DAIRY MODELS AND MODELING FOR POLICY ANALYSIS: CONCEPTS AND ISSUES

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FOREWORD

The papers in this publication were presented in a symposium in conjunction with a joint meeting of the three regional dairy marketing committees, NC-176, NE-153 and S-166. The symposium was held at The Ohio State University on October 29, 1985. The objective of the program was to share research on models and modeling aimed at finding solutions to the problems facing the dairy industry. The symposium was organized by Cameron S. Thraen, David E. Hahn and Robert L. Beck, respective chairpersons of the three committees. Many persons helped make the symposium a success. The Planning Committee wishes to thank all speakers, chairpersons and participants for contributing to the total program.

The proceedings were edited by Cameron Thraen and David Hahn. A special debt is owed to Marla Getty and Phyllis Seidel who typed and prepared this manuscript for publication.

Copies of this proceedings are available at a price of $8.50 per copy. To order, please contact:

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DIMX - A Dairy Industry Model*

by

Boyd M. Buxton**

A rapid succession of policy questions and issues during the early 1970's, prompted a need to develop an analytical model of the dairy industry. DIMX, a dairy industry model described in this paper, has its conceptual roots formed during the 1974 discussions on how dairy imports might impact the U.S. dairy industry. The analytical framework was formalized and used to evaluate the economic performance of the federal milk order program in 1977 (Dobson and Buxton), the impact of alternative classified pricing policies under the federal milk order program in 1978 (Fallert and Buxton), and the potential impacts of changing reconstituted milk provisions under federal milk orders in 1979 (Hammond, Buxton, and Thraen). In 1980 modifications were made to DIMX to evaluate alternative price support policies including supply control, deficiency payments, and alternative support prices. DIMX was used to prepare background analysis during the debate on the 1981 farm bill.

DIMX reflects many discussions and interactions between members of the dairy group in the Economic Research Service (ERS) and analysts from the Agricultural Stabilization and Conservation Service (ASCS), Agricultural Marketing Service (AMS) and the Foreign Agricultural service (FAS), all in the U.S. Department of Agriculture. The valuable input of Richard Fallert

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cannot be ignored nor can the many discussions between myself, Jerome Hammond and Cameron Thraen at the University of Minnesota. Cameron Thraen did the initial programming of DIMX.

A few comments about policy analysis in USDA is needed to better understand the logic behind the development of DIMX. In USDA analysts form ERS, ASCS, AMS, FAS and the world outlook board are represented on a dairy estimates committee. This committee is responsible for any official USDA projections on such things as milk production, consumption, prices, and government removals and costs. To reduce the possible confusion that would arise out of many different sets of projections a model was needed that could be consistent with the combined analysis and judgement of committee members. Therefore, a major objective for DIMX was to utilize the committee estimates a s a baseline then, as an analytical tool, analyze the impact of alternative policies as expected changes in the baseline production, consumption, prices, government removals, etc.

A second objective for DIMX was to develop a theoretical framework that could be used as a tool to do longer term research on a range of dairy policy issues such as classified pricing and reconstituted mild provisions under federal mild orders.

In this paper I briefly discuss milk pricing provisions of the mild order and price support programs followed by a geometrical and mathematical description of DIMX. Finally some of the special features of DIMX are discussed.

<u>Milk Pricing</u>

Pricing milk according to use (classified pricing) is a basic part of federal milk marketing orders and state milk control. Under classified pricing, there are three key prices in determining production and consumption of milk: (1) the "Class I" price paid by processors of fluid milk, (2) the "U.S. manufacturing milk" price paid by processors of manufactured dairy products, and (3) a weighted "all wholesale milk" price reflecting an average price received by all dairy farmers. The minimum difference between the Class I price and the U.S. manufacturing price (from here on referred to as the Class I differential) is a policy variable established under federal milk marketing orders. The price support program places a floor under all milk prices as the government stands ready to purchase manufactured dairy products at prices that result in farmers receiving the designated support price for raw milk. This link between classified pricing and price support policies and consumption, production, and prices is important in analyzing the broader implications of many milk marketing issues.

Geometric Description of DIMX

Static equilibrium (single period) - DIMX is a U.S. nine-region model of milk consumption and production. For simplicity, a three-region model for one time period is illustrated in Figure 1. The regional demand for fluid milk, which depends on the prevailing Class I price in that region, is represented by F_1, F_2 and F_3. The regional milk supply, which depends on the all wholesale milk price in each region, is represented by S_1, S_2, and S_3. Within each region, the demand for manufacturing milk is assumed to be infinitely elastic at the U.S. manufacturing milk price (p^m). The U.S. manufacturing milk price is determined by the intersection of the aggregate U.S. demand for manufacturing milk (M^d) and the total supply of

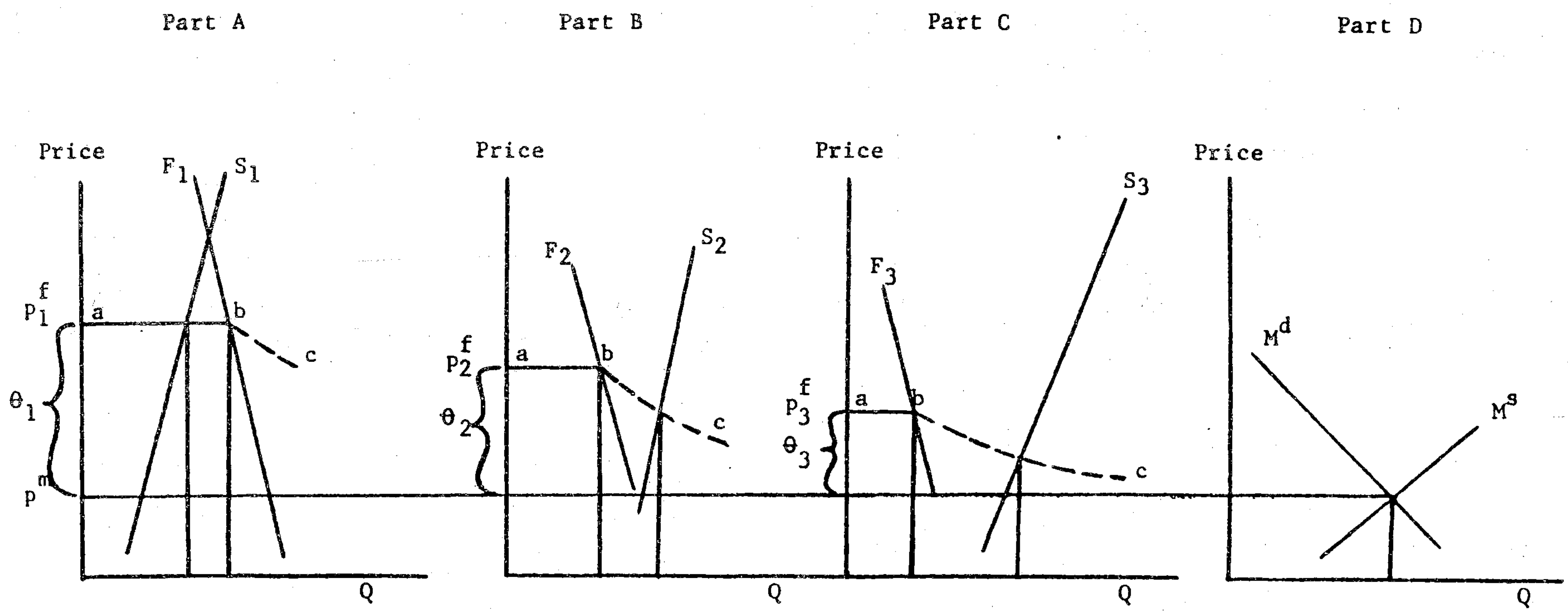

Figure 1. One period, three region partial equilibrium model of the U.S. dairy industry (DIMX).

milk available for manufacturing after the higher priced fluid demand is met (M^s).

Under federal milk marketing orders minimum Class I prices are set above the U.S. manufacturing price. The differential between these prices varies form one region to another. This is illustrated by different values of the Class I differential (θ_1, θ_2, and θ_3) in Figure 1. Without a change in Class I pricing policy, these differentials would be expected to remain fairly constant over time. DIMX is not a spatial model where Class I prices across regions endogenously reflect transportation costs. Regional Class I differentials are exogenously set in the model but cannot ignore transportation cost.

The average revenue to farmers per one hundred pounds of milk (the all wholesale milk price) reflects both Class I and manufacturing milk sales is illustrated with lines labeled abc in Figure 1. This average revenue can be written as:

$$(1)\quad P_i^w = \frac{P_i^f F_i + p^m(S_i - F_i)}{S_i}$$

where

i = 1 to 9 regions,

P_i^f = regional Class I milk price,

p^m = U.S. manufacturing milk price

F_i = regional milk used as Class I (including Class I milk shipped to other regions), and

S_i = regional milk production.

If the quantity of milk produced in a region increased relative to the quantity used as fluid, a larger proportion of the milk must be sold at the lower manufacturing price (P^m). Therefore, the average revenue would decline as illustrated by the bc segment of the abc curves in Figure 1. The average revenue curve (abc) becomes the effective demand curve facing producers in a given region. It is the intersection of this curve with the regional supply (S) that would determine the quantity of milk produced in each region.

The region illustrated in Part A of Figure 1 is deficit in fluid milk, therefore, the all wholesale milk price would be equal to the Class I price.[1] The quantity produced within that region would be determined by the intersection of the ab segment of the abc curve and S_1.[2] The horizontal distance between the intersection and the fluid demand curve is the quantity of fluid milk that would be shipped into that region from a surplus region(s). The regions illustrated in Parts B and C of Figure 1 produce more milk than is used as fluid.

The M^s curve in Part D of Figure 1 shows the quantity of milk <u>available</u> for manufacturing for all regions (after fluid demand has been met) at all possible manufacturing milk prices. The higher the manufacturing milk price, the greater will be the quantity of milk available for manufacturing.

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1 Because of seasonal variation in production, a region probably would have to import 20 percent or more of its fluid milk before it could utilize most of its own production as fluid Class I sales. Some of its milk production would be diverted to manufacturing during part of the year, causing the all wholesale milk price to be below the Class I price.

2 Implicit is that P_1^f in the deficit region exceeds the Class I milk price in the supply region (P_2^f or P_3^f) by the transportation cost between the regions. If the transportation cost is higher, then P_1^f would rise above the federal order price as high as the intersection of F_1 and S_1 at which point the region would be self sufficient (supply its own fluid needs).

This is because the resulting higher Class I prices would tend to decrease fluid consumption and the higher all wholesale milk prices would encourage production, leaving more milk available for manufacturing.

Comparative static equilibrium (multi-periods) - several time periods are represented in DIMX as a sequential set of one period partial equilibrium models. Up to 20 periods can be represented. Changes in population, tastes and preference, price of substitutes and other factors affecting demand would be shifting the demand curves over time. On the supply side, changes in feed and other input prices, returns from competing farm enterprises, and other factors affecting supply would be shifting the supply curves over time. There shifts along with specific Class I differentials established under federal milk marketing orders would generate a baseline series of annual equilibrium quantities and prices over several periods (Figure 2).

This baseline assumes expected inflation, feed costs, input prices, and other factors affecting the dairy industry. These baseline prices and quantities allow the supply and demand curves discussed above and illustrated in Figure 2 to be positioned for each period over the baseline. The slopes of the demand and supply curves are calculated from secondary supply and demand elasticity estimates.

The time periods are linked through secondary estimates of the lagged supply response to a deviation in the all wholesale milk price from the baseline price in the previous period(s). For example a policy to increase the support price from $P^o(t)$ to $P^s(t)$ in Period t of Figure 2 would be expected to shift the supply curve in periods II and III from S to S'. The two year response to the price increase is the sum of the movement along the

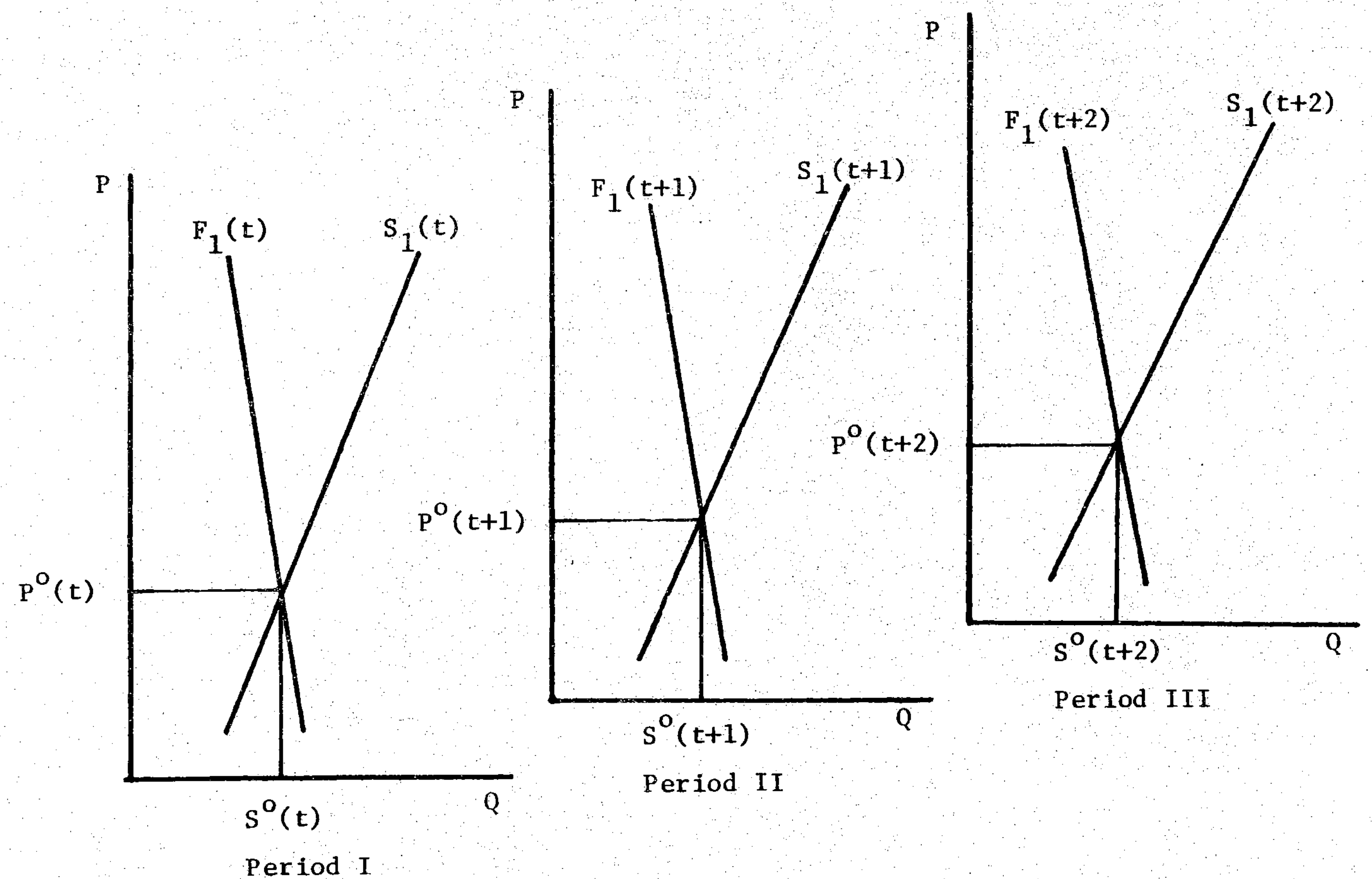

Figure 2. A three-period, one region partial equilibrium baseline for the U.S. dairy industry.

supply curve in period II (1st year responses) and the shift in period II (2nd year response) due to the previous year's price increase. The full three year response to a permanent increase in support price (Figure 3) is the sum of the movement along the curve in period III (1st year response), the shift in supply from S' to S" due to the price increase in period II and the shift in supply S to S' due to the price increase in period I. Four year supply elasticity estimates were made to be consistent with the DIMX model (Buxton).

Mathematical Description of DIMX

The more general model, of which the three--region model shown in Figure 1 is a special case, can be written in the following equations:

(2) $F_i(t) = a_i(t) + b_i(P_i{}^f(t)$

(3) $S_i(t) = c_i(t) + d_i(t)P_i{}^w(t)$

(4) $m^d(t) = e(t) + f(t)P^m(t)$

and identities:

(5) $P_i{}^f(t) = P^m(t) + \theta_i(t)$

(6) $P_i{}^w(t) = P^m(t) + \Upsilon_i(t)\theta_i(t)$

(7) $M^s(t) = \sum_{i=1}^{a} [S_i(t) - F_i(t)]$

where

i = 1 to 9 regions,

t = period,

$F_i(t)$ = fluid milk consumption,

$S_i(t)$ = total milk production

$M^d(t)$ = total U.S. manufacturing milk consumption,

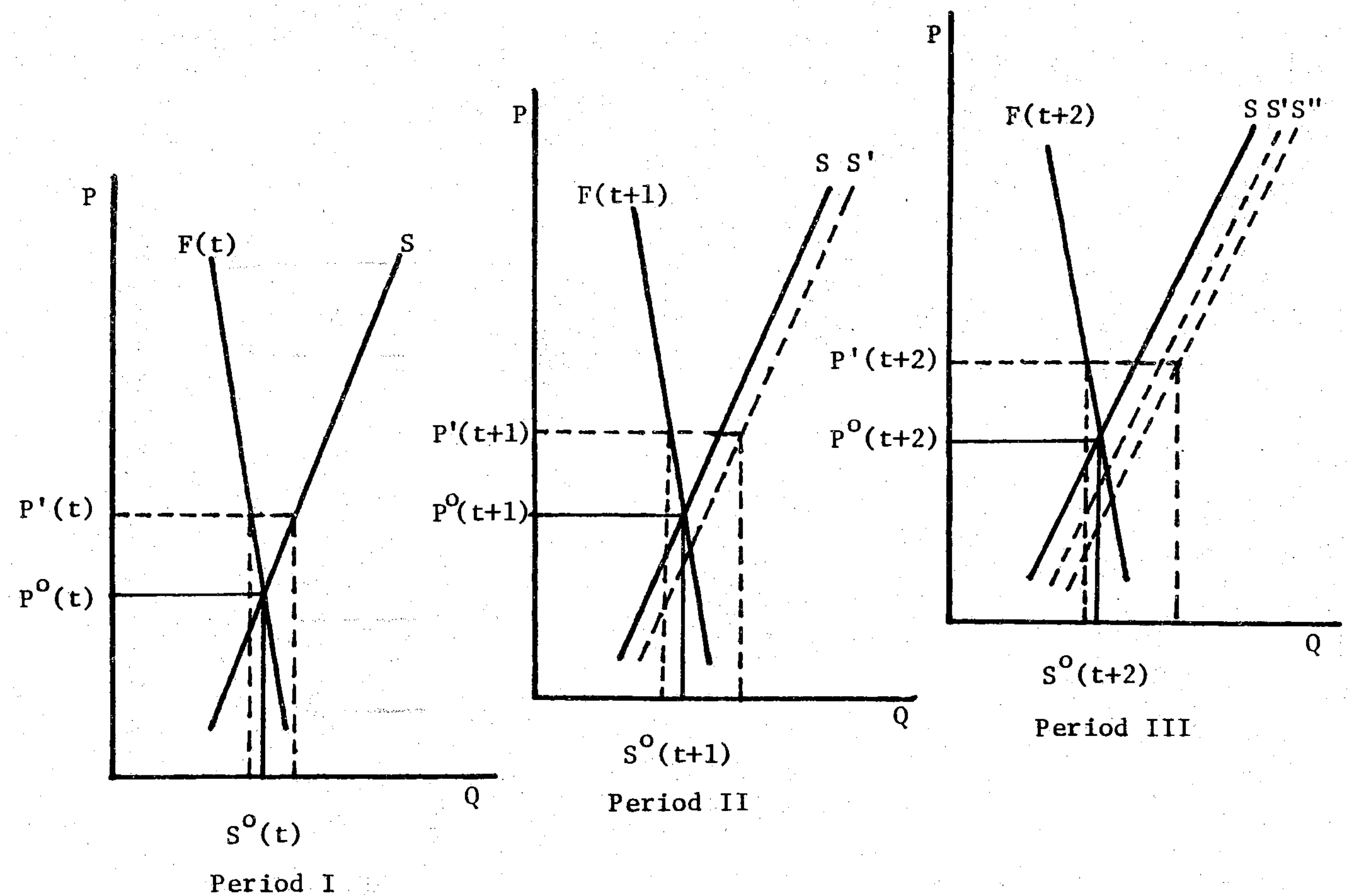

Figure 3. A three-period, one region partial equilibrium showing lagged responses to change in the all milk price.

$P^m(t)$ = U.S. average manufacturing milk price,

$P_i^f(t)$ = Class I milk price,

$P_i^w(t)$ = all wholesale milk price,

$M^s(t)$ = total milk available for manufacturing in the U.S.,

$\theta_i(t)$ = Class I milk price differential,

$Y_i(t)$ = percentage of milk used as Class I, and

$a_i(t)$, $b_i(t)$, $c_i(t)$, $d_i(t)$, $e(t)$, and $f(t)$ are intercept and slope coefficients for supply and demand equations.

Equation (6) is equivalent to equation (1) when Y is equal to the actual percentage of total milk used for fluid consumption.

Because less information is available on interregional milk shipments than on all wholesale milk prices, the percentage of Class I utilization is estimated from the all wholesale milk price, manufacturing milk price, and the Class I differential as:

$$Y_i(t) = \frac{P_i^w(t) - P^m(t)}{\theta_i(t)}$$

The equilibrium condition for each period is:

$$(8)\ M^d(t) = M^s(t)$$

The intercept and slope parameters of the model for the baseline are calculated using the baseline equilibrium prices and quantities and the estimates of demand and supply elasticity. The parameters for the supply and demand equations are calculated for each year. The slopes and intercepts of the fluid demand equations are estimated as:

$$b_i(t) = \frac{P_i^t(t)^o}{F_i(t)^o \eta_i^f(t)} = \text{slope}$$

and

$$a_i(t) = b_i(t)F_i(t)^o + P_i^f(t)^o = \text{intercept},$$

where:

o refers to baseline equilibrium quantity and price, and

$\eta_i^f(t)$ = elasticity of fluid demand in the ith region and tth period.

The slopes and intercepts of the supply equations were estimated as

$$d_i(t) = \frac{P_i^w(t)^d}{S_i(t)^d E_i^s(t)} = \text{slope}$$

and

$$C_i(t) = d_i(t)S_i(t)^o + P_i^w(t)^o = \text{intercept},$$

where:

o refers to baseline equilibrium quantity and price and

$E_i^s(t)$ = elasticity of milk production response in period t to a change in the all wholesale milk price in period t.

The slope and intercept of the aggregate U.S. demand for manufacturing milk were estimated as:

$$f(t) = \frac{P^m(t)^o}{M^d(t)^o \eta^m(t)} = \text{slope and } e(t) = -f(t)M^d(t)^o + P^m(t)^o = \text{intercept},$$

where:

o refers to forecasted equilibrium quantity and price and

$N_i^m(t)$ = elasticity of demand for manufacturing milk.

All the parameters of the model that are consistent with the baseline equilibrium prices and quantities have now been calculated. The model can be solved for the equilibrium U.S. manufacturing milk price in any period.

From the equilibrium condition (equation 8), the manufacturing price would be:

$$P^m(t) = \frac{e(t) + \sum_{i=1}^{a} [a_i(t) - c_i(t) - d_i(t)Y_i(t)\theta_i(t) + b_i(t)\theta_i(t)]}{\sum_{i=1}^{a} [d_i(t) - b_i(t)] - f(t)}$$

All other prices and quantities can then be calculated from this equilibrium manufacturing milk price. Changing anyone of the parameters will change the equilibrium prices and quantities form the baseline.

As illustrated in Figure 3 a lagged milk production response to a price change was built into the model. When the all wholesale mild price in any period deviated form that baseline, because of a policy or other change, it is assumed that the supply curve for the next four periods would shift (Figure 3).[3] The intercept for the supply curve in t + 1 would then be:

$$C_i(t + 1)' = C_i(t + 1) + \frac{E_i^L(t)[P_i^W(t) - P_i^W(t)^o]\, S_i(t)}{P_i^W(t)^o}$$

where:

$C_i(t + 1)$ = the supply intercept calculated from the baseline price and quantity and supply elasticity for period t + 1 and

$E_iL(t)$ = supply elasticity of a one-year lagged response to a deviation of the all wholesale milk price from the baseline equilibrium all whole milk price in t.

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[3] This shift is only due to the change in policy variable and is in addition to the effect of the exogenous supply shifters that are already reflected in the baseline supply equations.

If no policy change is introduced, the solution to the model will be the baseline equilibrium quantities since $P_i^W(t) - P_i^W(t)^o$ would be zero. The supply curve intercept three periods after a policy change was instituted would reflect the original intercept calculated from the baseline price and quantity plus the 2 shifts calculated from the deviation of the all wholesale milk price from the baseline all wholesale milk price for each of the prior 2 periods.

A second lagged supply response assumption could be selected. This lagged supply response to a deviation in milk prices from the baseline is a form of distributed lag. Results from a lag structure of up to five periods from the price change are reflected in the model.

The new intercept for the supply curve five periods after a policy change reflects the deviations in price form the base line price for the previous five periods as follows:

$$C_i(t + 5)' = C_i(t + 5) + \frac{E_i^{L1}[P_i^W(t + 4) - P_i^W(t + 4)^o]S_i^W(t + 4)}{P_i^W(t + 4)^o}$$

$$+ \; . \; . \; . \; . \; . \; . \; . \; .$$

$$+ \; . \; . \; . \; . \; . \; . \; . \; .$$

$$+ \frac{E_i^{L5}[P_i^W(t) - P_i^W(t)^o]S_i(t)}{P_i^W(t)^o}$$

where E_i^{L1}, E_i^{L2}, E_i^{L3}, E_i^{L4}, E_i^{L5} are the assumed lag response elasticities.

Special Features of DIMX

Special features have been built into DIMX to more accurately model real world conditions. Some of these features are described in this section.

Interregional milk shipments - the interregional shipments of milk from surplus to deficit markets are calculated in DIMX. However these shipments are not determined by a minimum cost spacial transhipment procedure. DIMX calculates the amount of outside milk needed by any deficit region based on seasonal variations in milk production and fluid milk consumption. If any region should require outside milk in the equilibrium solution, the sources of that milk are pre-specified based on an apriori judgement on the least cost supply areas for that region. Clearly the Northwest region would not be specified as a supply area for possible deficits in the Southeast.

Regional manufacturing milk prices - Regional differences in manufacturing milk prices can be reflected in DIMX as differentials from the U.S. manufacturing milk price.

Fat test of fluid milk sales - DIMX calculates the amount of whole milk needed to supply the indicated product pounds of fluid milk products consumed. This better reflects the real supply and demand situation than using milk equivalents on a strictly fat solids basis. The average fat test for fluid products and for whole milk by regions is exogenously determined for each of the nine regions. DIMX does all the remaining calculations to convert fluid products pounds to pounds of fluid milk on a milk equivalent basis and vice-versa.

Lagged supply response - In DIMX, the lagged response of milk production can be different for a price increase than for a price decrease.

Output variables - As indicated above, DIMX solves for the manufacturing milk price, regional Class I and blend prices, regional fluid demand and total milk supply, U.S. manufacturing milk demand, government removals and program costs. In addition DIMX solves for retail prices per half gallon milk, per pound of butter, and per pound of cheese. Also total consumer expenditures for fluid and manufactured products and total cash farm receipts to farmers are calculated for the baseline and each alternative policy. Finally the net social cost associated with any change relative to the baseline can be calculated.

Conclusions

An important advantage of DIMX is its flexibility to utilize the combined judgement of analysts and/or the results of econometric studies in a baseline set of prices and quantities. The implications for prices and quantities under alternative policies are calculated as a deviation from these baseline values. This approach eliminates justification for possible differences in baseline numbers resulting from different analyses but rather focuses attention on the net impact or change resulting form alternative policies. The model, like quadratic, separable and reactive programming approaches, must rely on research on the economic relationships that underpin supply, demand and prices in the dairy industry. A users guide to DIMX is available.

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DESCRIPTION AND APPLICATION OF A DAIRY SIMULATION MODEL*

by

James W. Richardson, Ronald D. Knutson, and Robert D. Yonkers**

Overview of the Model

The diary model (DAIRYSIM) was developed using the Farm Level Income Tax and Farm Policy Simulator (FLIPSIM V) as the central core. DAIRYSIM simulates the annual economic activities of a diary farm over a 10-year planning horizon. Because the model was developed as a tool for doing economic analyses of representative dairy farms in various regions, the model does not include the day-to-day details for each cow that managers must deal with in managing a dairy herd. However, the model incorporated most of the economic relationships the farm manager must deal with over the course of a year.

FLIPSIM V is a firm level, recursive, simulation model which simulates the annual production, farm policy, marketing, financial management, growth, and income tax aspects of a farm over a multiple year planning horizon (table 1). The computer program is capable of simulating a case farm situation for 1 to 10 years. The model recursively simulates a typical farm by using the ending financial position for year one as the beginning

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**Authors are respectively Associate Professor, Professor, and Graduate Research Assistant, Agricultural and Food Policy Center, Department of Agricultural Economics, Texas Agricultural Experiment Station, Texas A&M University System. Technical Article TA- ____ of the Texas Agricultural Experiment Station.

