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ABSTRACT. In perfectly competitive markets taxes and quotas are fully equivalent measures for environmental protection. Based on this regulators' revealed preference for quotas over that of fees finds its explanation in the procedures and spirits of political decision making. This paper offers another explanation: Ordinary welfare economic considerations make a quota preferable to a tax when regulating polluting firms in monopolistically competitive markets.

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PRICES vs. QUANTITIES IN MONOPOLISTIC COMPETITION

Arthur C. Pigou's (1947) study on public finance includes precise suggestions about the structure of policy measures for protection of the environment: the first-best tax on pollution equals the marginal environmental damage. Under a Pigouvian tax the consumers pay the social marginal cost of each item, the direct cost of resources and, in addition, the indirect cost of pollution. In these circumstances the market guides economy to its social optimum.

There are, though, other ways of establishing the proper economic incentives for abatement activities. Two alternative instruments have received extensive attention: unit subsidies and tradeable pollution quotas. With perfect knowledge the latter type of instrument is in principle a fully equivalent alternative to a tax, apart from matters relating to distributional issues. Or as Cropper and Oates (1992) put it: The regulator can, in short, set either "price" or "quantity" and achieve the desired result. Allocative equivalence and efficiency of the two types of instruments are established in an otherwise perfectly competitive economy. Concentrating upon the monopoly case Buchanan (1969) argues that a tax equal to the marginal external damage can be detrimental to welfare, see also Barnett (1980). This paper employes the monopolistic competitive model found in Spence (1976) and Dixit and Stiglitz (1977) to address the question of equivalence of the two types of instruments in markets with set-up costs.

The individual firm's revenue plays an important role regarding the socially optimal kinds and quantities of commodities because of the scale economies effects initiated by the set-up costs. Equality between the price and marginal cost, a condition for a socially optimal allocation, leaves the firm with negative profit. Allowing some degree of monopoly each firm can recover also the set-up costs, but then the price is above marginal costs. Spence (1976) and Dixit and Stiglitz (1977) show that product variety can be below the first best level in such circumstances. This paper examines taxes and quotas in such an economic environment. Since the market undersupplies product variety Buchanan's (1969) result weakly points to an optimal tax below the Pigouvian tax. And, and perhaps also more interesting, the relation between taxes and quotas can be expected to be quite complicated making quotas the preferred instrument. This is so since the firm's revenue and the number of firms, and thus product variety, are affected negatively by a tax withdrawing revenue from the firms. A quota following the grandfathering principle will not have the same detrimental effect on firm revenue and product variety. This tends to make quotas attractive relative to taxes showing that important policy conclusions follow the findings in Spence (1976) and Dixit and Stiglitz (1977).

Ever since the introduction of the Pigouvian tax the normative theory of environmental regulation has attracted enormous attention; for a survey of central results see Cropper and Oates (1992). Pigouvian taxes versus quotas is discussed in Weitzman (1974) and Roberts and Spence (1976) with focus on uncertain pollution control costs. Recently this line of thinking has been extended by Hoel and Karp (1999) to include asymmetric information. In this paper such kind of uncertainties is not the issue. Carraro et al. (1996) analyse environmental policy with a focus on taxes in an oligopolistic context. Set-up costs do not play a role here. This paper's focus on multiple market failures is found also in the recent discussion of Pigouvian taxes known as the double-dividend hypothesis; see for example Fullerton (1997), Bovenberg and de Mooij (1994) and Bovenberg and Goulder (1996). The idea here is that substitution of fees for other sources of revenues that carry sizeable excess burdens are probably socially beneficial.

I Taxation and Externalities in Monopolistic Competition

I.A The Market

The economic environment comprises a group of anonymous consumers, a monopolistically competitive industry with a total of n different goods. Product variety, the number of goods, is determined by the profit conditions in the industry. A numeraire good summarizes the remainder of the economy. Within the industry firms are characterized by scale economies and they are linked together by significant cross-elasticities. Following Dixit and Stiglitz (1977) scale economies are modelled by supposing that production involves a fixed set-up cost and constant marginal cost. Regarding the firms in the industry there are external costs in addition to the firms' private cost. External cost at the firm level is assumed to be a linear function of firm output. Following the polluters pay principle taxes against the external effects are levied at the firms. Apart from regulation and taxation the approach is close to Spence (1976) and Dixit and Stiglitz (1977) and the market in this economy is, thus, known to generate too little product variety relative to the socially optimum kinds of commodities.

The main concern here is the choice of taxes or quotas and in order to focus on this matter as simply as possible all commodities in the group have identical fixed and marginal costs. This symmetry also goes for external costs. Clearly, this is a somewhat restrictive assumption, for in problems involving fixed costs one may often encounter a natural asymmetry attributed to gradual physical differences in commodities. However, despite of this simplification some interesting results emerge. Owing to the symmetry assumption we can write the utility of consumption as:

$$u = G(m) + x_0, \quad G'(m) > 0 \quad G''(m) < 0 \tag{1a}$$

$$m = nax^B, \quad 0 < B < 1 \tag{1b}$$

with a and x being some constant and the output of a typical firm in the industry, respectively. As noted above n is the total number of commodities. With respect to B the assumption makes the industry's commodities less than perfect substitutes. Conventionally, G says that consumers' utility is increasing but at a decreasing rate in m, to be thought of as an index of congestion in the industry. Finally x_0 is the numeraire good which enters additively. This follows Spence (1976) and makes the welfare analysis of the industry amenable to a partial equilibrium approach.

