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The Royal Veterinary and Agricultural University
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Technological Change and Economies of Scale in Danish Agriculture.

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TECHNOLOGICAL CHANGE AND ECONOMIES OF SCALE IN DANISH AGRICULTURE

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Technological change and economies of scale in Danish agriculture

Abstract

This paper presents the results of an empirical analysis of technological change and economies of scale in Danish agriculture. The estimation is based on a multi-output, multi-input translog cost function and covers the period 1973-1995.

The results show that technological change varied considerably. The average rate of technological change was highest on arable farms (4.0% per year) and lowest on dairy farms (1.0% per year), with pig farms in between (2.2% per year). Alt three farm types showed an increasing trend in technological change over time, and technological changes were typically biased.

The elasticity of size was considerably above 1 for both small and large arable farms and dairy farms indicating a considerable economic incentive to increase the farm size. For pig farms only the small farms showed a clear incentive to increase the farm size.

The results support the hypothesis that policy measures have had a significant influence on the differences in development within the three farm types analysed.

Technological change and economies of scale in Danish agriculture

1. Introduction

Since Denmark joined the EEC in 1973, productivity in Danish agriculture has increased considerably. The Danish Institute of Agricultural and Fisheries Economics (DIAFE) has estimated that Total Factor Productivity (TFP) increased by 1.8% pr. year from 1973 to 1980, and by 3.2% from 1981 to 1993 (Hansen, 1990; 1995), with some differences between arable, dairy and pig farms.

The intention of this study is to further analyse the changes in productivity in Danish agriculture. Changes in productivity are primarily attributable to technological change (Hansen, 1990). However, changes in farm structure (in particular, the size of farms) and in efficiency may also have an influence. The objective is to empirically estimate technological changes and economies of scale *for different farm types* (arable, dairy and pig farms). This is interesting because the different farm types have been subject to different production conditions due to policy measures. The expectation is that this has an influence on technological development and the economic forces driving changes to farm structure (size).

There have been a number of previous empirical studies of changes in farm productivity and technology. Some of these studies have been based on econometric estimations of the cost function (the dual approach). Ryhänen (1994) analysed Finish dairy productivity; Glass & McKillop (1989; 1990) analysed the Irish agricultural productivity structure using cost functions; and Michalek (1988) analysed technological change in German agriculture. Capalbo (1988) compared a number of different econometric models (profit and cost functions) for estimating technological change in U.S. agriculture (Capalbo, 1988); and Kuroda (1988) estimated biased technological changes in Japanese agriculture. Most of these studies have been based on aggregate data.

An alternative way of analysing productivity growth is to use the Malmquist productivity index (Färe *et al*., 1992). Use of this index makes it is possible to decompose productivity changes into two components: technological change and technical efficiency change (Färe, Grosskopf, Norris and Zhang, 1994). This index was used by Tauer (1998) to estimate change in efficiency and technology for a sample of 70 New York dairy farms. The Malmquist index has the advantage of not requiring a specific functional form. However, the index precludes taking into account the effects of changes in scale (Färe and Grosskopf, 1998). Also, the Malmquist index approach requires panel data, which was not available in the present study.

In the present study, a cost function approach (dual approach) is used as a basis for estimating technological change. The estimations are based on representative average data for five different farm size groups, which makes it possible to consider economies of scale at the same time. This is of potential interest for comparing changes between the various farm types, because production conditions have been different for the different types of farm. For dairy farms, the growth of farm size has been limited by the EU milk quota system since 1984. On arable farms, the potential for growth is limited by access to land and by legal regulations concerning ownership of land. In pig production, there were no specific limitations related to the growth of farm firms, at least not up until the mid 1990s. These differences may have had an influence on development trends within the three farm types analysed.

2. Theoretical foundation

In the single-output case the (primal) rate of technical change (TC) is defined as:

$$\mathrm{TC} = \mathrm{TC}(x,t) = \frac{\partial \ln f(x,t)}{\partial t} \qquad (1)$$

where *f* is the production function, *x* is an *N*-vector of inputs, and *t* is time (Chambers, 1988, p. 205).

In the multiple-output case, the basis is a transformation function $Y(y,x)=0$, where *y* is an *m*-vector of outputs. Using the implicit function theorem we may define a production function

$$y_1 = F(y_2,\ldots,y_m,x_1,\ldots,x_N,t), \qquad (2)$$

where y_j is the production of output *j*. According to Antle and Capalbo (1988, p. 44), the (primal) rate of TC may in this case be defined as:

$$\mathrm{TC} = R_1 \frac{\partial \ln F}{\partial t} \qquad (3)$$

where R_1 is the revenue share for product y_1.

Measurement of TC may also be based on the cost function $C = C(w, y, z, t)$, where w is a price vector of variable inputs and z is a vector of quasi-fixed inputs. Antle and Capalbo (1988, p. 44) define $-\partial \ln C / \partial t$ as the dual rate of technical change (Dual TC). Assuming profit maximization, the dual TC in the multiple-output case is (Antle and Capalbo, 1988, pp 44-45):

$$-\frac{\partial \ln C}{\partial t} = \sum_{j=1}^{m} \frac{\partial \ln C}{\partial \ln y_j} R_1 \frac{\partial \ln F}{\partial t} \tag{4}$$

Thus, using the definition in (3), the primal rate of technical change (Primal TC) may be derived from the cost function in the multiple-output case as:

$$TC = R_1 \frac{\partial \ln F}{\partial t} = -\left(\frac{\partial \ln C}{\partial t}\right)\left(\sum_{j=1}^{m} \frac{\partial \ln C}{\partial \ln y_j}\right)^{-1} \tag{5}$$

The term $(\sum \partial \ln C / \partial \ln y_j)^{-1}$ in (5) is what Chambers (1988) terms the "elasticity of size" (p. 69); it is the reciprocal of the change in cost when output changes, and thus it is a measure of how much output changes when input is increased.

TC may be neutral or biased, depending on how the relationship between inputs is affected. TC is neutral, in the sense used by Hicks, when the expansion path is unaffected by the TC (Chambers, 1988, p. 207). Antle and Capalbo (1988) show how this approach is extended to the multi-output case. In the multi-output case a measure of bias can be obtained by analysing changes in optimal factor proportions (change in cost share). If the term $S_i(w, y, z, t)$ is used to describe the (cost minimizing) cost share of input x_i in total cost, then a dual measure of bias is (Antle and Capalbo, 1988, p. 45-46):

$$\mathrm{MB}_i = \frac{\partial \ln S_i}{\partial t} - \left[\sum_{j=1}^{m} \frac{\partial \ln S_i}{\partial \ln y_j} \Big/ \sum_{j=1}^{m} \frac{\partial \ln C}{\partial \ln y_j}\right] \frac{\partial \ln C}{\partial t} \tag{6}$$

The first term in (6) is the change in cost share due to technical change. However, this term does not give a correct measure of the Hicksian bias if the technology is non-separable, because change in scale may have an influence. The second of the two terms in (6) is the scale effect, by which the change in cost share is adjusted to give the correct bias measure MB_i. Thus, if MB_i is zero, TC is (Hicks) neutral. If the term is positive the TC is input *i* using, and vice-versa.

