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Information Systems, Incentives and the Timing of Investment.

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Abstract

The purpose of this paper is to study the effects of introducing information systems into a model featuring managerial incentive problems and investment opportunities that are mutually exclusive over time. In a principal-agent model in which a manager (agent) has superior information about investment costs, we introduce information systems, the signals from which are available to both the manager and the owner of the investment opportunity, which allow the owner to decrease the manager's informational advantage.

We examine (i) the characteristics of the optimal information systems; (ii) the effects of such information systems on the owner's investment and compensation choices and on the value of the investment opportunity to the owner; (iii) the effects of such information systems on the timing of investment; (iv) the effects of such information systems on the overall probability of investment; and (v) when the owner might want to improve the information system at a particular point in time.

1 Introduction

Recently, the theory of investment under uncertainty has undergone a revolution. This revolution stresses that, under circumstances where uncertainty is present, investment is irreversible, and an option exists as to when to accept an investment, the conventional net present value rule is incorrect (see good summaries of this work in, for example, Dixit and Pindyck, 1994, and Trigeorgis, 1996). Instead of setting a benchmark of zero against which the NPV of an opportunity must compete, the amended rule suggests that NPV must beat the value of the option to invest in the opportunity at some future date.

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The literature referred to above assumes a first-best world where the interests of investment decision-makers and the owners of investment opportunities are perfectly aligned. Further, it is implicitly assumed that the information relevant to the exercise of the investment opportunity is freely available to both investment decision-makers and the owners of investment opportunities. As a consequence, these analyses ignore the distribution of information and organizational context.

Another stream of literature emphasizes these features. In particular, in most organizations, information is decentralized, and better informed managers may not reveal relevant information unless they are properly compensated. For example, Antle and Eppen (1985) derive the optimal capital budgeting procedure when a manager has private information about one project's profitability at one point in time. The manager has a preference for slack consumption which arises when more resources are received from the owner than are needed to implement the investment opportunity. The owner is unable to detect when slack consumption occurs. As a consequence, the interests of owner and manager are not aligned with respect to investment decision-making. Antle and Eppen (1985) show that limiting the manager's information rents may involve the simultaneous existence of both capital rationing and budgetary slack. These results are extended to contexts where the manager has access to multiple projects in Antle, Bogetoft and Stark (1999a). Nonetheless, these analyses do not incorporate timing options.[1]

Antle, Bogetoft and Stark (1999b) extend Antle and Eppen (1985) to the consideration of an investment opportunity that can be accepted at one, and only one, of two possible dates.[2] The present value of the opportunity is known by both owner and manager. The manager has perfect information at each point in time at which the opportunity can be accepted concerning the cost of the project. At the earlier of these two points in time, owner and manager are similarly informed as to future project costs. In this model, the existence of the timing option distorts investment decision-making at both points in time. This is because the timing option gives the manager an option on slack consumption at the later date at which investment can occur that has to be taken into account when setting the optimal decision-making rule at the earlier date. The manager's slack option reduces the likelihood of investment at the earlier point in time below what it would be if the investment opportunity is a one-shot deal. Perhaps more surprisingly, the manager's slack option also reduces the likelihood of investment at the later point in time below what it would be if the investment opportunity were a one-shot deal. This effect arises because reducing the likelihood of investment at the later date also reduces the value of the manager's slack option and, as a consequence, the cost of investing at the earlier date.

[1] Other papers that study capital budgeting within a principal-agent framework include Baiman and Rajan (1995), Harris and Raviv (1996), Holmstrom and Weiss (1985), Rees (1986). Antle and Fellingham (1997) provides a useful summary of this type of literature.

[2] Antle and Fellingham (1990) analyse a two-period investment problem in which investment opportunities are available at both points in time.

Papers that emphasize the distribution of information and organizational context in analyzing investment decision-making tend to ignore the design of information systems the purpose of which is to reduce the information asymmetry between owner and manager.[3] The interaction between incentives, investment decision-making and information system design for a one-shot deal is considered, however, in Antle and Fellingham (1995). Within a setting again based upon that found in Antle and Eppen (1985), they explore the productive and distributional effects of alternative information systems about project profitability. The output of such information systems are available to both owner and manager. The owner's and manager's preferences over information systems are completely characterized for the case of uniformly distributed costs. These preferences are sometimes in conflict. To control the manager's slack, the owner prefers information systems that help distinguish costs, even in very profitable circumstances. To maximize his slack, the manager prefers information that help distinguish costs only in marginally profitable circumstances.

Nonetheless, little is known about the interaction between optimal investment rules, incentives, timing and information systems. As a consequence, the purpose of this paper is to study the effects of information system design on investment decision-making when there are incentive problems and when the timing of investment is at issue. Essentially we extend the analysis in Antle, Bogetoft and Stark (1999b) by introducing information systems and information system design considerations, and we extend, in part, the analysis in Antle and Fellingham (1995) by introducing timing issues.

We study a context in which a manager (agent) has superior information about the cost of an investment opportunity at time t_0, and at t_1 receives superior information about opportunity's cost in this period. The project can only be accepted once. In this set-up, we explore the design of costless information systems which generate imperfect information about investment costs at both t_0 and t_1 in the context of the owner's (principal's) planning and control problem, which is to develop optimal investment decision-making rules subject to the constraints created by the manager's strategic behavior. Unlike Antle and Fellingham (1995) we only consider the information system design from the point of view of the owner. Like Antle and Fellingham (1995), we make restrictive assumptions about the distribution of costs at t_0 and t_1 in order to achieve tractability and clarity of insight. Specifically, we assume that costs at t_0 and t_1 are uniformly distributed on the interval $[0, 1]$.

In particular, we examine:

1. the characteristics of optimal information systems;

2. the effects of such information systems on the owner's investment and compen-

[3] The value of information, although not the design of the information system, in an investment decision-making setting is considered by Gordon, Loeb and Stark (1990). Other studies have considered the value of information in a principal-agent setting without investment decision-making features. See, for example, Baiman and Evans (1983), Christensen (1981) and Penno (1984).

sation choices and on the value of the opportunity to the owner;

3. the effects of such information on the timing of investment;

4. the effects of such information on the overall probability of investment; and

5. when the owner might want to improve the information system at a particular point in time.

The remainder of the paper is organized as follows. Section 2 presents the basic assumptions and notation. Section 3 outlines the effects of information systems on the owner's profit. Section 4 characterizes the owner's most preferred information systems. Some applications of these results are provided in section 5. Section 6 concludes.

2 The Model

A risk neutral owner can invest in an opportunity at either t_0 or t_1, but not both. The investment project has a present value of 1 when undertaken. The investment must be implemented by a manager. The manager learns the investment costs, $c_0 \in C_0 = [0, 1]$ and $c_1 \in C_1 = [0, 1]$, immediately prior to t_0 and t_1 respectively. The owner does not know the costs at either time. The owner believes the investment costs in the two periods are independent and uniformly distributed. The manager shares these beliefs before becoming informed.