The demand functions for the industry's commodities follow immediately from (1): $p = G'(m)aBx^{B-1}$. Denoting by F and c the individual firm's fixed and constant marginal cost, respectively, firm profit under a tax, t, is: $\pi = G'(m)aBx^B - (t+c)x - F$. Market equilibrium is defined by equality between marginal revenue and cost and the zero-profit condition:

$$G'(m)\, aB^2 x^{B-1} = t + c \tag{2a}$$

$$G'(m)\, aBx^B = (t+c)\, x + F \tag{2b}$$

The interaction between the firms in the monopolistically competitive sector goes through the congestion index m. As the congestion index increases the inverse demand function of the individual firm shifts down. Implicit in the above equations is the presumption that $\partial m/\partial x = 0$. Here it is thus assumed that the industry is large, by the number of firms, in the sense that the manager of the individual firm does not need to be concerned with the reaction of other firms. Solving (2a) and (2b) we confirm that $dx(t)/dt < 0$ and $dm(t)/dt < 0$ (remember $G()$ is a concave function).

$$x(t) = \frac{BF}{(1-B)(t+c)} \tag{3a}$$

$$G'(m) = \frac{1}{aB}\frac{F}{1-B}\left[\frac{(1-B)(t+c)}{BF}\right]^B \tag{3b}$$

I.B Optimal Product Variety and Taxation

Turn now to the question of the optimum kinds and quantities of commodities and the optimal tax. There are two sources of market failure present in the economic environment. Firstly, there are scale economies effects. In relation to a comparison between the social optimum and the market equilibrium fixed cost has at least two implications. They restrict commodity variety and possibly also the volume of each produced commodity. And they are a source of non-competitive pricing. In the current model these forces contribute to suboptimality in the form of to little product diversity, cf. Spence (1976) and Dixit and Stiglitz (1977). The second source to market failure is production's external effect. Other things equal this implies that firm output is inefficiently large.

Firm output, however, contributes to the congestion index which is increasing with firm output. That is, the two sources of suboptimality in the market point in opposite directions.

To understand, more precisely, the nature of the bias involved consider the welfare function. Normalising to unity the price of the numeraire good, consumers spend $npx + x_0$ on the total consumption bundle and net-utility is therefore $G(m) - npx$. Total profits in the industry are $npx - n(cx + F)$. Including external costs and rewriting n in terms of m and x from (1a) the welfare function becomes:

$$W = G(m) - \frac{m}{ax^B}((y + c)x + F) \tag{4}$$

where yx is the externality from the individual firm's production activities. Maximizing (4) with respect to x and m the social optimum is:

$$x^* = \frac{BF}{(1 - B)(y + c)} \tag{5a}$$

$$G'(m^*) = \frac{(y + c)x^* + F}{ax^{*B}} \tag{5b}$$

From (5a) the output of the individual firm is decreasing in marginal external costs. The effect upon the optimal number of firms, n, is less clear. There are forces contributing to an increase as well as a decrease in n. A necessary condition for an optimal resource allocation is obviously that a given market congestion should be achieved at the least possible costs. Iso-congestion curves are defined by $\overline{m} = nax^B$ and are clearly convex to the origin. The isocost function for the industry is $\overline{TC} = n((y + c)x + F)$, its slope is negative and we have $\frac{\partial}{\partial y}\left(\frac{\partial n}{\partial x}\right) < 0$. That is, the trade off between the number of commodities and the output of each commodity changes in the favor of increased product variety as marginal external cost increases. The usual tangency condition, therefore, suggests that n increases with marginal external cost. Clearly, the increase in marginal external cost changes m in a downward direction[1], and this effect may dominate so that n decreases. To demonstrate consider the following example.

Example. Assume regarding consumers' valuation of consumption $G(m) = \omega m - \frac{1}{2}zm^2$. The first order condition for m reduces to $\omega - zm = ax^{-B}((y + c)x + F)$

[1] We have $G'(m) = a^{-1}x^{-B}((y + c)x + F)$. From (5a) we have $d/dx\left(a^{-1}x^{-B}((y + c)x + F)\right) = 0$ so that $dm/dy = (G''())^{-1}a^{-1}x^{1-B} < 0$.

so that $n = (az)^{-1} x^{-B} \left(\omega - ax^{-B}\left((y+c)\,x + F\right)\right)$. From this the precise condition for product variety to increase with y becomes:[2] $\omega > 2a^{-1}(1-B)^{B-1} F^{1-B} B^{-B}$. An economically meaningful solution (a positive number of firms) requires[3] $\omega > a^{-1}(1-B)^{B-1} F^{1-B} B^{-B}$. Thus we can have small ω values consistent with a positive n for which n is decreasing in y. But for large ω we have that the optimal number of firms in the industry increases with y.

The remarks on the trade off between the number of goods and their quantities as well as the example point to a Pigouvian tax strictly less than the external marginal cost. The explanation for this borrows clearly from the discussion of externalities in monopoly. Ignoring external effects a monopolist produces at a point where the price is strictly above marginal cost. Thus, the starting point is that production should be regulated in the upwards direction. Adding external effects to this picture may, once the externality is sufficiently important, call for lower production relative to the monopoly output but the tax will (obviously) never exceed the marginal external cost.

