fixed cattle breeding herd in the year under analysis. Some 6,265 observations were retained for analysis. It was also the case that records from successive years had to be linked longitudinally in order to describe fully activities relating to a single calendar year, where these activities are routinely reported in separate years.

The number of longitudinally paired observations retained per crop year is 1,917, 1,690, 1,398 and 1,260 for the years 1983-1986 inclusive. This corresponds to 1,502, 2,081, 2,493 and 189 observations by province, moving westward from Manitoba to British Columbia. These data are summarized in Table 2.

C. CONSTRUCTION OF OUTPUT AND INPUT PRICE INDICES

A Divisia-type price index is used which is known to be consistent in its properties with flexible functional forms such as a translog restricted profit function. (See Diewert, 1976.) In this case, care was taken in applying such an index to ensure it captures relative price differences across regions at a point in time, as well as capturing changes in regional prices over time. The index expresses all such changes relative to a common base, where the base is constructed using weighted average prices and shares across all Sub Provincial Areas in the year 1982.

The data requirements to construct eight of these price indices are substantial. They include prices of all of the components of each aggregate, plus the share of revenue or expenditure that each component occupies in its aggregate in each region in each year.

Descriptive statistics for the constructed price indices are given in Table 3.
Table 2: Descriptive Statistics by Year for Computed Left-Hand-Side and Capital Variables, Farm-Level Data, N = 6,265

<table>
<thead>
<tr>
<th>VARIABLE</th>
<th>MEAN</th>
<th>STANDARD DEVIATION</th>
<th>FIRST PERCENTILE</th>
<th>99TH PERCENTILE</th>
</tr>
</thead>
<tbody>
<tr>
<td>CROPS REVENUE ($)</td>
<td>52,515.18</td>
<td>75,920.17</td>
<td>0.00</td>
<td>396,970.19</td>
</tr>
<tr>
<td>CATTLE REVENUE ($)</td>
<td>69,938.53</td>
<td>258,841.45</td>
<td>0.00</td>
<td>1,273,375.90</td>
</tr>
<tr>
<td>INVTRY REVENUE ($)</td>
<td>1,184.70</td>
<td>503,704.73</td>
<td>-436,763.77</td>
<td>139,145.89</td>
</tr>
<tr>
<td>CUSTWK REVENUE ($)</td>
<td>11,154.19</td>
<td>100,595.70</td>
<td>0.00</td>
<td>167,943.23</td>
</tr>
<tr>
<td>MCHNRV EXPENDITURE ($)</td>
<td>-15,854.27</td>
<td>10,103.13</td>
<td>-85,280.00</td>
<td>0.00</td>
</tr>
<tr>
<td>CULTVN EXPENDITURE ($)</td>
<td>-15,541.05</td>
<td>25,113.97</td>
<td>-136,257.06</td>
<td>0.00</td>
</tr>
<tr>
<td>LVSTCK EXPENDITURE ($)</td>
<td>-37,892.11</td>
<td>189,767.92</td>
<td>-831,062.32</td>
<td>0.00</td>
</tr>
<tr>
<td>SUPPLY EXPENDITURE ($)</td>
<td>-16,571.94</td>
<td>55,231.88</td>
<td>-205,010.00</td>
<td>-200.00</td>
</tr>
<tr>
<td>TOTINP ALL INPUTS ($)</td>
<td>-85,859.37</td>
<td>239,521.37</td>
<td>-1,250,080.00</td>
<td>-2,907.26</td>
</tr>
<tr>
<td>PROFIT FARM AVERAGE ($)</td>
<td>48,933.23</td>
<td>509,145.91</td>
<td>-331,154.96</td>
<td>658,634.44</td>
</tr>
<tr>
<td>S1 CATTLE SHARE</td>
<td>1.76</td>
<td>34.47</td>
<td>-21.23</td>
<td>23.75</td>
</tr>
<tr>
<td>S2 INVTRY SHARE</td>
<td>-0.43</td>
<td>29.10</td>
<td>-8.47</td>
<td>10.48</td>
</tr>
<tr>
<td>S3 CROP SHARE</td>
<td>2.07</td>
<td>43.33</td>
<td>-17.73</td>
<td>14.37</td>
</tr>
<tr>
<td>S4 CUSTWK SHARE</td>
<td>0.11</td>
<td>2.33</td>
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<td>6.46</td>
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<td>199.46</td>
<td>1.00</td>
<td>724.86</td>
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<td>4,899.09</td>
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<tr>
<td>FASEF SAMPLE WEIGHT</td>
<td>21.15</td>
<td>20.76</td>
<td>0.84</td>
<td>98.00</td>
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<th>STANDARD DEVIATION</th>
<th>FIRST PERCENTILE</th>
<th>99TH PERCENTILE</th>
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<tbody>
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<td>30,343.20</td>
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<td>FASEF SAMPLE WEIGHT</td>
<td>21.15</td>
<td>20.76</td>
<td>0.84</td>
<td>98.00</td>
</tr>
</tbody>
</table>

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<sup>a</sup>The abbreviation (ac.) refers to land area measured in acres, (no.) refers to number of animals at July 1st, and ($) refers to current dollars/fiscal year.
D. ECONOMETRIC TECHNIQUE

The purpose of the econometric analysis is to estimate a series of net supply share equations (13) which are derived from a firm's restricted profit function (12). This will allow numerical evaluation of expressions (14)-(18).

1. The System of Linear Equations

A series of linear net output share equations is estimated, where output revenue and input expenditure, expressed as shares of restricted profit, are the dependent variables. All eight shares sum identically to one for every farm. All eight shares are explained by exactly the same set of independent or explanatory variables, which include eight prices, quantities of two types of capital, and sixteen cross-products relating them. Prices are expressed as natural logarithms of Divisia-type indices, and capital is expressed as the natural logarithm of the physical quantity of capital.

The system of equations possessing all of these characteristics is portrayed schematically in Figure 5 which incorporates the relationships and the definitions given earlier in this section, as well as portraying the set of homogeneity (adding-up) and symmetry restrictions. The symmetry and adding-up restrictions ensure that the regression errors across equations are correlated at each point in time. An unexpectedly large value of $S_1$, for example, ensures that some of the values $S_2-S_8$ will be correspondingly lower.

It is well known that under certain circumstances, these contemporaneous error covariances can be used to advantage in estimating the full system of equations relative to estimating each equation separately. One may employ the estimation method or technique referred to variously as Joint Generalized Least Squares (JGLS) or Zellner's Method of Seemingly Unrelated Regression (SUR). Estimates are made of the contemporaneous covariance of errors across equations, based on Ordinary Least Squares estimation of each equation separately. These error-covariance estimates are employed to estimate the entire system of equations in a second step of the estimation procedure, often subject to restrictions imposed across the system of equations. Some econometric computer packages allow several iterations in this process (referred to as Iterative SUR) updating the estimate of error covariance on each pass, although such software was not available for this project.
### Descriptive Statistics by Year

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<tr>
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<th>LABEL</th>
<th>N</th>
<th>MEAN</th>
<th>MINIMUM VALUE</th>
<th>MAXIMUM VALUE</th>
<th>RANGE</th>
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<td>INVENTORY PRICE</td>
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<td>CROP PRICE</td>
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<td>CUSTOM WORK PRICE</td>
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<td>0.990</td>
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<td>0.017</td>
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<td>P5</td>
<td>MACHINERY PRICE</td>
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<td>0.997</td>
<td>1.006</td>
<td>0.009</td>
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<td>1.022</td>
<td>0.036</td>
<td>0.014</td>
<td>25.038</td>
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</tbody>
</table>

**Year 1983**

| P1         | CATTLE PRICE        | 25| 1.030 | 1.021         | 1.050         | 0.028 | 0.008             | 25.757|
| P2         | INVENTORY PRICE     | 25| 1.093 | 1.058         | 1.147         | 0.089 | 0.030             | 27.324|
| P3         | CROP PRICE          | 25| 1.031 | 1.001         | 1.062         | 0.061 | 0.015             | 25.776|
| P4         | CUSTOM WORK PRICE   | 25| 0.907 | 0.899         | 0.920         | 0.021 | 0.008             | 22.664|
| P5         | MACHINERY PRICE     | 25| 1.046 | 1.037         | 1.069         | 0.013 | 0.005             | 26.091|
| P6         | CULTIVATION PRICE   | 25| 0.966 | 0.956         | 1.040         | 0.084 | 0.017             | 24.138|
| P7         | LIVESTOCK PRICE     | 25| 0.987 | 0.960         | 1.006         | 0.038 | 0.014             | 24.665|
| P8         | SUPPLY PRICE        | 25| 1.003 | 0.975         | 1.016         | 0.041 | 0.013             | 25.087|

