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A NEOCLASSICAL ANALYSIS OF THE DEMAND FOR CEREALS IN EGYPT

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"Abstract"

Direct and indirect translog production functions provide cost share equations which are flexible and are consistent with microeconomic theory. This paper combines recent developments in the technique of estimating production and consumer demand equation systems and in computing the standard errors of the elasticities accurately in studying the demand for cereal products in Egypt. The gain in accuracy from not using the approximation method--commonly used in the literature--is presented theoretically and empirically. The data employed in this paper are Egyptian and are for wheat, maize, and rice; these are interpreted as factor inputs that produce a specific product which we will refer to as "cereals". Our findings show that the Allen elasticities of substitution (AES) between wheat and maize and wheat and rice exhibit significant (statistically) substitution. On the other hand, the relationship between maize and rice is independent based, again, on a lack of statistical significance. Indeed, there is fairly convincing evidence that rice is functionally weakly separable from wheat and maize. The overall findings suggest that the neoclassical model offers considerable promise for modeling the household's demand for cereals in Egypt.

I. Introduction

It is generally agreed that the issue of substitutability/complementarity relationships among cereal products (wheat, maize, and rice) in Egypt has considerable policy significance, in view of the government's continuing effort to provide adequate cereals with the available resources. It turns out, however, that there is little agreement concerning the degree to which the components of cereals are substitutes, complements, or, of course, independent of each other and with this state of information on the demand for the product, it is clear that government policy would only accidently provide an optional solution to its allocation problem. It is, then, clear, that further econometric analysis of the structure of the consumer demand for cereals is in order, and to this end we propose employing the Neoclassical cost function in the present study. In this context, following the usual procedure, three crops have been choosen to represent the household's demand for cereals; these are wheat, maize and rice. These three are clearly the most important basic food crops in Egypt, with the first two accounting for nearly 40 percent of total food consumption.

Unfortunately, much of the empirical work dealing with the demand for food takes an excessively restrictive approach to modeling the consumer's behavior. This is to aggregate products to the level of "food" with the untested rationale that the components of "food" are homogenenous in providing food services and are distinct from the goods themselves. This separability, indeed is an especially unlikely and undesirable maintained hypothesis in this work.(1) Futhermore, the resulting lack of an explicit unifying theoretical framework, in addition to being methodologically unsatisfactory, renders it difficult to compare alternative findings obtained from different studies in a meaningfully consisent manner.(2)

The approach this study will take is based on a two-stage optimizing procedure. In the first stage the Egyptian householder determines the level of comsumption he requires from cereals. In the second stage, given this predetermined level, entrepreneurial decisions are made with respect to the determinations of the optimal levels of each of the individual cereal products. In this event--in which the cereal output level is held constant--it is reasonable to argue that the input levels are the endogeneous variables. Furthermore, one might reasonably assume that households are in competition with each other for cereals and hence that cereal prices can reasonably be taken as the exogeneous variables. With this specification, estimation of the cost function rather than the production function becomes an attractive alternative.

The objective of this paper is to derive a system of aggregate cereal products demand equations from a household model of production - maximizing behavior by exploiting recent developments in duality theory. The model is applied to annual household Egyptian data for 1964-79. Focal points in the study are the specification of the demand functions for the different kinds of cereals and the estimation of the degree to which they are substitutes for each other as measured by the Allen elasticities of Substitution (AES). It should also be pointed out that the empirical results, in addition to providing some interesting and useful information regarding different measures of substitutability among cereal demands also provide an explicit test of the neoclassical maximizing hypothesis which underlies the model. That is, in the context of the Translog functional form, we are able to test and to evaluate, over the given data, the appropriateness of neoclassical production theory for this problem. We consider this an equally important purpose of this paper in view of the stimulation it could provide to similar studies. An additional topic, and one in which we believe a contribution has been made, is to show the bias due to computing the standard errors of the AES using the approximation method.

The plan of this paper is as follows. Section II develops the econometric model which results in estimable demand functions for cereals derived from cost minimizing behavior, from which estimates of the AES and price elasticities may be obtained. Section III presents statistical tests of the restrictions on the translog parameters; these restrictions are symmetry, adding up, and weak separability. Section IV describes a method for obtaining estimates of the standard errors for the estimates of the elasticities.(3) Section V, then, describes the data and the estimation technique applicable to problems of this type. Section VI presents and discusses the final estimates of the relevant parameters. In section VII we summarize and conclude the paper.