With monopolistic competition things are slightly more complicated. The point is mentioned above: In the presence of external effects the trade off between the number of commodities and the volume of each commodity changes to the advantage the former. This shows that a tax can be problematic. A tax clearly accomplishes lower output at the firm level. But the tax also extracts revenue from the industry and this, other things equal, leaves room for fewer firms. The problem here derives from the zero profit condition in combination with fixed costs. It is straightforward to understand the problem. Use the expression for profit-maximizing output under a tax, $x(t) = \left((1-B)(t+c)\right)^{-1} BF$ and notice that the firm's total cost equals $F/(1-B)$ irrespective of the tax. A typical firm's revenue is $G'(m)\,aBx^{B}$ and the firm's adjustment towards lower production as a result of the tax will lower its revenue. Thus, each firm's demand curve should be shifted up for the zero-profit condition to be met. A decreasing value of the congestion index is the only way that this can happen. And the upper limit on the congestion index sets the limit for the number of commodities in the industry. It is easy to confirm that the best Pigouvian tax is characterized by $t < y$, since $dW/dt < 0$ for $t = y$.[4]

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[2] We have

$$n = \frac{1}{zax^{B}}\left[\omega - \frac{F}{(1-B)ax^{B}}\right]$$

so that

$$\frac{dn}{dy} = \left(-B\frac{1}{zax^{B}x}\left[\omega - \frac{F}{(1-B)\,ax^{B}}\right] + B\frac{1}{zax^{B}x}\frac{F}{(1-B)\,ax^{B}}\right)\frac{dx}{dy}$$

Since $dx/dy < 0$ the condition for $dn/dy > 0$ is $\omega > 2F\left((1-B)\,ax^{B}\right)$ or, using $x = (1-B)^{-1}(t+c)^{-1} BF$, that $\omega > 2a^{-1}(1-B)^{B-1} F^{1-B} B^{-B}$

[3] $n > 0$ for $\omega > \left((1-B)\,ax^{B}\right)^{-1} F$ or (cf. note 2) $\omega > a^{-1}(1-B)^{B-1}F^{1-B}B^{-B}$.

[4] Welfare is

II. Taxes versus Quotas

The argument pointing to the role of the zero-profit condition suggests that taxes and quotas can have fairly different allocative effects. A tax will certainly extract a revenue from the industry. Depending upon the type of quota system quotas may have a different effect upon firm profit and, thus, on product variety. If the quotas, or permits, are auctioned off, then of course firms must pay for the right to pollute as they would with a tax. But rather than introducing the quotas by auction they can be initiated with a one time distribution free of charge. Distribution can follow some form of grandfathering to allocate the quotas between firms based on historical performance. This is assumed here. Once a firm has been granted a quota it is marketable. Since firms are identical here, they are all given the same quota and they will not find it favorable to engage in selling or buying quotas so this will not be discussed further here.

The thesis on quotas' superiority is easily validated in the case of $B = (y+c)/(2y+c)$. In this particular case the first best solution for output per firm can be used as a quota and the market will subsequently guide the economy

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$$W = G(m(t)) - m(t)\,\frac{cz(t) + yz(t) + F}{az(t)^B} \tag{1}$$

Dropping "t" we have

$$\frac{dW}{dt} = \left[G'() - \frac{\alpha z + yz + F}{az^B}\right]\frac{dm}{dt} - m\frac{\partial}{\partial z}\left(\frac{cz + yz + F}{az^B}\right)\frac{dz}{dt} \tag{2}$$

Now, from

$$z = \frac{BF}{(1-B)(t+c)} \tag{3}$$

we have

$$\frac{\partial}{\partial z}\left(\frac{cz + yz + F}{az^B}\right) = 0 \tag{4}$$

for $t = y$. Thus,

$$\frac{dW}{dt}\Big|_{t=y} = \left(G'() - \frac{cz + yz + F}{az^B}\right)\frac{dm}{dt} \tag{5}$$

Now, we have from the zero profit condition that

$$G'(m) = \frac{cz + tz + F}{Baz^B} \tag{6}$$

Using $t = y$ and G' in (5) we have

$$\frac{dW}{dt} = \left(\frac{1}{B} - 1\right)\left(\frac{cz + yz + F}{az^B}\right)\frac{dm}{dt} < 0 \tag{7}$$

since $dm/dt < 0$.

to the optimal degree of market congestion.[5] As seen above this is impossible with a tax. A positive result on quotas relating only to a specific parameter configuration cannot, of course, command profound interest. Addressing the problems relating to tax revenue and product variety the first proposition compares a tax to a quota under the assumption that the two instruments implement identical firm output. The difference between the two instruments is, thus, the number of firms left in the regulated industry.

Proposition 1.
If $1 > B\,(1 + t/\,(c+t) + B\,(y-t)\,/\,(c+t))$ a quota $\bar{x} = x\,(t)$, where $x\,(t)$ is firm output under a tax, is to prefer to a tax.

The proof is found in the appendix. Now, it is a problem to proposition 1 that the optimal tax rate is dependent on y, B and c. This raises the question of the existence of the situation described in the proposition. To see whether the condition can be satisfied at all assume that $G\,(m) = m^s$, $0 < s < 1$. This allows us to solve for the optimal tax and give numerical examples. In the appendix the welfare maximizing tax is found to:

$$t = \frac{B^2 + (1-B)\,B\,(1-s)}{B^2 + (1-B)\,B\,(1-s) + (1-B)}\,(y+c) - c \tag{6}$$

Using this Table 1 lists the condition in proposition 1 for $y = 0.1c$, $y = 0.5c$, $y = c$ and some values of B and s and confirms that there is indeed parameter

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[5] Fix the quota at $\bar{x} = (1-B)^{-1}\,(y+c)^{-1}\,BF$. With this scheme the market supports a m-value given by

$G'\,(m) = \frac{c\bar{x}+F}{aB\bar{x}^B}$

Since $\bar{x} = x^*$, called x for now, optimal congestion is

$G'\,(m) = \frac{(y+c)x+F}{ax^B}$

The quota, thus, implements the social optimum if

$$\frac{cx+F}{aBx^B} = \frac{(y+c)\,x+F}{ax^B}$$

with $x = (1-B)^{-1}\,(y+c)^{-1}\,BF$. Using this in the above equation we have

$$\frac{B}{1-B}\frac{c}{y+c} + 1 = B\left(\frac{B}{1-B} + 1\right) \text{ or}$$

$$B = (2 + c/y)^{-1}\,(1 + c/y)$$

configurations making a quota the preferred option relative to taxes.[6]

