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**Abstract**

The contracts we consider in this paper must solve three problems: moral hazard, insurance and discrimination. The moral hazard problem is that of providing the agents with incentives to perform in a way that maximizes the profit to the principal, when the agent's actions are unobservable. The insurance problem is that of minimizing the cost of risk through risk minimization and risk sharing. The issue of discrimination is that of paying agents with different skills sufficiently to participate, without overcompensating other agents. We show how the principal may benefit from a strategic division of the agents into different tournaments or groups. The optimal number of groups from the principal's point of view is determined through a trade-off between moral hazard, insurance and discrimination issues.

## 1 Introduction

Tournaments are used in a number of agricultural contracts, especially livestock contracts. In tournaments (or relative performance) the reward to a producer is determined by comparing his performance to the performance of other producers. The previous papers on tournaments in agricultural contracts mainly focus on the issues of incentives (or moral hazard) and insurance (risk sharing). Knoeber (1989) focuses on the question of why tournaments dominate the broiler industry in the US. Goodhue (1999) and (2000) analyze how input provision can reduce information rents in the tournament based broiler contracts. Tsoulouhas and Vukina (1999) show how limited liability of the processor may hinder the use of relative performance schemes. They use this insight to explain why contracts based on relative performance standards are dominating in some industries, while other industries are dominated by fixed performance standards.

In this paper we focus on a simple and widely used linear cardinal tournament, where the producers are compensated according to their relative performance (see e.g. Olesen, 2001). The relative performance of a producer is determined by comparing his performance to the average performance in his group.

The use of tournaments has been rationalized mainly by two arguments. Shleifer (1983) rationalizes tournaments (or more precisely yardstick competition) as a way of motivating firms to reveal private information about production cost etc. Bogetoft (1997) extends this approach by combining Data Envelope Analysis and tournaments. This enables him to handle more general cost and production systems than the simple linear ones usually considered.

Holmström (1981) emphasizes the insurance problem and views tournaments as a way to extract information about common risk: "*forcing producers to compete with each other is valueless if there is no common underlying uncertainty*" (p. 335). According to Holmström tournaments serve as an insurance instrument, where common risk is shifted from the producer to the processor. This argument is also used by Knoeber (1989) to explain the use of tournaments in broiler contracts. Tsoulouhas and Vukina (2000) show that tournaments have positive welfare effects because they fascilitate risk sharing. Knoeber and Thurman (1995) compute the amount of common risk shifted from broiler producers to processors through tournaments.

The focus of this paper is on another feature of tournaments, the ability to discriminate between producers. We show that the combination of discrimination, incentives and insurance issues can explain sorting of producers in cardinal tournaments. We also show how strategic group division can increase the processor's profit[1].

Usually discrimination problems arise in adverse selection contexts, where the processor does not know the true types of the producers (precontractual hidden information). In this paper we analyze a different source of discrimi-

[1]Sorting producers in cardinal tournaments is relevant, since it improves the discrimination as we show below. In ordinal tournaments handicapping and sorting of producers can imprve the incentives, c.f. Knoeber and Thurman (1994).

nation problems, namely the case where the processor knows the producer's type but is unable to use this information directly in the contract (for reasons explained below). This forces the processor to compensate all producers according to the same rule where the payment depends on outcome only and not on the producer's type. The analysis of the discrimination in linear tournaments (Section 3) would give exactly the same results in an adverse selection context. However, the strategic division of the producers (section 4) can occur only if the processor knows the type of each producer.

Different circumstances may force the processor to compensate all producers according to one rule independent of the type of the producers.

First, the producers may possess some bargaining power and, through collective bargaining, force the processor to use only one contract. If the processor is prevented from using different payment schemes to different producers, the processor may be forced to raise the payment to all producers in order to attract a particular type of producers. An example of this situation is the case of the pea growers in Denmark producing for Danisco Foods. Due to efficiency considerations, the firm wants to use its limited capacity to contract with the growers on the best soils. The growers, however, have blocked the use of bonuses for growers with high soil quality through collective bargaining. This enables the growers to extract information rent (Olesen, 2000).

Second, legal restrictions such as anti discrimination clauses may prevent the processor from using all available information directly in the contract. This may force the processor to offer the same payment scheme to all producers.

Finally, transaction costs may give the processor incentives to simplify the contracts, see e.g. Milgrom and Roberts (1992) and Goodhue (2000).

This paper deviates from earlier studies of tournaments in agricultural contracts, because we assume that the producers are heterogenous in two dimensions - skills and reservation value. Most of the previous papers on the subject assume homogenous producers. The papers analyzing models with heterogeneous producers (e.g. Goodhue, 2000) assume that the producers have different skills, but that all producers have the same reservation value. However, in many cases high-skilled producers have better opportunities outside the contract relationship than low-skilled producers. Therefore, we relax the assumption that all producers have the same reservation value and allow high-skilled producers to have higher reservation value than low-skilled producers.

We analyze a case where a processor contracts with two types of producers, high-skilled producers with high reservation value and low-skilled producer with low reservation value. In this setting, the low-skilled producers expect to always lose a tournament to high-skilled producers and to receive a lower payment than high-skilled producers. However, a low-skilled producer wants to participate as long as the profit he receives within the tournament exceeds his reservation value. Often one type of producers (e.g. low-skilled producers) receive quasirent (i.e. a compensation exceeding their reservation value) when the contract is adjusted to meet the reservation value of another group of producers (e.g. the high-skilled producers). This paper demonstrates how dividing the producers into groups can reduce the quasirents.

