〔*Jpn. J. Rural Econ.* Vol.12, pp.1-13, 2010〕

# Risk Aversion and Optimal Input Utilization under State Contingent Technology

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The theory of production under uncertainty predicts that, in a single input case, a risk-averse farmer with fair insurance increases fertilizer and decreases pesticide. However, empirical studies do not always support the theoretical predictions. Chambers and Quiggin presented state contingent technology and a method to decompose the difference of optimal revenues between farmers with different risk attitudes to the pure-risk effect and the expansion effect. The theory has potential to explain the ambiguous results in the empirical studies. However, because their analyses only considered risk-averse vs. risk-neutral farmers and assumed some restrictive conditions on the technology, the implication was limited. This paper aims to address these weaknesses. An alternative method of decomposition is introduced to consider the degree of risk-aversion. Local property of marginal revenue-cost function is discussed to examine the sign of expansion effect under more general conditions of technology. This paper provides a theoretical basis for the ambiguity in the empirical studies.

*Key words*: uncertainty, pesticide, fertilizer, state contingent technology, degree of risk-aversion, pure-risk effect, expansion effect, marginal revenue-cost function.

## 1. Introduction

Optimal input utilization under uncertainty has been intensively studied in agricultural economics in the last three decades. It has been well analyzed theoretically that, in a single input case, a risk-averse farmer uses less (more) inputs than a risk neutral farmer if the input increases (decreases) the variability of outputs (e.g., Pope and Kramer [11] and Ramaswami [13]). Accordingly, an input which increases (decreases) the variability of outputs is called a risk-increasing (risk-decreasing) input. Most empirical studies estimating agricultural production technology under uncertainty showed that fertilizer is a risk-increasing input, while pesticide is a risk-decreasing input, which matches with our intuition (see Roumasset et al. [16] for fertilizer and Marra and Carlson [8] for pesticide).

Given the nature of these inputs, the theory predicts that a risk-averse farmer with a fair insurance will increase the utilization of fertilizer, but decrease the utilization of pesticide, because she will behave as a risk-neutral farmer (Nelson and Loehman [10]). However, empirical studies which estimated the effect of insurance on the factor demand of fertilizer and pesticide have not been able to produce clear-cut results as predicted (e.g., Horowitz and Lichtenberg [6] and Smith and Goodwin [17]).

Following them, some studies tried to explain the mixed results theoretically. Loehman and Nelson [7] showed that in the case of multi input production technologies, whether a risk-averse farmer uses more inputs than a risk-neutral farmer depends not only on the effect of input on variability of output but also the

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This research is a partial contribution of Impact of Climate Change of Agricultural Systems in Arid Areas (ICCAP) Project, administered by the Research Institute for Humanity and Nature (RIHN). I am grateful to the editorial board and two anonymous referees of this journal for their valuable comments and suggestions. I would also like to thank Dr. Seiichi Fukui, Dr. Atsuyuki Asami, and Dr. Yukio Muto of Kyoto University for their advice and help during the early stages of this study. Any errors in this paper are my own responsibility.

substitutability or complementarity between inputs. Roosen and Hennessy [14] concluded that additional restrictive conditions on the production technology are necessary to assure that an increase in the degree of risk-aversion reduces (increases) the optimal input utilization.

These studies assumed the stochastic production function (SPF) to represent the production technology under uncertainty. Chambers and Quiggin [4], on the other hand, studied the effect of fair insurance on the revenues and the input use under the state-contingent technology (SCT). They decomposed the effect on the optimal revenues to a part representing the difference of the degrees of riskiness (the pure-risk effect) and a part representing the difference of the expected values (the expansion effect), and showed that even if the pure-risk effect gives an incentive to increase (decrease) the input utilization, the total cannot be always determined: If the expansion effect gives the opposite incentive, the total difference of input utilization is ambiguous.

However, Chambers and Quiggin [4] still have two points of weakness. First, they did not consider the effect of degree of risk-aversion. Most of agricultural policies intended to promote or regulate input uses by mitigating uncertainty do not remove all uncertainty the farmers confront. It is necessary to understand the effect of degree of risk-aversion on the input utilization in order to examine the effect of such policies (Roosen and Hennessy [14]). Second, they assumed only constant absolute riskiness (CAR) technology or constant relative riskiness (CRR) technology, and did not consider if the CAR or CRR restriction was necessary to determine the property of pure-risk effect and expansion effect respectively. As a result, they cannot examine the total difference of input utilization under general condition of technology, and the implication of their theoretical analyses is limited.

This paper extends the results of Chambers and Quiggin [4]. The input utilization of farmers with different degrees of risk-aversion is compared under more general conditions of technology. To this end, first, this paper uses an alternative way to decompose the effect of degree of risk-aversion on the revenues into the pure-risk effect and the expansion effect. Second, instead of imposing CAR or CRR, this paper examines the local property of technology. The marginal revenue-cost function with a certain increment of revenues (denoted by function $D$) is classified to risk substitute and risk complement near the optimum. We will see that the decomposition works well when the effect of degree of risk-aversion is considered. The property of pure-risk effect does not depend on the technology considered in this paper. Consequently, the sign of incentive which the pure-risk effect gives for the input use depends on the type of input (whether it is a risk substitute or risk complement) at the optimal. In contrast, the property of expansion effect depends on the technology (whether $D$ is risk substitute or risk complement) at the optimal. Consequently, the sign of incentive which the expansion effect gives for the input use depends on the type of technology and the type of input (whether it is regressive or non-regressive) at the optimal. Because the total effect is the sum of pure-risk effect and expansion effect, the sign cannot be always determined. Therefore, this paper provides a theoretical explanation of the ambiguity of the effect of risk on input utilization.

This paper starts the following sections from the explanation of the setting of the analyses.

## 2. Setting

We consider farmers who produce products of $M$ kinds using inputs of $N$ kinds. The farmers face states of nature of $S$ kinds, which affect productivity and price of the products (e.g., precipitation, incidence of pests, depression, and etc.). A vector $\mathbf{x} \in \mathcal{R}_+^N$ denotes the bundle of inputs committed before the realization of a state of nature. Therefore, the states of nature represent the production and price uncertainty. A SCT transforms $\mathbf{x}$ into a matrix of state-contingent outputs $\mathbf{z} \in \mathcal{R}_+^{M\times S}$ of which each column vector $\mathbf{z}_i \in \mathcal{R}_+^M$ shows the outputs that would be produced if state of nature $i$ ($1 \le i \le S$) were to occur. Consequently, $\mathbf{z}$ represents the potential output the farmer might obtain through her management. From Chambers and Quiggin [3], the SCT is defined by the input set:

$$X(\mathbf{z}) = \{\mathbf{x} \in \mathcal{R}_+^N : \mathbf{x} \text{ can produce } \mathbf{z} \in \mathcal{R}_+^{M\times S}\}.$$

We denote the matrix of the product prices by $\mathbf{q} \in \mathcal{R}_{++}^{M\times S}$. Each column vector $\mathbf{q}_i \in \mathcal{R}_{++}^M$ shows the bundle of product prices that would be realized if state of nature $i$ were to occur. The state-contingent revenue vector $\mathbf{r} \in \mathcal{R}_+^S$ can be defined as the diagonal elements of $\mathbf{q}'\mathbf{z}$.

The element $r_i$ of $\mathbf{r}$ shows the revenue that the farmer would obtain if state of nature $i$ were to occur. We assume that the prices of all inputs $\mathbf{w} \in \mathcal{R}^N_{++}$ are given before the farmer decides her management. In a multi-products case, the concept of state-contingent revenue $\mathbf{r}$ reduces the dimension of state-contingent outputs $\mathbf{z}$ (i.e. $\mathcal{R}^{M\times S}_{+}$ to $\mathcal{R}^S_+$). The concept allows us to treat simultaneously the production uncertainty and the price uncertainty.