Table 1. Summary of the Options in the FLIPSIM V Computer Program

Decision Variables	Number or Types of Options Available to Analyst
Type of Analysis	--deterministic or stochastic
Number of years simulated	--1 to 10 years
If the model is run stochastic	--2 to 300 iterations
	--6 alternative distributions for selecting
	--4 alternative means for presenting the statistical analysis
	--4 alternative means for presenting cummulative distributions
Type of Farm:	
Crop farm	--1 to 10 crop enterprises
Beef cattle ranch	--0 to 5 beef enterprises
Dairy	--includes or does not include a dairy enterprise
Cropmix and Tenure:	
Cropmix	--constant or variable (LP or QP)
Full owner, tenant, or part owner	--any beginning equity level
Part owner	--cropshare or cash lease
Tenant operator	--cropshare or cash lease
Landlord	--cropshare or cash lease to the tenant
Marketing	--4 alternatives for marketing crops
Rental costs	--constant, increasing over time, or a function of land value
Farm Growth:	
Growth through land acquisition	--the farm may or may not grow
Means of cropland acquisition	--purchase and/or lease
Cropland availability	--2 options for land availability
Lever existing equity	--either no leverage possible or lever up to 50% of downpayment
Capability to change costs, yields, prices, machinery as farm grows	--yes or no
No. of larger size farms one can provide data for	--0 to 10
Machinery, Buildings and Breeding Stock:	
Number machines owned	--1 to 99 machines
Number machines leased	--1 to 50 machines
Means of disposal	--traded-in or cash sale
Depreciation	--straight line or double declining balance
Cost recovery	--straight line or accelerated (ACRS)
First year expensing	--yes or no
Reduce basis for ITC	--yes or no
General:	
Use surplus cash to prepay principal	--yes or no
Sell cropland to survive	--yes or no
Family living expenses	--can be calculated 14 different ways
Farmland market values	--either exogenous or endogenous
Farm Policies:	
Price support (recourse and/or nonrecourse)	--can be turned on or off each year, for each crop
Indirect FOR using price support for 1 year	--can be turned on or off each year, for each crop
Direct entry FOR bypassing the price support	--can be turned on or off each year, for each crop
Release FOR stocks at release price or trigger price	--can be turned on or off each year, for each crop
Direct entry FOR price different from loan rate	--can be turned on or off each year, for each crop
Target price (fixed or tied to loan)	--can be turned on or off each year, for each crop
Low yield dissaster program	--can be turned on or off each year, for each crop
FCIC crop insurance	--can be turned on or off each year, for each crop
Acreage diversion	--can be turned on or off each year, for each crop
Acreage set-aside	--can be turned on or off each year, for each crop
Marketing quota	--can be turned on or off each year, for each crop
Acreage allotment	--can be turned on or off each year, for each crop
Marketing certificate	--can be turned on or off each year, for each crop
Marketing loan	--can be turned on or off each year, for each crop
Number of years FOR interest is charged on the loan	--0 to 3 years
Adjust FCIC insurance premiums for loss records	--yes or no
Payment limitation	--yes or no
Scale program benefits by size (acres)	--yes or no
Scale program benefits by size (cash receipts)	--yes or no
Federal Income Tax Policies	
Maximum interest deductions	--yes or no
Cash vs. accrual accounting	--yes or no
Adjust income tax rates for changes in CPI after 1984	--yes or no
Reduce basis for ITC	--yes or no
Income tax provisions	--1982, 1984, or 1985
Stochastic Features:	
Types of distributions	--independent and multivariate normal
	--independent and multivariate empirical
	--independent and multivariate triangular
Random values for crops	--annual crop prices and crop yields
Random values for beef cattle	--annual prices for culled cows, heifers, steers, replacement heifers, culled herd bulls, stockers, and feeders.
Random values for dairy	--annual prices for milk, culled cows, replacement cows, calves, dairy feed, and annual milk production per cow
Variance over time	--relative variance for empirical and triangular pdfs can be constant or altered over time.

position for the second year, and so on. Accounting equations and identities constitute most of the computational components of the model. Only two econometric relationships with fixed parameters (land value and family living expenses) are included in the model. See Richardson and Nixon for a detailed description of the FLIPSIM V model.

The major economic relationships simulated annually in DAIRYSIM are: milk production, dairy herd management, crop production and harvest, feed purchase and/or sale, variable costs, fixed costs, debt repayment, machinery replacement and depreciation, marketing, cash receipts, farm policy, income taxes, self-employment taxes, herd and farm growth, and cash withdrawals for family living expenses. At the end of each year the farm's income statement, cashflow statement, and balance sheet are evaluated to determine the farm's ability to remain in business another year.

The model can be run for 10 years using the annual prices and milk production levels assigned by the analyst, or the 10-year planning horizon can be repeated 50 times using stochastic annual prices and milk production levels for the 10-year planning horizon. In the latter case, annual prices and milk production are drawn at random from multivariate probability distributions specified by the user. Using random values for prices and milk production drawn from known distributions, allows the analyst to incorporate the major economic risks associated with a dairy farm and thus permits one to estimate whether the dairy will be able to remain solvent for 10 years.

Description of DAIRYSIM

At the outset of each year in the planning horizon, the model determines the season average price for milk, replacement cows, calves, bulls, dairy feed, and crops produced on the farm, as well as the annual production of milk per cow and crop yields per acre. These values are selected randomly from probability distributions supplied by the analyst for these variables or are predetermined for each year by the analyst. The means these variables can be trended up and/or down over the planning horizon to incorporate changes in technology, farm policy, and the cattle cycle.

Next, the model simulates the economic activities for the dairy enterprise. A dairy enterprise in the model generally consists of a milking herd, replacement heifer calves under 12 months, replacement heifers over 12 months, and one or more herd bulls. (However, a California-type dairy consisting of only milk cows can also be simulated.) Annual cash receipts for milk are the product of annual production per cow, number of cows milked, and the season average price received for milk.

Monthly labor requirements for the herd are calculated for cows milked, dry cows, and replacement heifers each month. These labor requirements are combined with monthly labor requirements for crops on the farm to determine the total monthly labor requirements for the farm.

Cash receipts from the sale of calves and replacement heifers are calculated based on the replacement schedule, calving fraction, and prices provided by the analyst. The model sells a specified fraction (provided by the analyst) of all live calves at the per head price for baby calves. Half of the culled replacement heifers over 12 months of age are assumed to bring a price equal to 50% of the replacement cost for cows (culled for sickness or failure to breed), while the other half sells for 65% of the replacement cost for cows (sold as dairy replacements).

The non-labor, non-interest costs for the dairy enterprise are calculated next. The analyst provides the initial value for annual per head non-labor, non-interest cash costs for milk cows, dry cows, heifers, calves, and bulls and the annual inflation rate for these costs over the planning horizon. Total variable production costs are calculated as the sum of the products for these inflated costs and the number of head in each category. The total non-labor, non-interest variable production cost for the dairy is later added to the variable costs for producing and harvesting crops, prior to calculating interest costs on operating capital for the whole farm.

Purchased livestock (bulls and dairy cows) are eligible for depreciation (cost recovery under the 1981, 1982 and 1984 tax acts). Bulls and cows purchased prior to 1981 are depreciated using the analyst's information for purchase price, economic life (depreciation life), and salvage value. Bulls and cows placed into service on the farm after 1980 are cost recovered using the analyst's data for purchase price, year purchased, and the number of years to be cost recovered.

Once dairy cows and bulls reach the end of their respective economic lives, they are sold. Capital gains or losses on each animal are calculated and depreciation recapture is computed, if appropriate. The proceeds from these sales are treated as capital gain income. Cost recovery (depreciation) schedules for bulls and cows purchased as replacements or for increasing the herd size (growth) are established within the model. Based on analyst specified options, the model uses either a straight line or an accelerated five year cost recovery system for livestock. Investment tax credit and first year expensing are calculated for both bulls and cows, if the analyst elects these options.

The market value of all dairy animals on the farm at year end is estimated using season average stochastic (or deterministic) livestock prices and the number of head in each category. Cows over two years of age are valued at the price of replacement cows. These market values are used to update the farm's balance sheet at year end.

The model next updates the dairy herd for the following year. This involves solving the identities for the calf herd (birth, death, and sale) to determine the number of heifers entering the replacement herd; the replacement herd (death, sale, and breeding) to determine the number of replacements entering the milking herd; and the milk cow herd (culling and death) to determine the number of cows to be bought or sold, to achieve the analyst's desired herd size in the following year. These values are calculated using the number of head in each category and the replacement strategy information provided by the analyst.

All cows and bulls purchased as replacement stock or for herd growth are financed as intermediate-term assets. The minimum downpayment for livestock is calculated using the downpayment fraction specified by the analyst and the total value of the purchase.

The model next simulates all crop enterprises on the farm. Variable production costs for each crop are calculated by multiplying the per acre input costs by planted acreage for the respective crops. Labor costs are calculated as the sum of full-time labor charges plus the cost of part-time labor. Part-time labor needs are based on the difference between hours of monthly labor available from full-time employees and non-paid family members, and the monthly labor needs for all crops and the dairy enterprise.

Crop harvesting costs are the product of the out-of-pocket per unit harvesting costs, random (or predetermined) yield, and harvested acreage.

The model calculates property taxes based on the updated value of land and the property tax rate for farms in the study area. Other annual fixed costs are determined by the analyst. The model amortizes all outstanding loans assuming they are simple interest mortgages. Annual interest rates for existing debt on land, machinery, dairy cows, and operating loans are calculated using the annual interest rates provided by the analyst. Cash reserves and off-farm investments are allowed to earn a return each year based on specified annual interest rates. The market value of farm machinery is updated annually assuming the market value of used equipment decreases a given percentage (e.g. 1%) each year. The market value of cropland is estimated in one of two ways: (a) the annual percentage change in value can be predetermined by the analyst, or (b) the percentage change can be a function of the rate of return to the farm's production assets.

The model next depreciates each piece of equipment on the farm for income tax purposes. Equipment placed in use prior to 1981 is depreciated using either the straight-line or the double declining balance method. Equipment placed in use after 1980 is cost recovered assuming a 5-year life and the ACRS rules. Regular purpose and special purpose buildings are depreciated using ACRS rules or the double declining balance method where applicable. Equipment which has passed its economic life is replaced by trading the existing piece in on a new replacement. The cost of replacement equipment can be increased, decreased, or held constant over the planning horizon, based on values provided by the analyst. First year expensing and investment tax credit are calculated for all equipment purchases if these options are selected.

Cash receipts for crops sold is the product of harvested acres, fraction of the crop sold, and random (or predetermined) season average price and yield per acre harvested, less the landlord's share of the crop. If the CCC (or FOR) loan is available for the crop, it is placed under loan when that option provides greater receipts than selling the crop outright. Deficiency payments, set-asides, acreage diversions, quotas, and allotments can also be simulated for the individual crops.

After simulating the farm policies specified by the user, the model determines the farm operator's year-end financial position, calculates cash withdrawals for family living expenses and accrued income taxes. Year-end cash flow deficits are handled as follows: (a) grant a lien on crops in storage at the operating loan interest rate, (b) refinance long-term equity, (c) refinance intermediate-term equity, (d) and/or sell cropland. If the operator is unable to cover the deficit in one of these ways, the farm is declared insolvent. The farm may also be declared insolvent if its ending year equity ratios fall below the minimums specified by the analyst.

Personal income taxes and self-employment taxes are calculated assuming the operator was married, filing a joint income tax return, and itemizing personal deductions.[1] The regular income tax liability is computed using two methods: (a) income averaging (if qualified), and (b) the standard tax

[1]Depreciation recapture, capital gains and losses, investment tax credit, and depreciation allowances are explicitly accounted for in calculating the sole proprietor's accrued income tax liability. If there is a net operating loss from prior years, taxable income in the current year is appropriately reduced. If there is a net operating loss in the current year it is automatically carried forward. Net operating loss carryback is not permitted in the model.

tables. The model selects the tax strategy which results in the lower income tax liability.[2]

At the end of the year, the model updates the farm operator's balance sheet, cash flow statement. If the farm remained solvent, the model prepares to simulate the next year of the planning horizon. The steps in the simulation process described above are repeated for 10 years or until the farm is declared insolvent. After completing each iteration (10-year planning horizon), the model summarizes the information for numerous key output variables and reinitializes the analyst's beginning economic environment for the dairy farm being simulated. This insures that the dairy farm faces the same economic, policy, and physical relationships for each of the 50 iterations analyzed.

Applications of DAIRYSIM

The model is sufficiently general to be used for both farm management and farm policy analyses. The model can be used to address the following types of farm management problems that relate to the probable long-run survival and profitability of a dairy farm:

- using new feeding methods and dairy rations,
- growing vs. purchasing roughage,
- using bovine growth hormones, and other new technologies,
- evaluating alternative culling and replacement strategies,

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[2]All investment tax credit allowances were deducted from the regular tax liability and compared to the income tax liability under the alternative minimum tax. The operator paid the excess of the alternative minimum tax over the sum of the regular income tax liability and the regular minimum tax. Income tax rate schedules for 1981, 1982, 1983 and 1984 were included in the model, as well as a procedure to develop tax rate schedules for 1985-1990 based on changes in the Consumer Price Index.

- changing the herd size over time,
- raising vs. purchasing all replacements,
- restructuring debt,
- changing interest rates, inflation rates, and input prices,
- altering depreciation/replacement strategy for machinery, and
- altering depreciation strategy for purchased dairy cows and bulls.

DAIRYSIM can be used to estimate the profitability of a dairy farm for changes in policies and variables, such as:

- dairy support prices,
- supply control measures,
- checkoffs,
- general price level for milk, and
- new dairy programs.

Applications of DAIRYSIM to date include the following:

"Debt Servicing Capacity of Dairy Producers in Erath and Hopkins Counties, Texas." James W. Richardson and DeeVon Bailey, Contract Report for the Farm Credit Bank of Texas, Nov. 1982.

Firm level computer simulation model was used to predict probable effects of three alternative levels of debt and two credit availability scenarios on the economic well-being of dairy producers in Erath and Hopkins Counties, Texas. Input data was obtained from the Federal Land Bank of Texas, local Production Credit Association, and various Texas Department of Agriculture reports, as well as interviews with extension specialists and dairy producers in the areas.

"Effects of Alternative Dairy Support Programs on a Typical Dairy Farm in Erath County, Texas." Robert B. Schwart, Jr., James W. Richardson, DeeVon Bailey, and Robert D. Yonkers. Proceedings Eighth Southern Dairy Conference, Atlanta, Georgia, February 1983.

Economic impacts of five alternative dairy policy proposals on a typical diary farm in North Central Texas were analyzed using a firm level computer simulation model. Annual costs and prices were allowed to vary due to the policy options being analyzed, but the basic dairy operation was assumed not to change.

Technology, Public Policy, and the Changing Structure of American Agriculture: A Special Report for the 1985 Farm Bill. Office of Technology Assessment, OTA-F-272, March 1985, pp. 53-61 and 84-86.

DAIRYSIM was used to simulate eight typical dairy operations in three regions of the United States for 10 years under seven alternative policy scenarios and two different technology scenarios. Input data for the dairies was provided by Boyd M. Buxton (see "Economic, Policy, and Technology Factors Affecting Herd Size and Regional Location of U.S. Milk Production," report prepared for Office of Technology Assessment, U.S. Congress, June 1985).

"Impact of Evolving Bio and Information Technologies on the Structure of Dairy Farming: Some Policy Implications." Robert D. Yonkers, James W. Richardson, Ronald D. Knutson and Boyd M. Buxton, in review.

The economic activity of eight representative dairy farms over 10 years was simulated using a farm level dairy simulation model. Results indicate that emerging technologies and their rate of adoption will have a major affect on the structure of the dairy industry and traditional regional milk production patterns. Implications for new technology information dissemination programs and milk pricing policy are discussed.

Planned Expansion of DAIRYSIM

A macro simulation model of the U.S. dairy industry will be developed using functional relationships from existing models (e.g., FAPSIM, COMGEM) with current data. This econometric dairy model will be tested as a Monte Carlo simulation model for dairy policy and price analysis. Once the model performs under testing with stochastic processes, it will be linked to DAIRYSIM. This will provide a means of generating stochastic milk prices under alternative dairy policy scenarios for use in the farm level simulation portion of the enhanced model.

If adequate funding is available, DAIRYSIM will be expanded to allow for simultaneous simulation of multiple dairy farms (varying by size, debt structure, regions, etc.). This addition would provide feed back to the econometric dairy model, thus providing both aggregate and firm level estimates of alternative dairy policies. The resulting model would incorporate policy impacts directly, allow for risk in the analysis of dairy policy at the aggregate level.

THE FOOD AND AGRICULTURAL POLICY SIMULATOR: THE DAIRY-SUBMODEL

by

Larry E. Salathe, J. Michael Price, Kenneth E. Gadson and Robert C. Green*

Abstract

This article presents the structure, parameters, and validation statistics for the dairy-sector submodel contained in the U.S. Department of Agriculture's (USDA's) Food and Agricultural Policy Simulator (FAPSIM). This submodel endogenously estimates dairy cow numbers; milk production; farm-level milk prices; fluid milk consumption; and the supply, utilization, and prices of butter, cheese, nonfat dry milk, condensed and evaporated milk, and frozen milk products. It also endogenously estimates USDA purchases of manufactured dairy products and costs of Government dairy product purchases under alternative dairy price-support options.

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THE FOOD AND AGRICULTURAL POLICY SIMULATOR: THE DAIRY-SECTOR SUBMODEL

Introduction

U.S. dairy policy has been under continuous debate since 1972. During the mid-seventies, debate focused on dairy import quotas (1).[1] Recently, large Federal budget outlays resulting from dairy price-support operations have raised questions concerning the Government's role in the U.S. dairy industry. Because of Government involvement in the dairy sector through dairy price supports, dairy import quotas, and milk marketing orders and agreements, it is likely that dairy policies and programs will remain under considerable scrutiny. Researchers have developed a variety of economic models to examine and evaluate alternative dairy policies and programs (2, 3, 6, 8, 9). Such models have generally recognized interrelationships among the dairy, feed grain, and beef and veal sectors, but they have treated such sectors as exogenous. The failure to endogenize the beef and veal, and the feed grain sectors could result in substantial errors when researchers analyze dairy policies.

The U.S. Department of Agriculture's (USDA) Food and Agricultural Policy Simulator (FAPSIM) is an annual econometric model of the agricultural sector (11). FAPSIM consists of a set of individual commodity models for beef, pork, dairy, chickens, eggs, turkeys, corn, grain sorghum, barley, oats, wheat, soybeans, and cotton that are linked via common variables. The model estimates a price-quantity equilibrium solution that is

[1] Number in parenthesis refer to items in the References listed at the end of this article.

simultaneously consistent across all commodity sectors. This paper details the dairy sector of FAPSIM. The dairy submodel's structure, equation parameter estimates, validation statistics, and linkages to other FAPSIM submodels are presented.

Structure of the Dairy-Sector Submodel

The dairy-sector submodel explicitly recognizes the role of the Federal Government in milk marketing and pricing.[2] The Government supports the price of manufacturing milk (and of milk eligible for fluid consumption) by purchasing manufactured dairy products. The support level for manufacturing milk is set at some fraction of parity as determined by the Congress. This support level is then adjusted by a processing allowance to derive the price at which the Government will then purchase butter, cheese, and nonfat dry milk. These purchases remove manufactured dairy products from the commercial market and support the price of milk. When prices of manufactured products reach 110 percent of designated purchase levels, the Government may release accumulations of manufactured dairy products. Such releases increase supplies and lower milk prices. Because the Government supports milk prices by purchasing butter, cheese, and nonfat dry milk, Government purchases of such products depend on the level of supply and demand for each product.

The dairy submodel consists of four subcomponents: (1) milk supply, (2) milk price, (3) milk manufacturing, and (4) commercial demand. The

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[2] The model present draws upon earlier work by Novakovic and Thompson (6) and Salathe (9). Major structural differences between the model presented and previous studies are in the supply relationships for manufactured dairy products and Government stock specifications.

underlying structure of the model is summarized in seven flow charts (figures 1-7).

Milk Supply

The milk supply component (figure 1) contains equations for dairy cow slaughter, additions to the dairy cow herd, dairy cow numbers, milk production, milk fed to calves, milk sold to plants and dealers, and the supply of milk eligible for fluid consumption. An identity (equation) is used to determine the ending inventory of dairy cows based on beginning inventory (adjusted for death loss), dairy cow slaughter, and additions to the dairy cow herd. This identity is the following:

COWSNMC(+1) = 0.98 COWSNMC + COWSEMC - COWKSMC

where:

COWSNMC = the number of dairy cows on farms on January 1,

COWSEMC = the number of additions to the dairy cow herd during the year, and

COWKSMC = the number of dairy cows slaughtered during the year.[3]

It is assumed that 2 percent of all dairy cows die during each calendar year. Data on the actual number of dairy cow additions are not available. Therefore, it is also assumed that 60 percent of all dairy cow replacements over 500 pounds on January 1 are added to the dairy herd during the calendar year. Although both assumptions are open to debate, they are necessary if the dairy and beef and veal sectors submodels are to be linked. For example, historical estimates and future projections on dairy cow slaughter

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[3] This paper follows the convention that a variable name followed by a number in parenthesis represents that variable lagged or led by that number of time periods. For example, COWSNMC(+1) is $COWSNMC_{t+1}$, where t is time.

can be generated by use of the identity. A data series reflecting dairy cow slaughter is otherwise unavailable. Yet, without such a series, it is impossible to estimate either the contribution of dairy cow slaughter to total beef production or the effects of beef and milk prices on dairy cow slaughter.

Dairy cow slaughter and additions to the dairy cow here are hypothesized to be influenced by the price of milk, the price of cattle, the price of feed, and the stock of dairy cows. The ratio of the price of milk to the price of cattle, and the ratio of the price of milk to the price of feed reflect the relative profitability of keeping rather than selling dairy heifer calves and dairy cows. The price of feed is calculated as a weighted (reflecting average importance in dairy rations) average of the prices of corn, oats, grain sorghum, barley, wheat, and soybean meal. This variable links the dairy and the crops submodels.

Milk production per cow is a function of lagged milk production per cow, a time trend, and the ratio of milk price to the feed price. The time trend captures improvements in management practices over time such as improved culling and breeding practices. The ratio of milk price to the price of feed was included on the assumption that farmers reduce feeding rates during periods when milk prices are low relative to feed costs.

The fraction of milk eligible for fluid consumption has steadily increased over time. Salathe (9) found that at least a portion of the increase could be explained by the lagged difference between the producer prices for fluid and manufacturing grades of milk. Therefore, the supply of milk eligible for fluid consumption is hypothesized to be related to the lagged difference in producer prices for fluid and manufacturing grades of milk and to the quantity of milk sold to plants and dealers.

Milk Price

The milk price component (figure 2) is consistent with the pricing mechanism for Federal milk marketing orders. The Minnesota-Wisconsin manufacturing milk price series is the standard on which the Federal order system determines Class I and II milk prices. The Minnesota-Wisconsin manufacturing milk price is related to the wholesale prices of butter, cheese, and nonfat dry milk. The price of fluid-eligible milk is determined by weighting Class I and II prices by the proportion of fluid-eligible milk utilized as Class I and II.

The farm-level price of milk reflects both the relative proportion of milk produced as fluid and as manufacturing grades and their respective prices. The producer price of manufacturing milk is related to the wholesale prices of butter, cheese, and nonfat dry milk. The producer price of milk is calculated by weighting the prices of manufacturing and fluid-eligible milk by the proportion of milk produced as fluid-eligible and manufacturing grades.

Milk Manufacturing

The dairy submodel contains equations to predict supply, utilization, and prices for butter (figure 3), cheese (figure 4), nonfat dry milk (figure 5), frozen milk products (figure 6), fluid milk (figure 2), and condensed and evaporated milk (figure 7). It is hypothesized that the demand for milk to be processed into fluid milk, condensed and evaporated milk, and frozen milk products will be satisfied prior to the allocation of milk to butter, cheese, and nonfat dry milk production. The volume of milk available for manufacturing (milk production less that processed into fluid milk, condensed and evaporated milk, frozen milk products, and milk consumed by calves) explains production of butter, cheese, and nonfat dry milk. Production of butter, cheese, and nonfat dry milk is also affected by their respective wholesale price-proxies reflecting the relative profitability of producing each of these products. Production of evaporated and condensed milk is related to the prices of fluid and condensed and evaporated milk. Imports and exports of dairy products are exogenous.

Retail prices of the six dairy products are expressed as a function of their respective wholesale price and variables that reflect marketing costs. Explicit econometric equations do not need to be specified either for the wholesale prices of cheese, nonfat dry milk, and butter or for the retail price of condensed and evaporated milk as these equations can be derived from specified production, demand, and stock relationships.

Commercial Demand

Commercial demand for dairy products consists of exports, domestic consumption, stocks, and Government purchases. Exports and military consumption are exogenous. Civilian consumption of each dairy product is related to its own real price, the real price of competing products, real disposable income, and population growth. Commercial stocks of butter, cheese, and nonfat dry milk are related to their respective wholesale prices and to production.

Government purchases (placements) of dairy products have generally been specified as linear functions of the wholesale price and the Government support price (6). Such functional relationships ignore the discontinuity in Government purchases when market clearing prices are above the designated support price.

This problem is avoided here by computing Government purchases as the residual difference between supply and demand. Initially, a free-market clearing price is computed. This price is then compared with the price-support level, and if the free-market is above the price-support level, and below the release price, no action is taken. However, if the free-market price is below the price-support level, the market price is set equal to the price-support level, and the level of Government purchases is computed as the difference between supply and demand at the support price. A similar process is followed when the free-market price exceeds the release price for a particular dairy product.

Empirical Equations

The equation parameters of the dairy submodel were estimated using ordinary least squares. Three distinct time periods (1950-79, 1955-79, and 1960-79) were selected for parameter estimation.[4] The final set of equations selected represents the best set based on hypothesized parameter signs, significance of the parameter estimates, and the standard error of regression. Parameter estimates were compared over the three estimation periods. When parameter estimates did not vary substantially over the estimation periods, the equation using the longest data series was included in the submodel.

A few equations, while accurately predicting a particular variable over much of the estimation period, contained rather substantial errors for

[4] Selected equations have since been reestimated using later data. However, only the original estimates are presented.

selected years. The most notable errors were for dairy cow additions during the 1965-71 period and dairy cow slaughter during the 1965-69 period. Dummy variables were included only after alternative specifications were explored and found inferior. Table 1 defines the variables contained in the submodel. Tables 2 through 8 report the parameter estimates.

The dairy cow additions and slaughter equations indicate that increases in cattle (utility cow and calf) prices and in feed costs reduce the number of dairy cows. An increase in feeding costs negatively affects milk production per cow. The stock of dairy cows on farms 2 years earlier was included in the dairy cow additions equation as a proxy for the available supply of replacements.

Production of butter and cheese was found to be significantly related to the wholesale prices of butter, cheese, and nonfat dry milk and to the quantity of milk available for manufacturing. Producer milk prices were significantly related to the wholesale prices of butter, cheese, and nonfat dry milk. Nonfat dry milk production was positively related to butter production, but negatively related to cheese production.

Per capita civilian disappearance of fluid milk is a function of the ratio of the retail price of fluid milk relative to the consumer price index (CPI) for nonalcoholic beverages and is a function of the ratio of the retail price of fluid milk relative to the price of nonfat dry milk. Increases in both variables significantly reduce civilian disappearance of fluid milk. A time trend captures the decline in consumer preferences for fluid milk relative to nonalcoholic beverages during the estimation period. Per capita disposable real income was dropped from the regression because it was not statistically significant.

Per capita civilian disappearance of nonfat dry milk declines as the price of nonfat dry milk increases relative to the price of fluid milk. Unlike per capita civilian disappearance of fluid milk, there is a fairly strong positive relationship between per capita consumption of nonfat dry milk and real per capita disposable income.

Per capita civilian disappearance of butter declines significantly as the ratio of the retail price of butter increases relative to the retail price of margarine, but the disappearance of butter does not appear to be significantly affected by the level of real capita disposable income. A time trend reflects reduced consumption of foods high in cholesterol. Beginning in 1978, the downward trend in civilian disappearance of butter seems to have leveled off somewhat.

Per capita civilian disappearance of cheese is a function of the ratio of the retail price of cheese relative to the all-item CPI and to real per capita disposable income. The retail price of meat was dropped from the equation because it was not statistically significant. However, the demand for cheese seems to have shifted upward in 1973, immediately after the large increase in meat prices. It appears that consumers significantly increased their demand for cheese following the large increase in meat prices in 1972-73 and did not reduce their demand for cheese after meat prices leveled off.

Validation Statistics

Various procedures have been proposed for validating econometric models. These procedures generally involve examining the statistical characteristics of individual equations, as well as examining the predictive ability of the entire system of equations. The equations comprising the dairy submodel seem to contain parameters of appropriate sign and magnitude. However, such characteristics do not ensure that the entire system of equations will accurately predict future events. Since future events are unknown, researchers have proposed that model predictions for historical periods be used to examine a model's predictive ability.

A variety of validation statistics have been proposed to determine the predictive adequacy of econometric models.[5] The most widely used include: the mean absolute relative error (MARE), Theil's U1 and U2 statistics, and turning point error (TPE). The MARE is widely used because of its ease in calculation and interpretation. It can be interpreted as the mean error of the model's estimate for a particular variable. If the MARE equals zero, the model's estimate for a particular variable exactly equals that variable's historical data. The MARE is independent of measurement units.

A drawback of the MARE is that it does not possess an upper limit. Thus, Theil's U1 statistic was proposed as an alternative measure of a model's predictive ability. The value of this statistic equals zero if the model's estimates for a variable are exactly equal to that variable's historical data. The maximum value of Theil's U1 statistic is 1, which will occur either when negative proportionality exists between the model's estimates and the historical data or the model always predicts a value of zero for nonzero historical values or when the model predicts nonzero values for historical values that are zero.