The outline of the paper is as follows: A description of our basic model is given in Section 2, where the insurance and the moral hazard are introduced. In Section 3, we introduce the discrimination issue and solve the processor's problem when all producers compete in one big group. In Section 4 we analyze how the processor prefers to divide the producers. We consider the case of internally heterogenous groups in section 4.1, where every group includes both high and low-skilled producer. In Section 4.2 we address the question: how many groups does the processor prefer? In Section 4.3 we analyze the case where the producers are divided into internally homogeneous groups, so that high-skilled producers compete only with high-skilled producers. In Section 5 we illustrates the findings in a numerical example. Section 6 concludes the paper.

# 2 The Model

We consider a processor contracting with $n$ producers. The processor earns $p$ on each unit produced. The production of producer $i$ is given by a simple additive function

$$y_i = e_i + s_i + \varepsilon_i + \mu \tag{1}$$

where $e_i$ is producer $i$'s unobservable effort, $s_i$ is the skill of producer $i$. The output is affected by two independent random variables. The random variable $\mu \sim N(0, \Sigma^2)$ is a general disturbance affecting all producers, and $\varepsilon_i$ is a disturbance factor affecting only producer $i$ (idiosyncratic risk), $\varepsilon_i$ is independent identically distributed $N(0, \sigma^2)$.

For convenience, we assume that there are only two types of producers, $n_H$ producers with high skills ($s_H$) and $n_L$ producers with low skills ($s_L$), where $s_H > s_L$. We assume that it is always optimal for the processor to contract with all producers. We also assume that $n_L = n_H = \frac{n}{2}$, this symmetry enables us to normalize our model with $s_L = -\theta$ and $s_H = \theta$, giving average skills of zero.

We assume that the cost of effort is

$$\Psi(e_i) = \frac{1}{2}e_i^2$$

Furthermore, we assume that the processor is risk neutral and that the producers are risk averse and have a utility function of the form

$$u_i(x_i, e_i) = -\exp\left(-r\left[x_i - \Psi\left(e_i\right)\right]\right)$$

where $r$ is the absolute risk aversion, common to all producers, and $x_i$ is the payment to producer $i$.

Using the properties of the distribution of the uncertainty and the negative exponential utility function, the utility can be expressed in terms of certainty equivalence[2]

$$CE_i\left(x_i, e_i\right) = E\left(x_i\right) - \Psi\left(e_i\right) - \frac{1}{2}\beta^2 rVar(x_i)$$

The processor can observe neither the effort nor the uncertainty parameters, i.e. $e$, $\mu$, $\varepsilon_i$. He observes the skill $s_i$, but cannot use this information directly in the payment scheme.

The producers are compensated according to a linear tournament[3] given by

$$x_i = t + \beta\left(y_i - \bar{y}\right) \tag{2}$$

I.e., the producer is paid a base transfer $t$, common to all producers, plus a reward-factor $\beta$ times the relative performance, measured by his deviation from the average performance of the group $\bar{y} = \frac{1}{n}\sum_{j=1}^{n} y_j$.

[2]See for instance Holmström and Milgrom (1991) for a similar modelling approach.

[3]All of our results also hold for a linear yardstick contract of the form $x_i = t + \beta\left(y_i - \bar{y}_{-i}\right)$, where $\bar{y}_{-i} = \frac{1}{n-1}\sum_{j\neq 1} y_j$ (c.f. Schleifer, 1985). Under yardstisk contracts the intuition for group division is that a high-skilled producer will be compared to relatively more low-skilled producers than the number of high-skilled producers if the producers are divided into heterogenoeus groups (see Section 4.1.).

The payment to producer $i$, determined by the tournament, is

$$x_i = t + \beta \left[ (s_i - \overline{s}) + (e_i - \overline{e}) + (\varepsilon_i - \overline{\varepsilon}) \right]$$

where $\overline{s}$, $\overline{e}$, and $\overline{\varepsilon}$ are defined in the same way as $\overline{y}$. Note that $\overline{s} = 0$ due to our normalization. Note also that the tournament removes the common risk $\mu$ from the payment. This enables the processor to ensure the producers against common risk without reducing the incentives, c.f. Holmström (1982).

The certainty equivalence to producer $i$ is

$$CE = t + \beta \left[ s_i + (e_i - \overline{e}) \right] - \frac{1}{2} e_i^2 - \frac{1}{2} \beta^2 r Var\left( \varepsilon_i - \overline{\varepsilon} \right) \tag{3}$$

The processor chooses the contract parameters $t$ (base transfer) and $\beta$ (reward-factor) to maximize his profit under two constraints. The structure of the risk in our model, imply that $Var\left(\varepsilon_i - \overline{\varepsilon}\right) = \sigma^2 \frac{n}{n-1}$ since $\varepsilon_i$ is i.i.d. $N(0, \sigma^2)$.

The first constraint is that the producers must benefit (weakly) from participating. This is the individual rationality constraint (IR). Each producer has an outside opportunity giving him a certainty equivalence of $\overline{u}\left(s_i\right)$. We assume that $\overline{u}\left(s_H\right) \geq \overline{u}\left(s_L\right)$, such the processor must pay more to the high-skilled producers to attract them. Without loss of generality[4], we normalize the measure of the reservation values such that $\overline{u}\left(s_L\right) = -w$ and $\overline{u}\left(s_H\right) = w$. Hence, the average reservation utility is zero.

The second constraint is that the producers choose their efforts to maximize their own utilities. This is the incentive compatibility constraint (IC).

The processor's problem is[5]

$$\max_{e,t,\beta} \Pi = E\left( p \sum_{i=1}^{n} y_i - nt \right)$$

$$s.t.$$

$$CE_i\left(e_i\right) \geq CE_i\left(e_i'\right) \quad \text{for all } i, e' \tag{IC}$$

$$CE_i\left(e_i, s_i\right) \geq \overline{u}\left(s_i\right), \qquad s_i \in \{\ s_L,\ s_H\} \tag{IR}$$

The IC constraint is fulfilled, when the certainty equivalence is maximized. By concavity of the certainty equivalence, the sufficient first order

[4] The actual level of reservertion value only affect the base transfer $t$. Hence, for our analysis it is sufficient to consider the relative reservation values.