Using the cost function approach as described here means that it is not possible to consider possible changes in technical *efficiency* over time. Thus, the implicit assumption used in the following is that the efficiency does not change over the time period analysed. If this assumption does not apply, the technological change measured in the following will be biased in the sense that the estimated technological change will also include change in technical efficiency, and therefore would be too high if the efficiency did in fact increase over time, and too low if it decreased.

3. Model specification

The general form of a multi-output cost function is:

$$C = C(w, y, z, t)\,, \qquad (7)$$

where *C* is variable costs, *w* is an *n*-vector of variable input prices, *y* is an *m*-vector of outputs, *z* is a *p*-vector of quasi-fixed inputs, and *t* is time.

In the long run all inputs are considered variable. In that case *w* therefore would include prices of all inputs, and the vector *z* would be empty. Correspondingly, *C* would include total costs.

Even if we consider a relatively long time horizon, it may be wrong to consider all inputs as being truly variable inputs. The input *land* is often not available within an adequate time frame or in sufficient quantity to be considered a real variable input. Under Danish conditions the availability of land is to a large extent regulated by the state in the form of various restrictions[1], and the decision to buy land will often either not be an option, or will be a decision of a completely different character than when buying other inputs.

For these reasons, the model used in the following has land as a quasi-fixed input. All other inputs are considered variable. The model used in this paper includes five variable inputs (n=5), two outputs (m=2), and one quasi-fixed input (p=1). The variable inputs are fertilizers (x_1), purchased feed (x_2), labour (x_3), machinery (x_4), and other capital (x_5). The two outputs are livestock (y_1) and crops (y_2). The quasi-fixed input is land (z_1).

The choice of land as a quasi-fixed input means that the model is a partial static equilibrium model as described by Capalbo (1988, p. 160). This means that the firm or the sector is considered to be in equilibrium with respect to all inputs except land. Thus, there is no substitution between land and the other (variable) inputs, and adjustments are made within the framework of the land available. Given conditions on the Danish land market and the state regulations involved, this is considered a fair description of the medium term conditions for farmers in Danish agriculture.

The translog function (Jorgenson, Christensen and Lau, 1973) is chosen as the functional form of the cost function, i.e.

$$\begin{aligned}\ln C = {} & a_0 + \sum_{j=1}^{2} a_j \ln y_j + b_1 \ln z_1 + \sum_{i=1}^{5} d_i \ln w_i + vt \\ & + ½\sum_{j=1}^{2}\sum_{p=1}^{2} f_{jp} \ln y_j \ln y_p + \sum_{j=1}^{2} g_{j1} \ln y_j \ln z_1 \\ & + \sum_{j=1}^{2}\sum_{i=1}^{5} h_{ji} \ln y_j \ln w_i + \sum_{j=1}^{2} k_j \ln y_j t \\ & + ½ m_{11} \ln z_1 \ln z_1 + \sum_{i=1}^{5} n_{1i} \ln z_1 \ln w_i + q_1 \ln z_1 t \\ & + ½\sum_{i=1}^{5}\sum_{l=1}^{5} r_{il} \ln w_i \ln w_l + \sum_{i=1}^{5} s_i \ln w_i t + ½u(t)^2\end{aligned} \tag{8}$$

where the parameters $f_{jp}=f_{pj}$ for all j, p=1,2 and $r_{il}=r_{li}$ for all i, and l=1...5. w_i (i=1...5) are the

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[1] These restrictions include for instance limitations on the number of farms a farmer may own, limitations on the size of any given farm in hectares, and limitations on buying land if the distance from the existing farm is too great.

prices of the variable inputs x_i (i=1...5). The translog function has the advantage of being a flexible functional form (For further details see for instance Antle and Capalbo (1988), Appendix 2A).

The cost share S_i of input i can be derived from the translog cost function (8) as:

$$S_i = \frac{\partial \ln C}{\partial \ln w_i} = d_i + \sum_j^2 h_{ji} \ln y_j + n_{1i} \ln z_1 + \sum_{l=1}^5 r_{il} \ln w_l + s_i t \qquad (i = 1,\ldots,5) \qquad (9)$$

4. Data

The data used as the basis for the analysis are representative farm account date from specialised arable farms, dairy farms and pig farms taken from the farm account database of The Danish Institute of Agricultural and Fisheries Economics (DIAFE). This database includes for each year a representative sample of 1,800-2,000 farms, corresponding to around 2-3% of all Danish farms. Of this sample, around 700 are arable farms, 700 are dairy farms and 500 are pig farms with some variability in the number from one year to another. (For further details on the farm sample see the yearly Agricultural Accounts Statistics from DIAFE).

For each of the three farm types, the data used include average values of variables over farms within each of five (economic) size groups for the 23-year time period 1973-1995[2)]. Thus 5×23=115 observations were available for each of the three farm types. The size groups were constructed so that for each year, the number of farms in each size group was the same (around 700/5 for arable and dairy farms, and around 500/5 for pig farms). This means that the average size of the farms in each of the five size groups increases over time, corresponding to the general trend in farm structure. Thus the change in farm structure (size of farms) over time is explicitly taken into account, and the results in the following therefore describe the development of the whole agricultural *sector* (i.e. the development within *representative farms* as opposed to the development within *individual farms*). The size groups are shown in Appendix 1

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[2] The original plan was to use panel data. However, *representative* data of Danish agriculture were available from DIAFE only on the condition that data were aggregated as just described.

The farm account data were aggregated into two outputs: 1) animal products (aggregate of all animal products), and 2) crops (aggregate of all crops)); and five variable inputs: 1) fertilizers (aggregate of fertilizers, seed, chemicals, energy, etc.), 2) feed (aggregate of all purchased feed, veterinary services, and medicine), 3) labour (family labour and hired labour), 4) machinery (aggregate of machines and contractor service), and 5) (other) capital (aggregate of buildings, herd, and stocks). Aggregate measures of prices and quantities were calculated using Törnqvist indexes (for a detailed discussion of Törnqvist indexes see Diewert (1981)). Prices of individual inputs and outputs were taken from official Danish price statistics and thus assumed to be constant across farm groups. (Further details on price and quantity indices are available from the author upon request).