II. Households' Demand for Cereals

It is possible to develop an analytical model for the consumption of cereals based on neoclassical technology to fit the case of Egypt. In our analytical model we assume that the household faces a fixed product level which is derived from a set of cereals and seeks to minimize the costs of producing this mix. In this framework the idea is that cereal products are taken as conventional factor inputs, but with an important distinction. This is that rather than a conventional production problem we are visualizing the consumer of cereals as operating what is known as a "delivered output" production function. In this use we are not discussing "production" in the sense of its physical meaning and, indeed, it is quite natural to apply the model to the consumer. More specifically, the output (derived from the mix of cereals) that the Egyptian household seeks is achieved by mixing the specific inputs of wheat, maize, and rice, with alternative feasible arrangements of the technical processes involved. Indeed, this framework offers an analytically convenient representation of the features of the production technology that are relevant and interesting for purpose of economic analysis.

In what follows, we will assume that the household minimizes the cost of producing a given output subject to a production function constraint which includes wheat, maize, and rice as factor inputs. The first-order condition for a minimum, together with the assumption of a well-behaved neoclassical production function, permit one to derive the neoclassical cost function.[(4)] It is possible, therefore, to describe the nature of the production technology from the cost function by application of the duality theorems of Uzawa (1962), Shephard (1970), and McFadden (1978).[(5)] Moreover, we specify a translog cost function introduced by

Christensen, Jorgenson, and Lau (1971) to represent the outcome of the minimization process. This form does not restrict *a priori* the nature of the AES, and allows direct testing of the neoclassical properties.

To develop the model, let q denote an N-dimensional vector of commodities, let P denote the vector of corresponding prices, and let Y denote the household's "real cereals product" derived from a mix of wheat (W), maize (M), and rice (R) during the period under consideration. Then, the real product (Y) produced by any mix of the N different kinds of cereals may be specified by the function:

$$Y_t = \phi_t (q_1, q_2, \ldots, q_N), \qquad i = 1, 2, \ldots, N \qquad t = 1, 2, \ldots, n \tag{1}$$

A cost function corresponds to a homothetic production structure if and only if the cost function can be written as a separable function in output and factor prices (see Diewert [1974]). A homothetic production structure is further restricted to be homogeneous if and only if the elasticity of cost with respect to output is constant. Thus, in order to correspond to a well-behaved production function, a cost function must be homogeneous of degree one in prices; that is, for a fixed level of output total cost must increase proportionally when all prices increase proportionally. Therefore, given the prices of inputs the households are facing, total output value--or cost--of the cereals mix C = (Y, P) is given as:

$$C - \sum_{i}^{N} P_i \, q_i = 0 \qquad i = 1, 2, \ldots, N \tag{2}$$

It is, then, assumed that the household wishes to minimize (2) subject to some derived level of real output, $\overline{Y}$. More specifically, the household wishes to

$$\underset{\underline{q}}{\text{Min.}} \quad C = \sum_{i}^{N} P_i \, q_i, \qquad i = 1, 2, \ldots, N \tag{3}$$

Subject to

$$\bar{Y} = \phi\ (q_1, q_2, \ldots, q_N). \tag{4}$$

Finally, assuming certain regularity conditions are satisfied, the following first order conditions are necessary and sufficient for a minimum:

$$\phi_i/\phi_j = P_i/P_j, \tag{5}$$

$$i, j = 1, 2, \ldots, N.$$

$$\bar{Y} = \phi\ (q_1, q_2, \ldots, q_N) \tag{6}$$

where $\phi_i = \partial\phi/\partial q_i$.

Returning to the question of measuring the substitutability among alternative mixes of inputs, one is immediately confronted by a choice between functional forms that exhibit good behavior globally and those that possess flexibility and impose minimum *a priori* restrictions. Certain simple forms, such as the CD or the CES satisfy certain regularity conditions globally, but place unnecessarily stringent conditions on the possible values of the estimated AES.(6) Fortunately, recent work in duality (e.g. Diewert [1971], Fuss [1978], and McFadden [1978]) in conjuction with flexible functional forms indicate a way to bypass such undesirable restrictions.

If we assume that there exists a minimum cost function that meets all the regularity conditions, then we can move to specify a parametric form for this cost function that permits estimation of the AES with few restrictions. A Translog form seems especially appropriate in the case in hand for two main reasons: i) it allows direct estimation of the substitution elasticities and ii) it entials no *a priori* restrictions regarding either their values or their variability.