Table 1

	y = 0.1c			y = 0.5c			y = c		
B\s	0.25	0.50	0.75	0.25	0.50	0.75	0.25	0.5	0.75
0.25	0.28	0.38	0.74	0.35	0.73	0.34	0.40	0.96	0.98
0.50	0.60	0.61	0.62	0.86	0.88	0.93	1.04	1.08	1.15
0.75	0.84	0.84	0.85	1.09	1.10	1.10	1.26	1.27	1.28

The table shows that high y and s values makes it less likely that the sufficient condition in the proposition is satisfied. To understand the sufficient condition and the result in table 1 consider the welfare implications of changes in the congestion index:

$$\frac{dW}{dm} = sm^{s-1}\ ax^B - ((y+c)\,x + F) \tag{7}$$

The quota on x is set at $x = x(t) = (1-B)^{-1}(t+c)^{-1}BF$. Using this and the expression for the optimal tax inspection shows that the higher is y the lower is the optimal value of the congestion index and this clearly lessens the

[6] Write the optimal tax as

$$t = \frac{y+c}{A} - c$$

with

$$\frac{1}{A} = \frac{B^2 + (1-B)\,B\,(1-s)}{B^2 + (1-B)\,B\,(1-s) + (1-B)}$$

The necessary condition in proposition 1 is then

$$1 > B\left[1 + \frac{\frac{y+c}{A} - c}{\frac{y+c}{A}} + \frac{y - \frac{y+c}{A} + c}{\frac{y+c}{A}}\right]$$

or

$$1 > B\left[1 + 1 - \frac{c}{y+c} + A - 1\right]$$

or

$$1 > B\left[1 + \frac{y}{y+c}A\right]$$

This is the expression listed in table 1.

tax's detrimental effect upon the number of firms.[7]

In the sufficient condition above the difference between a tax and a quota is the number of firms left in the industry (the effect on output is by definition identical). The results in table 1 can be interpreted as saying that a quota leaves to many firms in the industry for high y and s values. This effect lowers welfare under the quota relative to the tax. Turning attention to the proposition's proof this point is illustrated. The parameter restrictions in proposition 1 are equivalent to the requirement that optimal product variety for a given output is higher than the quota's product variety for the same output. When this is the case the quota is obviously better than a tax: they implement the same output at the firm level but the quota brings the number of firms closer to the optimal number than a tax does.

The effect of a change in s is somewhat complicated. Output under the tax and the quota is by definition the same, $\overline{x} = x(t) = (1-B)^{-1}(t+c)^{-1}BF$. Thus, whether one or the other instrument is preferred depends upon the deviation from optimal product variety (for the given output). Denoting by $m^*(t)$ optimal market congestion given that output is $x(t)$ and by $\overline{m}(t)$ and $m(t)$ market congestion under a quota, and a tax, respectively, we have (cf.(7)):

$$sm^*(t)^{s-1} = \frac{(y+c)\,x(t)+F}{ax(t)^B} \tag{8}$$

$$s\overline{m}(t)^{s-1} = \frac{c\overline{x}+F}{aB\overline{x}^B} \tag{9}$$

$$sm(t)^{s-1} = \frac{(c+t)\,x(t)+F}{aBx(t)^B} \tag{10}$$

Assume that we have an s value so that $\overline{m}(t) = m^*(t) > m(t)$ and consider an increase in s. The increase in s, since it is followed by a decreasing tax rate and increasing output, increases the right-hand side of (8) since $x(t)$ is

[7] Notice first from the derivation of the optimal tax that $(y+c)/(t+c)$ is a function of only B and s. From the text

$$\frac{dW}{dm} = sm^{s-1} - \frac{(y+c)x+F}{ax^B}$$

with $x = (1-B)^{-1}(t+c)^{-1}BF$. Thus,

$$\frac{dW}{dm} = sm^{s-1} - \frac{\frac{B}{1-B}\frac{y+c}{t+c}F+F}{a\left(\frac{BF}{(1-B)(t+c)}\right)^B} = sm^{s-1} - (y+c)^B\,\frac{\frac{B}{F_B}\frac{y+c}{t+c}F+F}{a\left(\frac{BF}{1-B}\frac{y+c}{t+c}\right)^B}$$

which clearly makes dW/dm a decreasing function of y as $(y+c)/(t+c)$ depends only on B and s.

strictly lower than that minimizing $\left(ax(t)^B\right)^{-1}((y+c)x(t)+F)$.[8] But the right-hand side of (9) decreases since $x(t)$ is strictly larger than the output minimizing $\left(aB\overline{x}^B\right)^{-1}(c\overline{x}+F)$.[9] Thus, as s increases $\overline{m}(t)$ tends to increase whereas $m^*(t)$ tends to fall. Considering the right-hand side of (10) this is affected only through the tax. Since the tax falls, an increase in s tends to give a higher $m(t)$, thus, the difference between optimal and actual market congestion lowers making the tax, relatively speaking, more attractive.

Coming back to the expression for the optimal tax it cannot be expected that quotas will prove superior to taxes in all circumstances. In the case of $B \to 1$ the optimal tax, with $G(m) = m^s$, $0 < s < 1$ converges to y why firm output converges to the optimal output. As $B \to 1$ the number of firms with a tax converges towards the socially optimal number of firms showing that quota cannot do better than the tax. $B = 1$ is the case of perfect competition.

