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Linear Structural Models of Production under Price Uncertainty : A Mean-Standard Deviation Approach

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The linearization of structural models of random price production may be carried out by exploiting the mean-standard deviation approach under a location and scale parameter condition. As shown here, the linearization problem may be solved under widely employed assumptions on production technology (i. e., a homogeneous production function) and on the type of risk aversion (i. e., constant absolute risk aversion or constant relative risk aversion). The linear structural models proposed in this study are more practical than those developed using the expected utility approach, for several reasons. First, they remarkably reduce the cost of estimating agent risk parameters. Second, they facilitate the calculation of various analytic measures that are useful for understanding production behavior, such as the risk premium and the elasticity of supply. Third, they allow for geometric explanations of agent attitudes toward price uncertainty. These practical attributes would facilitate a structural examination of farmer production behavior in the face of price risk. Furthermore, since the location and scale parameter condition under which all the arguments in this study are made is satisfied in a large number of economic models, the structural model simplification procedure considered here would be effective for developing tractable structural models involving alternative types of randomness, such as yield and financial uncertainties.

Key words : mean-standard deviation approach, location and scale parameter condition, structural form approach, linearization, risk aversion, competitive firm, price uncertainty.

1. Introduction

The production behavior of farmers facing uncertainty is a major area of empirical research in the field of agricultural economics, because agricultural production necessarily involves risk in the form of price and yield uncertainties. Much attention has recently been paid to a particular empirical approach to investigating this behavior that has been called, alternatively, the "joint estimation approach" and the "structural form approach" (see, for example, Love and Buccola [17] ; Saha, Shumway, and Talpaz [25] ; Chavas and Holt [6] ; Saha [24], Isik and Khanna [15] ; and Eggert and Tveteras [7]).[1] This approach utilizes the first-order conditions resulting from the optimization of economic models to estimate directly the structural parameters that indicate agent risk preferences and production technologies. Previous research involving such structural models has made use, primarily, of one of two distinct decision-making criteria. The first of these is the expected utility (EU) approach, the axiomatic fundamentals of which were provided by von Neumann and Morgenstern [31] and Savage [28]. The second is the mean- standard deviation (MS) approach,

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which is extremely practical, and was employed by Markowitz [18] and Tobin [30].

At an early stage in the development of structural production models, Love and Buccola [17] adopted the EU hypothesis and developed a structural model of random yield production under constant absolute risk aversion (CARA). Saha, Shumway, and Talpaz [25] and Chavas and Holt [6] subsequently relaxed the CARA restriction in their models of random yield production, and of random yield and random price production, respectively. Needless to say, general models are most useful in explaining economic phenomena that result from various farmer attitudes toward risk ; however, structural production models are frequently generalizable only at the cost of tractability. In particular, optimizing the trade-off between generality and tractability is a problem in the case of structural models developed using the EU approach, because these commonly involve an expectation operator and an intrinsically nonlinear specification, the very two factors that complicate structural models. These troublesome factors make structural estimation a very costly process. For example, Saha, Shumway, and Talpaz [25] conducted numeric integral calculus within nonlinear optimization routines. In addition, due to the expectation operator, the estimation of some useful analytical measures, such as the risk premium and the elasticity of supply, requires numerical integral calculation. Although the EU approach undoubtedly has axiomatic fundamentals, its adoption tends to considerably reduce the tractability of structural models.

During the next stage of the evolution of structural estimation, the MS approach was adopted as an alternative to the EU approach. The recent popularity of the MS approach has arisen, primarily, because of the theoretical contributions made by Meyer [19]. He demonstrated that the MS approach is equivalent to the EU approach under a location and scale parameter (LS) condition, observing that this LS condition is actually satisfied in a wide range of economic models as a result of their structures. In addition, he successfully translated various EU-based behavioral hypotheses, such as Arrow [2] and Pratt's [23] risk aversion measures, into appropriate analogues under the MS approach and the LS condition.2) This breakthrough made possible the translation of many EU models into the MS framework, with no loss of accuracy. Moreover, these transformed MS models could be analyzed on the basis of EU-based behavioral hypotheses. Thus, the MS approach is not only practical, but is also flexible enough to provide full explanations for the LS class of economic models. Saha [24], Isik and Khanna [15], and Eggert and Tveteras [7] took advantage of Meyer's theoretical contributions in their development of structural models involving randomness. Having adopted the MS approach, they formulated theoretical models that meet the LS condition. Owing to their theoretical structure, these models are equivalent to corresponding EU models and can be interpreted using EU-based behavioral hypotheses. In addition, the structural models subsequently derived from these do not involve an expectation operator, one of the factors that generally complicates EU-based structural models. Although these models retain an intrinsic nonlinearity, traditionally the other complicating factor of EU-based structural models, they are far more tractable than their predecessors.