Chambers and Quiggin [3] showed that if the input set satisfies some properties—($X(\mathbf{z})$ is closed for all $\mathbf{z} \in \mathcal{R}^{M\times S}_+$, $\mathbf{0}_N \notin X(\mathbf{z})$ for $\mathbf{z} \geq \mathbf{0}_M$ and $\mathbf{z} \neq \mathbf{0}_M$, and $X(\mathbf{0}_M) = \mathcal{R}^N_+$)—then duality between the output side and input side exists. If so, the revenue-cost function, which is consistent with preference of the risk-averse farmer, can be drawn (Chambers and Quiggin[3, pp.143–145 and pp.165–166]).[1)] The revenue-cost function under the SCT is defined as

$$C(\mathbf{w},\mathbf{r},\mathbf{q}) = \min\left\{\mathbf{w}'\mathbf{x} : \ \mathbf{x} \in X(\mathbf{z}), \ \mathbf{q}_i'\mathbf{z}_i \geq r_i \text{ for all } 1 \leq i \leq S\right\}.$$

Because the revenue-cost function has similar properties to a multi-products cost function under certainty such as convexity in $\mathbf{r}$, the details are omitted (see Chambers and Quiggin[3, p. 166]). However, we emphasize the fact that free disposability of outputs (i.e. $\mathbf{z}^o \geq \mathbf{z} \Rightarrow X(\mathbf{z}) \subseteq X(\mathbf{z}^o)$ ) is necessary to assure the marginal cost of all state-contingent revenues be nonnegative (i.e. $\mathbf{r}' \geq \mathbf{r} \Rightarrow C(\mathbf{w},\mathbf{r}',\mathbf{q}) \geq C(\mathbf{w},\mathbf{r},\mathbf{q})$) (Assumption 1). Also, we assume that $C(\mathbf{w},\mathbf{r},\mathbf{q})$ is smoothly differentiable in all state-contingent revenues (Assumption 2).

We compare the input utilization between farmers who differ in their degrees of risk aversion. We consider farmer $A$ and farmer $B$: both are risk-averse, but farmer $B$ is more risk-averse than farmer $A$. The preferences of farmer $A$ and $B$ are respectively represented by $U(y)$ and $W(y)$, where $y$ denotes the profit the farmer gets. We assume that $U(y)$ and $W(y)$ are von Neumann-Morgenstern type utility functions (Assumption 3): both functions are positive, non-decreasing, and concave ($U'(y) \geq 0 \geq U''(y)$ and $W'(y) \geq 0 \geq W''(y)$). Therefore, this paper relies on the expected utility theory. From Mas-Colell, Whinston, and Green [9] (Proposition 6.C.2), the utility function $W(y)$ is more risk-averse than $U(y)$, if and only if there exists a positive, concave, and non-decreasing translation function $V[\cdot]$ that satisfies $W(y) = V[U(y)]$. We assume that such a function $V[\cdot]$ exists (Assumption 4).

The expected utility theory is restrictive because the preference does not allow the dependency of welfare between states of nature (Chambers and Quiggin [3]). Indeed, Chambers and Quiggin [4] assumed the generalized Schur-concave preference which allows the dependency. Recently, Chambers and Quiggin [5] presented some measures of degree of risk-aversion under the generalized preference structures. However, any useful properties of the preference such as the transformation function $V[\cdot]$ under expected utility, which globally assure that farmer $B$ is more risk-averse than farmer $A$, could not be derived. Because of this, this paper relies on the expected utility. However, when we consider risk-averse vs. risk-neutral, all discussions in this paper are maintained even if the generalized Schur-concave preference is assumed.

We assume that both farmers have the same subjective probability of the realization of each state of nature, as denoted by $p_i$ (Assumption 5). Then, both farmers calculate the expected value of the risky profile (utility, profit, and revenue) based on the same probability.

The optimal revenue profile and optimal profit profile of farmer $A$ are denoted respectively as $\mathbf{r}^A = (r^A_1,\ldots,r^A_S)'$ and $\mathbf{y}^A = (y^A_1,\ldots,y^A_S)'$. Similarly, $\mathbf{r}^B = (r^B_1,\ldots,r^B_S)'$ and $\mathbf{y}^B = (y^B_1,\ldots,y^B_S)'$ denote the optimal revenue profile and optimal profit profile of farmer $B$, respectively. We assume that both farmers have equal recognition of the goodness or badness of the state of nature (Assumption 6). If so, the states of nature can be ordered consistently according to their badness. Therefore, we limit the analyses in the case that $r_i \leq r_j$ and $y_i \leq y_j$ for all $i < j \leq S$ are satisfied, unless otherwise noted.[2)] The optimal revenue profile of farmer $A$ is given by solving the following problem:

$$\max_{\mathbf{r}} E[U(y)] = \sum_{i=1}^{S} p_i U\left(r_i - C(\mathbf{w},\mathbf{r},\mathbf{q})\right), \quad (1)$$

where $E[\cdot]$ denotes the operator of expectation. Consequently, if the problem described above has interior solutions, the first-order conditions are written as

$$p_i U'(y_i^A) - \frac{\partial C(\mathbf{w}, \mathbf{r}^A, \mathbf{q})}{\partial r_i} \sum_{j=1}^{S} p_j U'(y_j^A)$$
$$= 0, \quad \text{for all } 1 \leq i \leq S. \tag{2}$$

By summing up the first-order conditions for all $i$, we obtain

$$1 - \sum_{i=1}^{S} \frac{\partial C(\mathbf{w}, \mathbf{r}^A, \mathbf{q})}{\partial r_i} = 0. \tag{3}$$

The left-hand side of this equation shows the marginal change of her profit by adding identical and small amounts of revenue to all elements of $\mathbf{r}^A$. Consequently, eq.(3) shows that a risk-averse farmer has no possibility of expanding her expected profit with certainty. For that reason, Chambers and Quiggin [3] referred to eq.(3) as an arbitrage condition. Apparently, farmer $B$ should also satisfy the arbitrage condition at her optimal output profile (substitute $\mathbf{r}^A$ with $\mathbf{r}^B$ in eq.(3)). The arbitrage condition includes no terms that directly express the preference structure of the risk-averse farmer (i.e., $U(y)$ and $W(y)$). Therefore, all risk-averse farmers are assumed to have similar criteria for efficiency of revenue-cost. Indeed, all risk-averse farmers are as efficient in this sense as a risk-neutral farmer (see Chambers and Quiggin[3, p. 167]).

## 3. Analyses

### 1) Decomposition of the difference of revenue and profit profiles

The revenue-cost function under SCT satisfies Shephard's lemma (Chambers and Quiggin[3, p. 166]). Therefore, the factor demand function for input $k$ to produce revenue profile $\mathbf{r}$ at minimized cost is given as

$$x_k^*(\mathbf{w}, \mathbf{r}, \mathbf{q}) = \frac{\partial C(\mathbf{w}, \mathbf{r}, \mathbf{q})}{\partial w_k}.$$

The factor demand $x_k^*(\mathbf{w}, \mathbf{r}, \mathbf{q})$ depends only on the revenue vector $\mathbf{r}$ if all factor and product prices remain constant. Then, the difference of factor demand for input $k$ between farmer $B$ and farmer $A$, $x_k^*(\mathbf{w}, \mathbf{r}^B, \mathbf{q}) - x_k^*(\mathbf{w}, \mathbf{r}^A, \mathbf{q})$, will be determined by the form of factor demand function and the difference of the optimal revenue vector between them.

