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Estimating the Potential Gains from Mergers: The Danish Agricultural Extension Services.

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Abstract: We introduce simple non-parametric models to estimate the potential gains from merging production units. Three effects are distinguished. A merger may affect technical efficiency. It also affects the size of the operation which may or may not be advantageous depending on the return to scale properties of the underlying technologies. Lastly, it affects the mix of inputs available and the mix of outputs demanded. A merged unit face more "balanced" or "harmonic" input and output profiles which is typically advantageous. We use the model to estimate the potential gains from merging agricultural extension offices in Denmark.

Contents: 1. Introduction, 2. Literature, 3. DEA Models, 4. Measures of Merging Gains, 5. Decomposing Merging Gains, 6. The Danish Agricultural Extension Services, 7. Final Remarks, References.

1. Introduction

There are frequent reports of mergers and takeovers in the business press and it seems that mergers play an important role in the structural development of most sectors. There is also a considerable theoretical literature on the pros and cons of mergers as well as a number of academic studies trying to evaluate the effects of actual mergers ex post.

There are many reasons to merge. The internal or organizational reasons include the possibility to exploit economies of scale, economies of scope, risk sharing, scarce managerial skills etc. The external or market oriented reasons include the possibility to gain market power via size or scope or the facilitation of collusive behavior. There are also many obstacles to mergers, including the possible conflicts between different business cultures and public policies directed against the exercise of market power.

The aim of this paper is to focus on the *potential production economic effects* of mergers and in particular to discuss ways of quantifying these. We deviate from previous papers by estimating the potential gains a priori rather than the realized gains ex post. We deviate also by using a multiple input multiple output productions model as opposed to a more aggregate cost model. Lastly, we deviate by developing a framework where the potential gains can be decomposed and related to different strategic possibilities, viz. improvement of efficiency in individual firms, exchange of input and output via inter-firm markets, and genuine full scale merger.

We model the multiple input multiple output production processes using an activity analysis or so-called *Data Envelopment Analysis* (DEA) approach. This approach is easy to use and it has proved to be a flexible and powerful tool in a large number of empirical studies. A particular advantage is that it does not require prices on inputs or outputs. Our specific methodological contribution is to show how the effects of mergers can be captured and decomposed by DEA models.

In the application, we use our approach to evaluate the potential gains from mergers of *agricultural extension offices* in Denmark.

The outline of the paper is as follows. In Section 2, we relate our approach to some existing literature. In Section 3, we review some of the production models and productivity measures that have been suggested within the DEA paradigm. In Section 4, we purpose aggregate measures of the potential gains from a merger, and in Section 5 we discuss how these measures can be decomposed into technical efficiency, size and harmony effects. The application is discussed in Section 6, and final remarks are given in Section 7.

2. Literature

In the general economic literature, the primary theme has been how mergers affect costs and competition and which mergers may therefore be expected, cf. among others Perry and Porter(85) and Farrell and Shapiro(90). A recent and interesting issue is the strategic value of an early merger, cf. e.g. Nilssen and Sørgaard(1998).

The production economic effects of mergers are related to cost aspect, i.e. to the efficiency of production, the economies of scale and the economies of scope. The efficiency of production plans and the economies of scale has received considerable attention in the DEA literature, cf. the references below. Less focus has been given to the economies of scope, an exception being Färe, Grosskopf and Lovell (1994,Sec.10.4). Economies of scope prevail if joint production is cheaper than separate production. With two products, this means that $C(y_1, y_2)<C(y_1, 0)+C(0, y_2)$, where $C(y_1, y_2)$ is the minimal costs of producing products 1 and 2 in the amounts y_1 and y_2.[1] In this paper, we use the scope idea somewhat broader. We do not require the merged units to initially produce different types of products, just different product mixes. To emphasize this, we shall talk about economy of harmony or mix.

It is clear that one cannot predict or prescribe mergers based solely on the potential production economic gains. The competition effect of mergers as well as the strategic timing issue are relevant also. Furthermore, merger decisions involve many issues that are hard to capture in a formal model, including the business cultures in the units and the likes and dislikes among the leading managers. Subsequent to the application described below, several managers contacted us to discussed the different merger possibilities. They generally emphasized the multiplicity of relevant factors.

Another line of literature which is clearly relevant to the present paper is concerned with estimating the potential gains from resource reallocations. An early paper combining this question with DEA models is Lewin and Morey (1981). They discuss the decomposition of inefficiency in a hierarchical organization into what can be attributed to inefficiencies in the production units with given resources and misallocation of resources among the units at different levels of the organization. A recent contribution is Bärnlund, Chung, Färe and Grosskopf(1998). They estimate the potential gains from allowing certain inputs (pollution permits) to be tradable among the firms of an industry. This is done by calculating the profit under an existing distribution of the permits and comparing it to the profit that is possible when profits are maximized subject to the constraint that the sum of the pollution levels cannot exceed the sum of the old pollution permits. Our approach is related to this approach except that we do not have a single relevant output like profit (and therefore use Farrell type proportional changes). Additionally, we distinguish the gains associated with reallocation among similarly sized firms, called the harmony effect below, and the gains that are available by changing the size of the firms.

Kao and Yang(1989) use a DEA model to evaluate alternative organizations of the national forest districts in Taiwan. The need for fewer and larger districts give rise to four alternative district-plans. The new districts are merged from old ones, and the expected performance of the new units are evaluated by comparing the aggregation of the constituent districts to a district

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1 A key feature of most examples is the use of some sharable or quasi-public input in production, say a technological improvement which can be used in one area without affecting its use in another area.

production model based on the original ones. This is similar to the approach we use here to measure the overall gain of a possible merger. The aim is different, however, since Kao and Yang(1989) seek districts that are similar in terms of efficiency. They do so to provide a fair basis for subsequent competition and comparison, and they do not consider alternative ways of accomplishing this[2]. We seek a reorganization that maximizes the potential gains and we consider alternative ways to accomplish this ranging from learning from peers groups to genuine mergers.

We most mention also the notion of the structural efficiency of an industry. According to Farrell(1957,p.262) structural efficiency is "the extent to which an industry keeps up with the performance of its own best firms" and it can be measured by comparing the horizontal aggregation of the industry's firms with the frontier constructed from its individual firms. This is similar to the way we measure the aggregate gain from a merger. A related approach is the average unit approach suggested by Försund and Hjalmarsson(1979). In this approach the structural efficiency is estimated by taking the average of each type of input and each type of output and measure the associated average unit's distance to the frontier. This is similar to our concept of harmony used to capture the mix effects of a merger except that we correct for individual inefficiencies first.