Immediately prior to t_0, the owner installs a costless information system I, that will provide additional information at both t_0 and t_1 about the costs of investment at those times. The signal is available to both owner and manager and, hence, can be used for contracting purposes. The information system can be split into two elements: the information system at t_0, denoted I_0, and the information system at t_1, denoted I_1. Thus, $I = \{I_0, I_1\}$. For simplicity, we shall concentrate on information systems corresponding to order preserving partitions of the sets of possible costs. Thus, for example, an order preserving *m-partition* of C_0 is a partition into m subsets $I_{01} = [a_{00}, a_{01}]$, $I_{02} = (a_{01}, a_{02}]$, ..., $I_{0m} = (a_{0m-1}, a_{0m}]$, where $0 = a_{00} < a_{01} < ... < a_{0m} = 1$. Therefore, the owner, by observing $I_0 = \{I_{01}, ..., I_{0m}\}$, learns to which of the intervals $I_{0i} \in I_0$, $i \in \{1, ...m\}$, the cost c_0 belongs. This interval is denoted $I(c_0)$, and its index is denoted $i(c_0)$. Similar notation is used for I_1, an order preserving *n-partition* of C_1. The manager receives the same signals as the owner from the information system. We do not assume that the information systems at t_0 and t_1 have the same design and, in particular, the same number of elements.

We treat m and n as indicators of the level of detail of the information systems at t_0 and t_1. Specifically, as either m or n increases, so does the level of detail of the relevant information system.

The owner must transfer to the manager the funds required to carry out the investment. Let y_i denote the amount transferred from the owner to the manager at time t_i ($i = 0, 1$). We assume that the manager can consume any funds transferred in excess of those needed to carry out the investment. Hence, if investment is undertaken at time t_i, the manager enjoys the slack of $s_i = y_i - c_i$ at time t_i and of $s_j = y_j$, $j \neq i$ at t_j. It is often useful to think of slack as compensation and to consider the owner as paying the investment cost, c_i, and the manager's compensation, s_i.

The owner's objective is to develop investment and compensation strategies to maximize the net present value of the opportunity. We assume that both the owner and manager have the same time preferences as represented by the interest rate $\rho \geq 0$ with corresponding discount factor $k = 1/(1+\rho) \leq 1$.[4]

It is advantageous for the owner to ask the manager to report the costs c_0 and c_1 as he learns them. The owner may use the possibly manipulated reports in his choice of investments and transfer to the manager. We assume that the owner can commit to a contract covering both points in time at t_0. We will discuss the significance of, and justification for, this assumption below.

To formalize the model, we define contracts as consisting of cost- and information-dependent investment decision-making and compensation policies:

$$\begin{aligned} d_0[.,.] &: C_0 \times \{1, ..., m\} \rightarrow \{0, 1\} \\ d_1[.,.,.,.] &: C_0 \times C_1 \times \{1, ..., m\} \times \{1, ..., n\} \rightarrow \{0, 1\} \\ s_0[.,.] &: C_0 \times \{1, ..., m\} \rightarrow \Re_0 \\ s_1[.,.,.,.] &: C_0 \times C_1 \times \{1, ..., m\} \times \{1, ..., n\} \rightarrow \Re_0 \end{aligned}$$

where, dropping arguments, $d_i = 1$ indicates investment and $d_i = 0$ indicates no-investment at time t_i. Similarly, s_i denotes the manager's compensation at time t_i.

It is easy to ensure that the manager will not report a c_i outside $I(c_i)$, for some t_i, by simply setting $s_i = 0$ for such a report. Therefore, we focus on schemes that ensure no manipulation of reports within the $I(c_0)$ or $I(c_1)$ sets.

Given an information system, I, the owner selects the investment and compensation plans that maximize the net present value of the opportunity:

$$\begin{aligned} \int_{C_1}\int_{C_0} &[d_0(c_0, i(c_0))(1 - c_0) - s_0(c_0, i(c_0)) + k(d_1(c_0, c_1, i(c_0), i(c_1))(1 - c_1) \\ &- s(c_0, c_1, i(c_0), i(c_1)))]dc_0dc_1 \end{aligned} \tag{OF}$$

subject to four classes of constraints.

The first class of constraints refer to the investment plans:

$$d_0(c_0, i(c_0)) \in \{0, 1\} \text{ and } d_1(c_0, c_1, i(c_0), i(c_1)) \in \{0, 1\}, \ \forall c_0, c_1 \tag{1}$$

[4]We assume the manager's discount rate is the same as the owner's because this assumption implies neither party has a comparative advantage in storage, and helps isolate the effects of information systems on incentives and investment policy.

$$d_0(c_0, i(c_0)) + d_1(c_0, c_1, i(c_0), i(c_1)) \leq 1, \ \forall c_0, c_1. \tag{2}$$

This class of constraints has two subclasses. The first (1) restrict the investment policy to be an indicator variable, and so defines the decision as 'accept / don't accept'. Note that 'don't accept' is not the same outcome as reject at t_0 because the opportunity can be reconsidered at t_1. The second subclass of constraints (2) captures the real option nature of the investment by imposing mutual exclusivity.

The next class of constraints reflect the fact that the manager is unable to use his/her own resources to implement the investment opportunity. As a consequence, the owner must provide all the resources necessary for any implementation:

$$s_0(c_0, i(c_0)) \geq 0 \text{ and } s(c_0, c_1, i(c_0), i(c_1)) \geq 0, \ \forall c_0, c_1 \tag{3}$$

By requiring the manager's compensation to be nonnegative in each period, constraints (3) effectively ensure that the owner pays all the production costs.[5]

The final class of constraints induce the manager to reveal the investment cost truthfully at each point in time. Assuming that the owner can commit to his use of the manager's cost reports, the revelation principle (Myerson, 1979) implies that, without loss of generality, we may restrict attention to investment and compensation plans that induce truthful reporting. We break this class of constraints into two sets. The first set (4) ensures truthtelling at t_1 regardless of the manager's t_0 message. The second set (5) ensures truthtelling at t_0 provided the manager's tells the truth at t_1. One can verify that these constraints are equivalent to the full set of constraints guaranteeing truthtelling in both periods. In terms of our notation:

$$\begin{aligned} s_1(c_0, c_1, i(c_0), i(c_1)) &\geq s_1(c_0, c_1', i(c_0), i(c_1)) + d_1(c_0, c_1', i(c_0), i(c_1))(c_1' - c_1), \\ &\forall c_0 \text{ and } \forall c_1, \ c_1' \in I(c_1) \end{aligned} \tag{4}$$

$$\begin{aligned} s_0(c_0, i(c_0)) + k \int_{C_1} s_1(c_0, c_1, i(c_0), i(c_1)) dc_1 &\geq s_0(c_0', i(c_0)) + d_0(c_0', i(c_0))(c_0' - c_0) \\ + k \int_{C_1} s_1(c_0', c_1, i(c_0), i(c_1)) dc_1, \ \forall c_1 &\text{ and } \forall c_0, c_0' \in I(c_0) \end{aligned} \tag{5}$$