Quotas are not given their best chance in Proposition 1 (and table 1) since they are fixed by the output obtaining under a tax. Such a quota may be rather far away from the optimal quota. The following result allows the output with a tax and a quota to be different. Defining $\widehat{x}$ by $\widehat{x}^{-B}(c\widehat{x}+F) = x(t)^{-B}((c+t)x(t)+F)$, $x(t) = (1-B)^{-1}(t+c)^{-1}BF$ (see figure 1) the next proposition is:

Proposition 2.
If $\widehat{x} \geq x^*$, $x^* = (1-B)^{-1}(y+c)^{-1}BF$ then a quota $\overline{x} = \widehat{x}$ is better than a tax.

Proof: The quota is given by $\overline{x} = \widehat{x}$ with $\left(aB\widehat{x}^B\right)^{-1}(c\widehat{x}+F) = \left(aBx(t)^B\right)^B((t+c)x(t)+F)$. From this two things are observed. Firstly, $G'(\overline{m}) = G'(m(t))$ so that $\overline{m} = m(t)$. Secondly, $\overline{x} = \widehat{x} < x(t)$. By assumption $\overline{x} > x^*$ where x^* minimizes $\left(ax^B\right)^{-1}((y+c)x+F)$. Notice that $x(t)$ minimizes $\left(aBx^B\right)^{-1}((c+t)x+F)$ It, thus, follows that:

$$\frac{(y+c)\overline{x}+F}{a\overline{x}^B} < \frac{(y+c)x((t)+F)}{ax(t)^B}$$

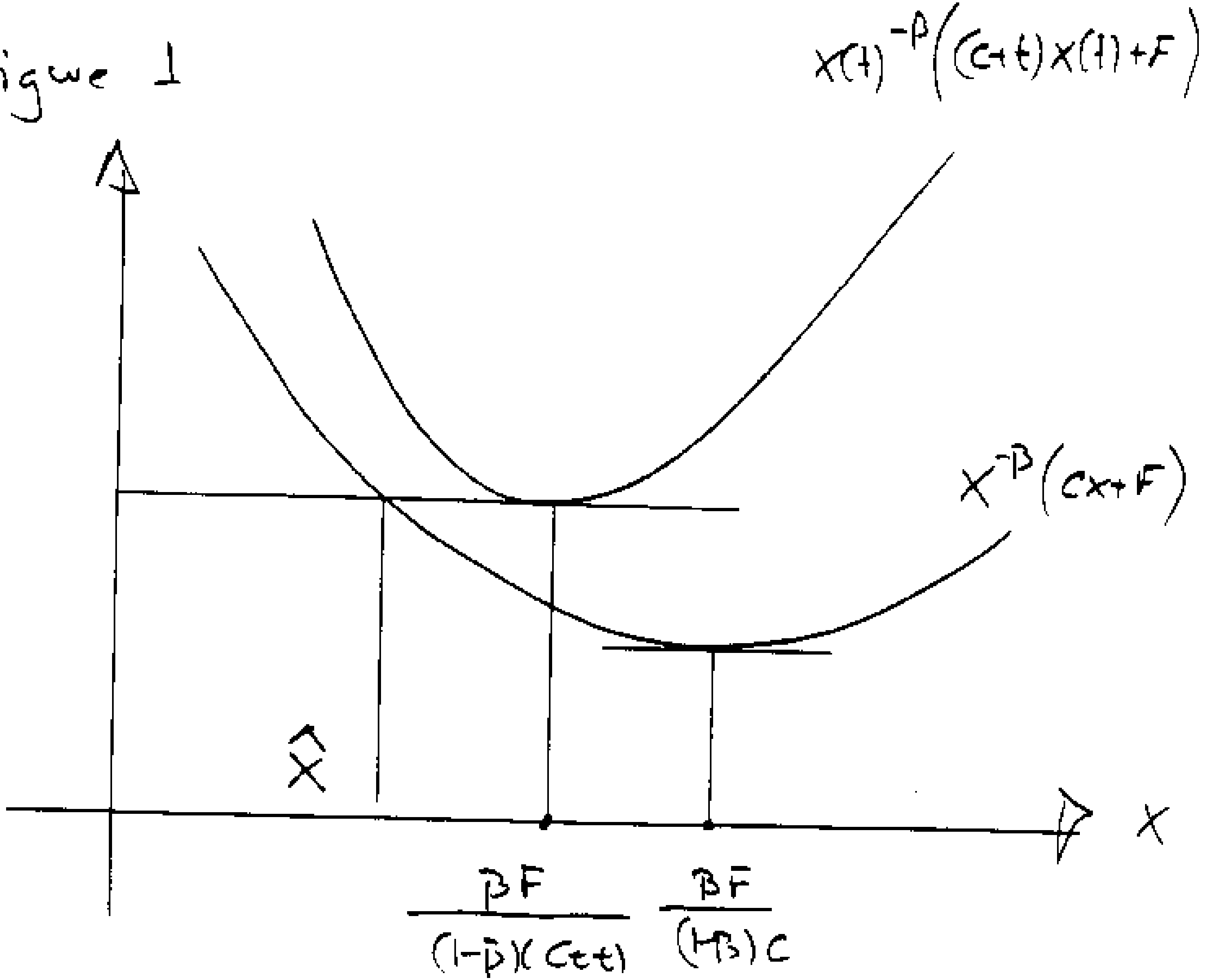
as $\overline{x} = \widehat{x}$, and from this

$$G(\overline{m}) - \overline{m}\frac{(y+c)\overline{x}+F}{a\overline{x}^B} > G(m(t)) - m(t)\frac{(y+c)x(t)+F}{ax(t)^B}$$

[8] Now $x = (1-B)^{-1}(y+c)^{-1}BF$ minimizes the right-hand side of (8). Since $x < x(t) = (1-B)^{-1}(t+c)^{-1}BF$ as $t < y$ it must be the case that the right-hand side of (8) increases with x at $x(t)$.