It is relevant to observe - as does Farrell(1957,p.261) - that the structural efficiency will in general fall short of the average of the individual efficiency of the firms. The reason is the curvature of the frontier or more precisely the convexity of the production possibility set. This is not a problem in our estimation of the potential gains from a merger. In fact, it is precisely this tendency for more aggregate or average units to be able to save more inputs or produce more outputs that we consider to be an important source of gains from a merger and which we call the harmony effects. Caution is called for, however, in ex post studies. Often the cost aspects of a merger is evaluated by comparing the average pre-merger efficiency with the post-merger efficiency of the new unit, cf. e.g. Akhaven, Berger and Humphrey(1997), Chapin and Schmidt(1999) and the references herein. In this case the positive effects of a merger may well be underestimated simply because the target for the merged unit is more demanding than the targets for it constituent parts. Put differently, even though this measure indicates increased slack, the merger may be advantageous from a point of view of production economics. Our harmony index emphasize this.

Lastly, let us mention that there are obvious technical resemblances between the merging issue and the role of aggregation in production theory. There is a large literature on the aggregation of variables and the separability of production processes. There are also a few papers explicitly linking these issues to the efficiency measurement problem, cf. Färe and Lovell(1988). Such studies may give conditions on the technology under which the different merger effects can be excluded, see also Bogetoft(1998b). We hope to pursues this issue in later research.

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2 See Bogetoft(1997) for some similar design considerations in a formal agency setting involving DEA .

3. DEA Models

Data Envelopment Analysis (DEA) can be used to model and evaluate productive units that perform similar tasks and for which measurements of inputs and outputs are available. For a text-book introduction to DEA, see Charnes, Cooper, Lewin and Seiford (1994).

Consider the case where each of n Decision Making Units (DMUs), $i\in I=\{1, 2, \ldots, n\}$, have transformed p inputs to q outputs. Let $x^i = (x_1^i,\ldots,x_p^i)\in \Re_0^p$ be the inputs consumed and $y^i = (y_1^i,\ldots,y_q^i)\in \Re_0^q$ the outputs produced in DMU^i, $i\in I$. Also, let T be the production possibility set

$$T = \{(x,y)\in \Re_0^{p+q} \mid x \text{ can produce } y\}$$

and let $x\rightarrow P(x)$ and $y\rightarrow L(y)$ be the associated production and consumption correspondences

$$P(x) = \{y|(x,y)\in T\} \qquad L(y) = \{x| (x,y)\in T \}$$

In many applications, the underlying production possibility set T is unknown. The DEA approaches therefore estimate T from the observed data points and evaluate the observed productions relative to the estimated technology. The estimate of T, the empirical reference technology T* with correspondences P*(.) and L*(.), is constructed according to the *minimal extrapolation* principle: T* is the smallest subset of $\Re_0^{p+q}$ that contains the actual production plans (x^i,y^i), $i\in I$, and satisfies certain technological assumptions specific to the given approach. The relative efficiency of DMU^i may then be measured in input or output space by

$$E^i = \text{Min}\{E\in\Re_0 \mid (Ex^i , y^i)\in T^*\} \quad \text{or} \quad F^i = \text{Max}\ \{F\in\Re_0 \mid (x^i , Fy^i)\in T^*\}$$

where E^i is the maximal contraction of all inputs and F^i is the maximal expansion of all outputs that are feasible in T*.

In the original constant returns to scale (crs) DEA model proposed by Charnes, Cooper and Rhodes (1978, 1979), and the decreasing returns to scale (drs) and (local) variable returns to scale (vrs) models developed by Banker (1984) and Banker, Charnes and Cooper (1984), it is assumed that for all $x', x''\in \Re_0^p$ and $y', y''\in \Re_0^q$

A1 *disposability*: $(x',y')\in T$ and $x''\geq x'$ and $y''\leq y' \Rightarrow (x'',y'')\in T$

A2 *convexity*: T convex

A3 *s-return to scale*: $(x', y') \in T \Rightarrow k(x', y') \in T$ for $k \in K(s)$

where s = "crs", "drs" or "vrs", and where $K(crs) = \Re_0$, $K(drs) = [0,1]$ and $K(vrs) = \{1\}$, respectively. For future use, we shall define also increasing return to scale (irs) as $K(irs) = [1, +\infty)$. It is easy to see, cf. e.g. the references above, that A1, A2 and A3(s) lead to the empirical reference technology

$$T^*(s) = \{(x, y) \in \Re_0^{p+q} \mid \exists\, l \in \Re_0^n : x \geq \sum_i \lambda^i x^i,\ y \leq \sum_i \lambda^i y^i,\ \lambda \in \Lambda(s)\}$$

where $\Lambda(crs) = \Re_0^n$, $\Lambda(drs) = \{\lambda \in \Re_0^n \mid \sum_i \lambda^i \leq 1\}$ and $\Lambda(vrs) = \{\lambda \in \Re_0^n \mid \sum_i \lambda^i = 1\}$.

The assumptions A1-A3 have been relaxed in the free disposability hull (fdh) model used by Deprins, Simar and Tulkens (1984), and the free replicability hull (frh) model briefly proposed in Tulkens(1993). The fdh model invokes only A1 and T*(fdh) therefore has the structure above with $\Lambda(fdh) = \{\lambda \in \Re_0^n \mid \sum_i \lambda^i = 1,\ \lambda^i \in \{0, 1\}\ \forall i\}$. The frh model invokes A1 and a replicability or (super) additivity assumption

A4 *additivity*: $(x',y') \in T$ and $(x'',y'') \in T \Rightarrow (x'+x'',y'+y'') \in T$

such that T*(frh) has the structure above with $\Lambda(frh) = \{\lambda \in \Re_0^n \mid \lambda^i \text{ integer } \forall i\}$.[3]

We believe that the additivity assumption from an applied point of view has advantages over the scaling and the convexity assumptions adhered to in microeconomic textbooks. The appeal of the additivity assumption is straightforward. If one DMU can produce y' using x' and another can produce y'' using x'', a unit with input x'+x'' should be able to produce at least y'+y'', since it can just operate as two independent divisions imitating the original ones. The convexity assumption lacks this "peer group" or "proved by way of examples" rationale. A convex combination is an addition of artificial units derived by down-scaling actual ones.