This study attempts a further simplification of structural production models by exploiting the MS approach under the LS condition. As a prominent example of a case in which the LS condition holds, Sandmo's [26] production theory under price uncertainty has been adopted, and an attempt is made to linearize the structural model to which it gave rise. In particular, this study demonstrates that the linearization problem may be solved under frequently employed conditions on production technology, i. e., a homogeneous production function, and on the type of risk aversion, namely CARA or constant relative risk aversion (CRRA). The linear structural models proposed in this study are more practical than those developed using the EU approach, for several reasons. First, they remarkably reduce the cost of estimating risk parameters, since they do not involve an expectation operator and are linear ; therefore, neither numeric integral calculus nor nonlinear estimation methods are required. Second, they facilitate the calculation of various useful measures, including the risk premium and the elasticity of supply, because numeric integral calculation methods are no longer necessary. Third, they make it possible to explain agent attitudes toward price uncertainty geometrically. In consequence, these models are, potentially, of practical use in the structural analysis of the production behavior of farmers who must manage under unstable agricultural prices. Furthermore, the LS condition under which all of the conclusions of this study obtain is satisfied in a large number of economic models. Thus, the procedure used to simplify the structural model considered would also be effective in the development of tractable structural models involving alternative random factors, such as yield and financial uncertainties.

The linear structural models of random price production considered in this paper are developed by means of a two-step procedure. After a brief review of the economic implications of the LS condition, Sandmo's EU-based random production theory is translated into an MS model satisfying the LS condition. As a result of this step, the expectation operator is excluded without a loss of accuracy. Next, a structural model is derived from the MS model, and the possibility of removing its intrinsic nonlinearity is considered. In the course of solving this linearization problem, special attention is paid to the issue of the specification of the utility function in the MS approach under the LS condition. Two functional forms are then proposed, one for each of the cases of CARA and CRRA. Finally, the practical merits of the linear structural models proposed in this paper are discussed.

2. MS-Based Random Price Production Theory

The LS condition considered in this paper is defined as a condition under which the distributions of random payoffs differ from one another only by location and scale parameters (Feller [10]). Meyer [19] demonstrated that, under this LS condition, an agent's expected utility can be represented using only the first two moments of his random payoff.[3] More concretely, assume that a random payoff π_i meets the LS condition, and denote x as the random variable obtained from the standardizing transformation of π_i, $x=(\pi_i-\mu_i)/\sigma_i$, where μ_i and σ_i denote the finite mean and the standard deviation of π_i, respectively. Under the LS condition on π_i, x is unique, regardless of which π_i is selected for the purpose of defining x. Therefore, the expected utility from $\pi_i(=\mu_i+\sigma_i x)$ for any agent with a von Neumann-Morgenstern utility function $U(\pi_i)$ and a cumulative distribution function for $x, F(x)$, can be written as the function of the first two moments of π_i, as described below:

$$\begin{aligned} EU(\pi_i) &= \int_a^b U(\mu_i+\sigma_i x)dF(x) \\ &\equiv V(\sigma_i, \mu_i), \end{aligned} \tag{1}$$

where a and b denote the endpoints of the interval containing the support of x. It should be noted here that, since the LS condition holds for the random payoff π_i as a whole, no restrictions are placed on the utility function $U(\pi_i)$ or the cumulative distribution function $F(x)$.

Meyer subsequently pointed out that the LS condition is actually satisfied in a wide variety of EU models, owing to the actual structure of these models, because the random payoff in a given EU model depends linearly on a random parameter that is unique to that model.[4] For example, in the context of Sandmo's [26] random price production theory, which has been frequently applied in the field of agricultural economics, the random payoff (profit) π may be formalized as:

$$\pi = pq-C(q, \mathbf{w}, \mathbf{k})-B, \tag{2}$$

where p denotes a random output price, q denotes output, $\mathbf{w}$ denotes a vector of variable input prices, $\mathbf{k}$ denotes a vector of fixed inputs, $C(q, \mathbf{w}, \mathbf{k})$ denotes a variable cost function, and B denotes a fixed cost. Since the random payoff π is a linear transformation of the unique random parameter p, the distributions of the random payoff π differ from one another only by the location parameter of p, namely $-C(q, \mathbf{w}, \mathbf{k})-B$, and the scale parameter q.[5] Thus, the LS condition is met through the formation of the random payoff π itself.

The implication of the above observation is that many EU models can be transformed by means of the MS approach, without imposing assumptions on the von Neumann-Morgenstern utility function (e. g., a quadratic utility function) or the probability density function of the random payoff (e. g., a normal distribution). Therefore, Sandmo's production model, which meets the LS condition, can equivalently be transformed by means of the MS approach, as follows :

$$\begin{aligned} &\underset{q}{Max}\, V(\sigma, \mu), \\ &\mu = \mu_p q - C(q, \mathbf{w}, \mathbf{k}) - B, \\ &\sigma = \sigma_p q, \end{aligned} \tag{3}$$

where $V(\sigma, \mu)$ denotes the utility function under the MS approach, where μ and σ, respectively, denote the mean and standard deviation of π, and where μ_p and σ_p, respectively, denote the mean and standard deviation of p. The equivalency of the new MS model (3) to the original EU model can be demonstrated algebraically via the application of standard calculus and Meyer's Properties 1, 2, 4, 5 and 6, which translate EU-based behavioral hypotheses into an MS framework satisfying the LS condition.[6, 7]

Property 1 : $V_\mu(\sigma, \mu) > 0$ if and only if $U_\pi(\pi) > 0$.