A more stringent test of the predictive ability of an econometric model is Theil's U2 statistic. This statistic equals zero when the model's

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[5] See (5) and (7) for in-depth discussions on historical validation of econometric models.

estimates for a particular variable exactly coincide with that variable's historical data. It equals 1 if the forecast error generated by the model for a variable equals the error generated by a no-change model (current-year values equal previous-year values). A value greater than 1 indicates that the model generates predictive errors exceeding those derived by a no-change model.

Another measure of the ability of a model to predict is the turning points error statistic (TPE). Errors in predicting turning points stem from two sources. First, the model may predict a turning point in a variable when one did not occur. Second, the model may fail to predict a turning point when one did occur. The TPE measures the relative frequency of the total number of turning point errors.

The dairy-sector submodel was validated over the 1966-79 period.[6] In the validation run, historical values were used for all nondairy-sector variables contained in FAPSIM. The dairy-sector submodel generated values for lagged endogenous variables. As a result, model errors over the historical period stem from two sources. The first source is a result of the inability of the model's equations to exactly predict economic events in the dairy sector in any particular year. The second source stems from the model's inability to exactly predict past (lagged) values for dairy-sector variables.

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[6] A Gauss-Seidel algorithm is used to solve the model's system of simultaneous equations (4).

Table 9 presents the validation statistics computed for the dairy-sector variables for the 1966-79 period.[7] Overall, the dairy-sector equations appear to predict with reasonable accuracy. Total cow numbers (COWSNMC) were predicted with an average error of less than 1 percent and with no turning point errors. Total milk production (MILAP) was predicted within about 1 percent. Over the 14-year (1966-79) period, the model predicted three turning points in milk production that did not occur. Two of those errors occurred in 1974 and 1975 when milk prices were increasing rapidly. However, as indicated by the MARE and by Theil's U statistics, the failure to predict such turning points did not lead to substantial prediction errors.

Milk prices are predicted with reasonable accuracy, as well as production, utilization, and prices of manufactured dairy products. Of the 44 variables, 27 are predicted within a 5-percent error on average over the 1966-79 period, and 26 have fewer than four turning point errors (table 9). Only seven variables have average errors exceeding 10 percent, and only five variables have Theil's U2 statistics exceeding 1.0.

Commercial stocks of evaporated and condensed milk, nonfat dry milk, and butter were all predicted with an average error exceeding 10 percent. Such errors were not unexpected as commercial stocks of these dairy products are small relative to total production (generally less than 0.5 percent) and tend to be quite volatile. Because such stocks comprise only a small

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[7] The validation statistics presented in table 9 for milk production and price are similar to those obtained when the entire FAPSIM model was validated (11).

portion of the demand for these dairy products, sizable prediction errors in these variables do not generally result in substantial errors in other variables.

The three additional variables with MARE exceeding 10 percent were USDA purchases of cheese (CHEGU), butter (BUTGU), and nonfat dry milk (MILGUND). However, if 1979 is ignored, the MARE of USDA purchases of cheese declines from 101 to 34 percent and the MARE of USDA purchases of butter declines from 50 to 22 percent. The large overestimates of Government purchases of butter and cheese in 1979 stem from an overestimate of milk production coupled with an underestimate of fluid milk consumption. Both those prediction errors caused the model to overestimate butter and cheese production, which in turn caused substantial overestimates of USDA purchases of butter and cheese.

The Theil U2 statistic and the TPE statistic suggest that the large errors predicted for USDA purchase of butter, cheese, and nonfat dry milk are somewhat misleading. First, the number of turning point errors are not substantial. Second, for both butter and nonfat dry milk, the model outperforms a no-change-from-previous-year forecast. Furthermore, such purchases were extremely volatile over the validation period and in many years were negligible. For example, USDA purchases of cheese ranged from less than 3.0 million pounds in 1973 to 148.0 million pounds in 1977. The MARE statistic will tend to be large in such circumstances as a 3.0 million-pound error in 1973 is treated as equivalent to a 148.0 million-pound error in 1977.

An additional validation test is to compare model predictions with actual data for periods not included in the estimation of model equations.

Therefore, a 1-year simulation for 1980 was performed.[8] The model estimated milk prices and production with less than a 1-percent error. The only substantial error occurred in the model's estimate of USDA cheese purchases: it exceeded its actual value by 106.0 percent. Again, the residual nature of this variable was the cause of the large error. In 1980, the model overestimated cheese production by 5.0 percent, and it underestimated civilian consumption of cheese by 6.8 percent. Together, these two errors caused the large overestimate of USDA cheese purchases. This finding suggests that although the supply and utilization of dairy products may be estimated with reasonable error, the residual nature of dairy product purchases may still result in rather substantial errors in predictions for USDA purchases.

Overall, the model performed adequately over the 1966-79 validation period and in 1980. The model demonstrated an ability to generate reasonable and accurate forecasts for a period characterized by rapidly changing milk prices.

Use of the Model

Policy Analysis

Salathe used FAPSIM to determine the probable effects of various dairy program options (10). Some of the alternatives are built directly in the model, while others have to be user applied in the course of the solution. The dairy support price variable is used to determine the level of support

[8] These are the results of the original validation test applied to the dairy model in 1981. Time constraints for this presentation did not allow for extension of the validation period. However, the ultimate test of a model is its usefulness. Overall, estimates provided by this model have proved to be acceptable through time.

for nonfat dry milk, butter, and cheese. The model estimates the levels of support for dairy products based on the CCC's standard procedure (13). Alternatively, the level of the support prices can be set exogenously. Any adjustments in the level of parity support or in the parity index formula must be applied exogenously to determine the level of the dairy support price.

A dairy target price variable is included in the model as an alternative price support option. There is a switch variable which allows for either the model to determine the target price in terms of a moving average of wholesale price received by farmers for all milk or for the user to set the target price exogenously. However, the model is not set up to determine either the level of direct payments to the producer or total government outlays for these payments.

One program option under consideration for the 1985 farm bill is to adjust the support level according to CCC net removals of dairy products. As net removals reach certain levels, this triggers a specified decline in the support level. This option is not built into FAPSIM and must be applied by the user in the course of the solution. The model is solved and the user checks the level of net removals. If the trigger level is hit in a particular year, the user exogenously lowers the support price in that year by the set amount and the model is resolved.

Other program options currently under consideration include increasing commercial exports through some sort of an export enhancement program. As exports and imports are exogenous, these must be applied by the user prior to model solution.

The dairy diversion program has to be exogenously applied to the model. The program results in changes in dairy cow slaughter, dairy cow placements, and milk production per cow. These adjustments have to be made exogenously. The producer payment (assessment) portion of this program was accounted for by adjusting the level of price support.

Technology Analysis

The dairy submodel is based on average technology (e.g.,feed efficiency, milk cow productivity) observed over the estimation period. This resulted in certain set of structural coefficients in the model. The effects of technological change obtained through bio-technology, genetic engineering, and changes in feed efficiency or rations is to change some of the structural coefficients of the industry. If this change is expected to be permanent, these adjustments in coefficients can be built into the structure of the model. In the short run, this option is not available to the user. The effects on animal inventory, milk production, and feed demand have to be determined exogenously. Initially the model is solved. The user then determines the level of adjustments that are necessary in order for the estimates to reflect the technological change. Then the model is resolved with constant level adjustments applied as add factors.

Conclusions

Mounting Government surpluses of manufactured dairy products and recent substantial Federal budget outlays for dairy price supports have renewed debate on the Government's role in the U.S. dairy industry. A variety of proposals have been formulated by policymakers, farmer groups, and the dairy industry to reduce the Government's role in milk pricing and marketing. The complexity of the dairy industry requires that a formal analytical framework be used to analyze and quantify the potential impacts of alternative proposals on dairy farmers, milk processing firms, and consumers.

The dairy submodel described here explicitly recognizes the role of the Government in supporting milk prices and marketing. Furthermore, the model captures the interrelationships among dairy products at both processing and consumer levels.

As an aggregate annual econometric model, it cannot be used to evaluate inter-year nor inter-regional market activities. Further, the model was not specified to directly address the effects of technological change obtained through bio-technology, genetic engineering, and changes in feed efficiency or rations. Finally the model cannot be used to evaluate the financial stress of individual producers.

The model endogenously estimates dairy cow numbers, milk production; farm level milk prices; fluid milk consumption; and the supply, utilization, and prices of butter, cheese, nonfat dry milk, condensed and evaporated milk, and frozen milk products. It also endogenously estimates USDA purchases of manufactured dairy products and costs of Government dairy product purchases under alternative dairy price-support options.

The dairy-sector submodel has been integrated into USDA's FAPSIM. FAPSIM estimates a simultaneous price-quantity equilibrium solution for a set of individual commodity models for beef, pork, dairy, chicken, eggs, turkey, corn, oats, barley, grain sorghum, wheat, soybeans, and cotton. FAPSIM can be used to explore the impacts of changes in dairy policies on crop and livestock producers as well as the impacts of changes in nondairy-sector variables (for example crop exports) on milk prices and production and on Government purchases of dairy products.

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Table 1--Dairy submodel variables

Variable	Definition
Endogenous:	
BUTCC	Civilian disappearance of butter, billion pounds
BUTGU	USDA purchases of butter, billion pounds
BUTHB	Beginning commercial stocks of butter, billion pounds
BUTHG	Beginning USDA stocks of butter, billion pounds
BUTIR	Retail price index of butter, 1967 = 1.0
BUTSP	Production of butter, billion pounds
CHECT	Civilian disappearance of cheese, billion pounds
CHEGU	USDA purchases of American cheese, billion pounds
CHEHB	Beginning commercial stocks of cheese, billion pounds
CHEHG	Beginning USDA stocks of American cheese, billion pounds
CHEIRAM	Retail price index of American cheese, 1967 = 1.0
CHESP	Production of cheese, billion pounds
COWKSMC	Number of milk cows slaughtered, million head
COWSEMC	Number of dairy cow replacements, million head
COWSNMC	Number of milk cows on farms, January 1, million head
DAIFC	Cash receipts from milk sales, billion dollars
DAIGP	Total cost of USDA dairy product purchases, million dollars
DARCPI	Retail price index of dairy products, 1967 = 1.0
MILAMCHEE	Wholesale price of American cheese at Wisconsin assembling points, 40-pound block, cents per pound
MILAP	Total milk production, billion pounds
MILASFM	Quantity of milk produced eligible for fluid market, billion pounds
MILBC	Quantity of milk fed to calves, billion pounds
MILBUT	Wholesale price of Grade A butter, Chicago, cents per pound
MILCCEC	Civilian disappearance of evaporated and condensed milk, billion pounds
MILCCFZ	Civilian disappearance of milk used in frozen dairy products, billion pounds
MILCCMC	Civilian disappearance of fluid milk plus cream, billion pounds
MILCCND	Civilian disappearance of nonfat dry milk, billion pounds
MILECLOP	Effective Class I milk price paid by dealers, dollars per cwt
MILGUND	USDA purchases of nonfat dry milk, billion pounds
MILHBND	Beginning commercial stocks of nonfat dry milk, billion pounds
MILHGND	Beginning USDA stocks of nonfat dry milk, billion pounds
MILHTEV	Ending stocks of evaporated and condensed milk, billion pounds
MILIR	Retail price index for fluid milk, 1967 = 1.0
MILIREV	Retail price index for evaporated milk, 1967 = 1.0
MILIRIC	Retail price index for ice cream, 1967 = 1.0

Table 1--Dairy submodel variables (continued)

Variable	Definition
MILOMP	Minimum Federal order price for Class I milk, dollars per cwt
MILMFG	Quantity of milk available for manufacturing, billion pounds
MILMWAT	Minnesota-Wisconsin manufacturing grade milk price, dollars per cwt
MILPF	Average price received by farmers for all milk sold to plants, dollars per cwt
MILPPFEMAT	Average price received by farmers for fluid eligible milk, dollars per cwt
MILPPMAT	Average price received by farmers for manufacturing grade milk, dollars per cwt
MILPWDR	Wholesale price index for nonfat dry milk, 1967 = 1.0
MILSPEC	Production of evaporated and condensed milk, billion pounds
MILSPECM	Production of evaporated and condensed milk, billion pounds of milk used
MILSPFZ	Production of frozen dairy products, billion pounds of milk used
MILSPND	Production of nonfat dry milk, billion pounds
MILSPPLTS	Quantity of milk sold to plants and dealers, billion pounds
Exogenous:	
BARPF*	Price received by farmers for barley, June-May, dollars per bushel
BUTCM	Military disappearance of butter, billion pounds
BUTDV	USDA unaccounted-for change in stocks of butter, billion pounds
BUTGG	USDA donations of butter, billion pounds
BUTMG	USDA exports of butter, billion pounds
BUTMI	Imports of butter, billion pounds
BUTMX	Exports of butter, billion pounds
CALPF*	Price received by farmers for calves, dollars per cwt
CATPFNF*	Price of utility cows, Omaha, dollars per cwt
CHECM	Military disappearance of cheese, billion pounds
CHEDV	USDA unaccounted-for change in stocks of American cheese, billion pounds
CHEGG	USDA donations of American cheese, billion pounds
CHEMG	USDA exports of cheese, billion pounds
CHEMI	Imports of cheese, billion pounds
CHEMX	Exports of cheese, billion pounds
CORPF*	Price received by farmers for corn, Oct.-Sept., dollars per bushel
DUMij	Dummy variable, 19ij = 1.0
DUMijkl	Dummy variable, 19ij - 19kl = 1.0
MARIR*	Consumer price index for margarine, 1967 = 1.0

Table 1--Dairy submodel variables (continued)

Variable	Definition
MILBCND	Nonfat dry milk fed to calves, billion pounds
MILCHCHSPP	USDA purchase price of American cheese, dollars per cwt
MILCIDF	Historical difference between Federal order minimum Class II milk price and Minnesota-Wisconsin manufacturing grade price, dollars per cwt
MILCMEC	Military disappearance of evaporated and condensed milk, billion pounds
MILCMFZ	Military disappearance of frozen dairy products, billion pounds
MILCMND	Military disappearance of nonfat dry milk, billion pounds
MILDVND	USDA unaccounted-for change in stocks of nonfat dry milk, billion pounds
MILGGND	USDA donations of nonfat dry milk, billion pounds
MILMGND	USDA exports of nonfat dry milk, billion pounds
MILMIEC	Imports of evaporated and condensed milk, billion pounds
MILMIFZ	Imports of frozen dairy products, billion pounds
MILMIND	Imports of nonfat dry milk, billion pounds
MILMXEC	Exports of evaporated and condensed milk, billion pounds
MILMXND	Exports of nonfat dry milk, billion pounds
MILNFDSPP	USDA purchase price of nonfat dry milk, dollars per cwt
MILOOP	Federal order over order payments for Class I milk, dollars per cwt
MILPFDIF	Historical difference between average price received by farmers for fluid eligible milk and weighted Federal order price for fluid eligible milk, dollars per cwt
MILSPPBUT	USDA purchase price of butter, dollars per cwt
OATPF*	Price received by farmers for oats, June-May, dollars per bushel
SOMPF*	Price of soybean meal, Decatur, 44 percent, Oct.-Sept., dollars per cwt
SORPF*	Price received by farmers for grain sorghum, Oct-Sept., dollars per bushel
WHEPF*	Price received by farmers for wheat, June-May, dollars per bushel
.GASIR	Consumer price index for regular and premium gasoline, 1967 = 1.0
.NPC	Total U.S. population, millions
.PC*	Consumer price index for all items, 1967 = 100
.PCNAL*	Consumer price index for nonalcoholic beverages, 1967 = 1.0
.TIME	Time trend 1950 = 50, 1951 = 51, and so forth
.WRHD	Dairy manufacturing industry wage rate, dollars per hour
.YPD$	U.S. personal disposable income, billion dollars

*Denotes variables that are exogenous to the dairy submodel, but endogenously computed by other FAPSIM submodels.

Table 2--Milk supply

Variable	Equation
COWSNMC(+1)	0.98 COWSNMC + COWSEMC - COWKSMC
COWKSMC	0.738171 + 0.326629 DUM6569 + 0.479213 DUM5758
	(2.41) (5.59) (6.21)
	- 0.149505 MILPF/FDD
	(-3.50)
	+ 0.102808 COWSNMC + 0.501987 COWSEMC - 0.754813
	(2.85) (2.33) (-1.62)
	MILPF/CATPFNF
	R^2 = 0.987
COWSEMC	0.203916 + 1.09718 MILPF(-1)/CALPF(-1) + 0.0841727
	(0.52) (1.74) (1.41)
	MILPF(-1)/FDD(-1)
	+ 0.142653 COWSNMC(-2) - 0.318917 DUM6571
	(18.82) (-6.02)
	R^2 = 0.961
$\frac{\text{MILAP}}{\text{(COWSNMC(+1)+COWSNMC)/2}}$	- 3.92481 + 0.135732 MILPF/FDD + 0.127848.TIME
	(-2.61) (2.38) (2.83)
	+ 0.424017 MILAP(-1)/(COWSNMC + COWSNMC(-1))/2
	(2.20)
	R^2 = 0.991
MILBC	- 0.381728 + 0.167949 COWSNMC
	(-5.87) (42.31)
	R^2 = 0.984

Table 2--Milk supply (continued)

Variable	Equation
MILSPPLTS / (MILAP-MILBC)	- 1.73964 + 0.0717014 .TIME -0.000473564 .TIME**2 (-17.00) (23.28) (-20.63) R^2 = 0.993
MILASFM / MILSPPTLTS	- 0.0433665 + 1.02736 MILASFM(-1)/MILSPPLTS(-1) (-1.24) (38.61) + 0.0236661 (MILPPFEMAT(-1) - MILPPMAT(-1)) (1.38) R^2 = 0.986
MILMFG	MILAP - MILBC - MILCCMC - MILSPFZ - MILSPECM
FDD	0.5563 CORPF(-1) + 0.0469 SORPF(-1) + 0.2565 OATPF(-1) + 0.0462 BARPF(-1) + 0.0102 WHEPF(-1) + 0.0839 SOMPF(-1)

Note: Numbers in parentheses are Student-t values.

Table 3--Milk price

Variable	Equation
MILPPMAT	$- 0.283616 + 0.0178284$ MILBUT $+ 0.599078$ MILPWDR (-1.31) (1.77) (3.15) $+ 0.0543683$ MILAMCHEE (5.13) $R^2 = 0.999$
MILMWAT	$- 0.226964 + 0.0114579$ MILBUT $+ 0.449113$ MILPWDR (-3.15) (3.34) (3.52) $+ 0.0663590$ MILAMCHEE (9.31) $R^2 = 0.999$
MILOMP	MILCIDF + MILMWAT
MILECLOP	MILOOP + MILOMP
MILPPFEMAT	MILPFDIF + [(MILECLOP)(MILCCMC)(MILSPPLTS)/ (MILAP - MILBC) + (MILMWAT)((MILASFM)- (MILCCMC)(MILSPPLTS)/(MILAP - MILBC))]/MILASFM
MILPF	[(MILPPFEMAT)(MILASFM) + (MILPPMAT)(MILSPPLTS - MILASFM)]/MILSPPLTS

Note: Numbers in parentheses are Student-t values.

Table 4--Butter sector

Variable	Equation
BUTSP	- 0.350572 + 1.22365 MILBUT/MILAMCHEE +0.0116949 MILMFG (-1.30) (6.31) (2.40) - 0.152769 MILAMCHEE/MILPWDR + 0.153427 DUM74 (-2.42) (2.40) R2 = 0.926
BUTCC/.NPC	0.0600122 - 0.00274512 BUTIR/MARIR +0.00114400 DUM7879 - 0.00080432 DUM74 (9.17) (-2.46) (3.12) (-1.61) - 0.152247 .TIME/.NPC (-8.93) R^2 = 0.869
BUTIR	- 0.0858682 + 0.0130207 MILBUT + 0.0413876 .WRHD + 0.101378 .GASIR (-3.36) (16.24) (4.12) (2.95) R2 = 0.996
BUTHB(+1)	0.0036095 + 0.0162062 BUTSP + 0.0156486 DUM7374 (0.32) (2.49) (2.49) R^2 = 0.203
MILBUT	$(-BUTSP + BUTCC + BUTHB(+1) - BUTHB + BUTMX + BUTCM - BUTMI + BUTHG(+1) - BUTHG)^{-1}$
BUTHG(+1)	BUTSP - BUTCC + BUTHG - BUTHB(+1) + BUTHB - BUTMX - BUTCM + BUTMI
BUTGU	BUTHG(+1) - BUTHG - BUTGG + BUTMG - BUTDV

Note: Numbers in parentheses are Student-t values.

Table 5--Cheese sector

Variable	Equation
CHESP	- 6.07091 + 0.111475 MILMFG + 3.12002 (-3.74) (10.79) (3.74) MILAMCHEE/MILBUT + 0.0101392 (0.60) MILAMCHEE/MILPWDR - 0.517856 DUM74 + (-3.22) 0.288983 DUM68 (2.15) R^2 = 0.966
CHECT .NPC	0.00307155 - 0.955747 CHEIRAM/.PC + 0.609481 (1.11) (-2.02) (7.68) .YPD$/(.NPC)(.PC) + 0.00368518 DUM7480 (6.90) R^2 = 0.990
CHEIRAM	0.0391632 + 0.0138097 MILAMCHEE + 0.0832134 .WRHD + (1.00) (4.20) (1.59) 0.0832052 .GASIR (1.13) R^2 = 0.995
CHEHB(+1)	- 0.139726 + 0.260058 CHEHB + 0.556479 CHESP (-3.23) (1.48) (3.06) R^2 = 0.581
MILAMCHEE	$(-CHESP + CHECT + CHEHB(+1) - CHEHB + CHEMX + CHECM - CHEMI + CHEHG(+1) - CHEHG)^{-1}$
CHEHG(+1)	CHESP - CHEHB(+1) - CHECT - CHEMX - CHECM + CHEMI + CHEHB + CHEHG
CHEGU	CHEHG(+1) - CHEHG + CHEGG + CHEMG - CHEDV

Note: Numbers in parentheses are Student-t values.

Table 6--Nonfat dry milk sector

Variable	Equation
MILSPND	0.220950 + 1.50162 BUTSP - 0.225588 CHESP
	(0.71) (8.62) (-4.44)
	R^2 = 0.961
$\frac{\text{MILCCND}}{\text{.NPC}}$	0.00667157 + 0.00140079 DUM73 -0.00243915
	(14.99) (5.07) (-10.95)
	MILPWDR/MILIR + 0.0515417 .YPD$/(.NPC)(.PC)
	(2.08)
	R^2 = 0.937
MILHBND(+1)	0.0420496 + 0.276756 MILSPND + 0.0647213 DUM74
	(2.27) (2.35) (2.65)
	R^2 = 0.301
MILPWDR	(-MILSPND + MILCCND + MILHGND(+1) - MILHGND - MILMIND - MILHBND + MILMXND + MILBCND + MILHBND(+1) + MILCMND)$^{-1}$
MILHGND(+1)	MILCCND + MILSPND + MILHGND - MILBCND + MILHBND - MILMXND + MILMIND - MILHBND(+1) - MILCMND
MILGUND	MILHGND(+1) - MILHGND + MILGGND + MILMGND - MILDVND

Note: Numbers in parentheses are Student-t values.

Table 7--Evaporated and condensed milk sector

Variable	Equation
MILSPEC	8.54493 - 0.112500 .TIME + 0.939724 MILIREV/MILIR (33.12) (-16.89) (3.40) R^2 = 0.975
MILCCEC / .NPC	0.0230599 + 0.00121912 DUM6568 - 0.00241843 (13.12) (4.06) (-2.15) MILIREV/MILIR - 0.459281 .YPD$/(.NPC)(.PC) (-5.37) R2 = 0.980
MILHTEV(+1)	- 0.0291461 + 0.0546571 DUM6667 + 0.0862268 MILSPEC (-1.82) (3.35) (9.68) R^2 = 0.862
MILIREV	(-MILSPEC + MILCCEC + MILHTEV(+1) - MILMIEC + MILMXEC + MILCMEC - MILHTEV)$^{-1}$
MILSPECM	0.313912 + 1.96209 MILSPEC (6.63) (75.60) R^2 = 0.997

Note: Numbers in parentheses are Student-t values

Table 8--Frozen desserts and fluid milk sector

Variable	Equation
MILCCFZ .NPC	0.0730505 - 1.90300 MILIRIC/.PC - 0.093076 .YPD$/ (7.28) (-3.46) (-0.61) (.NPC)(.PC) R^2 = 0.740
MILIRIC	2.35231 + 0.335003 .WRHD + 0.0423319 MILECLOP - (9.32) (5.50) (1.79) 0.0382222 .TIME (-8.44) R^2 = 0.982
MILSPFZ	MILCMFZ - MILMIFZ + MILCCFZ
MILCCMC .NPC	2.45628 - 0.0915642 MILIR/.PCNAL - 0.0470187 (10.67) (-7.86) (-2.54) MILIR/MILPWDR - 6.02686 .TIME (-9.75) R^2 = 0.960
MILIR	0.221189 + 0.0491676 .WRHD + 0.105076 MILECLOP (14.85) (3.37) (13.24) R^2 = 0.997
DARCPI	- 0.039374 + 0.671257 + 0.102841 BUTIR + 0.190153 (-4.80) (39.59) (11.69) (14.60) CHEIRAM + 0.0775998 MILIRIC (10.26) R^2 = 0.999
DAIGP	((BUTGU)(MILSPPBUT) + (CHEGU)(MILCHCHSPP) + (MILGUND)(MILNFDSPP)).10
DAIFC	290.148 + 9.97787 (MILPF)(MILSPPLTS) (10.42)(282.07) R^2 = 0.999

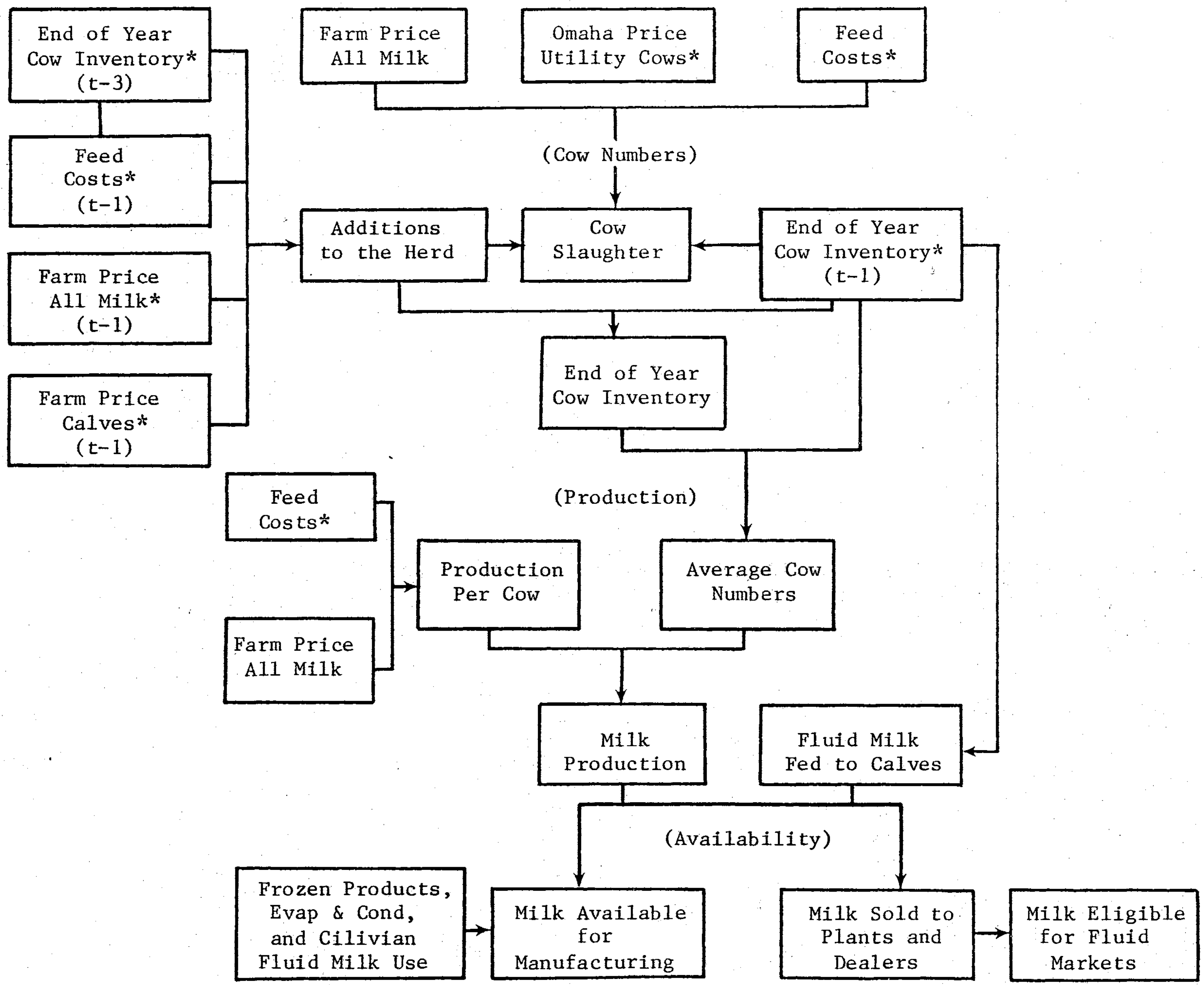
Note: Numbers in parentheses are Student-t

Table 9--Validation statistics, 1966-79

Variable	Mean Absolute relative error	Theil U_1 statistic	Theil U_2 statistic	Turning point error
	Percent			
BUTCC	4.55	.505	1.113	.357
BUTGU	50.86	.403	.790	.286
BUTHB	43.77	.315	.540	.500
BUTIR	6.06	.382	.885	.071
BUTSP	6.28	.520	.951	.500
CHECT	2.72	.250	.542	.071
CHEGU	101.34	.569	1.420	.143
CHEHB	9.17	.355	.598	.357
CHEIRAM	3.83	.247	.505	.143
CHESP	3.39	.268	.572	.071
COWKSMC	2.58	.215	.445	.429
COWSEMC	3.17	.668	1.296	.286
COWSNMC	0.87	0.174	0.329	0.000
DAIFC	4.98	.336	.735	.214
DAIGP	47.69	.580	1.393	.214
DARCPI	3.06	.221	.450	.143
MILAMCHEE	7.36	.382	.790	.214
MILAP	1.03	.320	.619	.214
MILASFM	1.86	.516	.819	.214
MILBC	2.18	.394	.867	.143
MILBUT	6.64	.379	.942	.429
MILCCEC	2.64	.217	.437	.143
MILCCFZ	1.45	.449	.862	.214
MILCCMC	1.95	.531	1.394	.429
MILCCND	4.63	.238	.513	.286
MILECLOP	4.68	.311	.615	.214
MILGUND	54.33	.304	.552	.357
MILHBND	27.33	.282	.486	.500
MILHTEV	14.26	.241	.445	.286
MILIR	2.83	.217	.420	.143
MILIREV	4.32	.221	.459	.214
MILIRIC	2.92	.180	.367	.143
MILMFG	3.33	.203	.407	.286
MILMWAT	6.39	.344	.703	.143
MILOMP	4.93	.327	.644	.286
MILPF	5.34	.332	.673	.143
MILPPFEMAT	5.31	.340	.697	.143
MILPPMAT	6.04	.327	.668	.143
MILPWDR	4.56	.277	.497	.143
MILSPEC	3.08	.231	.424	.214
MILSPECM	3.12	.233	.426	.214
MILSPFZ	1.45	.167	.319	.357
MILSPND	9.08	.424	.743	.571
MILSPPLTS	1.18	.315	.620	.214

1/ The number of turning point errors divided by 14, the total number of possible turning point errors.