[5] We use $\beta \left[ \sum_{i=1}^{n} \left(y_i - \overline{y}\right) \right] = 0$.

condition is

$$\frac{\partial CE_i}{\partial e_i} = \beta \frac{n-1}{n} - e_i = 0$$

In optimum all producers choose the same level of effort $e_i = \beta \frac{n-1}{n}$, which is lower than the reward-factor. The reason is that the producers take account for the fact that if a producer increases his effort he also increases the mean output in his group, to which he is being compared.

## 2.1 Homogeneous Producers: The Moral Hazard Effect

It is useful to consider the case where all producers are identical, i.e. $s_i = 0$ and $\overline{u}_i(s_i) = 0$ for all $i$, as a benchmark case. In this case, the processor does not impose any distortion in the producers effort levels to discriminate. Identical producers face the same incentives and provide the same effort and therefore obtain the same expected output. Thus, the IR constraint reduces to

$$t \geq \frac{1}{2}e_i^2 + \frac{1}{2}\beta^2 r\sigma^2 \frac{n}{n-1} + \overline{u}_i$$

i.e. the base transfer must exceed the producers' cost of providing effort plus the risk premium plus the reservation value. Substituting $\overline{u}_i = 0$, and $\beta = e\frac{n}{n-1}$ (where $e$ is the common value of $e_i$), and $t = \frac{1}{2}e^2 + \frac{1}{2}re^2\sigma^2\frac{n}{n-1}$ into the objective function gives

$$\max_e \frac{\Pi}{n} = p(\theta + e) - \frac{1}{2}e^2 - \frac{1}{2}e^2 r\sigma^2 \frac{n}{n-1}$$

The objective function now reflects the usual moral hazard problem. To increase the effort level $e$ the processor must expose the producers to more risk which increases the risk premium. This optimization problem has the following (sufficient) first order condition

$$e^* = \frac{p}{1 + r\sigma^2 \frac{n}{n-1}} \tag{4}$$

This is a classic result in the moral hazard literature; the processor does not induce the first best level of effort ($e = p$), because this would expose the producers to too much risk, see e.g. Holmström (1979). An increase in the variance $\sigma^2$ or the absolute risk aversion $r$ increases cost of the risk. Thus, the processor induces a lower level of effort when $\sigma^2$ or $r$ is high. As the number of producers increases, the risk premium decreases and the optimal level of effort increases. This is so, because the processor obtains more precise information about the effort $e_i$ since the noise from $\overline{\varepsilon}$ can be eliminated more and more (the law of large numbers).

# 3 Heterogeneous Producers: The Discrimination Effect

After introducing the moral hazard issue, we now look at the discrimination issue. As mentioned in the introduction, we consider situations where the processor is restricted to use the same $t$ and $\beta$ for all producers in one group, i.e. he cannot relate the payment directly to the skills. Therefore, the processor can only meet the IR constraint for one type of producers by altering $\beta$ and $t$ for all producers.

The payment to the two types of producers differ due to the difference in their skills and resulting output. The IR constraint must hold for both types of producers, i.e.

$$CE_L\left(e_L, s_L\right) \geq \overline{u}\left(s_L\right) \quad \text{and} \quad CE_H\left(e_H, s_H\right) \geq \overline{u}\left(s_H\right)$$

Using (3) these IR constraints can be written as

$$t - \theta e \frac{n}{n-1} - \frac{1}{2}e^2 - \frac{1}{2}e^2 r\sigma^2 \frac{n}{n-1} \geq -w \tag{IR-L}$$

and similarly

$$t + \theta e \frac{n}{n-1} - \frac{1}{2}e^2 - \frac{1}{2}e^2 r\sigma^2 \frac{n}{n-1} \geq w \tag{IR-H}$$

Notice that the high skilled-agents are rewarded for their skills and that the low-skilled agents are penalized. Notice also that both the left- and the right-hand side of the IR constraint for the high-skilled producers are larger than for the low-skilled producers. Thus, either of the constraints can be binding in a given situation.

The optimal base payment $t$ depends on the more demanding of the two individual rationality constraints. If the IR constraint for the low-skilled producers is the more demanding constraint, the processor chooses

$$t = t_L = -w + \theta e \frac{n}{n-1} + \frac{1}{2}e^2 + \frac{1}{2}e^2 r\sigma^2 \frac{n}{n-1}$$

and the high-skilled producers earn quasirents. If the IR constraint for the producers with high skills are the more demanding, the reverse is true and the base payment is

$$t = t_H = w - \theta e \frac{n}{n-1} + \frac{1}{2}e^2 + \frac{1}{2}e^2 r\sigma^2 \frac{n}{n-1}$$

Our model does not tell per se which of the IR constraints is the more demanding. We therefore have three cases to consider. Case A, where the low-skilled producers receive quasirents. Case B, where the high-skilled producers receive quasirents. And finally Case C, where none of the producers receive quasirents.

## 3.1 Case A: quasirents to low-skilled producers

We now look at the situation where the payment exactly meets the reservation value of the high-skilled producers, such that high-skilled producers receive no quasirents. We use that $\beta = e$ and $t = t_H$. We denote the level of effort in this setting $e^H$. The processor maximizes his profit per agent

$$\frac{\pi^H}{n} = \max_{e^H} \left[ pe^H - \frac{1}{2}\left(e^H\right)^2 - \frac{1}{2}\left(e^H\right)^2 r\sigma^2 \frac{n}{n-1} - \left( w - \theta e^H \frac{n}{n-1} \right) \right]$$

$$sub : w \geq \theta e^H \frac{n}{n-1}$$

The constraint $w \geq e^H \theta \frac{n}{n-1}$ ensures that the IR constraint for the low-skilled producer is fulfilled. In Case A, the IR constraint is not binding for the low-skilled producers. If the constraint binds, none of the producers receive quasirents (Case C).