**Year 1984**

| P1         | CATTLE PRICE        | 25| 1.080 | 1.059         | 1.097         | 0.038 | 0.015             | 27.004|
| P2         | INVENTORY PRICE     | 25| 1.150 | 1.103         | 1.198         | 0.094 | 0.024             | 28.754|
| P3         | CROP PRICE          | 25| 1.046 | 1.000         | 1.005         | 0.077 | 0.018             | 26.159|
| P4         | CUSTOM WORK PRICE   | 25| 0.923 | 0.902         | 0.954         | 0.052 | 0.020             | 23.081|
| P5         | MACHINERY PRICE     | 25| 1.080 | 1.067         | 1.090         | 0.022 | 0.009             | 27.006|
| P6         | CULTIVATION PRICE   | 25| 0.997 | 0.990         | 1.117         | 0.037 | 0.009             | 27.364|
| P7         | LIVESTOCK PRICE     | 25| 1.095 | 1.000         | 1.164         | 0.063 | 0.013             | 26.135|
| P8         | SUPPLY PRICE        | 25| 1.045 | 1.001         | 1.064         | 0.038 | 0.015             | 27.004|

Table 3: Descriptive Statistics by Year and Across Years for Output and Input
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</tr>
<tr>
<td>P1 CATTLE PRICE</td>
<td>25</td>
<td>1.074</td>
<td>1.052</td>
<td>1.099</td>
<td>0.047</td>
<td>0.017</td>
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<td>P2 INVENTORY PRICE</td>
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<td>1.170</td>
<td>0.063</td>
<td>0.017</td>
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<td>0.099</td>
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<td>0.932</td>
<td>0.057</td>
<td>0.012</td>
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<tr>
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<td>0.010</td>
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<tr>
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<td>0.056</td>
<td>0.012</td>
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<tr>
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<td>1.011</td>
<td>0.992</td>
<td>1.091</td>
<td>0.100</td>
<td>0.019</td>
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<tr>
<td>P7 LIVESTOCK PRICE</td>
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<td>1.059</td>
<td>1.021</td>
<td>1.138</td>
<td>0.117</td>
<td>0.028</td>
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<tr>
<td>P8 SUPPLY PRICE</td>
<td>25</td>
<td>1.059</td>
<td>1.016</td>
<td>1.078</td>
<td>0.062</td>
<td>0.013</td>
<td>26.475</td>
</tr>
</tbody>
</table>

| **YEAR=1966**        |     |        |               |               |       |                   |      |
| P1 CATTLE PRICE      | 25  | 1.198  | 1.156         | 1.239         | 0.083 | 0.026             | 29.939 |
| P2 INVENTORY PRICE   | 25  | 1.287  | 1.235         | 1.353         | 0.118 | 0.038             | 32.181 |
| P3 CROP PRICE        | 25  | 0.602  | 0.629         | 0.709         | 0.080 | 0.027             | 17.054 |
| P4 CUSTOM WORK PRICE | 25  | 1.006  | 0.901         | 1.048         | 0.066 | 0.027             | 25.158 |
| P5 MACHINERY PRICE   | 25  | 1.040  | 1.032         | 1.108         | 0.075 | 0.015             | 26.008 |
| P6 CULTIVATION PRICE | 25  | 1.009  | 0.974         | 1.094         | 0.119 | 0.025             | 25.237 |
| P7 LIVESTOCK PRICE   | 25  | 0.853  | 0.793         | 0.906         | 0.193 | 0.051             | 21.322 |
| P8 SUPPLY PRICE      | 25  | 1.066  | 1.039         | 1.118         | 0.079 | 0.031             | 26.662 |

<table>
<thead>
<tr>
<th><strong>DESCRIPTIVE STATISTICS ACROSS ALL YEARS</strong></th>
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<th></th>
<th></th>
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<td>P5 MACHINERY PRICE</td>
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<td>0.040</td>
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<td>P6 CULTIVATION PRICE</td>
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<td>0.026</td>
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<tr>
<td>P7 LIVESTOCK PRICE</td>
<td>125</td>
<td>0.997</td>
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<td>1.138</td>
<td>0.345</td>
<td>0.088</td>
<td>126.601</td>
</tr>
<tr>
<td>P8 SUPPLY PRICE</td>
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<td>0.975</td>
<td>1.110</td>
<td>0.144</td>
<td>0.033</td>
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</tr>
</tbody>
</table>
Shares = f(Intercept, Prices, Capital, (Prices•Breeding Herd), (Prices•Land), Dummy Variables)

\[
S_i = \alpha_i + \sum_{j=1}^{2} \gamma_{ij} \ln p_j + \sum_{j=1}^{2} \delta_{ij} \ln k_j + \sum_{j=1}^{2} \sum_{k=1}^{2} \delta_{ijk} \ln p_j \ln k_k + \sum_{j=2}^{25} \theta_{ij} \text{Region}_j + \sum_{j=1}^{3} \rho_{ij} \text{Year}_j
\]

<table>
<thead>
<tr>
<th>Cattle</th>
<th>Inventory</th>
<th>Crops</th>
<th>Custom Work</th>
<th>Machinery</th>
<th>Cultivation</th>
<th>Livestock</th>
<th>Supplies</th>
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<tbody>
<tr>
<td>(\alpha_1)</td>
<td>(\alpha_2)</td>
<td>(\alpha_3)</td>
<td>(\alpha_4)</td>
<td>(\alpha_5)</td>
<td>(\alpha_6)</td>
<td>(\alpha_7)</td>
<td>(\alpha_8)</td>
</tr>
<tr>
<td>(P_1, \ldots, P_8)</td>
<td>(P_1, \ldots, P_8)</td>
<td>(K_1 K_2)</td>
<td>(K_1 K_2)</td>
<td>(K_1 K_2)</td>
<td>(K_1 K_2)</td>
<td>(K_1 K_2)</td>
<td>(K_1 K_2)</td>
</tr>
</tbody>
</table>

\(\sum_{i=1}^{8} S_i = 1\)

\(\sum_{i=1}^{8} \alpha_i = 1\)

\(\sum_{j=1}^{8} \gamma_{ij} = 0\)

\(\sum_{j=1}^{8} \delta_{ij} = 0\)

\(\sum_{j=1}^{8} \delta_{ijk} = 0\)

\(\sum_{j=2}^{25} \theta_{ij} = 0\)

\(\sum_{j=1}^{3} \rho_{ij} = 0\)

\(j = 1, \ldots, 8\)  \(j = 1, \ldots, 8\)  \(j = 2, \ldots, 25\)  \(j = 1, 2, 3\)

\(\gamma_{ij} = \gamma_{ji}\)  \(i \neq j\)

\(\delta_{ijk} = \delta_{jik}\)  \(i \neq j\)

Figure 5: Schematic Representation of the System of Equations to be Estimated
In order to employ the two-stage JGLS estimation method the estimated error-covariance matrix must be non-singular. This will not be the case when all eight share equations are estimated simultaneously subject to symmetry and adding-up restrictions. To ensure non-singularity, any one of the eight share equations is dropped, although its coefficients may be inferred as residuals by reference to the constraints imposed. McGuire et al., (1968, p.1205) show that JGLS estimates will be invariant to which equation is dropped under certain conditions. Accordingly, in this analysis, the eighth input share, which is associated with Services and Supplies, is omitted, as indicated by the horizontal dashed line in Figure 5. In most cases, the system of seven equations is subsequently re-estimated dropping the seventh (input) share instead of the eighth, so that a complete set of standard errors can be obtained for the eighth share. The estimates so obtained are identical.

Although considerable information about the short-run or within-season production structure can be recovered from the system of eight net supply share equations (13), this system does exclude some parameters found in (12). In particular, the estimated values of $\beta_i$ ($i = 1, 2$) and $\phi_{ij}$ ($i, j = 1, 2$) will be required to evaluate numerically the derivatives described by (14) and (15). These derivatives characterize optimal firm response to price uncertainty.