Figure 1. Flow Chart of Fluid Milk Supply



*Denotes variables which are either predetermined or exogenous in the dairy model.

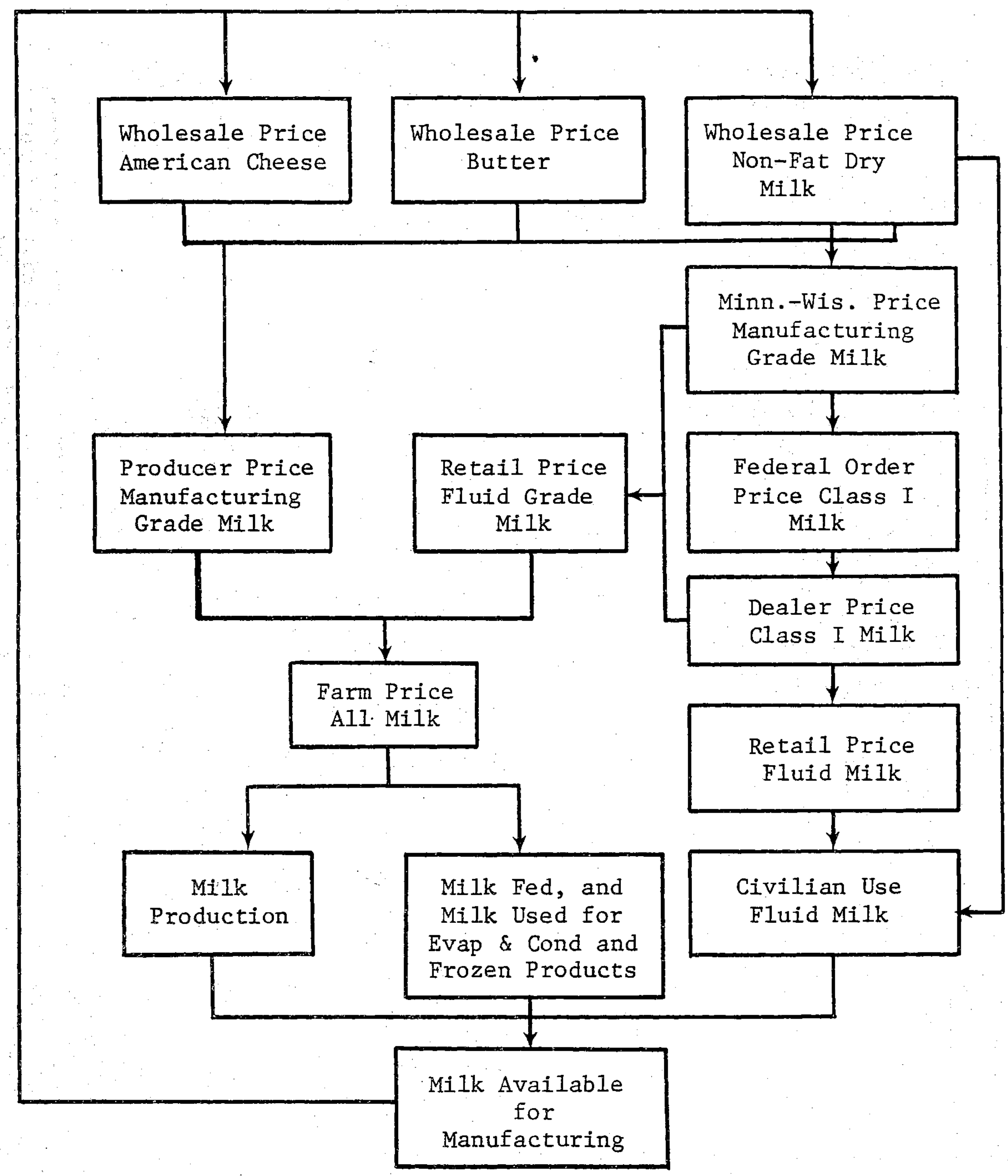
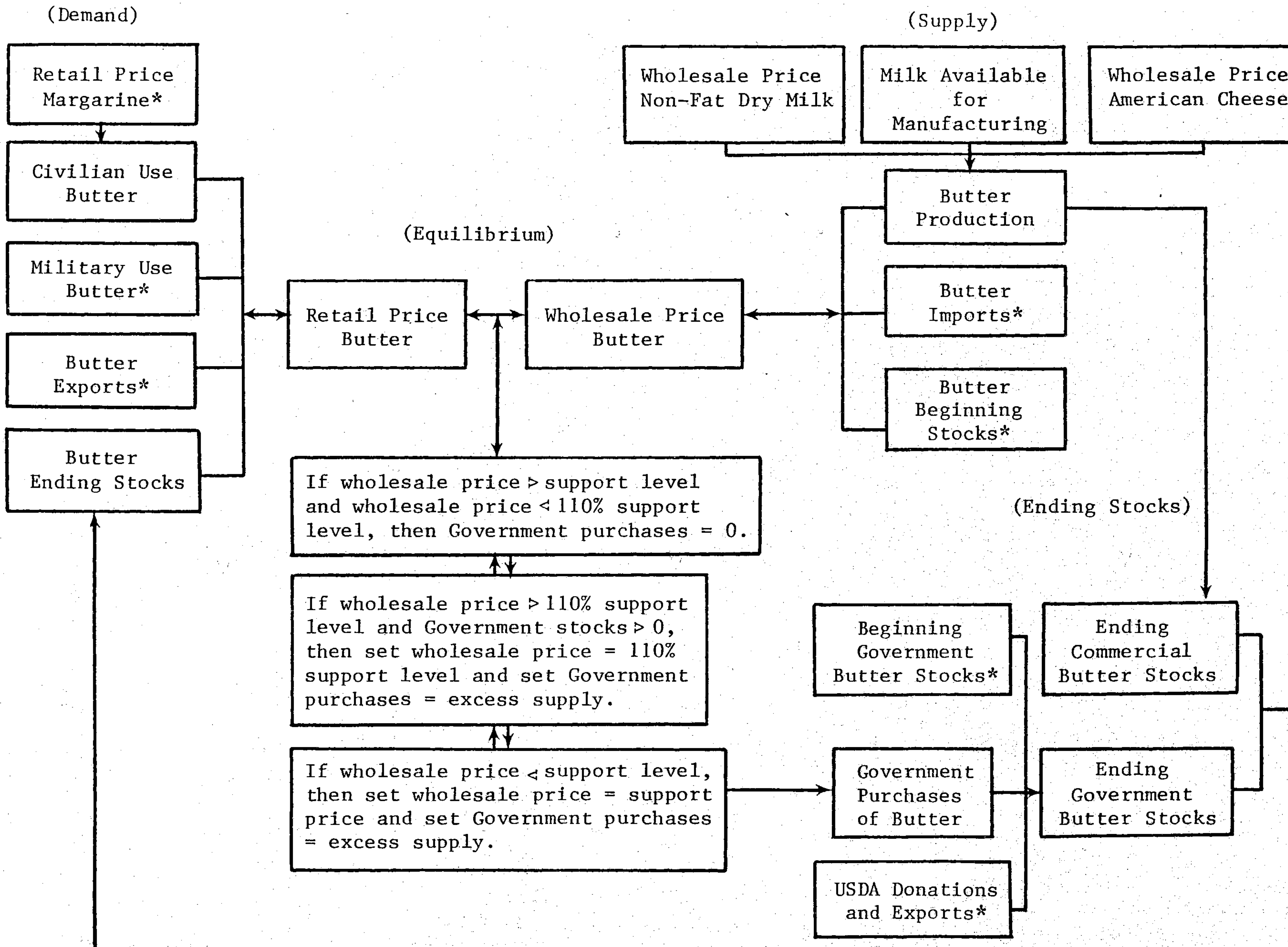


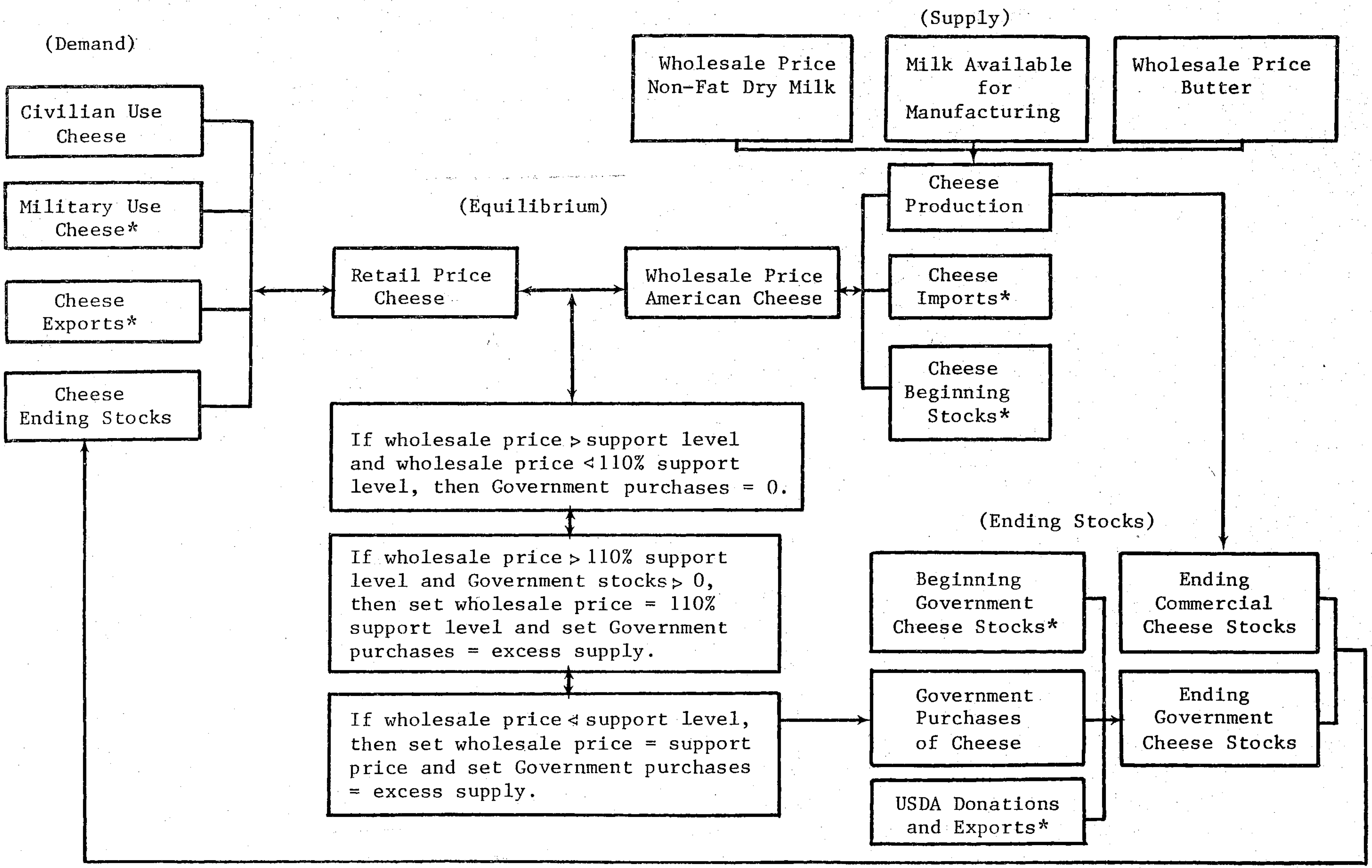
Figure 2. Flow Chart of the Price Linkages

Figure 3. Flow Chart of the Butter Sector



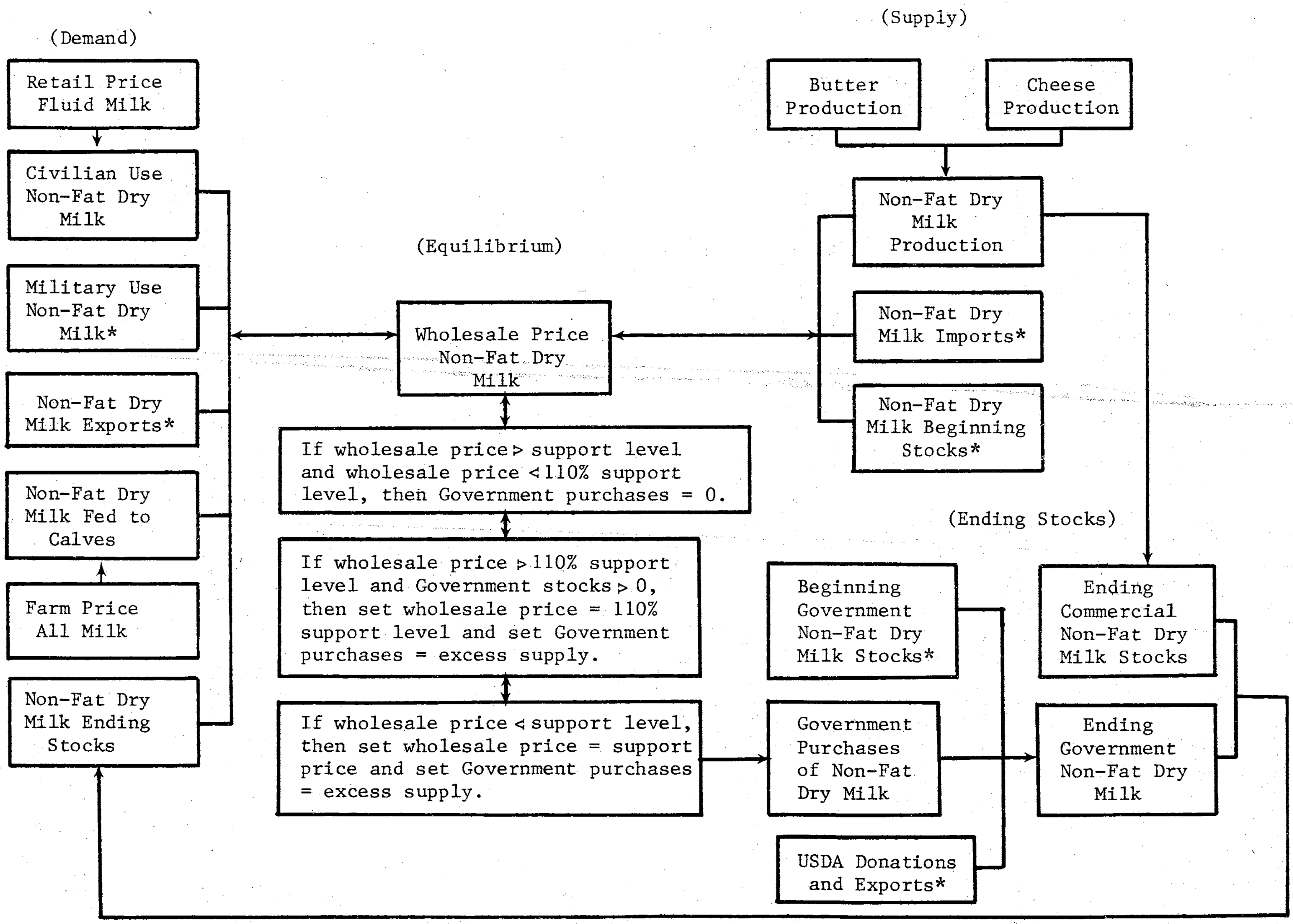
*Denotes variables which are either predetermined or exogenous in the dairy model.

Figure 4. Flow Chart of the Cheese Sector



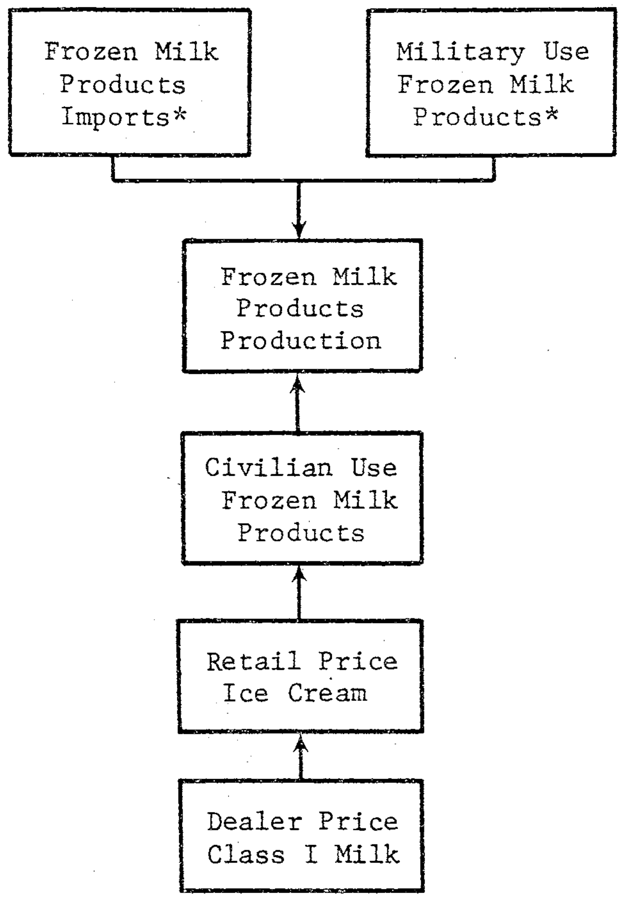
*Denotes variables which are either predetermined or exogenous in the dairy model.

Figure 5. Flow Chart of the Non-Fat Dry Milk Sector



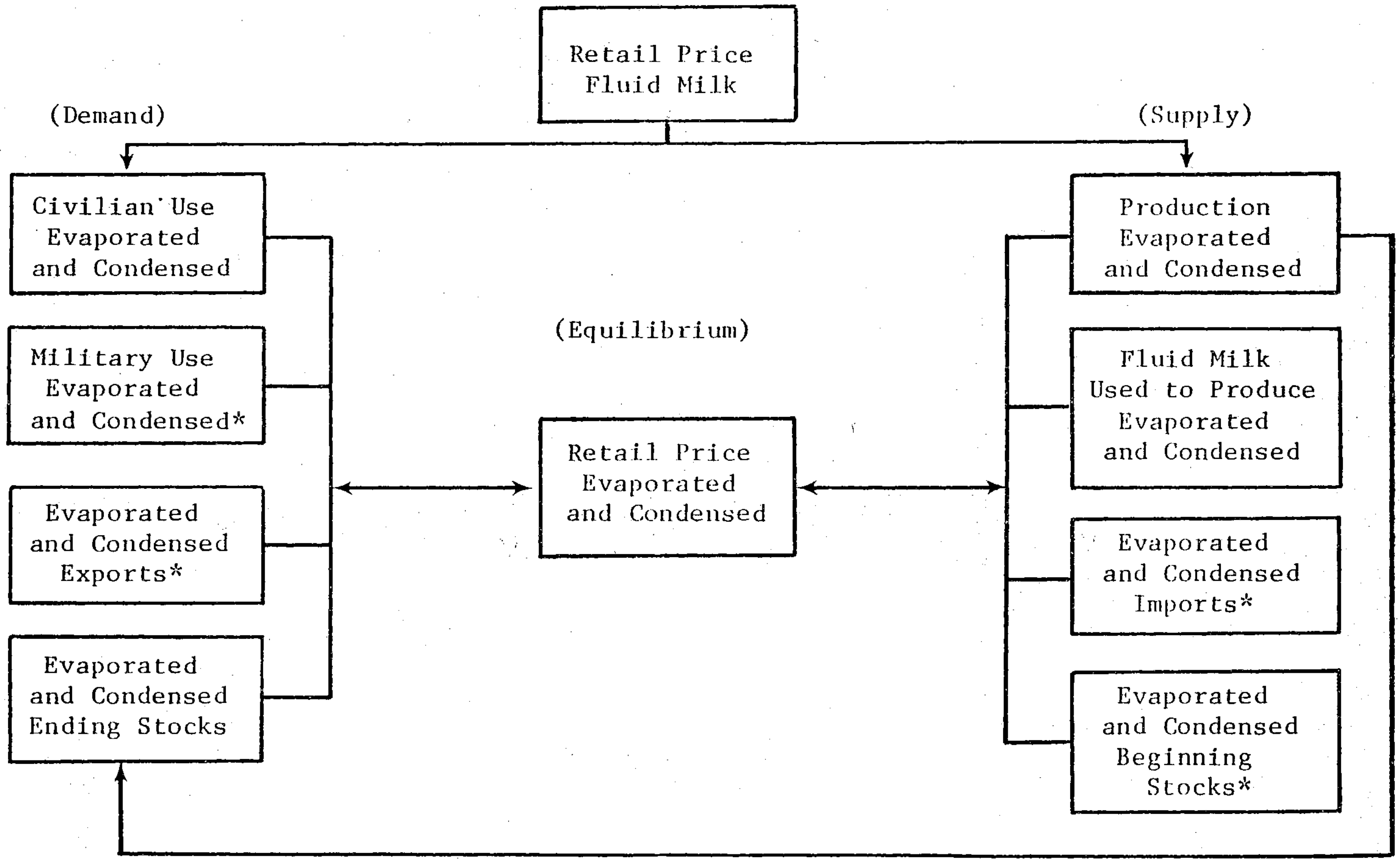
*Denotes variables which are either predetermined or exogenous in the dairy model.

Figure 6. Flow Chart of Frozen Milk Products



*Denotes variables which are either predetermined or exogenous in the dairy model.

Figure 7. Flow Chart of the Evaporated and Condensed Milk Sector



*Denotes variables which are either predetermined or exogenous in the dairy model.

RATIONAL EXPECTATIONS AND AGRICULTURAL POLICY: AN ECONOMETRIC APPLICATION TO THE U.S. DAIRY ECONOMY

by

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ABSTRACT

The conceptual and econometric applications of the rational expectations paradigm for modelling producers' expectations are derived for a simple macro-economic model of the dairy producing sector. It is demonstrated that the parameters of the estimated reduced-form equations are functions of the specific dairy price-support rule in effect.

RATIONAL EXPECTATIONS AND AGRICULTURAL POLICY: AN ECONOMETRIC APPLICATION TO THE U.S. DAIRY ECONOMY

Agricultural policy in the United States has had a long history of promoting the production of specific commodities while simultaneously protecting agricultural producers from low prices by means of price-support programs. The federal dairy price-support program has provided producers with a minimum annual price for over three decades.

A question of central importance with regard to the dairy support program concerns the evaluation and assessment, on a historical basis, of the economic behavior of the dairy economy under alternative hypothetical price support policies.

Previous economic models and analyses of the dairy price-support program have been based on the conceptual paradigm of static profit maximization, which excludes any account of risk preference, and have relied either implicitly or explicitly on the ad hoc notion of adaptive expectations or partial adjustment models to impart dynamic elements to their econometric models (Chou, Dahlgren, Heien).

The fact that producers' expectations play a central role in determining optimal production and input use, and that price supports modify these expectations, necessitates that we specify how this interaction occurs (Nerlove). The rational expectations hypothesis (REH) has been put forth as an expectations model which can fulfill this need in a consistent and logically appealing manner. REH postulates that producers learn to expect prices as given by the conditional expectations of the economic system within which they must make their input and output decisions (Muth).

Correctly modeling changes in exogenous policy variables which may modify these conditional expectations, such as the price-support level, requires that the equations describing how producers formulate their expectation of endogenous variables and the linkage with exogenous policy variables become central elements in the complete economic model (Sargent, 1980). The purpose of this paper is to briefly review and illustrate the econometric implications of REH and to demonstrate how REH may be incorporated into an econometric model of the aggregate dairy economy for policy evaluation. The original work by the author incorporates a more detailed and comprehensive analysis of the price-support policy (Thraen, 1981).[1]

Rational Producer Expectations and Policy Evaluation:

From the foregoing arguments, it should be apparent that price-supports and producer expectations of price supports are instrumental in determining dairy producer decisions. In this section, we will examine the relationship between the REH formulation of producers' price expectations and changes in the government's rules for establishing price supports.

Consider the following structural simultaneous equation model, in which anticipated or expected values of certain endogenous variables are included (Wallis, 1980; Fisher, 1982).

(1.1) $$B(L)y_t + Ay_t^* + T_1x_{1t} + T_2x_{2t} = U_t$$

where y_t is a vector of g endogenous variables, y_t^* is a vector h of expected endogenous variables, $(h \leq g)$, x_{1t} is a k_1 element vector of exogenous variables and x_{2t} is a $(k-k_1)$ vector of "known" exogenous variables. $B(L) = B_o + B_1L + \ldots + B_rL^r$ is the matrix polynomial lag function $(L^r y_t = y_{t-r})$ which allows for lagged endogenous variables. The matrix dimensions are $B\sim(gxg)$, $A\sim(gxg)$, $T_1\sim(gxk_1)$ and $T_2\sim(gx(k-k_1))$.

The producer, under the REH, formulates his anticipations of the h variables y_t^* as conditional expectations, conditioned on the structure of the relevant economic system describing the economy, i.e., the model in (1.1). Thus y_t^* is defined as $y_t^* = E(y_t | \Omega_{t-1})$ where Ω_{t-1} is the producer's information set based on (1.1).

From (1.1) we can rearrange terms

(1.2) $$B_o y_t + A y_t^* = - \{B_1 y_{t-1} + \dots + B_r y_{t-r}\} - T_1 x_{1t} - T_2 x_{2t} + U_t$$

and applying the conditional expectations operator E

(1.3) $$E(B_o y_t + A y_t^*) = E\{-B_1 y_{t-1} + \dots + B_r y_{t-r} - T_1 x_{1t} - T_2 x_{2t} + U_t\}$$

and given that $E(y_t | \Omega_{t-1}) = y_t^*$

(1.4) $$y_t^* = -(B_o + A)^{-1}\{T_1 E\{x_{1t} | \Omega_{t-1}\} + T_2 X_{2t} + B_1 y_{t-1} + \dots B_r y_{t-r}\}$$

where $E\{x_{1t} | \Omega_{t-1}\} = \hat{x}_{1t}$ is the expectation of the exogenous variables x_{1t} and all other variables are either known or predetermined. Note at this point the substantive difference between the REH formulation of y_t^* as expressed in (1.4) and equivalent formulations of expectations models widely used in econometric modeling, i.e., naive and adaptive respectively,

(1.5) naive $y_t^* = y_{t-1}$

(1.6) adaptive $y_t^* = (1-\lambda)\sum_{i=o}^{\infty}\lambda^i y_{t-i}$.

It is apparent that these models are consistent with the REH model only if we are willing to impose substantial zero-order restrictions on the elements of the matrices B(L), A, T_1, T_2.[2]

Substituting (1.4) into (1.1) yields a simultaneous structural equation system in forecast and observable variables

(1.7) $$B(L)y_t - A(B_o + A)^{-1}\{T_{1t}\hat{x}_{1t} + T_2 x_{2t} + B_1 y_{t-1} + \dots + B_r y_{t-r}\}$$

$$+ T_1 x_{1t} + T_2 x_{2t} = U_t.$$

The reduced form of the structural system becomes

$$(1.8)\quad y_t = \Pi_1\hat{x}_{1t} + \Pi_2 x_{2t} + \sum_{i=1}^{r} \Pi_{2+i} y_{t-1} + \Pi_{r+3} x_{1t} + \Pi_{r+4} x_{2t} + V_t$$

where $\Pi_1 = B_o^{-1}A(B_o + A)^{-1}T_1$, $\Pi_2 = B_o^{-1}A(B_o + A)^{-1}T_2$, $\Pi_3 = B_o^{-1}A(B_o + A)^{-1}B_1$

$\Pi_{2+i} = B_o^{-1}A)B_o + A)^{-1}B_i$, $\Pi_{r+3} = -B_o^{-1}T_1$, $\Pi_{r+4} = -B_o^{-1}T_2$ and $V_t = B_o^{-1}U_t$.