The quasi-fixed input *land* includes the sum of owned and rented land. Due to differences in the quality of land across the country, the input *land* was measured in *quality-corrected* number of hectares. Thus, before aggregating the data from the individual farms, DIAFE was asked to multiply the number of hectares of each farm by a quality index. This index was estimated using the price statistics from the Danish tax authorities (Told & Skat: *Ejendomssalg*) on regional trade prices of agricultural farms within the size group 30-60 hectares. Average data from the period 1984-1991 were used, and the estimated relative prices were used as an index of land quality. The estimated quality indexes for the six regions are as follows: Zealand: 1,36; Lolland, Falster and Funen: 1.44; Southern Jutland: 0.90; Eastern Jutland: 1.02; Western Jutland: 0.80; Northern Jutland: 0.91

5. Estimation

The parameters of the cost function (8) were estimated on the basis of the model system including the cost function (8) and four of the five share equations in (9) using Iteratively Seemingly Unrelated Regression (ITSUR)[3]. The estimation was performed imposing the normal symmetry conditions (Antle and Capalbo, 1988, p.75). Separate estimations were performed for specialized arable farms, dairy farms and pig farms.

Due to the high correlations between some of the explanatory variables, the estimation was

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[3] The SYSLIN Procedure in the SAS system for Windows, version 6.12

performed on a linear transformation of the model. Thus prices and costs were transformed into real prices (imposing homogeneity in input prices) using the price of machinery as the numeriare, and outputs into yield (output y_1 and y_2 were divided by the number of hectares z_1). After estimation, the original parameters were "recaptured" by simple "backward" transformation. The parameter estimates and standard errors for the three cost functions are presented in Appendix 2.

The estimation gave a very high degree of explanation, with system-weighted[4)] R^2 of 0.9896, 0.9759 and 0.9502 for the three model systems arable, dairy and pigs respectively.

The residuals were tested for first-order autocorrelation using a Durbin-Watson test on all 5×23 years, i.e. 115 residuals from the ITSUR estimation. The Durbin-Watson test statistic *dw* (Verbeek, 2000, p. 449, p. 95) was 1.4766 (0.0021) for arable farms, 0.8871 (0.0001) for dairy farms, and 0.6604 (0.0001) for pig farms (probability level in parentheses). These results show that especially the residuals from dairy and the pig farm were significantly autocorrelated, indicating either an incorrect functional form, omitted variables or missing dynamic specification (Verbeek, 2000, p. 90). Concerning dairy farms plots of the residuals against time show that there is a significant change in the time series in the years after 1984 - the year when the milk quota system was introduced. Therefore it was contemplated to improve the estimation efficiency for the dairy model by including a dummy variable for the time period after 1984. However, this change of model was given up (see the discussion in Section 7). Concerning pig farms, plots of the residuals against time indicate that cyclical price movements might be the major course for autocorrelation. Thus a possible improvement (which was not attempted hear) of the econometric model would be to include dynamic specifications of decision-making and production in pig production.

The residuals were also tested for autoregressive conditional heteroscedasticity (see Verbeek, 2000, p. 265) [5)]. For arable farms there were no significant autoregressive conditional heteroscedasticity. However, both dairy and pig farm showed significant autoregressive conditional heteroscedasticity. This means that the variance of the residuals vary over observations (time) according to an autoregressive model. Together with the autocorrelation mentioned above this

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[4)] See definition of System weighted R^2 in SAS/ETS User's Guide, Version 6, second ed. (1993), p. 849

[5)] The test was performed using the ARCHTEST-option in PROC AUTOREG. See SAS/ETS User's Guide, Version 6, second ed. (1993), p. 201

indicates that including dynamic specifications in the model probably would improve the estimation efficiency.

The functional form of the model was also tested. First, the model was tested for homotheticity using an F-test. The hypothesis tested is that all h_{ji} (i=1…5; j=1,2) are equal to zero (cost shares S_i in (9) independent of production y_j). With 8 and 530 DF, the F-values for arable farms, dairy farms and pig farms were 6.94, 17.52 and 9.50 respectively, which indicates highly significant parameter values. Thus, the hypothesis of homotheticity was rejected. This is equivalent to rejecting the hypothesis of linear input expansion paths.

The estimated models were checked for monotonicity and curvature conditions. For the translog cost function it is only possible to make local checks of these conditions. The following checks were all based on average values of the variables. For C to be a cost function it has to be (1a) non-decreasing and (1b) concave in w_i and (2a) non-decreasing in y_j. Under decreasing returns to scale the cost function is (2b) convex in y_j (Chambers, 1988).

Condition (1a) implies that the cost shares S_i are positive. The five cost shares for each of the three farm types were calculated using (9). Thirteen of the fifteen cost shares S_i were positive while two (S_1 (fertilizers) for both dairy and pig farms) were negative.

The concavity condition (1b) implies that the parameter r_{ii} is less than the cost share S_i. This condition was fulfilled in ten out of the fifteen cases, the exceptions being i=1 (fertilizers) for both dairy and pig farms, i=3 (labour) for both arable farms and pig farms, and i=5 (other capital) for arable farms.

Due to the transformation of variables mentioned above, the term $\partial \ln C / \partial \ln y_j \; (j = 1{,}2)$ in (5) cannot be derived directly from the estimated cost function (8). As the model was estimated using yields (output per hectare y_1/z_1 and y_2/z_1) as regressors, the derivative of ln$C$ with respect to lny_i in (8) only reflects change in cost when *yield* changes. The change in cost when output changes due to changes in use of land (z_1) is captured by the derivative of ln$C$ with respect to lnz_1 in (8). Therefore, to fully capture the change in cost when output changes the term $\partial \ln C / \partial \ln y_j \; (j = 1{,}2)$ in (5) has to be replaced by the term $d \ln C / d \ln y_j \; (j = 1{,}2)$, estimated as:

$$\frac{d\ln C}{d\ln y_j}=\frac{\partial\ln C}{\partial\ln z_1}\cdot\frac{\partial\ln z_1}{\partial\ln y_j}+\frac{\partial\ln C}{\partial\ln y_j}\quad (j=1,2) \tag{10}$$

In the following, elasticity of size refers to the term ($\sum d\ln C/d\ln y_j$)$^{-1}$.

The term $\partial\ln z_1/\partial\ln y_j$ $(j = 1,2)$ in (10) cannot be captured from the estimated translog cost function, and thus has to be estimated separately.

To facilitate this, consider the inverse cost function

$$z_1 = z_1(y_1, y_2, \overline{C}, w_1,..., w_5) \tag{11}$$

where $\overline{C}$ is the cost per hectare ($\overline{C} = C / z_1$).