In order to recover estimates of the six additional parameters which are unique to (12), a separate single equation model will be estimated by weighted least-squares methods after estimation of (13). It will take the form:

$\ln g(p; K) = \sum_{i=1}^{8} \hat{\alpha}_i \ln p_i - \frac{1}{2} \sum_{i=1}^{8} \sum_{j=1}^{8} \hat{\gamma}_{ij} \ln p_i \ln p_j$

$- \sum_{i=1}^{8} \sum_{j=1}^{2} \hat{\delta}_{ij} \ln p_i \ln K_j - \frac{1}{2} \sum_{i=1}^{8} \sum_{j=1}^{8} \sum_{k=1}^{2} \hat{\delta}_{ijk} \ln p_i \ln p_j \ln K_k$

$- \sum_{i=1}^{8} \sum_{j=2}^{25} \hat{\theta}_{ij} \ln p_i \ln Region_j - \sum_{i=1}^{8} \sum_{j=1}^{3} \hat{\rho}_{ij} \ln p_i \ln Year_j$

$= \alpha_0 + \sum_{i=1}^{2} \beta_i \ln K_i + \frac{1}{2} \sum_{i=1}^{2} \sum_{j=1}^{2} \phi_{ij} \ln K_i \ln K_j$
subject to:

\[ \phi_{ij} = \phi_{j1} \quad \forall \ i = 1,2 \quad j = 1,2 \quad i \neq j, \]  

where \( \hat{\alpha} \) refers to the JGLS parameter estimate from the system of equations (13).

2. **Weighted Least-Squares Analysis**

A final consideration in the choice of econometric technique concerns the treatment of sample design and the property of the NFS data that they are not selected randomly, but result from a stratified survey. In the NFS, there is a higher sampling rate from strata which consist of farms that are large (in specific dimensions, including number of cattle) than from strata which consist of farms of all sizes. The availability of sampling weights allows the researcher to employ weighted least-squares analysis with these data. The available weights are employed in all of the analysis reported here.

V. **ECONOMETRIC RESULTS**

Presentation and evaluation of the empirical results obtained from performing this regression analysis involve a number of steps. These include reviewing the sign and significance of individual coefficient estimates, reviewing the degree to which the proposed model conforms to the available data, and then examining the extent to which the estimated model conforms with *a priori* beliefs about rational profit-maximizing behaviour in the short run. Ideally, a set of estimates which performs well according to the above criteria would be useful not only for testing the specific hypotheses which describe firm behaviour in the face of price uncertainty, but for providing information about numerous other characteristics of the underlying production structure. (See Shumway, 1989, for example.)

A. **ESTIMATION OF THE TRANSLOG NET SUPPLY SHARE EQUATIONS**

In estimating the most general translog model (with 6,265 observations), the estimated coefficients are not generally statistically significant and do not correspond well to *a priori* beliefs about economic behaviour. This seems to be an unfortunate outcome of estimating the general model described by (13) (and as portrayed in Figure 5) using micro data. As a result, it is not possible with this approach to evaluate in a meaningful way the hypotheses under consideration. A review of those regression results suggests some
methodological and data issues, including specific micro-data concerns, which may be detrimental to the estimation procedure employed. As a result, alternative specifications of the model are presented and estimated.

In reviewing both the data and the results, a number of issues come to light which may be detrimental to the estimation procedure employed. They include the existence of single observations for which restricted profit, as defined, is arbitrarily small or negative. Further, in attempting to measure behaviour of restricted profit where behaviour is thought to be restricted by physical capital (such as the size of the breeding herd at the start of the season) it may be the case that such restrictions are only meaningful above some threshold number of animals. Other relevant data issues include multicollinearity and inaccurate data.

There are a number of suggested revisions to the system first estimated which might result in estimates which conform more closely to the behaviour being modelled. In consideration of these, the original system of equations will be estimated for a subset of data which excludes all firms with negative restricted profits or with arbitrarily small restricted profits. Further, farms are excluded if they report fewer than some threshold size of breeding herd, or if they are Feedlots.15

B. ESTIMATION OF TRANSLOG SHARES ON A SUBSET OF THE DATA

The changes made in estimating this system of equations have to do with selecting some observations for analysis and excluding the rest. In particular, observations are excluded from analysis if any of the following conditions is true:

1) the farm is a feedlot according to a classification scheme such as the ones employed by McKenzie and Plaunt (1989) or Ehrensaft (1987),

2) the farm has a calculated value of restricted profit which is not positive,

15 Alternative approaches include (i) estimation of translog shares with grouped data (the system of equations is re-estimated with the NFS data aggregated into cells), and (ii) estimation employing a generalized quadratic profit function (the system of equations is re-specified using an alternate functional form). These results are reported in Horbulyk (1989).
iii) the farm has a breeding herd (cows plus bulls) of less than thirty head.

iv) any of the net output shares calculated for the farm is greater than 2.5 in absolute value.

As a result of imposing these criteria the number of observations available for analysis falls to 2,119. Each of these criteria applied individually would have excluded (i) 573 (9.1%), (ii) 1,213 (19.4%), (iii) 2,636 (42.1%) and approximately (iv) 1,627 (25.9%) (respectively) of the original 6,265 observations.

There are two evident changes in the data as a result of excluding these observations. Over the original sample, the (weighted mean) net output share associated with inventory change is negative, which implies that inventories were reduced on balance. Over the subsample of 2,119 observations, this share is positive (that is, on balance inventories were augmented). This subsample of firms employs both capital inputs (breeding herd and land) in greater quantities, on average, than the original sample.

The results obtained in the analysis of the 2,119 data points using the functions (13) show stronger measures of association on an equation-by-equation basis and for the system as a whole. Tighter data selection criteria have been invoked to address the problems associated with negative restricted profit, arbitrarily small restricted profit, and firms operating below a threshold level of fixed factors.

It is also evident that despite improvements in various measures of fit, a considerable number of individual system parameters are not statistically significant at even the 10% level of significance. Another of the data limitations which was encountered with firm-level data is multicollinearity. Examination of such measures as Pearson correlation coefficients for these data indicate that there remains a substantial degree of correlation of right-hand-side variables, which appears almost identical to that for the original sample.

An established approach to reducing estimated standard errors, so as to determine better the influence of individual parameters of interest, is to test and impose (if warranted) the restriction that some subset of coefficients is jointly equal to zero.
C. ESTIMATION WITH MODEL SELECTION RESTRICTIONS

At this juncture, an alternative approach to estimating the system of equations (13) is to impose (and test) restrictions on the choice of model per se holding constant the issue of data selection. In particular, it will be instructive to consider a model similar in form to the system of equations (13) where the number of coefficients to be estimated in each equation is reduced in a specific fashion.

The approach which will be followed here is to test a set of restrictions which would restrict some of the coefficients estimated to equal zero. In seeking to impose such restrictions, one will want to avoid restricting any specific parameters which, a priori, are thought to have an important influence on the behaviour being modelled. In such a case, restricting those parameters' (to equal zero) may introduce an omitted-variables bias, even though the parameters may not be statistically significant over this sample of data.

Accordingly, it is proposed to test the restriction that all the estimated parameters with t statistics less than one (in absolute value) are jointly equal to zero. The parameters excepted from this group will be the own-price terms \( P_1 \) in Share\(_1\) and the own price-capital product terms \( P_1 \cdot K_j \) \((j = 1, 2)\) in Share\(_1\) as each is thought to be important a priori. The former determine the slope of the short-run net supply functions, while the latter describe the effect of capital use on short-run net output. If such a null hypothesis is not rejected, then these restrictions will be imposed and the system of equations re-estimated.

The choice of \(|t| < 1\) is thought to be a fairly conservative rule, the use of which may reduce the potential for omitted-variables bias, while at the same time reducing the standard errors of the estimates. The decision to impose such a model selection experiment on this set of data \((N = 2,119)\) is based on the relatively severe limitations which were identified with the larger data set \((N = 6,265)\).

Testing the null hypothesis that such a group of parameters is jointly equal to zero across the system of JGLS results (seven equations), yields an F statistic of 1.145. (In all, 149 of the 432 coefficients (34%) are set to
zero in this process.) The associated probability that the null is true is 0.139. An identical test result is obtained when either share equation seven or eight is dropped from the system. Thus, the hypothesis may not be rejected at the 5% or 10% level of significance. On the basis of this result, it is decided to re-estimate the system of equations (13) over the 2,119 observations with these restrictions in place.