4. Measures of Merging Gains

Let us assume that it makes "organizational sense" to merge the J-DMUs, i.e. the DMUs with

[3] DEA models partially relaxing the convexity assumptions are suggested in Bogetoft (1996) and Petersen (1990).

indexes $j \in J \subseteq \{1, 2, ..., n\}$. In our application we merge DMUs that are close in a geographic sense since here proximity to customers are crucial. In other cases, it may be more important to have the same owners or to have similar organizational cultures in order for a merger to be meaningful.

The merged unit is denoted DMU^J. Direct pooling of the inputs and outputs gives a unit which has used $\sum_{j\in J} x^j$ to produce $\sum_{j\in J} y^j$. This corresponds to having a completely decentralized, flat organization with the divisions corresponding the J-units.

A radial input based measure of the *potential overall gains from merging* the J-DMUs is therefore

$$(P_1) \qquad E^J = \text{Min}\{E \in \Re_0 \mid (E[\textstyle\sum_{j\in J} x^j], \sum_{j\in J} y^j) \in T\}$$

E^J is the maximal proportional reduction in the aggregated input $\sum_{j\in J} x^j$ that allows the production of the aggregated output profile $\sum_{j\in J} y^j$. If $E^J < 1$, we can save by merging. If $E^J > 1$, the merger is costly.

Similarly, an output based measure of the potential overall gains from merging the J-DMUs could be

$$(P_2) \qquad F^J = \text{Max}\ \{F \in \Re_0 \mid (\textstyle\sum_{j\in J} x^j, F[\sum_{j\in J} y^j]) \in T^*\}$$

F^J is the maximal proportional expansion of the aggregate output $\sum_{j\in J} y^j$ that are feasible in a (merged) unit with aggregate input $\sum_{j\in J} x^j$. If $F^J > 1$, we can gain by merging. If $F^J < 1$, the merger is costly.

If we insert DEA estimates of the production possibility set we get the following operational measures of the potential merging gains

$$(P_3) \qquad \begin{aligned} &\min_{E^J,\lambda}\ E^J \\ &\text{s.t.}\quad E^J[\textstyle\sum_{j\in J} x^j] \geq \sum_{i\in I} \lambda^i x^i \\ &\qquad\quad [\textstyle\sum_{j\in J} y^j] \leq \sum_{i\in I} \lambda^i y^i \\ &\qquad\quad \lambda \in \Lambda(k) \end{aligned}$$

and

$$(P_4) \qquad \begin{aligned} &\max_{F^J,\lambda}\ F^J \\ &\text{s.t.}\quad [\textstyle\sum_{j\in J} x^j] \geq \sum_{i\in I} \lambda^i x^i \\ &\qquad\quad F^J[\textstyle\sum_{j\in J} y^j] \leq \sum_{i\in I} \lambda^i y^i \\ &\qquad\quad \lambda \in \Lambda(k) \end{aligned}$$

We observe that neither of these programs may have feasible solutions. In such cases we define $E^J = +\infty$ and $F^J = -\infty$, respectively. Intuitively, the programs may be infeasible for two reasons. First, the merged unit may be large and the return to scale properties may not favor large units. This may be the case in the drs, vrs and fdh model. Second, the merged unit may involve an input mix that ise not very productive or an output mix that is hard to produce. This may be the case in the fdh model. We shall return to the scaling and mixture effects of mergers in more details below.

A general sufficient condition for the merger to be (weakly) advantageous is that the technology satisfies the J-additivity condition

$$\sum_{j\in J} T \subseteq T$$

In such cases it is possible to produce $\sum_{j\in J} y^j$ using $\sum_{j\in J} x^j$ given that it was possible to produce y^j using x^j, $j\in J$. This leads to the following simple observation

Lemma 1 A sufficient condition for (P_1) and (P_2) to have feasible solutions is the additivity assumption A4. In particular, the crs and frh technologies lead to feasible (P_3) and (P_4) programs.

Proof: By induction, the additivity assumption A4 implies the J-additivity assumption $\sum_{j\in J} T \subseteq T$ which in turn implies the existence of solutions to (P_1) and (P_2) as observed above. The crs model satisfies A4 since it satisfies A2 and A3(crs) which implies A4. (Indeed, the crs model can alternatively be defines by A1, A3(crs) and A4). The frh is defined by A1 and A4 and hence satisfies A4 as desired. Q.E.D.

We note that the J-additivity condition $\sum_{j\in J} T \subseteq T$ is equivalent to the either of the conditions

$$\sum_{j\in J} L(y'^j) \subseteq L(\sum_{j\in J} y'^j)$$

$$\sum_{j\in J} P(x'^j) \subseteq P(\sum_{j\in J} x'^j)$$

for arbitrary $(x'^j,y'^j)\in T$, $j\in J$. A sufficient condition for the merger of the J-DMUs to be weakly advantageous is of course that either of these conditions hold for the given values of (x^j,y^j), $j\in J$.

Since merging advantages are often expressed in cost terms, we further note that the first of these conditions implies *subadditivity* of the cost function $C(y,w) = \text{Min}\{wx \mid x\in L(y)\}$ for all possible input prices $w\in \Re_0^p$

$$\sum_{j\in J} C(y'^j,w) \geq C(\sum_{j\in J} y'^j,w)$$

and that the latter implies *superadditivity* of the revenue function $R(x,p) = \text{Max}\{py \mid y\in P(x)\}$

for all possible output prices $p\in \Re^q_0$

$$\sum_{j\in J}R(x'^j,p) \le R(\sum_{j\in J}x'^j,p)$$

Under convexity assumptions, we have by duality theory that the latter conditions are in fact equivalent to the former, c.f. also Färe, Grosskopf and Lovell(1994,ch.10.4.)

5. Decomposing Merging Gains

Our measures of the potential overall merger gains, E^J and F^J, encompass several effects. In this section, we decompose the overall effects into technical efficiency, scale and scope effects and we discuss the organizational relevance of this decomposition.

Some or all of the units in J may be technically inefficient and this may be captured in E^J and F^J. Although a merger may bring in new management which may facilitate the elimination of such inefficiencies, it may also be possible to reduce technical inefficiencies through other means, e.g. by imitating the better performers, sometimes referred to as the peer group. To avoid compounding the effects, therefore, it is useful to adjust the overall merger gains for the *technical efficiency effect*. To do so, we can project the original units to the production possibility frontier and use the projected plans as the basis for evaluating the remaining gains from the merger.