When farmer $A$ and farmer $B$ differ in the degree of risk aversion, their optimal profit (revenue) profiles will differ in degree of riskiness. As described in Rothschild and Stiglitz [15], the degree of riskiness of a risky profile is appropriately defined by comparison to another risky profile with the same expected value. The optimal production plans of farmer $A$ and $B$, however, generally result in different levels of expected revenue and profit as well as the degree of riskiness. Therefore, we cannot directly compare the degree of riskiness of their optimal profit profiles or revenue profiles.

Chambers and Quiggin [4] suggested that, by introducing an appropriate revenue vector $\tilde{\mathbf{r}}$ that has the same expected value as the optimal revenue vector of farmer $B$ and which reflects the degree of riskiness of the optimal revenue vector of farmer $A$, we decompose the difference of the optimal revenue vectors between them, $\mathbf{r}^B - \mathbf{r}^A$, to two parts: $\tilde{\mathbf{r}} - \mathbf{r}^A$ and $\mathbf{r}^B - \tilde{\mathbf{r}}$. Because $\mathbf{r}^B$ and $\tilde{\mathbf{r}}$ have the same expected value, if $\tilde{\mathbf{r}}$ correctly reflects the degree of riskiness of $\mathbf{r}^A$, the part $\mathbf{r}^B - \tilde{\mathbf{r}}$ may purely represent how the optimal revenue profiles of farmer $A$ and $B$ differ in the degree of riskiness. Chambers and Quiggin [4] referred to $\mathbf{r}^B - \tilde{\mathbf{r}}$ as the pure-risk effect. On the other hand, the part $\tilde{\mathbf{r}} - \mathbf{r}^A$ may represent the difference of expected values of the optimal revenue profiles. Chambers and Quiggin [4] referred to $\tilde{\mathbf{r}} - \mathbf{r}^A$ as the expansion effect. As described above, because the factor demand depends only on the revenue vector, we get

$$\begin{aligned} & x_k^*(\mathbf{w}, \mathbf{r}^B, \mathbf{q}) - x_k^*(\mathbf{w}, \mathbf{r}^A, \mathbf{q}) \\ & \quad = \left\{ x_k^*(\mathbf{w}, \mathbf{r}^B, \mathbf{q}) - x_k^*(\mathbf{w}, \tilde{\mathbf{r}}, \mathbf{q}) \right\} \\ & \quad + \left\{ x_k^*(\mathbf{w}, \tilde{\mathbf{r}}, \mathbf{q}) - x_k^*(\mathbf{w}, \mathbf{r}^A, \mathbf{q}) \right\}. \end{aligned} \tag{4}$$

The difference of input uses between farmer $A$ and $B$ can also be decomposed to the part associated with the pure-risk effect, $x_k^*(\mathbf{w}, \mathbf{r}^B, \mathbf{q}) - x_k^*(\mathbf{w}, \tilde{\mathbf{r}}, \mathbf{q})$, and the part associated with the expansion effect, $x_k^*(\mathbf{w}, \tilde{\mathbf{r}}, \mathbf{q}) - x_k^*(\mathbf{w}, \mathbf{r}^A, \mathbf{q})$. When each part has the same sign, each part gives the same incentive for input utilization, and we can determine which farmer uses more input, farmer $A$ or farmer $B$.

The remaining problem is what revenue vector should be defined as $\tilde{\mathbf{r}}$. Chambers and Quiggin [4] presented two candidates, although they suggested that there are infinite methods of decomposition. The left-hand-side of Fig.1 illustrates the two candidates in the case of two states of nature, where a revenue vector on the bisector gives the same revenue to all states of nature, a revenue vector on the fair-odds line has the same expected value as a reference rev-

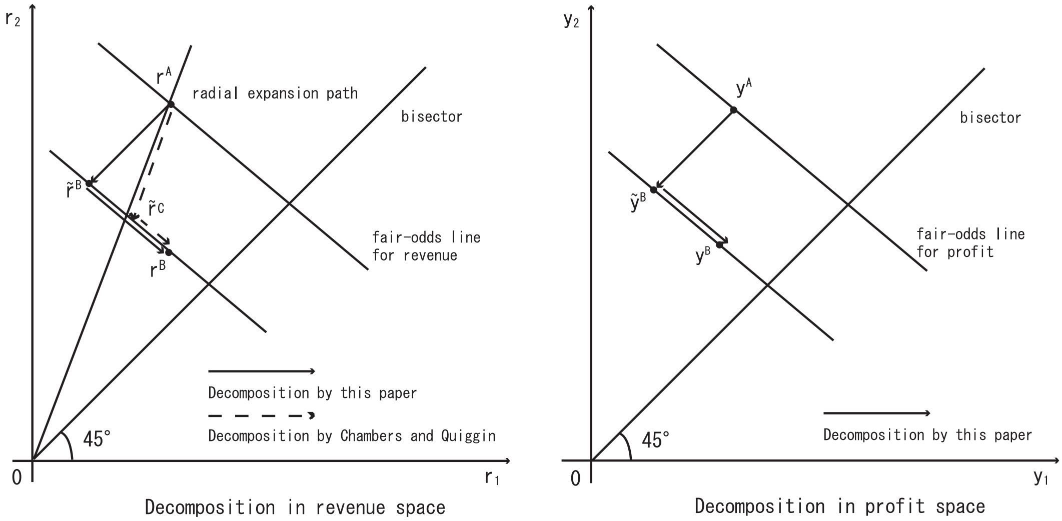


**Figure 1. Decomposition in revenue and profit space**

enue vector (e.g. $\mathbf{r}^B$), and a revenue vector on the radial expansion path is a proportional expansion or a contradiction of a reference vector (e.g. $\mathbf{r}^A$). The first candidate is the intersection point (denoted by $\tilde{\mathbf{r}}^C$) of the radial expansion path from $\mathbf{r}^A$, with the fair odds line for $\mathbf{r}^B$. The second candidate is the intersection point (denoted by $\tilde{\mathbf{r}}^B$) of the line which is parallel to the bisector and passes through $\mathbf{r}^A$, with the fair odds line for $\mathbf{r}^B$. Let $\Delta\mathbf{r}^B = (\Delta r_1^B, \ldots, \Delta r_S^B)'$ denote the difference of the optimal revenue vector between farmer $A$ and farmer $B$ ($\mathbf{r}^B - \mathbf{r}^A = \Delta\mathbf{r}^B$). Each intersection point $\tilde{\mathbf{r}}^C$ and $\tilde{\mathbf{r}}^B$ can be written as

$$\tilde{\mathbf{r}}^C = \mathbf{r}^A + \frac{\mathbf{r}^A}{E[r^A]}E[\Delta r^B], \text{ and} \qquad (5)$$

$$\tilde{\mathbf{r}}^B = \mathbf{r}^A + E[\Delta r^B]\mathbf{1}_S, \qquad (6)$$

where $E[\Delta r^B] = \sum_{i=1}^S p_i \Delta r_i^B$ and $E[r^A] = \sum_{i=1}^S p_i r_i^A$, and where $\mathbf{1}_S$ denotes $S$ dimensional vector with all elements equal to unity.

Chambers and Quiggin [4] adopted the first candidate for additional analyses, because they thought the first candiate would be most familiar to most economists. Note that $\sum_{i=1}^S p_i r_i^A / E[r^A] = 1$. Hence, eq.(5) says that $r_i^A / E[r^A]$ can be interpreted as the weight of each state of nature to obtain the intersection point of the first candidate. The first candidate requires weighting each state of nature depending on the optimal revenue profile of farmer $A$. In contrast, the weight to obtain the intersection point of the second candidate depends only on the probability of each state of nature (i.e. the weight is independent of the behavior of farmer $A$). This property of the second candidate eases the following analyses. Furthermore, the second candidate can be easily applied to the comparative statics that are familiar in the SPF literature (e.g. Loehman and Nelson [7] and Ramaswami [13]).[3)] For these reasons, this paper uses the second candidate for additional analyses.