The first order condition[6] for maximum profit generated by one producer is

$$\frac{d\pi^h(e)}{de^H} = p - e^H - \underbrace{e^H r\sigma^2 \frac{n}{n-1}}_{\text{moral hazard effect}} + \underbrace{\theta \frac{n}{n-1}}_{\text{discrimination effect}} = 0$$

[6] The second order condition is always fulfilled, since: $\frac{d^2\pi^h}{d(e^H)^2} = -1 - \frac{n}{n-1}r\sigma^2 < 0$.

⇔

$$e^H = \frac{p + \theta \frac{n}{n-1}}{1 + r\sigma^2 \frac{n}{n-1}} \tag{5}$$

The first order condition shows that the discrimination effect $\theta \frac{n}{n-1}$ is positive[7]. *I.e. the processor induces a higher level of effort in order to reduce quasirents to the low-skilled producers.* Stronger incentives means higher penalty to low-skilled producers for low performance.

Figure 1 below illustrates the situation. The figure displays the payment $(x)$ as a function of output $(y)$. The producers are rewarded according to their relative skills, hence the high-skilled producers are paid more for the same level of effort than the low-skilled producers. The low-skilled producers receive a higher payment than their IR constraint require - i.e. quasirents.

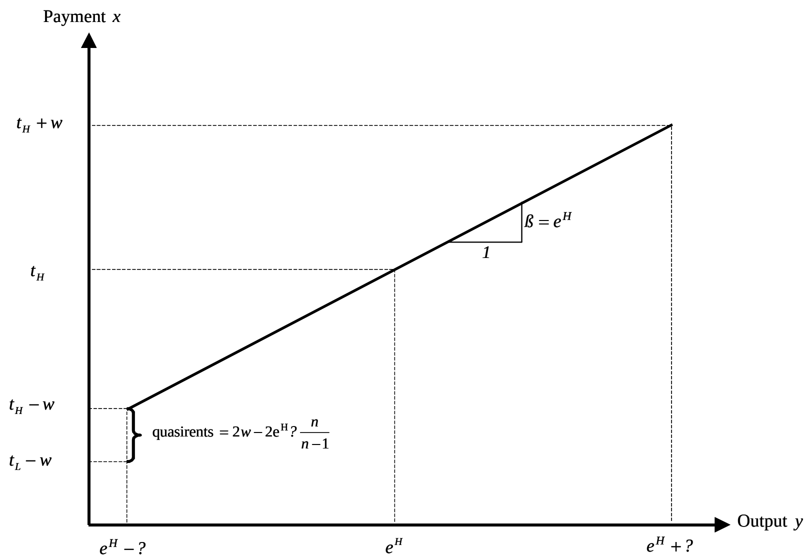


**Figure 1: Quasirents to low-skilled agents**

## 3.2 Case B: quasirent to high-skilled producers

Next, we consider the situation where the payment exactly meets the low-skilled producers' reservation value, i.e. they receive zero quasirents. In this

[7] A common feature of adverse selection models is that the principal distort the level of effort to reduce quasirents, see for example Laffont and Tirole (1993).

section we assume that the high-skilled producers receive positive quasirents (we return to the situation where none of the producers receive quasirents in Case C). We denote the level of effort in this case $e^L$. The processor maximizes profit per agent

$$\frac{\pi^L}{n} = \max_e \left[ pe^L - \frac{1}{2}\left(e^L\right)^2 - \frac{1}{2}\left(e^L\right)^2 r\sigma^2 \frac{n}{n-1} - \left(-w + \theta e^L \frac{n}{n-1}\right)\right]$$

$$s.t. \quad w \leq \theta e^L \frac{n}{n-1}$$

The constraint $w \leq \theta e^L \frac{n}{n-1}$ ensures that the IR constraint is fulfilled for the high-skilled producers. In Case B, the constraint holds with inequality, and the maximization problem has an internal solution of

$$e^L = \frac{p - \theta\frac{n}{n-1}}{1 + r\sigma^2 \frac{n}{n-1}} \tag{6}$$

*When the high-skilled producers receive quasirents, the processor induces a lower level of effort*, i.e. the discrimination effect is negative. The reason is that reducing the incentives also reduces the benefits for having high skills. The situation is illustrated in Figure 2 below. The figure is constructed in the same way as Figure 1. In Figure 2, the high-skilled producers receive quasirents.

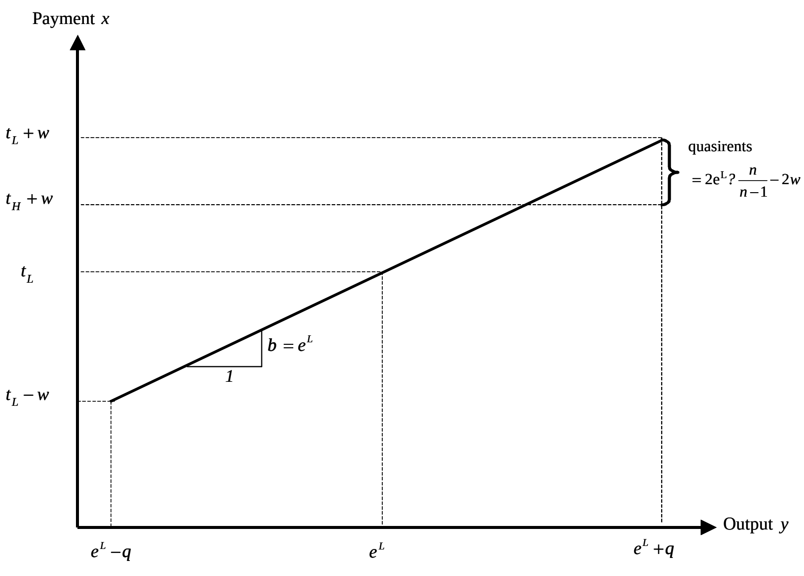


**Figure 2: Quasirents to high-skilled agents**

The figure shows that reducing the incentives decreases the quasirent to the high-skilled producers, therefore the discrimination effect decreases the optimal level of effort.