Thus, for example, we may project (x^j,y^j) into (E^jx^j,y^j) for all $j\in J$, where $E^j=E^{\{j\}}$ is the standard efficiency score for the single DMU^j , and use the projected plans (E^jx^j,y^j), $j\in J$, as the basis for calculating the *adjusted overall gains* from the merger

$$(P_5) \qquad E^{*J} = \text{Min}\{E\in \Re_0 \mid (E[\textstyle\sum_{j\in J}E^jx^j], \sum_{j\in J} y^j) \in T\}$$

The output based measure F^J can be adjusted in a similar manner. Letting $T^J = E^J/E^{*J}$ we have

$$E^J = T^J * E^{*J}$$

where $T^J\in [0,1]$ indicates what can be saved by individual adjustments in the different units in J.

Since the individual units can be projected in many ways, there are many possibilities to construct merging measures that are adjusted for technical inefficiencies at the individual level. In the E^{*J} program above, we could for example use an output based projection of the individual units instead of the input based projection suggested. Also, we could supplement the proportional reductions with non-proportional slack adjustments or we could introduce non-radial projections.

Assuming that individual technical inefficiencies has been dealt with, we are left with the two most interesting production economic effects of a merger.

One is the *scaling* or *size effect*. A merger leads to a unit operating at a large scale which may or may not be advantageous depending on the scale properties of the underlying technology. Figure 1 below illustrates a case of a positive size effect from merging DMU^1 and DMU^2.

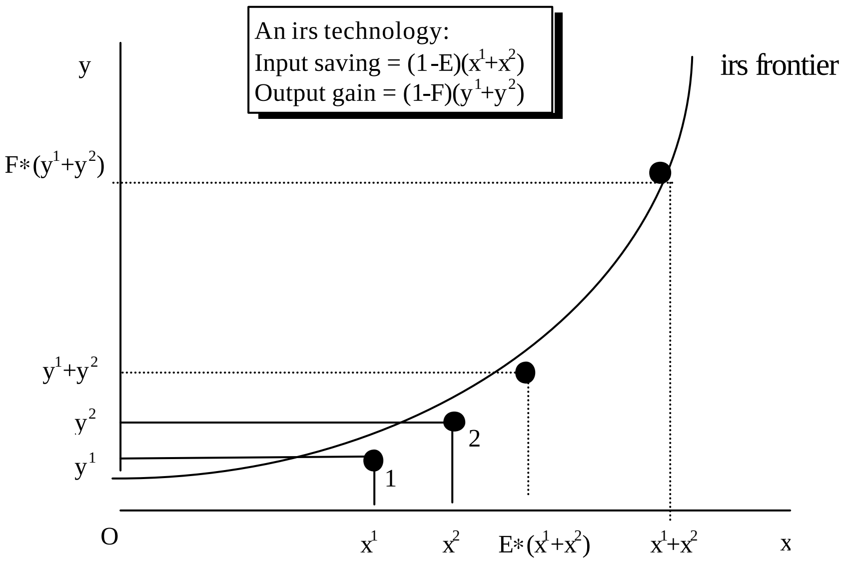
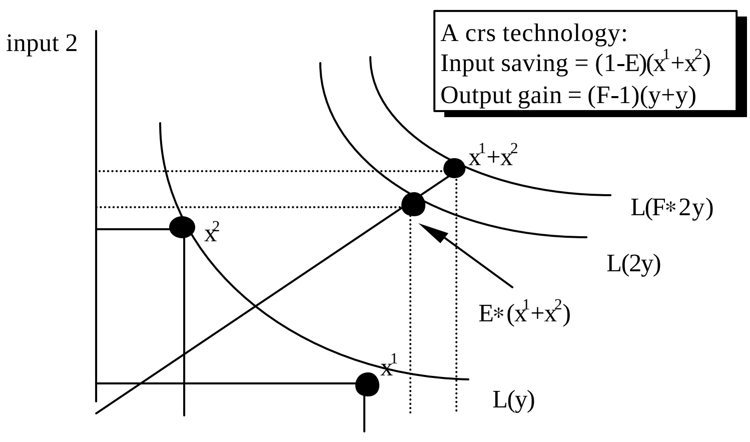


Figure 1. The Size Effect of Merging

The other main production economic effect of a merger is that it leads to other input and output mixes which may be advantageous by taking us into more "productive" directions of the product space. We shall refer to this as the *harmony, scope* or *mixture effects*. Figure 2 illustrates a case with positive harmony effect.



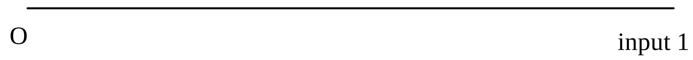
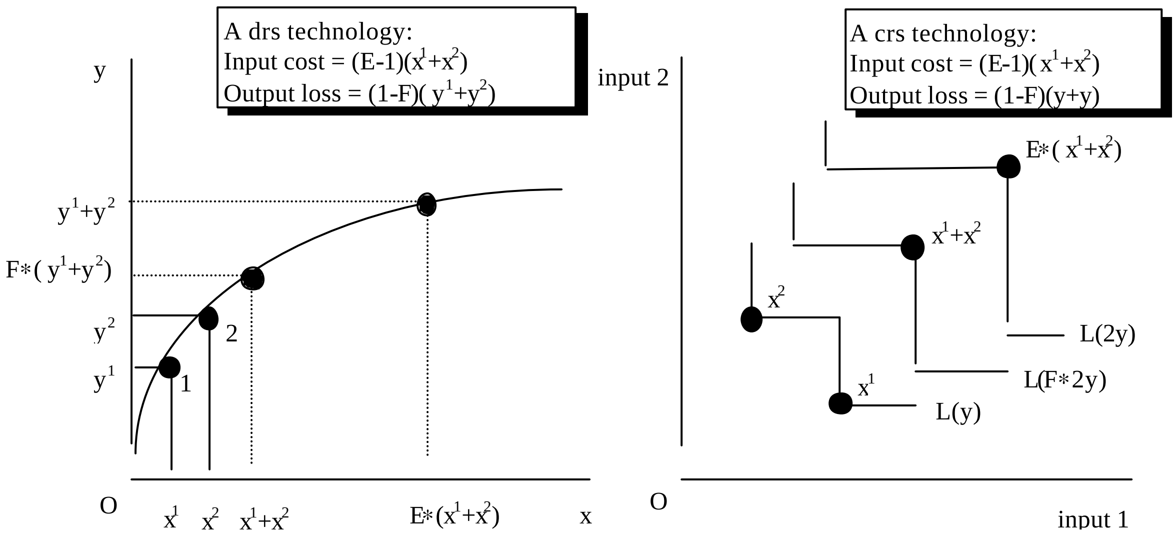


Figure 2. The Harmony Effect of Merging

Without further assumptions about the technology, we cannot put signs on the size and the harmony effects. We have already illustrated cases where they are positive. Negative size and harmony effects are illustrated in Figure 3. A case of opposing effects will be is illustrated in Figure 4 below.