Chambers and Quiggin [4] defined only the decomposition in revenue space. The difference of optimal revenue profiles between farmer $A$ and $B$ originates from their difference of preference on profit profiles. To clarify the difference of revenue profiles between farmer $A$ and $B$, at first, it is useful to examine the difference of optimal profit profiles between them. In addition to the decomposition in revenue space, we define $\tilde{\mathbf{y}}^B$ as the intersection point of the line which is parallel to the bisector and passes through $\mathbf{y}^A$, with the fair odds line for $\mathbf{y}^B$. Then, we decompose the difference of profit profiles between farmer $A$ and $B$, $\mathbf{y}^B - \mathbf{y}^A$, to the pure-risk effect in profit space, $\mathbf{y}^B - \tilde{\mathbf{y}}^B$, and the expansion effect in profit space, $\tilde{\mathbf{y}}^B - \mathbf{y}^A$. The right side of Fig.1 illustrates this decomposition in profit space in the case of two states of nature. Let define $\Delta\mathbf{y}^B = \mathbf{y}^B - \mathbf{y}^A$. From the definition of $\tilde{\mathbf{y}}^B$, we obtain

$$\tilde{\mathbf{y}}^B = \mathbf{y}^A + E[\Delta y^B]\mathbf{1}_S. \qquad (7)$$

### 2) Properties of the pure-risk effect and the expansion effect in profit space

Before the analyses of the pure-risk effect and expansion effect in revenue space, the statements on the properties of the expansion effect and the pure-risk effect in profit space are presented. The properties play an important role in analyzing the pure-risk effect and expansion effect in revenue space. Let $\preceq_\pi$ denote risk ordering in the sense of Rothschild and Stiglitz [15].[4)] Consequently, risk ordering $\mathbf{y}^o \preceq_\pi \mathbf{y}'$ means that $\mathbf{y}^o$ is less risky than $\mathbf{y}'$, where $\mathbf{y}^o$ and $\mathbf{y}'$ have the same expected value. In addition, $\mathbf{y}^o \preceq_\pi \mathbf{y}'$ is defined if and only if $E[U(y^o)] \geq E[U(y')]$ is satisfied for all concave and non-decreasing expected utility functions. Under Assumption 1 to 6, we can derive the properties of the expansion effect in profit space, $\tilde{\mathbf{y}}^B - \mathbf{y}^A$, and the pure-risk effect in profit space, $\mathbf{y}^B - \tilde{\mathbf{y}}^B$.

**Proposition 1** Under Assumption 1 to 6, the expansion effect in profit space is non-positive:

$$\tilde{\mathbf{y}}^B - \mathbf{y}^A = E[\Delta y^B]\mathbf{1}_M \leq \mathbf{0}_M.$$

**Proof** See Appendix.

**Proposition 2** Under Assumption 1 to 6, the degree of riskiness of the profit profile $\mathbf{y}^B$ is lower than that of the profit profile $\tilde{\mathbf{y}}^B$:

$$\mathbf{y}^B \preceq_\pi \tilde{\mathbf{y}}^B.$$

**Proof** Proposition 1 and free disposability of outputs assure that the profit profile $\tilde{\mathbf{y}}^B$ is always producible by disposing equivalent amount of revenue to $-E[\Delta y^B]$ from all elements of $\mathbf{y}^A$. Farmer $B$ maximizes her expected utility $E[W(y)]$. Then, the expected utility at her optimal profit profile $\mathbf{y}^B$ is expected to be higher than that of $\tilde{\mathbf{y}}^B$, that is, $E[W(y^B)] \geq E[W(\tilde{y}^B)]$. Furthermore, by the definition of $\tilde{\mathbf{y}}^B$, $\mathbf{y}^B$ and $\tilde{\mathbf{y}}^B$ have the same expected value.□

### 3) Properties of the pure-risk effect and the expansion effect in revenue space

Here, we examine the pure-risk effect and the expansion effect in revenue space. The difference of optimal profit profile between farmer $B$ and farmer $A$ is given by

$$\Delta\mathbf{y}^B = (\mathbf{r}^B - \mathbf{r}^A) - \left\{C(\mathbf{w}, \mathbf{r}^B, \mathbf{q}) - C(\mathbf{w}, \mathbf{r}^A, \mathbf{q})\right\}\mathbf{1}_S.$$

Therefore, from eq.(6) and (7), $\tilde{\mathbf{y}}^B$ is expressible as

$$\tilde{\mathbf{y}}^B = \tilde{\mathbf{r}}^B - C(\mathbf{w}, \mathbf{r}^B, \mathbf{q})\mathbf{1}_S.$$

Using this equation, the property of the pure-risk effect in profit space (Proposition 2) is rewritten as

$$\mathbf{r}^B - C(\mathbf{w}, \mathbf{r}^B, \mathbf{q})\mathbf{1}_S \preceq_\pi \tilde{\mathbf{r}}^B - C(\mathbf{w}, \mathbf{r}^B, \mathbf{q})\mathbf{1}_S.$$

Because the risk-ordering $\preceq_\pi$ has translation invariance and positive homogeneity (Artzner et al. [1]), we can clarify the pure-risk effect in revenue space, i.e.,

$$\mathbf{r}^B \preceq_\pi \tilde{\mathbf{r}}^B. \tag{8}$$

The pure-risk effect in revenue space has the same property as the pure-risk effect in profit space: $\mathbf{r}^B$ is less risky than $\tilde{\mathbf{r}}^B$. This means that a more risk-averse farmer always has an incentive to decrease the riskiness of the optimal revenue profile of a less risk-averse farmer. Chambers and Quiggin [4] proved this property by comparing risk-averse and risk-neutral farmers and by assuming CAR and CRR. We can derive the same properties by comparing farmers with different degrees of risk aversion. This property is maintained regardless of the structure of technology, if both farmers have equal recognition of the badness of the state of nature (Assumption 6).

Next, we consider the expansion effect in revenue space. Proposition 1 says that the expansion effect in profit space, $\tilde{\mathbf{y}}^B - \mathbf{y}^A$, is non-positive. From this property, however, nothing can be inferred about the property of the expansion effect in revenue space. Nor did Chambers and Quiggin [4] give any conditions of SCT to identify the sign of the expansion effect in revenue space.