D. SINGLE EQUATION ESTIMATION OF (19)

The single equation model (19) was estimated independently of (13) in order to recover estimates of the six slope coefficients which are unique to (12) (i.e., $\beta_1, \beta_2, \phi_1, \phi_{12}, \phi_{21}$ and $\phi_{22}$). This required the use of parameter estimates from (13) to construct the new dependent variable for (19). The same data set ($N = 2,119$) was employed to construct the new dependent variable.

The $R^2$ value associated with this regression is 0.296. On the basis of the reported $F$ statistic, one may reject the null hypothesis that all nonintercept coefficients are jointly zero. Each estimated parameter is statistically discernible at the 95% confidence level on the basis of a two-tailed test.\(^{16}\)

E. SUMMARY OF TRANSLOG ESTIMATES USING A SUBSET OF THE DATA

As a description of the overall goodness of fit and local compliance with theoretical regularity conditions, the following findings may be summarized. Based on prior model selection tests (zero restrictions), the value of the system weighted $R^2$ is 0.138. Prior to such restrictions being imposed, equation-by-equation OLS estimates (without cross-equation restrictions) rejected the null hypothesis that all nonintercept coefficients were jointly equal to zero, at the 5% level of significance. No joint test of

\(^{16}\) The left-hand-side variable in (19) is constructed, in part, from the JGLS estimates from the system of equations (13). Further, the price variable, $p_2$, associated with changes in cattle inventory, is constructed from forecasts which derive from a univariate time-series analysis. To the extent that these estimates and forecasts are themselves inexact, the standard errors which are reported in their subsequent use will be too low, and the $t$ statistics, for example, too high. This may influence the interpretation of results presented.
homogeneity and symmetry was undertaken at the JGLS stage, although homogeneity was subsequently imposed.

The JGLS estimates, subject to cross-equation and model selection (zero) restrictions, reject symmetry of own-price terms at the 5% level of significance, conditional on homogeneity. (There is a 0.02% probability the null is true.) These estimates reject symmetry of price-capital product terms associated with the land use but not those associated with the breeding herd, at the 10% level of significance when homogeneity restrictions are imposed. All three symmetry restrictions are imposed on the estimates described here.

Two theoretical regularity conditions of a restricted profit function are concavity in fixed factors and convexity in prices. The estimated system of equations is not consistent with concavity of the restricted profit function in fixed factors at the sample mean nor at the point of approximation. Such a condition is not tested at other points nor is it subsequently imposed.

The estimated model is not consistent with convexity of the restricted profit in prices at any of the three points which were checked. It was decided not to check this condition over the entire range of sample points due to (i) the relatively large size of the sample (N = 2,119) and, (ii) the relative complexity of the modified Hessian matrix elements (as defined in (17) and (18)). It is not known whether convexity in prices would be rejected at other sample points, especially in a statistical test of such an hypothesis. Such a statistical test would require introducing other estimation algorithms and is beyond the scope of the current research.

Given that data problems have been identified (including multicollinearity, which is endemic to cross-sectional price data) it is indeed possible that concavity in fixed factors and/or convexity in prices would be rejected statistically by the estimates described here over some or all of the range of the sample data. This is an important caveat to the interpretation of estimated results which follows.

Failure of curvature conditions can be due to violation of the underlying
profit-maximizing postulate.\footnote{Antle and Capalbo (1988, p.33) cite the hypothetical example of government production controls on agriculture, under which cost minimization might be a more relevant firm objective. Although delivery quotas exist on some Western Canadian grains, they need not restrict production because even when binding on deliveries, feed uses and sales for feed use are permitted. This sector is otherwise relatively free of such production controls.}

...But researchers must be aware that violations of curvature or other theoretical properties by an econometrically estimated model may simply be due to sampling error or to other problems, such as data errors, model misspecification, or simultaneous equation bias.

(Antle and Capalbo, 1988, p.76)

Rejection of theoretical regularity conditions is a commonplace occurrence of econometric production studies, particularly those which estimate restricted profit functions.

Given these outcomes, it appears to be prudent, in general, for the researcher to identify the specific data and other issues (such as multicollinearity, missing or unavailable data, inaccurate micro data, and institutional restrictions on firm behaviour) which might be contributing to the (local or global) non-compliance of estimated values with these theoretical conditions. In some circumstances it is possible to impose such conditions, including homogeneity, symmetry, and convexity. Where such regularity conditions are not subsequently imposed, the fact that they have not been satisfied should remain as an important caveat on the interpretation of those results.

The next section interprets the above results in light of firm behaviour. The discussion illustrates the application of estimates this type to determination of optimal firm behaviour in the short run. Although the results are supportive of specific types of adjustment, they are accompanied by a number of caveats about their statistical significance. One cannot conclude that these results will be robust to changes in the data sample or in the functional forms employed. As these are the first empirical results to address these specific firm adjustments, it is not possible to draw comparisons with them from the literature.
VI. INTERPRETATION OF RESULTS

This section will address several aspects of firm behaviour which may be inferred from the parameter estimates. First, the short-run own-price and cross-price elasticities of supply and demand are presented and interpreted in terms of the short-run production flexibility of firms. Information is also provided about the estimated shadow value of capital and about the total elasticity of cattle supply. Then the special role of fixed factors in the two-stage production process is considered, specifically in terms of their effect on the magnitude of the short-run elasticities. The econometric estimates are interpreted to describe how a risk-neutral firm would optimally respond to changes in price variability, including changes associated with enrolment in a participatory stabilization scheme. The section concludes with some comments about the way the short-run firm behaviour described in this paper might be affected by the introduction of a stabilization scheme.

A. RESTRICTED ELASTICITIES OF CHOICE

The restricted Marshallian own- and cross-price elasticities of supply and factor demand (generally denoted elasticities of choice) describe the percentage change in net output (quantity) corresponding to a one percent change in a net output price, subject always to a constant level of use of the fixed factors. These elasticity measures are also referred to as short-run output-variable or short-run uncompensated elasticities. They may be evaluated at a point or across some range of values, and they have the property that cross-price elasticities need not be symmetric.

Table 4.a reports such restricted Marshallian elasticities as evaluated at the mean of the sample data. These elasticities are evaluated using the estimated coefficient values, the estimated net supply share values (at the sample mean observation) and some of the mean data values. Given the complexity of the individual terms compared to those associated with a conventional translog which is flexible to the second order, it has not proven possible to compute standard errors associated with each elasticity value. Accordingly, a caveat in the interpretation of individual elasticity signs and magnitudes will be the absence of accompanying statistical measures of significance.
Table 4: Estimated Values of the Restricted Marshallian Own- and Cross-Price Elasticities of Supply and Demand

a. $\sigma_{ij}(p;K)$ Evaluated at the Mean of the Sample Data

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Cattle Sales</th>
<th>Invtry Change</th>
<th>Crops</th>
<th>Custom Work</th>
<th>Mchnry Inputs</th>
<th>Cultvn Actvty</th>
<th>Lvstck Actvty</th>
<th>Supply Service</th>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>P1</td>
<td>1.998</td>
<td>-25.789</td>
<td>0.106</td>
<td>3.971</td>
<td>4.012</td>
<td>-3.109</td>
<td>-2.758</td>
<td>1.017</td>
</tr>
<tr>
<td>P2</td>
<td>-2.268</td>
<td>30.725</td>
<td>0.106</td>
<td>-2.358</td>
<td>0.202</td>
<td>0.852</td>
<td>0.089</td>
<td>0.201</td>
</tr>
<tr>
<td>P3</td>
<td>0.086</td>
<td>0.987</td>
<td>0.151</td>
<td>0.831</td>
<td>1.074</td>
<td>-0.582</td>
<td>-0.096</td>
<td>0.831</td>
</tr>
<tr>
<td>P4</td>
<td>0.325</td>
<td>-2.192</td>
<td>0.083</td>
<td>17.922</td>
<td>6.735</td>
<td>-0.434</td>
<td>1.272</td>
<td>-3.363</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Cattle Sales</th>
<th>Invtry Change</th>
<th>Crops</th>
<th>Custom Work</th>
<th>Mchnry Inputs</th>
<th>Cultvn Actvty</th>
<th>Lvstck Actvty</th>
<th>Supply Service</th>
</tr>
</thead>
<tbody>
<tr>
<td>P5</td>
<td>-1.299</td>
<td>-.744</td>
<td>-0.425</td>
<td>-26.678</td>
<td>-17.504</td>
<td>4.264</td>
<td>3.810</td>
<td>-.546</td>
</tr>
<tr>
<td>P6</td>
<td>0.728</td>
<td>-2.267</td>
<td>0.167</td>
<td>1.242</td>
<td>3.083</td>
<td>0.019</td>
<td>-.317</td>
<td>-.766</td>
</tr>
<tr>
<td>P7</td>
<td>0.646</td>
<td>-.238</td>
<td>0.028</td>
<td>-3.643</td>
<td>2.756</td>
<td>-.317</td>
<td>-1.784</td>
<td>-.238</td>
</tr>
<tr>
<td>P8</td>
<td>-.215</td>
<td>-.483</td>
<td>-0.215</td>
<td>8.713</td>
<td>-.357</td>
<td>-.693</td>
<td>-.215</td>
<td>4.305</td>
</tr>
</tbody>
</table>