3a. Size Effect

3b. Harmony Effect

Figure 3. Negative Size and Harmony Effects

We note that both the size and harmony effects are reflected in the additivity condition $\sum_{j\in J} T \subseteq T$, or equivalently the conditions $\sum_{j\in J}P(x^j) \subseteq P(\sum_{j\in J}x^j)$ or $\sum_{j\in J}L(y^j) \subseteq L(\sum_{j\in J}y^j)$. Positive size effects correspond to strict inclusions with proportional input (or output) profiles while positive mixture effects corresponds to strict inclusions with input (or output) profiles pointing in different directions. The size effect is possible in both a single and a multiple product environment while the harmony effect is only possible in a multiple product environment.

In Figures 1-3, the size and harmony effects could easily be distinguished since we assumed constant return to scale (leaving no room for size effects) when we illustrated the harmony effect and we assumed a single input single output technology (leaving no room for harmony effects) when we illustrated the size effects. In general, however, it is less obvious how to distinguish the two effects. We shall return to some of the degrees of freedom below. First, however, we outline a decomposition which we find natural and useful.

We propose to capture the *harmony gains* by examining how much of the average input could have been saved in the production of the average output, i.e. by the measure H^J

$$(P_6) \qquad H^J = \mathrm{Min}\{H \in \Re_0 \mid (H[|J|^{-1}\sum_{j\in J}E^j x^j], |J|^{-1}\sum_{j\in J}y^j) \in T\}$$

where |J| is the number of elements in J. We look at the average input and average output since we do not want the expansion of size to come into play yet[4]. Note that $H^J < 1$ would indicate a savings potential due to improved harmony while $H^J > 1$ would indicate a cost of harmonizing the inputs and outputs.

Next, we capture the *size gains* by asking how much could have been saved by operating at the full scale rather than the average scale, i.e. by the measure S^J

$$(P_7) \qquad S^J = \mathrm{Min}\{S \in \Re_0 \mid (S[H^J \sum_{j\in J}E^j x^j], \sum_{j\in J}y^j) \in T\}$$

Again, the re-scaling may be advantageous, $S^J < 1$, if we have economies of scale, and costly, $S^J > 1$, if the return to scale properties do not favor larger units.

As previously, we could approximate the production possibility sets with DEA-like minimal extrapolation models. This lead to *linear programming versions* of the E^{*J} , H^J , and S^J programs above. Since this in a straightforward operation which has already been illustrated above we leave out the details here.

Proceeding like above we get

$$E^{*J} = H^J * S^J$$

and by $E^J = T^J * E^{*J}$ we get our *basic decomposition*

$$E^J = T^J * H^J * S^J$$

This corresponds to a decomposition of the basic merger index E^J into a technical efficiency index T^J, a harmony index H^J and a size index S^J. The technical efficiency measure T^J captures what can be gained by making the individual units efficient. The remaining potentials to save, E^{*J}, are created by the harmony effect, H^J , and the size effect, S^J.

Figure 4 below illustrates the decomposition in a case with positive harmony effect and negative size effect.

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4 Using the average is most relevant if the units in J are not too different in size to begin with. If the sizes differs considerably, we may be picking up scale effects, e.g. if some units are larger than and some are smaller than the "optimal scale size" as defined by Banker(1984).

The decomposition of the potential gains from merging DMUs into a technical efficiency measure, a harmony measure and a size measure is important because full scale mergers are typically not the only organizational option available and it may be that alternative organizational changes may be easier to implement. In particular, we suggest that the following may guide the *restructuring*:

Low technical efficiency measure T^J:
One could let the inefficient DMUs learn from the practices and procedures of the more efficient ones. If the problem is not a lack of skills but rather motivation, one could improve the incentives, e.g. by using relative performance evaluation and yardstick competition based on the technical efficiency measures, cf. Bogetoft(1994,95,97). Of course, if the problem is scarcity of management talent, it may still be necessary to make a genuine merger to transfer control to the more efficient administrative teams and hereby improve the managerial efficiency (X-efficiency).

Low harmony measure H^J:
One could consider reallocating the inputs and outputs among the DMUs to create more productive input mixes and more easily produced output profiles. The can be done inside a hierarchy, by long term contracts or perhaps by creating a market for key inputs and outputs, cf. also Brännlund, Chung, Färe and Grosskopf(1998).

Low size measure S^J:
In this case, full scale mergers may be the only alternative. If we need large amount of fixed capital, highly specialized staff, long run-lengths or simply a critical mass to obtain sufficient returns from scale, it may be relevant to merge. Also, and perhaps most importantly, this may be relevant if the reallocation through contracts or a market are associated with too many transaction costs to make it attractive, cf. the general discussion of the size of the firm in the industrial organization literature, e.g. Tirole(1988).

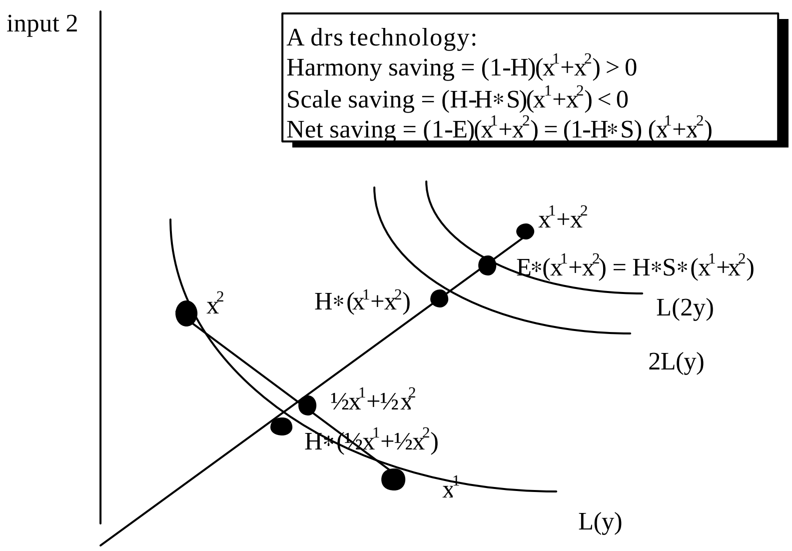

Figure 4. Positive Harmony and Negative Size Effects

The decomposition developed above is only one (natural we believe) possibility to define and distinguish between technical efficiency, the size and the harmony effects. A similar decomposition is possible in the output space, but it will not in general lead to the same quantitative measures of the different effects. Also, there is typically some possibility to substitute between the harmony and size effects.