Recall that all farmers under SCT satisfy the arbitrage condition, $1 - \sum_{i=1}^S \partial C(\mathbf{w}, \mathbf{r}, \mathbf{q})/\partial r_i = 0$ by their optimum production plans. By defining a function $D(\mathbf{w}, \mathbf{r}, \mathbf{q}) = \sum_{i=1}^S \partial C(\mathbf{w}, \mathbf{r}, \mathbf{q})/\partial r_i$, we get

$$D(\mathbf{w}, \mathbf{r}^B, \mathbf{q}) - D(\mathbf{w}, \mathbf{r}^A, \mathbf{q}) = 0. \tag{9}$$

The function $D(\mathbf{w}, \mathbf{r}, \mathbf{q})$ refers to the marginal revenue-cost when all revenues equally increase (i.e., a revenue the farmer gets is incremented certainly regardless of the realized state of nature). Therefore, this paper calls $D(\mathbf{w}, \mathbf{r}, \mathbf{q})$ a marginal revenue-cost function. However, please note that the function is not the marginal revenue-cost with respect to an increase of revenue corresponding to one state of nature. By using this method to decompose the difference of the optimal revenue profiles to the pure-risk effect and the expansion effect, eq.(9) can also be decomposed to

$$\left\{D(\mathbf{w}, \mathbf{r}^B, \mathbf{q}) - D(\mathbf{w}, \tilde{\mathbf{r}}^B, \mathbf{q})\right\} + \left\{D(\mathbf{w}, \tilde{\mathbf{r}}^B, \mathbf{q}) - D(\mathbf{w}, \mathbf{r}^A, \mathbf{q})\right\} = 0. \quad (10)$$

This equation states that the change of marginal-revenue cost associated with expansion effect in revenue space ($D(\mathbf{w}, \tilde{\mathbf{r}}^B, \mathbf{q}) - D(\mathbf{w}, \mathbf{r}^A, \mathbf{q})$) is expected to be balanced out by the change of marginal revenue-cost associated with the pure-risk effect in revenue space ($D(\mathbf{w}, \mathbf{r}^B, \mathbf{q}) - D(\mathbf{w}, \tilde{\mathbf{r}}^B, \mathbf{q})$). Here, we introduce a definition of risk substitute and risk complement technologies based on the properties of marginal revenue-cost function.

**Definition 1** For an ordering of riskiness $\preceq_\pi$, marginal revenue-cost function is a risk substitute (complement) at $\mathbf{r}^o$ if $\mathbf{r}^o \preceq_\pi \mathbf{r}' \Rightarrow D(\mathbf{w}, \mathbf{r}', \mathbf{q}) \leq D(\mathbf{w}, \mathbf{r}^o, \mathbf{q})$ $(D(\mathbf{w}, \mathbf{r}', \mathbf{q}) \geq D(\mathbf{w}, \mathbf{r}^o, \mathbf{q}))$.

If revenue-cost function is twice differentiable, the following lemma can be derived.[5)]

**Lemma 1** marginal revenue-cost function is a risk substitute at $\mathbf{r}^o$ only if

$$\frac{1}{p_i}\frac{\partial D(\mathbf{w}, \mathbf{r}, \mathbf{q})}{\partial r_i} - \frac{1}{p_j}\frac{\partial D(\mathbf{w}, \mathbf{r}, \mathbf{q})}{\partial r_j} \geq 0 \quad \text{for all } i < j \leq S.$$

**Lemma 2** marginal revenue-cost function is a risk complement at $\mathbf{r}^o$ only if

$$\frac{1}{p_i}\frac{\partial D(\mathbf{w}, \mathbf{r}, \mathbf{q})}{\partial r_i} - \frac{1}{p_j}\frac{\partial D(\mathbf{w}, \mathbf{r}, \mathbf{q})}{\partial r_j} \leq 0 \quad \text{for all } i < j \leq S.$$

Suppose that a farmer plans to move from a high risk revenue profile to a less risky one. The farmer will increase the revenues in bad states of nature as well as decrease the revenues in good states of nature, while keeping the expected revenue constant. By doing so, the farmer receives higher (lower) marginal revenue-cost, if the marginal revenue-cost function satisfies the risk substitute (complement). In this sense, the property of marginal revenue-cost function defines the relation between the technology and uncertainty. Lemma 1 and 2 say that if the response of marginal revenue-cost to an increase of the revenue in a relatively bad state of nature divided by the probability of the state is always larger (smaller) than the response in relatively good state of nature, such technology is a risk substitute (complement). The risk substitute (complement) technology in Definition 1 requires that the sum of the increases of marginal revenue-cost with the increased revenues in bad states is larger (smaller) than sum of the decreases of it with the decreased revenues in good states (i.e. the marginal revenue-cost function is elastic in bad (good) states of nature, but inelastic in good (bad) states of nature). This requirement for the risk substitute (complement) technology will be satisfied if Lemma 1 (Lemma 2) is satisfied.

Combining eq.(10) and Definition 1, we can derive the property of the expansion effect in revenue space.

**Proposition 3** Under Assumption 1 to 6, the expansion effect in revenue space is non-positive (non-negative) if marginal revenue-cost function is a risk substitute (a risk complement) at $\mathbf{r}^B$.

**Proof** Because the pure-risk effect in revenue space satisfies $\mathbf{r}^B \preceq_\pi \tilde{\mathbf{r}}^B$, Definition 1 says that $D(\mathbf{w}, \mathbf{r}^B, \mathbf{q}) - D(\mathbf{w}, \tilde{\mathbf{r}}^B, \mathbf{q})$ is non-positive (non-negative) if $D(\mathbf{w}, \mathbf{r}, \mathbf{q})$ is a risk complement (risk substitute) at $\mathbf{r}^B$. From eq.(10), we can see that $D(\mathbf{w}, \tilde{\mathbf{r}}^B, \mathbf{q}) - D(\mathbf{w}, \mathbf{r}^A, \mathbf{q})$ is non-negative (non-positive) if $D(\mathbf{w}, \mathbf{r}, \mathbf{q})$ is a risk complement (risk substitute) at $\mathbf{r}^B$. Recall that revenue vector $\tilde{\mathbf{r}}^B$ is derived by adding $E[\Delta r^B]$ to all elements of $\mathbf{r}^A$ (eq.(6)) and that $D(\mathbf{w}, \mathbf{r}, \mathbf{q})$ shows the marginal change of revenue-cost when the same marginal revenue is added to all elements of the revenue profile. The expansion effect in revenue space, $\tilde{\mathbf{r}}^B - \mathbf{r}^A$, has the same sign as $D(\mathbf{w}, \tilde{\mathbf{r}}^B, \mathbf{q}) - D(\mathbf{w}, \mathbf{r}^A, \mathbf{q})$ because, if not, farmer $A$ would be able to expand her profit by producing the revenue profile $\tilde{\mathbf{r}}^B$.□

Propositions 1 and 2 say that a more risk-averse farmer always decreases the expected profit of a less risk-averse farmer in order to decrease the riskiness of the profit profile. Therefore, a more risk-averse farmer always confronts the tradeoff between the expected profit and the degree of riskiness. Eq.(8) and Proposition 3 say that, in the case of the pure-risk effect and the expansion effect in revenue space, she might not confront such a tradeoff. Therefore, she might expand expected revenue together with a decreased degree of riskiness. Such a possibility disappears (appears with certainty) if the

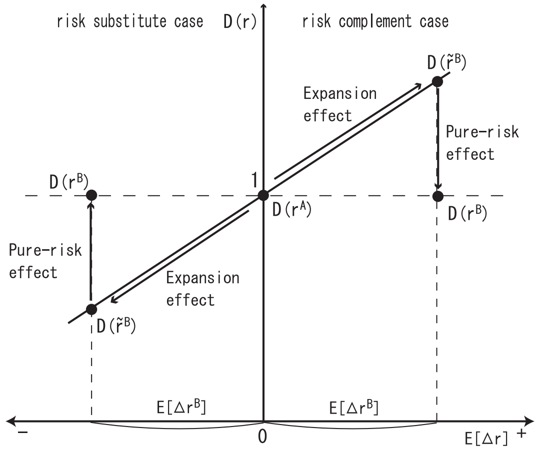


**Figure 2. Restoration of arbitrage condition**

marginal revenue-cost function is a risk substitute (complement) near the optimum.