[9] The right-hand side of (10) is minimized for $x = (1-B)^{-1}c^{-1}BF > x(t)$. Thus, the right-hand side of (10) is decreasing in x at $x(t)$.

Figure 1



$x(t)^{-\beta}\left((c+t)x(t)+F\right)$
$x^{-\beta}(cx+F)$
$\hat{x}$
x
$\frac{\beta F}{(1-\beta)(c+t)}$
$\frac{\beta F}{(1-\beta)c}$

q.e.d.

Figure 1 about here

The idea of the proof can be said to reverse the reasoning of proposition 1. In that proposition the quota and the tax implement the same output at the firm level but market congestion differs between the two instruments. Here the quota and the tax involve identical market congestion but firm output is lower under the quota. This is the same as there being more firms under the quota compared to the tax illustrating once again the important role of the zero-profit condition. As for proposition 2 it is unclear whether the basic assumption, $\hat{x} \geq x^*$, can be satisfied. To show this, use once again $G(m) = m^s$, $0 < s < 1$ for which the optimal tax is derived explicitly (equation (6)). The requirement of proposition 2 is:

$$\left.\frac{cx+F}{aBx^B}\right|_{x=x^*} > \left.\frac{(c+t)\,x+F}{aBx^B}\right|_{x=x(t)} \tag{11}$$

for $x^* = ((1-B)(y+c))^{-1} BF$ and $x(t) = ((1-B)(t+c))^{-1} BF$. Manipulations show this inequality to be satisfied for[10]

$$\left(\frac{y+c}{t+c}\right)^B \left(1 - B\frac{y}{y+c}\right) > 1 \tag{12}$$

Using the optimal tax, table 2 shows that the condition in proposition 2 can be met. Table 2 shows the condition in proposition 2 is more likely to be satisfied

[10] Using $x^* = ((1-B)(y+c))^{-1} BF$ and $x(t) = ((1-B)(t+c))^{-1} BF$:

$$\left(\frac{B}{1-B}\frac{c}{y+c}F + F\right)\left(\frac{BF}{(1-B)(t+c)}\right)^B > \left(\frac{B}{1-B}F + F\right)\left(\frac{BF}{(1-B)(y+c)}\right)^B$$

or

$$\left(\frac{y+c}{t+c}\right)^B \left(\frac{B}{1-B}\frac{c}{y+c} + 1\right) > \frac{1}{1-B}$$

or

$$\left(\frac{y+c}{t+c}\right)^B \left(B\frac{c}{y+c} + 1 - B\right) > 1$$

or

$$\left(\frac{y+c}{t+c}\right)^B \left(1 - B\frac{y}{y+c}\right) > 1$$

as s increases. An increase in s lowers the tax rate and this increases the right-hand side of (10). Alternatively, as the tax goes down $\hat{x}$ goes up making it more likely that the condition is satisfied, cf. figure 1. The reason that the condition is less likely to be satisfied as y increases is that the tax increases with y. This forces $\hat{x}$ down and eventually it will reach and pass x^* so that the assumption in the proposition is no longer satisfied.

Table 2

	y = 0.1c			y = c			y = 2c		
B\s	0.25	0.50	0.75	0.25	0.50	0.75	0.25	0.50	0.75
0.25	1.03	1.52	1.52	0.92	1.36	1.36	0.88	1.29	1.30
0.50	1.40	1.46	1.50	1.28	1.34	1.41	0.98	1.02	1.08
0.75	1.17	1.19	1.21	1.10	1.11	1.13	0.73	0.74	0.75

The discussion centered around proposition 1 and 2, and the examples employing the utility function $G(m) = sm^s$ illustrate some aspects of the role of the zero-profit condition in relation to the regulation of firms' detrimental environmental effects. Proposition 3 (proved in the appendix) is the paper's most general conclusion in the sense that it does not rely on a specific utility function.

Proposition 3.
If $B \leq (1+2\,y/c)^{-1}(1+y/c)$ a quota implements higher welfare than a tax.

The condition in proposition 3 is sufficient for a quota to be preferable to a tax. As a matter of fact, our previous analysis has identified a case like $B = 0.75$ and $y = c$ violating the condition of the proposition but with quotas giving higher social welfare than a tax (cf. table 2). On the other hand, and as noted already: for $B \to 1$ the best tax support the social optimum so that one cannot hope to establish that quotas are in all circumstances better than taxes. This would also be remarkable, since, to follow Spence's (1976) opening remarks on his paper on set-up costs and product variety: a paper can have a simply stated goal where the pursuit of it is less easy than the stating of the goal.

III. Conclusions

According to Cropper and Oates (1992) regulators prefer environmental regulation in the form of quotas rather than that of taxes. If the two types of instruments are allocatively alike, as they are under perfect competition, the explanation for this can appeal to the nature of the policy making process, for example that direct control is liked over indirect measures like a tax. This paper's result offers the simple explanation that quotas are better in some circumstances. And if they are then this can be a much more straightforward explanation for regulators liking for quotas.

The basic explanation for quotas' superiority is that a market solution considers profit at the appropriate margin, while the social optimum reflects the consumer's surplus. When product variety is explicitly valued this gives a bias towards quotas since they leave more revenue in the industry compared to a tax. In this way more firms survive under quota regulation and this enhances product variety. This result is novel since the comparison of prices and quantities along the lines of Weitzman (1974) has entirely different focus.

Of course, due to the nature of the model, there are several obvious extensions to be pointed to. Firstly one can note that quotas follow the grandfarthering principle. The conclusions would surely be changed if firms were to buy quotas. It is clearly also of interest to investigate the possibilities of recycling the tax revenue to the industry. At least in theory this removes an important drawback of taxes. Whether competition legislation allow for such action is another matter.