This function has its own interpretation, as shown in Appendix 3. Under standard assumptions concerning production technology, the function has the following properties:

$$\partial z_1 / \partial y_j \geq 0 \quad (j = 1, 2); \qquad \partial z_1 / \partial\overline{C} \leq 0; \qquad \partial z_1 / \partial w_i \geq 0 \quad (i = 1...5) \tag{12}$$

A translog was chosen as the functional form of (11). However, with only one model to estimate (compared to five in estimating the cost function), the number of degrees of freedom was relatively low, and the precision of the parameter estimates correspondingly low. Therefore the model was reduced excluding most of the (non-significant) cross product and quadratic terms. The final model estimated has the following form:

$$\begin{aligned}\ln z_1 = \alpha + \sum_{j=1}^{2}\beta_j \ln y_j + \beta_3 t + ½\sum_{i=1}^{2}\sum_{j=1}^{2}\beta_{ij}\ln y_i \ln y_j \\ + \sum_{j=1}^{2}\beta_{1t}\ln y_j t + \beta_{tt}t^2 + \gamma\ln\overline{C} + \theta(\ln\overline{C})^2 + \sum_{i=1}^{5}\delta_i \ln w_i\end{aligned} \tag{13}$$

This model was estimated applying the usual symmetry restrictions $\beta_{ij}=\beta_{ji}$ (j, i=1,2). The estimation was performed on a transformed model, in which costs and prices were divided by w_4 (price of machinery). The restrictions in (12) were *not* imposed. The parameter estimates had - with few exceptions - the correct sign and the parameter estimates with a wrong sign were not different from zero at a significant level. However, the model obviously had problems in separating the effect of labour (w_3) and machinery (w_4) prices. In this context this was not considered a serious problem, as the aggregate effect of the two was in accordance with the properties in (12).

The estimation performed well, with R^2 of 0.9934, 0.9873 and 0.9822 for the three models arable, dairy and pigs respectively.

The monotonicity and curvature conditions concerning C as a function of y_j may now be tested. Condition (2a) that C is non-decreasing in y_j implies that the value at the right hand side in (10) is positive. The value of the right hand side of (10) was estimated for each of the two y's and for each of the three farm types. For j=1 (animal products) this term was clearly positive (0.295 for arable farms, 0.981 for dairy farms, and 0.869 for pig farms). For j=2 (crops) the term was positive for arable farms (0.361) and around zero for dairy farms (0.010) and pig farms (-0.021).

The convexity condition (2b) implies that $g_{j1}(\partial \ln z_1/(\partial \ln y_j) + \beta_{jj}(\partial \ln C/(\partial \ln z_1) + f_{jj}$ is greater than the right hand side of (10). This condition was fulfilled in only one out of the six cases (j=2 (crops) on pig farms). These results indicate increasing returns to scale.

Having estimated (13), the term $\partial \ln z_1 / \partial \ln y_j \; (j = 1,2)$ to be used in (10) can be derived directly from (13).

6. Results

Based on the estimated parameters, the dual rate of technological change, elasticity of size, and (primal) technological change were calculated according to (5) (with $\partial \ln C / \partial \ln y_j$ replaced by $d \ln C / d \ln y_j$), using the following method.

First, predicted values were calculated for all observations (all farm groups and years). On the basis of these predicted values, weighted averages were calculated over years (results in Table 2), over farms (results in Table 3), and finally over both farms and years (results in Table 1). The weights used for dairy and pig farms were production of livestock products (y_1), and for arable farms, production of crop products (y_2).

Table 1. Rate of technological change per year and elasticity of size. Weighted averages over farms and years.

	Dual tech. change	Elasticity of size	Technological change
Arable farms	0.025 (0.002)*	1.573	0.040
Dairy farms	0.008 (0.003)*	1.142	0.010
Pig farms	0.021 (0.004)*	1.039	0.022

* Standard deviations in parentheses

The results in Table 1 show that the rate of technological change was highest on arable farms (4.0% per year) and lowest on dairy farms (1.0% per year), with pig farms in between (2.2% per year).

These results are in good accordance with the results on Total Factor Productivity (TFP) found by Hansen (1995). Hansen (1995) used 1981-1993 data and found that for the Danish agricultural sector as a whole, TFP increased by 3.2% pr. year from 1981 to 1993. This number is not directly comparable to the numbers in Table 1 because changes in TFP not only include the effect of technological changes but also the effect of increasing farm size. Thus technological changes are normally lower than TFP. A simple average of the results in Table 1 shows an average technological change of 2.4%, which is clearly lower than the TFP of 3.2%. Concerning the individual sectors, Hansen (1995) estimated the increase in TFP per year to be 3.1% in the crop sector, 2.3% in the dairy sector, and 3.4% in the pig sector. The results in Table 1 are in accordance with Hansen's results in the sense that the estimated technological changes are lower than TFP, except for arable farms.

To make further comparison, Glass and McKillop (1990) estimated the average technological change per year to be 1.33% in Irish agriculture 1953-1986 and Ryhänen (1994) estimated technological change in Finnish dairy farms to be 1.3% per year using 1965-1991 data. The results are not directly comparable as the time period analysed is different. However, the estimated technological change is at a comparable level.

Elasticity of size was greater than 1 on all three farm types, indicating a potential economic benefit from increasing average farm size. It is interesting to note that pig farms, which had the most liberal conditions for growth of farm size during the period, also had an elasticity of close to 1, indicating that the potential for economies of scale were (almost) fully exploited. On the other hand, arable farms, where growth is highly restricted by access to land, had a high elasticity of size, indicating a high potential economic benefit from an increase in farm size. The result indicates that on arable farms a 1.0% increase in all variable inputs (and a corresponding, proportional increase in land) would have increased production by almost 1.6%.

Table 2. Rate of technological change per year and elasticity of size. Weighted averages over years

Arable farms

Farm size	Dual tech. change	Elasticity of size	Technological change
1 (Small)	0.011	1.713	0.020
2	0.014	1.611	0.023
3	0.016	1.566	0.025
4	0.025	1.591	0.039
5 (Large)	0.027	1.537	0.042

Dairy farms

Farm size	Dual tech. change	Elasticity of size	Technological change
1 (Small)	0.042	0.817	0.034
2	0.027	0.937	0.025
3	0.021	1.013	0.021
4	0.011	1.154	0.013
5 (Large)	-0.008	1.447	-0.011

Pig farms

Farm size	Dual tech. change	Elasticity of size	Technological change
1 (Small)	0.036	1.463	0.052
2	0.032	1.289	0.042
3	0.029	1.153	0.033
4	0.022	1.072	0.023
5 (Large)	0.014	0.956	0.014

The results in Table 2 show that for arable farms the amount of technological change was greatest on large farms, while for dairy and pig farms the technological change was greatest on small farms. The explanation of the results for arable farms may be that technological change (development of new (labour saving) machinery) focuses on large-scale production, and/or that the motivation/ability to implement new technologies in general is greatest on large farms. For dairy and pig farms, improved technology was apparently not of a form that was specifically advantageous to large-scale production. The very small amount of technological change on large dairy farms (-1.1%) indicates that the large dairy farms were seriously restricted in their ability/motivation to invest in new technology. This is further indicated by the very high elasticity of size (1.447), which shows that the dairy farms in the large group had a considerable incentive to increase the farm size.