The CAR technology assures the negative expansion effect in the global space of revenue (Chambers and Quiggin [4]). Proposition 3 says that, when CAR restriction is relaxed, the sign of expansion effect in revenue space is determined by the structure of technology (marginal revenue-cost function) near the optimum. Farmer $B$ always has an incentive to decrease the riskiness of the revenue profile of farmer $A$. To do so, however, the arbitrage condition is violated (i.e. the farmer loses cost efficiency). If the marginal revenue-cost function is a risk substitute (complement), a decrease in the riskiness of the revenue profile increases (decreases) the value of the marginal revenue-cost function. To keep the arbitrage condition, farmer $B$ should decrease (increase) the expected revenue. Hence, the expansion effect in revenue space becomes non-positive (non-negative). Fig.2 illustrates this process of restoration of the arbitrage condition by farmer $B$. In other words, if the marginal revenue-cost function is a risk substitute (complement), farmer $B$ has an incentive to decrease (increase) the expected revenue in order to decrease the riskiness of the revenue profile.

Because Chambers and Quiggin [4] did not analyze the property of expansion effect in local space, they required CAR restriction on the technology. In contrast, Definition 1 is given in the local space of revenue. The sign of the expansion effect in revenue space can be examined by testing whether the SCT satisfies Lemma 1 or Lemma 2 near the optimum. Because of this, we can analyze the effect of risk-aversion on input use, even if the sign of expansion effect is not defined in global space.

### 4) Effect of degree of risk-aversion on input utilization

Now, we can analyze the effect of the degree of risk-aversion on the input utilization. First, we discuss the incentives for the input utilization which the pure-risk effect and the expansion effect in revenue space give to a more risk-averse farmer. Then, by combining the results, the effect of degree of risk-aversion on the input utilization is examined.

We start from the discussion about the incentives that the pure-risk effect gives. Because the property of the pure-risk effect in revenue space is equivalent to Chambers and Quiggin [4] regardless to the structure of technology, the incentives that the pure-risk effect gives can be also examined by using their result. Chambers and Quiggin[3,4] defined a risk complement (risk substitute) input by comparing factor demand for input $k$ to produce $\mathbf{r}^o$ and $\mathbf{r}'$, the riskiness of which is ordered by $\mathbf{r}^o \preceq_\pi \mathbf{r}'$.

**Definition 2(Chambers and Quiggin [3], Definition 4.1)** For an ordering of riskiness $\preceq_\pi$, input $k$ is a risk complement (risk substitute) at $\mathbf{r}^o$ if $\mathbf{r}^o \preceq_\pi \mathbf{r}' \Rightarrow x_k^*(\mathbf{w}, \mathbf{r}', \mathbf{q}) \geq x_k^*(\mathbf{w}, \mathbf{r}^o, \mathbf{q})$ $(x_k^*(\mathbf{w}, \mathbf{r}', \mathbf{q}) \leq x_k^*(\mathbf{w}, \mathbf{r}^o, \mathbf{q}))$.

An increase of the riskiness of the revenue profile increases (decreases) factor demand of input $k$, if the input is a risk complement (substitute). In other words, by using more risk complement (substitute) input, the riskiness of the revenue profile increases (decreases). Because a more risk-averse farmer decreases the riskiness of the revenue profile of a less risk-averse farmer, the more risk-averse farmer uses less (more) input for which the demand decreases (increases) with a decrease of the riskiness of the revenue profile. From eq.(8) and Definition 2, we can derive the following result.

**Result 1** Under Assumption 1 to 6, the pure-risk effect in revenue space gives a more risk-averse farmer an incentive to decrease (increase) the optimal input utilization of a less risk-averse farmer, if input $k$ is a risk complement (risk substitute) at $\mathbf{r}^B$.

Next, we examine the difference of input utilization associated with the expansion effect in revenue space. When the technology does not define the sign of expansion effect in the global space of revenue, the sign depends on the structure of technology (i.e. marginal revenue-cost function) near the optimum. First, suppose that marginal revenue-cost function is a risk substitute near the optimum. In this case, the expansion effect in revenue space is non-positive (Proposition 3). Therefore, a more risk-averse farmer decreases the expected revenue of a less risk-averse farmer. If the demand of input decreases (increases) with a decrease of expected revenue, the more risk-averse farmer decreases (increases) the input utilization. In contrast, suppose that marginal revenue-cost function is a risk complement near the optimum. In this case, the expansion effect in revenue space is non-negative (Proposition 3). Therefore, a more risk-averse farmer increases the expected revenue of a less risk-averse farmer. If the demand of input increases (decreases) with an increase of expected revenue, the more risk-averse farmer increases (decreases) the input utilization.

In order to justify the above conjecture, we introduce a definition of input, which is modified from Chambers and Quiggin [4] according to the decomposition in a direction parallel to the bisector.[6)]

**Definition 3** Input $k$ is non-regressive (regressive) in a direction parallel to the bisector if factor demand for input $k$ increases (decreases) in response to an expansion of $\mathbf{r}$ in a direction parallel to the bisector and decreases (increases) in response to a contraction of $\mathbf{r}$ in a direction parallel to the bisector.

Note that a non-regressive input can exist in a non-limited space of revenue. In contrast, a regressive input is a special case of technology because farmers would always be able to expand their revenue by decreasing the utilization of such an input if a regressive input can exist in a non-limited space of revenue.

From Proposition 3, the sign of expansion effect in revenue space can be tested. If the sign is determined, Definition 3 derives the following results.

**Result 2** Suppose that the marginal revenue-cost function is a risk substitute at $\mathbf{r}^B$ under Assumption 1 to 6. The expansion effect in revenue space gives a more risk-averse farmer an incentive to decrease (increase) the optimal input utilization of a less risk-averse farmer if the input is non-regressive (regressive).

**Result 3** Suppose that the marginal revenue-cost function is a risk complement at $\mathbf{r}^B$ under Assumption 1 to 6. The expansion effect in revenue space gives a more risk-averse farmer an incentive to increase (decrease) the optimal input utilization of a less risk-averse farmer if the input is non-regressive (regressive).

By combining Result 1 to 3, we can examine the effect of the degree of risk-aversion on the input utilization. Eq.(4) says that, if the pure-risk effect and the expansion effect in revenue space give the same incentives to a more risk-averse farmer, we can assure that she uses less or more input than a less risk-averse farmer. Result 2 and 3 suggest that the incentive the expansion effect gives depends on the sign of expansion effect. Even if we know the properties of input (risk complement or substitute and non regressive or regressive), the effect of a degree of risk-aversion on the input utilization is ambiguous unless the sign of expansion effect can be determined. The sign depends on the whole structure of technology near the optimum (i.e. marginal revenue-cost function). Therefore, it is necessary to examine the structure of technology before inquiring about the properties of input.

First, suppose that the marginal revenue-cost function is a risk substitute near the optimum. Then, the expansion effect in revenue space is non-positive (Proposition 3). In this case, we can apply Result 1 to the incentive the pure-risk effect gives and Result 2 to the incentive the expansion effect gives. As summarized in Case I of Table 1, four possibilities exist for the structure of incentives. We can find two cases (Case I-1 and Case I-2) in which each effect gives the same incentives.

**Corollary 1** Suppose that marginal revenue-cost function is a risk substitute at $\mathbf{r}^B$ under Assumption 1 to 6. A more risk-averse farmer uses less (more) input $k$ than a less risk-averse farmer if input $k$ is a risk complement (substitute) and is also non-regressive (regressive) at $\mathbf{r}^A$.

Next, suppose that marginal revenue-cost function is a risk complement. Then, the expansion effect in revenue space is non-negative (Proposition 3). In this case, as summarized in Case II of Table 1, we can apply Result 1 and Result 3. We can find two cases (Case II-1 and Case II-2) in which each effect gives the same incentives.