To emphasize the substitution possibility, consider the following modified definition of the harmony effect

$$(P_7) \qquad H_\alpha^J = \text{Min}\{H \in \Re_0 \,|\, (H[\alpha\textstyle\sum_{j\in J} E^j x^j], \alpha\sum_{j\in J} y^j) \in T\}$$

where $\alpha \in [0,1]$ is a scalar that defines the activity level at which we calculate the harmony gain. Above, we used $\alpha=|J|^{-1}$, i.e. we used a simple mean size to determine the harmony gain. We could still define the size effects as in (P_6) except that we should use H_α^J instead of H^J. Now, by varying α we get different values for the harmony and size effects. It is straightforward to prove the following simple lemma.

Lemma 2 H_α^J is independent of, weakly increasing in or weakly decreasing in $\alpha \in [0,1]$ when T is a constant return to scale (crs), decreasing return to scale (drs) or increasing return to scale (irs) technology.[5]

Proof: Assume first that all programs have feasible solutions. Now, since (H_α^J $[\alpha\sum_{j\in J}E^jx^j]$, $\alpha\sum_{j\in J}y^j$) $\in T$, so is (H_α^J $[\alpha'\sum_{j\in J}E^jx^j]$, $\alpha'\sum_{j\in J}y^j$) $\in T$ for $\alpha' \in \Re_0$, $\alpha' \in [0, \alpha']$ and $\alpha' \in [\alpha,+\infty)]$ when T exhibits crs, drs and irs properties respectively. This implies that H_α^J decreases when α reduces in a drs technology and that H_α^J decreases when α increases in a irs technology. In the crs case, it shows that H_α^J weakly decreases when α varies and since this holds for arbitrary α, it implies that H_α^J is constant. Cases where the program may not be feasible follows similarly. Q.E.D.

Lemma 2 emphasize the possibility to make alternative decompositions. If we choose a low value of α in a drs technology, we assign some size effect to the harmony component. In the irs case, a low value of α would lead us to assign some of the harmony effect to the size

5 In a vrs technology, we get a non-monotonic H_α^J. For small value, it decreases, then it is constant and than it increases in $\alpha \in [0,1]$. The constancy occurs when ($\alpha\sum_{j\in J}E^jx^j$, $\alpha\sum_{j\in J}y^j$) is projected into a most productive scale size plan, cf. Banker(1984). The proof follows the general outline above.

component. We leave it to future research to determine more general technological properties that are necessary for the decomposition to be independent of α.

The introduction of a modified harmony index is only one way to vary the decomposition. A more fundamental alteration would be to calculate harmony and scaling effects without presuming technical efficiency. Organizationally, this may be relevant since it may be easier to initialize a reallocation of resources than a change in the internal culture, tradition and routines. Technically, the difficulty is that the rates of substitution are usually only considered to be well defined on the frontier. Still, by making proper assumptions about the technology, methodologically sound "off-the-frontier-reallocation" gains may be calculated. Some initial work along these lines are Bogetoft and Färe(1999).

6. The Danish Agricultural Extension Services

In Denmark, agricultural advisory services are provided by 71[6] extension offices. These offices serve different geographical regions. The offices are operated as co-operatives owned by the farmers in the corresponding regions. The regions have a long tradition of cooperation. The regional organizations are part of a national organization which operates a supra-advisory office, The National Agricultural Advisory Centre at Skejby, from which the local offices can buy standardized computer-programs, expert-help etc.

In this section, we are going to evaluate the individual offices. We expect to find high relative efficiency levels because of the similarity of the technologies and the widespread cooperation of the offices. At the same time we expect potential gains from mergers, in particular from the harmony effect. The reason is that the farmers' demand for extension services may change relatively fast due to new market conditions and environmental regulations while the qualifications and structures of the extension offices may adopt slower due to union restrictions, recruitment difficulties etc.

The data for this study includes a rather detailed description of all the activities in the economic sections of the 71 offices in the year Oct. 1994-Sept. 1995. Due to confidentiality clauses, we cannot reveal the names or location of these offices. For the purpose of this analysis, we have aggregated the information into a description of the production process in terms of 4 inputs and 4 outputs, namely

Inputs:
HELABOR: Number of employees with higher education (academic staff)
LELABOR: Number of employees with less education (technicians etc)

[6] Two of these offices, numbered 52 and 47, actually share advisors, and they are therefore treated as one, numbered office 47, in this study.

EDB: Computation costs
BUILDCOST: Office rent and other costs

Outputs:
ECOACC: Number of external accounts (financial statements) produced
PRODACC: Number of internal accounts produced
BUDGETACC: Number of budgets produced
TOTOACC: Number of other services produced, e.g. manure plans, strategic plans

We note that the data do not capture the ultimate extension output, namely improved farm performance. The National Agricultural Advisory Centre is presently developing quality measurement techniques that could improve the output description used here.

Summary statistics for the 4 inputs and 4 outputs are provided in Table 1 below. 1 US$ is approximate 6.5 Dkr.

Table 1. Summary statistics of the 70 advisory offices

Labor size	No.	econacc (acc. No.)	prodacc (acc. No.)	budgacc (acc. No.)	totoacc (acc. No.)	helabor (full-time)	lelabor (full-time)	EDB (100 kr)	Buildcost (100 kr)
		averages in groups							
<10,00	7	188,43	75,57	32,43	78,71	2,09	4,37	1947,82	3577,17
10,00-19,99	27	431,70	240,33	67,85	165,63	5,75	9,73	4089,98	9239,44
20,00-29,99	16	660,44	362,94	107,13	266,31	10,28	14,08	6631,51	14813,25
30,00-39,99	10	807,20	456,50	128,90	364,00	12,40	19,98	8309,37	17227,83
40,00-49,99	2	1064,50	576,00	165,50	437,00	13,80	28,66	7431,58	17822,79
50,00-59,99	6	1235,67	520,33	166,83	576,83	17,60	39,42	9618,38	28314,19
>60,00	2	1635,50	836,50	314,50	1182,00	27,75	44,55	19585,24	41861,53
Total	70	44428,00	23337,00	7023,00	19623,00	646,79	1101,24	425005,93	973049,17
Average	-	634,69	333,39	100,33	280,33	9,24	15,73	6071,51	13900,70
Min	-	76,00	19,00	19,00	10,00	1,00	2,50	1189,85	1200,73
Max	-	1760,00	923,00	401,00	1216,00	31,00	49,60	20707,88	48189,33
STDEV	-	351,66	190,95	61,91	234,57	5,84	10,83	3560,40	9053,47

To investigate the efficiency of the individual offices, we initially calculated input based DEA scores for each of them. The efficiency distributions in both crs and vrs technologies are reported in Figure 5 below.