Ryhänen (1994) found the same result in Finland. He concludes that: “In milk production it has not been possible to utilize the economies of size in full during the last two decades” … “a 1.0% increase in output increases costs about 0.6%. So unit costs decrease as production is expanded. Therefore, increasing the size of dairy farms should be allowed that the advantages related to economies of size can be utilized” (p. 594).

In the case of arable and pig farms, the results for elasticity of size are as expected, with a greater potential for economies of scale on small farms than on large farms. However, the results are just the opposite for dairy farms. An obvious explanation may be that the (Danish administration of the) milk quota system introduced in the EU in 1984 was a much greater disadvantage to the farms in the large size group, due to the fact that they had limited ability to grow because of the quota restrictions. Furthermore, the difficulty which large farms had in acquiring land for roughage production probably also had an influence.

While even the large arable farms still had an economic incentive to increase their size (elasticity of size greater than 1), the large pig farms had an elasticity of size close to 1 (0.956), indicating that there was no economic incentive to increase the farm size further.

Table 3. Rate of technological change per year and elasticity of size. Weighted averages per year over farms

Arable farms

Year	Dual tech. change	Elasticity of size	Technological change
1973-77	-0.012	1.237	-0.015
1978-82	0.004	1.400	0.006
1983-87	0.015	1.484	0.023
1988-92	0.031	1.658	0.051
1993-95	0.039	1.792	0.071

Dairy farms

Year	Dual tech. change	Elasticity of size	Technological change
1973-77	-0.037	1.399	-0.050
1978-82	-0.018	1.302	-0.022
1983-87	0.003	1.221	0.003
1988-92	0.032	1.043	0.033
1993-95	0.059	0.941	0.055

Pig farms

Year	Dual tech. change	Elasticity of size	Technological change
1973-77	-0.003	1.120	-0.004
1978-82	0.006	1.113	0.007
1983-87	0.015	1.038	0.016
1988-92	0.025	1.021	0.026
1993-95	0.031	1.043	0.033

The results in Table 3 show the predicted values over five sub-periods from 1973 to 1995. It is clear that the rate of technological change increased considerably over the time period. However, the estimation method used does not allow too much weight to be attached to the results for the individual sub-periods. The results should only be used to show the trend.

The results support those found earlier by Hansen (1995); technological change was at a very low level in the first 5-10 years after joining the EEC in 1973. The explanation may be that there were considerable product price increases during this period, and consequently there was no external pressure to improve productivity. Over the longer time period examined there was a considerable increase in technological change, especially for arable and dairy farms.

The elasticity of size figures show that for arable farms economies of scale has increased over the period, indicating an even greater economic advantage obtainable through being able to increase farm size at the end of the period than earlier. On dairy farms elasticity of size decreased over the time period, and at the end of the time period, the figure was close to 1. This result indicates that over time the dairy farmers have been able to re-allocate resources to adjust to the consequences of the quota system. On pig farms elasticity of size was almost constant (close to 1) indicating that there had not been any serious limitation on adjustment of the farm size.

It has not been possible to make any direct statistical test of significance of the estimated (primal) technological change. However, to give an indication, the standard deviations of the dual rate of technological change are shown in Table 1. The results of normal t-tests show that the dual rates of technological change in Table 1 are all significantly different from zero at the 5% test level.

Hicksian bias of technological change was measured using the measure MB_i shown in (6). Calculation of this measure of bias was based on predicted values of cost and cost shares, using the average values of the observations.

Table 4. Measure of Hicksian bias (MB_i) based on average values of variables

	Fertilizers	Feed	Labour	Machinery	Capital
Arable	0.048	-0.006	-0.012	0.005	-0.014
Dairy farms	-0.035	-0.019	0.008	-0.009	0.002
Pig farms	-0.015	-0.004	-0.006	-0.002	-0.001

Table 4 shows that during the period considered, technological change on arable farms was fertilizer using, feed saving, and labour saving. On dairy farms technological change was fertilizer saving, feed saving, labour using and machinery saving. On pig farms technological change was fertilizer saving, feed saving, labour saving, and machinery saving.

The figures are relatively low, except for fertilizers, where MB_i (the change in the share of cost of fertilizers in total costs, assuming constant costs) is 0.048, indicating a 4.8% increase per year in the cost share of fertilizers, assuming constant costs. Thus, the change in technology (including change in crop structure) was certainly based on increased intensity of use of fertilizers. On the other hand, the negative figures for fertilizers for dairy and pig farms indicate fertilizer-saving technological development. This may be due to better utilisation of manure from animal production, and a substitution of chemical fertilizers for manure. However, it may also just be the consequence of the development of farm structure towards more specialised production; when arable farms reduce the number of animals per hectare and the dairy and pig farms increase the number of animals per hectare as is the case in Denmark, the relative use of fertilizers will increase for arable farms and decrease for dairy and pig farms. The subject is interesting given the ongoing debate on ground-water pollution from the use of fertilizers. However, the results presented here are uncertain due to the fact that the cost shares for fertilizers did not pass the check of being positive.

To compare, Ryhänen (1994) found technological change in Finnish dairy farms to be (purchased) feed-saving. Glass and McKillop (1990) found that in Irish agriculture technological change was labour-saving and fertilizer-using.

No formal test of significance of the results in Table 4 has yet been made, as the variances were not calculated. However, the results clearly indicate that fertilizer was the input most affected by technological changes. On the other hand it is important to note that the (Hicksian) measures of bias in Table 4 do not necessarily indicate the actual change in the relative use of the inputs. Even with Hicks-neutral technological changes (no change in the expansion path), the share of an input in total cost may change due to changes in production. This is illustrated by comparing the results in Table 4 with the results in
Table **5**.

Table 5. Change in cost share over time ($\partial S_i/\partial t$) (standard error in parenthesis)

	Fertilizers	Feed	Labour	Machinery	Capital
Arable	0.005 (0.001)	-0.001 (0.001)	-0.004 (0.001)	0.001 (0.001)	-0.002 (0.001)
Dairy farms	0.008 (0.001)	-0.007 (0.001)	0.004 (0.001)	-0.006 (0.001)	0.000 (0.000)
Pig farms	0.006 (0.001)	-0.004 (0.001)	0.000 (0.001)	-0.003 (0.001)	0.000 (0.000)

The results in Table 5 show the change in cost share over time. All three types of farms showed significant increases in the share of fertilizer costs, indicating that the technological change was *not* cost-neutral. Thus the technological change was neither Hicks-neutral nor cost-neutral (see Chambers (1988, p. 218-220) on the relationship between Hicks neutrality and cost neutrality). However, it should be taken into consideration that the estimated model is neither non-decreasing nor concave in the price of fertilizers input for both dairy and pig farms. Therefore one should be careful not to draw definite conclusions.