b. Interpretation of Restricted Own- and Cross-Price Elasticities of Supply

<table>
<thead>
<tr>
<th>Quantity Price</th>
<th>Cattle Sales</th>
<th>Inventory Change</th>
<th>Crops</th>
<th>Custom Work Misc. Livestock</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cattle Sales</td>
<td>elastic</td>
<td>substitutes</td>
<td>complements</td>
<td>complements</td>
</tr>
<tr>
<td>Expected Cattle Price</td>
<td>substitutes</td>
<td>elastic</td>
<td>complements</td>
<td>substitutes</td>
</tr>
<tr>
<td>Crops</td>
<td>complements</td>
<td>complements</td>
<td>inelastic</td>
<td>complements</td>
</tr>
<tr>
<td>Custom Work Misc. Livestock</td>
<td>complements</td>
<td>substitutes</td>
<td>complements</td>
<td>elastic</td>
</tr>
</tbody>
</table>
For interpretation purposes, the table of reported elasticities (Table 4.a) may be segregated into quadrants, which correspond to (a) restricted own- and cross-price elasticity of supply, (b) restricted own- and cross-price elasticity of factor demand, (c) restricted elasticity of input demand with respect to output prices, and (d) restricted elasticity of output supply with respect to factor prices. The first such quadrant will be described.

The diagonal elements of Table 4.b show that supply of each of the outputs is elastic in the short run, except for crops. Once the land area is fixed for a given season, (and presumably seeded), apparently there is an inelastic supply response of crops to within-season price changes. This is in contrast to cattle sales and inventory, which appear to exhibit production flexibility in the short run.

The own-price elasticity associated with changes in inventory of cattle is extremely large, (30.7) which is consistent with the theoretical model. In the model the year-end decision to sell or keep some or all of the herd is thought to be conditioned largely on comparison of current versus (discounted) future prices. Except for the influence of price and quality differences among cattle sub-classes—which provide the firm with a separate portfolio choice problem—it is hard to show in a simple model why firms do not make an all-or-nothing inventory decision. Such a decision by firms would be consistent with an infinite own-price elasticity at some point. By modelling inventory adjustment in this fashion, one anticipates that the estimated elasticity may be very large.

The off-diagonal elements of the above chart show whether the output pairs are complements or substitutes within a production season (as the elasticities are positive or negative respectively). Each column refers to a specific output.

As expected, the quantities of cattle sold versus kept in inventory are substitute activities. The elasticity of cattle sales (from current production) with respect to expectations of future prices is considerably lower in absolute value (-2.3) than the elasticity of inventory change with respect to changes in the current market price of cattle (-25.8). This is consistent with firms having less flexibility to adjust within-season
production (when expectations change) than to adjust the sell-versus-keep proportions, once current market prices are observed. There may also be at work a process by which, within a season, firms filter or discount news about possible changes in future prices, and thus adjust to changes in expected prices more cautiously.

Ordinarily, one might think of all other outputs as being gross substitutes, whereas the chart shows many to be complements. Complementary short-run behaviour is consistent with jointness in production, such as cattle grazing on crop aftermath, and cattle feeding on lower quality grains which are produced but not marketed directly. This complementarity of outputs is also consistent with some farm households deciding to allocate more labour on the farm when a single output price rises (by reducing hours worked off the farm). This labour allocation, in turn, augments production of all outputs. Farm household decisions about farm versus off-farm employment are external to the restricted profit function model estimated in this paper except where they are captured by Custom Work activities), but have been the subject of numerous other studies. (See, for example, Lopez (1984) and Singh, Squire and Strauss (1986).)

The other quadrants of Table 4.a will not be interpreted in similar detail, although the following observation is made. Two of the own-price elasticities of demand are positive, although the objective of profit-maximization suggests they should be negative (through convexity of the restricted profit function in prices). Their positive signs follow directly from the observation that two of the diagonal elements of the Hessian matrix of the profit function with respect to input prices are not of the expected sign (as evaluated at the sample mean). This is a necessary condition for convexity in prices to hold.

To summarize the reported elasticities of choice, it appears that most are consistent with a priori beliefs about the short-run production technology in general, and with the relative responsiveness of current cattle sales and cattle inventory adjustments in particular. They show that within the production period, once capital commitments in the form of breeding herd size and available land area are made, firms do have some capacity to adjust production to input and output price changes.
The reader is cautioned that these reported elasticity values are generated from a model which does not satisfy convexity of the profit function in prices or concavity in fixed factors at the sample mean, and that, in particular, two of the own-price elasticities of demand do not have the expected sign.

B. ESTIMATES OF THE TOTAL ELASTICITY OF CATTLE SUPPLY

Gordon (1984, p.118) notes that in a model such as this, the own-price elasticity of supply of cattle sales portrays a partial response and is defined under the assumption that expectations about future prices are not influenced by changes in current prices. If one accommodates a change in price expectations whenever the current price changes, then one can calculate a (short-run) total elasticity of cattle supply. As a theoretical and empirical issue, it will be possible for such an elasticity to be positive or negative. Firms may choose to sell fewer cattle in the current period in response to an increase in the current price provided that there is a sufficiently large corresponding increase in (the present value of) the expected price next period.

In the following illustration, allow $x_1$ to refer to the quantity of cattle sales, while $p_1$ and $p_2$ refer to the current price and the (discounted) expectation of the price next year. One obtains a measure of the total elasticity of cattle supply by means of:

\[
\frac{dx_1}{dp_1} = \frac{\partial x_1}{\partial p_1} + \frac{\partial x_1}{\partial p_2} \frac{\partial p_2}{\partial p_1}, \text{ which gives:}
\]

\[
\frac{dx_1}{dp_1} \frac{p_1}{x_1} = \left( \frac{\partial x_1}{\partial p_1} \frac{p_1}{x_1} \right) + \left( \frac{\partial x_1}{\partial p_2} \frac{p_2}{x_1} \right) \left( \frac{\partial p_2}{\partial p_1} \frac{p_1}{p_2} \right)
\]

Total Elasticity of Supply = Direct Elasticity of Supply + Cross-Price Elasticity of Supply (Expectation of Supply)

\[
\eta_{11}(p;K) = \sigma_{11}(p;K) + \sigma_{12}(p;K) + \left( \frac{\partial p_2}{\partial p_1} \frac{p_1}{p_2} \right).
\]

In this case, one may evaluate a restricted or short-run total elasticity of cattle supply by employing the calculated values of $\sigma_{11}(p;K)$ and $\sigma_{12}(p;K)$.
from Table 4 (as evaluated at the mean of the sample data).

\[
(24) \quad \eta_{11} = 1.998 + (-2.268) \left( \frac{\partial p_2 / \partial p_1}{(1.086)/(1.156)} \right) \\
= 1.998 - 2.131 \left( \frac{\partial p_2 / \partial p_1}{3p_2 / 3p_1} \right)
\]

Examination of (24) reveals that any value of $\partial p_2 / \partial p_1 < 0.938$ will ensure that $\eta_{11}$ is strictly positive.

One source of estimates of the value of $\partial p_2 / \partial p_1$ is the sample auto-correlation coefficients estimated in the ARIMA models used to generate the series of price expectations, $p_2$. These models employ monthly data, so that the parameters associated with lag 12 (or, more generally, lags 11 through 13) will indicate the effect on the current price of a change in the price last year ($\partial p_2 / \partial p_1$).