In its general form the Translog cost function may be written as

$$\log C = \log \alpha_0 + \sum_i \alpha_i (\log P_i) + \tfrac{1}{2} \sum_i \sum_j \beta_{ij} (\log P_i)(\log P_j)$$
$$+ \sum_i u_i (\log Y)(\log P_i) + u (\log Y) + \gamma/2 (\log Y)^2 \quad (7)$$

Because of the assumption that real output levels are constant at $\overline{Y}$ we have, in effect, assumed linear homogeneity in output. This implies that

$$u_i = 0 \ ,$$
$$u = 1 \ ,$$
$$\gamma = 0 \ , \quad (8)$$

as maintained hypotheses. In this event the translog cost function is written as:

$$\log C = \log \alpha_0 + \sum_i \alpha_i \log P_i + \tfrac{1}{2} \sum_i \sum_j \beta_{ij} \log P_i \log P_j \quad (9)$$
$$i, j = 1, 2, \ldots, N.$$

where α and β are the parameters of the model. Linear homogenity in prices along with symmetry then imply the following restrictions on these parameters:

$$\beta_{ij} = \beta_{ji} \ ,$$
$$\sum_j \alpha_j = 1 \ ,$$
$$\sum_j \beta_{ij} = \sum_i \beta_{ij} = 0 \quad \forall_{i,j}. \qquad \forall_{i,j} \quad (10)$$

Differentiating (9) logarithmically and applying Shephard's Lemma yields three factor input cost share equations as linear functions of the logarithms of the factor prices. To do this, first partially differentiate the cost function with respect to the factor prices, that is, calculate

$$\partial C/\partial P_i = q_i \quad (11)$$

This result, known as Shephard's Lemma (Shephard [1953]), is conveniently expressed in logarithmic form for the translog cost function as

$$\partial \log C/\partial \log P_i = P_i q_i/C = S_i \quad , \tag{12}$$

where S_i indicates the cost share of the i^{th} factor input. The translog cost function, therefore, yields the following linear cost share equations (with Y fixed to be constant at $\bar{Y}$):

$$S_i = \alpha_i + \sum_j \beta_{ij} \log P_j \quad , \qquad i, j, = 1, 2, \ldots, N. \tag{13}$$

These costs share equations form the basis of our empirical estimations.

III. Hypothesis Testing

Once the model is estimated the information exists to verify the appropriateness of the model. For one thing the implications of real cereals output maximization that are not maintained hypothesis, i.e., monotonicity, positivity, and curvature conditions can and should be verified. In particular, we can examine the non-negativity condition (which implies that the values of the fitted demand functions be non-negative; $S_i \geq o$); the neoclassical monotonicity requirement (that $\partial \log C/\partial \log P_i > o$, $i = 1, 2, \ldots, N$); and the necessary and sufficient curvature conditions. For this requirement the well-known Allen-Uzawa elasticities of substitution must provide a negative semi-definite matrix of rank equal to at most (N-1). In turn, negative semi-definiteness requires placing alternating sign restrictions on the first N-1 principle minors of the N-dimensional matrix. A necessary condition, indeed is that the own elasticities of substitution all be non-positive (see Berndt, Diewert, and Darrough [1977]).

A second area of concern consists of the investigation of the theoretical and functional form restrictions of equality (or adding up), symmetry, and weak separability. These tests can be preformed directly.[7] The empirical test of adding up comes from the accounting identity that the input shares sum to unity and hence that the sum of the changes in shares (in response to a price change) for a given input must be zero. In particular, the translog cost function must meet the following equality conditions

$$\sum_i \alpha_i = 1 \tag{14}$$

$$\sum_i \beta_{ij} = 0 \qquad i, j, = 1, 2, \ldots, N. \tag{14'}$$

Here (14) is maintained and (14') is used to test for adding up. To do this we will test the two restrictions

$$\begin{aligned} \beta_{11} + \beta_{12} + \beta_{13} &= 0 \\ \beta_{21} + \beta_{22} + \beta_{23} &= 0 \end{aligned} \tag{15}$$

in the two estimated equations (for wheat and maize). For symmetry, quite simply, partial differentiation of (9) produces the following symmetry condition

$$\partial^2 \log C/\partial \log P_i \, \partial \log P_j = \partial^2 \log C/\partial \log P_j \, \partial \log P_i = \beta_{ij} = \beta_{ji} , \tag{16}$$

$$(i, j, = 1, 2, \ldots, N)$$

This leaves us with the restriction that

$$\beta_{12} = \beta_{21} \; . \tag{17}$$

Lastly, we consider weak separability. The motivation for providing this test is a practical one, involving Egyptian cereals themselves. Thus, even though wheat, maize, and rice are mixed together to produce (or provide) what we are calling the "real cereals output" necessary for consumption units, in fact wheat and maize can be mixed first--and separately from rice--in the production process of bread and then bread and rice may be mixed again to satisfy the household's needs for cereals output (in this case bread is a kind of "intermediate" product). In this event rice might actually be functionally separable from both wheat and maize. It is appropriate to test for this possibility in order to strengthen our grip on the empirical cereals problem (in Egypt).