### 3.3 Case C: no quasirents

When neither Case A nor Case B applies, the processor eliminate all quasirents. We refer to this as Case C. The relevant interval for Case C is

$$\frac{p-\theta\frac{n}{n-1}}{\frac{n-1}{n}+r\sigma^2}\theta \leq w \leq \frac{p+\theta\frac{n}{n-1}}{\frac{n-1}{n}+r\sigma^2}\theta \tag{7}$$

In this interval the processor always chooses the level of effort $e^C = \frac{n-1}{n}\frac{w}{\theta}$, which induces perfect discrimination between the two types of producers, such that all quasirents are eliminated. In Case C there is no freedom in the choice of $\beta$ and $t$. The processor basically has to solve two independent equations with two independent variables, hence $e^C = \frac{n-1}{n}\frac{w}{\theta}$.

Outside the interval (7) there is imperfect discrimination and one type of producer receive quasirents. When one type of producer receive quasirents, the level of effort determined in Case A or Case B applies. Thus, we have the following function for the level effort $e$

$$e = \begin{cases} e^H = \frac{p+\theta\frac{n}{n-1}}{1+r\sigma^2\frac{n}{n-1}} & \text{if } \; w \geq \theta e^H \frac{n}{n-1} \quad \text{(Case A)} \\ e^C = \frac{n-1}{n}\frac{w}{\theta} & \text{otherwise} \quad \text{(Case C)} \\ e^L = \frac{p-\theta\frac{n}{n-1}}{1+r\sigma^2\frac{n}{n-1}} & \text{if } \; w \leq \theta e^L \frac{n}{n-1} \quad \text{(Case B)} \end{cases} \tag{8}$$

Figure 3 maps the three different cases.

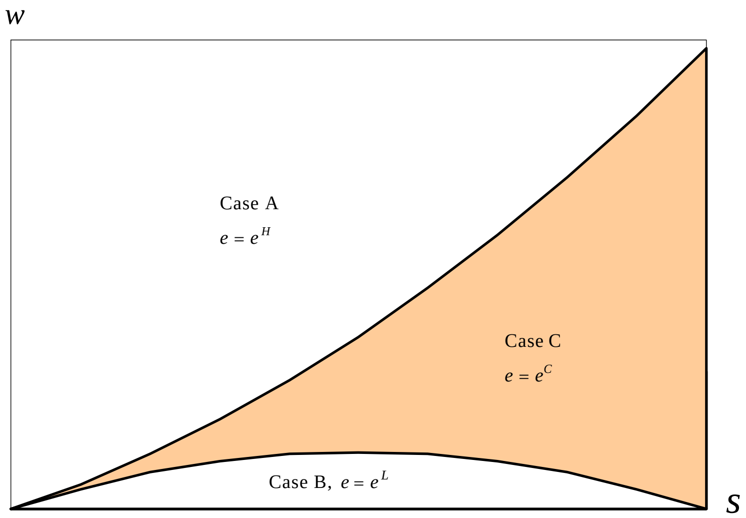


**Figure 3: Mapping of case A, B and C.**

Notice that $e^L < e^* < e^H$, i.e. outside the interval where no producer earns quasirents, the level of effort is always distorted. In the interval where no producer earns quasirents (Case C), *the level of effort may be distorted both upwards and downwards* due to the discrimination effect, i.e. $e^C \gtrless e^*$.

# 4 Group Division

So far, we have considered only cases where all producers compete in one group (tournament). However, the processor may use his knowledge about the producers' types to divide them into smaller groups. In particular, the processor may reduce the quasirents if he divides the producers into more than one group.

We assume that the level of effort is in the interval where it is optimal to discriminate perfectly between the two types of producers and eliminate all quasirents - i.e. Case C[8].

There are two distinct ways the groups can be divided. The processor can choose to make internally heterogeneous groups where producers with high skills are mixed with producers with low skills. The other approach is to make internally homogeneous groups where all producers in one group have the same skills.

[8] I.e. we assume: $\frac{p-\frac{n}{n-1}\theta}{1-\frac{1}{n}+r\sigma^2}\theta \leq w \leq \frac{p+\frac{n}{n-1}\theta}{1-\frac{1}{n}+r\sigma^2}\theta$.

## 4.1 Heterogeneous Groups

When there is perfect discrimination, the reward factor is $\beta = \frac{w}{\theta}$, hence the reward (penalty) for having high skills (low skills) is independent of the group size. However, a producer has more impact on the average performance in his group when the group is small. This reduces the producers incentives to provide effort, thus the level of effort changes according to the size of the group. The distortion in the level of effort can either increase or decrease when the producers are divided into groups.