A further important extension, following Spence (1976), is to study the biases against particular kinds of commodities allowing firms' cost and role to consumers to differ. The bias is against commodities whose revenues capture smaller fractions of the contributed (social) surplus. Such an approach naturally allows for a richer discussion of taxes versus tradeable quotas. If the two instruments affects the difference between the commodities' gross benefits and revenue then they would have different impact.

As a final conclusion attention is directed to proposition 1 showing that a quota restricting firm output to the level implemented by a tax is preferable to the tax in a range of circumstances. This result will possibly extend the policy oriented result of Oates and Strassman (1984). With reference to the literature on environmental tax in monopoly markets Oates and Strassman (1984) shows that it is relatively safe to ignore the monopoly effects and go for simple competitive Pigouvian tax. An approach like this is relevant when the regulator fails to have the information or the authority needed to correct for the monopoly effects. Future research can examine the conditions for which the regulator should opt for some simple quota not demanding precise knowledge about demand and cost conditions.

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Appendix

I. Proof of Proposition 1:

The quota is given as $\overline{x} = x(t) = (1-B)^{-1}(t+c)^{-1}BF$. The congestion index for a quota and a tax are:

$$G'(\overline{m}) = \frac{c\overline{x}+F}{aB\overline{x}^B} \tag{A.1A}$$

$$G'(m(t)) = \frac{(t+c)\,x(t)+F}{aBx(t)^B} \tag{A.1B}$$

The aim is to demonstrate that

$$G(\overline{m}) - \overline{m}\frac{(y+c)\overline{x}+F}{a\overline{x}^B} \geq G(m(t)) - m(t)\frac{(y+c)\,x(t)+F}{ax(t)^B} \tag{A.2}$$

Since $\overline{x} = x(t) = (1-B)^{-1}(t+c)^{-1}BF$, we will write simply x for now. Due to concavity of $G()$: $\overline{m} > m(t)$. Also, from concavity of $G()$:

$$\frac{G(\overline{m}) - G(m(t))}{\overline{m} - m(t)} > G'(\overline{m}) \tag{A.3}$$

or

$$\frac{G(\overline{m}) - G(m(t))}{\overline{m} - m(t)} > \frac{c\overline{x}+F}{aB\overline{x}^B} \tag{A.4}$$

If it is possible to show

$$\frac{c\overline{x}+F}{aB\overline{x}^B} \geq \frac{(c+y)\overline{x}+F}{a\overline{x}^B} \tag{A.5}$$

then the quota $\overline{x} = x(t)$ is better than a tax since:

$$\frac{G(\overline{m}) - G(m(t))}{\overline{m} - m(t)} > \frac{(c+y)\,x+F}{ax^B} \tag{A.6}$$

or

$$G(\overline{m}) - \overline{m}\frac{(c+y)\,x+F}{ax^B} > G(m(t)) - m(t)\frac{(c+y)\,x+F}{ax^B} \tag{A.7}$$

with $x = x(t)$.

Notice that (A.5) implies $G'(\overline{m}) \geq G'(m^*(\overline{x}))$ or $\overline{m} < m^*(\overline{x})$ where $m^*(\overline{x})$ is optimal produced variety given the output $\overline{x}$.

Now (A.5) reduces to

$$c\overline{x} + F > B((c-y))\overline{x} + F \tag{A.8}$$

or, using the expression for x:

$$\frac{cBF}{(1-B)(c+t)} + F > B\left(\frac{(c+y)BF}{(1-B)(c+t)} + F\right) \tag{A.9}$$

$$\frac{cBF}{(1-B)(c+t)} + F > B\left(\frac{B}{1-B}\frac{c+t}{c+t}F + F + \frac{B}{1-B}\frac{y-t}{c+t}F\right) \tag{A.10}$$

Cancelling F

$$\frac{cB}{(1-B)(c+t)} + 1 > B\left(\frac{1}{1-B} + \frac{B}{1-B}\frac{y-t}{c+t}\right) \tag{A.11}$$

or

$$\frac{B}{1-B}\frac{c}{c+t} + \frac{1-2B}{1-B} > B\frac{B}{1-B}\frac{y-t}{c+t} \tag{A.12}$$

or

$$\frac{Bc}{c+t} + 1 - 2B > B^2\frac{y-t}{c+t} \tag{A.13}$$

or

$$1 > 2B + B^2\frac{y-t}{c+t} - B\frac{c}{c+t} \tag{A.14}$$

or

$$1 > B\left[1 + 1 - \frac{c}{c+t} + B\frac{y-t}{c+t}\right] \tag{A.15}$$

This reduces to

$$1 > B\left[1 + \frac{t}{c+t} + B\frac{y-t}{c+t}\right] \tag{A.16}$$

This proves proposition 1.

II. The optimal tax where $G(m) = m^s$, $0 < s < 1$.
With a tax, t, the equilibrium is

$$x = \frac{BF}{(1-B)(t+c)} \tag{A.17}$$

$$sm^{s-1} = \frac{(t+c)x + F}{Bax^B} \tag{A.18}$$

Welfare is

$$W = m^s - m\frac{(y+c)x + F}{ax^B} \tag{A.19}$$

From (A.19):

$$\frac{dW}{dt} = \left[sm^{s-1} - \frac{(y+c)x+F}{ax^B}\right]\frac{dm}{dt} - m\frac{\partial}{\partial x}\left[\frac{(y+c)x+F}{ax^B}\right]\frac{dx}{dt} \tag{A.20}$$