Across all of the ARIMA models estimated, the largest positive values of any estimated parameters associated with lags 11, 12 and 13 are 0.487, 0.344 and 0.135, respectively. As each of these values satisfies the condition $\partial p_2 / \partial p_1 < 0.938$, it follows that the total elasticity of cattle supply will be positive when evaluated at the mean of the sample data.

The mean estimate of $\partial p_2 / \partial p_1$ for the prices of D1,D2 Cows at Edmonton is 0.234, which by (24) gives $\eta_{11} = 1.50$. The mean estimate of $\partial p_2 / \partial p_1$ for the prices of Steer Calves at Edmonton is 0.252, which yields a value, $\eta_{11} = 1.46$. These estimated values are consistent with cattle sales in the current period increasing in response to increases in the current price, even when price expectations are allowed to change. These results may be compared with a similar estimate of 1.41 provided by Gordon for the years 1978-1981 (1984, p.150).

C. ESTIMATES OF THE SHADOW VALUE OF CAPITAL

As one indicator of whether the econometric estimates provide a reasonable description of cattle production in Western Canada during the years 1983 through 1986, one may employ the estimates to determine the values $\partial g / \partial K_k (k = 1, 2)$, using expression (16). These values represent the expected

\[\text{50}\]

\[\text{18} \text{Gordon (1984) reviews other empirical research which purports to show a negative total supply response to own price in cattle production.}\]
marginal value product of each type of capital (or equivalently, the shadow value of capital), assuming that its (fixed) level of use could be increased at the margin within the average season.

Evaluated at the sample mean, the estimated shadow value of an acre of land is $14.25, and the shadow value of an additional beef cow is $202. This says that for the "average" producer, the use of one more acre of land in a given year would contribute $14.25 more to revenues than to variable costs. With hindsight, this is the largest amount such a producer should have been willing to pay to rent more land, for example.

The shadow value of $202 for cows implies that keeping a marginally larger herd would have made a net contribution to restricted profit in this amount. That net contribution would have taken the form of the difference between the revenue from an extra calf sold (plus some increase in the revenue from the sale of culled cows) and the extra costs of feeding and raising these animals. Thus, the shadow value corresponds to the flow value of services provided by additional breeding stock, which would normally be less than the capital value at market.

Although there is no active and observable rental market for breeding stock, some data are available on land rental rates. Annual surveys conducted by the Alberta Department of Agriculture for the years 1984 and 1985 suggest farm land rental rates in that province were on average somewhat higher (Alberta Department of Agriculture, 1984 and 1985). Reported rents for crop land of various qualities generally range from less than ten dollars to sixty dollars per acre, for a one year lease where the landlord pays the property taxes. Although fifteen dollars per acre is a commonly quoted rate for crop land, the mean appears to be closer to thirty dollars per acre. Rates for pasture land are lower, and generally are not quoted on an area basis.

D. HYPOTHESES ABOUT OPTIMAL FIRM RESPONSE TO OUTPUT PRICE UNCERTAINTY

It remains to evaluate the optimal response of the risk-neutral firm to changes in the variability of output prices, based on the econometric estimates. As shown in Section II, this response will depend on whether larger levels of use of the fixed factors (capital) increase or decrease the firm's short-run elasticities of choice. Of particular interest is the firm's
adjustment to increased variability of current and future prices for cattle.

Firms will increase their use of a specific fixed factor (breeding herd or land area) in response to increased input-price or output-price variability, if the expected marginal value product function for that factor is convex in the price concerned. (Strict) convexity is evident if \( \frac{\partial^3 g(p;K)}{\partial K \partial p_i^2} \) is (strictly) positive. Opposite signs indicate concavity of the marginal value product function, and a corresponding decrease in the level of use of the fixed factor.

Optimal firm behaviour may also be influenced by increased correlation of net output prices with each other, be they input or output prices. The determinant of optimal firm response to increased price correlation will be the effect that increased levels of fixed factors will have on restricted cross-price elasticities of supply or demand. For example, if increasing the size of the breeding herd increases the cross-price elasticity of cattle supply with respect to the price of crops, then risk-neutral firms would wish to increase herd size in response to any increase in the correlation of cattle and crop prices. This characteristic of the firm's technology is captured in a positive sign for the third-order derivatives of the form \( \frac{\partial^3 g(p;K)}{\partial K \partial p_i^2} \) \((i \neq j)\).

Expressions (14) and (15) characterize the sign of \( \frac{\partial^3 g(p;K)}{\partial K \partial p_i^2} \) and \( \frac{\partial^3 g(p;K)}{\partial K \partial p_i \partial p_j} \) in terms of the estimated parameters, the estimated net supply shares, and the sample data, and will be evaluated at the mean of the sample data.

The calculated values of the various third-order derivatives should be interpreted in the sense of partial changes. This is particularly relevant where the two fixed factors, land and breeding herd, may be gross substitutes or complements. For example, it may be the case that, ceteris paribus, an increase in the size of the breeding herd will increase the firm's short-run own-price elasticity of output supply for cattle. If so, then one would expect the herd size to be increased if firms foresaw an increase in the variability of cattle prices. However, once the level of use of the other fixed factor, land, was allowed to adjust, one would expect that it would also
increase if it were complementary with the breeding herd, and to decrease if
they were gross substitutes (Epstein, 1978, p.258). The estimates which have
been generated from (12) and (13) are short-run in nature, do not describe
fully this relationship between fixed factors. Thus, they should not be
interpreted as characterizing all of the factor adjustments which could occur.

Hypotheses which define the direction of the firm's optimal response
cannot be formally tested in a statistical sense given the absence of standard
errors for the expressions which are to be signed. Accordingly, the results
reported below must be interpreted subject to the limitation that they may not
be significantly different than zero at the 5% or 10% level of significance.

Table 5 reports the numerical values which are obtained by evaluating the
expressions (14) and (15) with respect to each fixed factor using the
econometric estimates described above. Tables 5.a and 5.b will be described
in turn, reviewing both the own-price and cross-price effects.

1. Optimal Adjustment of the Size of Breeding Herd

a. Own-Price Effects

The effect of the size of the breeding herd on the responsiveness of each
net output to a change in its own price is given by the sign of the diagonal
elements in the Table 5.a. Among the outputs, all of the signs are positive,
except for the sign associated with Crop production when estimated at the
sample mean. These signs indicate that all except one of the expected value
of marginal product functions are convex in the output prices.

An interpretation of this result is that a marginal increase in the size
of a firm's breeding herd will increase that firm's short-run own-price
elasticity of supply for Cattle Sales, Cattle Inventory adjustments, and
Custom Work and Other Livestock. Equivalently, if there is an increase
(decrease) in the variability of any of those output prices, then the expected
marginal value of investments in the breeding herd will increase (decrease)
and an expected-profit-maximizing firm will choose to employ more of that
fixed factor.

Provided the production technology is such that other variable inputs
which are employed in the short run are not technically competitive with the
Table 5: Estimated Values of the Third-Order Derivatives of the Form (14) and (15)

a. \( \frac{\partial^3 g(p,K)}{\partial K \partial p_i \partial p_j} \) Evaluated at the Mean of the Sample Data

\( K_1 = \text{BREEDING HERD SIZE} \)

<table>
<thead>
<tr>
<th></th>
<th>Cattle Sales</th>
<th>Invtry Change</th>
<th>Crops</th>
<th>Custom Work</th>
<th>Mchnry Inputs</th>
<th>Cultvn Actvty</th>
<th>Lvstck Actvty</th>
<th>Supply Service</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>P1</td>
<td>P2</td>
<td>P3</td>
<td>P4</td>
<td>P5</td>
<td>P6</td>
<td>P7</td>
<td>P8</td>
</tr>
<tr>
<td>P1</td>
<td>1877.43</td>
<td>-1356.2</td>
<td>445.110</td>
<td>61.4284</td>
<td>-509.64</td>
<td>118.068</td>
<td>-411.43</td>
<td>-113.62</td>
</tr>
<tr>
<td>P2</td>
<td>-1356.2</td>
<td>1011.18</td>
<td>-232.85</td>
<td>-39.812</td>
<td>582.678</td>
<td>-47.204</td>
<td>-9.9940</td>
<td>-8.4220</td>
</tr>
<tr>
<td>P5</td>
<td>-509.64</td>
<td>582.678</td>
<td>209.042</td>
<td>-1119.1</td>
<td>1363.22</td>
<td>-222.03</td>
<td>-530.88</td>
<td>21.8120</td>
</tr>
<tr>
<td>P6</td>
<td>118.068</td>
<td>-47.204</td>
<td>-89.521</td>
<td>472.799</td>
<td>-222.03</td>
<td>-567.16</td>
<td>308.016</td>
<td>39.0839</td>
</tr>
<tr>
<td>P7</td>
<td>-411.43</td>
<td>-9.9940</td>
<td>-138.38</td>
<td>401.473</td>
<td>-530.88</td>
<td>308.016</td>
<td>439.222</td>
<td>32.0206</td>
</tr>
</tbody>
</table>

b. \( \frac{\partial^3 g(p,K)}{\partial K^2 \partial p_i \partial p_j} \) Evaluated at the Mean of the Sample Data