A technology is separable with respect to a partitioning of the factors into two subgroups provided that the ratio of marginal products between pairs of inputs in one subgroup, G, are independent of the composition of inputs in the other subgroup, $\overline{G}$ (Leontief [1947]). Thus, if the production function is denoted ϕ (q), where q is a vector of inputs quantities, then this condition can be written as

$$\phi_j \cdot \phi_{iK} - \phi_i \cdot \phi_{jK} = 0 \quad , \text{ for } i, j \in G \text{ and } K \in \overline{G}. \tag{18}$$

where $\phi_j = (\partial\phi/\partial q_i)$ and $\phi_{iK} = (\partial^2\phi)/(\partial q_i \; \partial q_K)$. In the case of the trans-log cost function we may rewrite this condition in terms of the cost shares, S_i, and the parameters of the function, β_{ij}. Thus, a necessary and sufficient condition for wheat and maize to be weakly separable from rice in the production process at all points in factor space is

$$S_1 \cdot \beta_{23} - S_2 \cdot \beta_{13} = 0 \quad . \tag{19}$$

Where wheat, maize, and rice represent the order respectively. Since the cost shares are positive, such conditions can be satisfied if and only if $\beta_{23} = \beta_{13} = 0$.(8) The behavioral implication of separability in this study is that the household makes the optimal input decision in two stages. In the first stage, expenditure is allocated between wheat and maize on the one hand and rice on the other, then in the second stage, expenditure is allocated between wheat and maize. Therefore, changes in the composition of wheat and maize do not alter the optimum input mix between bread (wheat and maize mixture) and rice in our model.

To sum up this section, the estimated model is consistent with a well-defined production function if and only if the symmetry and adding up conditions hold. The test for both, in addition to indirect tests of non-negativity, monotonicity, and the curvature requirements provide a check on the validity of the underlying model. These tests can also be regarded as prior information which guarantees that the parameter estimates from each equation are consistent.

IV. Elasticities and their asymptotic standard errors

When parameter estimates are in hand the quantities typically of interest in a demand study may be obtained from these estimates; these are, of course, the elasticities of substitution and the own and cross-price elasticities. Uzawa (1962) has show that the Allen partial elasticities of substitution (AES) can be computed from the cost function by the formula

$$\sigma_{ij} = CC_{ij}/C_iC_j \quad , \tag{20}$$

where $C_i = (\partial C)/(\partial P_i)$, and $C_{ij} = (\partial^2 C)/(\partial P_i \, \partial P_j)$. For the translog cost function we have that

$$\sigma_{ij} = (\beta_{ij} + S_iS_j)/S_iS_j \qquad i \neq j \tag{21}$$

$$\sigma_{ii} = [\beta_{ii} + S_i(S_i - 1)]/S_i^2 \qquad i = j \tag{21^1}$$

Similarly, the price elasticities (E_{ij}) for factor inputs follow directly from the elasticities of substitution as in

$$E_{ij} = \sigma_{ij} \, S_j \tag{22}$$

Since $\hat{\sigma}_{ij}$ and $\hat{E}_{ij}$ are linear transformations of the parameter estimates and prices whose econometric properties are known, the economic properties of the elasticities are known as well. A common error in the literature, however, is to assume that shares calculated at the mean may be considered constant and hence that the asymptotic standard errors are[9]

$$SE\ (\hat{\sigma}_{ij}) = \left[\left(\frac{1}{\bar{S}_i\bar{S}_j}\right)^2 \cdot Var\ (\beta_{ij})\right]^{\frac{1}{2}} \quad . \tag{23}$$

In fact, these standard errors are based on the assumption that the cost shares are nonstochastic; such a specification contradicts the assumptions used in the estimation even though it might hold asymptotically.[10] In this study, alternatively, we choose to follow the recent approach of Gallant (1982) in which a method for computing standard errors is provided. The explanation of this technique follows.

Recall the first and second partial derivatives of the Translog cost function

$$(\partial/\partial P_i)\ C\ (P) = \alpha_i + \sum_j \beta_{ij} \log P_j . \tag{24}$$

$$(\partial^2/\partial P_i \partial P_j)\ C\ (P) = \beta_{ij} \tag{25}$$

and let

$$\theta = (\alpha_1, \ldots, \alpha_N, \beta_{11}, \beta_{12}, \ldots, \beta_{NN})', \text{ [11]} \tag{26}$$