**Corollary 2** Suppose that the marginal

**Table 1. The incentives for the input utilization that each effect in revenue space gives a more risk-averse farmer and the effect of an increase of risk-aversion on the input utilization**

| | Pure-risk effect | | Expansion effect | | Effect of an increase of risk-aversion[5)] |
|---|---|---|---|---|---|
| | Property of input[1)] | sign[2)] | Property of input[3)] | sign[4)] | |
| Case I: Marginal revenue-cost function is risk substitute | | | | | |
| Case I-1 | Complement | − | Non-regressive | − | − |
| Case I-2 | Substitute | + | Regressive | + | + |
| Case I-3 | Complement | − | Regressive | + | ? |
| Case I-4 | Substitute | + | Non-regressive | − | ? |
| Case II: Marginal revenue-cost function is risk complement | | | | | |
| Case II-1 | Complement | − | Regressive | − | − |
| Case II-2 | Substitute | + | Non-regressive | + | + |
| Case II-3 | Complement | − | Non-regressive | + | ? |
| Case II-4 | Substitute | + | Regressive | − | ? |

Notes: 1) The column shows whether the input is a risk complement or a risk substitute.
2) The column shows the sign of the incentive the pure-risk effect gives a more risk-averse farmer.
3) The column shows whether the input is non-regressive or regressive.
4) The column shows the sign of the incentive the expansion effect gives a more risk-averse farmer.
5) The column shows the sign of the effect of an increase of risk-aversion on the input utilization.

revenue-cost function is a risk complement at $\mathbf{r}^B$ under Assumption 1 to 6. A more risk-averse farmer uses less (more) input $k$ than a less risk-averse farmer if input $k$ is a risk complement (substitute) and is also regressive (non-regressive) at $\mathbf{r}^A$.

In total, we have eight combinations for the structure of incentives which each effect gives.[7)] Chambers and Quiggin [4] showed the possibilities of these eight combinations when the technology exhibits CRR. However, because they did not examine the condition of technology to determine the sign of the expansion effect in revenue space, Result 2 and 3 could not be derived. Consequently, Corollary 1 and 2 could not. In contrast, this paper does not rely on the assumption of CRR to derive these eight combinations. Moreover, this paper provided the formal method (Proposition 3) to divide these eight combinations into the cases in which the expansion effect in revenue space is non-positive (Case I) and the cases in which it is non-negative (Case II). Because of this, we can derive Corollary 1 and 2.

## 4. Conclusion

This paper presented an examination of optimal input utilization of risk-averse farmers under SCT based on the approach presented by Chambers and Quiggin [4]. Unlike Chambers and Quiggin [4], this paper considered the effect of the degree of risk-aversion on the input utilization. The decomposition of the revenue profile to the pure-risk effect and the expansion effect worked well. This paper also relaxed the restrictions on the technology such as CAR and CRR. Definitions of technology sufficient to determine the sign of expansion effect near the optimum were presented. The definitions allow us to examine the effect of the degree of risk-aversion on the input utilization, even if the sign of expansion effect is not defined in the global space of revenue.

As suggested in Chambers and Quiggin [4], the difficulty of analyzing the optimal input utilization under uncertainty mainly comes from ambiguity of the sign of expansion effect in revenue space. This paper provided a theoretical explanation of this ambiguity: The sign of expansion effect depends on whether the marginal revenue-cost function is a risk substitute or complement near the optimum. Therefore, the effect of the degree of risk-aversion on the input use depends on the whole structure of technology as well as the type of input. This conclusion provides a theoretical basis for the mixed results in empirical studies, and also implies that empirical analyses on the type of input will not be enough to consider this issue empirically.

Two important theoretical questions remain. First, the expected utility was assumed in this paper. In order to consider the effect of the degree of risk-aversion under generalized preferences, the properties of the preferences which globally assure the difference of degree of risk-aversion should be developed. Second, this paper assumed that all farmers have the same recognition of the badness of the state of nature. If this assumption is removed, the decomposition of the difference of revenue profiles does not work well. Other approaches are necessary to analyze the difference of input utilization under the condition that farmers may have different recognition of the badness of the state of nature.

1) The state-contingent revenue is an indirect representation of SCT. Hence, the conditions of SCT required to derive the revenue-cost function come from the conditions to derive the cost function defined on the state-contingent output space. See chapter 4 of Chambers and Quiggin [3] for more details. Under the SPF, a cost function that has consistency with all expected utility functions does not exist unless some strict assumptions are imposed on the production function structure. See Chambers and Quiggin [2] and Pope and Chavas [12] for more details.

2) When both farmers have the same probability $p_i$, whether both farmers have equal recognition of the badness of the states of nature or not depends on the structure of production technology and the range of revenue considered (Chambers and Quiggin[3, pp.147–149]). Mathematically, $(\partial C(\mathbf{w},\mathbf{r},\mathbf{q})/\partial r_i)/(\partial C(\mathbf{w},\mathbf{r},\mathbf{q})/\partial r_j) \geq p_i/p_j$ for all $1 \leq i \leq S$ is necessary. If technology exhibits CAR or CRR, both farmers have same recognition of the badness of the states of nature in a non-limited space of revenue (Chambers and Quiggin[3, pp.153–154]).

3) Consider a farmer with constant absolute risk aversion. The degree of absolute risk aversion is denoted by $\eta$. The marginal change of input utilization with a marginal increase of $\eta$ can be decomposed to

$$\begin{aligned}&\frac{\partial x_k^*(\mathbf{w},\mathbf{r}^*,\mathbf{q})}{\partial\eta}\\&\quad= cov\left[\frac{1}{p_i}\frac{\partial x_k^*(\mathbf{w},\mathbf{r}^*,\mathbf{q})}{\partial r_i},\frac{\partial r_i^*}{\partial\eta}\right]\\&\quad+E\left[\frac{1}{p_i}\frac{\partial x_k^*(\mathbf{w},\mathbf{r}^*,\mathbf{q})}{\partial r_i}\right]E\left[\frac{\partial r_i^*}{\partial\eta}\right],\end{aligned}$$

where $\mathbf{r}^*$ denotes the optimal revenue profile of the farmer. Loehman and Nelson [7] decomposed the marginal certainty equivalent with respect to $\eta$ in the same manner. $E[\partial r_i^*/\partial\eta]$ shows the marginal change of expected revenues in a direction parallel to the bisector, which corresponds to the expansion effect of the second candidate. Then, the second term shows the change of input utilization associated with the expansion effect. After some manipulations, we can see that the first term shows the change of input utilization associated with the pure-risk effect of the second candidate.

4) The notation $\preceq_\pi$ followed Chambers and Quiggin [3].

5) The proof is omitted because it might be readily apparent using a similar method for the proof of property of a risk complement (risk substitute) input. See Chambers and Quiggin[3, p. 139].

6) Chambers and Quiggin [4] defined non-regressive (regressive) input in a direction of radial expansion path from $\mathbf{r}^A$ because they defined the expansion effect in revenue space in a direction of radial expansion path from $\mathbf{r}^A$.

7) More precisely, more possibilities exist because input $k$ might be neither a risk complement nor a risk substitute. Additionally, it is possible that input $k$ is neither non-regressive nor regressive, and that marginal revenue-cost function is neither a risk complement nor a risk substitute. In these cases, we cannot assess the effect of the degree of risk-aversion on the input utilization.