Note that in (1.8), the exogenous variables x_{1t} appear as both forecast or expected values $\hat{x}_{1t}$ and as non-forecast values x_{1t} . This suggests that by imposing the REH, the endogenous variables y_t are determined by both the producers expectations of the exogenous variables and their actual realized values. An alternative argument would suggest that if x_{1t} needs to be forecast at all, then $x_{1t} = \hat{x}_{1t}$ and the endogenous variables depend upon only the forecast values of these x_{1t} exogenous variables. If we accept the first argument, then the endogenous variables will be determined by current and lagged values of the exogenous variables, whereas with the second argument, only lagged values of the x_{1t} variables will appear in the reduced form equations.

A third alternative is to recognize that the reduced form equations are simply algebraic constructs which do not have a behavioral economic interpretation. In this case, if producers' expectations of an endogenous variable depend upon expectations of more fundamental exogenous variables, then when the rational expectation is substituted into the original structural equation, all of these expected exogenous variables are entered as expectations and not as known values. Again, the final form of the structural equation will contain only lagged values of the expected variables.

To complete the specification of the reduced form model (1.8), we need to postulate a model for x_{1t}. Note that the imposition of the REH onto the structural model has nothing to do with how we formulate the forecasting model for x_{1t}. The implications of REH are focused exclusively on the endogenous variables in the economic system.

To proceed with the modeling of x_{1t} we can move along two lines of reasoning. If a particular variable of the vector x_{1t} is itself an endogenous variable in another economic system, and assuming that the producer has full information on that system also, we can impose the REH onto that system and repeat the same steps as detailed above. Following this line of reasoning, the particular economic model we are studying would include determining variables from many other economic systems in addition to those bearing directly on our own system.

A second line of reasoning, and one which is most often used in the REH literature, is to assume that the producers in our model do not have full information of the structure of all of the other systems, and therefore, use much more simplistic forecasting rules for these exogenous variables. Such a model or forecasting rule is usually given as a vector autoregressive moving average (ARMA) model of varying degrees of complexity (Wallis, 1980; Fisher, 1982).

A simple form of this model is the first-order autoregressive model,

(1.9) $$x_{1t} = \Phi x_{1_{t-1}} + \varepsilon_t$$

where ε is a white noise process, assumed to be independent of V_t. The optimal one-step-ahead forecast for this model is $E(x_{1t} | \Omega_{t-1}) = \hat{x}_{1t} = \Phi x_{t-1}$. On substituting (1.9) into (1.8) we have the final form equations

(1.10) $$y_t = (\Pi_1 + \Pi_{r+3}) \Phi \hat{x}_{1t} + (\Pi_2 + \Pi_{r+4}) x_{2t} + \sum_{i=1}^{r} \Pi_{2+i} y_{t-i} + V_t$$

Equations (1.9) and (1.10) represent the system of equations to be estimated. From this development of the final form equations and the specification that producers' expectations are formed rationally, it is apparent that changes in the "structure", i.e., Φ, which generates the forecast values of x_{1t}, as well as the "structure", i.e., the fundamental parameters comprising the Π_i matrices, determine the values of the endogenous variables.

A Digression on Expected Price, Price Supports and Producers Output Decisions

Dairy producers operate in an economic environment which can be characterized by its asset owning nature. Dairy cows represent unique capital assets which generate a stream of revenues from joint outputs of livestock (new capital) and milk. Because a dairy farmer must make substantial capital investments today, in order to capture net revenues tomorrow and on into the future, his expectations of market prices, for both inputs and outputs play a central role in deciding on the desirability of owning the dairying assets. Specifically, the values of an asset (V_t) can be expressed as

$$V_t' = \frac{E(R_t)}{k_t} \tag{2.1}$$

where $E(R_t)$ is the expected return to the asset and k is the capitalization factor. $E(R_t)$ includes all net revenues while k_t includes both market factors as well as individual risk discount factors.

The value of $E(R_t)$ for a specific period depends upon the diary farms expectations of market price, production level and variable input costs.

Assuming that production and input costs can be taken as known, the only non-deterministic variable is market price.

Within the current U.S. policy structure for dairy, producers are paid a weighted average or blend price for milk. This price reflects the distribution of milk sold, at two different prices in two separate markets. Specifically, the blend price can be expressed as

$$(2.2) \quad P_t^B = \frac{P_t^f F_t + P_t^m M_t}{TMS_t}$$

where P_t^f ≡ fluid milk price, F_t is fluid use, P_t^m ≡ manufacturing milk price, M_t ≡ manufacturing use and TMS ≡ total milk sold. In addition, the two prices are linked by the relationship

$$(2.3) \quad P_t^f = P_t^m + \theta_t$$

where θ_t is a specified differential between P_t^m and P_t^f, established under the Federal Milk Marketing Order program.

By using (2.3) and substituting into (2.2), the blend price can be expressed as

$$(2.4) \qquad P_t^B = P_t^m + Y_t\theta_t.$$

where $$Y_t = \frac{F_t}{TMS_t}.$$

From this derivation it is apparent that a dairy producer's expectations of the blend price are fundamentally expectations of the manufacturing price, fluid utilization and the price differential θ, i.e.,

$$_{t-i}E_i(P_t^B) = {_{t-i}E_i}\{P_t^m + Y_t\theta_t\} = {_{t-i}E_i}\{P_t^m\} + {_{t-i}E_i}\{Y_t\theta_t\}$$

where $_{t-i}E_i$ is the expectations operator at a prior time t-i.

First, consider the term ${}_{t-i}E_i\{Y_t\theta_t\}$. If Y_t is taken as a known variable, then the expectation of this term is

$$Y_t \; {}_{t-i}E_i\{\theta_t\}.$$

Therefore, the producers expected market blend price is

$$(2.5) \quad {}_{t-i}E_i\{P_t^B\} = {}_{t-i}E_i\{P_t^m\} + Y_t \, {}_{t-i}E_i\{\theta_t\}.$$

From this, it is apparent that expected revenues from milk production depend on the producer's expected manufacturing price ${}_{t-i}E_i\{P_t^m\}$ and the utilization weighted expectation of the pricing differential $Y_t \, {}_{t-i}E_i\{\theta_t\}$

Because P_t^m is not a freely varying market price, but instead a price which is limited from below by the price support program, the dairy producer must also formulate his expectation of the government set price support level P_t^s. If P_t^m, unaffected by a guaranteed minimum price, is assumed to be normally distributed, then the linkage between ${}_{t-i}E_i\{P_t^m\}$ and P_t^s can be shown. The producers price expectation is transferred from P_t^m to a weighted average price P_t. This price is a combination of the expected minimum price P_t^s and the expected price which the producer would realize if the actual market price is higher than the support price P_t^s. Formally,

$$(2.6) \quad E(P_t) = \int_0^{P^s} N(p;\, \bar{P}^m,\, \sigma^2_{\bar{p}m})\, dp \; P_t^s + \int_{P^s}^{\infty} N(p;\, \bar{P}^m,\, \sigma^2_{\bar{p}m}))\; p\, dp$$

where $\bar{p}^m$ and $\sigma^2_{\bar{p}m}$ are the first and second moments of the price distribution. The first term on the right-hand side of (2.6) gives the probability weighted value of the support price P_t^s, while the second term is the expected value of the addition to P^s, given some positive probability that the market price will be above the support price. If this latter probability is zero, then the expected market price is the support or expected support price $E\{P_t^s\}$ and the expected blend price is

$$E\{P_t^B\} = E\{P_t^s\} + Y_t E\{\theta_t\}$$

With these price relationships, we can see that the dairy producer's expected market blend price is more fundamentally determined by his expectations of the level of price-support $E\{P^s\}$, the expected level of price differential $E\{\theta\}$ and the assessed probability that market prices will exceed the prevailing support price P_t^s.

<u>The Dairy Production Sub-Model</u>

From (2.1) the explicit objective of the diary firm can be characterized as attempting to choose the time path of capital stock K_t so as to ensure a maximum value of expected net returns to the dairy enterprise:

$$\text{(3.1)} \quad \underset{K_{t+j},\, K_{t+j+1},\ldots}{\text{Maximize}} \quad V_t = E_t \sum_{i=0}^{\infty} b^j \{P_{t+j}^m\, m_{t+j}\, K_{t+j} - C_{t+j}^F\, K_{t+j} - W_{t+j}\, L_{t+j} - q_{t+j}\, (K_{t+j} - K_{t+j-1}) - \frac{d}{2}\, (K_{t+j} - K_{t+j-1})^2 \}$$

where the gross income from milk output of the diary herd stock, which is equal to the price of milk times the number of milking animals, multiplied by average yield:

(i) $GI_{t+j}^m = p_{t+j}^m \bullet K_{t+j} \bullet m_{t+j}$;

and the total feed cost of the diary herd (K_{t+j}):

(ii) $TC_{t+j}^F = C_{t+j}^f\, K_{t+j}$;

and the cost of animals added to the diary herd in (t+j):

(iii) $CA_{t+j} = q_{t+j}(K_{t+j} - K_{t+j-1})$;

and the labor cost defined at wage rate W_{t+j} :

(iv) $LC_{t+j} = W_{t+j} \bullet L_{t+j}$;

the capital stock adjustment cost:

(v) $CAC_{t+j} = \frac{d}{2} (K_{t+j} - K_{t+j-1})^2$.

The solution to this problem which satisfies the boundary (transverality) condition is:

$$K_{t+j} = \beta K_{t+j-1} - \frac{1}{d} \sum_{i=0}^{\infty} b^i E_{t+j} \{q_{t+j+1} - bq_{t+j+1+i} + C^F_{t+j+1} - P^m_{t+j+1} m_{t+j+1}\} \quad \text{(vi)}$$

where the expectations operator E_{t+j} is reintroduced and b is the discount factor. Given specific stochastic processes for $\{q_{t+j+1}\}$, $\{C^F_{t+j+1}\}$, $\{P^m_{t+j+1}\}$ and $\{m_{t+j+1}\}$; expressions for $E_{t+j}\, q_{t+j+1}$, $E_{t+j}\, C^F_{t+j+1}$, $E_{t+j}\, P^m_{t+j+1}$, and $E_{t+j} m_{t+j+1}$ can be calculated and substituted into (vi) to yield an expression for optimal capital stock K_{t+j} in terms of observable variables.

The conceptual equations from the production sub-model are:

Capital Stock Equation:

(3.2)... $K_t = g_1(K_{t-1}, E_{t-1}(P^m_t \mid \Omega_{t-1}), E_{t-1}(q_t \mid \Omega_{t-1}), \sigma^2_{dt}, \sigma^2_{at}, U_{1t})$,

Domestic Production Equation:

(3.3)... $Q^{dp}_t = g_2(K_t, Z_t, U_{2t})$,

Rational Expectations formulation:

(3.4)... $E_{t-1}(P^m_t \mid \Omega_{t-1}) = g_3(E_{t-1}(P^s_{gt} \mid \Omega_{t-1}), E_{t-1}(X_t \mid \Omega_{t-1}), U_{3t})$,

Price Support Rule:

(3.5)... $P^s_{gt} = g_4(P^s_{gt-1}, U_{4t})$, where:

K_t = A measure of dairy capital stock in period t,

$E_{t-1}(P^m_t \mid \Omega_{t-1})$ = the rational expectation of market price in period t, conditioned on the information set Ω_{t-1},

$E_{t-1}(q_t \mid \Omega_{t-1})$ = the rational expectation of the market price of capital in period t, conditional on the information Ω_{t-1},

$E_{t-1}(P^s_{gt} \mid \Omega_{t-1})$ = the rational expectation of the level of dairy price support in period t, conditional on the information set Ω_{t-1},

σ^2_d = A measure of the "riskiness" of dairying as an economic activity,

σ^2_a = A measure of the "riskiness" of an alternative economic activity, other than dairy,

Q^{dp}_t = total annual domestic production of milk in period t,

Z_t = A vector of exogenous variables which helps explain short-run fluctuations in domestic production in period t,

P^s_{gt} = the U.S. Federal dairy price support level in period t,

X_t = relevant economic variables contained in the producer's information set Ω_{t-1} which helps form the expectation on market price.

The following four equation model is postulated as characterizing the diary production sub-model of a more complete model of the dairy economy. The first equation is the price identity, the second is the capital stock equation, the third is the production relationship and the fourth is an aggregate demand specification.

(3.6) $P_t - P^m_t - \lambda\theta = 0$

$$(3.7) \quad \beta_{11}K_t + \alpha_{13}P_t^{m*} + \gamma_{11}PC_t + \gamma_{14}P_{gt}^{s} + \gamma_{15}k_{t-1} + \gamma_{16}\sigma_D^2 + \gamma_{17}(\overline{\lambda\theta})_t + I = U_{1t}$$

$$(3.8) \quad \beta_{21}K_t + \beta_{22}Q_t + I = U_{2t}$$

$$(3.9) \quad \beta_{32}Q_t + \beta_{33}P_t^{m} + \gamma_{32}Y_t + \gamma_{33}P_t^{s} + I = U_{3t}$$

where the parameter matrices are

$$B = \begin{bmatrix} 1 & 1 & 0 \\ \beta_{21} & 1 & 0 \\ 0 & \beta_{32} & 1 \end{bmatrix}, \quad A = \begin{bmatrix} 0 & 0 & \alpha_{13} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad T_1 = \begin{bmatrix} \gamma_{11} & 0 & 0 & \gamma_{14} \\ 0 & 0 & 0 & 0 \\ 0 & \gamma_{32} & \gamma_{33} & 0 \end{bmatrix}$$

$$T_2 = \begin{bmatrix} \gamma_{15} & \gamma_{16} & \gamma_{17} & \gamma_{18} \\ 0 & 0 & 0 & \gamma_{28} \\ 0 & 0 & 0 & \gamma_{38} \end{bmatrix}.$$

The variable vectors are

$$y = \{K_t,\ Q_t,\ P_t^{m}\}$$

$$y^* = \{K_t^*,\ Q_t^*,\ P_t^{m*}\}$$

$$x_{1t} = \{PC_t, Y_t, P_t^{s}, P_{gt}^{s}\}$$

$$x = \{K_{t-1}, \sigma_D^2, \overline{\lambda\theta}_t, 1\}$$

where k_t ≡ index of productive capital (i.e., herd stock) Q_t ≡ domestic milk production, P_t^m market price of milk, K_t^*, Q_t^* and P_t^{m*} are expectations on these variables respectively. The anticipated exogenous variables are PC_t ≡ feed price concentrate, Y_t ≡ real consumer disposable income, P_t^s index of substitute prices, P_{gt}^s ≡ government price support level. The exogenous variables taken as known are K_{t-1} ≡ last periods capital capacity,

σ_D^2 ≡ a measure of economic risk in dairying, $\overline{\lambda\theta}$ ≡ the utilization weighted class I differential.

With this model specification, we have the special relationship that capital capacity K_t is in part influenced by producers' expectations of the federal dairy price-support level and domestic milk production is then determined by the chosen level of capital capacity.

By carrying out the matrix multiplications and inversions indicated by the general matrix equation

$$(3.10) \quad y_t = B^{-1}A(B+A)^{-1}T_1\hat{X}_{1t} - B^{-1}T_1X_{1t} + B^{-1}A(B+A)^{-1}T_2X_{2t} - B^{-1}T_2X_{2t} + B^{-1}U_t,$$

the dairy producers' aggregate capital stock equation can be expressed as

$$(3.11) \quad K_t = \{\alpha_{13} - \beta_{21}\alpha_{13} + \beta_{21}\beta_{32}\alpha_{13}/\eta - \gamma_{18}\} + \beta_{21}\beta_{32}\alpha_{13}\gamma_{15}\eta - \gamma_{15})K_{t-1} + (\beta 21\beta 32\alpha 13\gamma 11\eta)\ \hat{PC}_t - \gamma 11PC_t + (\alpha 13\gamma 32\eta)\hat{Y}_t +)\alpha 13\gamma 33)\hat{P}_t^s + (\beta_{21}\beta_{32}\alpha_{13}\gamma_{14}\eta)\ \hat{P}_{gt}^s - \gamma_{14}P_{gt}^s + (\beta_{21}\beta_{32}\alpha_{13}\gamma_{16}\eta - \gamma_{16})\overline{\sigma}_D^2 + (\beta_{21}\beta_{32}\alpha_{13}\gamma_{17}\eta - \gamma_{17})\ \overline{\lambda\theta}_t + V_t.$$

As stated earlier, an important problem at this point, and one which has received little attention in the applied REH literature, is how to deal with the repetition of some of the exogenous variables. More specifically, any exogenous variable which appears in the structural form of the capital stock equation, along with the expected price variable, will show up in the estimable equation as both the expectation of that variable and the current value itself. Thus, we can see that in equation (3.11) we have both $\hat{PC}_t$ and PC and $\hat{PC}_t$ and $\hat{P}_{gt}^s$ and p_{gt}^s.

The usual practice has been to ignore this question and to estimate the equation with both the expectation and the current value. This does not seem to be reasonable. If the value of the exogenous variable needs to be forecast to derive the expected market price, then it is only reasonable to assert that it does not belong on the structural equation as a known variable and that in this form the equation is misspecified. More appropriately, these variables, and in particular, PC_t and P^s_{gt} should originally appear as anticipated exogenous variables PC^*_t and P^{s*}_{gt}. In this way, we can reasonably combine terms in equation (3.11) to get

$$K_t = \lambda_0 + \lambda_1 K_{t-1} + \lambda_2 \hat{P}^s_{gt} + \lambda_3 \hat{PC}_t + \lambda_4 \hat{Y}_t + \lambda_5 \hat{P}^s_t + \lambda_6 \sigma^2_D + \lambda_7 (\overline{\lambda\theta}_t) + V_t \tag{3.12}$$

where the λ_i's represent the parameters in equation (3.11) with respect to each exogenous variable.

The only remaining question concerns the particular form which the forecasting equations for P^s_{gt}, PC_t, Y_t, P^s_t should take. Following the simplest form, we propose unvariate autoregressive models ARIMA (1,0,0) such that

$$\begin{aligned} P^s_{gt} &= \phi_1 P^s_{gt-1} + \xi_{1t} \text{ and } \hat{P}^s_{gt} = \hat{\phi}_1 P^s_{gt-1} \\ PC_t &= \phi_2 PC_{t-1} + \xi_{2t} \text{ and } \hat{PC}_t = \hat{\phi}_2 PC_{t-1} \\ Y_t &= \phi_2 Y_{t-1} + \xi_{3t} \text{ and } \hat{Y}_t = \hat{\phi}_3 Y_{t-1} \\ P^s_t &= \phi_4 P^s_t + \xi_{4t} \text{ and } \hat{P}^s_t = \hat{\phi}_4 P^s_{t-1} \end{aligned} \tag{3.13}$$

where ξ_{it} is a stochastic variable with $E(\xi_{it}) = 0$, $E(\xi_{it}\xi_{it-1}) = 0$.

Substituting (3.13) into (3.12) we arrive at the REH form of the capital stock equation

$$K_t = \lambda_0 + \lambda_1 K_{t-1} + \lambda_2 \phi 1 P^s_{gt-1} + \lambda_3 \phi 2 PC_{t-1} + \lambda_4 \phi 3 Y_{t-1}$$

(3.14)

$$+ \lambda_5 \phi_4 P^s_{t-1} + \lambda_6 \overline{\sigma}^2_D + \lambda_7 (\overline{\lambda\theta})_t + V_t$$

The exogenous policy variable in this equation is P^s_g , therefore, this equation, along with the forecast rule for P^s_{gt} yields the basis for linking K_t to the policy parameter ϕ_1

The rational expectation implications of a change in price support can be seen by examining the partial derivative of K_t with respect to $\hat{P}^s_{gt}$. This derivative is given by

(3.15) $\delta K_t = \lambda_2 \delta \hat{P}^s_{gt}$

and $\delta E(P^s_{gt} | \Omega_{t-1}) = \delta \hat{P}^s_{gt} = \phi_1 \delta_1 P^s_{gt-1}$

so we have

(3.16) $\delta K_t = \lambda_2 \{\phi_1 \delta P^s_{gt-1} + P^s_{gt-1} \delta\phi_1\}$

$$= \lambda_2 \phi_1 \delta P^s_{gt-1} + \lambda_2 P^s_{gt-1} \; \delta\phi^1$$

The interpretation of this last equation is that the change in K_t with respect to $\delta E(P^s_{gt} | \Omega_{t-1})$ is given by $\lambda_2 \phi_1$ only as long as the $\delta\phi_1 = 0$. Therefore, any change in the expected level of price-supports which <u>implies</u> a different ϕ_1, i.e., $\hat{P}_{gt} = \phi_1{}' P^s_{gt-1}$, is accounted for in the capital stock equation by both terms and not just the $\lambda_2\phi_1$ term. This would manifest itself in the capital stock equation (3.14) by a change in the parameter $\lambda_2\phi_1$. Suppose that the federal authority in charge of establishing the price support rule shifts from a policy of continually increasing price-supports, represented by

$$(3.17)\quad P^s_{gt} = \phi_1 P^s_{gt-1} + \xi_t \text{ with } \phi_1 > 1$$

to a policy designed to gradually phase out price-supports, represented by

$$(3.18)\quad P^s_{gt} = \phi_1' P^s_{gt-1} + \xi_t \text{ with } \phi_1' < 1$$

New levels of capital stock K_t would be determined by changes in both the level of price-supports over time and the value of the parameter $\lambda_2\phi_1$. This would become $\lambda_2\phi_1' \neq \lambda_2\phi_1$ to reflect producer anticipation of the new "structure" of the support policy.

In contrast to the more traditional models of policy impacts, not only does the exogenous variable P^s_g change but also the parameter of the producers capital stock equation changes to reflect the shift in government policy. Also notice that the kinds of policy evaluations which can be undertaken are severely constrained by the adoption of the rational expectations viewpoint. Having chosen a new value for the policy parameter ϕ, we are constrained to specify each new level of price-support P^s_{gt+1} such that it is consistent with the policy equation (3.18).

In addition to altering the interpretation of policy evaluation, the rational expectations hypothesis also has another economic and econometric implication. Recalling equation (3.14), we can see that market price does not appear as an explanatory variable in determining capital stock. Rational expectations does not imply that K_t is independent of market prices. K_t is determined by expected market prices, which are determined by more fundamental economic variables (Wallis, 1981).

The Econometric Model and Policy Evaluation

The evaluation of the impact of price-supports on prices, production and consumption under the REH requires that we specify more than alternative levels of the support price from one period to the next. What is required is that we specify a policy rule, i.e., an explicit form for equation (3.17). In this way, the level of price support in period t is linked in a logical way to the level in period t-1.

Recalling the discussion on producer expectations and their relationship to the reduced-form parameters, the estimate of ϕ from the data on price supports 1949-1978, along with the estimate of the parameter on lagged price-support in the capital capacity equation allows us to estimate the policy invariant component of the reduced form coefficient. The estimated equations for (3.14) and (3.17) are presented in equations (3.19) and (3.20).[3]

Dairy Capital Stock:

$$(3.19)\ldots \quad K(t) = \underset{(4.46)}{18255.57} + \underset{(5.61)}{0.56}K(t-1) + \underset{(3.15)}{2.99}\, p_g^s(t-1)$$

$$- \underset{(-4.33)}{1.58}\, ACP(t-1) + \underset{(2.13)}{26.68}\left(\frac{\sigma_d^2}{\sigma_a^2}\right)$$

$R^2 = 0.84$ F = 36.56 Durbin = "h" = +0.68

where ACP is the average annual cull cow price and the other variables have already been defined.

Price-Support Policy Rule:

$$(3.20)\ldots \quad P_g^s(t) = \underset{(38.83)}{1.067611}\, P_g^s(t-1)$$

R^2 = .98 F= 1516.1 D/W "d" = 1.23

As an example of the implications of the REH and the AR(1,0,0) forecasting rule for the period 1949-1978, consider the estimated parameters on ϕ from (3.20) and $P_g^s(t-1)$ from (3.19). With this estimated AR(1) forecasting "rule" the implied structurally invariant parameter is:

$$B = 2.99/1.067611$$
$$= 2.80$$

Any other historical time path of price-supports implies a different rule, i.e., AR (ϕ) parameter and hence a different value of B. In order to be consistent with the view that expectations are formed rationally, it is not possible to evaluate diary price-support policy by simply specifying hypothetical levels of price-support from one year to another and calculating a level for the endogenous capital stock K. By adopting the REH perspective we are constrained, when making hypothetical policy evaluation, to alter, in a logical fashion, both the support rule parameter, ie., the value of ϕ, and those of the reduced form to generate hypothetical behavior for the endogenous variables. The traditional method of policy analysis, that of setting the policy variable to alternative, arbitrary levels from period to period is inconsistent with this reasoning. Such a policy would imply an autoregressive parameter close to zero with a very large error-term variance. Under such an implied structure, producers would be unable to form any reasonably forecasts off the policy variables, and such a variable would logically not be a determinant in optimal economic decisions.

What this discussion suggests for actual policy evaluation is that we must carefully consider the usefulness and validity of econometric policy evaluations such as "what happens if we set the level of price support to zero in 1949 and maintain it there through 1978?" Clearly, the implied behavior of endogenous variables resulting from such a policy evaluation would have to be viewed with substantial skepticism. Instead, we must pose the question in a more reasonable manner, "What are the economic implications of a price-support rule which, historically, would have maintained a constant or possibly a more rapidly declining level of support from 1949 through 1978?" To answer this question, we would select a value of ϕ such that the price-support declined rapidly, for example, from 1949 onward. We would then use the invariant estimate of B to calculate a new parameter for $P^{s}_{g(t-1)}$. Using this in equation (3.19), we would estimate capital stock in each year consistent with the new price-support rule.[4]

Conclusions

The concept of REH constitutes a phenomenon which is both logically simple and empirically complex. Its simplicity lies in the fact that applied econometricians have been for a long time constructing equilibrium models within which the REH has been implicitly embedded. Once recognized, however, the REH is now as easily incorporated explicitly into these models. The intent of this paper was to develop and explore the conceptual and econometric implications of REH in an aggregate econometric model of the U.S. Dairy economy. This development illustrates the nature of the constraints which must be placed on future policy models in dairy and elsewhere, if the econometrician's view of the world is to be consistent with the concept of rational economic agents. The view of the world developed here is clearly not the most complex one which could conceivably be taken. If the endogenous variables are anticipated in a rational manner, then what constitutes a rational model for exogenous variables? Clearly the more complex the model posited for a variable such as P_g^s , the more intricate and complex the econometric model becomes. Notice also that I have said nothing about testing the econometric model in a manner which would allow the rejection of the REH (Hoffman and Schmidt). This constitutes yet another area of research which the applied econometrician must undertake if he/she is to develop maximum confidence in the descriptive and prescriptive performance of his/her models.

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Notes

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2. Wallis (1981) in an unpublished paper points out that these models can in fact be considered "rational" expectations if in fact the implied restriction on the parameter space are valid.

3. Coefficients in parenthesis are "t" values based on 28df.

4. Note that there is nothing in the rational expectations hypothesis which rules out the case in which the authorities decide to set ϕ - 0, which would occur when a program was simply cancelled. However, in a situation such as this, $\phi = 0$ is econometrically equivalent to setting $P^{s}_{gt=1} = 0$ for all ti. Note that the question of policy evaluation with with this type of policy change is difficult to address because the implications of the REH become indistinguishable from that of the naive models.

REACTIVE PROGRAMMING: ONE TOOL FOR DAIRY POLICY ANALYSIS

by

Roger A. Dahlgran*

Abstract

This paper discusses the assumptions, logical structures, and solution algorithms of transportation and reactive programming models. Because of the similarities between these two types of models, an understanding of transportation models is used to generate an understanding of reactive programming models. The key features of the author's reactive programming model, which was used to analyze the interregional impacts of U.S. dairy market regulation, are then discussed. A critical appraisal of the model with suggested improvements is then offered.

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REACTIVE PROGRAMMING: ONE TOOL FOR DAIRY POLICY ANALYSIS

Reactive programming is a mathematical programming procedure developed by T. E. Tramel and A. D. Seale, Jr. (see Tramel and Seale, 1959, 1963b; Seale and Tramel, 1963; and Tramel, 1965) and is but one member of a larger family of closely related mathematical programming procedures. Reactive programming is a generalization of the transportation model which, because of its coefficient matrix structure, is a special form of a linear programming model of product flows. The solutions to transportation and reactive programming models are useful in studying interregional trade in agricultural commodities, because they give, under various types of competition, the spatial distribution of prices, the spatial arrangement of production and consumption, and the interregional product flows.

This paper discusses the use of reactive programming in dairy policy analysis. The origins of, and early studies employing reactive programming will first be examined. Because reactive programming is a generalized form of the transportation model, the assumptions, logical structure and general solution algorithm for the transportation model will be discussed first. Attention will then focus on the assumptions, logical structure, and general solution algorithm for reactive programming models. The author's reactive programming model of the U.S. dairy industry will then be discussed. Finally, a critical appraisal of this model will be offered. This report should offer the reader an understanding of reactive programming's assumptions, logical structure and solution algorithm. An understanding of the strengths and weaknesses of the author's model can also be obtained from this report.

History and Applications of Reactive Programming

Since its introduction, reactive programming has been used to study interregional trade in cabbage (Allen and Seale), watermelons (Seale and Allen, 1959), snap beans (Seale and Allen, 1960), and milk (Riley and Blakley; Blakley and Riley; Dahlgran, 1980; and Whipple). Its use in the analysis of milk pricing indicates more the magnitude of dairy policy problems than the special appropriateness of reactive programming for studying dairy policy. Blakley and Riley (see also Riley and Blakley) were the first to use reactive programming in dairy policy analysis. They projected welfare changes in 31 federal order markets under alternative class I pricing systems. Subsequently, reactive programming was used by Dahlgran to model site prices and milk flows of a hypothetically unregulated U.S. dairy industry. By comparing the prices and quantities of the unregulated solution with the prevailing regulated equilibrium, both regional changes in producer and consumer surpluses, and welfare losses caused by regulation were computed. A final recent reactive programming analysis of dairy policy was done by Whipple. Whipple studied site prices and regional activity levels of the U.S. dairy industry under various policies toward reconstituted fluid milk.