The resulting mathematical program is rather complex. Results in Antle, Bogetoft and Stark (1999b) are helpful in simplifying the analysis. A simple extension of their results suggests that optimal investment and compensation rules can be specified by a vector of $m + n$ different cost targets. m cost targets are defined for t_0, denoted by the vector $\mathbf{c}_0^T = (c_{01}^T, c_{02}^T, ..., c_{0m}^T)$, and n cost targets are defined at t_1, denoted

[5] These constraints are the only ones that refer to the absolute size of the manager's compensation. Implicitly, we are assuming that the manager has a reservation utility of 0, where utility equals the present value of compensation. The constraints on the size of compensation ensure that the manager at least receives a utility from employment of 0. As a consequence, a separate constraint is redundant.

by the vector $c_1^T = (c_{11}^T, c_{12}^T, ..., c_{1n}^T)$. The cost targets can be given the following interpretation. First, if the information observed at t_0 is I_{0i}, c_{0i}^T is the highest cost such that all $c_0 \in I_{0i}$ equal to or less than it lead to investment at t_0. Second, if the information observed by the owner at t_1 is I_{1i}, c_{1i}^T is the highest cost such that all $c_1 \in I_{1i}$ equal to or less than it lead to investment at t_1 if investment did not take place at t_0Formally,

$$d_0(c_0, i(c_0)) = \begin{cases} 1 & \text{if } \; a_{0i(c_0)-1} < c_0 \leq c_{0i(c_0)}^T \\ 0 & \text{otherwise} \end{cases}$$

$$d_1(c_0, c_1, i(c_0), i(c_1)) = \begin{cases} 1 & \text{if } d_0(c_0, i(c_0)) = 0, \; a_{1i(c_1)-1} < c_1 \leq c_{1i(c_1)}^T \\ 0 & \text{otherwise} \end{cases}$$

The targets also fully characterize the optimal compensation policies. Again, a simple extension of the results in Antle, Bogetoft and Stark (1999b) shows that the optimal compensation plan, as a function of the targets, is:

$$s_0(c_0, i(c_0)) = d_0(c_0, i(c_0))[c_{0i(c_0)}^T - c_0 + k \int_{C_1} s_1(c_0, c_1, i(c_0), i(c_1)) dc_1]$$

$$s_1(c_0, c_1, i(c_0), i(c_1)) = d_1(c_0, c_1, i(c_0), i(c_1))[c_{1i(c_1)}^T - c_1]$$

The form of the optimal compensation is revealing. The manager's compensation at t_1, s_1, equals the excess of the target cost over the actual cost, when investment takes place at t_1. This form of the manager's compensation is a direct implication of the truthtelling conditions. Getting the manager to reveal the investment cost at t_1 entails paying him all the cost savings relative to the t_1 target cost for the information interval in which the true cost lies (i.e., $c_{1i(c_1)}^T - c_1$). If no investment takes place at t_1, the manager's compensation is 0.

The manager's compensation at t_0, s_0, is more complicated. If investment is to take place at t_0, giving the manager the cost saving relative to the t_0 target cost is not sufficient to get the manager to reveal the cost of investment at that time. When investment is undertaken at t_0, the manager's option on slack at t_1 is destroyed. Therefore, as pointed out by Antle, Bogetoft and Stark (1999b), to provide the manager with incentives to truthfully reveal the cost of investment at t_0, the manager must be compensated for the loss of expected slack at t_1. These two effects are given in the formula for the optimal s_0. The term $c_{0i(c_0)}^T - c_0$ gives the manager the cost savings at t_0. $k \int_{C_1} s_1(c_0, c_1, i(c_0), i(c_1)) dc_1$ gives the manager the present value of his expected slack at t_1. If no investment takes place at t_0, the manager's compensation is 0.

It is important to note that, from the manager's perspective at t_0, the manager *always* receives any expected slack from possible investment at t_1. If investment occurs at t_0 (i.e., $d_0(c_0, i(c_0)) = 1$), the manager receives the present value of expected slack from possible investment at t_1 in his t_0 compensation. If investment does not occur at t_0 (i.e., $d_0(c_0, i(c_0)) = 0$), the manager receives expected slack through the t_1 investment problem. This means that *the manager benefits dollar-for-dollar by any slack created at* t_1.

Nonetheless, the form of compensation contract at t_0 (in particular, the existence of a valuable option on expected slack at t_1 held by the manager at t_0) depends crucially on the assumption that the owner can commit to the manager over both periods. Suppose, as an alternative, the owner could commit to the firing of the manager after t_0 if investment does not take place and the hiring of a strictly different manager at t_1The manager would possess no t_0 option on expected slack at t_1 if this is the case. Nonetheless, committing to thiscourse of action is only unequivocally economically rational if there are no costs associated with changing managers. There will be costs if the existing manager possesses advantages over other available managers in the labor market. Further, if the existing manager does possess such advantages, it is not necessarily advantageous to commit to replacing him with complete certainty if investment does not take place at t_0. We justify our commitment assumption, therefore, on the basis that the managerial labor market is not sufficiently complete to ensure that managers can be replaced costlessly and that the costs of replacing managers are sufficiently high to discourage such a course of action.

Because the cost targets determine the compensation, for a given information system, $I = \{I_0, I_1\}$, the optimal investment and compensation policies can be determined by optimizing over the $n + m$ cost targets. To determine the optimal information system, we need an additional $n + m - 2$ variables corresponding to the possible division points between $a_{00} = 0$ and $a_{0m} = 1$ and between $a_{10} = 0$ and $a_{1n} = 1$. As we shall see in the next two sections, however, the optimal information system from the owner's point of view is actually characterized by two parameters corresponding to the highest costs at which investment occurs at t_0 and t_1 respectively.