Then first and second order partials are linear functions of the form

$$(\partial/\partial P_i)\ C\ (P) = g_i'\theta \tag{27}$$

$$(\partial^2/\partial P_i \partial P_j)\ C\ (P) = h_{ij}'\theta \tag{28}$$

where g_i, h_{ij} and θ are vectors which are defined later. Using the previous notation, an elasticity of substitution and its derivative with respect to θ are

$$\sigma_{ij}(\theta) = 1 \cdot (g_i'\theta)^{-1} (g_j'\theta)^{-1} (h_{ij}'\theta) \tag{29}$$

$$(\partial/\partial\theta)\ \sigma_{ij}\ (\theta) = (g_i'\theta)^{-1} (g_j'\theta)^{-1} h_{ij}$$

$$-(g_i{}'\theta)^{-2}\,(g_j{}'\theta)^{-1}\,(h_{ij}{}'\theta)g_i$$

$$-(g_i{}'\theta)^{-1}\,(g_j{}'\theta)^{-2}\,(h_{ij}{}'\theta)g_j \qquad (30)$$

and for i=j

$$\sigma_{ii}(\theta) = 1 + (g_i{}'\theta)^{-2}\,(h_{ii}{}'\theta)\,-(g_i{}'\theta)^{-1} \qquad (31)$$

$$(\partial/\partial\theta)\sigma_{ii}(\theta) = (g_i{}'\theta)^{-2}\,h_{ii} - 2(g_i{}'\theta)^{-3}(h_{ii}{}'\theta)g_i + (g_i{}'\theta)^{-2}\,g_i \quad . \qquad (32)$$

Let $\hat{\Omega}$ denote the estimated variance - covariance matrix of $\hat{\theta}$; then an estimate of, say, an elasticity of substitution is obtained by evaluating $\sigma_{ij}\,(\theta)$ at $\hat{\theta}$,

$$\hat{\sigma}_{ij} = \sigma_{ij}\,(\hat{\theta}), \qquad (33)$$

and its standard error is computed as

$$SE\,(\hat{\sigma}_{ij}) = [(\partial/\partial\theta')\sigma_{ij}\,(\hat{\theta})\,\hat{\Omega}\,(\partial/\partial\theta)\,\sigma_{ij}(\hat{\theta})]^{\frac{1}{2}} \qquad (34)$$

We can follow a similar procedure for all other elasticities.(12)

In addition, the gain from computing the asymptotic standard errors directly without approximation can be explained if we link both estimations, the approximated and the non-approximated method, to each other. There are two ways to do this, however. First, we present the estimates of the elasticities of substitution in table (3) below with the corresponding two computations of the asymptotic standard errors. The reader can verify that the bias in the approximation is actually upwards. This is not surprising since the approximation method treats the shares as constant and observed while the correct method actually ought to take the predicted shares which have the least variance. Second, the mere fact that the "approximated standard errors" of the elasticities of substitution are not identical with the "non-approximated standard errors" implies that the former method is less accurate. It is instructive, however, to obtain an explicit expression for the gain in accuracy obtained by using the correct method. Verification of this statement is presented below in the case of the translog cost function.

For

$$\theta = (\alpha_1, \ldots, \alpha_N, \beta_{11}, \beta_{12}, \ldots, \beta_{NN})' \quad , \tag{35}$$

we need to define g_i' and h_{ij}' in (27) and (28). The g_i' s and h_{ij}' s should be constructed in such a way that if they are multiplied by θ they give the S_i' s and the corresponding parameters, β_{ij}, respectively. Let

$$g_i' = \begin{cases} (1\ 0\ 0\ \ln P_1\ \ln P_2\ \ln P_3\ 0\ 0\ 0\ 0\ 0\ 0), & i = 1 \\ (0\ 1\ 0\ 0\ 0\ 0\ \ln P_1\ \ln P_2\ \ln P_3\ 0\ 0\ 0), & i = 2 \\ (0\ 0\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ \ln P_1\ \ln P_2\ \ln P_3), & i = 3 \end{cases} \tag{36}$$

and

$$h_{ij}' = \begin{cases} (0\ 0\ 0\ 1\ 0\ 0\ \ 0\ 0\ 0\ \ 0\ 0\ 0) & \text{when } i,j = 1,1 \\ (0\ 0\ 0\ 0\ 1\ 0\ \ 0\ 0\ 0\ \ 0\ 0\ 0) & \text{when } i,j = 1,2 \\ \vdots & \vdots \\ (0\ 0\ 0\ 0\ 0\ 0\ \ 0\ 0\ 0\ \ 0\ 0\ 1) & \text{when } i,j = 3,3 \end{cases} \tag{37}$$

Then, recalling the elasticity of substitution of

$$\sigma_{ij}(\theta) = 1 + (g_i'\ \theta)^{-1}\ (g_j'\ \theta)^{-1}\ (h_{ij}'\ \theta) \qquad i \neq j \tag{38}$$