On both dairy farms and pig farms, changes in production increased the proportion of fertilizer cost to total cost even though the technological change was fertilizer saving under the Hicks definition.

7. Discussion

The estimation of the three models performed relatively well. There was a very high degree of explanation, and most of the parameters had the expected sign. Estimation of the model on arable farms caused almost no problems. However, estimation of the models on dairy and pig farms involved significant problems. The monotonicity and the curvature conditions concerning fertilizer input were violated. And the residuals showed significant autocorrelation, indicating problems with the models.

Different ways of reducing autocorrelation were considered. A formal F-test of the residuals using dummies for each size group showed that there was a clear covariance between years (residuals from farms in the largest and the smallest size groups deviate significantly from the three middle groups). This may not be surprising, as the extreme groups typically catch the “outliers”. However, a new estimation with dummy variables included for the two extreme size groups did not improve the precision of estimation, and the estimated technological change was only marginally influenced.

To reduce autocorrelation in the dairy model, it was contemplated to include a dummy variable for the time period *after* introduction of the milk quota system in 1984. Although this would reduce autocorrelation, it would not necessarily improve the economic explanation. One should also consider that farmers would adjust to the quota system over time. The dummy variable would therefore only be relevant for a certain time period. Choice of this (unknown) time period would in any case be rather arbitrary.

Another possibility would be to include the milk quota as a capital asset in the model. However, this would be relevant only if the milk quota could be sold and purchased at some market price. In Denmark this has *not* been the case until recently. Before 1997 it was hardly possible to acquire milk quota, except by renting and buying land. However, if the farmer already had a (basis) quota, 33% of the quota acquired by buying or renting land was confiscated by the authorities. Before 1997 it was also possible to buy and sell milk quotas directly. But the price was controlled by the authorities, and was fixed at such a low level that almost no milk quota was traded in this way.

After 1997 a market has been established for trading milk quotas. Therefore, future analysis using data after 1997 should be based on a model including milk quotas as a tradable asset (input).

Concerning the monotonicity condition it is clearly a problem that the estimated cost function for both dairy farms and pig farms are (locally) *decreasing* in the price of fertilizers (aggregate of fertilizers, seed, chemicals, energy, etc.). There are a number of possible explanations. First of all it is a problem that the (real) price of fertilizers is correlated with some of the other explanatory variables. This makes it difficult to efficiently separate the effect of the individual variables. Secondly, the possible substitution between fertilizers and animal manure may not have been taken sufficiently into account. During the time period considered the efficiency of using animal manure has improved. It is possible that this improvement has not been totally accounted for in the model and the data used. Therefore future analysis may be improved by more explicit modelling of the production of animal manure and the use of animal manure as a substitute for fertilizers.

Concerning the curvature condition on input prices (a cost function is concave in input prices), this condition was violated in a few cases. When the monotonicity condition is not met (as with fertilizers in the dairy and pig models) it is not relevant to consider the curvature condition. Therefore the curvature condition in input prices was efficiently violated in only three out of the fifteen cases. This is of course a problem, but such a failure is often seen in empirical applications involving the translog form (Antle and Capalbo, 1988, p. 76).

The choice to consider land as a quasi-fixed input involved problems that might have been avoided if instead land had been considered a variable input. In a model with both input (here land) and output (here livestock and crops) as explanatory variables it is difficult to avoid multicollinearity – especially with aggregate data. This was also the case here, and therefore is was necessary to estimate both the cost function (10) and an inverse cost function (13). This two-step estimation could have been avoided by considering land as a variable input.

There are no objective criteria for determining whether land should be considered fixed or variable input. The choice made here was based on the fact, that Danish farmers normally consider land as a fixed input – at least in the short and medium run. Due to regulation of the Danish land market, land is often not available, and even in the long run there may be restrictions involved.

On the other hand, land is to some extent a variable input because it is possible to rent land. One could therefore ague that due to this possibility of renting land, the input land should be considered a variable input.

Which of the two approaches is the most "correct" may of course be discussed. In any case it could be interesting to re-estimate the model with land considered a variable input. However, this was not done here, as the relevant prices of land were not available.

8. Conclusion

There was a positive level of technological change in Danish agriculture from 1973 to1995. On average, the technological change levels for arable farms, dairy farms and pig farms were 4.0%, 1.0% and 2.2% respectively. The greatest average rate of technological change was observed on the largest arable farms, while the rate of technological change was very low on large dairy farms. The rate of technological change increased for all farm types over the period analysed.

The estimated elasticities of size show that there were economic incentives to increase the size of farms for both arable and dairy farms (elasticity of size greater than 1). For pig farms elasticity of size was close to 1, indicating that pig farms were able to adjust farm size in accordance with the economic incentives.

Using the Hicks measure of bias, the results show that the technological change was biased. On arable farms the technological change was fertilizers using, and feed saving, while on dairy and pig farms technological change was fertilizer saving and feed saving. However, the cost share of fertilizers increased on both arable, dairy and pig farms.

In the introduction the hypothesis was put forward that due to different production conditions, there would probably be differences in rates of technological change and economic performance between the three farm types. The results presented in this paper support this hypothesis. The rate of technological change was clearly greater on arable and pig farms than on dairy farms. On the other hand, while the largest farms showed the greatest amount of technological change in the case of arable farms, the opposite was the case for pig farms. As far as economies of scale were concerned, even large arable farms had considerable economic advantages from further increasing the size, while for pig farms this was the case only for farms with a size below average. For dairy farms the economic potential from increasing the farm size was greatest for the largest farms, which clearly indicates that these farms were seriously limited in their economic performance by the milk quota system.

The general conclusion to be drawn is that the amount of technological development was different on arable farms, dairy farms and pig farms over the period studied. The potential economic benefit from increasing farm size was also different for arable farms, dairy farms and pig farms, reflecting differences in economic conditions.

The high level of potential benefit from increasing farm size for arable farms may be explained by the restrictions on land acquisition during the time period considered. However, despite these restrictions, it was apparently advantageous to introduce modern production technology (machinery) in crop production, as the rate of technological change was relatively high. This was not the case with dairy farms. The large dairy farms in particular displayed a very low rate of technological change. At the same time they had a high potential economic benefit from increasing farm size. This probably reflects the milk quota restriction, but perhaps also the scarcity of land for roughage production. The combination of the very low level of technological change and the high elasticity of size indicates that technological change in dairy production was closely related to investments in increasing the farm size; or, to put it in another way, that the amount of technological change was low because the possibility or the economic motivation to increase farm size through new investments was low.