When the producers compete in two heterogeneous groups, the level of effort is

$$e^{he} = \frac{n-2}{n}\frac{w}{\theta} \tag{9}$$

The difference in the level of effort compared to the case where all producers compete in one big group is

$$e^{C} - e^{he} = \frac{n-1}{n}\frac{w}{\theta} - \frac{n-2}{n}\frac{w}{\theta} = \frac{1}{n}\frac{w}{\theta} > 0$$

i.e. *the level of effort is always lower if the producers compete in two heterogeneous groups than if all producers compete in one group.*

The processor may gain from dividing the producers into two heterogeneous groups, if the effort induced when all producers compete in one big group is distorted upwards relative to $e^*$ (i.e. $e^C > e^*$). On the other hand, if the level of effort is distorted downwards relative to $e^*$ when all producers compete in one big group (i.e. $e^C < e^*$), dividing the group will only make things worse - for two reasons. First, the level of effort is always lower when the producers compete in two heterogeneous groups instead of one group, i.e. the distortion in the level of effort increases when the producers are divided into two groups. Second, the uncertainty increases when the producers are divided into two groups. In other words, *it is relevant to divide the producers into two groups only if the level of effort is distorted upwards* when all producers compete in one big group (i.e. $e^C > e^*$).

When all producers compete in one big group, the processor earns a profit per agent of

$$\frac{\pi^C}{n} = pe^C - \frac{1}{2}\left(e^C\right)^2 - \frac{1}{2}\left(e^C\right)^2 r\sigma^2 \frac{n}{n-1}$$

When the producers compete in two heterogeneous groups, the processor obtains a profit per agent of

$$\frac{\pi^{he}}{n} = pe^{he} - \frac{1}{2}\left(e^{he}\right)^2 - \frac{1}{2}\left(e^{he}\right)^2 r\sigma^2 \frac{n}{n-2}$$

The difference in profit is (using (8))

$$\begin{aligned}\frac{\pi^{C} - \pi^{he}}{n} &= pe^{C} - \frac{1}{2}\left(e^{C}\right)^2 - \frac{1}{2}\left(e^{C}\right)^2 r\sigma^2 \frac{n}{n-1} \\ &\quad - \left[pe^{he} - \frac{1}{2}\left(e^{he}\right)^2 - \frac{1}{2}\left(e^{he}\right)^2 r\sigma^2 \frac{n}{n-2}\right]\end{aligned}$$

$\Leftrightarrow$

$$\frac{\pi^{C} - \pi^{he}}{n} = \frac{w}{\theta}\left[\frac{1}{n}p - \left(\frac{1}{2}r\sigma^2 + \frac{n-\frac{3}{2}}{n^2}\right)\left(\frac{w}{\theta}\right)\right]$$

We therefore have the following result: *the processor chooses to have the producers competing in*

$$\begin{aligned}&\text{one group if } \ w \leq \frac{p\theta}{1-\frac{3}{2n}+\frac{1}{2}r\sigma^2} \\ &\text{two groups if } \ w > \frac{p\theta}{1-\frac{3}{2n}+\frac{1}{2}r\sigma^2}\end{aligned} \qquad (10)$$

There is always a fraction of the interval where no producers earns quasirents (i.e. Case C) where the processor prefers to have only one group of producers. The reason is that the lower boundary for Case C is below the point where the processor is indifferent between one group and two groups[9]. On the other hand the processor only divides the producers into two groups if the difference in skills ($\theta$) is large enough[10], otherwise the processor always prefers one group in Case C.

[9] In case C, we have (cf. (7)): $w \geq \frac{p-\frac{n}{n-1}\theta}{1+\frac{n}{n-1}r\sigma^2}\frac{n}{n-1}\theta$. The point $w = \frac{p\theta}{1-\frac{3}{2n}+\frac{1}{2}r\sigma^2}$ is always relevant in case C, because $\frac{p\theta}{1-\frac{3}{2n}+\frac{1}{2}r\sigma^2} > \frac{p-\frac{n}{n-1}\theta}{1+\frac{n}{n-1}r\sigma^2}\frac{n}{n-1}\theta$ .

[10] In case C, we have (cf. (7)): $w \leq \frac{p+\frac{n}{n-1}\theta}{1+\frac{n}{n-1}r\sigma^2}\frac{n}{n-1}\theta$. Hence, $g = \frac{n}{2}$ can only occur if $\theta \geq p\frac{n-1}{n}\frac{\frac{1}{n}+r\sigma^2}{2-\frac{3}{n}+r\sigma^2}$.

The higher the price $p$, the larger the interval of $w$ (the difference in the reservation values) where the processor prefers that the producers compete in one group. The reason is that effort is more valuable if the price is high.

If the number of producers goes up, the interval where the processor prefers two groups increases. The reason is that the uncertainty only increases very little, when a large number of producers are divided into two groups. Therefore, the gain from a lower distortion in the level of effort can be obtained through a small increase in the risk premium.

Another interesting result is that *high variance or high risk aversion increases the interval where the processor prefers two groups.* When the producers are divided into two groups, two opposite effects come into play. First, the uncertainty increases due to the law of large numbers. Second, the incentives weaken. The latter effect dominates the first in the risk premium (the incentives are raised to the second power when calculating the risk premium). When the producers compete in two heterogeneous groups instead of one big group, the risk premium decreases by

$$\frac{1}{2}\left(\frac{n-1}{n}\frac{w}{\theta}\right)^2 r\sigma^2\frac{n}{n-1}-\frac{1}{2}\left(\frac{n-2}{n}\frac{w}{\theta}\right)^2 r\sigma^2\frac{n}{n-2}$$
$$=\frac{1}{2}r\sigma^2\left(\frac{w}{\theta}\right)^2\frac{1}{n}>0$$

This expression shows that the reduction in distortion more than outweights the increase in uncertainty.

## 4.2 How Many Groups?

We have shown that the processor may benefit from dividing the producers into two heterogeneous groups. This reasoning can be repeated. The processor may prefer to have four heterogenous groups rather than just two groups and so on. In this section we address the question: what is the optimal number of groups from the processors point of view?