Property 2 : $V_\sigma(\sigma, \mu) < 0 (=0)$ if and only if $U_{\pi\pi}(\pi) < 0 (=0)$.

Properties 1 and 2 jointly determine the sign of the slope of the indifference curve of $V(\sigma, \mu)$, which is denoted $S(\sigma, \mu) = -V_\sigma(\sigma, \mu)/V_\mu(\sigma, \mu)$.[8] Thus, $S(\sigma, \mu)$ is positive (zero) if the agent is risk-averse (risk-neutral) (Meyer's Property 3).

Property 4 : $V(\sigma, \mu)$ is concave if and only if $U_\pi(\pi) > 0$ and $U_{\pi\pi}(\pi) \leq 0$.

Property 5 : $S_\mu(\sigma, \mu) < 0 (=0, >0)$ if and only if absolute risk aversion is decreasing (constant, increasing).

Property 6 : $S_t(t\sigma, t\mu) < 0 (=0, >0)$ if and only if relative risk aversion is decreasing (constant, increasing).

As a result of this translation of the EU model into an MS model satisfying the LS condition, one of the factors typically complicating EU-based structural models, namely the expectation operator, has been excluded without a loss of accuracy.

3. Linearization of the Structural Model

So that a structural model may be derived from the MS model (3), it is assumed that (3) has an interior solution. Under this assumption, the first-order condition characterizing the optimum is given by :[9]

$$\mu_p - S(\sigma, \mu)\sigma_p = C_q(q, \mathbf{w}, \mathbf{k}). \tag{4}$$

When equation (4) is divided by average variable cost $C(q, \mathbf{w}, \mathbf{k})/q$ and is modified through the addition of an error term ε that is associated with optimization error, a structural model of random price production is obtained :

$$\frac{\mu_p q}{C(q, \mathbf{w}, \mathbf{k})} = \eta + S(\sigma, \mu)\frac{\sigma}{C(q, \mathbf{w}, \mathbf{k})} + \varepsilon, \tag{5}$$

where $\eta = \partial \ln C(q, \mathbf{w}, \mathbf{k})/\partial \ln q$ is the output elasticity of variable cost. In the above model (5), the left-hand term $\mu_p q/C(q, \mathbf{w}, \mathbf{k})$ and part of the second term on the right-hand side, namely $\sigma/C(q, \mathbf{w}, \mathbf{k})$, are observable given actual values for μ_p, σ_p, q and $C(q, \mathbf{w}, \mathbf{k})$.[10] On the other hand, the output elasticity of variable cost, η, and the slope of the indifference curve of the utility function in the MS framework, $S(\sigma, \mu)$, are generally unknown functions to be estimated. Thus, the structure of model (5) is determined by the functional specifications chosen for η and $S(\sigma, \mu)$; moreover, it is clear that this formulation may be reduced to a linear model if η is specified to be a constant function (i. e., a parameter) and $S(\sigma, \mu)$ is specified to be a function that is linear in parameters. The following discussion demonstrates that these specifications may be selected under widely employed assumptions on production technologies and types of risk aversion.

The parameterization of η may be obtained easily under the following assumption.

Assumption 1 : The production function is well behaved and homogeneous of degree m in the short-run.[11]

Under this assumption, the corresponding variable cost function may be written as :

$$C(q, \mathbf{w}, \mathbf{k}) = q^{\frac{1}{m}} g(\mathbf{w}, \mathbf{k}), \tag{6}$$

where $g(\mathbf{w}, \mathbf{k})$ denotes a non-decreasing and linear homogeneous function in $\mathbf{w}$.[12] The cost function (6) yields $\eta = 1/m$; that is, η is the reciprocal of the degree of homogeneity of the underlying production function. Therefore, η is a parameter that is independent of q, $\mathbf{w}$ and $\mathbf{k}$.

The linearization of (5) may be achieved through a linear specification of $S(\sigma, \mu)$, which, in turn, may be achieved under either of the following two assumptions.

Assumption 2.1 : The agent is a risk averter with constant absolute risk aversion (CARA).

Assumption 2.2 : The agent is a risk averter with constant relative risk aversion (CRRA).

In choice problems not involving uncertainty, an ordinal utility function is sufficient; that is, the specification of the utility function has no intrinsic meaning. Ordinal utility theory, however, does not hold for the MS approach under the LS condition. As is apparent from Meyer's Property 4 (i. e., the concavity condition), the utility function $V(\sigma, \mu)$ has a cardinal meaning and, therefore, its functional form must be appropriately specified prior to a discussion of whether it is possible to specify the slope of the indifference curve $S(\sigma, \mu)$ as a function that is linear in parameters.