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Appendix 1.

Table 6. Average gross output in DKK ’000 (current prices) for every farm size category and for every year.

	Arable					Dairy					Pig				
Year	Size 1	Size 2	Size 3	Size 4	Size 5	Size 1	Size 2	Size 3	Size 4	Size 5	Size 1	Size 2	Size 3	Size 4	Size 5
1973	28	73	144	255	687	66	123	185	249	420	60	112	172	298	671
1974	37	85	160	262	642	81	148	216	280	489	60	119	202	338	749
1975	42	109	190	300	660	102	178	253	335	574	82	165	249	417	901
1976	44	109	196	325	730	115	206	292	395	681	83	163	284	457	1059
1977	49	131	240	397	876	146	253	359	505	824	104	235	419	624	1281
1978	50	133	250	427	946	176	311	445	609	1014	116	260	425	646	1283
1979	44	114	229	426	1130	204	331	439	571	1037	137	290	439	669	1451
1980	42	114	220	440	1093	216	369	491	644	1234	150	313	507	784	1543
1981	51	139	274	571	1399	250	437	582	803	1551	192	422	657	1058	2176
1982	65	158	302	610	1749	267	496	690	976	1742	221	472	764	1151	2656
1983	57	143	301	659	1765	281	530	748	1091	1972	240	560	961	1508	3295
1984	69	164	337	735	2069	283	550	782	1114	1918	268	639	1067	1689	3294
1985	63	171	422	894	2437	369	712	952	1207	2040	291	720	1141	1769	3621
1986	67	183	440	831	2282	350	697	914	1174	1985	267	695	1048	1586	3213
1987	50	124	311	653	1903	359	678	903	1175	1980	223	574	969	1523	3226
1988	55	140	295	636	1975	385	742	991	1297	2216	265	679	1105	1802	3453
1989	60	146	320	722	2324	420	835	1105	1470	2516	329	870	1501	2330	4423
1990	57	152	317	710	2360	397	785	1045	1422	2430	364	849	1439	2303	4460
1991	57	153	338	782	2450	395	796	1072	1479	2373	368	962	1672	2549	5206
1992	47	125	268	674	2243	433	818	1077	1464	2397	338	878	1559	2430	5186
1993	52	135	273	584	2132	495	900	1167	1597	2456	268	716	1401	2192	4637
1994	56	140	285	667	2251	536	906	1216	1677	2544	343	854	1537	2297	4796
1995	71	168	331	698	2557	554	925	1287	1735	2636	440	1108	1782	2566	5072

Appendix 2

Table 7. Parameter estimates and standard errors of the three cost functions.

	Arable model		Dairy model		Pig model	
Parameter	**Estimate**	**Std.Error**	**Estimate**	**Std.Error**	**Estimate**	**Std.Error**
a_0	0.089	0.018	0.791	0.101	0.629	0.086
a_1	0.067	0.039	0.643	0.382	0.235	0.217
a_2	-0.041	0.069	-0.069	0.114	0.055	0.114
b_1	0.918	0.011	0.971	0.022	0.842	0.029
d_1	0.110	0.010	-0.133	0.013	-0.125	0.016
d_2	0.117	0.007	0.197	0.011	0.267	0.014
d_3	0.321	0.013	0.497	0.012	0.345	0.012
d_4	0.304	0.010	0.295	0.012	0.377	0.013
d_5	0.148	0.008	0.144	0.005	0.135	0.006
v	-0.009	0.004	-0.067	0.009	-0.031	0.011
f_{11}	0.052	0.039	-1.128	0.876	0.037	0.298
f_{12}	0.064	0.078	0.097	0.237	-0.144	0.119
h_{11}	-0.024	0.007	-0.076	0.021	-0.084	0.020
h_{12}	0.016	0.003	0.115	0.018	0.057	0.016
h_{13}	-0.000	0.005	-0.125	0.016	-0.022	0.008
h_{14}	0.016	0.003	0.114	0.017	0.046	0.013
h_{15}	-0.008	0.003	-0.027	0.006	0.002	0.005
g_{11}	-0.164	0.094	0.931	1.026	0.213	0.515
k_1	-0.008	0.004	0.047	0.018	-0.001	0.016
f_{22}	-0.094	0.188	-0.030	0.124	0.230	0.120
h_{21}	0.063	0.014	0.036	0.008	0.003	0.013
h_{22}	-0.034	0.007	-0.036	0.007	0.007	0.010
h_{23}	-0.001	0.011	0.022	0.006	-0.008	0.005
h_{24}	-0.017	0.008	-0.029	0.006	0.010	0.008
h_{25}	-0.011	0.007	0.007	0.002	-0.012	0.003
g_{21}	0.064	0.202	-0.044	0.308	-0.249	0.246
k_2	0.010	0.010	-0.001	0.006	-0.008	0.009
m_{11}	0.106	0.292	-1.220	1.354	0.144	0.975
n_{11}	-0.042	0.019	-0.024	0.025	-0.011	0.038
n_{12}	0.043	0.009	0.002	0.021	0.035	0.031
n_{13}	-0.048	0.014	-0.026	0.019	-0.073	0.016
n_{14}	0.040	0.010	0.019	0.020	0.050	0.025
n_{15}	0.008	0.008	0.029	0.008	-0.001	0.010
q_1	-0.005	0.011	-0.018	0.022	0.025	0.029
r_{11}	0.098	0.029	0.092	0.016	0.150	0.028
r_{12}	-0.015	0.013	-0.031	0.013	-0.114	0.022
r_{13}	-0.022	0.022	-0.026	0.013	0.037	0.016
r_{14}	-0.042	0.015	-0.027	0.014	-0.101	0.020
r_{15}	-0.020	0.013	-0.007	0.006	0.028	0.009
s_1	0.005	0.001	0.008	0.001	0.006	0.001
r_{22}	0.063	0.011	0.019	0.012	0.153	0.020
r_{23}	-0.110	0.017	0.020	0.012	-0.111	0.017
r_{24}	0.083	0.015	-0.005	0.012	0.124	0.019
r_{25}	-0.021	0.009	-0.004	0.005	-0.052	0.009
s_2	-0.001	0.001	-0.007	0.000	-0.004	0.001
r_{33}	0.404	0.049	0.119	0.026	0.293	0.033
r_{34}	-0.187	0.040	-0.033	0.024	-0.200	0.027
r_{35}	-0.087	0.019	-0.080	0.009	-0.019	0.011
s_3	-0.003	0.001	0.004	0.001	0.000	0.001
r_{44}	0.182	0.044	0.074	0.026	0.254	0.029
r_{45}	-0.036	0.016	-0.009	0.008	-0.076	0.011
s_4	0.001	0.001	-0.006	0.001	-0.003	0.001
r_{55}	0.163	0.016	0.100	0.007	0.119	0.008
s_5	-0.002	0.001	0.000	0.000	0.000	0.000
u	-0.003	0.001	-0.006	0.001	-0.002	0.001

Appendix 3.