\( K_2 = \text{AVAILABLE LAND AREA} \)

<table>
<thead>
<tr>
<th></th>
<th>Cattle Sales</th>
<th>Invtry Change</th>
<th>Crops</th>
<th>Custom Work</th>
<th>Mchnry Inputs</th>
<th>Cultvn Actvty</th>
<th>Lvstck Actvty</th>
<th>Supply Service</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>P1</td>
<td>P2</td>
<td>P3</td>
<td>P4</td>
<td>P5</td>
<td>P6</td>
<td>P7</td>
<td>P8</td>
</tr>
<tr>
<td>P5</td>
<td>6.35523</td>
<td>-25.986</td>
<td>-14.028</td>
<td>57.2750</td>
<td>71.42879</td>
<td>43.1070</td>
<td>4.31070</td>
<td>2.58348</td>
</tr>
<tr>
<td>P7</td>
<td>12.0659</td>
<td>-.44753</td>
<td>5.03902</td>
<td>17.636</td>
<td>74.2879</td>
<td>43.1070</td>
<td>145785</td>
<td>-12.784</td>
</tr>
</tbody>
</table>
fixed factors (i.e. $f_{12} \geq 0$), then it follows that the level of production will also increase in response to the firm's decision to keep a larger breeding herd.

Based on the negative sign associated with the variability of Crops prices, it would appear that maintaining a larger breeding herd will reduce the short-run own-price elasticity of supply of Crops. *Ceteris paribus*, a risk-neutral firm would reduce the size of its breeding herd in response to increased variability of Crops prices, so as to preserve (or augment) its short-run ability to adjust Crop supply. A conventional wisdom is that firms will increase cattle production when grain prices are low, as cattle represent an alternative marketing channel which provides added value to the grain produced on the farm. The suggestion here that smaller breeding herds should accompany increased variability of grain prices is not necessarily contrary to the conventional view which associates smaller herd size with higher grain prices. One view relates to price dispersion and the other to price level.

Among the variable inputs, all of the own-price effects are positive with the exception of Cultivation Activities. Recall that Cultivation Activities and Supply and Services (inputs) had positive restricted own-price elasticities of factor demand, evaluated at the sample mean, so that the signs of their third-order derivatives should also be evaluated with caution.

As written, the own-price effects on variable inputs suggest that greater investment in the breeding herd will increase the short-run elasticity of factor demand for three of the inputs. Therefore, an expected-profit-maximizing firm would increase its herd size if it faced an increase in the variability of any of these three factor prices. The rationale for this behaviour appears to be that investments in the breeding herd are investments in short-run flexibility for most variable input (and output) activities.

The negative sign associated with Cultivation inputs suggests that, *ceteris paribus*, firms with larger breeding herds have less short-run flexibility to respond to changes in the price of Cultivation inputs. This is consistent with the result that larger breeding herds reduce the short-run own-price elasticity of Crop supply, where the level of Crop production bears an obvious technical connection with the level of Cultivation inputs used.
b. Cross-Price Effects

The sign of the off-diagonal elements of Table 5.a may be interpreted with respect to firm response to increased or decreased correlation of pairs of net output prices. For example, the more highly correlated that current and future cattle prices become, the smaller will be the optimal size of the breeding herd. In such a situation, a larger herd appears to reduce the cross-price elasticity of supply of cattle sales with respect to future cattle prices. This is in contrast to the added short-run flexibility it provides when only one of the two prices becomes more variable.

If cattle prices and crop prices become more highly correlated, then the positive cross-price effect in Table 5.a suggests a larger breeding herd will be optimal. For example, when cattle and grain prices become more (negatively) correlated and follow opposite price cycles, a firm will be better poised to deal with output price uncertainty by having a larger breeding herd. Conversely, where the (anticipated) price of cattle next year becomes more highly correlated with crop price this year, then holding a smaller breeding herd of cattle now is indicated. Presumably, this reflects the balance between maintaining a lower stock when crop prices are more variable and a higher stock when future cattle prices are more variable.

Similar interpretations of Table 5.a are possible for changes in the degree of correlation between an input price and an output price, and between input prices. For example, if cattle prices were to become more highly correlated with any of the variable factor prices, except one, then firms should reduce the size of the breeding herd to preserve short-run flexibility. The one exception is Cultivation inputs. Recall that a smaller breeding herd is prescribed when the Cultivation input price becomes more variable. The advisability of lowering herd size to deal with increased dispersion of these input prices also appears to influence the optimal capital adjustment when these Cultivation prices become more highly correlated with cattle prices.

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19 One might associate higher correlation of current and future prices with more pronounced price cycles in the cattle industry, for example.
2. Optimal Adjustment of the Available Land Area

a. Own-Price Effects

Employing the values reported at the mean of the sample data in Table 5.b, it appears that when the variability of current and future cattle prices is reduced by a participatory stabilization program, the area of land in use will increase. Conversely, if price variability increases, firms would choose to reduce the area of land they commit to use. In this case, operating from a smaller land base appears to allow the firm greater within-season flexibility to adjust the supply of cattle to changing prices. Presumably, the within-season responsiveness of cattle output is not augmented by having greater potential access to grazing lands or feed produced on-farm.

Within a specific season, it may be the case that cattle production does not rely on land as an intensive input at the margin. A firm wishing to take advantage of an unexpectedly high cattle price in the short-run may well choose to delay the delivery date, or to improve the feed ration for all cattle it plans to market. Neither of these adjustments necessarily requires a larger land area, per se.

Conversely, where the price of crops becomes more variable, Table 5.b suggests that firms should increase their level of use of land as a fixed factor. This appears to be a case where it is easier to reduce production or deliveries if prices should unexpectedly fall, than to increase production or deliveries (from a fixed land base) if prices should suddenly rise. Faced with greater uncertainty, firms choose to increase short-run flexibility by using more land.

On the input side, having a larger land area helps firms deal with increased own-price variability of Machinery Use and Cultivation Activities. When the increased price uncertainty affects Livestock Production activities or Supplies and Services, firms will commit to use less land.

b. Cross-Price Effects

According to Table 5.b, if the current prices of cattle and crops become more highly correlated then firms will choose to reduce the area of land they use. Although a larger land area appears to help firms respond to crop price
uncertainty, it reduces responsiveness to cattle prices. The latter effect appears to dominate when the two prices become more highly correlated. The opposite adjustment in land area is prescribed when future cattle prices and current crops prices become more highly correlated. In this case it appears that the effect first mentioned above dominates.

When the two cattle price series (present versus future) become more highly correlated, firms should increase their use of land. This contrasts with the land area reduction which would accompany increased variability of either cattle price alone. Perhaps when the two cattle price series become more highly correlated, the firm's short-run adjustments depend more on its ability to switch between crop and cattle production than on its ability to influence the timing of cattle sales (through inventory adjustments). In this case, augmenting land use will increase the short-run own-price elasticity of crop supply, which appears to increase the cross-price elasticity as well.

Having identified the optimal direction of adjustment in the use of each fixed factor to various price changes, it will now be of interest to relate these observations to the impact of a stylized participatory stabilization scheme of the type described in Section II.