Two sets of conditions must be fully satisfied in the specification of $V(\sigma, \mu)$. One of these comes from Meyer's Properties 1, 2, 4, 5 and 6,[13] while the other derives from the Arrow-Pratt risk aversion measures. The former is straightforward. For example, Property 1 restricts $V(\sigma, \mu)$ to be increasing in μ, while Property 2 specifies that it must be decreasing in σ, if the agent is a risk averter. Similarly, Property 4 stipulates that the relevant Hessian matrix with respect to μ and σ is negative semi-definitive if the agent is a risk averter. Properties 5 and 6 restrict the slope of the indifference curve, $S(\sigma, \mu)$. In particular, Property 5 restricts $S(\sigma, \mu)$ to be decreasing (constant, increasing) in μ when the agent's absolute risk aversion is decreasing (constant, increasing), while Property 6 restricts it to be decreasing (constant, increasing) along rays through the origin when the agent's relative risk aversion is decreasing (constant, increasing). In contrast, the second set of conditions requires careful consideration, because it is implicitly imposed by the conditions imposed by Meyer's Properties 5 and 6. As mentioned by Saha [24], the Arrow-Pratt definition of risk aversion severely restricts the relationship between absolute and relative risk aversion. More formally, let $A(\pi)$ and $R(\pi)=\pi A(\pi)$ respectively denote absolute and relative risk aversion. Then the differentiation of $R(\pi)$ yields $R_{\pi}(\pi)=A(\pi)+\pi A_{\pi}(\pi)$. If absolute risk aversion is decreasing (DARA), i. e., $A(\pi)>0$ and $A_{\pi}(\pi)<0$, then the sign of $R_{\pi}(\pi)$ is not determined. In other words, DARA does not restrict the type of relative risk aversion. However, when the absolute risk aversion measure is constant (CARA) or increasing (IARA), i. e., when $A(\pi)>0$ and $A_{\pi}(\pi)\geq 0$, the sign of $R_{\pi}(\pi)$ is restricted to being positive. That is, increasing relative risk aversion (IRRA) is indicated, and decreasing relative risk aversion (DRRA) and CRRA are ruled out. As summarized in Table 1, the combination of absolute and relative risk aversion is uniquely determined, except that relative risk aversion is not restricted under DARA and absolute risk aversion is not restricted under IRRA.[14] Under the EU formulation, attention need not be paid to this relationship, since it is automatically fulfilled in the specification of a von Neumann-Morgenstern utility function. However, its fulfillment is not guaranteed in the MS approach under the LS condition; therefore, this relationship must be explicitly taken into consideration in the specification of the utility function $V(\sigma, \mu)$. For example, since CARA implies IRRA, both of the conditions that Meyer's Properties 5 and 6 impose on $V(\sigma, \mu)$ must be fulfilled simultaneously. Thus, the slope of the indifference curve $S(\sigma, \mu)$

Table 1. Relationships among Arrow-Pratt risk aversion measures (Saha [24])

	DRRA	CRRA	IRRA
DARA	Feasible	Feasible	Feasible
CARA	Infeasible	Infeasible	Feasible
IARA	Infeasible	Infeasible	Feasible

must be increasing along rays through the origin, as well as constant in μ.

The following discussion provides the paper's main results regarding the functional specification of $V(\sigma, \mu)$. These results are consistent with Assumptions 2. 1 (CARA) and 2. 2 (CRRA) under the LS condition. An attempt is then made to derive further results that yield a specification of $S(\sigma, \mu)$ that is linear in parameters. These modifications convert the structural model (5) into a linear model.

1) Constant absolute risk aversion (CARA)

An additively separable and partially linear functional form, such as

$$V(\sigma, \mu) = \alpha\mu + h(\sigma), \tag{7}$$

where α denotes a positive parameter and $h(\sigma)$ denotes a monotonically decreasing and strictly concave function in σ, is appropriate under Assumption 2. 1 and the LS condition (see the Appendix for the derivation of (7)). However, the above functional form (7) may be further restricted so that the slope of the indifference curve $S(\sigma, \mu)$ is linear in parameters. In particular, this study focuses on the following functional form :

$$V(\sigma, \mu) = \mu - \sum_{i=1}^{n} \frac{\beta_i}{i!}\sigma^i, \tag{8}$$

where the $\beta_i (i=1, 2, 3, \cdots, n)$ denote parameters that are restricted to $\beta_i \geq (i=1, 2, 3, \cdots, n)$ and $\beta_i > 0$, for at least one $i (i=2, 3, 4, \cdots, n)$. As may be easily confirmed, this functional form (8) belongs to the family of forms specified in (7), and the slope of its indifference curve, $S(\sigma, \mu) = \sum_{i=1}^{n} \beta_i \sigma^{i-1}/(i-1)!$, is linear in the parameters $\beta_i (i=1, 2, 3, \cdots, n)$ (see the Appendix for the derivation of (8)).

Through a substitution of the slope of this indifference curve for (5), a structural model of random price production may be obtained.

$$\text{Model I :} \quad Y = \eta + \sum_{i=1}^{n} \beta_i X_i + \varepsilon, \tag{9}$$

where $Y = \mu_p q / C(q, \mathbf{w}, \mathbf{k})$ and $X_i = \sigma^i / \{(i-1)! C(q, \mathbf{w}, \mathbf{k})\} (i=1, 2, 3, \cdots, n)$. In Model I, the dependent variable Y and the explanatory variables $X_i (i=1, 2, 3, \cdots, n)$ are observable given actual values of μ_p, σ_p, q and $C(q, \mathbf{w}, \mathbf{k})$. In addition, as has been previously discussed, η is a parameter under Assumption 1. Thus, the linearization of the structural model may be readily achieved under Assumptions 1 and 2. 1.