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(Received September 26, 2008; accepted November 18, 2009)

## Appendix

Assume that $E[\Delta y^B] > 0$. Then, the expansion effect in profit space, $\tilde{\mathbf{y}}^B - \mathbf{y}^A$ is positive (see eq.(7)). Here, we consider a profit vector $\mathbf{y}^U = (y_1^U, \ldots, y_S^U)'$ that satisfies $E[y] = E[y^B]$ and $E[U(y)] = E[U(y^A)]$ and a profit vector $\mathbf{y}^W = (y_1^W, \ldots, y_S^W)'$ that satisfies $E[y] = E[y^B]$ and $E[W(y)] = E[W(y^A)]$. If $\mathbf{y}^A$ and $\mathbf{y}^B$ are the optimal profit profiles of farmer $A$ and $B$, respectively, the following inequalities are expected to be satisfied:

$$E[U(y^B)] \leq E[U(y^U)] = E[U(y^A)], \text{ and}$$
$$E[W(y^B)] \geq E[W(y^W)] = E[W(y^A)].$$

Therefore, we obtain the following risk ordering:

$$\mathbf{y}^B \preceq_\pi \mathbf{y}^W \text{ and } \mathbf{y}^U \preceq_\pi \mathbf{y}^B.$$

Furthermore, from the transitivity of risk ordering (Rothschild and Stiglitz [15]), we obtain

$$\mathbf{y}^U \preceq_\pi \mathbf{y}^W. \tag{A1}$$

We define $\Delta\mathbf{y}^W$ as $\mathbf{y}^W - \mathbf{y}^A$. We assume that $\Delta\mathbf{y}^W$ is reasonably small, and that the change of expected utility of $W(y)$ associated with the change of profit profile from $\mathbf{y}^A$ to $\mathbf{y}^W$ can be approximated by $E[W'(y^A)\Delta y^W]$. Because of Assumption 6, we obtain the following equation from the definition of $\mathbf{y}^W$:

$$\begin{aligned}\sum_{i=1}^{S} p_i W'(y_i^A)\Delta y_i^W &\\ = cov(W'(y_i^A), \Delta y_i^W)&\\ + E[W'(y^A)]E[\Delta y^W] = 0.&\end{aligned} \tag{A2}$$

From the definition of $\Delta\mathbf{y}^W$, $E[\Delta y^W] = E[\Delta y^B] > 0$. Moreover, $E[W'(y^A)]$ is positive because of the non-decreasing of $W(y)$. Hence, the covariance terms of eq.(A2) is expected to be negative. This condition is apparently satisfied if $\Delta y_i^W$ monotonically increases as the state of nature $i$ increases, because the utility function $W(y)$ is concave and $y_i^A \leq y_j^A$ for all $i < j \leq S$.

Recall that there exists a positive and concave function $V(\cdot)$ which satisfies $W(y) = V[U(y)]$. Hence, eq.(A2) can be decomposed alternatively as follows:

$$\sum_{i=1}^{S} p_i V'(U(y_i^A))U'(y_i^A)\Delta y_i^W \\ = cov(V'(U(y_i^A)), U'(y_i^A)\Delta y_i^W) \\ +E[V'(U(y^A))]E[U'(y^A)\Delta y^W] = 0. \tag{A3}$$

The term $E[U'(y^A)\Delta y^W]$ shows the change of expected utility of $U(y)$ associated with the change of profit profile from $\mathbf{y}^A$ to $\mathbf{y}^W$. Because eq.(A1) implies $E[U(y^A)] = E[U(y^U)] \geq E[U(y^W)]$, $E[U'(y^A)\Delta y^W]$ is expected to be negative. In addition, $E[V'(U(y^A))]$ is positive because of the non-decreasing of $V(U(y))$. Hence, the covariance term of eq.(A3) is expected to be positive. The covariance term can be rewritten as

$$cov(V'(U(y_i^A)), U'(y_i^A)\Delta y_i^W) \\ = \sum_{i=1}^{S} p_i V'(U(y_i^A)) \Big\{U'(y_i^A)\Delta y_i^W \\ -E[U'(y^A)\Delta y^W)]\Big\} \\ = \sum_{i=1}^{S-1} \Big[V'(U(y_i^A)) - V'(U(y_{i+1}^A))\Big] \\ \sum_{j=1}^{i} p_j \Big\{U'(y_j^A)\Delta y_j^W - E[U'(y^A)\Delta y^W)]\Big\}. \tag{A4}$$

Suppose that $\Delta y_i^W$ monotonically increases as the state of nature $i$ increases, and that eq.(A2) is satisfied. If $\Delta y_i^W \geq 0$ for all $i$, eq.(A2) cannot be satisfied. Hence, there exists one state of nature $\bar{s}$ that satisfies $\Delta y_i^W \leq 0$ for all $i \leq \bar{s}$ and $\Delta y_i^W \geq 0$ for all $i \geq \bar{s}+1$. Here, we show that the term $U'(y_i^A)\Delta y_i^W - E[U'(y^A)\Delta y^W)]$ changes the sign from negative to positive only one time as the state of nature increases and does not change the sign any more. Because $U'(y_i^A)$ is positive and decreasing as the state of nature $i$ increases, $U'(y_i^A)\Delta y_i^W$ is negative and increasing in the state of nature until $i \leq \bar{s}$. Furthermore, $E[U'(y^A)\Delta y^W)]$ is negative. Hence, there exists one state of nature $\underline{s}$ $(\leq \bar{s})$ that satisfies $U'(y_i^A)\Delta y_i^W - E[U'(y^A)\Delta y^W)] \leq 0$ for all $i \leq \underline{s}$ and $U'(y_i^A)\Delta y_i^W - E[U'(y^A)\Delta y^W)] \geq 0$ for all $\underline{s}+1 \leq i \leq \bar{s}$. Because $U'(y_i^A)\Delta y_i^W$ is positive for all $i \geq \bar{s}+1$, $U'(y_i^A)\Delta y_i^W - E[U'(y^A)\Delta y^W)]$ cannot be negative for all $i \geq \bar{s}+1$. Therefore, $U'(y_i^A)\Delta y_i^W - E[U'(y^A)\Delta y^W)] \geq 0$ for all $i \geq \underline{s}+1$.

Because $U'(y_i^A)\Delta y_i^W - E[U'(y^A)\Delta y^W)]$ is negative for all $i \leq \underline{s}$, clearly $\sum_{j=1}^{i} p_j\{U'(y_j^A)\Delta y_j^W - E[U'(y^A)\Delta y^W)]\} \leq 0$ for any $i \leq \underline{s}$. Now suppose that $\sum_{j=1}^{i} p_j\{U'(y_j^A)\Delta y_j^W - E[U'(y^A)\Delta y^W)]\} > 0$ for some $i \geq \underline{s}+1$. If so, $U'(y_i^A)\Delta y_i^W - E[U'(y^A)\Delta y^W)]$ must be negative for some $i \geq \underline{s}+1$ because $\sum_{j=1}^{S} p_j\{U'(y_j^A)\Delta y_j^W - E[U'(y^A)\Delta y^W)]\} = 0$. But, this contradicts the fact that $U'(y_i^A)\Delta y_i^W - E[U'(y^A)\Delta y^W)] \geq 0$ for all $i \geq \underline{s}+1$. Clearly $\sum_{j=1}^{i} p_j\{U'(y_j^A)\Delta y_j^W - E[U'(y^A)\Delta y^W)]\} \leq 0$ for all $i \leq S$ (also see Ramaswami [13], Proposition 2). Hence, from eq.(A4) and concavity of $V(U(y_i^A))$ in the state of nature, we see that $cov(V'(U(y_i^A)), U'(y_i^A)\Delta y_i^W)$ is negative. However, this statement contradicts the statement of eq.(A3). Therefore, the assumption $E[\Delta y^B] > 0$ is an incorrect assumption.□

The scalar transformation function $V[\cdot]$ under the expected utility is critical for this proof (see eq.(A3)). Under the generalized preferences, intuitively, the dependency of welfare between states of nature might require vector transformation functions even if such functions can be derived. Therefore, Proposition 1 might not be satisfied under the generalized preferences. In order to precisely discuss this issue, the properties of the generalized preferences which globally assure the difference of degree of risk-aversion should be developed.