Let $g$ denote the number of heterogeneous groups. The level of effort in the interval where no producer earns quasirent, Case C, is

$$e(g)=\frac{n-g}{n}\frac{w}{\theta} \tag{11}$$

The processor's profit per agent expressed as a function of $g$ is

$$\frac{\pi(g)}{n}=pe(g)-\frac{1}{2}e(g)^2-\frac{1}{2}e(g)^2 r\sigma^2\frac{n}{n-g}$$

Solving the first order condition[11] and ignoring integer problems gives the optimal number of groups

$$g = n + \frac{1}{2}nr\sigma^2 - np\frac{\theta}{w}$$

There has to be at least one group, and each group must have at least two producers for the tournament to work. Thus, the optimal number of groups is

$$g = \begin{cases} 1 & \text{if } w \leq \frac{p\theta}{\frac{n-1}{n}+\frac{1}{2}r\sigma^2} \\ n - np\frac{\theta}{w} + n\frac{1}{2}r\sigma^2 & \text{otherwise} \\ \frac{n}{2} & \text{if } w \geq \frac{p\theta}{\frac{1}{2}+\frac{1}{2}r\sigma^2} \end{cases} \tag{12}$$

*The optimal number of groups is found through a trade-off between moral hazard, insurance, and discrimination issues.* When the moral hazard issue dominates due to a high price $p$, the processor prefers to have the producers competing in few and larger groups. The reason is that the level of effort is higher in larger groups.

When the discrimination effect dominates, the processor chooses to divide the producers into small groups to increase the discrimination. The discrimination issue is dominating when the difference in the reservation value ($w$) is large relative to the difference in the skills ($\theta$).

When the insurance issue dominates, the processor prefers to divide the producers into small groups. In this way he can lower the risk premium through weaker incentives - even though the uncertainty actually increases. The reason is that the decrease in the incentives more than outweigh the increase in uncertainty as described in section 3.1.

## 4.3 Homogeneous Groups

We now consider homogeneous groups where all high-skilled producers compete in one group and all low-skilled producers compete in another group. If the processor can pay different base payment (i.e. different $t$) to groups, he can discriminate perfectly between the two types of producers without distorting the level of effort. However, there is a cost of doing so, which is

[11] The first order condition for $g$ is: $\frac{d\pi(g)}{dg} = -p\frac{w}{\theta} + \frac{1}{n}\frac{w^2}{\theta^2}(n-g) + \frac{1}{2}r\sigma^2\frac{w^2}{\theta^2} = 0$ The second order condition for $g$ is fulfilled since: $\frac{d^2\pi(g)}{dg^2} = -\frac{1}{n}\left(\frac{w}{\theta}\right)^2 < 0$ .

an increase in the uncertainty since the number of producers in each group decreases.

When the producers are divided into two internally homogeneous groups, the processor's profit per agent is

$$\frac{\pi^{ho}}{n} = pe^{ho} - \frac{1}{2}\left(e^{ho}\right)^2 - \frac{1}{2}\left(e^{ho}\right)^2 r\sigma^2 \frac{n}{n-2}$$

where $e^{ho} = \frac{p}{1+\frac{n}{n-2}r\sigma^2}$.

When the processor divides the producers into two homogeneous groups with different base payment, his profit changes by

$$\frac{\pi^C - \pi^{ho}}{n} = p\left(e^C - e^{ho}\right) - \frac{1}{2}\left[\left(e^C\right)^2 - \left(e^{ho}\right)^2\right] - \frac{1}{2}r\sigma^2\left[\frac{n}{n-1}\left(e^C\right)^2 - \frac{n}{n-2}\left(e^{ho}\right)^2\right]$$

This expression can be positive as well as negative. In our model the processor, under some circumstances, prefers to have all producers competing in one big group, while under other circumstances it is preferable to have the producers compete in one big group.

In many cases the processor can not use different base payments to different groups, due to legal restrictions, negotiations, etc. (c.f. the Introduction). It is less favorable to divide the producers into internally homogeneous groups, when the processor must use the same base payment to both groups. The reason is that the low-skilled producers always receive quasirents[12], when the producers compete in two homogeneous groups receiving the same base payment.

When the producers compete in two internally homogeneous groups with the same base payment, the effort is $\widehat{e}^{ho} = e^{ho}$. Thus, the processor's profit per agent is

$$\frac{\widehat{\pi}^{ho}}{n} = pe^{ho} - \frac{1}{2}\left(e^{ho}\right)^2 - \frac{1}{2}r\left(e^{ho}\right)^2 \frac{n}{n-2}\sigma^2 - w$$

When the processor is required to pay the same base payment to both groups, his profit decreases by $\widehat{\pi}^{ho} - \pi^{ho} = nw$. Hence, when the base payment is the same in both groups, there are fewer instances where the processor prefers two homogeneous groups rather than one group. However, if $\pi^C - \pi^{ho} > nw$ the processor prefers two homogeneous groups rather than one big group including all producers - even though he must pay the same base payment to both groups.

[12] The quasirents to a low skilled agent equal the difference in reservation value, i.e.$2w$.

# 5 Numerical Example

In this section we illustrate our results through a simple numerical example. Consider a case with four producers. We assume that $P = 1$, and $r\sigma^2 = 0.3$, and $w = 1$.

We start by considering $\theta = 1$. In this case the effort levels are

$$e^H = \frac{p + \theta\frac{n}{n-1}}{1 + r\sigma^2\frac{n}{n-1}} = \frac{1 + \frac{4}{4-1}1}{1 + \frac{4}{4-1}0.3} = 1.6667$$

and $e^L = -0.2381$, and $e^C = 0.75$.