2) Constant relative risk aversion (CRRA)

Although additive separability and partial linearity play a crucial role in the specification of (7), their functional properties are not applicable under Assumption 2. 2, as described in the following corollary :

Corollary 1 : $V(\sigma, \mu)$ is 1) non-additively separable, 2) nonlinear in μ and σ, and 3) homothetic, if the agent is a risk averter with CRRA under the LS condition (see the Appendix for the proof).

Instead, the third part of the above Corollary 1 is significant here, because a large fraction of the functional forms that have been developed and exploited in economic analyses belong to this homothetic family. Therefore, candidates for the specification of $V(\sigma, \mu)$ can be chosen from this family. For example, consider the constant-elasticity-of-substitution (CES) function. A modified version of the CES function is given by

$$V(\sigma, \mu) = (\mu^{\delta} - \gamma\sigma^{\delta})^{\frac{1}{\delta}}, \tag{10}$$

where γ and δ denote parameters that are restricted as $\gamma > 0$ and $\delta > 1$, and that are appropriate under Assumption 2. 2 and the LS condition.[15, 16] Then, this functional form (10) may be further restricted so as to yield a slope for the indifference curve $S(\sigma, \mu)$ that is linear in parameters. Since its slope derived from (10), $S(\sigma, \mu) = \gamma\mu^{1-\delta}\sigma^{\delta-1}$, is linear in γ but nonlinear in δ, the value of δ may be assigned arbitrarily a priori, as long as $\delta > 1$. This study focuses on the case of $\delta = 2$, which implies

$$V(\sigma, \mu) = (\mu^2 - \gamma\sigma^2)^{\frac{1}{2}}. \tag{10'}$$

Through a substitution of the slope of the indifference curve of (10′), $S(\sigma, \mu) = \gamma\mu^{-1}\sigma$, for (5), another structural model of random price production may be obtained.

$$\text{Model II :} \quad Y = \eta + \gamma Z + \varepsilon, \tag{11}$$

where $Z = \sigma^2 / \mu C(q, \mathbf{w}, \mathbf{k})$. The dependent variable Y and the explanatory variable Z are observable given actual values of μ_p, σ_p, q and

$C(q, \mathbf{w}, \mathbf{k})$. Furthermore, η is a parameter under Assumption 1. Thus, the structural model of random price production likewise may be successfully linearized under Assumptions 1 and 2. 2.

4. Discussion

Models I and II, which are linear structural models of random price production, were developed under the assumptions of CARA and CRRA ; moreover, they have been widely employed in EU-based empirical studies (e. g., Love and Buccola [17] ; Bontems and Thomas [3]). For example, the exponential function (CARA ; Freund [11]) and the power function (CRRA ; Hansen and Singleton [13]) are well-known specifications of von Neumann-Morgenstern utility functions. Compared to EU-based models based on these functional forms, the linear models considered in this study have several practical advantages.

First, they greatly reduce the cost of estimating agent risk preferences regarding random price production, since they do not involve an expectation operator and are linear. Therefore, their estimation requires neither numeric integral calculus nor nonlinear estimation methods. In fact, these models are probably the simplest structural models of random price production that have so far been developed.

Second, these models facilitate the calculation of various useful tools that are commonly employed to analyze agent production behavior, such as the risk premium and the elasticity of supply. The risk premium, which is denoted by ρ_I in Model I and by ρ_{II} in Model II, may be obtained by substituting the functional forms (8) and (10′) for the second term on the left-hand-side of (4) :

$$\rho_I = \sum_{i=1}^{n} \beta_i \frac{\sigma^i}{(i-1)!q} , \tag{12}$$

$$\rho_{II} = \gamma \frac{\sigma^2}{\mu q} . \tag{13}$$

These variables represent marginal willingness-to-pay for the insurance that secures expected profit. In the case of the elasticity of supply, for example, the expected price, price-risk, and safety-income elasticities may be derived from a comparative-static analysis of the models. The expected price elasticity of supply, denoted by ϕ_I in Model I and by ϕ_{II} in Model II, and the price-risk elasticity of supply, denoted by ζ_I in Model I and by ζ_{II} in Model II, may be respectively obtained as :

$$\phi_I = \frac{\mu_p q}{\eta(\eta-1)C(q, \mathbf{w}, \mathbf{k}) + \sum_{i=2}^{n} \beta_i \frac{\sigma^i}{(i-2)!}} , \tag{14-a}$$

$$\zeta_I = -\frac{\sum_{i=1}^{n} \beta_i \frac{i\sigma^i}{(i-1)!}}{\eta(\eta-1)C(q, \mathbf{w}, \mathbf{k}) + \sum_{i=2}^{n} \beta_i \frac{\sigma^i}{(i-2)!}} , \tag{14-b}$$

and

$$\phi_{II} = \frac{\mu_p q (1+\gamma\mu^{-2}\sigma^2)}{\eta(\eta-1)C(q, \mathbf{w}, \mathbf{k}) + \gamma\mu^{-3}\sigma^2(\mu^2-\gamma\sigma^2)} , \tag{15-a}$$

$$\zeta_{II} = -\frac{2\gamma\mu^{-1}\sigma^2}{\eta(\eta-1)C(q, \mathbf{w}, \mathbf{k}) + \gamma\mu^{-3}\sigma^2(\mu^2-\gamma\sigma^2)} . \tag{15-b}$$