Interpretation of the inverse cost function

Consider a farmer producing one output *y* using one variable input *x* and one fixed input z_1. Prices of *y* and *x* are *p* and *w* respectively. Assuming profit maximizing behaviour, the relation between use of fixed input z_1 and production *y* is determined by the so-called pseudo scale line (Debertin, 1986, p. 124) *aa* shown in Figure 1, where $y^1 < y^2 < y^3$ are isoquants of *y*.

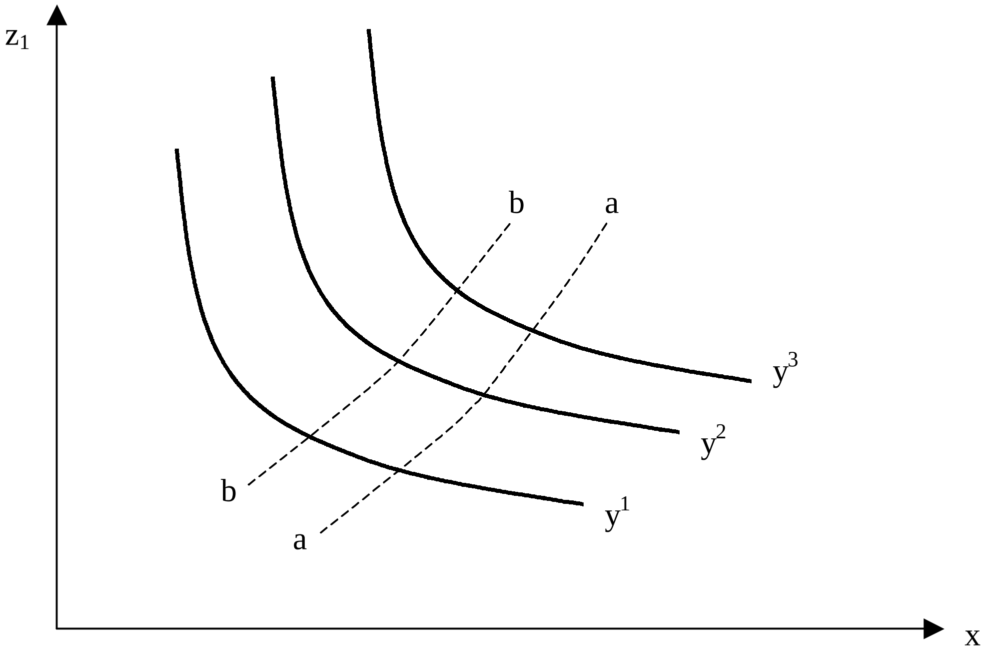

Figure 1. Relationship between the use of fixed input z_1, variable input *x*, and production *y*

The pseudo scale line *aa* is the relation between z_1, *y* and *x* where:

$$\frac{\partial y(x, z_1)}{\partial x} p = w \qquad \text{(i)}$$

If we relax the assumption of profit maximizing behaviour and only assume cost minimizing behaviour, the relation between z_1, *y* and *x* will be to the left of *aa* – for instance at *bb* in Figure 1 – where

$$\frac{\partial y(x,z_1)}{\partial x}p \geq w \qquad \text{(ii)}$$

Given the (variable) cost per hectare $\overline{C}$, the cost minimizing relationship between z_1 and y may be indirectly determined by the dual relationship

$$z_1 = z_1(y,\overline{C},w) \qquad \text{(iii)}$$

The function (iii) may be generalized to include more outputs and inputs. If y is a vector (y_1, y_2) and w is a vector ($w_1 \ldots w_5$), then the cost minimizing relationship between the variable inputs is determined by:

$$\frac{\partial y_k(x,z_1)}{\partial x_i} = \frac{\partial y_k(x,z_1)}{\partial x_j} \qquad \text{(all i, j, k)} \qquad \text{(iv)}$$

Given the (variable) cost $\overline{C}$ per hectare, the cost minimizing relationship between z_1 and y_1, and y_2, may be indirectly determined by the dual relationship

$$z_1 = z_1(y_1,y_2,\overline{C},w_1,\ldots,w_5) \qquad \text{(v)}$$

which is identical to (11). Thus the function z_1 in (11) is the amount of land necessary to produce y, given efficient use of the variable inputs. Correspondingly $\partial z_1/\partial y_j$ is the change in the use of land when production y_j changes and variable input is still used efficiently.

Under standard assumptions concerning production technology, the function z_1 has the following properties:

$$\partial z_1/\partial y_j \geq 0 \ ; \quad \partial z_1/\partial\overline{C} \leq 0 \ ; \quad \partial z_1/\partial w_i \geq 0$$

Appendix 4

Table 8. Parameter estimates and standard errors of the three inverse cost functions (13)

	Arable model		Dairy model		Pig model	
Parametre	Estimate	Std.Error	Estimate	Std.Error	Estimate	Std.Error
α	0.049	0.077	0.913	0.283	0.305	0.196
β_1	0.381	0.069	1.004	0.105	0.475	0.101
β_2	0.240	0.096	-0.096	0.067	-0.014	0.083
β_3	0.007	0.008	-0.017	0.007	0.009	0.010
β_{11}	0.058	0.077	-0.220	0.214	0.171	0.084
β_{12}	-0.106	0.093	0.113	0.145	-0.092	0.088
β_{22}	0.193	0.120	-0.028	0.109	0.133	0.102
β_{1t}	-0.009	0.005	-0.003	0.006	-0.016	0.007
β_{2t}	-0.011	0.006	0.001	0.005	-0.002	0.006
β_{tt}	0.001	0.001	-0.000	0.000	0.001	0.000
γ	-1.285	-	-0.738	-	-0.594	-
θ	0.686	0.370	0.780	0.270	-0.058	0.205
δ_1	0.117	0.104	0.047	0.075	0.205	0.103
δ_2	0.109	0.124	-0.084	0.057	-0.041	0.126
δ_3	2.025	0.310	1.686	0.353	0.931	0.314
δ_4	-1.285	-	-0.995	-	-0.469	-
δ_5	0.319	0.145	0.084	0.090	-0.032	0.115

Note: As the parameters of model (13) are estimated based on transformed variables (see the text after (13)), γ and δ_4 are recaptured by backward transformation involving both w_4 and $\overline{C}$. The values of γ and δ_4 in the table are estimated based on average values of w_4 and $\overline{C}$.