Transportation Models and Reactive Programming

Because reactive programming generalizes the transportation model, understanding the transportation model contributes to understanding reactive the programming model. The transportation model is presented in a generic form in Table 1. This table shows m producing points and n consuming points. The cost of transporting the commodity from producing point i to consuming point j is represented by t_{ij}. This cost is generally a function of the distance between the two points. Fixed quantities, $q_{i.}$, i=1,2,...m, are assumed to be available at the production sites and another set of fixed quantities, $q_{.j}$, j=1,2 ...n, are assumed to be needed at the consumption sites. The objective of the model is to satisfy the fixed demands out of available supplies by allocating product flows, q_{ij}, so as to minimize the transportation cost. Mathematically, the problem is

$$\text{Minimize} \quad \Sigma_i \Sigma_j t_{ij} q_{ij}$$

$$\text{Subject to} \quad \Sigma_i q_{ij} \geq q_{.j}$$

$$\Sigma_j q_{ij} \leq q_{.i}$$

Under these assumptions, a solution will exist only if the total amount available for consumption exceeds the total consumption demand, i.e.

$$\Sigma_i q_{i.} > \Sigma_j q_{.j}$$

The equilibrium price under these conditions will be zero, but the prices at the individual supply and demand points will differ from zero because of locational advantages or disadvantages.

To solve the transportation model, four matrices and two vectors must be defined. The matrices are the transportation cost matrix, $[t_{ij}]_{mxn}$; the product flow matrix, $[q_{ij}]_{mxn}$; the equilibrium cost matrix, $[e_{ij}]_{mxn}$; and the shadow price matrix, $[s_{ij}]_{mxn} \equiv [t_{ij}] - [e_{ij}]$, which gives the cost of using inactive routes. One vector, $[u_i]$, of length m designates product values at the production sites; and another vector, $[v_j]$, of length n designates product values at the consumption sites. The solution algorithm is (for greater detail, see Bressler and King, pp. 93-100)

1. Select an initial allocation of supplies among destinations.

2. Construct the equilibrium cost matrix by entering t_{ij} for e_{ij} when $q_{ij} \neq 0$ for all i and j.
3. Set the product value to zero at one production or consumption site (i.e. set one u_i or v_j equal to zero).
4. Calculate all remaining u_i, v_j and e_{ij} values for the equilibrium cost matrix by rearranging the identity, $e_{ij} \equiv v_j - u_i$ for all i and j, so as to find one unknown at a time.
5. Calculate the cost of inactive routes as $[s_{ij}] \equiv [t_{ij}] - [e_{ij}]$.
6. Check for optimality. If any element in the shadow price matrix is negative, then the difference between product values at consumption site j and production site i exceeds transportation costs from site i to site j. A reallocation of supplies will reduce total transportation costs. A new allocation of supplies must be selected and the entire process repeated.

To generalize the transportation model into a reactive programming model, relationships between product values and quantities at the consumption and production sites are substituted into the transportation model, replacing the assumed fixed quantities and the resulting product values. The algorithm requires the functional relationships between quantities and prices to be written in price dependent form. Mathematically, $p_{i.} = h_i(q_{i.})$ and $p_{.j} = f_j(q_{.j})$ are, respectively, substituted for the u_i and v_j of the transportation model. The quantities produced and/or consumed at each site adjust to remove arbitrage opportunities as the algorithm iterates toward a solution. King and Ho (pp. 5-6) provide a general description of the procedure.

"An initial set of supply and demand quantities is selected and a linear programming subroutine is used to allocate supplies among the markets. A market price is calculated from the demand function for each of the consuming areas. By subtracting transportation costs from these market prices, net shipping point prices are obtained for the shipments in the initial allocation. A new level of output for the first shipping area is selected consistent with the average net revenue received. This new quantity is then allocated among markets in such a way as to maximize net returns, given the market prices and previous shipping patterns of all other shippers.

This same process is repeated for the second shipping area, given the behavior of all other shipping areas. The interactive routine continues until it is not profitable for any shipping area either to change the level of output or to reallocate supplies."

Fortunately, a potential user of reactive programming can easily obtain efficient and usable source code so that he or she does not have to convert this sketchy description of the computational algorithm into a working computer program. Work on modifying and improving the reactive programming code has been ongoing at the Department of Economics and Business, North Carolina State University since the early seventies under the direction of Richard King. Interested users can obtain the most current version of the code by inquiring there.

Reactive Programming Applied to the U.S. Dairy Industry

Dahlgran and Whipple analyzed U.S. dairy policy using reactive programming models. Dahlgran's model will be described because Whipple's study uses Dahlgran's model with a few revisions. Table 2 is a representation of Dahlgran's reactive programming model of the U.S. dairy industry. In this formulation, milk is supplied at 35 points, which correspond generally to the geographic centers of the states except that contiguous states that produce small amounts of milk are combined. Milk is demanded at 47 fluid milk consumption sites and four manufactured milk consumption sites. The 47 fluid milk consumption sites correspond to the larger state and federal order markets and aggregations of smaller contiguous markets. The four manufactured milk consumption sites are the Northeast, South, North Central and West. These manufactured milk consumption site aggregations are justified by assuming that raw milk utilized for manufactured products will be transported in product form making for negligible transportation costs within these rather large regions but transportation cost between regions will not be negligible. Further, this aggregation reduces the size of the model to be solved.

To compute transportation costs, geographic coordinates corresponding to the geographic center of production areas and the major population center of consumption areas, were used to first establish distances. The formula used for computing the distance between two geographic points was (Tramel and Seale, 1963a)

$$d_{ij} = 3958.62 \times \{\frac{\pi}{2} - \sin^{-1}[\sin y_1 \sin y_2 + \cos y_1 \cos y_2 \cos(x_1 - x_2)]\}$$

where y_1 = latitude of the supply point in radians,

x_1 = longitude of the supply point in radians,

y_2 = latitude of the demand point in radians, and

x_2 = longitude of the demand point in radians.

The distances between production and fluid milk consumption points were computed and then substituted into the 1976 fluid milk transportation cost function estimated by Lough,

$$t_{ij} = 7.87 + 0.218\ d_{ij}$$

where t_{ij} = bulk transport cost of fluid milk in cents per hundredweight, and

d_{ij} = distance computed according to the Trammel and Seale formula.

Because milk utilized for manufactured dairy products was assumed to be transported in manufactured product form, transportation costs for manufactured products were computed as 0.060673 times the cost of transporting raw milk over the corresponding distance. This assumption is consistent with the procedure employed by Hallberg, et al.

The final set of assumptions in this model is embodied in the procedure used to construct of the price-quantity relationships at the production and consumption points. First, supply and demand functions were estimated. Farm level fluid and manufacturing milk demand functions and grade A and grade B milk supply functions were fit to monthly data for a sample of fourteen milk markets. Because significant differences in the regional fluid milk demand elasticities, in the regional manufacturing milk demand elasticities, and in the regional supply elasticities could not be detected, a fluid milk demand elasticity of -0.112 was used for all fluid milk consuming points, a manufacturing milk demand elasticity of -0.352 was used for all manufacturing milk consuming points, and a total milk supply elasticity of 1.19 computed as the weighted average of the grade A and grade B milk supply elasticities was used for all milk supply points.

Next, it was assumed that U.S. price support purchases were exogenous and purchased entirely at the North Central manufacturing milk consuming point. Third, it was assumed that in an unregulated environment, any milk not going directly into fluid utilization nor used to maintain the surplus of fluid grade milk required to satisfy fluid milk demand during the peak demand period of the year, will be grade B. This grade B milk was assumed to be utilized in manufactured products and priced at the equilibrium grade A milk price less the difference between the grade A and grade B costs of production. This cost of production differential was assumed to be $0.15 per hundredweight (Ippolito and Masson, citing Bartlett, p. 37).

Finally, the fluid milk demand functions,

$$q_{.j} = \phi j \, p_{.j}^{-0.112} \qquad j \leq 47$$

the manufacturing milk demand functions,

$$q_{.k} = \theta_k \, (p_{.j} - 0.15)^{-0.352} \qquad k=1,2,3,4; \; j = k + 47$$

and the total milk supply functions,

$$q_{i.} = \gamma_i \, p_{i.}^{1.19} \qquad i = 1,2, \ldots 35$$

were inverted and solved for ϕ_j, θ_k, and t_i so that the functions passed through 1976 price and quantity points for each market. The assumptions described in the previous paragraph were incorporated into this demand structure by (1) subtracting exogenous price support purchases from manufacturing utilization in the North Central region before inverting the manufacturing demand function for that region, and (2) substituting the fluid grade milk price less fifteen cents for the manufacturing milk price in the manufacturing milk demand functions. The model solves for the grade A milk price, which implies the grade B, or manufacturing milk, price. The implied grade B and manufacturing milk prices must be derived while intrepreting the solution of the reactive programming model.

A Critical Appraisal of the Model

Referring to Table 2, it can be seen that all of the pieces are now in place so the reactive programming model can be solved. Before concluding, however, the potential user of this model should be made aware of the shortcomings and limitations of the model. First, it should be emphasized that the model constructed is useful only in performing one task, modeling a unregulated equilibrium in the U.S. dairy markets in 1976 under the assumptions set forth in the previous section. The model can be revised to reflect unregulated dairy market equilibrium under different assumptions, such as at a different time, with different transportation costs or different price support purchase levels carried out in a different region or regions. But an attempt to use the model to do other tasks such as model regulated equilibrium under different policies would result in the application of a misspecified model to the problem. The reason the model cannot be used to model regulated dairy market equilibrium is that the algorithm does not recognize the classified pricing and pooling provisions of federal and state order markets. A revision of the computer code to incorporate these complexities would be substantial.

A second limitation of the model is the number of product considered. On the supply side, actual producers can produce either grade A or grade B milk. The model considers only total milk production and assumes that in its unregulated environment, any milk in excess of the seasonal peak in fluid demand will be grade B. This assumption probably seems rather cavalier to some dairy policy analysts. On the demand side, the model allows only demands for milk for fluid and manufacturing utilization. A more complete specification, allowing manufacturing milk demand to be derived from the demands for cheese, butter, ice cream, and evaporated milk at retail could probably be accomplished more easily by other methods.

The final limitation of this model concerns the supply and demand elasticities used for the supply and demand functions in the model. Other researchers seem to have more confidence in these results than does the author. These elasticities were estimated from monthly data for fourteen milk markets. The estimation procedure resulted in some elasticities of the wrong sign and/or with low levels of significance. These estimated elasticities were then examined for consistent regional and market size patterns. Since no patterns were found, the incorrectly signed elasticity estimates were disregarded and the remaining elasticities were averaged to get the values used. A reformulation of the supply model using duality concepts with a distributed lag model and estimated with annual aggregate U.S. data (Dahlgran, 1985) gives the elasticity plot shown in Figure 1. According to this figure, any supply elasticity could have been used in the reactive programming model and would have been correct. However, a length of run, or period of adjustment, corresponds to the elasticity chosen. Figure 1 shows the period of adjustment corresponding to an elasticity of 1.19 is about 16 years. But, the analysis of the results of the reactive programming model assumed that the elasticities used had a one year period of adjustment. Referring again to Figure 1, a better annual elasticity estimate would be 0.2, and to fully capture the dynamic adjustment of the dairy industry, the continuum shown in Figure 1 should be incorporated into the reactive programming model.

To conclude, the reactive programming model discussed here, like most models, is useful for a rather limited purpose. It is useful only in modeling unregulated dairy market equilibrium. However, the techniques used and components described can be used in other models that incorporate transportation of dairy products, and the response of regional production, consumption and prices to different dairy polies.

Table 1. A generic transportation model and solution procedure.

Production points	Consumption points 1	2	....	n	Supplies	Algorithm
1	t_{11}	t_{12}	....	t_{1n}	$q_{1.}$	Given:
2	t_{21}	t_{22}	....	t_{2n}	$q_{2.}$	t_{ij} = transport cost from
.	.	.		.	.	point i to point j
m	t_{m1}	t_{m2}	....	t_{mn}	$q_{m.}$	$q_{i.}$ = supplied at point i
Demands	$q_{.1}$	$q_{.2}$	....	$q_{.n}$		$q_{.j}$ = demanded at point j
	Product flow matrix					
1	q_{11}	q_{12}	....	q_{1n}		1. Allocate $q_{i.}$ through q_{ij}
2	q_{21}	q_{22}	....	q_{2n}		to $q_{.j}$ so that
.	.	.		.		$\Sigma_i\ q_{ij} \geq q_{.j}$
m	q_{m1}	q_{m2}	....	q_{mn}		and $\Sigma_j\ q_{ij} \leq q_{.i}$
	Equilibrium cost matrix					
1	e_{11}	e_{12}	....	e_{1n}	u_1	2. Initialize one u_i or v_j
2	e_{21}	e_{22}	....	e_{2n}	u_2	to zero.
.	.	.		.	.	3. $e_{ij} = t_{ij}$ when $q_{ij} \neq 0$
m	e_{m1}	e_{m2}	....	e_{mn}	u_m	for all i,j.
	v_1	v_2	....	v_n		4. $e_{ij} \equiv v_j - u_i$ for all i,j.
	Shadow price matrix					
1	s_{11}	s_{12}	....	s_{1n}		5. $s_{ij} = t_{ij} - e_{ij}$
2	s_{21}	s_{22}	....	s_{2n}		6. If $s_{ij} < 0$, then go to 2.[a]
.	.	.		.		
m	s_{m1}	s_{m2}	....	s_{mn}		

a/ This step determines if the transportation cost is less than the price difference between production and consumption centers. If such conditions exist, profitable arbitrage opportunities are still available so supplies can be reallocated.

Table 2. A schematic representation of Dahlgran's reactive programming dairy model.

Prod'n Points	Fluid Milk Consumption Points 1	2	...	47	Manufactured Milk Consumption Points 48	...	51	Total Prod'n	Supply Functions
1	$t_{1,1}$	$t_{1,2}$	...	$t_{1,47}$	$t_{1,48}$	...	$t_{1,51}$	$q_{1.}$	$p_{1.}=g_1(q_{1.})$
2	$t_{2,1}$	$t_{2,2}$	...	$t_{2,47}$	$t_{2,48}$	...	$t_{2,51}$	$q_{2.}$	$p_{2.}=g_2(q_{2.})$
.	.	.		.	.		.	.	.
.	.	.		.	.		.	.	.
.	.	.		.	.		.	.	.
35	$t_{35,1}$	$t_{35,2}$	...	$t_{35,47}$	$t_{35,48}$	...	$t_{35,51}$	$q_{35.}$	$p_{35.}=g_{35}(q_{35.})$
Total Cons'n	$q_{.1}$	$q_{.2}$	...	$q_{.47}$	$q_{.48}$	...	$q_{.51}$		
AR Fns	$p_{.1}=f_1(q_{.1})$	$p_{.2}=f_2(q_{.2})$	...	$p_{.47}=f_{47}(q_{.47})$	$p_{.48}=h_1(q_{.48})$	...	$p_{.51}=h_4(q_{.51})$		

Major assumptions:

1. Transportation costs, t_{ij}, a function of distance, d_{ij}.

$$t_{ij} = \begin{cases} 7.87 + 0.218 \; d_{ij} & \text{for } j \le 47 \\ 0.465 + 0.01323 \; d_{ij} & \text{for } j \ge 48 \end{cases}$$

$$d_{ij} = 3958.62 \times \{ \frac{\pi}{2} - \sin^{-1}[\sin y_1 \sin y_2 + \cos y_1 \cos y_2 \cos(x_1 - x_2)]$$

(x_1,y_1) = (longitude,latitude) of the supply point in radians

(x_2,y_2) = (longitude,latitude) of the demand point in radians

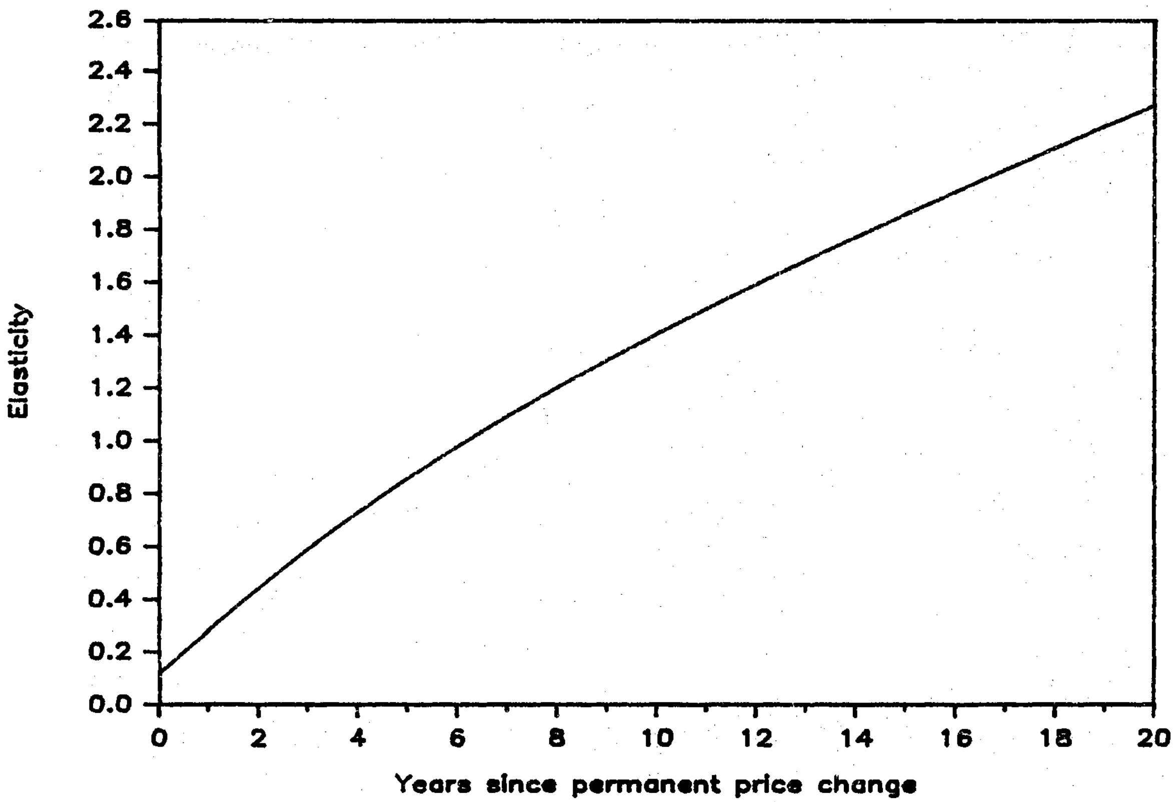
2. Demand and supply functions estimated from time series data.

a. Fluid milk demand $\quad f_j^{-1} = \phi_j \, p_{.j}^{-0.112} \quad j \le 47$

b. Manufacturing milk demand $\quad h_k^{-1} = \theta_k \, (p_{.j} - 0.15)^{-0.352} \quad k=1,2,3,4; \; j = k+47$

c. Total milk supply $\quad g_i^{-1} = \gamma_i \, p_{i.}^{1.19} \quad i = 1,2, \ldots 35$

Figure 1. Estimated elasticity of total milk production with respect to a permanent change in milk prices



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A QUADRATIC PROGRAMMING MODEL FOR THE DAIRY INDUSTRY

M. C. Hallberg*

This paper discusses the application of a spatial equilibrium model for the U.S. dairy industry for which quadratic programming was used to obtain the solution. The formulation and implementation are fully described in the report by Hallberg, et. al. In this paper I merely sketch the model in summary fashion, discuss some policy applications and results, and outline some ways in which it might fruitfully be modified.

I should say at the outset that I am not necessarily a devotee of quadratic programming. I am of the opinion, however, that spatial issues are <u>very</u> important in the dairy industry in part because of Federal Order pricing practices and in part because of recent and potentially further shifts in regional milk production. Hence I believe we <u>must</u> utilize spatial models to give guidance to dairy policy-makers. In fact, as I have argued elsewhere, in view of the importance of the spatial dimension in all of agricultural marketing, I am somewhat surprised by the fact that there are so few researchers building and using models of this type today in preference to non-programming econometric models.

More to the point of the current topic, I do think that within the price-quantity ranges we normally work, linear approximations to supply and demand functions are quite good enough. And as Takayama and Judge have

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shown when supply and demand functions are linear, the quadratic programming algorithm will produce unambiguous solutions to the spatial equilibrium problem.

Does quadratic programming offer advantages over reactive programming for these types of problems? I will beg the question by letting a reactive programmer answer. In general, however, I have found the QP formulation to be quite convenient when I needed to place a set of special restrictions on the problem so as to make it consistent with reality and/or consistent with the policy alternatives under considerations.

The Model

The model presented here was intended first and foremost to generate a solution or set of solutions consistent with a specified welfare criterion and certain real-world conditions felt to be critical determinants of this welfare criterion. Thus, the model was intended to generate solutions which are to be used as a norm for comparison with real-world results and/or certain base solutions generated with known pre-conditions. No claim is made of <u>reproducing</u> real-world results.

The welfare criterion chosen here is that of perfect competition in space and form. The model generates <u>equilibrium</u> solutions. It contains no dynamic elements so at best it can be used for comparative static analyses.

The model is a modification of the one-product spatial equilibrium model originally formulated by Samuelson and made operational by Takayama and Judge. The explicit welfare criterion is to maximize <u>net social payoff</u> which Samuelson defines to be the sum of (aggregate) producers' and (aggregate) consumers' surplus minus interregional shipment costs. Here equilibrium is achieved through the device of maximizing net social payoff.

If one feels uncomfortable with a welfare criterion involving Marshallian surpluses, he or she may view this model as a simple operating mechanism for achieving competitive equilibrium in space.

For the more brave, the model does allow calculation and comparison of regional as well as aggregate consumers' and producers' surplus. As in any model generating a competitive equilibrium solution, however, this one fails to deal with the equity question except to say that more "net social payoff" is better than less. Tradeoffs among producers, among consumers, and/or between producers and consumers can only be addressed imperfectly by examining the solutions. The model does not and cannot say that more producers' surplus in, say, the Midwest is preferred to more producers' surplus in the West, nor for that matter to more consumers' surplus in the Midwest.

Model Assumptions

The model assumes that producers supply raw milk and consumers consume this milk in the form of processed fluid milk and manufactured dairy products. As implemented it does not distinguish between Grade A and Grade B milk.

The point of trade is considered to be at the farm level so that the demand for all dairy products must be translated into a demand for raw milk. Thus, demand for fluid milk at the farm level is derived from the retail demand for fluid products, and demand for manufacturing milk at the farm level is derived from the retail demand for manufactured dairy products.

Treating the point of trade to be the farm level means that all marketing and processing activity (other than transportation) is ignored. If we assume constant marketing margins everywhere--a not too unrealistic assumption given the ranges of quantity variations within which we are likely to be working--this is not likely to be a serious omission. We assume all milk is processed in the region where it is produced and that processors ship the finished product directly to retailers in the consuming regions. Since quantities are expressed in raw milk equivalents, transportation costs for raw milk used in fluid products and for raw milk used in manufactured products is derived from the cost of transporting the finished product.

Finally the model assumes that producers respond to a blend price which is a weighted average of the farm price of fluid and of manufacturing milk. Thus, we assume all milk produced in the U.S. is pooled into region-wide pools and that producers are paid out of the their associated regional pool.

Model Equations

The problem to be solved can be written as follows:

$$\max NSP = \Sigma_j[a_j^f P_j^f + 0.5b_j^f (P_j^f)^2 + a_j^m P_j^m + 0.5b_j^m (P_j^m)^2]$$

$$- \Sigma_i[c_i P_i^b + 0.5d_i (P_i^b)^2] - \Sigma_i\Sigma_j[t_{ij}^f X_{ij}^f + t_{ij}^m X_{ij}^m]$$

subject to :

(1) $P_j^f - P_i^f \leq t_{ij}^f$ for all i and j,

(2) $P_j^m - P_i^m \leq t_{ij}^m$ for all i and j,

(3) $P_j^b - P_j^m \geq D_j$ for all j, and

(4) $P_j^b + w_j^f P_j^f + w_j^m P_j^m$ for all j.

Constraint set (1) and (2) are the standard arbitrage constraints of Takayama-Judge. Constraint set (3) represents the price-differential constraints. Although not quite appropriate, we might think of the D_j in equation (3) as the Federal Order Class I differentials. Constraint set (4) consists of the blend price relations. Here w_j^f is the proportion of all milk in region j consumed as fluid milk and $w_j^m = 1 - w_j^f$.

Clearly this is a quadratic programming problem and, by Kuhn-Tucker theory, its solution (when expressed in the price domain) is given by the solution to the following system of (matrix) equations:

Fluid transport equations

(5) $T_f = A_f \; P_f + SL_f$

Manufacturing transport equations

(6) $T_m = A_m \; P_m + SL_m$

"Class I differential" equations

(7) $D = P_f - P_m + SL_d$

Blend price equations

(8) $\emptyset = W_f \; P_f + W_m \; P_m \; P_b$

Fluid demand equations

(9) $a_f = b_f \; P_f - A_f' \; X_f - W_f \; V - Z$

Manufacturing demand equations

(10) $a_m = b_m \; P_m - A_m' \; X_m - W_m \; V + Z$

Supply equations

(11) $c_b = d_b \; P_b + V$

Complementary relations from Kuhn-Tucker theory

(12) $\emptyset = SL_f' X_f$

(13) $\emptyset = SL_m' X_m$

(14) $\emptyset = SL_d' Z$

(15) $P_f, P_m, P_b, X_f, X_m, Z \geq 0$

(16) V unrestricted

Here the Ø are vectors with all elements zero and the SL are vectors of slack activities. Slack activities are also implied on the demand and supply equations, but since they are known a priori to be zero they are omitted in the above formulation. X, V, and Z are vectors of LaGrangian multipliers. From the specification given it is clear that the X's are vectors of transport activities, V is a vector whose elements represent total milk supplied by the respective regions, and Z is a vector whose elements represent the amount of surplus fluid milk in the respective regions used to satisfy, wholly or in part, manufacturing milk demand in the same region. D is the vector of "Class I" differentials. In our application we set all elements of D equal to what was considered the smallest reasonable differential. All other vectors and matrices should be clear from the nature of the problem specified in equations (1) thru (4).

Solution Algorithm

This problem is quadratic in the objective function, but unfortunately non-linear in the constraints due to the existence of the blend price equations. That is, since W_f and W_m are functions of X_f and X_m, equation (8) is a non-linear function of the P_b and X_s. Thus any standard quadratic programming algorithm will not solve this problem directly. We have resolved this dilemma by adopting the following iterative scheme:

(1) select trial values (guesses) for the w_i^f and w_i^m based on past history.

(2) solve the quadratic programming problem using these trial values,

(3) reestimate the w_i^f and w_i^m based on the solution obtained in step (2),

(4) if the new trial values of w_i^f and w_i^m are not equal to the old trial values, repeat steps (2) and (3), otherwise the optimal solution has been found.

This procedure is somewhat cumbersome but it is fairly easy to implement. Furthermore convergence is surprisingly fast--on problems involving 28 producing/consuming regions plus 11 consuming regions we got convergence to within a reasonable degree of accuracy within 6 to 8 iterations.

To solve the programming problems we used the quadratic algorithm developed by Cutler and Pass. This algorithm is somewhat limited as to the size of problem it can solve. In fact given the computer available at the time we solved the above problem, we severely taxed the algorithm's limits. A more satisfactory algorithm today may be the Stanford product known as MINOS. MINOS is a FORTRAN-based computer program system designed to solve a general class of large-scale optimization problems. MINOS will not only solve larger problems, it will handle general nonlinearities in <u>both the objective function and in the constraints</u> both the objective function and in the constraints directly. Hence it should facilitate obtaining solutions to problems of the type described here.

Analyses Possible

The analyses possible with the model outlined are fairly straight forward. We can, for example, trace the impacts of changing the following parameters:

(1) demand and supply elasticities --- to determine how sensitive the model is to errors in parameter specification, and to contrast "short-run" and "long-run" solutions,

(2) supply intercepts --- to study the implications of changing regional comparative advantages,

(3) the w_i --- to examine the implications of different regional pooling procedures and/or of merging regional orders,

(4) "class I differentials"" --- policy variables established by Federal Orders, and

(5) transportation costs --- to study the impacts of introducing new transportation technology.

Some Results

Full details on the results generated using this model are presented in a Northeast Regional publication (see Hallberg, et. al.). I will not attempt to reproduce the discussion of that report here, but merely summarize some of what I believe were the highlights.

Gainers and Losers

Application of the model verified that under equilibrium there would be gainers and losers. Here we measured gains and loses in terms of gross producer returns and gross consumer expenditures so as to stay away from the Marshallian surpluses that incite so many, and (as Cochrane suggests) are probably unintelligible to policy-makers anyway.

	Regional Losers	Regional Gainers
Producers	Northeast, South Atlantic Lake States, Southwest	South Central, Plains, Mountain, Southwest, Northwest, Corn Belt
Fluid Milk Consumers	South Central, Plains, Mountain, Southwest, Northwest, Northeast	Lake States, South Atlantic
Manufactured Product Consumers	South Atlantic, Northeast	Corn Belt

Price-Basing Points

One of the more interesting results relates to the idea of multiple price-basing points. The Class I differential in each Federal Order is determined by adding to the smallest differential a fixed amount for every 100 miles from Chicago to a city centrally located in the Order under consideration. This rule is not reflected in the differentials produced by the model. Here the economics of the situation were allowed to determine the appropriate differentials as well as the appropriate price-basing point or points for fluid milk. The model results suggest that one such point would be located in the Northeast, one in the Southeast, and one in the Lake States (and probably one more in the Far West), and that milk prices and fluid differentials across the country should be lined up with this set of multiple pricing points rather than with the single one at Eau Claire, Wisconsin. These three points were singled out and identified as price-basing points on the basis of the fact that the equilibrium fluid differential and fluid price are at or near a minimum in these regions. This, in turn, is a reflection of the fact that substantial pools of surplus milk would exist (in equilibrium) in each of these regions. Unfortunately, however, Federal Milk Marketing Orders include provisions which are based on the erroneous idea (undoubtedly correct in their early history) that Eau Claire, Wisconsin, is the proximity of the sole surplus milk production area in the country.