The "approximated variance" of the elasticity of substitution is given by

$$\mathrm{Var}_A\ (\hat{\sigma}_{ij}) = (g_i'\ \theta)^{-1}\ (g_j'\ \theta)^{-1}\ h_{ij}'\ \ \hat{\Omega}\ h_{ij}\ (g_j'\ \theta)^{-1}\ (g_i'\ \theta)^{-1} \tag{39}$$

The "non-approximated variance" of the elasticity of substitution can be written as

$$\mathrm{Var}_N\ (\hat{\sigma}_{ij}) = [(g_i'\ \theta)^{-1}\ (g_j'\ \theta)^{-1}\ h_{ij}' - \Lambda']\ \hat{\Omega}\ [h_{ij}\ (g_i'\ \theta)^{-1}\ (g_j'\ \theta)^{-1} - \Lambda] \tag{40}$$

in which case

$$\Lambda = (g_i' \theta)^{-2} (g_j' \theta)^{-1} (h_{ij}' \theta) g_j + (g_i' \theta)^{-1} (g_j' \theta)^{-2} (h_{ij}' \theta) g_j \quad (41)$$

and hence

$$Var_A (\hat{\sigma}_{ij}) - Var_N (\hat{\sigma}_{ij}) = [2 \Lambda' \hat{\Omega} h_{ij} (g_j' \theta)^{-1} (g_i' \theta)^{-1} - \Lambda' \hat{\Omega} \Lambda] \quad (42)$$

It is interesting to note that the last expression introduces the gain in accuracy due to the use of the non-approximation method.

V. Data and Estimation technique

Egypt shares with many underdeveloped countries the problems of availability and quality of the data. Although the sources of the data used here are official the quantity index and the price index have a long way to go to meet the standard one is used to for developed countries. The best available data, in any event, are those used and discussed in Ghazal (1982) but extended by three years. The data are time series from 1964 to 1979; the prices are scaled with all prices at 1964 equal to zero in the logarthmic form. Finally, shares, S_i, are deflated by the price index and by population to give real share per capita.

Following conventional practice, we assume that errors in the process of cost minimization give rise to additive stochastic errors with the cost-share equations. The errors are likely to be correlated, because random deviations from the maximization process should simultaneously affect the three markets for wheat, maize, and rice. This contemporaneous correlation may be corrected by using--as adapted in this study--Zellner's Efficient Method (ZEF), as suggested by Zellner (1962). Since the cost shares sum to unity, the contemporaneous covariance matrix of the disturbances is singular, and one equation must be dropped to estimate the system (rice is dropped here). The parameter estimates of the deleted equation can be recovered from the restrictions imposed.[13]

VI. The Empirical Results

The translog form (13) for three factor inputs has eight parameters to be estimated. The rest can be obtained from the restrictions imposed. The estimates for all eight parameters are presented in the first column of table (1) along with their asymptotic standard errors. We adopt the following subscripts for ease in reporting our results; (W) for wheat; (M) for maize; and (R) for rice.

The monotonicity condition and the non-negativity condition are satisfied for all data points in our sample. Similarly, computation of the bordered Hessians for our estimates indicates that the translog cost function satisfies the curvature condition. For overall validity of the translog cost approximation, the restrictions are tested and then imposed. Table (1) displays the parameter estimates for the constrained fit in columns (2)-(5). They are symmetry, adding up, both symmetry and adding up, and finally separability. All the estimates are significant with the exception of β_{MR} in all models.

In table (2) we provide critical levels for a variety of possibilities along with the test statistic for each set of restrictions. The reader is free to evaluate our test statistics with reference to this table. However, the results clearly indicate that the null hypothesis implied by symmetry, adding up, both symmetry and adding up, and separability cannot be rejected at $P \leq .05$. According to the separability test, the findings confirm and support that rice must be treated as an independent input to the production process of cereals. This also explains the non-significance of

Table (1)

Cost Function Parameter Estimates 1964-1979
(asymptotic standard errors in Parentheses)

Parameter	Model				
	Unrestricted	Symmetry	Adding up	Both	Separability
α_W	.4851 (.0100)	.4838 (.0087)	.4971 (.0039)	.4976 (.0034)	.4841 (.0098)
α_M	.3673 (.0078)	.3663 (.0068)	.3528 (.0030)	.3529 (.0031)	.3625 (.0076)
β_{WW}	.1817 (.0684)	.1916 (.0562)	.1064 (.0365)	.0994 (.0308)	.1829 (.0683)
β_{WM}	-.1417 (.0610)	-.1526 (.0432)	-.0817 (.0443)	.0742 (.0253)	-.1549 (.0521)
β_{WR}	-.0132 (.0315)	-.0098 (.0285)	-.0192 (.0311)	-.0251 (.0262)	
β_{MW}	-.1605 (.0531)	-.1526 (.0432)	-.0697 (.0283)	-.0742 (.0253)	-.1549 (.0521)
β_{MM}	.1949 (.0473)	.1894 (.0421)	.1292 (.0344)	.1321 (.0334)	.1280 (.0405)
β_{MR}	-.0667 (.0244)	-.0671 (.0244)	-.0595 (.0242)	-.0579 (.0237)	