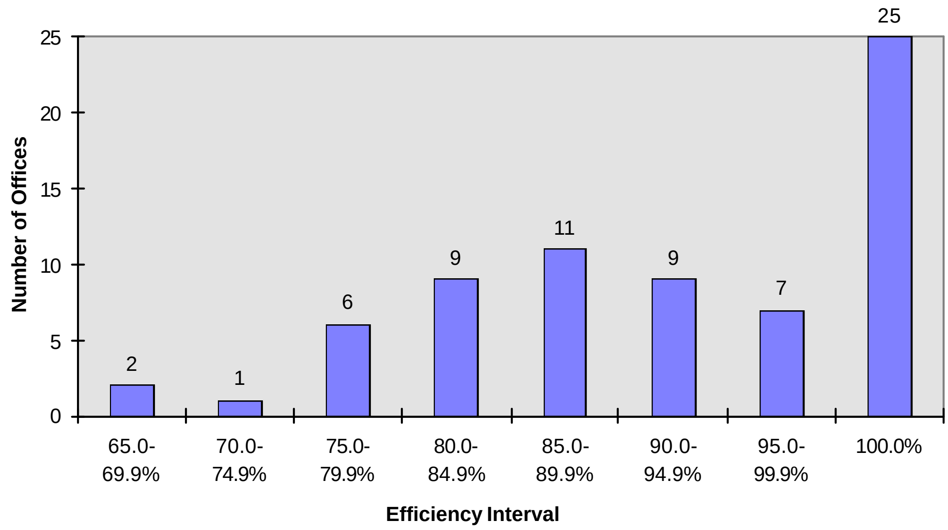

Figure 5a E^j Efficiency Distribution of 70 Offices
in CRS Technology (mean = 91.25%, STDEV = 9.11%)

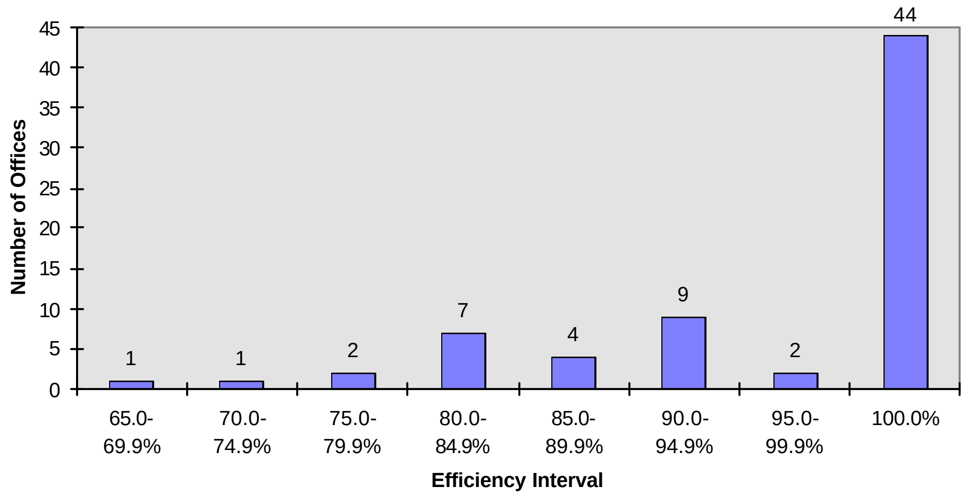

Figure 5b E^j Efficiency Distribution of 70 Offices
in VRS Technology (mean = 94.98%, STDEV =7.87%)

We see that if the technology is modeled using a constant return to scale DEA model, the estimated average saving is 8.75%. In the varying return to scale model, the average saving potential is 5.02%. Informal comparison with other DEA based productivity studies show that these numbers are in no way extreme - given the number of DMUs and the number of inputs and outputs.

Many of the offices are located close to each other and a lot of the services are delivered via phone and computers. For the purpose of considering potential mergers, we have therefore examined what potentially could be saved by merging offices located within a driving distance of 50 kilometers (approximate 30 miles) from each others. This leads to a total of 458 possible mergers involving two or three offices. We have tested the merging gains from all of these combinations using both crs and vrs DEA-models as the basic production model. The distribution of the merging gains are reported in Table 2 below, where we have left out the cases with no potential gains.

Table 2 shows that there are considerable potential gains from merging. Assuming a crs technology and assuming that we have first corrected for individual inefficiencies, we see that 409 of the possible 458 E^{*J} scores was less than 1. Furthermore, about 100 of these mergers generate a saving which are of approximately the same size, 8-10 %, as the average gains from individual improvements reported above. Under a vrs technology, the gains from merging (as the gains from individual improvements) are considerably less since the scale effects generally works against mergers.

Table 2. Distribution of merging efficiency measures (<100%) under CRS and VRS

Efficiency	CRS technology		VRS technology	
interval in %	E^J	E^{*J}	E^J	E^{*J}
55,00-59,99	2	0	0	0
60,00-64,99	2	0	0	0
65,00-69,99	1	0	0	0
70,00-74,99	19	0	2	0
75,00-79,99	50	4	0	0
80,00-84,99	112	14	13	1
85,00-89,99	118	23	15	3
90,00-94,99	83	96	29	12
95,00-99,99	36	272	30	25
Total	423	409	89	41

To further examine the most promising mergers, Table 3 list the 25 mergers leading to the lowest E^J scores under the crs assumption. Again, this illustrates that there are non-trivial production economic gains from mergers in the Danish extension sector.

Finally, we list the 25 most promising mergers in a vrs technology in Table 4. In this case, the decomposition of the total gain also show that there are generally quite a bit to be gained in terms of the harmony effects. The size effect, however, generally works against the mergers. Only in the merger of the rather small offices 56 and 65, code A10, do we get a positive size effect.