Altering the "Class I Differentials"

Altering the "class I differentials" did produce fairly significant impacts: consumer expenditures would increase particularly in the Corn Belt, Lake States, Northeast, and South, and producer receipts would increase particularly in the South, Northeast, and Corn Belt. We were surprised, however, that the aggregate impacts were not greater.

National Order

This policy was pursued by setting all w_i^f equal. The results were a 2.7 percent increase in producer receipts and a 10.2 percent increase in consumer expenditures. In general a considerably greater amount of fluid milk entered interregional trade so that transportation costs increased markedly, and the geographic structure of prices changed so that only one fluid milk price-basing point in the Lake States area existed. All this suggests the undesirability of a National Order as defined here.

MODEL MODIFICATIONS

I should like to close by considering some modifications to the model that might be worth pursuing. Of course, parameter estimates, shipping points, regional aggregations, etc. are always candidates for discussion in such models. Rather than spend time here, though, I will try to stimulate discussion by suggesting two rather substantial modifications in the structure of the model itself. Perhaps you will disagree with me as to their importance or as to the feasibility of incorporation into the model outlined here.

Processing Capacity

The model outlined assumes that all milk produced in the region is processed in the region and enters into interregional trade in processed form. This is a simplifying assumption that as near as I can tell does little damage. Nevertheless unprocessed milk does move between regions as in the case of the Reserve Standby Pool Cooperative. Furthermore we know that processing capacity in the various regions is not unlimited.

Thus it may be useful to introduce regional processing capacity constraints (which for analytical purposes could be alternately relaxed and tightened) and unprocessed milk shipment activities so as to add more realism and to trace the impacts on geographic price structures.

Dynamic Considerations

The model as presently conceived is a static, annual model. Thus we are prevented from looking at seasonal issues, and we cannot deal with storage problems. The latter is to me a serious limitation and one today's modellers might fruitfully spend some time contemplating.

We have the technology available today to produce a milk product that is storable and transportable at much lower costs than storing and transporting raw or conventionally packaged fluid milk. I am referring to both UHT milk and to a product from which the water has been removed (even removed on the farm!) and which can later be reconstituted as fluid milk at the destination. There could be very substantial regional consequences of this technology---consequences agricultural economists should be tooling up to deal with!

I am not yet convinced that the model framework presented here is the best one with which to deal with this issue, but it seems to me to offer a viable possibility. Clearly the model would have to be converted into a multi-period one---perhaps a four-quarter model---so that it generates equilibrium in space, form, <u>and</u> time. Storage activities as well as activities which permit moving milk into and out of storage would also be needed. Finally fluid milk activities might need to be defined in terms of two different forms of fluid milk---the traditional form and the new technology form---especially in formulating the problem so one can examine issues relating to the transition from the current technology to the new technology.

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AN INTERREGIONAL DAIRY MARKETING AND POLICY MODEL USING SEPARABLE PROGRAMMING TECHNIQUES

By Howard McDowell*

The dairy industry is among the most regulated in U.S. agriculture and is characterized by complex economic organization. Considerable dairy marketing and policy research has been done over the years in the areas of regulation, supply and demand analysis, market organization, and interregional trade. Work has been done concerning the reasons for regulation, and possible the loss in social welfare due to regulation itself preventing the evolution of the industry towards a competitive market. On the other hand certain economic conditions suggest that noncompetitive markets would exist without regulation, however, the nature of the regulation may need to change.

The economic model presented in this paper is based upon the interregional trade models that generate competitive equilibrium prices and quantities by maximizing the areas commonly known as producers' and consumers' surplus. The regulatory constructs of classified prices, pooling, and price support are incorporated through the use of constraints. The economic model is specified as a nonlinear programming problem having linear and nonlinear constraints. The model is solved by using separable programming methods and standard mainframe linear programming software. A matrix generator converts supply and demand functions, blend price functions, transportation costs, and other constraints into an input file for the linear programming package. The generator uses standard fortran

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programming language to convert a fundamentally nonlinear problem into a linear approximation of the problem.

The first reason for taking this approach is the perceived need to develop a conceptual economic model of dairy markets that is general enough to easily handle alternative assumptions concerning competitiveness and market regulations at a regional level of disaggregation. To the extent that regulations can be translated into price or quantity constraints or restrictions, or into a function of endogenous and exogenous variables, the model appears to be capable of handling alternative regulatory and competitive assumptions. Prices and quantities are determined endogenously, subject to constraints reflecting regulations. The economic model is consistent with those developed by Dahlgran, and Ippolito and Masson.

The second reason for the approach is that through the use of separable programming techniques, close linear approximation may be achieved with standard linear programming algorithms. This method tends to be more stable and consistent than other programming methods in finding optimal values of variables.

The perception is that the research agenda in dairy marketing and policy includes the evaluation of alternative regulations and competitive assumptions. It is also perceived that continuity in research through increased generality is desirable because of the enhanced comparability of the results. It is expected that continued refinements of the approach will take place through the modification and further development of the matrix generator.

Economic Model

In this section a general economic model of the dairy industry is specified. Specific assumptions are made and both short-run and long run models are discussed.

General Economic Problem

The United States dairy industry is treated as a sector isolated from the rest of the economy under the assumption that all other prices and consumers' incomes remain constant. The only prices permitted to change are prices of raw milk and processed dairy products. The United States is divided into n regions. Milk is consumed within each region in the forms of fluid and manufactured dairy products. Both Grade A and Grade B milk production are accounted for where applicable. Milk utilized in fluid products is restricted to Grade A, produced under stricter sanitation conditions and at a higher average cost than Grade B. Each region is assumed to be a separate market, and for Grade A milk, regulated by a federal milk marketing order or similar state regulations. Except for government regulation, the industry is assumed to be purely competitive.

The marketing margin includes costs of route assembly of raw milk, processing raw milk into consumer products, distribution of the product, and the interregional shipment of the product in its cheapest form. In the case of interregional shipments of Grade A milk for fluid use, milk is assembled, shipped in raw bulk form to its destination, and processed in the region of consumption. Soft manufactured products, including ice cream, yogurt, and cottage cheese are closely associated with the fluid industry and are usually treated in the same way through the marketing chain.

Interregional trade of hard manufactured products, including butter, non-fat dried milk powder, and hard cheese, take place in the processed product form. Milk is assembled, processed into final form, then shipped to its destination for final distribution.

Model Assumptions

The model development thus far has concentrated upon two types of movement of Grade A milk. The first is the interregional shipment of Grade A milk for fluid use in a short-run context. In this case milk flows are attracted by increasing fluid milk prices that reflect scarcity of local supplies. The second Grade A milk movement is represented by a shift of Grade A milk supplies, from one market or pool to another, based upon the relationships of blend prices and transportation costs. The mathematical model specified in this paper is the short-run model.

The demand functions are assumed to be at the plant level. It is assumed that all milk is assembled, processed and distributed regionally at a constant average cost, and that interregional shipping takes place at constant average costs determined by distances between import and export locations. Since distribution and processing costs are assumed to be constant in each region, these costs are subtracted from retail to derive a plant level demand function for milk in fluid use in each region. In the case of insufficient regional supplies of Grade A milk for fluid deand, interregional shipment of Grade A milk in raw bulk form may take place.

It would be desirable to specify the regional demands for the major manufactured products and let the model solve for optimal milk production, processing locations, and product shipment. This, however, is beyond the scope of the research at this stage. The demand for milk in manufactured products is considered to be a national demand, that is met by Grade B milk, and Grade A milk beyond fluid use.

It is assumed that the demand for fluid and manufactured products are unrelated, and have zero cross-price elasticities. Furthermore, it is assumed that the demands for dairy products have no income effect, or have zero income elasticities.

Substitutability in production is not feasible. Due to sanitation regulations, each production unit owned by a firm may be Grade A or Grade B, but not both. These sanitation regulations result in different fixed and variable production costs. Grade B milking facilities need not meet the minimum standard necessary for Grade A production, resulting in different capital costs which are fixed in the short run. Differences in variable costs are due to additional cleanliness restrictions for equipment in Grade A production. Producers could switch from Grade B to Grade A, or from Grade A to Grade B, depending on average revenues relative to average production costs. Contractual and health regulations, however, limit this type of decision to the long run. In the long run, capital is a variable input, and cross-price effects are relevant. The model specified in this study makes no endogenous provision for capital investment or transfer between Grade A and Grade B. It is assumed that supply functions for Grade A and Grade B milk are known for each region.

The pricing of milk is influenced by two federal programs, price support and federal milk marketing orders. The dairy price support program is administered by the Agricultural Stabilization and Conservation Service (ASCS). Product sales and purchases are managed by the Commodity Credit Corporation (CCC). Through the purchase of surplus manufactured dairy products at guaranteed prices, manufacturing milk prices are effectively supported at a minimum level.

Federal milk marketing orders, administered by the Agricultural Marketing Service, regulate some 80 percent of the Grade A milk marketing in the U.S. All Grade A milk is assumed to be sold in regulated markets with classified pricing of milk according to its use, and pooling of revenues for producers according to prices within their own market. The classified pricing scheme assigns differential minimum prices to Grade A milk in fluid use, Class I, and in manufactured use, Class II. All manufacturing uses of Grade A milk are assumed to be in one class. The base price for the federal order classified pricing system is the monthly average price paid to dairy farmers in the Minnesota-Wisconsin region for milk in manufactured use. The minimum Class II price for all federal markets in month t is the base price (M-W) for month t. The minimum Class I price in all federal order markets in month t is the M-W price in month t-2 plus a differential.

Pooling provides that processors pay into a pool, the Class I and Class II prices for Grade A milk in the respective uses according to each processors' sales. Producers in each regulated market receive the same market blend price, the weighted average price for market sales. Once the average M-W price in any month t is known, the minimum Class I price in month t + 2 is known. That is, market participants know with certainty the minimum fluid prices a month in advance, since the average M-W is known at month's end and the differential is constant. This information coupled with an estimate of the local manufactured milk price would yield an estimate of the average revenue curve for the following month. The support price places a floor under the M-W price.

The blend price paid to Grade A dairy farmers is a weighted average price of both Class I and Class II sales. The minimum Class I price applies to both intraregional and interregional sales. Therefore, the appropriate fluid demand curve is the horizontal summation of the local demand for fluid milk, and all excess demand functions from other regions, in terms of the local fluid milk price.

The blend price in region i is defined as,

$$pa_i = (p1_i \sum_j xa1_{ij} - p2_i\, xa2_i)xa_i ,$$

where,

$p1_i$ is the minimum price for milk in fluid use in region i,

$E_j xa1_{ij}$ is the sum of all fluid milk shipments from region i,

$p2_i$ is the price for mile in manufacturing use in region i,

$xa2_i$ is the Grade A milk from region i manufacturing use,

xa_i is the Grade A milk produced in region i.

It is appropriate at this point to discuss some issues concerning the time period of the model. One issue is that of the relevant supply inducing price. It is highly unlikely that the supply of milk in any given month is affected by the current blend price. It is more likely that supply response is lagged at least one quarter. The blend price for period t is determined by Class I and Class II demand quantities and prices in period t, and the quantity of Grade A milk supplied in period t. But the Grade A quantity supplied is a function of an earlier price or set of prices.

In a monthly or quarterly time frame, increasing fluid prices attract milk flows from exporting regions to regions of tight supply. As supplies tighten relative to demand, Class I prices increase to the point of providing the incentive for exporting region to allocate milk to the importing region for Class I use. For a very short-run model to become operational, the issue of lagged supply response must be addressed. This would move the model into dynamic setting. The literature in agricultural economics includes a number of poly-period programming problems, and it is possible that this work will be extended to multiple periods or dynamic specification. An extensive dynamic modeling effort has been made by Novakovic et al.

If the model specification is annual, the analytics become considerably easier and are relevant to the degree that pertinent results may be obtained from a static model. Annual average prices and quantities are generated by annual fluid and manufacturing milk demand functions, annual supply function, and classified pricing and pooling to generate equilibrium prices and quantities. A model of longer run could provide comparative static results of annual average prices and quantities in response to policy changes and longer-run elasticities. Specifying an annual model, however, presents problems in handling the seasonal nature of interregional flows of milk for fluid use. One method of handling this problem might be to estimate an excess demand for deficit regions as a function of each region's annual average blend price. Another, more ad hoc, method is to estimate an annual reserve requirement of Grade A milk as a percentage of fluid use in each region. Care would be necessary to prevent such a requirement to be applied outside of its reasonable bounds.

An alternative view of interregional trade is more relevant over this longer run. Within a year it is true that interregional shipments are made of Class I milk. These shipments appear to be a relatively small proportion of total Class I sales. In 1982, 16.5 million pounds of Class I milk was shipped from Wisconsin as compared to 40.8 billion pounds of Class I sales in all federal orders. This obviously does not include all shipments but does illustrate the point. The more relevant question in the longer term is whether the differences in blend prices exceed transportation costs to the extent that milk supplies are shifted from one pool or market to another. A shipment between regions in the long-run model constitutes a shift in supply from one region to another. This is of major concern in the discussion of drastically changing Class I differentials in the federal order system.

Nonlinear Programming Specification

The mathematical model is a specific version of the general multi-region, multiproduct model specified by Takayama and Judge (1971:107-127). Modifications of the general problem are in the form of constraints reflecting the government programs. The short-run model is specified.

The plant-level demand functions for fluid and manufacturing milk, and farm-level supply functions for Grade A and Grade B milk for each region are defined below.

(1) $y1_i = y1_i(p1_i)$, the quantity of fluid milk demanded in region i is a function of its own price.

(2) $y2_i = y2_i(p2_i)$, the quantity of manufacturing milk demanded in region i is a function of its own price.

(3) $xa_i = xa_i(ra_i)$, the quantity of Grade A milk supplied in region i is a function of its own price. (I-2)

(4) $xb_i = xb_i(rb_i)$, the quantity of Grade B milk supplied in region i is a function of its own price.

In inverse form these functions are written as follows:

(1) $p1_i = d1_i(y1_i)$

(2) $p2_i = d2_i(y2_i)$ (I-3)

(3) $ra_i = sa_i(xa_i)$

(4) $rb_i = sb_i(xb_i)$

It is assumed that the commodities are non-inferior, that demand functions slope downward. Supply functions are assumed to be upward sloping, increased production takes place at increasing supply prices. Transportation costs are constant for all shipments between any 2 regions, and within any region. Shipments are divided into three groups, Grade A to fluid use, Grade A to manufactured use, and Grade B to manufactured use. Shipments for manufactured use are restricted to intraregional.

Let the shipment quantity and the shipment cost vectors for a given region i, i = 1, ..., n, be specified as follows:

$$X^i_{ij} = (xa1_{ij}, xa2_{ii}, xb_{ii})' \quad j = 1, \ldots, n, \qquad \text{(I-4)}$$

$$T_{ij} = (ta_{ij}, tb_{ii}) \quad j = 1, \ldots, n, \qquad \text{(I-5)}$$

where:

x denotes the quantity shipped,

t denotes the cost of the shipment,

$a1_{ij}$ denotes Grade A in fluid use shipped from region i to j,

$a2_{ii}$ denotes Grade A in manufactured use shipped within region i,

b_{ii} denotes Grade B in manufactured use shipped within region i.

For any region i total transportation costs are incurred and defined by

$$T_{ij}X^i_{ij} = \Sigma t_j a1_{ij} xa1_{ij} + ta2_{ii} xa2_{ii} + tb_{ii} xb_{ii} \quad j = 1, \ldots, n. \qquad \text{(I-6)}$$

The transport cost over all regions is written as

$$TX_{ij} = \Sigma_i T_{ij} X^i_{ij} \quad i,j = 1, \ldots, n \tag{I-7}$$

Since the aggregate quasi-welfare function, (I-8) below is the sum of functions that are strictly concave in X and Y, and the transportation cost functions are strictly linear, the aggregate quasi-welfare function is also strictly concave.

Because of zero cross-price elasticity, the aggregate welfare function may be written as the sum independent single-variable definite integrals. This is true because the variable of integration has an effect upon only a single demand or supply function.

The aggregate welfare function is maximized subject to the following pricing constraints due to regulation, and the commodity balance constraints requiring that excess supplies be non-negative for feasibility.

$p1_j - p1^\circ_j \geq 0$, the price for milk in fluid use in region j, $p1_j$, is at least as great as the minimum price $p1^\circ_j$, set by the federal order, j = 1, ..., n.

$p2_j - p2^\circ_j \geq 0$, the price for milk in manufactured use in region j, $p2_j$ is at least as great as the support price $p2^\circ_j$, j = 1, ..., n.

$rb_i - tbi_i - p2_i \geq 0$, the supply price for Grade B milk in region i, rb_i, is at least as great as the price for manufacturing milk in region i less the assembly cost, tb_{ii}, i = 1, ..., n.

$xa_i - \sum_{j=1}^{n} xa1_{ij} - xa2_{ii} \geq 0$, the quantity of Grade A milk produced in region i, xa_i, is at least as great as the sum of all its Class I shipments, $\sum_{j=1}^{n} xa1_{ij}$, and milk in Class II use, $xa2_{ii}$.

$xb_i - xb_{ii} \geq 0$, the quantity of Grade B milk produced in region i, xb_i, is at least as great as the amount shipped xb_{ii}.

$\sum_{i=1}^{n} xa1_{ij} - y1_j \geq 0$, the sum of Class I Grade A shipments from all regions to the j^{th}, $\sum_{i=1}^{n} xa1_{ij}$, must be at least as great as fluid consumption in the j^{th} region, $y1_i$, i, j = 1, ..., n.

$xb_{ii} + xa2_i - y2_j - y2^{\circ}_j \geq 0$, the sum of Grade B shipments, and Class II Grade shipments xb_{ii} and $xa2_i$ respectively, must be at least or as great as the quantity of manufacturing milk demanded $y2_j$, and quantity of manufacturing milk demanded by the government $y2^{\circ}_j$, in region j, where i = j for all i = 1, ..., n.

$ra_i - ta_{ii} - [p1^{\circ}_i \sum_{j=1}^{n} xa1_{ij} + p2_i xa2_i)xa_i^{-1}] \geq 0$, the supply price for Grade A milk in region i, ra_i, is at least as great as the region i blend price, enclosed in [] (also labeled pa_i) less the assembly cost ta_{ii}, for all i, j = 1, ..., n.

At this point the quasi-welfare function is specified as a Lagrangian to be maximized with respect to quantities consumed, produced, and shipped, subject to the constraints specified above. It is pointed out that the model specified here is a short-run model. The interregional shipments are of Class I milk. In the long-term specification, the shipments would be of Grade A milk in region i to a pool in region i.

$$\underset{Y,X,X_{ij},\lambda}{\text{Max } Wt} = \Sigma_j [\int_0^{y1_j} d1_j(y1_j)dy1_j + \int_0^{y2_j} d2_j(y2_j)dy2_j] \tag{I-8}$$

$$-\Sigma_i [\int_0^{xa_i} sa_i(xa_i)dxa_i + \int_0^{xb_i} sb_i(xb_i)dy2_j]$$

$$+ \Sigma_j p^{\circ}2_j y^{\circ}2_j - \Sigma_i \Sigma_j ta_{ij} xa1_{ij} - ta_{ii} xa2_{ii} - tb_{ii} xb_{ii}$$

$$+ \sum_{j=1}^{n} \lambda 1_j \; [p1_j - p^{\circ}1_j]$$

$$+ \sum_{j=1}^{n} \lambda 2_j \; [p2_j - p^{\circ}2_j]$$

$$+ \sum_{j=1}^{n} \lambda 3_i \; [rb_i - tb_{ii} - p2_i]$$

$$+ \sum_{j=1}^{n} \lambda 4_i \; [xa_i - \sum_{j=1}^{n} xa1_{ij} - xa2_{ii}]$$

$$+ \sum_{j=1}^{n} \lambda 5_i \; [xb_i - xb_{ii}]$$

$$+ \sum_{j=1}^{n} \lambda 6_j [\sum_{i=1}^{n} xa1_{ij} - y1_j]$$

$$+ \sum_{j=1}^{n} \lambda 7_j \; [xb_{jj} + xa2_{jj} - y2_j - y^{\circ}2_j]$$

$$+ \sum_{j=1} \lambda 8_i \; [ra_i - ta_{ii} - \{(p^{\circ}1_i - p2_i)\Sigma xa1_{ij} xa_i^{-1} + p2_i\}]$$

If it can be assumed that there are vectors $X°$, $Y°$, $X°_{ij}$ in the opportunity set that satisfy all inequality constraints as strict inequalities, if the objective function is (strictly) concave, and the constraint functions are convex, then the Kuhn-Tucker conditions are necessary and sufficient for a (strict) local maximum. By the local-global theorem (Intrilligator, 1971:15), if the opportunity set is a nonempty, compact, convex set, and the objective function is continuous and concave over the opportunity set, then a local maximum is a global maximum. If the objective is strictly concave, the solution is unique.

The objective function has been shown to be continuous and concave. The set of all nonnegative quantities up to some finite quantity greater than feasible is a convex set, and any linear constraint is convex. The only nonlinear constraints are the blend price constraints. It is asserted that the blend price function slopes downward at a decreasing rate and is therefore convex. Given the assumptions and theorems, a set of vectors, X°, Y°, and X°_{ij} may be found that maximize the objective function.

Linear Programming Problem

Following the technique of Duloy and Norton (1975), the nonlinear programming problem is restated as a separable linear programming problem. The general procedure is to segment the range of each integral in the objective of the Lagrangian (I-8), into a finite number of quantity steps. The value of the objective at each of the quantity steps is the value of the integral evaluated at zero and the quantity step. A convexity constraint on each of the functions assures feasibility. Additional constraints are required to transfer prices and quantities into the constraints placed on the problem by government regulation.

Objective Function

Recalling the regional supply and demand functions from (I-3), and the assumptions regarding cross price elasticities and a single manufacturing milk demand, the supply and demand functions may be stated as follows.

(1) $p1_j = d1_j(y1_j)$ for $j = 1, \ldots, n$

(2) $p2 = d2(y2)$

(3) $ra_i = sa_i\ (xa_i)$ for $i = 1, \ldots, n$ (II-1)

(4) $rb_i = sb_i\ (xb_i)$ for $i = 1, \ldots, n$

Now let the activities associated with the objective function be defined in association with quantities that will enter the solution if the activity is in the solution:

s_{jt}, the fluid milk demand activity in the j^{th} region at quantity level t, corresponding to the fluid quantity $y1_{jt}$, $j=1,\ldots,n$, $t=1,\ldots,m$.

q_t, the manufacturing milk demand activity at quantity level t, corresponding to the manufacturing milk quantity $y2_t$, $t=1,\ldots,m$.

u_{it}, the Grade A milk supply activity associated with quantity xa_{it}, for regions $i=1,\ldots,n$, and quantity level $t=1,\ldots,m$.

v_{it}, the Grade B milk supply activity associated with quantity xb_{it}, for regions $i=1,\ldots,n$, and quantity level $t=1,\ldots,m$.

The coefficients associated with each of the activities are calculated by evaluating the appropriate supply or demand function integral, at the particular level t of the quantity. Expressions for these calculations are given below.

$$
\begin{aligned}
&(1)\ \Delta 1j_t = \int_o^{y1_{jt}} [d1_j(y1_j)]dy1_j \\
&(2)\ \Delta 2_t = \int_o^{y2_t} [d2(y2)]dy2 \\
&(3)\ \Gamma a_{it} = \int_o^{xa_{it}} [sa_i(xa_i)]dxa_i \\
&(4)\ \Gamma b_{it} = \int_o^{xb_{it}} [sb_i(xb_i)]dxb_i
\end{aligned}
\qquad \text{(II-2)}
$$

The linear portion of the original objective function need not be modified. Therefore, the separable linear programming problem objective may be stated as follows.

$$
\begin{aligned}
\text{Max } W = {} & \Sigma_j\Sigma_t s_{jt}\Delta 1_{jt} + \Sigma_t q_t \Delta 2_t - \Sigma_i[\Sigma_t u_{it}\Gamma a_{it} - \Sigma_t v_{it}\Gamma b_{it}] \qquad \text{(II-3)} \\
& + p2°y2° - \Sigma_i\Sigma_j ta_{ij}xa1_{ij} - \Sigma_i ta_{ii}xa2_{ii} - \Sigma_i tb_{ii}xb_{ii}
\end{aligned}
$$

This function is to be maximized with respect to s, q, u, v, y2°, and the shipments x_{ij}, subject to the necessary constraints.

Institutional Constraints

The institutional constraints affecting the dairy sector embodied in the Lagrangians of the nonlinear programming problem are modified only slightly for the linear specification. Most modifications stem from the assumption of a single manufacturing milk market. This market has also been expanded to include beginning stocks and imports on the supply side, and ending stocks and exports on the demand side. The other major modification is that the constraints involving the Grade A milk supply price and the Grade A blend price are treated differently and discussed separately.

The constraints listed below are identical to those in the nonlinear specifications.

$p1_j - p1_j^\circ \geq 0$, the price for milk in fluid use in region j, $p1_j$, is at least as great as the minimum price $p1^\circ_j$, $j = 1, \ldots, n$.

$p2 - p2^\circ \geq 0$, the price for milk in manufactured use p2, is at least as the minimum price $p2^\circ$.

$rb_i - tb_{ii} - p2 \geq 0$, the supply price for Grade B milk in region i, rb_i, is at least as great as the price for manufacturing milk less the assembly cost, tb_{ii}, $i = 1, \ldots, n$.

$xa_i - \sum_{j=1}^{n} xa1_{ij} - xa2_{ii} \geq 0$, the quantity of Grade A milk produced in region i, xa_i, is at least as great as the sum of all its Class I shipments, $\sum_{j=1}^{n} xa1_{ij}$, and milk in Class II use, $xa2_{ii}$.

$xb_i - xb_{ii} \geq 0$, the quantity of Grade B milk produced in region i, xb_i, is at least as great as the amount shipped xb_{ii}.

$\sum_{i=1}^{n} (xb_{ii} + xa2_{ii}) + (BS - ES) + (IM - EX) - y2 - y2^\circ \geq 0$, the sum of Grade B shipments, and Class II Grade A shipments, beginning stocks, and imports, xb_{ii}, $xa2_{ii}$, BS, and IM, are greater than or equal to ending stocks, exports, consumer demand, and government purchases of manufacturing milk, ES, EX, y2, and $y2^\circ$.

Separable Programming Constraints

This group of constraints is required by the stepwise linearization of a functional form. These constraints include those associated with the objective function itself, and those associated with the blend price function.

The constraints associated with the demand and supply functions are straight forward. Each step in the objective function is associated with a particular quantity. The model, however, also places restrictions on prices. For each quantity step in a function, its corresponding price is also calculated. Each supply or demand equation requires three constraints, quantity, price and convexity. These constraints are specified below.

Grade A supply:

$\Sigma_t u_{it} xa_{it} - xa_i \geq 0$, the quantity of Grade A milk produced at a positive cost, $\Sigma_t u_{it} xa_{it}$, is greater than or equal to xa_i, the amount of Grade A milk demanded.

$\Sigma_t u_{it} ra_{it} - ra_i \geq 0$, the supply price of Grade A milk as calculated by activity steps along the supply function, $\Sigma_t u_{it} ra_i$, may not be less than ra_i, the supply price to be used in other restrictions.

$1 - \Sigma_t u_{it} \geq 0$, the level of any particular activity or the sum of activity levels cannot be greater than one.

Grade B supply:

Similar to Grade A supply,

$\Sigma_t v_{it} xb_{it} - xb_i \geq 0$,

$\Sigma_t v_{it} rb_{it} - rb_{it} \geq 0$,

$1 - \Sigma_t v_{it} \geq 0$.

Fluid demand:

$y1_j - \Sigma_t s_{jt} y1_{jt} \geq 0$, the quantity of fluid milk consumed $\Sigma_t s_{jt} y1_{jt}$, may be no greater than the quantity supplied, $y1_j$.

$-p1_j + \Sigma_t s_{jt} p1_{jt} \geq 0$, the price of fluid in the demand function $\Sigma_t s_{jt} p1_{jt}$, is at least as great as the price used in other constraints in the model.

$1 - \Sigma s_{jt} \geq 0$, the level of any particular activity or sum of activity levels cannot be greater than one.

Manufacturing demand:

$y2 - \Sigma_t q_t y2_t \geq 0$,

$p2 - \Sigma_t q_t p2_t \geq 0$,

$1 - \Sigma_t q_t \geq 0$.

Pooling Constraints

The constraint requiring the Grade A supply price ra_i, to be at least the blend price is a bit different. The demand for milk facing Grade A farmers is the demand for fluid milk at prices above the minimum Class I, and the blend price for quantities produced beyond that which satisfies fluid demand at the minimum Class I price. The fluid demand portion of the demand is taken care of in a set of constraints discussed above. The blend price function must be segmented into separable activities, each activity associated with a particular quantity of Grade A milk supplied and blend price.