3 The Effects of Information System Design and Investment Decision-Making Policy

To explore these effects, it is useful to begin with a broad analysis of how the t_0 and t_1 profit and slack affect the owner's overall profit. Let Π_0 and S_0 represent expected profit and compensation respectively, for some combination of information system design and cost targets. Let Π_1 and S_1 represent expected profit and compensation respectively, conditional on investment not taking place at t_0, for some combination of information system design and cost targets. By profit we mean revenues net of target cost transfers. Let $p(I)$ represent the probability of investment taking place at t_0, given the combination of information system design and cost targets. The net present value of the opportunity to the owner, Π, then is given by:

$$\Pi = \Pi_0 - p(I)kS_1 + k(1 - p(I))\Pi_1 \tag{6}$$

To see this, observe that with probability $p(I)$, investment takes place at t_0. If investment takes place at t_0, the owner transfers the cost target plus an amount equal to kS_1 to the manager. The first two terms on the right hand side of the equation capture the effect of the revenues less these total transfers. Investment is considered at t_1 with probability $(1 - p(I))$. Conditional on investment not taking place at t_0, profits are Π_1. Hence, the last expression on the right hand side of the equation captures the contribution of possible investment at t_1 to the value of the opportunity to the owner.

Inspecting equation (6) reveals some interesting insights into the owner's preferences. Straightforwardly, for a given probability of investing at t_0, the owner prefers, *ceteris paribus*, higher Π_0 and Π_1. The owner is indifferent to the size of S_0, other than to the extent it affects Π_0. Perhaps less straightforwardly, for a given probability of investing at t_0, the owner prefers, *ceteris paribus*, a lower S_1.

The owner's desire to maximize profit at t_0 and t_1 is not surprising. Both directly increase the value of the owner's investment opportunity, with an increase in t_1 profit increasing the value of the owner's option to wait. More surprisingly, the owner also cares about the minimization of t_1 slack. Reducing S_1 eases the owner's incentive problem with the manager at t_0, regardless of the level of Π_1. This reflects an important effect of the *manager's* timing option.

We can contrast these preferences with those that hold for the one-shot deal setting analyzed in Antle and Fellingham (1995). In a static world (i.e. in a one period model) analyzed there, the owner is interested in the manager's slack only to the extent that it comes *at the expense of* profit. That is, the owner is not interested in the manager's slack, given a fixed level of profit and likelihood of investment. In a multi-period model, however, slack in later periods represents an option to the manager, and this option makes it more difficult to control incentives in the initial period. The enlarged set of preferences identified above are related to the timing option associated with the investment opportunity and the consequent linking of the periods. We now examine in detail how these expanded preferences feed through into information system design.

4 The Owner's Optimal Information System

In the previous section, we have identified the owner's general concerns in the design of the information system, the setting of investment policy and the compensation scheme. We shall now show how this pins down the design of the owner's optimal information systems and investment policies. All mathematical proofs are given in the appendix.

We initially show that the owner only rations capital in the highest interval of both I_0 and I_1. Further, as a consequence, the owner needs to simultaneously think about the highest cost at both t_0 and t_1 below which production will take place and the partitioning of the cost space below these cost levels. We provide the following theorem on these issues.

Theorem 1 *For given m and n, there exist $a_0^* \in (0,1]$ and $a_1^* \in (0,1]$ which fully characterize both the optimal information system design and the associated optimal investment policy such that:*

$$a_{00} = 0, \quad a_{0j} = \frac{j}{m} a_0^* \quad for \quad j = 1, ..., m-1, \quad a_{0m} = 1$$

$$c_{0j}^T = \frac{j}{m} a_0^* \quad for \quad j = 1, ..., m$$

and

$$a_{10} = 0, \quad a_{1j} = \frac{j}{n} a_1^* \quad for \quad j = 1, ..., n-1, \quad a_{1n} = 1$$

$$c_{1j}^T = \frac{j}{n} a_1^* \quad for \quad j = 1, ..., n$$

With this theorem, we show that to characterize the owner's most preferred overall information system and associated investment policy, we need specify only two parameters, a_0^* and a_1^*. The interpretation of a_0^* (a_1^*) is that it is the maximum cost leading to investment at t_0 (t_1). We also show that the optimal information system makes the highest production interval and the cost intervals below this production interval equally wide at each point in time.

This is similar, but not identical, to the optimal information system and investment policy developed for the one period, static investment problem in Antle and Fellingham (1995). In Antle and Fellingham (1995), the information regions are also equally wide, but the size of the no-investment region (i.e. of $(c_{0m}^T, 1]$ or $(c_{0n}^T, 1]$ in our terminology) is as wide as the cost intervals. As will be illustrated below, we do not get this last result here because the owner's aim is not just to maximize profit at t_1 but to minimize the associated slack as well. As pointed out above, in the static case, minimization of slack is not desirable *per se* because the principal does not care about incentive provisions in other periods.[6]

This difference between our analysis and that of Antle and Fellingham (1995) with respect to optimal information system design and investment policy is crucially

[6] In the extreme case of $n = m = 1$, we revert to the case considered in Antle, Bogetoft and Stark (1999b).

dependent upon an aspect of our assumption of commitment. The aspect here is that renegotiation does not occur after t_0 and prior to t_1, should investment not occur at t_0. Given our specification of the overall problem, which does not prevent the owner from adopting the Antle and Fellingham (1995) information system design and investment policy, clearly, such a design is not optimal, as evaluated at t_0. Nonetheless, if renegotiation were to take place, such a design would be adopted (i.e., the opportunity at t_1 would be treated as a one-shot deal if renegotiation were allowed).[7] We justify our assumption that no renegotiation takes place by assuming that the owner can credibly endow a third-party with the authority to fine the owner should renegotiation take place, and the size of the fine is sufficient to discourage such behavior.[8]

In our multiperiod model, the effects of allowing renegotiation in the absence of the ability to commit are fairly straightforward, as indicated above. This is not true in other settings, however. For example, a number of other papers (e.g., Arya, Glover and Sunder, 1998; Demski and Frimor, 1999, and Gigler and Hemmer, 1998) establish settings where accurate reporting of information is not necessarily desirable at some intermediate stage in the multiperiod problem and as a consequence, reported information is garbled. The rationale underlying these results is that the garbling damps down the effects of renegotiation and the inability to commit to a multiperiod contract.

Returning to the analysis, and given the results of Theorem 1, we have:

$$S_1 + \Pi_1 = a_1^*(1 - \frac{a_1^*}{2})$$

$$\Pi_1 = a_1^*(1 - \frac{(n+1)a_1^*}{2n})$$

and

$$\Pi_0 = a_0^*(1 - \frac{(m+1)a_0^*}{2m})$$

Inserting these expressions into equation (6) gives the following expression (OF) for the value of the investment opportunity to the owner:

$$OF = a_0^*(1 - \frac{(m+1)}{2m}a_0^*) + ka_1^*(1 - \frac{(n+1)}{2n}a_1^*) - ka_0^*a_1^*(1 - \frac{a_1^*}{2}) \tag{7}$$

[7] We assume in the argument above that both information system design and investment policy are open to renegotiation. Nonetheless, an alternative approach would be to assume that information system design is a particular form of commitment device (e.g., the design of an accounting system is standardised in advance of its use) which then only leaves investment policy subject to renegotiation.