From (A.17):

$$\frac{dx}{dt} = -\frac{BF}{(1-B)(t+c)^2} \tag{A.21}$$

Using (A.17) in (A.18):

$$sm^{s-1} = \frac{F}{1-B}\frac{1}{aB}\frac{(1-B)^B(t+c)^B}{(BF)^B} \tag{A.22}$$

From (A.22):

$$\frac{dm}{dt} = \frac{m}{s-1}\frac{B}{t+c} \tag{A.23}$$

Finally

$$\frac{\partial}{\partial x}\left[\frac{(y+c)x+F}{ax^B}\right] = \frac{(1-B)(y+c) - \frac{BF}{x}}{ax^B} \tag{A.24}$$

Using (A.18) for sm^{s-1}, (A.23) for dW/dt, (A.24) for $\partial/\partial x\left(\left((y+c)x+F\right)/ax^B\right)$, and $(A.21)$ for dx/dt:

$$\frac{dW}{dt}=\left[\frac{(t+c)\,x+F}{Bax^{B}}-\frac{(y+c)\,x+F}{ax^{B}}\right]\frac{m}{s-1}\frac{B}{t+c}+m\frac{(1-B)\,(y+c)-\frac{BF}{x}}{ax^{B}}\frac{BF}{(1-B)\,(t+c)} \tag{A.25}$$

Setting $dW/dt=0$ (A.25) comes down to

$$\frac{1}{s-1}\left[(t+c)\,x+F-B\left((y+c)\,x+F\right)\right]+\frac{(1-B)\,(y+c)-\frac{BF}{x}}{(1-B)\,(t+c)}BF=0 \tag{A.26}$$

Using the definition of x:

$$\frac{1}{s-1}\left[\frac{BF}{1-B}+F-B\left(\frac{B}{1-B}\frac{y+c}{t+c}+F\right)\right]+\frac{(1-B)\,(y+c)-(1-B)\,(t+c)}{(1-B)\,(t+c)}BF=0 \tag{A.27}$$

or

$$\frac{1}{s-1}\left[\frac{1}{1-B}-\frac{B^{2}}{1-B}\frac{y+c}{t+c}-B\right]+B\frac{y+c}{t+c}-B=0 \tag{A.28}$$

Solving:

$$\frac{y+c}{t+c}=1+\frac{1-B}{B^{2}+(1-B)\,(1-s)} \tag{A.29}$$

or

$$t=\frac{B^{2}+(1-B)\,B\,(1-s)}{B^{2}+(1-B)\,B\,(1-s)+(1-B)}\,(y+c)-c \tag{A.30}$$

Proof of proposition 3: For $B=(1+2\,y/c)^{-1}\,(1+y/c)$ the first best solution can be achieved, (cf. note 5). To prove that a quota is better than a tax in the remainder of cases, let:

$$\overline{x}=x^{*}=\frac{BF}{(1-F)\,(y+c)} \tag{A.31}$$

From proposition 2 we concentrate on $x^{*}>\widehat{x}$. Notice that since $t<y$ (cf. note 4) we have $x\,(t)>x^{*}$ and then from figure 1 that $G'\,(\overline{m})<G'\,(m\,(t))$ or $\overline{m}>m\,(t)$. $\overline{m}$ and $m\,(t)$ are defined by:

$$G'(\overline{m}) = \frac{c\overline{x} + F}{aB\overline{x}^B} \tag{A.32}$$

$$G'(m(t)) = \frac{(c+t)\,x(t) + F}{aBx(t)^B} \tag{A.33}$$

If

$$G(\overline{m}) - \overline{m}\frac{(y+c)\overline{x} + F}{a\overline{x}^B} \geq G(m(t)) - m(t)\frac{(y+c)\,x(t) + F}{ax(t)^B} \tag{A.34}$$

a quota is preferable to a tax. Since x^* minimizes, $(ax^B)^{-1}((y+c)\,x + F)$, we have:

$$G(\overline{m}) - \overline{m}\frac{(y+c)\,x^* + F}{ax^{*B}} \geq G(\overline{m}) - \overline{m}\frac{(y+c)\overline{x} + F}{a\overline{x}^B} \tag{A.35A}$$

Of course, we also have

$$G(m(t)) - m(t)\frac{(y+c)\,x^* + F}{ax^{*B}} > G(m(t)) - m(t)\frac{(y+c)\,x(t) + F}{ax(t)^B} \tag{A.35B}$$

Thus, the proposition follows if we prove

$$G(\overline{m}) - \overline{m}\frac{(y+c)\,x^* + F}{ax^{*B}} \geq G(m(t)) - m(t)\frac{(y+c)\,x^* + F}{ax^{*B}} \tag{A.36A}$$

or

$$\frac{G(\overline{m}) - G(m(t))}{\overline{m} - m(t)} \geq \frac{(y+c)\,x^* + F}{ax^{*B}} \tag{A.36B}$$

From concavity

$$\frac{G(\overline{m}) - G(m(t))}{\overline{m} - m(t)} > G'(\overline{m}) \tag{A.37}$$

But $G'(\overline{m}) = (aBx^{*B})^{-1}(cx^* + F)$. Thus, we must show

$$\frac{cx^* + F}{aBx^{*B}} \geq \frac{(y+c)\,x^* + F}{ax^{*B}} \tag{A.38A}$$

or

$$cx^* + F \geq B\left((y+c)\,x^* + F\right) \tag{A.38B}$$

or

$$\frac{cBF}{(1-B)\,(y+c)} + F \geq B\left(\frac{BF}{1-B} + F\right) \tag{A.38C}$$

or

$$\frac{B}{1-B}\frac{c}{y+c} + 1 \geq \frac{B}{1-B} \tag{A.38D}$$

or

$$B < (1 + 2\,y/c)^{-1}\,(1 + y/c) \tag{A.38E}$$

q.e.d.