Needless to say, these elasticities represent agent supply responses to the mean and the standard deviation of the output price. The safety-income elasticity of supply, denoted by τ_{II}, may be derived from Model II as

$$\tau_{II} = \frac{\gamma\mu^{-2}\sigma^2 L}{\eta(\eta-1)C(q, \mathbf{w}, \mathbf{k}) + \gamma\mu^{-3}\sigma^2(\mu^2-\gamma\sigma^2)} , \tag{15-c}$$

where L is the amount of safety income.[17] The elasticity τ_{II} would be a meaningful measure by which to evaluate the effect of a direct payment policy on supply under price uncertainty. These analytical tools can be calculated easily using the estimates of the parameters η, $\beta_i (i=1, 2, 3, \cdots, n)$, and γ, and the actual values of μ_p, σ_p, q and $C(q, \mathbf{w}, \mathbf{k})$. The simplicity of these calculations contrasts sharply with the complexity of analogous ones in the cases of EU-based structural models, where more complex procedures, such as numerical integral calculation, are commonly required to calculate these measures.

Third, these models make it possible to explain agent attitudes toward price uncertainty geometrically. For example, under the additional assumptions $\beta_i = 0 (i=1, 2, \cdots, k-1, k+1, \cdots, n \ ; \ k \fallingdotseq 1)$, Model I reduces to a two-variable model :

$$\text{Model I}' : \ Y = \eta + \beta_k X_k + \varepsilon \ \ (k=2, 3, 4, \cdots, n). \tag{9′}$$

A scatter plot of (X_k, Y) indicates whether an agent is risk-averse. If X_k is positively correlated with Y (i. e., $\beta_k>0$), then the agent is risk-averse. In contrast, if X_k is not correlated with Y (i. e., $\beta_k=0$), then the agent is risk-neutral, for the reason that the relationship between X_k and Y decides the sign of the slope of the indifference curve upon which Model I′ is based, i. e., $S(\sigma, \mu)=\beta_k \sigma^{k-1}/(k-1)!$. If $\beta_k>0$, then this slope is positive ; when $\beta_k=0$, it is instead equal to zero (Meyer's Property 3). In particular, the case of $k=2$ is noteworthy because, if a positive correlation between X_2 and Y is observed, the angle of the scatter plot can itself be interpreted as a measure of absolute risk aversion. That is, if $k=2$, the utility function (8) upon which Model I′ is based becomes a linear mean-variance model, $V(\sigma, \mu)=\mu-\frac{\beta_2}{2}\sigma^2$; therefore, the parameter β_2 may be interpreted as a measure of absolute risk aversion.[18] Furthermore, the intercept of the Y-axis indicates the existence of economies of scale in production because, as has been already discussed, η represents the reciprocal of the degree of homogeneity of the production function. The judgment criterion in this case is whether the intercept is greater than (diseconomy of scale) or less than (economy of scale) one. Such a geometric analysis would be useful as a preliminary examination of agent production behavior under price uncertainty.

5. Conclusion

This study has considered the linearization of structural models of random price production by exploiting the MS approach under the LS condition ; it has also shown that such linearization can be achieved under widely employed assumptions on production technology (i. e., a homogeneous production function) and on the type of risk aversion (i. e., CARA and CRRA). As has been previously noted, the linear structural models proposed in this study excel, especially in tractability, and they would be useful in performing a practical structural analysis of farmer production behavior under price uncertainty. Despite the fact that this study has focused on the linearization of structural models of random price production, the simplification procedure proposed should be applicable also to other models. For many production theories, the LS condition holds, and the functional specification of the MS approach discussed here would be applicable. An extensive application of this simplification procedure to other models remains for future research.

1) The reduced-form approach, which directly specifies supply and derived demand functions, has been adopted as well (e. g., Chavas and Holt [5]; Pope and Just [22]; Appelbaum and Ullah [1]). Although the approach is convenient for understanding agent responses to exogenous variables, it does not facilitate the derivation of structural parameters indicating attitudes toward risk.

2) Sinn [29] independently studied the economic implications of the LS condition. He referred to a set of random variables for which the LS condition holds as a linear distribution class.

3) The normal distribution condition, which has been often cited as a sufficient condition for the EU and MS approaches to be consistent with one another, is a special case of the LS condition. This relationship is apparent from the fact that normality is preserved under a location and scale shift.

4) Sandmo's [26] random price production theory, the extended theories (e. g., Holthausen [14]; Feder [8]; Feder, Just, and Schmitz [9]), and Tobin's [30] portfolio theory are all prominent examples of frameworks satisfying the LS condition.

5) Sandmo's [26] production model can be interpreted as a model in which the location and scale shift parameters are decided endogenously.

6) Functions with subscripts denote partial derivatives, and all functions are assumed to be differentiable.

7) In proving the necessity of Properties 2 and 4, it is implicitly assumed that the second derivative of the von Neumann-Morgenstern utility function $U_{\pi\pi}(\pi)$ does not change sign depending on the level of the payoff π ; that is, the situation that was discussed by Friedman and Savage [12], namely the coexistence of insurance and a lottery, is ruled out.