[8] Similar arguments concerning commitments not to fire the manager at t_0 and not renegotiate at t_1 if investment does not take place at t_0 can be found in Antle, Bogetoft and Stark (1999b).

Therefore, a_0^* and a_1^* are the joint solutions to the following first order conditions:

$$\partial OF/\partial a_0^* = (1 - \frac{(m+1)}{m}a_0^*) - ka_1^*(1 - \frac{a_1^*}{2}) = 0 \tag{8}$$

$$\partial OF/\partial a_1^* = k[(1 - \frac{(n+1)}{n}a_1^*) - a_0^*(1 - a_1^*)] = 0 \tag{9}$$

Further, to ensure that these joint solutions represent a maximum for the problem, the second-order conditions need to be satisfied. Koo (1977) demonstrates that necessary second-order conditions for a maximum are:

$$\partial^2 OF/\partial a_0^{*2}, \partial^2 OF/\partial a_1^{*2} < 0$$

Straightforward further (partial) differention reveals that these conditions hold.[9] Koo (1977) also demonstrates that a further sufficient condition for a maximum is that:

$$\mid J \mid = \left| \begin{array}{cc} \partial^2 OF/\partial a_0^{*2} & \partial^2 OF/\partial a_0^* \partial a_1^* \\ \partial^2 OF/\partial a_1^* \partial a_0^* & \partial^2 OF/\partial a_1^{*2} \end{array} \right| > 0$$

We state the following lemma.

Lemma 2 *For the problem above, $\mid J \mid > 0$. Therefore, solutions to first-order conditions given in equations [8] and [9] represent a maximum to the owner's problem.*

We then have the following theorem.

Theorem 3 *The value of the investment opportunity increases with both m and n. Further, a_0^* increases with m and decreases with n, whereas a_1^* decreases with m and increases with n. In addition, the overall probability of investment:*

$$a_0^* + a_1^*(1 - a_0^*)$$

increases with m.[10]

It is not particularly surprising that the value of the investment opportunity increases with m and n. Clearly, the owner could replicate the result for an m-partition by setting one of the intervals in an $m+1$-partition equal to the null set. The same argument applies for n. As a consequence, the value of the opportunity must be at least *weakly* increasing in m and n.

[9] $\partial^2 OF/\partial a_0^{*2} = -\frac{(m+1)}{m} < 0; \partial^2 OF/\partial a_1^{*2} = k(a_0^* - \frac{(n+1)}{n}) < 0.$

[10] The first-order conditions also reveal that both a_0^* and a_1^* are dependent upon k, the discount rate. The dependency of optimal investment policy at both points in time on the discount rate, in the absence of information systems and in the presence of incentive problems, has been pointed out by Antle, Bogetoft and Stark (1999b). This paper reveals that optimal information system design at both points in time is similarly dependent upon the discount rate. In Theorem 5, we show that $\frac{da_0^*}{dk} < 0$ and $\frac{da_1^*}{dk} > 0$.

We note that increasing the number of partitions at any point in time also increases the (possibly conditional) probability of investment at that point in time and decreases the (possibly conditional) probability of investment at the other point in time. This result is not so obvious and is the result of a number of different effects, some of which are partially offsetting.

Consider the situation at t_0 initially. Increasing m reduces the cost of investing at t_0 by reducing slack payments for any fixed a_0^*, *ceteris paribus.* This suggests that an increase in a_0^* could be attractive to the owner. Nonetheless, such an increase also increases the expected payment resulting from motivating the manager to forego the slack option if investment is to take place at t_0 - a feature that makes the increase less attractive. To counteract this effect the owner can reduce the manager's slack option by reducing a_1^*, although this also reduces expected profits at t_1. Thus, there are countervaling influences on the attractions of changing investment policy at either point in time as a consequence of changing m. The net effect produces an increase in a_0^* with a reduction in a_1^* to reduce the manager's slack option.

Now consider the effect of increasing n. Here, the situation is similarly complex. *Ceteris paribus*, increasing n reduces the value of the manager's slack option. This increases expected profits at t_1. This makes investing at t_1 more attractive to the owner and suggests that an increase in a_1^* could be attractive. It also, however, makes investing at t_0 more attractive because of the reduction in the manager's slack option. Nonetheless, increasing a_1^* increases both expected profits and slack payments at t_1, holding n fixed. This last effect then increases the value of the manager's slack option, reducing the attraction of investing at t_0. There are, again, countervaling influences on the attractiveness of changing investment policy at either point in time as a consequence of increasing n. The balance favours increasing a_1^* and decreasing a_0^*.

Increasing m increases the overall probability of investment. Nonetheless, increasing n has an ambiguous effect. The derivative of the overall probability of investment with respect to n is given by the following expression:[11]

$$(1-a_1^*)\frac{da_0^*}{dn}+(1-a_0^*)\frac{da_1^*}{dn}=\frac{a_1^{*2}}{n^2\mid J\mid}[k(-1+3a_1^*-\frac{3a_1^{*2}}{2})+\frac{1}{m}]$$

Therefore, the sign of the derivative with respect to n is given by the sign of $k(-1+3a_1^*-\frac{3a_1^{*2}}{2})+\frac{1}{m}$. From Theorem 2, as m gets larger, a_1^* gets smaller. If a_1^* gets small enough and m large enough, then $k(-1+3a_1^*-\frac{3a_1^{*2}}{2})+\frac{1}{m}$ can be negative. An example of this effect is when $m=10$ and $k=1$. In this case, increasing n from 1 to 2 results in a decrease of a_0^* from .742 to .593, an increase in a_1^* from .205 to .449, and a fall in the overall probability of investment from .795 to .775.

This is an interesting effect. In a one-period world, decreasing incentive problems increases the probability of investment. Here, increasing n decreases incentive prob-

[11] The expression on the right hand side is a result of substituting for the appropriate derivatives given in the Appendix and a certain amount of rearranging using the first first-order condition.

lems at t_1 but *decreases* the overall probability of investment. This is related to the relative strength of the incentive problems at t_0 and t_1. In particular, for the overall probability of investment to decrease with n, incentive problems need to be much decreased at t_0 relative to t_1. In the example, a high value of m relative to n has achieved this outcome.

5 Illustrative Examples and Further Results

We gain some idea of how a_0^* and a_1^* change as m and n increase, and the associated changes in the owner's wealth and overall probabilities of investment, in Table 1. Further, the examples lead to a number of questions to which we respond with additional theorems. For the examples, we set $k = 1$. It is worth bearing in mind that, in the absence of incentive problems, investment would take place if t_0 cost is less than .5, and, if investment has not taken place at t_0, would take place for all costs at t_1. Hence, the overall probability of investment is 1In this situation, the owner's wealth is .625. We can use these features of the first-best solution as benchmarks when evaluating the results in the table below.