In the short-run model, the blend price function is necessarily predicted since the price is not calculated until after the market clears each month. A predicted value in all but one case is the manufactured milk price p2. The exception is when the support price is so high that it becomes the effective manufacturing or Class II milk price. In the short-run interregional shipments of Class I milk must also be estimated. This involves an iterative procedure discussed below. For each region the estimated blend price is calculated by the following sequence, given p2 and the differential d_i in each region.

$$p1_i = p2 + d_i$$

$$xa1_i^\circ = y1_i^\circ \qquad \text{(II-4)}$$

$$pa_i = [p1_i^\circ xa1_i^\circ + p2(xa_i - xa1_i^\circ)]/xa_i,$$

where,

$xa1_i^\circ = y1^\circ_i = y1_i(p1_i^\circ)$ is the demand for fluid milk in region i, not including Class I sales to other regions.

The activities and constraints associated with the pooling activity are defined as:

w_{it}, the average revenue or pooling activity in the j^{th} region at quantity level t, corresponding to the Grade A quantity xa_{it}, $i = 1, \ldots, n$, $t = 1, \ldots, m$.

$xa_i = \Sigma_t w_{it} xa_{it} \geq 0$, or the quantity of Grade A milk pooled, $\Sigma_t w_{it} xa_{it}$, and receiving the greater of $p1_i$ or pa_i, may be no greater than the quantity supplied, xa_i.

$ra_i - \sum_t w_{it} pa_{it} \geq 0$, ra_i, the supply price of Grade A milk, may be no less than the blend price pa_i as calculated above in (II-4).

$1 - \sum_t w_{it} = 0$, the levels of any particular activity or sum of activities must be equal to one.

Notice that the convexity constraint for the pooling activity is unlike the others in that it is a strict equality constraint as opposed to inequality. This is in response to problems resulting from the fact that the federal milk marketing program of classified pricing and pooling of Grade A milk is not consistent with the objective function, maximizing the sum of consumers' and producers' surplus. If the Grade A supply function were to intersect the fluid demand function at a price higher than the minimum Class I price, the pooling constraints would be of no consequence. If however, the supply function intersected with the blend price function at an average revenue, pa_i, that is greater than p2 but less than p1°, the effect on the objective function of producing Grade A milk for manufacturing use is negative. This is because the marginal cost of production, ra_i, is greater than the marginal revenue, p2. In the nonlinear programming specification, the assumption of continuity would assure that the constraint would hold, and that Grade A production would take place such that the marginal cost would equal the average revenue or blend price. The fact that the separable programming activities are by definition noncontinuous, and given an inequality convexity constraint, results in the situation where Grade A milk would be produced at a level sufficient to supply the fluid markets with $p1_i \geq ra_i$ over all regions. The pooling constraints would hold as inequalities with all w_{it} activities at zero levels. By equating $\Sigma_t w_{it}$ with one, the price constraint,

$$ra_i - \Sigma_t w_{it} pa_{it} \geq 0,$$

becomes effective, forcing the pooling and classified pricing system to be effective.

The estimated blend price function for each region is calculated based upon the Class I price, the estimated Class II price, and an estimation of the fluid milk demand within each region alone. In the case of the model solving for positive short-run interregional shipments or a Class II price not equal to that estimated, an interative procedure is necessary to reach equilibrium. In this case the fluid quantity demanded in any exporting region i, must be adjusted by the amount shipped. Class I sales are calculated as,

$$xa1_i^\circ = \Sigma_j xa1_{ij} \qquad i, j = 1, \ldots, n.$$

This results in a horizontal shift in the blend price function by $\Sigma_j xa1_{ij}$, at $p1_i^\circ$. The model is then solved with the adjusted blend price function. Due to the absence of perfectly elastic supply curves, the increase in Grade A production will be less than $\Sigma xa1_{ij}$.

In the case of $p1_i \geq p1^\circ_i$ in the exporting region, and always in the importing region no adjustment is necessary for interregional shipment because the supply price would equal the Class I price less transportation charges.

Full Specification.

Given the objective and constraints, the linear programming specification of the domestic dairy industry an interregional trade model follows below in (II-5).

$$
\begin{aligned}
\text{Max } W = {} & \Sigma_j \Sigma_t s_{jt} \Delta 1_{jt} + \Sigma_t q_t \Delta 2_t - \Sigma_i [\Sigma_t u_{it} \Gamma a_{it} - \Sigma_t v_{it} \Gamma b_{it}] \\
& + p2^{\circ} y2^{\circ} - \Sigma_i \Sigma_j ta_{ij} xa1_{ij} - \Sigma_i ta_{ii} xa2_{ii} - \Sigma_i tb_{ii} xb_{ii} \qquad \text{(II-5)} \\
& + \Sigma_j \lambda 1_j (p1_j - p2 - d_j) \\
& + \lambda_2 (p2 - p2^{\circ}) \\
& + \Sigma_i \lambda_{3i} (rb_i + tb_{ii} - p2) \\
& + \Sigma_i \lambda_{4i} (xa_i - \Sigma_j xa1_{ij} - xa2_{ii}) \\
& + \Sigma_i \lambda_{5i} (xb_i - xb_{ii}) \\
& + \Sigma_j \lambda_{6j} (\Sigma_i xa1_{ij} - y1_j) \\
& + \lambda_7 [\Sigma_i (xb_{ii} + xa2_{ii}) + (BS - ES) + (IM - EX) - y2 - y2^{\circ}] \\
& + \Sigma_i \quad \lambda_{i8x} (\Sigma_t u_{it} xa_{it} - xa_i) \\
& + \qquad \lambda_{i8r} (\Sigma_t u_{it} ra_{it} - ra_i) \\
& + \qquad \lambda_{i8c} (i - \Sigma_t u_{it}) \\
& + \Sigma_i \quad \lambda_{i9x} (\Sigma_t v_{it} xb_{it} - xb_i) \\
& + \qquad \lambda_{i9r} (\Sigma_t v_{it} rb_{it} - rb_i) \\
& + \qquad \lambda_{i9c} (1 - \Sigma_t v_{it}) \\
& + \Sigma_j \quad \lambda_{j10y} (y1_j - \Sigma_t s_{jt} y1_{jt}) \\
& + \qquad \lambda_{j10p} (-p1_j + \Sigma_t s_{jt} p1_{jt}) \\
& + \qquad \lambda_{j10} c (1 - \Sigma_t s_{jt}) \\
& + \qquad \lambda_{11y} (y2 - \Sigma_t q_t y2_t) \\
& + \qquad \lambda_{11p} (p2 - \Sigma_t q_t p2_t) \\
& + \qquad \lambda_{11c} (1 - \Sigma_t q_t) \\
& + \Sigma_i \quad \lambda_{i12x} (xa_i - \Sigma_t w_{it} xa_{it}) \\
& \qquad \lambda_{i12r} (ra_i - \Sigma_t w_{it} pa_{it}) \\
& \qquad \lambda_{i12} (\Sigma_t w_{it} - 1)
\end{aligned}
$$

Kuhn-Tucker Conditions

1. $\frac{aw}{as_{jt}} = \Delta 1_{jt} - \lambda_{j10y} y1_{jt} + \lambda_{j10p}\ p1_{jt} - \lambda_{j10c} \le 0 \qquad \frac{aw}{as_{jt}}\ s_{jt} = 0$ (II-6)

2. $\frac{aw}{aq_{t}} = 2_{t} - \lambda_{11y} y2_{t} - \lambda_{11p} p2_{t} - \lambda_{11c} \le 0 \qquad \frac{aw}{aq_{t}}\ q_{t} = 0$

3. $\frac{aw}{au_{it}} = -\ \Gamma a_{it} + \lambda_{i8x} xa_{it} + \lambda_{i8r} - \lambda_{i8c} \le 0 \qquad \frac{aw}{au_{it}}\ u_{it} = 0$

4. $\frac{aw}{av_{it}} = \Gamma b_{it} + \lambda_{i9x} xb_{it} + \lambda_{i9r} rb_{it} - \lambda_{i9c} \le 0 \qquad \frac{aw}{av_{it}}\ v_{it} = 0$

5. $\frac{aw}{aw_{it}} = \lambda_{i12x} xa_{it} - \lambda_{i12r} pa_{it} + \lambda_{i12c} \le 0 \qquad \frac{aw}{aw_{it}}\ w_{it} = 0$

6. $\frac{aw}{ay2^{\circ}} = p2^{\circ} = \lambda 7 \le 0 \qquad \frac{aw}{ay2^{\circ}}\ y2^{\circ} = 0$

7. $\frac{aw}{axa1_{ij}} = -ta_{ij} - \lambda 4_{i} + \lambda_{6j} \le 0 \qquad \frac{aw}{axa1_{ij}}\ xa1_{ij} = 0$

8. $\frac{aw}{axa2_{ii}} = ta_{ii} xa2_{ii} - \lambda 4_{i} + \lambda_{7} \le 0 \qquad \frac{aw}{axa2_{ii}}\ xa2_{ii} = 0$

9. $\frac{aw}{axb_{ii}} = -tb_{ii} xb_{ii} - \lambda 5_{i} + g_{7} \le 0 \qquad \frac{aw}{axb_{ii}}\ xb_{ii} = 0$

10. $\frac{aw}{a\lambda} \le 0$ and $\frac{aw}{a\lambda}\ \lambda = 0$ for all λ

Solution Computation

The computations are made through the use of a fortran computer program developed by Paul Chang and Terry L. Roe, and documented by McDowell (1982). A new subroutine generating price and quantity rows, and the blend price function was developed by McDowell for use with the general program. The program is used in conjunction with Control Data Corporation's APEX-I linear programming algorithm. The program is nearing operational status in conjunction with IBM mainframe computers and the MPS linear programming algorithm.

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FEDERAL MILK MARKETING ORDER ISSUES AND RESEARCH NEEDS*

by

Richard F. Fallert**

Background

The basic legislation of Federal milk marketing orders traces to the Agricultural Marketing Agreement Act of 1937 and to some extent the preceding Agricultural Adjustment Acts of 1933 and 1935. This basic legislation stemmed from the perceived need of providing milk producers some assistance in achieving and maintaining a degree of bargaining power over the prices they received for milk. The major objectives of the program were to attain parity prices, provide orderly marketing, and assure consumers an adequate milk supply at reasonable prices.

Federal milk marketing orders set minimum prices that must be paid by processors to dairy farmers or their cooperatives for Grade A (fluid grade) milk in markets where producers have elected to come under Federal orders. The 44 Federal milk marketing orders operating January 1, 1985, regulate the handling and pricing of about 70 percent of all milk sold to plants and dealers, and about 81 percent of the Grade A milk marketed in the United States. About 85 percent of the Nation's milk supply is Grade A and about 45 percent of all Grade A milk sold is used for fluid milk products. Only

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*Based primarily on researchable questions and issues raised by a group of dairy economists meeting at "Dairy Modeling in the 1980's: Symposium on Current Research," Columbus, Ohio. October 9, 1985.

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milk of Grade A quality (meeting the higher standard for fluid milk products) is regulated by Federal orders.

Two major provisions of Federal milk orders are:

- Classified pricing of milk according to use, and
- Pooling or combining all revenue from the sale of regulated milk from which a single uniform or blend price is paid to producers.

Minimum class prices are established for all of the 44 Federal marketing orders on the basis of specified relationships to the average price of manufacturing grade milk in Minnesota and Wisconsin (M-W price), so they will automatically reflect changes in support prices when market prices are at or below support. With a few minor exceptions, Federal order prices for Grade A milk used in manufactured products are set at or near the M-W price base. Minimum prices for milk used in Class I (fluid milk products) are higher by fixed differentials unique to each Federal order.

The geographical structure of Class I differentials corresponds closely to a basing point system with Eau Claire, Wisc., as the base. Moving from Eau Claire, minimum order Class I prices increase at a rate of about 15 cents per cwt per 100 miles, which is less than half of current actual transportation costs. Actual Class I prices paid by handlers usually exceed the minimum order prices in most markets by the amount of over-order payments negotiated between cooperatives and fluid milk processers. This price premium reflects the fine-tuning of prices to cover transportation costs not covered in Federal order minimum prices; additional costs of standardizing milk to customers' needs in form, time, and place; and, in some cases, a pure negotiated price premium that may not be cost-related.

The basic structure of Class I differentials (especially the portion designed to reflect transportation costs between markets) was last changed in 1968. In general, the differentials increase with the distance from the Upper Midwest, the largest source of Grade A milk supplies in excess of regional fluid needs.

General Setting

In the early days (generally prior to 1968), Federal milk marketing orders were local in nature and primarily concerned with generating an adequate supply of Grade A milk for fluid milk markets plus adequate weekly and seasonal reserves. Over time, however, more and more Grade A milk became associated with fluid milk markets and a higher proportion of regulated Grade A milk was used in manufactured dairy products. At the present time it is unclear what the specific objectives of Federal milk orders are except as stipulated in generalized terms such as "orderly marketing," "parity prices," "interests of consumers," and "adequate supply," which generally lack clarity. More recent literature also refers to such terms as assuring "equity among handlers," promoting "constructive competition," and "stabilizing fluid milk prices and markets." Other explicit or implicit objectives could be "maximizing or enhancing returns to specific groups of milk producers through price discrimination," "minimizing costs to consumers," or some "compromise" between the two. Then there is the possible objective of "individual market self sufficiency in milk supplies for fluid milk markets" vs. "optimizing the overall system" based on competitive advantage and/or comparative advantage in inputs markets, milk production, assembly, processing, and distribution.

If analysts were to evaluate the market performance of the overall milk marketing order system based on the tenets of location and optimization theory, the scorecard might show only mediocre scores. Without a well-defined objective function it is difficult to formulate policy and/or to evaluate the overall performance of a pricing and marketing system.

Research Questions and Issues

The group of dairy economists meeting at the Ohio dairy modeling symposium and addressing the milk order issue began the discussion by listing the first researchable questions and issues that came to mind. Some of these were:

- What are the merits of a single Class I price basing point in Eau Claire, Wisconsin vs. a multi-basing point or a base pricing zone across the northern area of the country stretching from Minnesota through the Midwest and on to the Northeast?
- What should the Class I price differential be at the basing points or basing zone?
- What should the Class I price surface (Class I differentials) be at points distant from the basing points or basing zone?
- What are appropriate intra-order transportation allowances?
- What are the costs and benefits (pros and cons) of local, regional, or national orders?
- Should Class I differentials be cost-based or is pure textbook price discrimination still appropriate and in vogue?
- What is the purpose and effectiveness of the minimum Class I price under orders vs. the purpose and role of over-order charges and the associated effective Class I price structure?
- What is the role of reconstituted milk as a means of balancing fluid milk supplies and optimizing the overall system?

- To what extent do allocation and compensatory payment provisions in Federal milk marketing orders hinder the use of reconstituted milk and stifle attainment of milk production, processing, and overall efficiency?

- What impact do market orders and market order provisions have on optimal location of milk production, processing, and distribution?

- What are the intramarket, intermarket, and regional location impacts of milk marketing orders in terms of gross returns to producers and consumer expenditures? What are the overall aggregate impacts?

- Are milk market orders and order provisions flexible enough to accommodate population shifts from the frost belt to the sunbelt, the changing demands of consumers, and the dynamics of emerging milk production and marketing technology (UHT milk, reconstituted milk, membrane technology, Bovine growth hormones, nutritional supplements, etc.)?

- What are the regional and aggregate implications and effects of emerging milk production, processing, marketing, and distribution technology?

- Are marketing orders neutral in their effects on the past, present and future location of milk supplies, location of processing facilities, and rates of innovation in the milk production and processing subsectors?

- To what extent do milk orders add stability to milk prices and markets? What are the costs and benefits of added stability to producers, processors, and consumers?

- To what extent is there regional or aggregate undue price enhancement resulting from orders?

- What are the effects of milk orders on producer cooperatives (structure and competitive effects)?

- What are the economic effects (costs and benefits) of classified pricing, administered pricing, and pooling and sharing of returns among producers?

- What are the tradeoffs between marketwide pooling of producer returns and compensating individual cooperatives and processors who provide marketwide services to the fluid milk market? (Are there any new alternatives to current pooling requirements and current pooling systems--balancing service credits, transportation credits, two-tier pooling systems, standby pools, etc.?)

- What would be the costs and benefits of component pricing under orders?

More Philosophical Questions and Issues

After thinking through, listing, and discussing the common set of issues and questions surrounding milk orders, the discussion took on a more questioning and philosophical tone. Some of these questions were:

- Do the milk production, processing, and marketing conditions of the 1980's still warrant an institutionalized administered pricing and marketing system for milk that was initiated over 50 years ago?

- What are the inherent characteristics of milk, its products, and milk markets that warrant extensive Government involvement in milk pricing and marketing?

- To what extent are milk marketing orders stifling and/or promoting technological innovation, product development, marketing innovation, and optimal location and types of milk production, processing, and distribution?

- Will the new set of emerging technologies in the milk production, processing, and distribution subsectors alter the inherent characteristics of milk and its products such that there will be less of a need for Government involvement in the dairy sector?

- In this emerging futuristic environment of shifting location of consumers (generally away from the lowest-cost areas of milk production), changing location of milk production, and an emerging set of new technologies, what are the likely effects of milk marketing orders on economic efficiency, costs, productivity, and other goals of society?

Other Observations and Recent Events

Dairy Sector in the United States and World Agricultural Setting

One of the goals of Federal milk marketing orders is to assure an adequate supply of milk. However, a problem for the mid-1980's is how to reduce the amount of excess resources in the dairy industry. This problem then readily translates into both a regional and aggregate resource adjustment issue along with an associated issue of equity among dairy farmers in different areas under an administered pricing system.

Much of the 1982, 1983, and 1985 dairy legislation evolved from an attempt to address the problems of excess milk supply and large Government purchases and costs primarily resulting from the high level of minimum price support with midyear adjustments from 1977 to 1980. One might suspect, however, that the subtle way in which dairy price supports, in conjunction with Federal milk marketing orders with marketwide pooling provisions and liberal pool plant requirements work together to affect milk supplies and the dynamics of the milk marketing system. Thus, the phenomenon of relatively high milk prices--coupled with reduced risk and uncertainty, lack of alternative uses for farm resources, and low feed prices--have resulted in over 10 percent more milk than consumers have been willing to buy at the supported prices.

Excess Agricultural Capacity and the U.S. Competitive Position in World Markets

Research related to Federal milk market orders may also be indirectly influenced by such issues as the world glut in dairy and food supplies, declining U.S. exports of agricultural products, the U.S. competitive position in international commodity markets, financial stress in the agricultural sector, and excess resources in the overall agricultural plant. A number of these issues could affect the overall industry structure and competitive advantage of dairying in different regions of the Nation.

The 1985 Omnibus Farm Bill

On December 18, 1985, Congress agreed on a compromise farm bill and President Reagan signed it into law on December 23. Many of the dairy provisions of the final bill are still subject to technical and legal interpretations and promulgation of rules and regulations for administering the law. As a legislative precedent, the Congress has stipulated (increased) Class I differentials in 35 of 44 Federal orders. The increases range from $1.03 per cwt in the Southeastern Florida order to no increase in the Michigan Upper Peninsula order as well as no increases in several other orders in the Mountain and Pacific regions. The weighted average increase in the Class I differential is about 30 cents per cwt. Even though these stipulated minimum Class I differentials may be in place for some time, it does not reduce the basic need for evaluating the pricing, pooling, and allocation provisions of orders.

Another provision of the 1985 Omnibus Farm Bill that could affect regional and aggregate milk supplies and indirectly affect Federal milk orders is the "Milk Production Termination Program" or herd buyout. The Secretary is to establish such a program not later than April 1, 1986, and it is authorized through September 30, 1987. It is likely that producer participation in this program will vary significantly among regions as was the case with the previous milk diversion program.

The Gramm-Rudman bill is another uncertainty that could affect agriculture programs (including dairy) and, again, indirectly affect milk orders. The Agricultural legislation also established a National Commission on Dairy Policy to study the likely effects of emerging technologies on milk supplies and on the structure of the industry--especially its effects on small- and medium-sized family dairy farms. Concern of the Congress about the effects of agricultural programs and other factors on farm structure could lead to questions about the appropriatness of historic size-neutral dairy programs.

Even though there is a longer term 1985 Omnibus Farm Bill in place with direct impacts on Federal milk order provisions, there are still numerous uncertainties that could raise important issues related to orders. A body of research capital providing farmers, industry participants, and policymakers with useful information on the effects of milk orders and specific milk order provisions on economic efficiency, costs, productivity, and other goals of society will still be in great demand.

DAIRY MARKETING ISSUES AND MODELING NEEDS RELATED TO INDUSTRY STRUCTURE

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The Dairy Modeling Symposium made a valuable contribution toward significant future research. It brought together information about existing models, and it helped interested researchers in their ongoing efforts to improve modeling applications to emerging problems of the diary industry.

One set of emerging problems involves future industry structure. In order to incorporate structural characteristics it is appropriate to focus initially on static models. As such models include more realistic considerations, they can be reformulated to accommodate dynamic aspects.

Major structural characteristics that may be important to take into account in dairy models are the following:

(1) Form of organization, that is, whether the firms are cooperatives or proprietary firms and the sizes and relative importance of each.

(2) Types of vertical arrangements that prevail and how they might affect industry operating behavior.

(3) Relative sizes and locations of processing plants.

(4) Variations in assembling, processing and distribution costs.

(5) Alternative farm enterprise locations, combinations and their effects on milk production costs.

The purpose in considering structural factors is to determine what constraints or behavior rules should be incorporated in models. It will also be important to ascertain how particular modifications in industry structure and behavior might affect outcomes. Also, by including regional as well as industry characteristics, changes in regional comparative advantage can be estimated.

In addition to various industry structure and spatial dimensions, social and political objectives are also important in real life and should be taken into account. They influence actual industry performance, often considerably. Through identifying and including social and political goals as constraints or variables, the costs and benefits of alternatives can be appraised. Examples of such factors are support prices, milk marketing orders, health regulations, farm size distributions, tax policies and foreign trade policies. Effects of regional incentives, such as state or municipal subsidies that affect particular locations of processing plants, sometimes play an important role and should be addressed.

In the modeling effort, it is also necessary to consider the accuracy and reliability of the coefficients, or the basic building blocks. Examples include fluid milk processing costs by location, size of plant, industry structure, transportation costs, and supply and demand elasticities. Any model, whether simple or elaborate, can give results of practical value only if its components and rules are realistic. As is well known, estimates of distortions or X-inefficiency have meaning only if models represent actual conditions with reasonable accuracy.

Finally, in order for models to be useful in providing guidance for private investment decisions and public policy analysis, it is necessary to be able to incorporate the dynamic ongoing effects of economic and institutional influences that last over time. Among such influences are changes in government policies regarding prices and price relationships among regions, changes in marketing order provisions, evolutions in farm and industry structure, trends in verticle coordination, price transmission process changes, plant cost changes, transportation changes and the nature and persistence of impediments to change. Knowledge of the time path of important changes in process, of their effects, and of the probable resistances to such changes, is critical from the standpoint of practical use of models. The potential value of research to build more realistic models for private and public practical applications in the diary industry can be of substantial value not only to various segments of the industry but also to society.

PRICE SUPPORTS AND MODELING:
RESULTS OF PANEL DISCUSSION

Richard A. King, Reactor

This report on the panel discussion of price supports is organized around four ideas: objectives of dairy policy, the role of price supports, methods for evaluating policy alternatives and directions for future research.

Objectives of Dairy Policy

The objectives of dairy policy are not easily stated. One is the notion of a safety net that will assure the survival of the family farm. A second is to provide a reasonable level of price stability and protection of farm asset values. Some view dairy policy as a study in avarice, extracting what the political process will bear to the benefit of the dairy industry at the expense of the consumers and taxpayers.

There was concern expressed that we may expect too much from the agricultural policy formulation process. It is not politically neutral. There is need for continuity in programs over time, and for recognition of the policy formation environment out of which dairy programs emerge.

The Role of Price Supports

For many years price supports have played an important role in the dairy industry. This role is reflected in the central position price supports play in the dairy models that have been developed. The question was raised whether price supports have been emphasized to the exclusion of other potential policy instruments that might be worthy of consideration.

Price supports often are treated as exogenous to the system. Consideration might be given to methods for adjusting price support levels within the system. Feedback mechanisms, perhaps, are being short circuited. Some method for adjusting price supports to meet changing economic conditions such as new feed costs and returns to alternative enterprises might be developed. Noneconomic variables may need to be incorporated into model design.

Methods for Evaluation Policy Alternatives

In evaluating alternative policy alternatives many models rely on estimation of equilibrium prices and quantities given regional price differentials. The question was raised how such equilibrium values might be made more relevant to politicians who may use other criteria in the evaluation process.

Among the problems considered were the difficulties associated with evaluation of what might be regarded as unacceptable policy option. The unpredictable nature of the policy making process might be more fully understood through cooperative work with political scientists. Alternative methods for allocating specified levels of treasury outlays might be investigated.

Directions for Future Research

Recent debates in Congress suggest that more attention might be given to the distributive effects of present policies that rely heavily on price supports as a mechanism to the exclusion of more specifically targeted measures such as directed income maintenance schemes.

Would the analysis of radically different dairy programs influence the professional incentives extended to young economists? Should price supports continue to be the driving force in dairy sector models? Should short-term actions such as the recent milk diversion program and the herd buyout proposal be viewed as feedback responses to unrealistic price support levels?

Social impacts of adjustments of resources exiting from the dairy industry might be investigated. In particular, this might require more comprehensive models that link the dairy industry to other sectors which compete for the use of these resources.

Production controls represent one extreme with efforts to get the government out of farming another extreme. Political dimensions might be introduced in more fully evaluating a range of policy options.

IMPLICATIONS OF BIO AND INFORMATION TECHNOLOGY FOR DAIRY MODELING EFFORTS: RESULTS OF PANEL DISCUSSION

by

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Recent studies of the potential for an accelerated rate of technological infusion in the dairy industry have created a great deal of interest in the potential for major changes in the milk supply-demand balance, costs of production, the structure of dairy farms, and regional mild production patterns. For dairy research economists, questions arise on the usefulness of econometric models on past supply and demand relationships. What is the potential for altering these models to consider the impact of technological change? Are other modeling farms involving techniques such as aggregating the results of typical farm simulations that apply new technologies more useful than econometric models in analyzing the potential impact of technological change?

Some Technological Definitions

Biotechnology involves the application of recombinant DNA to produce new life forms, new production inputs, or new products. For example, a Cornell University study has found the potential for a 20-40% increase in output per cow due to the injection of a synthetic bovine growth hormone (Kalter et al.). While most of the news regarding biotechnology involves the use of biotechniques to produce new inputs (such as BGH) for production, equally good prospects exist for modifying product forms or actually developing new products. For example, biotechnology holds the long-term potential for changing the fat-protein composition of milk or, better yet, removing the cholesterol from milk.

Information technology involves the application of computers to management decisions. such decisions may involve least cost feed formulation, diagnosis of disease problems and prescription of remedies, or optimum breeding decisions. Complex "expert" computer systems are being developed that integrate each of these decisions into a single management decision software package.

Other types of technological change are also on the drawing boards or in developmental stages. Examples include reverse osmosis and microfiltration of milk. The most important effect is to reduce the cost of transporting fluid milk.

Institutional Changes in Technology Development

One of the primary reasons for accelerating rates of technological change in agriculture lies in recent institutional changes regarding patent rights. In 1980 the U.S. Supreme Court made new life forms patentable. IN the same year, Congress enacted a law that allowed computer software to be copyrighted.

These legal changes opened the floodgate for increased public and private research to develop new technologies from which patent or copyright rents could be captured. The fruits of these investments will, in all probability, first be experienced in animal agriculture, particularly dairy. This is the case for two primary reasons:

- More is known about the physiological, biochemical, and genetic aspects of animals.

- Many of the animal agriculture biotechnology developments are a spinoff from dramatically increases human research through government agencies such as the National Institute of Health.

Implications for Modeling

Conceptually, production technology has the primary effect of shifting the supply schedule to the right. Problems arise in specifying the specific magnitude of shift and the impacts on the elasticity of supply. Not enough is known about the nature of the adoption process to specifically identify the effects of different types of technological change.

As a starting point for analysis, the specific magnitude of the impact of a new technology may not be all that critical. It is important to be able to explicitly identify the direction of effect; whether the effect is one shot (BGH) or continuous (genetic improvement); the scale effects of the new technology; and any resulting regional bias.

The structural and regional impacts of new technology may be particularly important. For example, it is anticipated that rather complex changes in producer management and feeding strategies will be required to obtain maximum benefits from BGH. Larger, more progressive dairies are more likely to have the management skills required for optimum application of BGH technology. As a result, BGH might be expected to be structurally biased toward larger scale dairymen, thus favoring dairying in the West, Southwest, and Florida. Similar impacts might be anticipated with the application of computer based expert dairy farming system technology. Such firm and regional biases are critical to proper assessment of the implications for technological change for dairying--whether at the farm level, transportation, or processor level.

Farm level simulations such as those undertaken by OTA and Texas A&M (see paper infra by Richardson et.al.) may be logical beginning points for obtaining insight into the impacts of new technology. Initial research establishing the efficiency of new bio or information technology developments are generally completed on test farm situations. Care must be taken to translate such laboratory tests into farm level applications and effects. Economic analyses such as those completed at Cornell University are absolutely critical to constructing appropriate simulation models, but need to be replicated in several different farm situations.

Policy Implications

Rapid technological change holds the potential for large increases in milk production over a relatively short time period. While ideally policy changes might be made that anticipate such developments, realistically Congress generally does not operate in an anticipatory manner.

The best that can be hoped for is probably the use of indexing milk prices to factors that reflect technological change. Such factors might include the level of milk production, cost of production, CCC purchases, CCC stocks, or the level of Class I utilization in federal order markets. Similar adjustments would need to be made in state milk marketing orders.

Conclusions

The years ahead promise more rapid technological change in dairying than in any other agricultural enterprise. The analytical tools of dairy economists will need to be sharpened to keep pace with the implications of technological change in dairying.