Again, these numbers illustrate that the gains from mergers can be significant. The A13 merger, for example, suggest a potential gain of 15.88% from a merger even though the individual units cannot be improved. This gain is the result of a potential harmony improvement of 17.30% and a potential size loss of 1.72%.

Table 3. Merging efficiencies of the top 25 must promising mergers under CRS, in %

Code	Merger	E^J	T^J	E^{*J} ($=H^J$)
A1	Office 46 and 49	58,20	68,61	84,83
A2	Office 46, 49 and 53	59,77	76,09	78,55
A3	Office 46, 49 and 50	62,03	75,95	81,67
A4	Office 46 and 53	62,87	79,44	79,14
A5	Office 33, 34 and 35	68,32	82,93	82,38
A6	Office 46, 49 and 51	70,05	79,87	87,71
A7	Office 02 and 06	70,34	74,74	94,11
A8	Office 46 and 47	71,37	84,13	84,83
A9	Office 46, 47 and 50	71,73	84,66	84,73
A10	Office 33, 37 and 39	71,84	88,75	80,95
A11	Office 49 and 53	72,28	76,36	94,66
A12	Office 68 and 71	72,30	73,45	98,43
A13	Office 33 and 34	72,32	81,22	89,04
A14	Office 46, 51 and 53	72,64	85,27	85,19
A15	Office 46 and 50	72,93	81,50	89,48
A16	Office 33 and 37	73,20	90,43	80,95
A17	Office 33, 37 and 40	73,45	89,67	81,91
A18	Office 33 and 35	73,49	91,16	80,62
A19	Office 34, 35 and 44	73,58	84,08	87,51
A20	Office 33, 37 and 44	74,09	91,20	81,24

A21	Office 03 and 06	74,45	81,25	91,63
A22	Office 04 and 06	74,76	77,14	96,92
A23	Office 02, 03 and 06	74,93	80,91	92,61
A24	Office 34 and 44	74,95	84,60	88,59
A25	Office 33, 39 and 40	75,07	91,90	81,69

Table 4. Merger efficiencies of the top 25 must promising mergers under VRS, in %

Code	Merger combination	E^J	T^J	E^{*J}	H^J	S^J
A1	Office 46 and 49	73,88	84,41	87,53	83,29	105,09
A2	Office 68 and 71	74,59	75,42	98,90	97,72	101,21
A3	Office 56 and 57	80,38	85,62	93,88	93,39	100,52
A4	Office 57 and 71	81,07	76,87	105,46	99,58	105,90
A5	Office 28 and 32	81,68	81,95	99,67	94,08	105,94
A6	Office 57 and 68	82,13	83,35	98,54	96,73	101,87
A7	Office 64 and 67	82,52	84,19	98,02	90,44	108,38
A8	Office 12 and 31	82,61	91,97	89,82	87,27	102,92
A9	Office 56 and 64	83,31	85,99	96,88	89,23	108,57
A10	Office 56 and 65	83,38	96,04	86,82	89,61	96,89
A11	Office 12 and 32	83,95	88,52	94,84	94,55	100,31
A12	Office 12 and 28	84,09	81,03	103,77	97,67	106,25
A13	Office 47 and 48	84,13	100,00	84,12	82,70	101,72
A14	Office 12, 31 and 32	84,35	92,99	90,71	87,10	104,14
A15	Office 28 and 29	84,43	80,76	104,54	96,89	107,90
A16	Office 28 and 31	85,00	87,26	97,41	91,86	106,04
A17	Office 57, 68 and 71	85,10	78,48	108,43	97,54	111,16
A18	Office 17 and 22	85,18	86,61	98,35	98,35	100,00
A19	Office 59 and 64	85,84	82,44	104,13	96,60	107,80
A20	Office 13 and 32	85,87	89,53	95,91	94,04	101,99

A21	Office 46 and 53	86,33	92,77	93,06	78,92	117,92
A22	Office 28, 31 and 32	86,39	88,90	97,18	88,74	109,51
A23	Office 17 and 19	86,77	86,18	100,68	94,01	107,09
A24	Office 22 and 23	87,42	93,16	93,84	89,18	105,23
A25	Office 12 and 17	87,54	82,93	105,56	94,03	112,26

Leading up to the study period, there had been some re-organization of the advisory offices. Offices in several regions had in fact been merged. Subsequently, more offices have merged, including some of the combinations identified above. Since we cannot reveal the identity of the offices, we can also not reveal which offices have subsequently merged.

7. Final Remarks

In this paper, we introduced simple non-parametric models to compute the potential gains from merging Decision Making Units. We decomposed the gains into technical efficiency, size and harmony effects. A merger may force the units to perform more efficiently on an individual basis. It also affects the scale of operation which may or may not be advantageous depending on the return to scale properties. Finally, it affects the mix of inputs available and outputs demanded. A merged unit faces a more balanced or harmonic input and output profiles which is typically advantageous.

The decomposition allows us to identify alternative means of improving performance. If the technical efficiency is low, gains are possible by learning the practices of peer units and by introducing incentive schemes to motive efficiency. If the harmony index is low, improvements are possible by re-allocating resources, either within a hierarchy or through an inter-unit market for inputs and outputs. If the size index is low, a genuine merger may be called for to enable the optimal specialization, run-lengths etc.

The methodology was illustrated by computing gains from merging neighboring advisory offices in Denmark. We showed that considerable production economic gains from mergers can be expected. In many cases, the gains from individual improvements and from improved harmony effects were of the same order of magnitude.

There are numerous relevant extensions of the research reported here.

On the theoretical side, it is relevant to consider alternative decompositions and to identify technological regularities that suffices to make the decompositions unique. More generally, it is important to study what organizational changes to introduce in a post-productivity analysis and to discuss how the analysis could be tailored in the first place to support such changes, cf. also Bogetoft(1998a).

On the applied side, our framework is particularly relevant in those cases where it is important to keep a multiple input multiple output description of the production process. This may be the case quite generally since a merger probably requires the units to match and complement each other in several dimensions.

More specifically, we suggest that the framework is relevant in natural resource planning and regulation. Farms and forests are subject to an increasing number of environmental restrictions in most countries. In Scandinavia, such constraints are often refereed to as harmony constraints since they concern the balance or harmony between different aspects of the environments. The approach of this paper can be used to evaluate alternative designs of such restrictions, including the use of farm specific or tradable requirements. Also, it can be used to predict likely responses to such regulations. One way to meet the increasing number of constraints is to balance what one has in excess with what another lacks through a merger.

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