Table 1

Effect of m and n on Information System Design, Owner's Wealth, and Overall Probability of Investment ($k = 1$)

m	a_0^*	n	a_1^*	*Owner's Wealth*	*Probability of Investment*
1	.341	1	.397	.356	.603
1	.293	2	.586	.414	.707
1	.275	3	.685	.448	.772
2	.477	1	.343	.396	.657
3	.554	1	.308	.418	.692
5	.446	5	.735	.530	.853
5	.426	10	.852	.562	.915
10	.491	5	.718	.541	.856
10	.466	10	.842	.571	.916
10	.455	50	.965	.604	.981
50	.504	10	.832	.581	.917
50	.491	50	.962	.613	.981

As suggested by Theorem 3, as m increases, so does the overall probability of investment. Further, increases in m or n, holding the other parameter constant, produce opposite effects in a_0^* and a_1^*. As m and n increase, the value of the opportunity to the owner increases towards the first-best value. When $m = n = 1$, the value of the opportunity is less than 60% of the first-best value and the overall probability of investment is just over 60% of the first-best value. When $m = n = 5$, the value of the opportunity has risen to over 80% and the overall probability to over 85% of the first-best value. When $m = n = 50$, there are only small differences between the investment policies followed and the first-best policies, with little difference between the first- and second-best cases in terms of the value of the opportunity. We also note that, although not shown explicitly, a_0^* and a_1^* do not split the top interval of their respective information systems in half, as in Antle and Fellingham (1995).

The examples above make clear that it is not automatically the case that information system design will be identical between periods, even if $m = n$. Therefore, we now ask whether the information system specification can *ever* be identical between periods when $m = n$. Given the theorems above, for this to be the case requires $a_0^* = a_1^*$. Theorem 4 gives the answer.

Theorem 4 *If $m = n$, then $a_0^* \neq a_1^*$ unless $(m-2)k^2 + 6k - 4 = 0$.*

The implication of Theorem 4 is that the information system is identical in both periods only in special circumstances. An example of when identical information systems in each period are optimal is when $m = 3$ and $k = .605551$This value of k corresponds to a cost of capital of over 65%. In this case, $a_0^* = a_1^* = .5657$. As can be gathered by inspecting the condition in Theorem 3, as m increases, the value of k which allows the optimality of identical information systems in each period decreases and, hence, the implied cost of capital that must be in use to allow the optimality of identical information systems in each period increases.

The examples also suggest that there are systematic differences in the changes to the value of the opportunity to the owner as a result of increasing m versus n. In particular, Table 1 suggests that if $m = n$ the owner prefers to increase n rather than m. We provide in Theorem 5 some general conditions under which the owner prefers to increase n rather than m, or m rather than n.

Theorem 5 *If $m = n$, for 'high' values of k, and low values of the cost of capital, the owner prefers to increase n rather than m. For 'low' values of k, and high values of the cost of capital, the owner prefers to increase m rather than n. The definition of 'high' and 'low' depends upon the common value of m and n.*

Theorem 5 gives some sense of in which period it is best to invest in a more detailed information system, *ceteris paribus*. It states that if an equally detailed information system (i.e., $m = n$) is in place at both points in time, the owner has a preference for increasing the level of detail at t_1 rather than t_0 if k is sufficiently high. This is the case in Table 1. Nonetheless, if the cost of capital is high enough and, hence, k is low enough, this preference reverses. The intuition for this result is as follows. As k decreases, the importance of the timing option becomes less, both as an alternative investment opportunity to investing now and as a distortion to t_0 investment caused by the manager's slack option. As a consequence, the owner becomes more and more interested in investing at t_0 rather than t_1 and, hence, more and more interested in acquiring more detail on costs at t_0 as opposed to t_1.

6 Conclusion

In this paper we investigate the design of costless information systems in a setting where a manager has superior information about the cost of an investment opportunity at an earlier point in time and, at a later point in time, receives superior information about opportunity's cost in this period. The project can only be accepted once. Within a principal-agent framework, we then explore the design of information systems which generate imperfect information about investment costs at both points in time.

We identify the characteristics of the optimal information systems and associated investment policies. In particular, restricting our attention to partitions of the sets of costs at both points in time, we show that the information system and investment policy at each point in time is completely characterised by a single parameter and the number of intervals in the partition of the cost set. The parameter defines the highest cost below which investment always occurs. The optimal information system results in a partitioning of the cost into equal size intervals other that in the top interval. Further, the highest production region at each point in time is of equal size to the intervals below it.

We also study the effects of increasing the level of detail of the information system at either point in time on the value of the opportunity to the owner, on the timing of investment and on the overall probability of investment. First we show that increasing the level of detail at either time strictly increases the value of the opportunity to the owner. Second, we show that increasing the level of detail at a given point in time increases the probability of investment at that time and decreases it at the other point in time. Third, we show that increasing the level of detail at the first point in time increases the overall probability of investment. Increasing the level of detail at the later point in time, however, has an ambiguous effect on the overall probability of investment.

We then derive the circumstances under which the optimal information system and investment policy are identical at both points in time. The circumstances appear specialized and unlikely to hold as a matter of course. As a consequence, optimal information system design generally will differ between periods - a standardized system is unlikely to be optimal. Further, we derive results relating to the question of when it is more attractive to invest in an increased level of detail at one point in time versus the other. In particular, when the information systems at both points in time have the same level of detail then for low costs of capital it is best to increase the level of detail at the later point in time, whereas for high costs of capital it is better to increase the level of detail at the earliest point in time.

In summary, the design of the costless information systems defines the relative strengths of the incentive problems at both points in time and, hence, also defines the desirability of investing at any particular point in time. Increasing the level of detail of the information system at either point in time increases the welfare of the owner. It is not clearcut, however, which period should be emphasised in terms of the level of detail in the information system. This, even in the simple setting of the current paper, depends upon interest rates. Further, information system design will, in general differ from one period to the next, suggesting that such design issues are highly contextual.

Obviously, our analysis of information system design in the context of an investment opportunity with real option features is highly simplified. It only features two investment decision points. The only real option considered is the opportunity to time the acceptance of an investment opportunity. Uncertainty about costs is independent across the two points in time. Further, costs are distributed uniformly. Certainly, the specific design of the information systems at both points in time depends upon this latter assunption. As a consequence, further specific insights into the design of information systems can be gained from studying expanded real options settings, including the consideration of other real options. Nonetheless, the current paper makes a start at investigating the design of information systems in real options settings.