E. IMPLICATIONS FOR PARTICIPATORY STABILIZATION SCHEMES

Under the assumptions which have been made about participatory stabilization schemes, such as the National Tripartite Stabilization Program for Beef, these programs have the effect of narrowing the dispersion of total market returns to participating producers. The maximum price which would be obtained in any period by a non-participant is lowered for participants by virtue of the per period premium which is paid. Similarly, the minimum price which would otherwise be received is augmented by stabilization payments, net of premiums, to participants. Generally, the dispersion of total market returns is not reduced in a symmetric fashion relative to the expected mean, even in the case where the program is actuarially fair.20

20Two principal differences between the NTPSP and the stylized stabilization scheme characterized in the analysis of Section II are that the NTPSP is not self-financing (that is, it does not provide actuarially fair insurance against market risk) and that the NTPSP has separate program components which cover cow-calf and slaughter cattle production.
It is assumed that the multi-period production technology associated with cattle production will expose each firm to some degree of market risk. The possibility of financial loss in any period is thought to depend as much or more on the variability of input and output prices as on any other technical risks associated with the production process, per se.

The focus of this paper is on the short-run or within-season behaviour of firms. Although cattle producers may be exposed to market risk which spans many years or production periods, it is thought that an important part of this risk—perhaps the largest part—is associated with short-run price movements. These occur within the interval when the firm has necessarily made precommitments to use certain fixed factors, such as the breeding herd and the land base.

Consider the case of a risk-neutral producer faced with the decision to subscribe voluntarily to such a program. Presumably, the choice will be based on a subjective assessment of whether or not the program presents an expectation of return which is at least as favourable as a fair bet. Moreover, to be attractive, the program would have to provide a better expectation of return than other available risk-management strategies. These might include participation in futures or forward markets, self-insurance through a savings or investment plan, greater diversification of outputs, and so on.

One of the findings of this research is that, within a production season, firms do have short-run supply responses for cattle which are elastic. That is, based on the mean observation, \( \sigma(p;K) > 1 \) for all cattle production, whether the animals are sold that year or held over in inventory for sale in a future period. Moreover, firms also exhibit elastic demands for the group of variable inputs used to produce cattle, which includes feed and supplements, veterinary supplies, and replacement or feeder animals.

Stated another way, the short-run supply curves associated with cattle production are not vertical or negatively sloped on the basis of the reported estimates, nor are the factor demand curves vertical. The production technology exhibits some flexibility in the short run, such that output responds to realized prices which are higher or lower than those expected.
Based on the results reported in Table 5.a, for a risk-neutral firm the optimal response to a mean-preserving decrease in the dispersion of output prices would be a reduction in the size of the breeding herd. Conversely, increased investment in the breeding herd would increase the short-run own-price elasticity of supply for such firms. An increase would be the optimal response if the dispersion of prices increased in a mean-preserving way, such as if the firm withdrew from participation in the stabilization program.

These reported values are consistent with a reduction in optimal breeding herd size in response to joining a participatory stabilization scheme, provided that (i) firms are risk-neutral, and (ii) the change in the dispersion of total market returns is mean-preserving (equivalently, the program is perceived to be actuarially fair, representing a fair bet to producers).

While such an hypothesis cannot be tested formally on the basis of these data, this particular result is supportive of there being a contraction of cattle herds when price variability is reduced. That is, the supply curve of the firm (as in Figure 1.b) may shift to the left due to this reduction in the use of a fixed factor.

For firms which are risk-averse, or for programs which offer an expectation of return which is more favourable than a fair bet, there would likely be separate upward pressures on the optimal level of use of the fixed factor (and on output supply in general), which are associated with risk aversion and the expected revenue gain, per se.

Nonetheless, these results provide empirical support for the proposition that the technology of cattle production may encourage risk-neutral firms to keep larger herds and to produce more cattle during periods of greater price variability. This challenges the notion that all stabilization programs which affect production will necessarily increase it. These appear to be the first empirical results on this point which have been reported, so it is not yet possible to draw comparisons with them from the literature. One should not conclude that these results will be robust to changes in the data sample or in the functional forms employed. Although the results are supportive of specific types of adjustment, they are accompanied by a number of caveats.
about their statistical significance.

Whereas the results of this section show the qualitative response of levels of use of the fixed factor to changes in the variability or correlation of specific net output prices, there are a number of reasons why one should be more cautious in characterizing the response of production in total.

First among these are the "real world" likelihoods that firms may not behave as if risk-neutral, and that programs may appear to offer an expectation of return greater than a fair bet. Also, if there is free exit and entry, and if market prices are influenced by the collective responses of individual firms (such as an eventual increase in cattle prices when breeding herds are reduced), then one must take the effects of such price changes into effect when determining long-run output response. (See Horbulyk, 1989, Chapter 4.)

Another difficulty in characterizing unambiguously firm output response from the results reported here, is that there are multiple outputs and more than one fixed factor. As shown in this section, there would be opposing partial adjustment effects in the level of each fixed factor, and there exists the further possibility of complementarity or substitutability in the use of both.

Based on Table 5, the direction of optimal response in the use of the fixed factor land to changes in present and future cattle price variability is opposite to the direction of response of herd size. Both responses are based on partial adjustments which do not show how output would be affected if both occurred simultaneously. Further, neither response incorporates the possibility that there may be complementarity or substitution between the two fixed factors. For example, if the breeding herd and the land area are gross complements, then a decision to decrease the size of the breeding herd might suggest an accompanying decrease in the area of land used. This decrease could offset the increase in land area which would be optimal if land area alone were to be adjusted.

Casual observation of cattle production practices in Western Canada suggests that the specific decision to keep larger (or smaller) breeding herds is sufficient to ensure greater (or lesser) amounts of cattle production and
sales in the long run, independent of whether the commensurate adjustment in land use by firms is an increase or a decrease. These issues of long-run production response to combined adjustments in the level of fixed factors, cannot be resolved fully by reference to estimated characteristics of the short-run production technology or profit function.

One final implication of the reported results for the operation of participatory stabilization schemes concerns the distinction between programs which support output prices and programs which support the margin of price above estimated variable costs. Programs which support the market price will, in general, reduce the dispersion of total market return, as described above. Ceteris paribus, risk-neutral firms will adjust to such programs by reducing their investment in the breeding herd.

Programs which support the margin by which revenues exceed estimated cash costs not only reduce the dispersion of total market return, but they tend to dampen the effect on the firm of fluctuations in the cost of the principal inputs used in cattle production. (That is, the effects of peak grain prices are reduced, for example, through larger payments to support the defined margin.) The cross-price effects described in Table 5.a suggest that firms reduce the optimal size of breeding herd when cattle prices and cattle production costs become more highly correlated. The effect of margin stabilization programs for participating producers is to increase the perceived correlation between the total market return they receive (including net stabilization payments) and variable input prices. This is another case where the optimal adjustment by the risk-neutral firm to such a program would be to decrease the size of the breeding herd.

The empirical findings presented here are accompanied by the caveat that the short-run elasticities, and responses in these elasticities to changes in the level of fixed factors, are generated from an econometric model which does not satisfy convexity of the profit function in prices or concavity in fixed factors at the sample mean. Further, two of the own-price elasticities of factor demand do not have the expected sign.

21 The NTPSP for feeder calves is an example of the former, whereas the programs for feeder cattle and slaughter cattle are examples of the latter.
Similarly, the positive or negative responses in the level of use of fixed factors and cattle supply are not hypotheses which have been formally tested in a statistical sense. For example, reported results do not include upper and lower bounds for a confidence interval within which the true value of each of these estimates would be expected to fall with a stated degree of confidence. Accordingly, one cannot reject on statistical grounds the hypothesis that the true value of any particular elasticity value might be zero, or of the opposite sign.

Given the data which are currently available, there are several directions in which this work could be extended.

One line of inquiry would seek to quantify more fully the (qualitative) adjustments to changing price variability which are described in this paper. The approach of such research might be to estimate the proportional changes in the levels of use of fixed factors associated with stated changes in the dispersion of output prices. For specific groups of firms, these proportional factor changes could be used to evaluate associated output changes. One could then simulate the output effects of alternate stabilization program designs, for example.

Another area for future empirical research concerns industry-level response or adjustment. By examining the data from all firms, including those which enter and exit cattle production over the five year period 1983-1987, it may be possible to characterize patterns of supply response, for example, which are not observable in short-run estimates.

In a broader context, estimates of the production structure of the cattle industry might serve as a basis for quantifying some of the welfare consequences of alternate stabilization initiatives. This could include comparisons of stabilization schemes with recent proposals for "decoupled" programs (Gilson, 1988; Warley, 1988), which are designed to be neutral in their effect on output and trade.
REFERENCES