Table (2)

Test statistics (Chi-square) for restrictions on the translog cost function(14)

Hypotheses	Degrees of Freedom	Test Statistic	Critical Values of χ^2		
			.05	.025	.01
1. Symmetry	1	.087	3.841	5.024	6.635
2. Adding up	2	2.290	5.991	7.378	9.210
3. Both	3	2.250	7.815	9.348	11.345
4. Separability	2	5.595	5.991	7.378	9.210

of β_{MR} in all models.

Table (3) reports the estimated Allen partial elasticities of substitution evaluated at thre· points (1965, 1969, and 1976). They are presented along with their asymptotic standard errors computed twice; the first standard error reported in parentheses is computed according to the correct method shown above while the second standard error reported in square brackets is computed according to the approximation method which tends to bias the estimate of the asymptotic standard errors upwards. The reader can easily verify this by observing the differences in table (3).

Egyptian households exhibit partial elasticities of substitution significantly different from zero at the 5 percent level between wheat and maize and between wheat and rice. The estimates are not, however, significantly different from zero between maize and rice; this result is not surprising, of course. The AES estimates indicate that wheat should be considered a substitute for maize and rice in the household production process. Needless to say, the relationships between maize and rice should be interpreted as independent based on their asymptotic standard errors. Moreover, the substitution elasticities of wheat with rice often exceeds those between wheat and maize. The latter findings suggests that the household's adjustment to changes in relative prices is higher with rice than with maize; that is maize shows less substitutability with wheat than with rice. Given that wheat is the most essential factor in any meal, then rice demand behaves as if it is the first candidate for adjustment in response to changes in the price structure of cereals; nonetheless, it is independent of wheat and maize taken together in the production process.

Table (4) reports the demand elasticities from the derived demand

Table (3)

Estimated elasticities of substitution*

Elasticities of Substitution	Year		
	1965	1969	1976
σ_{WM}	.5589 (1308) [.1508]	.5594 (.1296) [.1502]	.5733 (.1264) [.1427]
σ_{WR}	.6915 (.2751) [.3200]	.6911 (.2731) [.3212]	.6687 (.2922) [.3599]
σ_{MR}	-.101435 (.3701) [.4634]	-.0696 (.3605) [.4378]	-.0715 (.3693) [.4439]
σ_{WW}	-.5758 (.1063) [.1128]	-.5934 (.1044) [.1194]	-.6150 (.1044) [.1254]
σ_{MM}	-.8191 (.2537) [.3187]	-.8093 (.2473) [.2988]	-.7758 (.2308) [.2569]
σ_{RR}	-2.0036 (1.034) [1.1334]	-2.0063 (.9286) [1.1638]	-1.9887 (1.1379) [1.2903]

*The asymptotic standard errors are in parentheses while the approximated asymptotic standard errors are in brackets.

Table (4)

Estimated own and cross price elasticities
(asymptotic standard errors in parentheses)

Elasticities	Year		
	1965	1969	1976
E_{WM}	.1838 (.0435)	.1870 (.0435)	.2015 (.0435)
E_{MW}	.2857 (.0648)	.2818 (.0640)	.2837 (.0640)
E_{WR}	.1104 (.0435)	.1119 (.0424)	.1028 (.0435)
E_{RW}	.3535 (.1410)	.3481 (.1389)	.3309 (.1456)
E_{MR}	-.0162 (.0591)	-.0118 (.0583)	-.0109 (.0565)
E_{RM}	-.0334 (.1212)	-.0233 (.1200)	-.0251 (.1296)
E_{WW}	-.2944 (.0529)	-.2989 (.0519)	-.3043 (.0519)
E_{MM}	-.2695 (.0866)	-.2705 (.0848)	-.2727 (.0800)
E_{RR}	-.3201 (.1729)	-.3249 (.1679)	-.3058 (.1808)

equations with their asymptotic standard errors; they are evaluated at three points, as before, 1965, 1969, and 1976. In general our estimates are consistent with the expectations of economic theory. In particular, all signs are as expected and all of the estimates are significantly different from zero except when the relation between maize and rice is involved. Of course, on the basis of these results, it is apparent that the translog cost function (on the Egyptian data) as well as neoclassical production theory are appropriate means to study this problem.