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7 Appendix

Proof of Theorem 1

Proof. First, we prove that there is no rationing in an element of the partition at either t_0 or t_1 other than the highest. Consider I_0. Suppose $a_{00} \leq c_{01}^T < a_{01}$. Then reduce all a_{0j}'s, for $m-1 \geq j \geq 1$ and c_{0j}^T's, for $m \geq j \geq 2$, by $a_{01} - c_{01}^T$. This has no impact on the agent's slack in either period nor the probability of production in either period but improves t_0 profit by reducing the costs of production. Given equation (6), this increases the value of the project to the owner. Now consider the second lowest interval. If there is rationing, follow the same procedure as above for this cost interval and those above. If there is not, leave the cost target and upper interval bound unchanged. Repeat this process sequentially moving up the intervals in the partition until the highest interval is reached. All rationing will have been eliminated in all but the highest interval leaving the owner strictly better off with the agent's welfare unchanged. Similar arguments can be applied to I_1 to show there will be no rationing in the any interval but the highest.

By implication then, there will be two values, a_0^* and a_1^* below which production will take place at t_0 and t_1 respectively and above which production will not take place. Given these upper production bounds, the question then becomes, how should the information system be designed to maximize the owner's wealth. Consider an arbitrarily chosen a_0^*. Consider any given division and recall that production is fixed. Hence, as implied by equation (6), the owner's only remaining concern is to minimize expected slack and thereby increase expected profit. Now, slack S_0 is given by:

$$S_0 = [\sum_{j=1}^{m-1} \frac{1}{2}(a_{0j} - a_{0j-1})^2] + \frac{1}{2}(a_0^* - a_{0m-1})^2$$

which by convexity and symmetry is clearly minimized by having equally wide production regions. Therefore:

$$a_{00} = 0, \quad a_{0j} = \frac{j}{m}a_0^* \quad for \quad j = 1, ..., m-1, \quad a_{0m} = 1$$

$$c_{0j}^T = \frac{j}{m}a_0^* \quad for \quad j = 1, ..., m$$

Now consider an arbitrarily chosen a_1^*. Again production at t_1 is fixed. Further, the owner wishes to minimize slack at t_1 even if it has no effect on profit. Nonetheless, minimizing slack also has the effect of increasing profit at t_1, which the owner also desires. Expected slack at t_1 is given by:

$$S_1 = [\sum_{j=1}^{n-1} \frac{1}{2}(a_{1j} - a_{1j-1})^2] + \frac{1}{2}(a_1^* - a_{1n-1})^2$$

which, again, by convexity and symmetry is clearly minimized by having equally wide production regions. Thus:

$$a_{10} = 0, \quad a_{1j} = \frac{j}{n} a_1^* \quad for \quad j = 1, ..., n-1, \quad a_{1n} = 1$$

$$c_{1j}^T = \frac{j}{n} a_1^* \quad for \quad j = 1, ..., n$$

■

Proof of Lemma 2

Proof. $\mid J \mid = \left| \begin{matrix} \partial^2 OF/\partial a_0^{*2} & \partial^2 OF/\partial a_0^* \partial a_1^* \\ \partial^2 OF/\partial a_1^* \partial a_0^* & \partial^2 OF/\partial a_1^{*2} \end{matrix} \right|$. We have that:

$$\partial^2 OF/\partial a_0^{*2} = -\frac{(m+1)}{m}$$

$$\partial^2 OF/\partial a_1^{*2} = -k(a_0^* - \frac{(n+1)}{n})$$

$$\partial^2 OF/\partial a_0^* \partial a_1^* = \partial^2 OF/\partial a_1^* \partial a_0^* = -k(1 - a_1^*)$$

Therefore, by multiplying out, rearranging, and substituting from equation [8], we have:

$$\mid J \mid = \frac{(m+1)(n+1)}{mn} - (1+k) + 3(1 - \frac{(m+1)}{m} a_0^*)$$

Suppose $\mid J \mid \leq 0$. This implies that:

$$a_0^* \geq \frac{m}{(m+1)} \left[\frac{2 + k + \frac{(m+1)(n+1)}{mn}}{3} \right] > \frac{m}{(m+1)}$$

But, equation [8] suggests that feasible values of a_0^* are bounded above by $\frac{m}{(m+1)}$. Hence, we have a contradiction and, therefore, $\mid J \mid > 0$. ■

Proof of Theorem 3

Proof. The value of the opportunity to the owner is given by (equation [7]):

$$OF = a_0^*(1 - \frac{(m+1)}{2m} a_0^*) + k a_1^*(1 - \frac{(n+1)}{2n} a_1^*) - k a_0^* a_1^*(1 - \frac{a_1^*}{2})$$

Now note that:

$$\frac{dOF}{dm} = \frac{\partial OF}{\partial a_0^*} \frac{\partial a_0^*}{dm} + \frac{\partial OF}{\partial a_1^*} \frac{\partial a_1^*}{dm} + \frac{\partial OF}{\partial m} = \frac{\partial OF}{\partial m}$$

because at the optimal values for a_0^* and a_1^*, $\frac{\partial OF}{\partial a_0^*} = \frac{\partial OF}{\partial a_1^*} = 0$. Similarly,

$$\frac{dOF}{dn} = \frac{\partial OF}{\partial a_0^*} \frac{\partial a_0^*}{dn} + \frac{\partial OF}{\partial a_1^*} \frac{\partial a_1^*}{dn} + \frac{\partial OF}{\partial n} = \frac{\partial OF}{\partial n}$$

We then use the signs of $\frac{dOF}{dm}$ and $\frac{dOF}{dn}$ as indicators of the impact of unit increases in m and n. Note that although m and n can only take integer values, such a constraint is not built into the functions describing the value of the opportunity and the first-order conditions. Therefore, if $\partial OF/\partial m$ and $\partial OF/\partial n$ are positive for all values of m and n, the value of the opportunity will change by a strictly positive amount between successive integer values for m and n. We then derive that:

$$\frac{\partial OF}{\partial m} = \frac{a_0^{*2}}{2m^2} > 0, \forall m$$

and

$$\frac{\partial OF}{\partial n} = \frac{ka_1^{*2}}{2n^2} > 0, \forall n$$

Now, we wish to identify the effects of varying m and n on a_0^* and a_1^*. The first-order conditions for the optimal a_0^* and a_1^* are given in the text in equations (8) and (9) as:

$$\partial OF/\partial a_0^* = (1 - \frac{(m+1)}{m}a_0^*) - ka_1^*(1 - \frac{a_1^*}{2}) = 0$$
$$\partial OF/\partial a_1^* = k[(1 - \frac{(n+1)}{n}a_1^*) - a_0^*(1 - a_1^*)] = 0$$