The relevant level of effort is $e^C = 0.75$, since $\theta e^H\frac{n}{n-1} = 1 \cdot 1.6667 \cdot \frac{4}{4-1} = 2.2223 > w$, and $\theta e^L\frac{n}{n-1} = -0.\,3175 < w$. The profit (per producer) to the processor is

$$\pi^C = 1 \cdot 0.75 - \frac{1}{2}(0.75)^2\left(1 + 0.3 \cdot \frac{4}{4-1}\right) = 0.3563$$

Table 1 below shows the level of effort and the resulting profit (per producer) to the processor for $\theta = 1$, $\theta = 0.75$, and $\theta = 0.25$, when the producers compete in:

- one big group
- two heterogeneous groups
- two homogeneous groups, receiving different base payment
- two homogeneous groups, receiving the same base payment

| | Case | Effort | processor's profit |
|---|---|---|---|
| $\theta = 1$ | | | |
| One group | C | $e^C = 0.75$ | $\pi^C = 0.3563$ |
| Heterogeneous groups | C | $e^{he} = 0.5$ | $\pi^{he} = 0.3000$ |
| Homogeneous groups, diff. $t$ | | $e^{ho} = 0.625$ | $\pi^{ho} = 0.3125$ |
| Homogeneous groups, same $t$ | | $e^{ho} = 0.625$ | $\widehat{\pi}^{ho} = -0.6875$ |
| | | | |
| $\theta = 0.75$ | | | |
| One group | C | $e^C = 1$ | $\pi^C = 0.3000$ |
| Heterogeneous groups | C | $e^{he} = 0.6667$ | $\pi^{he} = 0.3111$ |
| Homogeneous groups, diff. $t$ | | $e^{ho} = 0.625$ | $\pi^{ho} = 0.3125$ |
| Homogeneous groups, same $t$ | | $e^{ho} = 0.625$ | $\widehat{\pi}^{ho} = -0.6875$ |
| | | | |
| $\theta = 0.25$ | | | |
| One group | A | $e^C = 0.9524$ | $\pi^e = -3.3$ |
| Heterogeneous groups | A | $e^{he} = 0.9375$ | $\pi^{he} = -1.2$ |
| Homogeneous groups, diff. $t$ | | $e^{ho} = 0.625$ | $\pi^{ho} = 0.3125$ |
| Homogeneous groups, same $t$ | | $e^{ho} = 0.625$ | $\widehat{\pi}^{ho} = -0.6875$ |

**Table 1: Effort and resulting profits for different group divisions.**

The processor prefers that the producers compete in one group, when $\theta = 1$. When $\theta = 0.75$ the processor preferers to divide the producers into two heterogeneous groups rather than having the producers compete in one group. However, the processor can do even better by diving the producers into homogeneous groups, if he can pay differnt base payment to different groups.

For $\theta = 0.25$ the low-skilled producers recieve quasirents, when the producers compete in one group (Case A). In this situation the processor prefers to divide the producers into homogeneous groups. This holds even if the processor cannot use different base payment to the two homogeneous groups.

Table 2 summarizes our findings

| Assumption | Results | Best division |
|---|---|---|
| $\theta = 1$ | $\pi^C > \pi^{ho} > \pi^{he} > \widehat{\pi}^{ho}$ | One group |
| $\theta = 0.75$ | $\pi^{ho} > \pi^{he} > \pi^C > \widehat{\pi}^{ho}$ | Two homogeneous groups different base payment |
| $\theta = 0.25$ | $\pi^{ho} > \widehat{\pi}^{ho} > \pi^{he} > \pi^C$ | Two homogeneous groups different base payment |

**Table 2: Optimal group division.**

When $\theta = 1$ and $\theta = 0.75$ the producers do not receive quasirents. When $\theta = 0.25$ the producers earn quasirents, if the processor can not pay different base payment to the two homogeneous groups. In this case, the processor divides the producers into heterogeneous groups and the low-skilled producers receive quasirents of $2w = 2$. If the producers compete in one group, the low-skilled producers would receive quasirents of $2\left(w - \theta e^H \frac{n}{n-1}\right) = 1,3651$. Hence, when the processor cannot pay different base payment to different groups, both the processor and the producers are better off if group division is allowed. Thus, it may be beneficial for the producers to force the processor to use the same contract to all producers, but allow the processor to divide the producers into groups.

# 6 Conclusion

We have developed a model combining moral hazard, risk sharing, and discrimination issues in linear tournaments. We show that the processor can benefit by strategic division of the producers into tournaments or groups when the processor posses some information about the producers' type, which he cannot use directly in the contract.

The discrimination effect causes distortion in the level of effort. Consider the case, A, when the low-skilled producers receive quasirents. In this case, it is optimal for the processor to use stronger incentives and implement a higher level of effort. Stronger incentives reduces the quasirents to low-skilled producers via a stronger punishment. In a different case, B, the high-skilled producers receive quasirents. Here it is optimal for the processor to use weaker incentives. This distorts the level of effort downwards and reduces the quasirents to the high-skilled producers, since these producers benefit less from having better skills. In a third case, C, where none of the producers receive quasirents, the discrimination effect may lead to either weaker or stronger incentives.

The processor can use the division of producers into groups strategically. If the processor distorts the level of effort upwards in case C, he may gain in two ways from dividing the producers into more heterogeneous groups. First, the distortion in the level of effort falls, since the level of effort is lower in smaller groups. Second, the risk premium decreases due to weaker incentives.

We have shown that the processor may benefit from dividing the producers into internally homogeneous groups (groups where high-skilled producers compete only with other high-skilled producers etc.). The potential gain from dividing the producers into homogeneous groups is highest if the processor can use different base payment to different groups.

Our analysis emphasizes a controversial aspect of tournaments. A processor can use tournaments to discriminate between heterogeneous producers, especially if he uses his authority to divide the producers into groups. The discrimination in tournaments can increase the processor's profit - but often at the expense of the producers. This may explain why producers in many cases resist the use of tournaments in agricultural contracts.

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