8) As in traditional consumer theory, the slope of the indifference curve $S(\sigma, \mu)$ is obtained through the implicit differentiation of $V(\sigma, \mu)$, for $V(\sigma, \mu)=const.$

9) The second-order condition is given by :

$$-C_{qq}(q, \mathbf{w}, \mathbf{k})-\sigma_p\{S_\mu(\sigma, \mu)(\mu_p-C_q(q, \mathbf{w}, \mathbf{k}))+S_\sigma(\sigma, \mu)\sigma_p\}<0.$$

10) Strictly speaking, μ_p and σ_p are exogenous variables in a subjective sense; however, they have been estimated in the past (e. g., Saha,

Shumway, and Talpaz [25]; Chavas and Holt [6]).

11) Strict concavity on the production function ($m<1$) is not imposed here, since such a condition is not always necessary for the existence and uniqueness of an optimal solution in Sandmo's [26] production model.

12) See Chambers [4, pp. 68-77] for details regarding the duality between the production function and the cost function under Assumption 1.

13) Property 3 does not need to be considered in the specification of $V(\sigma, \mu)$, because it automatically applies if Properties 1 and 2 hold.

14) The relationships presented in Table 1 can be explained using a similar discussion, if a given type of relative risk aversion is assumed. For example, in the case of DRRA ($A(\pi)>0$ and $R_\pi(\pi)<0$), $A_\pi(\pi)$ is restricted to being negative, which implies DARA.

15) Although the sign of the parenthetical expression $(\mu^\delta - \gamma\rho^\delta)$ in (10) must be positive, this fact can be checked a posteriori through the use of estimation techniques and actual data.

16) During the refereeing process of this article, Nelson and Escalante [21] modified the linear mean-variance model to $V(\sigma, \mu) = -(\mu^2 - \phi\sigma^2)^{-1}$ ($\phi>0$); moreover, they showed that this model represents CRRA under the LS condition.

17) In the case of CRRA that implies DARA (see Table 1), supply is increasing in safety income; in addition, in the case of CARA, it is independent of safety income. See Sandmo [26][27].

18) See Freund [11] for the derivation of the linear mean-variance model, and for the interpretation of the risk parameter. Nakashima [20] used Model I$'$($k=2$) to analyze Japanese rice production empirically for the period of 1995-1997.

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Appendix

1. Constant Absolute Risk Aversion (CARA)

If the agent is a risk averter of type CARA under the LS condition, then $V(\sigma, \mu)$ must fully meet the following conditions :

$$V_\mu(\sigma, \mu) > 0, \tag{A1}$$
$$V_\sigma(\sigma, \mu) < 0, \tag{A2}$$
$$V_{\mu\mu}(\sigma, \mu) \leq 0, \tag{A3-a}$$
$$V_{\sigma\sigma}(\sigma, \mu) \leq 0, \tag{A3-b}$$
$$V_{\mu\mu}(\sigma, \mu)V_{\sigma\sigma}(\sigma, \mu) - V_{\mu\sigma}^2(\sigma, \mu) \geq 0, \tag{A3-c}$$
$$V_{\mu\sigma}(\sigma, \mu)V_\mu(\sigma, \mu) - V_{\mu\mu}(\sigma, \mu)V_\sigma(\sigma, \mu) = 0, \tag{A4}$$
$$V_{\sigma\sigma}(\sigma, \mu)V_\mu(\sigma, \mu) - V_{\mu\sigma}(\sigma, \mu)V_\sigma(\sigma, \mu) < 0. \tag{A5}$$

Conditions (A1)-(A3), which are at all times imposed when the agent is risk-averse under the LS condition, derive from Properties 1, 2, and 4, respectively. Condition (A4) derives from Property 5 when $S_\mu(\sigma, \mu) = 0$ is expressed in terms of $V(\sigma, \mu)$. Similarly, condition (A5) may be derived from Properties 5 and 6 by expressing $S_t(t\sigma, t\mu) > 0$ in terms of $V(\sigma, \mu)$, setting $t=1$, and then substituting (A4) for the inequality.

The functional specification of $V(\sigma, \mu)$ may be carried out using the signs of the differential coefficients that fully satisfy conditions (A1)-(A5). Although this procedure relies on a trial-and-error method, it allows the objective to be accomplished in three steps. First, a rough outline of $V(\sigma, \mu)$ may be drawn using (A1) and the signs of the derivatives $V_{\mu\mu}(\sigma, \mu) = 0$, $V_{\mu\sigma}(\sigma, \mu) = 0$, and $V_{\sigma\sigma}(\sigma, \mu) < 0$, which satisfy (A3-a), (A3-b), (A3-c), (A4), and (A5). For example, the combination of (A1) and $V_{\mu\mu}(\sigma, \mu) = 0$ indicates that $V(\sigma, \mu)$ is linearly increasing in μ ; in addition, $V_{\mu\sigma}(\sigma, \mu) = 0$ indicates that it is additively separable. These inferences together imply the form :