Let z represent an arbitrarily chosen parameter from the two parameters. Let the left hand sides of two first-order conditions be represented by the functions $A(a_0^*(z), a_1^*(z), z)$ and $B(a_0^*(z), a_1^*(z), z)$ respectively. Therefore, re-expressing a_0^* and a_1^* as $a_0^*(z)$ and $a_1^*(z)$ to recognise that these variables are functions of the parameters m and n, we have:

$$\partial OF/\partial a_0^*(z) = A(a_0^*(z), a_1^*(z), z)$$

and

$$\partial OF/\partial a_1^*(z) = B(a_0^*(z), a_1^*(z), z)$$

Let A_i (B_i) be the partial derivative of A (B) with respect to the i'th argument of the function. Therefore:

$$A_1 = \partial^2 OF/\partial a_0^*(z)^2 = -\frac{(m+1)}{m}$$
$$B_2 = \partial^2 OF/\partial a_1^*(z)^2 = -k(a_0^* - \frac{(n+1)}{n})$$
$$A_2 = B_1 = \partial^2 OF/\partial a_0^*(z)\partial a_1^*(z) = \partial^2 OF/\partial a_1^*(z)\partial a_0^*(z) = -k(1 - a_1^*)$$

If $z = m$, then

$$A_3 = -\frac{a_0^*}{m^2}$$
$$B_3 = 0$$

whereas if $z = n$,

$$A_3 = 0$$
$$B_3 = -\frac{ka_1^*}{n^2}$$

Now, using the chain rule (see, for example, Protter and Morrey, 1964, chapter 4), it is the case that:

$$A_1\frac{da_0^*}{dz} + A_2\frac{da_1^*}{dz} + A_3 = 0$$
$$B_1\frac{da_0^*}{dz} + B_2\frac{da_1^*}{dz} + B_3 = 0$$

which leads to the matrix equation:

$$\begin{bmatrix} A_1 & A_2 \\ B_1 & B_2 \end{bmatrix}\begin{bmatrix} \frac{da_0^*}{dz} \\ \frac{da_1^*}{dz} \end{bmatrix} = \begin{bmatrix} -A_3 \\ -B_3 \end{bmatrix}$$

We observe that, in the notation of above, J can also be defined as

$$J = \begin{bmatrix} A_1 & A_2 \\ B_1 & B_2 \end{bmatrix}$$

Therefore, the matrix equation above can be restated as:

$$J\begin{bmatrix} \frac{da_0^*}{dz} \\ \frac{da_1^*}{dz} \end{bmatrix} = \begin{bmatrix} -A_3 \\ -B_3 \end{bmatrix}$$

'Dividing' both sides of this equation by J gives matrix equation [10] which below provides the basis for identifying the effects of varying m and n on a_0^* and a_1^*:

$$\begin{bmatrix} \frac{da_0^*}{dz} \\ \frac{da_1^*}{dz} \end{bmatrix} = \frac{1}{\mid J \mid}\begin{bmatrix} B_2 & -A_2 \\ -B_1 & A_1 \end{bmatrix}\begin{bmatrix} -A_3 \\ -B_3 \end{bmatrix} \tag{10}$$

Applying matrix equation [10], gives:

$$\frac{da_0^*}{dm} = \frac{-k}{\mid J \mid}(a_0^* - \frac{(n+1)}{n})\frac{a_0^*}{m^2} > 0$$
$$\frac{da_1^*}{dm} = \frac{-k}{\mid J \mid}(1 - a_1^*)\frac{a_0^*}{m^2} < 0$$
$$\frac{da_0^*}{dn} = \frac{-k^2}{\mid J \mid}(1 - a_1^*)\frac{a_1^*}{n^2} < 0$$
$$\frac{da_1^*}{dn} = \frac{k}{\mid J \mid}(1 + \frac{1}{m})\frac{a_1^*}{n^2} > 0$$

The overall probability of investment is:

$$a_0^* + a_1^*(1 - a_0^*)$$

Therefore, the impact of changing m on the overall probability of investment is:

$$(1 - a_1^*)\frac{da_0^*}{dm} + (1 - a_0^*)\frac{da_1^*}{dm}$$

Using the expressions above for the derivatives shows that the overall probability of investment increases with m. ■

Proof of Theorem 4

Proof. Let $m = n$. If the constraint $a_0^* = a_1^*$ is imposed upon the solution to the two first-order conditions from Theorem 3, then equality of the two first-order conditions requires that:

$$a_0^* = a_1^* = \frac{(1-k)}{(1-\frac{k}{2})}$$

Putting these expressions into one of the first-order conditions gives an expression equal to:

$$\frac{((m-2)k^2 + 6k - 4)}{m(2-k)^2}$$

It is only when the numerator equals 0 that the first-order condition will equal 0 and, hence, identical information systems will be optimal. Otherwise, identical information systems at both points in time will be sub-optimal. ■

Proof of Theorem 5

Proof. First, we demonstrate the effect of k on a_0^* and a_1^*Specifically, we prove that $\frac{da_0^*}{dk} < 0$ and $\frac{da_1^*}{dk} > 0$. We use the same methods as in the Proof of Theorem 3. Therefore:

$$\frac{da_0^*}{dk} = \frac{-1}{|J|}(1 + \frac{1}{n} - a_0^*)a_1^*(1 - \frac{a_1^*}{2}) < 0$$
$$\frac{da_1^*}{dk} = \frac{1}{|J|}(1 - a_1^*)a_1^*(1 - \frac{a_1^*}{2}) > 0$$

Now let $m = n$. From Theorem 3, we know that:

$$\frac{\partial OF}{\partial m} = \frac{a_0^{*2}}{2m^2}$$

and

$$\frac{\partial OF}{\partial n} = \frac{ka_1^{*2}}{2n^2}$$

Therefore, if $m = n$, whether $\frac{\partial QF}{\partial m} > \frac{\partial QF}{\partial n}$, or *vice versa*, depends upon whether $a_0^{*2} > ka_1^{*2}$, or *vice versa*. From the first part of the Proof, we know that lowering k increases a_0^* and decreases a_1^*.

Now we prove that if $k = 1$, $a_0^* < a_1^* = ka_1^*, \forall m = n$. Let $a_0^* = a_1^* + x$. Substituting for a_0^* in the first-order conditions, substituting one from the other and some rearranging suggests that:

$$x = \frac{\frac{-a_1^{*2}}{2}}{(a_1^* + \frac{1}{m})} < 0$$

which gives the result.

Hence, for a given common $m = n$, as k is decreased, eventually the point will be reached at which the situation switches from $a_0^{*2} < ka_1^{*2}$ to $a_0^{*2} > ka_1^{*2}$. This point, therefore, is at some $k^* < 1$. k^* defines the boundary between 'high' and 'low', for the given common $m = n$. k^*, as defined, will only be approximate as an indicator of where the changeover occurs because, in looking at differentials, we are treating m and n as continuous variables and considering an infinitessimally small increase in either, whereas m and n are integer variables and we have to consider unit changes in them. ■