VII. Conclusions

In general, it is difficult to establish the validity of the neoclassical framework for modeling the household's demand for any aggregated commodities on the basis of the direct evidence alone. Generally, one finds problems because of model misspecification or because of the performance of particular functional forms. The way to proceed, of course, is to appraise the validity of the model and its particular functional form; among the methods that are available to achieve this objective the translog (flexible form) is the one adopted in this study. As it turns out, in the case of Egyptian cereals, our findings suggest that neoclassical production theory does offer an exceptionally useful conceptual setting within which to model the households' demand for cereals. In addition the separability test and the study of the bias in the asymptotic standard errors of the AES, as computed by the approximation method, are the most important aspects of this study.

It seems safe to say that while the empirical results of this paper are of considerable general interest they are also of value to the Egyptian authorities in conducting their agricultural policy . Even so, we do not think that the details of our findings are of general interest, and so we have not gone beyond the standard theoretical topics in this paper. It is also true that future research in this area depends on, and must await, the development of better price and quantity Dividia indices for cereals and their prices. One can safely conjecture, however, on the basis of the results reported in this paper, that a neoclassical technology would be an appropriate way to greet these new data, upon their arrival.

Footnotes

*We wish to thank Michael Wohlgenant and Elssayed Elssamadisy for helpful comments. All remaining errors are, of course, our responsibility.

(1) Analysis of the implications of the separability assumption as a maintained hypothesis in flexible functional forms is found in C. Blackorby et.al. (1977). Separability tests, using flexible forms involve fairly strong assumptions themselves. The translog functional form as one example - used below - can only model additive separability or homothetic separability. Indeed, a refutation of separability using the translog may be merely a rejection of the Cobb-Douglas form of a truly separable preference ordering.

(2) This literature is also deficient with respect to its use of specific functional forms. Simple functional forms that are consistent with the neoclassical theory of consumer behavior suffer a severe problem of being additive. On the otherhand, more advanced functional forms such as loglinear demand equations or differential demand equations (Rotterdam) show more flexibility than the simple ones. However, these latter forms have to be not only additive but also homothetic, to represent preferences which are consistent with utility or production maximization. Nonetheless, they severely restrict the substitution possibilities among commodities.

(3) This part is considered extremely important because almost all economic literature in the context of flexible functional forms ignores it or adopts an approximation .ethod which contains fallacious assumptions. Exceptions are Gallant (1982) and Ewis-Fisher (1982).

(4) The recent application of duality theory in economics has numerous applications to the study of production and cost relationships. Using these theorems (see Diewert [1974]), and given regularity conditions, there exist cost and production functions which are dual to each other. The specification of a production function implies a particular cost function. Choice of either approach has to be made on econometric grounds.

(5) For further details concerning these theorems, and some empirical application see Blackorby and Russell (1976), Fuss and McFadden (1978), Berndt and Wood (1975) and Christensen and Greene (1976).

(6) The Cobb-Douglas specification imposes the restriction that the AES of each pair of goods is constant and equal to unity; the CES, in turn, allows the AES to differ from unity, but forces it to be the same for different pairs of commodities (inputs).

(7) The tests of symmetry and adding up are very important. They refer to the validity of the maximization process and hence to the validity of neoclassical theory. Taken together these restrictions imply linear homogeneity. This is important since its acceptence is needed to justify a successful aggregation across households.

(8) For more details about linear and non-linear separability restrictions, the reader is referred to Berndt and Christensen (1973) and Blackorby and Russell (1976).

(9) $S_i \; \forall i$ are random variables and hence one should not take them either from observations or treat them as fixed. They have to be predicted from the model. One obvious reason is that the variance of the prediction is less than the variance of the observation.

(10) See Dennis and Smith (1978, p. 804).

(11) In computing the standard errors of the constrained parameters we have made use of the constraints:

$$\alpha_3 = 1 - \alpha_1 - \alpha_2 \;;$$
$$\beta_{33} = -(\beta_{13} + \beta_{23}) \;;$$
$$\beta_{32} = -(\beta_{12} + \beta_{22}) \;;$$
$$\beta_{31} = -(\beta_{11} + \beta_{21}) \;.$$

(12) The SAS program used to compute these estimates and the standard errors directly for given prices, $\hat{\theta}$, and $\hat{\Omega}$ is available on request from the authors.

(13) The estimates obtained here will not be invariant with respect to which equation is deleted. Kmenta and Gilbert (1968) and Dhrymes (1970) have shown that iteration of the Zellner estimation produces results in maximum-likelihood estimates which are invariant with respect to which equation is deleted (Barten [1969]). In our experiment, a small number of iterations did not show significant changes to justify the cost of more iterations or any risk of non-covergence. Therefore, only the ZEF estimate is reported below.

(14) A test of both symmetry and adding up simultaneously may be considered redundent here since each is accepted individually. However, it is presented for completeness.

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