$$V(\sigma, \mu) = \alpha\mu + h(\sigma), \tag{A6}$$

where α denotes a positive parameter and $h(\sigma)$ denotes some function of σ. Second, form (A6) must be restricted so that it meets the remaining conditions, (A2) and $V_{\sigma\sigma}(\sigma, \mu) < 0$. This step may be easily carried out by imposing the restriction that $h(\sigma)$ be monotonously decreasing and strictly concave. Thus, form (A6), in conjunction with the restrictions $\alpha > 0$, $h_\sigma(\sigma) < 0$, and $h_{\sigma\sigma}(\sigma) < 0$, fully meets conditions (A1)-(A5). Therefore, a functional form belonging to this family represents CARA under the LS condition (e. g., Freund's [11] linear mean-variance model). The third step involves a further restriction of (A6) that yields a slope for the indifference curve $S(\sigma, \mu)$ that is linear in parameters. Since the $S(\sigma, \mu)$ derived from (A6) may be written as

$$S(\sigma, \mu) = -\frac{h_\sigma(\sigma)}{\alpha}, \tag{A7}$$

the desirable result is obtained if an arbitrary number in the range $\alpha > 0$ is selected for the denominator a priori, and if the numerator is specified to be a linear function of its parameters. This study focuses on the case in which $\alpha = 1$ and $h(\sigma)$ is specified to be a polynomial function, so that its first derivative is linear in parameters. Now consider the n^{th}-order Taylor series approximation of $h(\sigma)$. By expanding $h(\sigma)$ and evaluating σ at $\sigma^0=0$, the following function may be obtained :

$$h(\sigma) \cong h(\sigma^0)+\sum_{i=1}^{n}\frac{h^{(i)}(\sigma^0)}{i!}\sigma^i.$$

Defining $h(\sigma^0)=\beta_0$, $h^{(i)}(\sigma^0)=-\beta_i(i=1, 2, 3, \cdots, n)$ then yields

$$h(\sigma) \cong \beta_0-\sum_{i=1}^{n}\frac{\beta_i}{i!}\sigma^i, \tag{A8}$$

where the $\beta_i(i=0, 1, 2, \cdots, n)$ are parameters. Since, as has already been mentioned, $h(\sigma)$ must meet the restrictions $h_\sigma(\sigma)<0$ and $h_{\sigma\sigma}(\sigma)<0$, the parametric restrictions, $\beta_i\geq 0(i=1, 2, 3, \cdots, n)$ and $\beta_i>0$, for at least one $i(i=2, 3, 4, \cdots, n)$, are imposed on (A8). Note also that, as the parameter β_0 does not play an important role, $\beta_0=0$ is assumed a priori. The substitution of (A8), $\alpha=1$, and $\beta_0=0$ for (A6) yields a functional form that is relevant to CARA under the LS condition :

$$V(\sigma, \mu) = \mu-\sum_{i=1}^{n}\frac{\beta_i}{i!}\sigma^i, \tag{A9}$$

where the above-mentioned parametric restrictions on $\beta_i(i=1, 2, 3, \cdots, n)$ are imposed.

2. Constant Relative Risk Aversion (CRRA)

If the agent is a risk averter of type CRRA under the LS condition, then $V(\sigma, \mu)$ must satisfy conditions (A1)-(A3), as well as the following conditions :

$$V_{\mu\sigma}(\sigma, \mu)V_\mu(\sigma, \mu)-V_{\mu\mu}(\sigma, \mu)V_\sigma(\sigma, \mu)>0. \tag{A10}$$

$$\frac{\partial}{\partial t}\left(-\frac{V_\sigma(t\sigma, t\mu)}{V_\mu(t\sigma, t\mu)}\right)=0. \tag{A11}$$

Condition (A10) may be derived from Property 5 by expressing $S_\mu(\sigma, \mu)<0$ in terms of $V(\sigma, \mu)$ (see Table 1). Condition (A11) is merely an alternate expression of the condition $S_t(t\sigma, t\mu)=0$ (Property 6).

As discussed in the text, the three parts of Corollary 1 follow from an examination of these conditions, as follows :

Proof of the first part of Corollary 1 : Suppose that $V(\sigma, \mu)$ is additively separable, i.e., $V_{\mu\sigma}(\sigma, \mu)=0$. Then, condition (A10) reduces to $V_{\mu\mu}(\sigma, \mu)V_\sigma(\sigma, \mu)<0$. This condition reduces further to $V_{\mu\mu}(\sigma, \mu)>0$, as a consequence of condition (A2). However, this inequality contradicts condition (A3-a), and the desired result is therefore obtained. ■

Proof of the second part of Corollary 1 : Suppose that $V(\sigma, \mu)$ is linear in either μ or σ and, thus, that $V_{\mu\mu}(\sigma, \mu)=0$ or $V_{\sigma\sigma}(\sigma, \mu)=0$. Then condition (A3-c) reduces to $V^2_{\mu\sigma}(\sigma, \mu)\leq 0$, which implies $V_{\mu\sigma}(\sigma, \mu)=0$. However, this means that $V(\sigma, \mu)$ is additively separable, which contradicts the first part of Corollary 1. Therefore, the desired result is obtained. ■

Proof of the third part of Corollary 1 : This follows directly from condition (A11) and Lau's [16] Lemma. ■